PROGRESS TOWARDS A QUASI-3D MMF:
Partially Prognostic Tests

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OUTLINE

TECHNICAL DESIGN:

- FORMULATION OF NORMAL FLUX IN THE QUASI-3D NETWORK

- SOLVING AN ELLIPTIC EQUATION IN THE QUASI-3D NETWORK

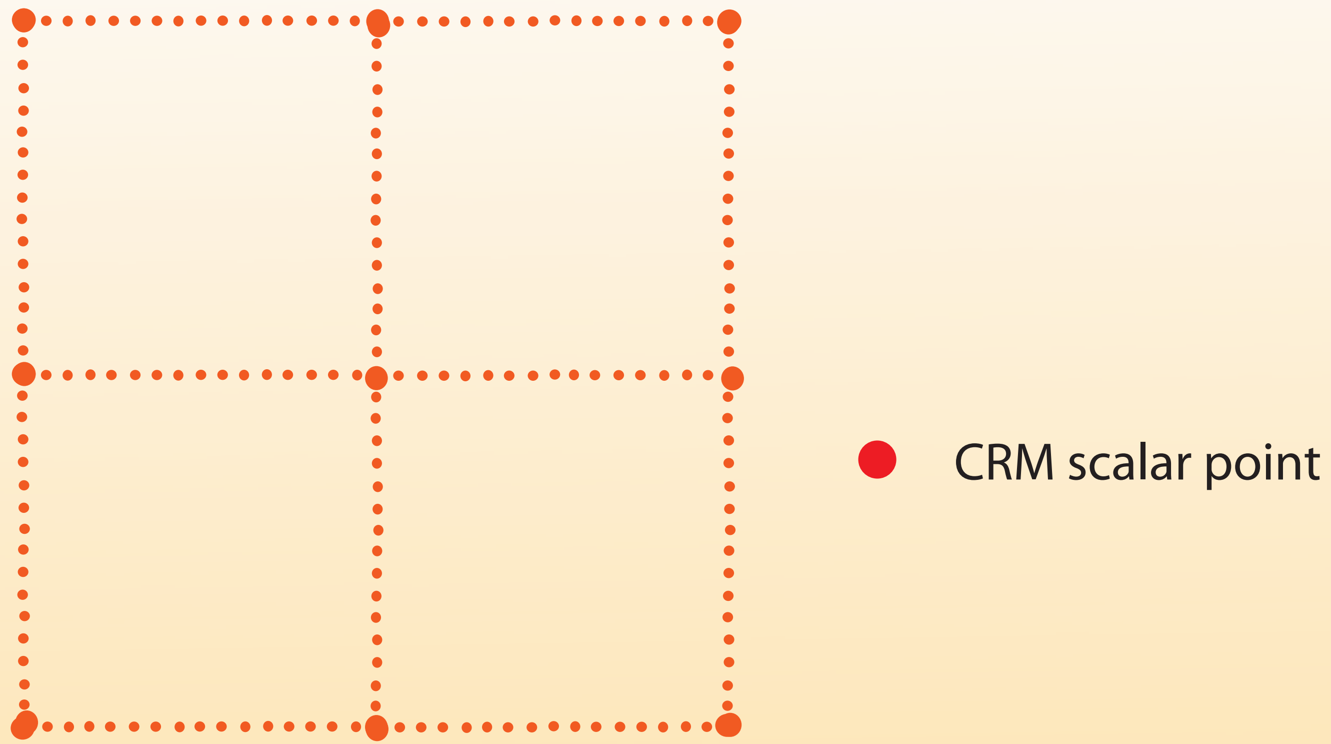
  The model we use is based on the 3D vorticity equation with an anelstic approximation and solves an elliptic equation for w.

PARTIALLY PROGNOSTIC TESTS:

- CALCULATION OF WIND COMPONENTS FROM PRESCRIBED VORTICITY FIELDS

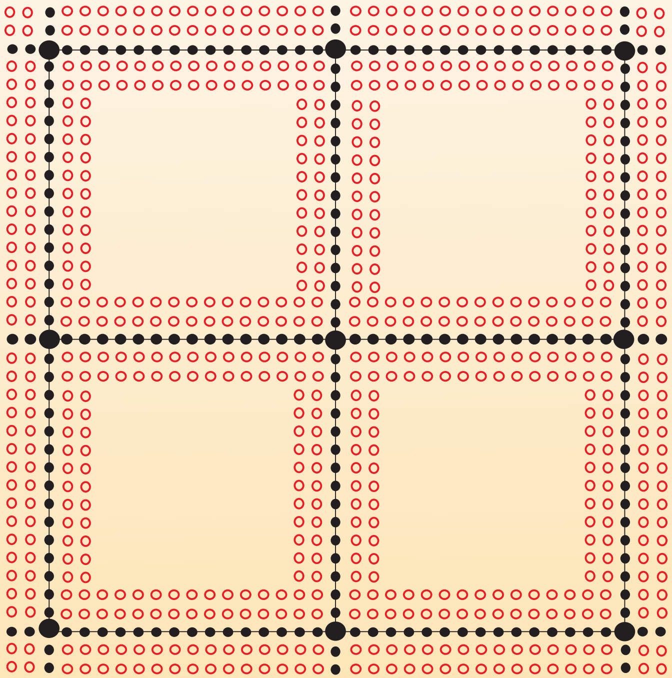
- ADVECTION WITH PRESCRIBED WINDS

Quasi-3D MMF



- We apply the same 3D algorithm for dynamics, advection and physics to all grid points.
- Then, except at the intersection points, we have to "estimate" advection in the direction normal to the grid-point arrays.
- We thus introduce "ghost points" along the grid-point arrays.

Decomposition of Fields



ghost point

To estimate the values of a prognostic variable, q, at ghost points and fluxes from/to theses points, we decompose the q field as

$$q = \overline{q} + q'$$

where

$\overline{q}$: Background field obtained by interpolation of GCM grid-point values, typically representing synoptic-scale fields

q' : Deviation of q from $\overline{q}$, typically representing the fields associated with clouds and their mesoscale organizations

Algorithms for estimating q' at ghost points and fluxes from/to these points are guided by considerations of stability, conservation property, and suppression of spurious trend.

