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Radiative-convective disequilibrium

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ABSTRACT

Radiative-convective systems inevitably seek a statistical equilibrium, but there is no reason to believe that this equilibrium must be steady. The obvious temporal variability of convection in nature, on many time scales, is clearly linked, at least in part, to forcing by transient large-scale dynamical processes (e.g. easterly waves) and transient radiative processes (e.g., the diurnal and seasonal cycles). The possibility remains, however, that under some conditions radiation and convection can interact nonlinearly to produce spontaneous temporal fluctuations on "large scales". Recent numerical simulations with both parameterized and model-resolved two-dimensional convection provides evidence that this can, indeed, happen. Radiative-convective oscillations on both short (days) and long (weeks) time scales have been reported. Highly idealized models have been devised to interpret the results. Work is under way to determine to what extent such "free" radiative-convective oscillations are relevant to the transient convective activity that we see in nature.

INTRODUCTION

Prompt a meteorologist with the words "radiative-convective," and he or she will respond with "equilibrium." Ever since the work of Manabe and his colleagues in the 1960's (e.g., Manabe and Wetherald, 1967), conventional wisdom has held that in the absence of large-scale dynamical forcing the tropical atmosphere would asymptotically approach a steady state in which radiative cooling is balanced primarily by latent heat release in convective clouds, and convective precipitation is balanced by surface evaporation. The tendency for radiative cooling and surface evaporation to increase the moist convective available potential energy (CAPE) would be balanced by the warming and drying associated with cumulus convection, which tend to decrease the CAPE.

While such a balance is inevitable in a time-averaged sense, it need not hold on a moment-by-moment basis. Although Manabe and Wetherald's model did, in fact, approach a steady state, it relied on prescribed distributions of

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relative (or, alternatively, absolute) humidity. More recent studies of radiative convective equilibrium have been based on models that include an explicit hydrologic cycle. The results from these more complex and physically complete models provide evidence that under certain conditions free oscillations can spontaneously develop in radiative-convective systems. This can be described as radiative-convective disequilibrium. In the remainder of this paper, we occasionally refer to radiative-convective equilibrium (RCE), but with the understanding that the equilibrium may be only statistical, rather than truly steady.

This paper provides a review or synthesis of previously published results exhibiting radiative-convective disequilibrium, and also briefly presents a few new results that will be discussed more extensively elsewhere. The possible relevance of radiative-convective disequilibrium to observed convective transients and large-scale circulations is also discussed.

ONE-DIMENSIONAL MODELS WITH PARAMETERIZED CONVECTION

One way to simulate interactions between radiation and convection, in the absence of large-scale dynamics, is to construct a one-dimensional numerical model that includes prognostic equations for temperature and moisture as functions of height and time only. The physical processes included in such a model must include radiation, convection and turbulent exchange with the Earth's surface. Each of these processes must be parameterized somehow. The resulting model is essentially a single-column version of an atmospheric general circulation model.

Hu (1992) and Hu and Randall (1993; hereafter "HR") presented results from two such one-dimensional models. The first, which they imaginatively called "Model 1", included moist convective adjustment (Manabe et al., 1965) to parameterize the effects of cumulus convection, Newtonian cooling as an idealized representation of the effects of longwave radiation, and a bulk aerodynamic formula to determine the surface fluxes of sensible and latent heat.

HR also presented a "Model 2", which is essentially a single-column version of the Colorado State University general circulation model. Full parameterizations of both longwave and solar radiation were included (Harshvardhan et al., 1987), with radiatively interactive cloudiness (Harshvardhan et al., 1989; Randall et al., 1989). Cumulus convection was parameterized following the method of Arakawa and Schubert (1974). A variable-depth boundary layer was also incorporated (Suarez et al., 1983). In short, Model 2 was vastly different, and vastly more complicated, than Model 1. The external parameters of both models included the sea surface temperature (SST) and the surface wind speed.