Global Stability : Uniform current with $\overline{q'}^{\,i} = 0$

When a 3rd-order advection scheme is used, the array sum of $q'^{\,2}$ is bounded if

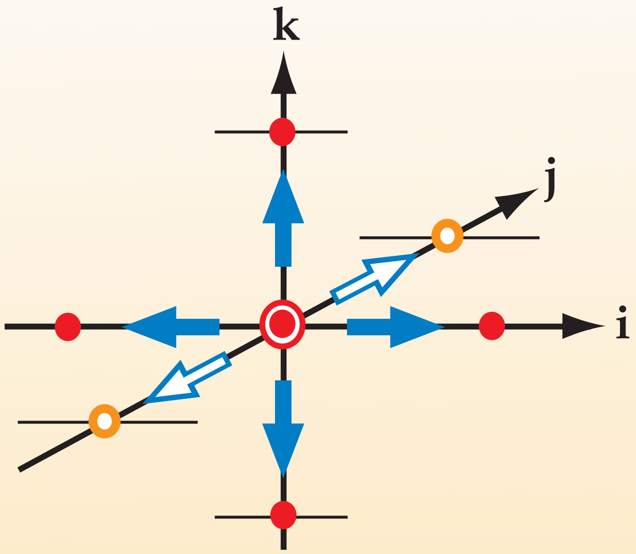
$$
\begin{aligned}
\Delta_j q' &= a_1 + b_1\, \Delta_i q' \\
\delta_j^2 q' &= a_2 + b_2\, \delta_i^2 q' \\
\Delta_j \delta_j^2 q' &= a_3 + b_3\, \Delta_i \delta_i^2 q' \\
\delta_j^4 q' &= a_4 + b_4\, \delta_i^4 q'
\end{aligned}
$$

$\uparrow$ Estimated differences in the normal direction

$\uparrow$ Predicted differences in the tangential direction

- All parameters are cloud-regime dependent and slowly vary in time.

Local Stability : Three-dimensionally variable current



When q' is positively (negatively) perturbed, the divergence of the 3D flux of q' must not decrease (increase).

Estimated flux does not produce positive feedback on a perturbation if

$$\delta_j^2 \hat{q}' = a_2 + b_2\, \delta_i^2 \hat{q}'$$

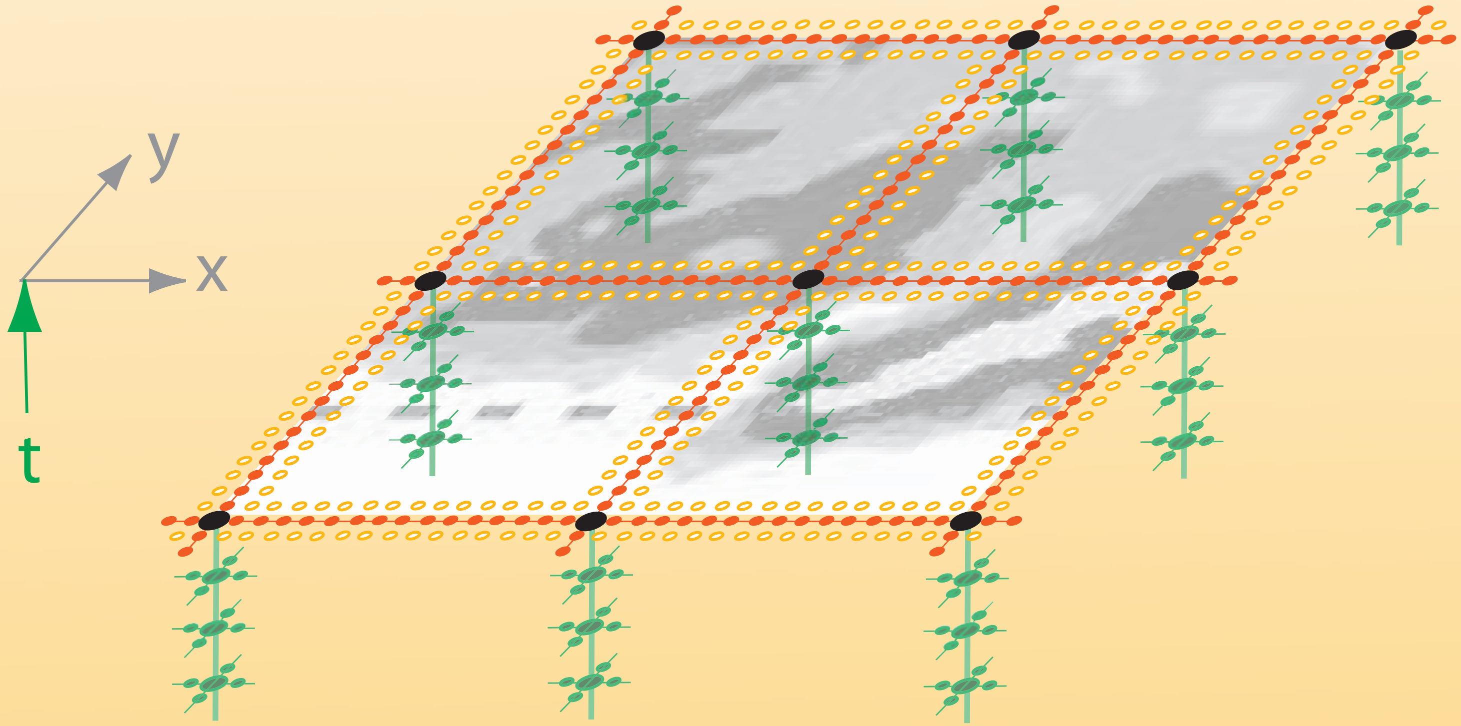
$$\delta_j^4 \hat{q}' = a_4 + b_4\, \delta_i^4 \hat{q}'$$

with $b_2 \geq 1$ and $b_4 \geq 1$

Determination of the Parameters

Hypothesis:

- Cloud regimes have longer spatial and temporal scales than individual clouds.
- The parameters can be statistically estimated from the history of the intersection and neighboring points.



Determination of the Parameters (continued.)

Parameters are statistically determined by analyzing the history data at all intersection points.

Parameter a

$$a_1 = \overline{\Delta_j \widehat{q'}}^{\,i} \qquad a_2 = \overline{\delta_j^2 \widehat{q'}}^{\,i} \qquad a_3 = \overline{\Delta_j \delta_j^2 \widehat{q'}}^{\,i} \qquad a_4 = \overline{\delta_j^4 \widehat{q'}}^{\,i}$$

Parameter b

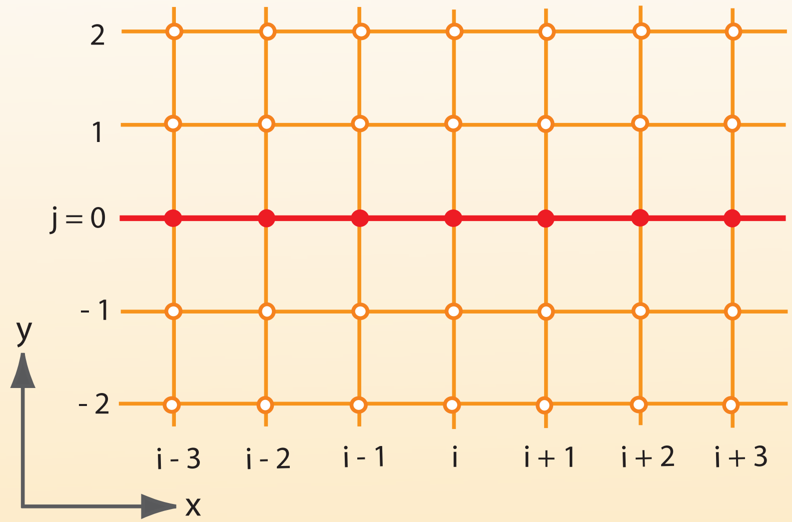
Linear regression: $Y = bX$

where

X: first/third-order finite differences of q' in x-direction

Y: first/third-order finite differences of q' in y-direction

Estimation of Values at Ghost Points



Parameters and the differences in tangential direction are known.

The differences in normal direction are estimated.

$$\hat{q}'_{j=1} = q' + \frac{1}{2}\left(\Delta_j \hat{q}' + \delta_j^2 \hat{q}'\right) \quad \hat{q}'_{j=-1} = q' + \frac{1}{2}\left(-\Delta_j \hat{q}' + \delta_j^2 \hat{q}'\right)$$

$$\hat{q}'_{j=2} = q' + \Delta_j \hat{q}' + 2\delta_j^2 \hat{q}' + \frac{1}{2}\Delta_j\left(\delta_j^2 \hat{q}'\right) + \frac{1}{2}\delta_j^2\left(\delta_j^2 \hat{q}'\right)$$

$$\hat{q}'_{j=-2} = q' - \Delta_j \hat{q}' + 2\delta_j^2 \hat{q}' - \frac{1}{2}\Delta_j\left(\delta_j^2 \hat{q}'\right) + \frac{1}{2}\delta_j^2\left(\delta_j^2 \hat{q}'\right)$$

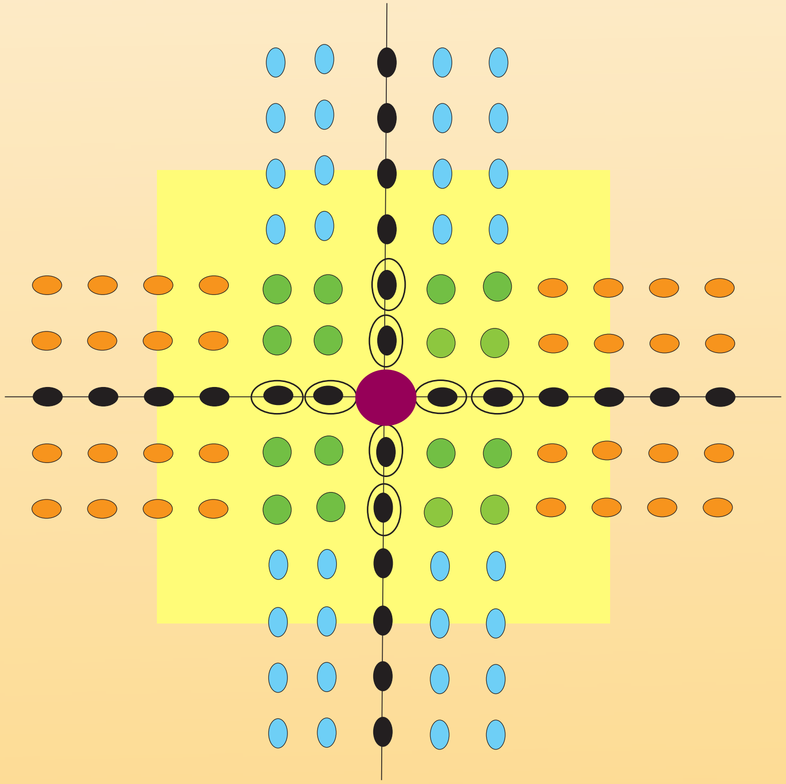
Correction of the Ghost-point Values near Intersection

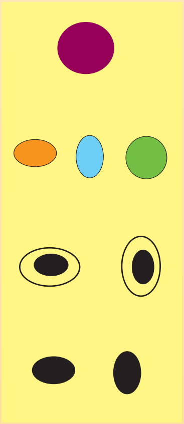
$C_{i,j}$: the correction at the data point (i,j) on the net

$c_{i',j'}$: the correction at the ghost point (i′,j′)

$$c_{i',j'} = \sum_{i,j} C_{i,j}\, e^{-\left(r_{i,j;i',j'}/r_o\right)^2} \Bigg/ \sum_{i,j} e^{-\left(r_{i,j;i',j'}/r_o\right)^2}$$

where $r_{i,j;i',j'}$ is the distance between the points and r_o is prescribed.