Both models produced spontaneous oscillations of both high and low frequency, when the SST was warm and the surface wind speed was weak; under other conditions the solutions were steady. As discussed by Hu (1992), the stationary equilibria become unstable for some values of the parameters. The existence of this instability determines whether or not unsteady solutions will occur. Figure 1 shows an example of an unsteady solution, obtained with Model 2. The precipitation rate fluctuates dramatically, with a spectral peak near a period of 50 days. HR analyzed this low-frequency component of the oscillations, and found that the basic mechanism works as follows. During a period of weak convection, radiation cools the air aloft, while moisture accumulates near the surface. This leads, eventually, to high humidities and a steep lapse rate. This destabilization allows convection to intensify and deepen. More intense convection dries the boundary layer, allowing stronger surface evaporation, and it also warms the air aloft. These convective feedbacks stabilize the sounding, forcing convection to weaken. The cycle then repeats. Convection never actually stops during the cycle; it only gains and loses intensity. Periods of intense precipitation are characterized by high cloud tops, while periods of weaker precipitation have lower cloud tops. Similar fluctuations of moist convective stability were observed in the tropics by Hendon and Liebmann (1990).

The period of the oscillation is controlled by the radiative relaxation time, for which a realistic value is believed to be on the order of 20 days, and by the time required for surface evaporation to moisten the lower troposphere; the latter is of course inversely related to the prescribed surface wind speed. HR

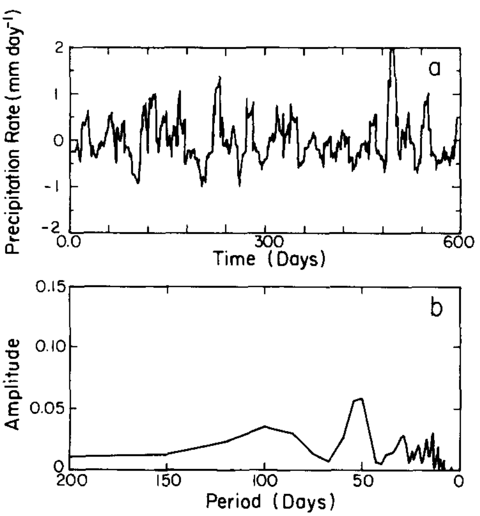

Fig. 1. Results obtained with Model 2 of Hu and Randall (1993): (a) Simulated precipitation rate mm day^{-1}, after application of an 11-day running mean, for a 600-day period. (b) The spectrum of the precipitation rate.

found that the oscillation is particularly strong and regular when the SST is warm (above about 300 K) and the surface wind speed is weak. These are precisely the conditions that prevail, climatologically, in the warm pool region of the tropical Western Pacific.

HR hypothesized that the observed tropical low-frequency or "intraseasonal" oscillations discovered by Madden and Julian (1971, 1972) are forced by a localized oscillating heat source which corresponds to the simulated radiative-convective oscillations produced by their models. The oscillating heat source is spatially localized (e.g. in the tropical Indian Ocean and/or the tropical Western Pacific) because conditions for its operation are favorable there, i.e. the SST is warm and the low-level winds are weak. As shown by Salby and Garcia (1987) and Garcia and Salby (1987), a low-frequency tropical heat source excites waves that propagate slowly and have structures similar to the observed Madden–Julian waves. Yasunari (1979), Weickmann and Khalsa (1990) and Hsu et al. (1990), among others, have presented observational evidence that the heat source associated with the Madden–Julian Oscillation (hereafter, MJO) does not propagate with the wave, but remains localized over the warm water of the eastern-hemisphere tropics.