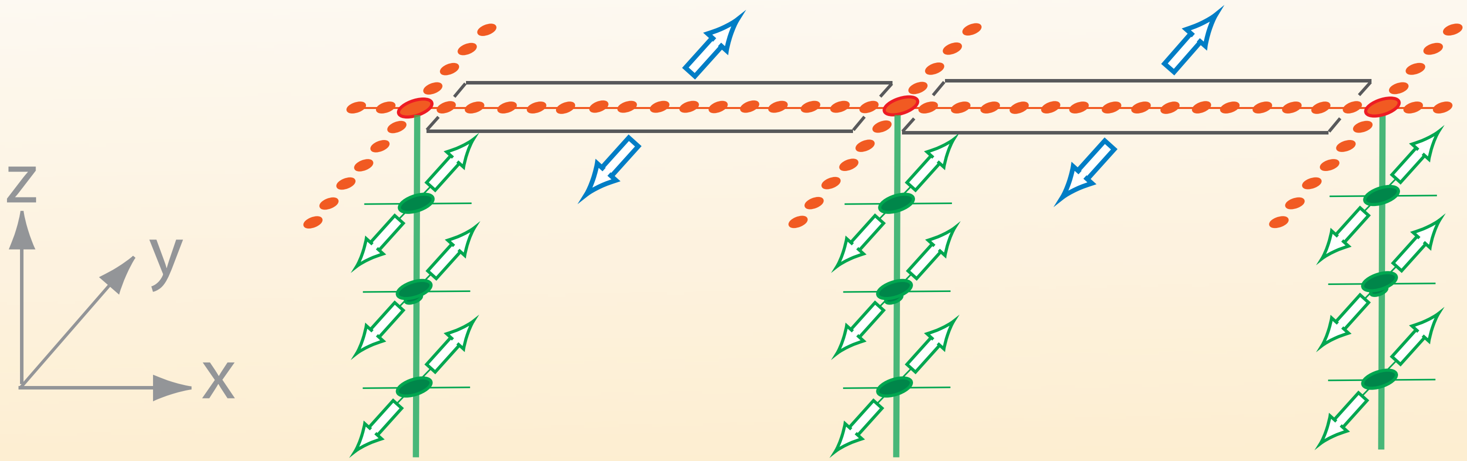
Intrsection point

Ghost points to be corrected using $c_{i',j'}$

Data points with non-zero correction $C_{i,j}$

Data points with zero correction

Conservation of the Vertically-integrated Network Mean



(Approximate) conservation is achieved by requiring the mean divergence of the flux from/to ghost points is equal to the mean divergence of the flux at the intersection points averaged in time over the analysis period.

$$\left(\delta_j G\right)_{i,j,k} \equiv \left(\delta_j G^*\right)_{i,j,k} - \left[\overline{\left(\delta_j G^*\right)_{i,j,k}}^{\,i} - \overline{\left(\delta_j G\right)_{I,J,k}}^{\,INT}\right]$$

corrected divergence

estimated divergence

estimated divergence averaged over the array

divergence averaged in time at the intersection

A Relaxation Method for Solving the Elliptic Equation

The elliptic equation is converted to a parabolic equation whose equilibrium solution is the solution of the elliptic equation.

$$\mu \frac{\partial w}{\partial t} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) w + \frac{\partial}{\partial z}\left[\frac{1}{\rho_0} \frac{\partial}{\partial z} (\rho_0 w) \right] + \rho_0 \frac{\partial \eta}{\partial x} - \rho_0 \frac{\partial \xi}{\partial y}$$

where μ defines the time scale for adjustment toward anelastic balance.

Discretization

Use a partially backward-implicit scheme for the horizontal derivative term and a fully backward-implicit scheme for the vertical derivative term.

$$\left[\frac{1}{\rho_{k-1/2}} \left(\frac{\mu}{\Delta t} + \frac{2}{\Delta x^2} + \frac{2}{\Delta y^2} \right) + \frac{1}{\Delta z} \left(\frac{1}{\rho_k \Delta z} + \frac{1}{\rho_{k-1} \Delta z} \right) \right] (\rho w)^{n+1}_{i,j,k-1/2} - \frac{1}{\Delta z} \left[\frac{(\rho w)^{n+1}_{i,j,k+1/2}}{\rho_k \Delta z} + \frac{(\rho w)^{n+1}_{i,j,k-3/2}}{\rho_{k-1} \Delta z} \right]$$

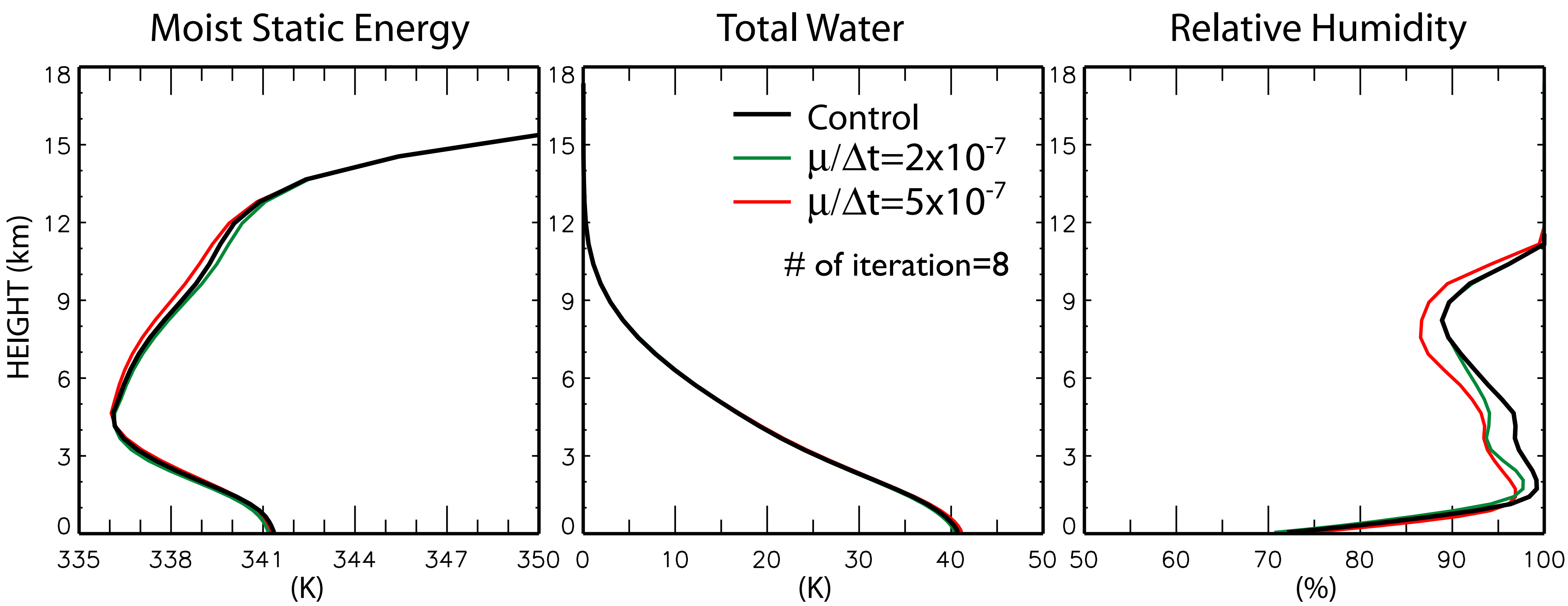
$$= \left[\frac{\mu}{\Delta t} + \frac{2}{(\Delta x)^2} + \frac{2}{(\Delta y)^2} \right] w^n_{i,j,k-1/2} + \frac{X^n_{i,j,k-1/2}}{(\Delta x)^2} + \frac{Y^n_{i,j,k-1/2}}{(\Delta y)^2} + F^{n+1}_{i,j,k-1/2}$$

where X and Y are the second-order finite differences in x- and y-directions.