Satoh and Hayashi (1992) described a one-dimensional model that is intermediate in complexity between HR's Models 1 and 2. Their cumulus parameterization made use of a convective mass flux, assumed to be independent of height, whose magnitude was determined by a cumulus kinetic energy balance condition. A neutral buoyancy condition was used to determine the cloud-top height. The surface fluxes of sensible heat and moisture were parameterized with a bulk aerodynamic method. Satoh and Hayashi used an idealized radiative transfer parameterization for a grey, non-scattering atmosphere, and did not include the radiative effects of clouds. The radiative cooling rate was affected by the predicted temperature profile, but was independent of the predicted water vapor distribution. A planetary radiation balance was enforced by requiring that the outgoing longwave radiation at the top of the atmosphere be in balance with a prescribed incident solar radiation, although they did not consider absorption or scattering of solar radiation.

Their model exhibited spontaneous fluctuations of both low and high frequency (see their fig. 8). In particular, the simulated cloud-top height fluctuated between about 200 mb and 600 mb. They argued that deep convective clouds quickly stabilize the sounding, in effect shutting themselves off. Shallow clouds carry on until radiative cooling at the upper levels destabilizes the sounding enough to allow deep convection to resume. Satoh and Hayashi did not report a detailed analysis of the fluctuations.

Rennó (1992) presented results from several one-dimensional radiative-convective models, distinguished by their different cumulus parameterizations. He obtained steady state solutions in some cases, but many of his results showed time-dependent behavior. He stated that the unsteady behavior arises when convective clouds "overstabilize" the sounding, removing all CAPE. Then, over a period of time, the CAPE builds up again, due to surface evaporation and radiative cooling aloft. This leads to a new round of convection. Rennó did not discuss the physical realism of these oscillations or their possible relevance to natural phenomena.

The basic mechanism of the fluctuations, as described by Rennó, is very similar to that discussed by HR and Satoh and Hayashi (1992). All three studies agree that a cycle of radiative destabilization, followed by convective stabilization, can occur in radiative-convective systems, under some circumstances. We refer to these as radiative-convective (RC) oscillations.

Bladé and Hartmann (1993) found free fluctuations of cumulus heating in a simple three-dimensional dynamical model that they applied to study the MJO. Their model includes a Newtonian relaxation to a prescribed temperature distribution; this can be interpreted as a crude parameterization of radiative cooling. They used a relaxation time of 15 days. They did not include an explicit moisture variable. Convection was parameterized with a "moisture convergence" assumption, in which the convective heating rate was assumed to be proportional to the large-scale mass convergence at the 750 mb level. In effect, they assumed that moisture convergence is proportional to mass convergence. When divergence was predicted at 750 mb, the convective heating was set to zero. To avoid some problems, the zonally averaged convective heating was also set to zero.

Wave-CISK can occur, in their model, when the SST is warm enough and the static stability is weak enough. Their model produces low-frequency fluctuations of the convective heating rate, which are associated with fluctuations of the static stability of the column (see their fig. 11). The mechanism is this: When the static stability is low, CISK leads to convection. The large-scale circulations forced by the convection act to increase the static stability, in effect shutting off the CISK mechanism and thus, indirectly, shutting off the convection. Bladé and Hartmann call this the "discharge–recharge" mechanism.

The discharge–recharge oscillations found by Bladé and Hartmann are superficially similar to the RC oscillations described by HR, Sato and Hayashi, and Rennó. Both types of oscillation involve fluctuations of the static stability and convection, with stronger convection occurring in concert with weaker static stability. In the RC mechanism, however, the static stability directly regulates convection by controlling the existence of CAPE, and fluctuations of the large-scale motion field play no role. In contrast, the concept of CAPE does not directly enter the convection parameterization of Bladé and Hartmann, and a time-varying large-scale motion field is essential in their model, both for increasing the static stability and for producing the moisture convergence that is assumed to drive convection.

Although the paper of Bladé and Hartmann does not discuss RCE per se, and although their model does not predict moisture, they do argue for essentially self-excited, low-frequency convective oscillations, and suggest that these may be relevant to the observed MJO.

CUMULUS ENSEMBLE MODELS

In recent years, "cumulus ensemble models" (CEM's) have been developed by several groups (e.g., Yamazaki, 1975; Soong and Tao, 1980; Krueger, 1988). CEM's have sufficient horizontal and vertical resolution to resolve individual clouds, yet large enough domains and long enough integration times to simulate many coexisting clouds over many cloud life times. Periodic horizontal boundary conditions are typically used. Some CEM's include radiative transfer parameterizations; examples are discussed below. Current computers allow only two-dimensional CEM's, but three-dimensional versions will become computationally feasible within a few years. Recently, several authors have applied CEM's to the problem of RCE.

Nakajima and Matsuno (1988) investigated RCE with a CEM in which the horizontal grid spacing was 1 km, and the vertical grid spacing ranged from 300 m near the surface to 1200 m near the top, at the 22.6 km level. The horizontal domain size was 512 km. Integrations lasted only 50 simulated hours; while this is much longer than the lifetime of an individual convective cloud, it is much shorter than simulations reported in more recent studies, as discussed below.

Radiative cooling was represented by two terms. The first was a relaxation of the horizontally averaged temperature to a prescribed vertical profile, with a damping time of 1/5 day. The second was a prescribed cooling rate 2 K day^{-1} up to 10 km, zero above 15 km, and a linear interpolation between 10 km and 15 km. The surface fluxes were computed using a bulk aerodynamic formula; the wind speed used was that predicted in the plane of the domain, plus a 3 m s^{-1} component in the direction normal to the plane of the domain; the minimum value of the wind speed was thus 3 m s^{-1}.

Several experiments were performed with the model, to determine the effects of microphysical processes on the results. In the most realistic case, with full Kessler (1969) microphysics, the model spontaneously produced organized convective systems, with lifetimes longer than 10 h.

Held et al. (1993) presented results from a CEM with a horizontal grid size of 5 km, a vertical grid size of 200 m and a domain 640 km wide and 26 km tall. They adopted a Kessler (1969) microphysics parameterization, with a modification to allow for the effects of snow. A sponge layer was provided at the top of the model. The solar and radiative transfer parameterizations follow the work of Ramaswamy and Kiehl (1985). The diurnal cycle was suppressed; a daily mean insolation was used. The predicted clouds were radiatively active. The radiative temperature tendencies were updated once every

6000 s. The surface wind used to compute the evaporation rate was constrained to be not less than 5 m s^{-1}.

Held et al. (1993) presented results for the final 40 days of an 80-day integration, with an SST of 30°C. They found a strong oscillation of the mean winds, associated with momentum transport by vertically propagating waves. The period of the oscillation was about 60 days. They performed an experiment in which the domain-averaged wind was forced to vanish, so that the oscillation of the winds could not occur. The convection then "localized" into a single strong convective disturbance. In a further experiment, Held et al. (1993) suppressed this localization by imposing a time-independent vertical shear of the mean flow. Their results (see their fig. 8) show strong fluctuations of the mean relative humidity, with time scales in the order of 10 days.

Sui et al. (1993) reported simulations with a CEM in which the horizontal domain size was 768 km and the vertical domain was 21.5 km high. The horizontal grid size was 1.5 km and the vertical grid size ranged from 200 m near the Earth's surface to 1 km above 15 km. The surface wind used to compute the evaporation rate was constrained to be not less than 4 m s^{-1}. Radiation was parameterized following the work of M.D. Chou. The simulated clouds were radiatively active. Sui et al. (1993) did not attempt to study RCE; they imposed a domain-averaged vertical motion designed to mimic that observed in the tropical Western Pacific region. They performed a 52-day integration. Although they did not emphasize the temporal variability in their results, their fig. 5 shows very noticeable fluctuations of the two-dimensionally averaged equivalent potential temperature and water vapor mixing ratio.

At Colorado State University, we have recently simulated RCE using the CEM developed by S. Krueger and A. Arakawa at UCLA (Krueger, 1988), modified to incorporate the solar and longwave radiation parameterizations of Harshvardhan et al. (1987); see Xu et al. (1993) for a discussion. The model is two-dimensional. Simulated clouds are radiatively active. The domain is 18 km deep. The horizontal grid size is 2 km, and the vertical grid size ranges from 300 m near the ocean to 1500 m near the top of the model. We have performed simulations with domain sizes of 128 km and 256 km, for 100 simulated days, and find little difference in the domain-averaged statistics of daily means.