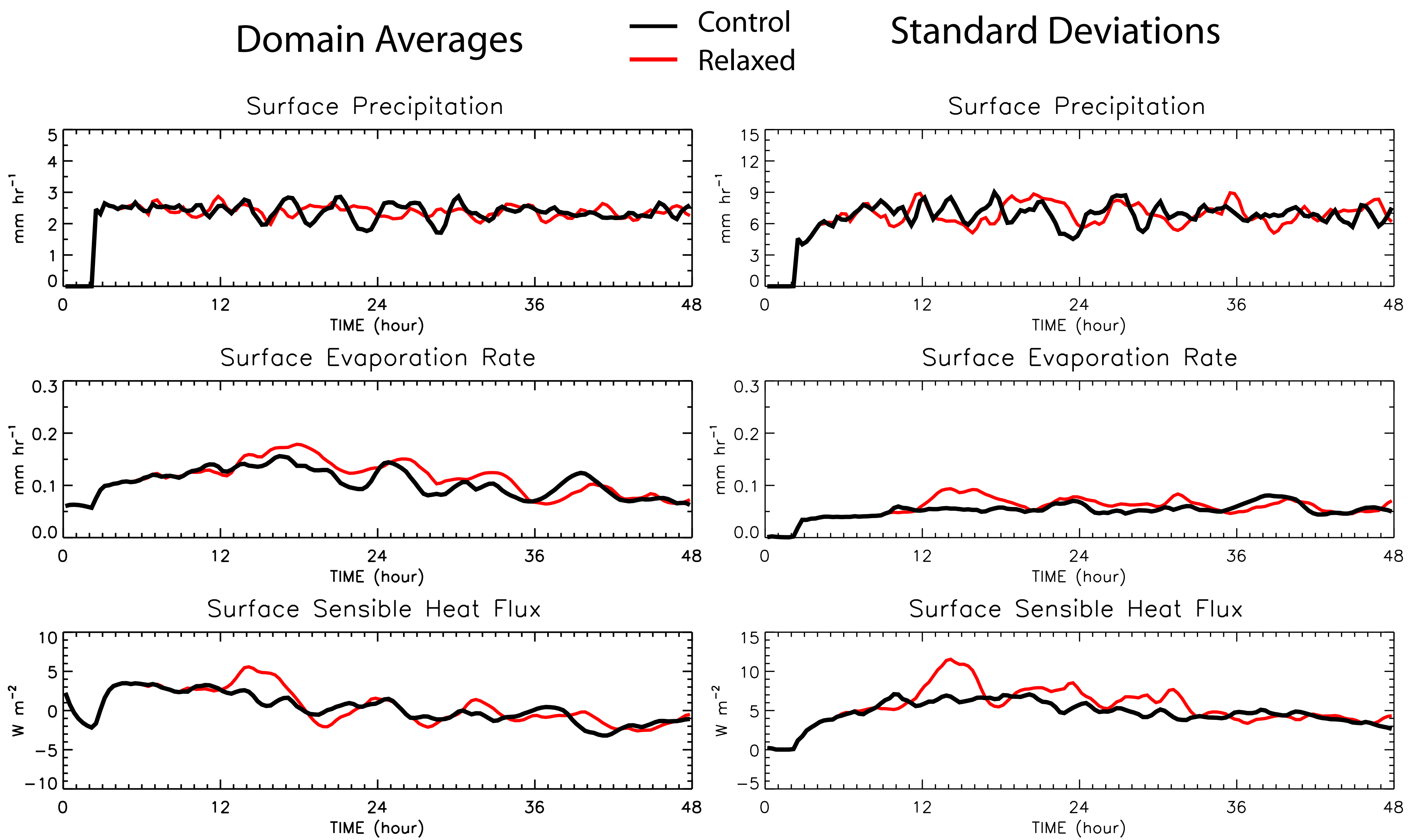
Relaxaion Method vs. Direct Method

Domain and Time Averaged Variables

(last 12 hour-average)



Relaxaion Method vs. Direct Method (Continued.)



Solving the Elliptic Equation in the Quasi-3D Network

The second-order finite difference in the normal direction is estimated as in the advection problem.

$$\left[\frac{1}{\rho_{k-1/2}}\left(\frac{\mu}{\Delta t}+\frac{2}{\Delta x^2}+\frac{2}{\Delta y^2}\right)+\frac{1}{\Delta z}\left(\frac{1}{\rho_k \Delta z}+\frac{1}{\rho_{k-1}\Delta z}\right)\right](\rho w)^{n+1}_{i,j,k-1/2}-\frac{1}{\Delta z}\left[\frac{(\rho w)^{n+1}_{i,j,k+1/2}}{\rho_k \Delta z}+\frac{(\rho w)^{n+1}_{i,j,k-3/2}}{\rho_{k-1}\Delta z}\right]$$

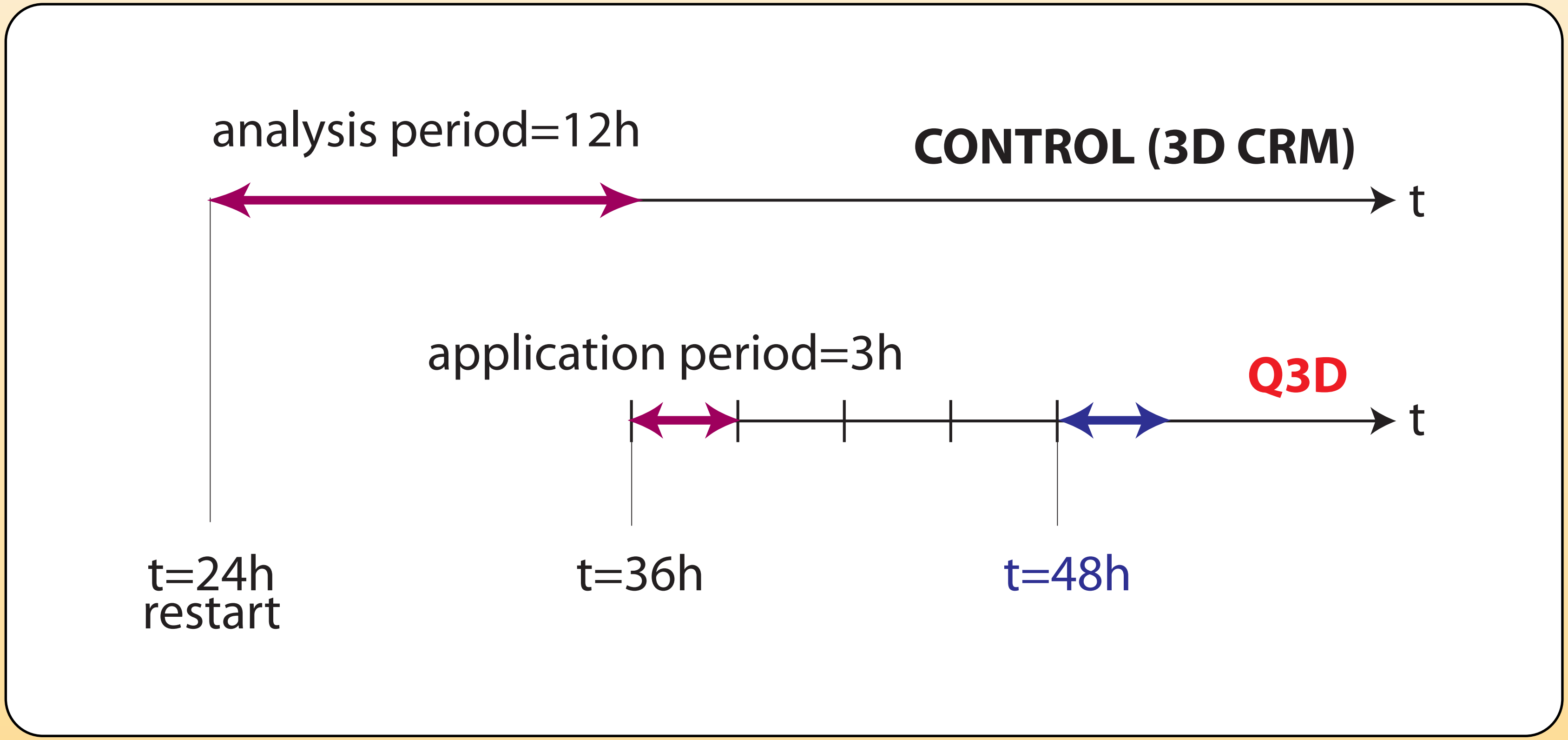
$$=\left[\frac{\mu}{\Delta t}+\frac{2}{(\Delta x)^2}+\frac{2}{(\Delta y)^2}\right]w^n_{i,j,k-1/2}+\frac{X^n_{i,j,k-1/2}}{(\Delta x)^2}+\frac{\left[\bar{Y}^n_{i,j,k-1/2}+\left(Y'^n_{i,j,k-1/2}\right)_{est}\right]}{(\Delta y)^2}+F^{n+1}_{i,j,k-1/2}$$

$$\left(Y'^n_{i,j,k-1/2}\right)_{est}\equiv a+bX'^n_{i,j,k-1/2}+C(r)\left(Y'^n_{I,j,k-1/2}-a-bX'^n_{I,j,k-1/2}\right)$$

where a and b are parameters, the subscript I denotes the nearest intersection point, and C(r) is a prescribed function of the distance from the intersection point satisfying C(0)=1.

Partially Prognostic Tests

- **Calculation of wind components from prescribed vorticity fields.**
  – Using the prescribed statistics of the history at the intersection and neighboring points.
- **Advection with prescribed winds and potential temperature.**
  – Tracer and different phases of water with physics



Model

A three-dimensional anelastic model based on the vector vorticity equation

by Joon-Hee Jung and Akio Arakawa (2006), Submitted to JAS

Control Run

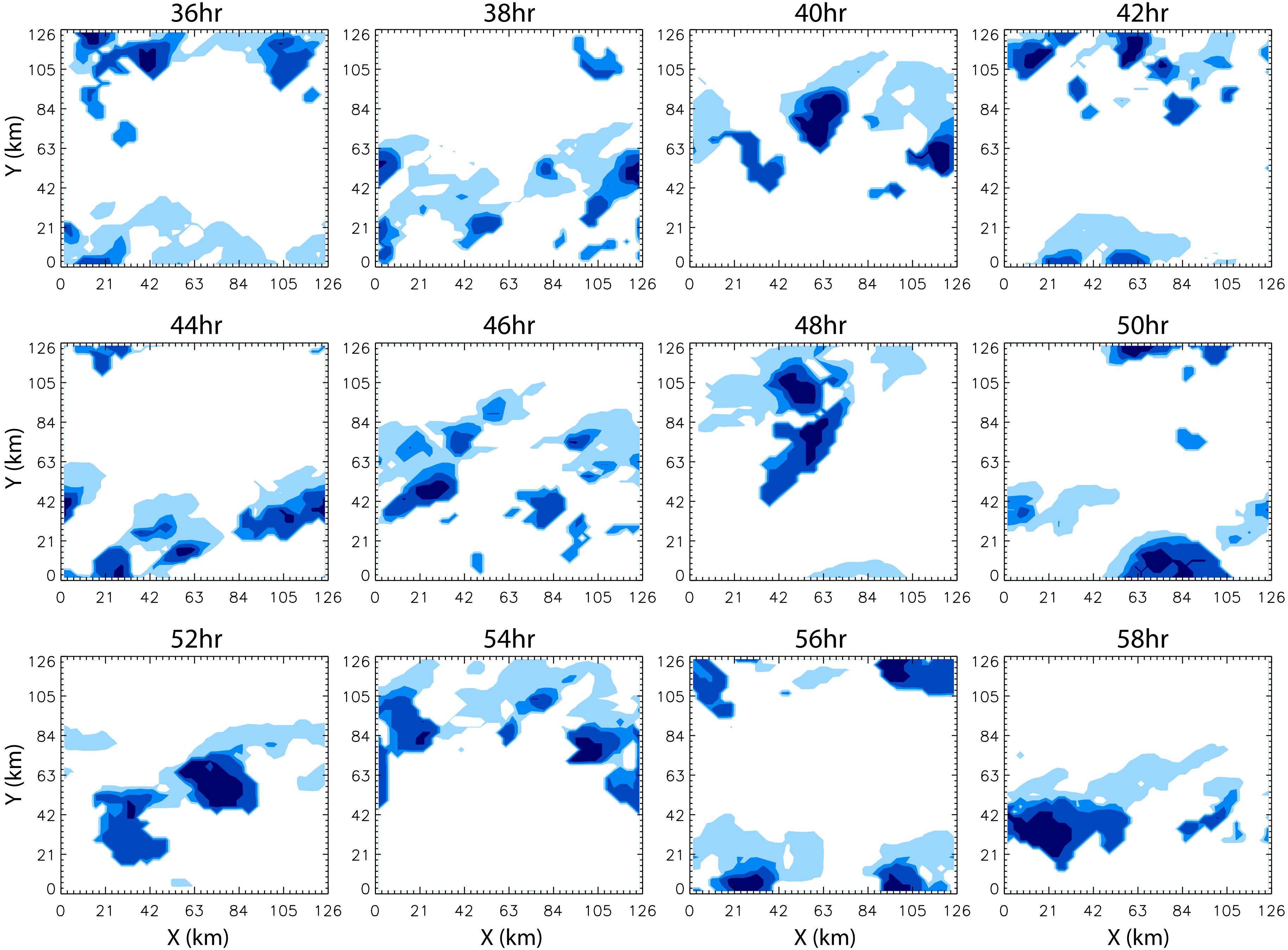
- **Domain size:** 126 km x 126 km x 18 km (height)
- **Horizontal resolution:** 3 km
- **Vertical resolution:** 34 layers with a stretched vertical grid
- **Lower-boundary:** ocean surface with a fixed temperature
- **Idealized tropical condition:** based on the GATE Phase-III mean sounding and wind profile during TOGA COARE
- **Large-scale forcing:** prescribed advective tendency
- **Perturbation:** small, random temperature perturbations into the lowest model layer