Only a brief account is given here; a full discussion of these simulations will be given elsewhere. Figure 2 shows the temporal variations of the daily-mean, domain-averaged precipitation rate (upper panel) and precipitable water (lower panel) in a three-hundred-day simulation in which the SST is about 302 K. After an initial adjustment period lasting approximately 20 days, both variables show strong high-frequency oscillations associated with the inertial period (about 3 days). In addition, there are obvious longer-period fluctuations in the precipitable water, whose range of variation is about 10 % of its mean value of 45 mm. Rapid decreases of the precipitable water are associ-

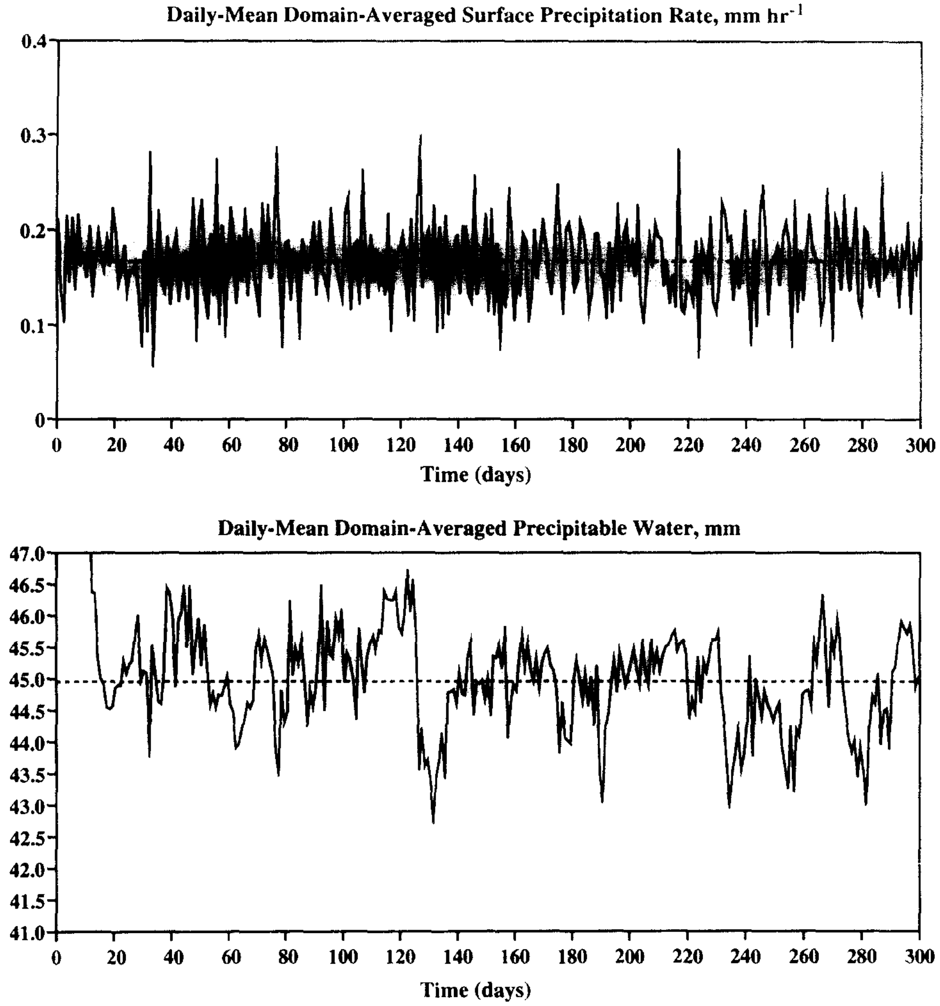

Fig. 2. The time evolution of the daily mean domain averaged precipitation (upper panel) and precipitable water (lower panel) in a three-hundred-day simulation with the CEM developed by Krueger (1988). The dashed line in each panel indicates the time-mean value.