Q3D Run

Share the same configurations with those of Control

- **Net size:** 63 km
- **Rayleigh type damping on deviation fields**

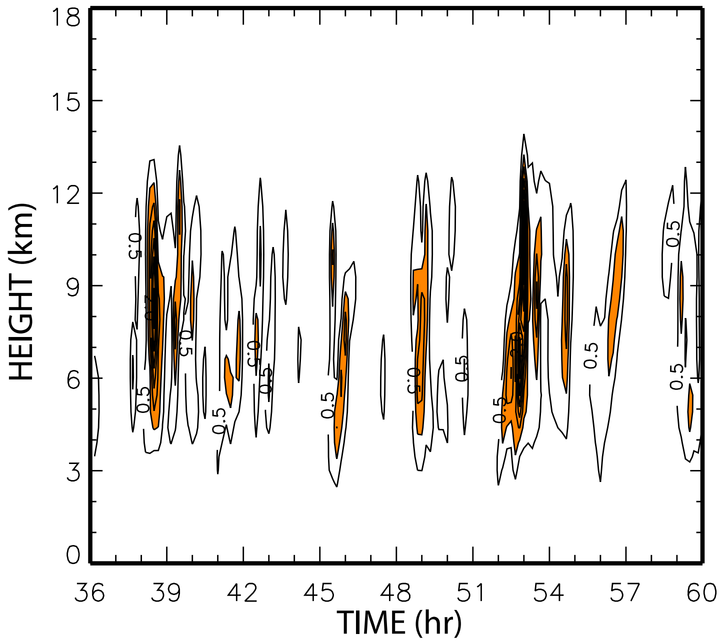
CONTROL
Cloud Top Temperature



w'^2 : Network Average

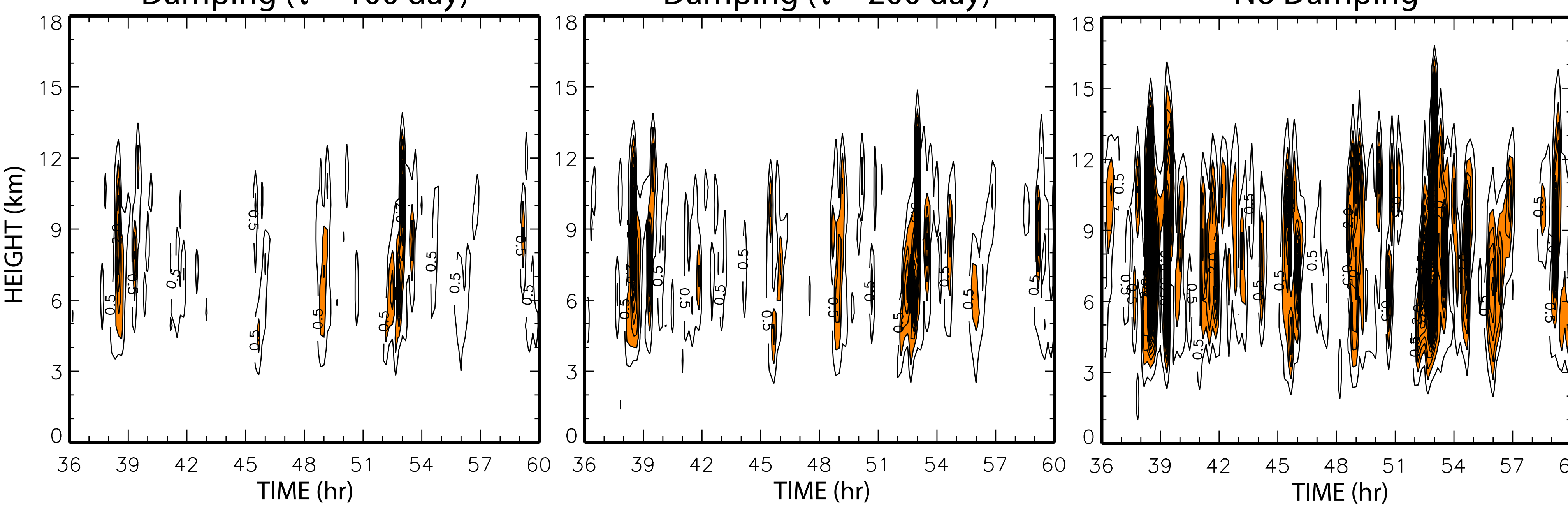
$$\mu\frac{\partial w}{\partial t} = \left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)w + \frac{\partial}{\partial z}\left[\frac{1}{\rho_0}\frac{\partial}{\partial z}(\rho_0 w)\right] + \rho_0\frac{\partial\eta}{\partial x} - \rho_0\frac{\partial\xi}{\partial y} - \frac{1}{\tau}(w-\bar{w})$$