ated with positive "spikes" in the precipitation rate. The range of variation of the precipitation rate is about equal to its mean value, which is 0.17 mm h^{-1} (about 4 mm day^{-1}). Spectral analysis of the precipitable water (not shown) reveals a fairly sharp and isolated peak at a period of about 40 days, as well as a weak high-frequency peak associated with the inertial oscillation. The spectrum of the precipitation rate is more complex and exhibits no distinct low-frequency maxima. Further analysis, however, shows that the amplitude of the diurnal cycle of the precipitation is strongly modulated with a period of tens of days.

Our plans include repeating these calculations with altered dynamics designed to completely eliminate the inertial oscillation.

Because the CSU CEM is two-dimensional, there is a tendency for energy to anti-cascade from smaller spatial scales to larger spatial scales (Fjortoft, 1953). Larger spatial scales are generally associated with longer time scales, so there may be some tendency for low-frequency fluctuations in a two-dimensional model to be artificially produced by the two dimensionality itself. We find it difficult to believe, however, that significant power at periods as long as 40 days can be generated in this way, in a domain only 128 km wide. The similarity between the results of the one-dimensional and two-dimensional models also argues against this interpretation.

INTERPRETIVE MODELS

Attempts have been made to interpret the unsteady numerical results obtained with one-dimensional RCE models and CEM's, as discussed above, using highly idealized models. Hu (1992) devised an analytical radiative-convective model that turned out to be a nonlinear oscillator. He assumed that convection acts to maintain a linear relationship between the low-level relative humidity and the lapse rate, as indicated in Fig. 3. This assumption

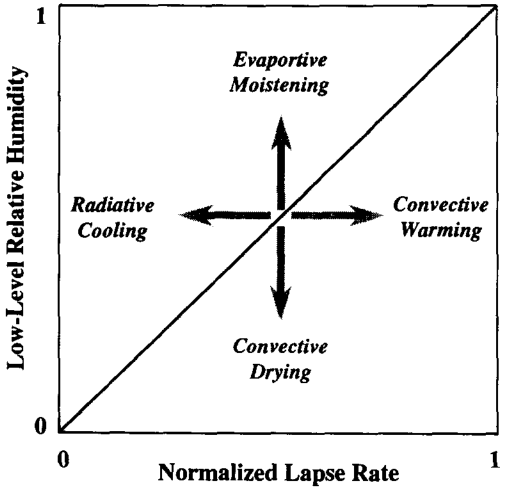

Fig. 3. Diagram illustrating a closure assumption used in a highly idealized model devised by Hu (1992). The horizontal axis is the tropospheric lapse rate of temperature, normalized so that zero denotes the dry adiabat and one denotes the moist adiabat. The vertical axis is the low-level relative humidity. Arakawa and Chen (1987) showed that convective atmospheres tend to fall along this line.

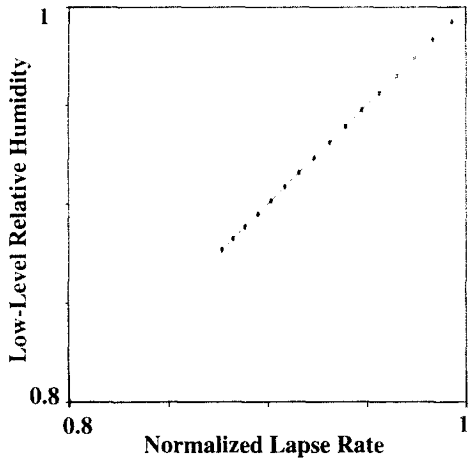

Fig. 4. A low-frequency oscillation obtained in a highly idealized radiative convective model proposed by Hu (1992). The results are plotted in the phase space defined in Fig. 3. The solution marches back and forth along the line.

was based on the observational results reported by Arakawa and Chen (1987; see their fig. 10). Surface evaporation tries to drive the system up away from the line, and radiative cooling aloft tries to drive it to the left. Convection opposes these destabilizing influences by drying and warming. The interplay among evaporation, radiation and convection dictates movement along the line. Convection is assumed to cease when the low-level relative humidity falls below a critical value.

Hu's model produces low frequency oscillations, as shown in Fig. 4. Destabilization by radiation and surface evaporation alternates with episodes of rapid convective stabilization. Further study is needed to establish more clearly whether or not the solutions obtained with Hu's idealized model truly "explain" the unsteady results obtained with the parameterized one-dimensional models and CEM's discussed in the previous two sections. Nevertheless, the model does illustrate the possibility that unsteady behavior in radiative-convective systems can be interpreted using very simple models.

CONCLUDING DISCUSSION

A variety of numerical modeling studies, using extremely diverse models and by several different groups, suggest that "radiative-convective equilibrium" is a misnomer for what happens when radiation and convection interact over a warm tropical ocean, in the absence of large-scale dynamical forcing. Simulations show that the large-scale-average state can undergo strong fluctuations, on time scales ranging from a few days to several weeks.

In the CEM's, the horizontal scale over which the fluctuations occur is the horizontal domain size of the model, and experiments suggest that the results are not sensitive to a doubling of the domain size. What if we drastically increase the domain size, say to 10^6 km? Will domain-average fluctuations of the same amplitudes and periods occur, regardless of how large the domain is made?

A similar question applies with more force to the one-dimensional models, in which the convection is parameterized. Cumulus parameterizations do not include an explicit dependence on the horizontal widths of the grid cells in which they are applied. When a cumulus parameterization is used inside a numerical model, information about the grid size is communicated indirectly to the parameterization, e.g. through the magnitude of the grid-cell averaged vertical motion. For example, if a cumulus parameterization is applied in models with progressively finer grid sizes, the peak vertical velocities will correspondingly intensify, as finer-scale circulations are resolved. The parameterization should respond by producing stronger grid-scale precipitation rates on the finer grid; see Xu and Arakawa (1992), for a discussion of this point.

For a one-dimensional model without imposed large-scale vertical motion, however, there is no way to infer the horizontal spatial scale represented by the grid-averaged quantities. A naive interpretation might be that these values must, therefore, represent averages over arbitrarily large cells. It seems doubtful, though, that horizontal averages over arbitrarily large cells can exhibit temporal fluctuations with time scales of days or tens of days. How could fluctuations in widely separated regions of a very large domain maintain a constant phase relationship with each other?

The answer, of course, is that constant or even coherent phase relationships can only be maintained over a distance equal, at most, to the period of the fluctuation times the "signal velocity" of a process that carries information horizontally. For the case of horizontally propagating gravity waves or even Kelvin waves, the horizontal distance that can be covered in a period of 10 days is many thousands of kilometers, so it is certainly possible, in principle, for a spatially coherent radiative-convective fluctuation to occur over a region the size of a CEM domain, or even the size of the warm pool in the tropical Western Pacific Ocean. A one-dimensional model cannot represent such horizontally propagating signals, and so cannot determine horizontal scale.

Of course, it is not straightforward to associate the model-simulated free radiative-convective fluctuations discussed in this paper with the phenomena of nature. We observe neither radiative-convective equilibrium nor radiative-convective disequilibrium; large-scale dynamical processes that can promote or inhibit convection are strong and ubiquitous. Can we then regard the observed state as a radiative-convective disequilibrium, modified by large-scale dynamical influences? Hu and Randall (1993) have hypothesized that the low-frequency components of free radiative-convective fluctuations are the driving force for the Madden–Julian Oscillation. Free radiative-convective fluctuations at higher frequencies may be relevant to other observed dynamical phenomena in the tropics. Further research is needed to investigate these possibilities.

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