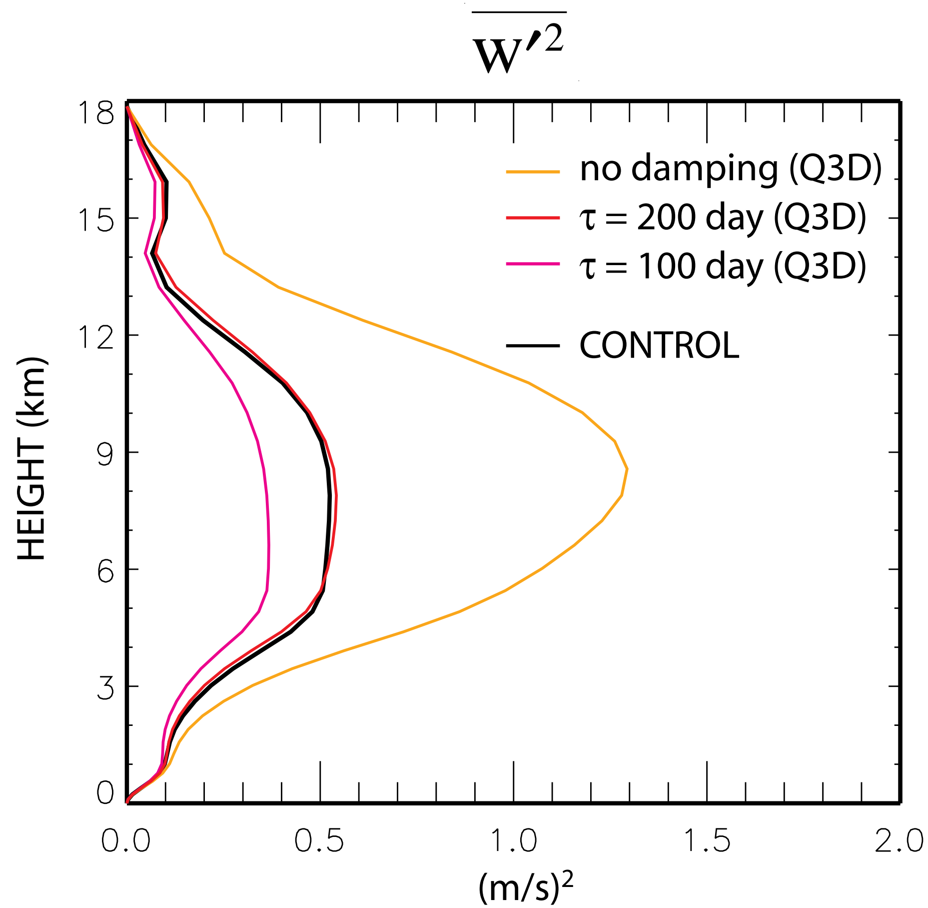
CONTROL (3D)



Q3D

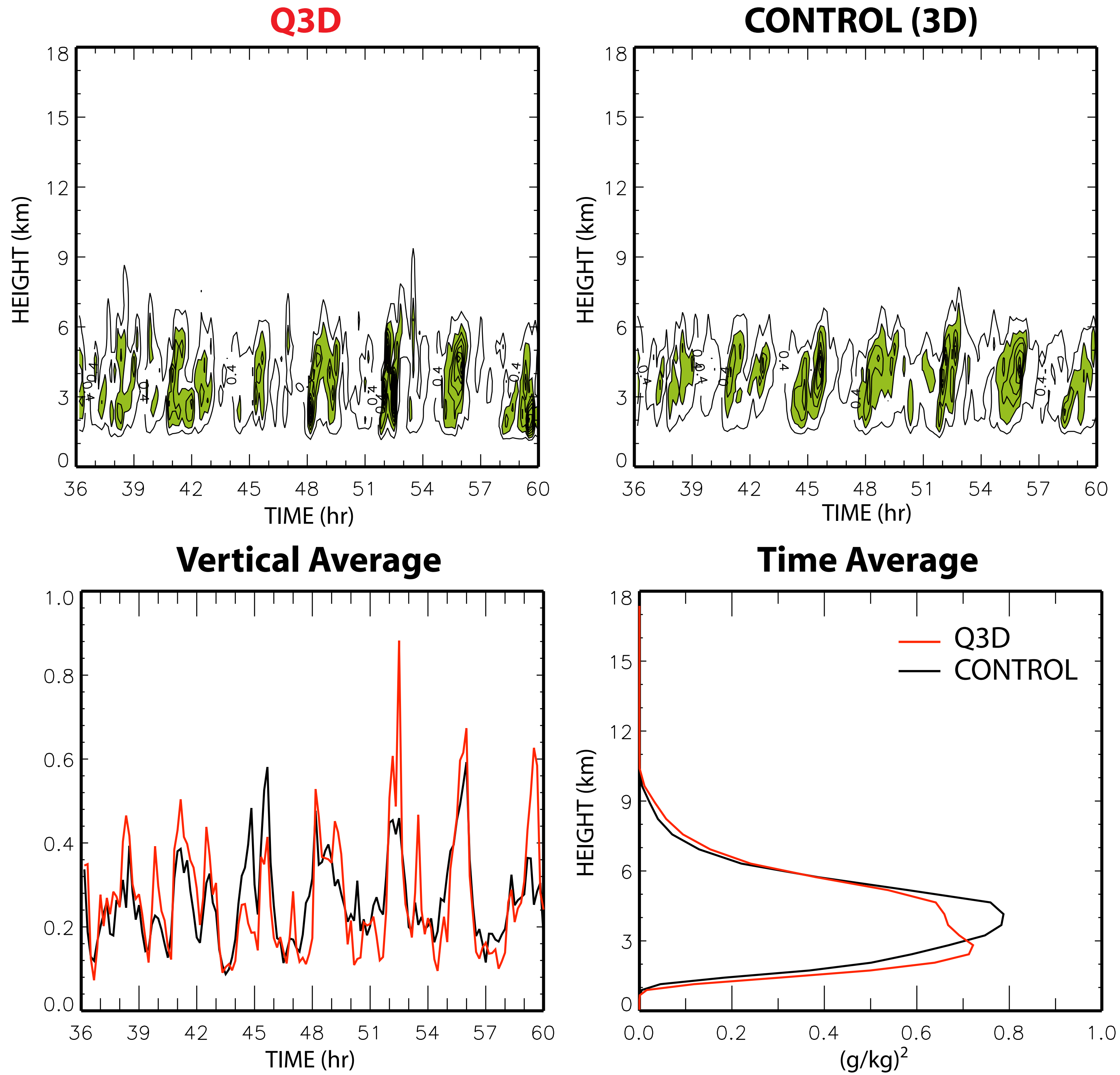
Damping (τ = 100 day) | Damping (τ = 200 day) | No Damping

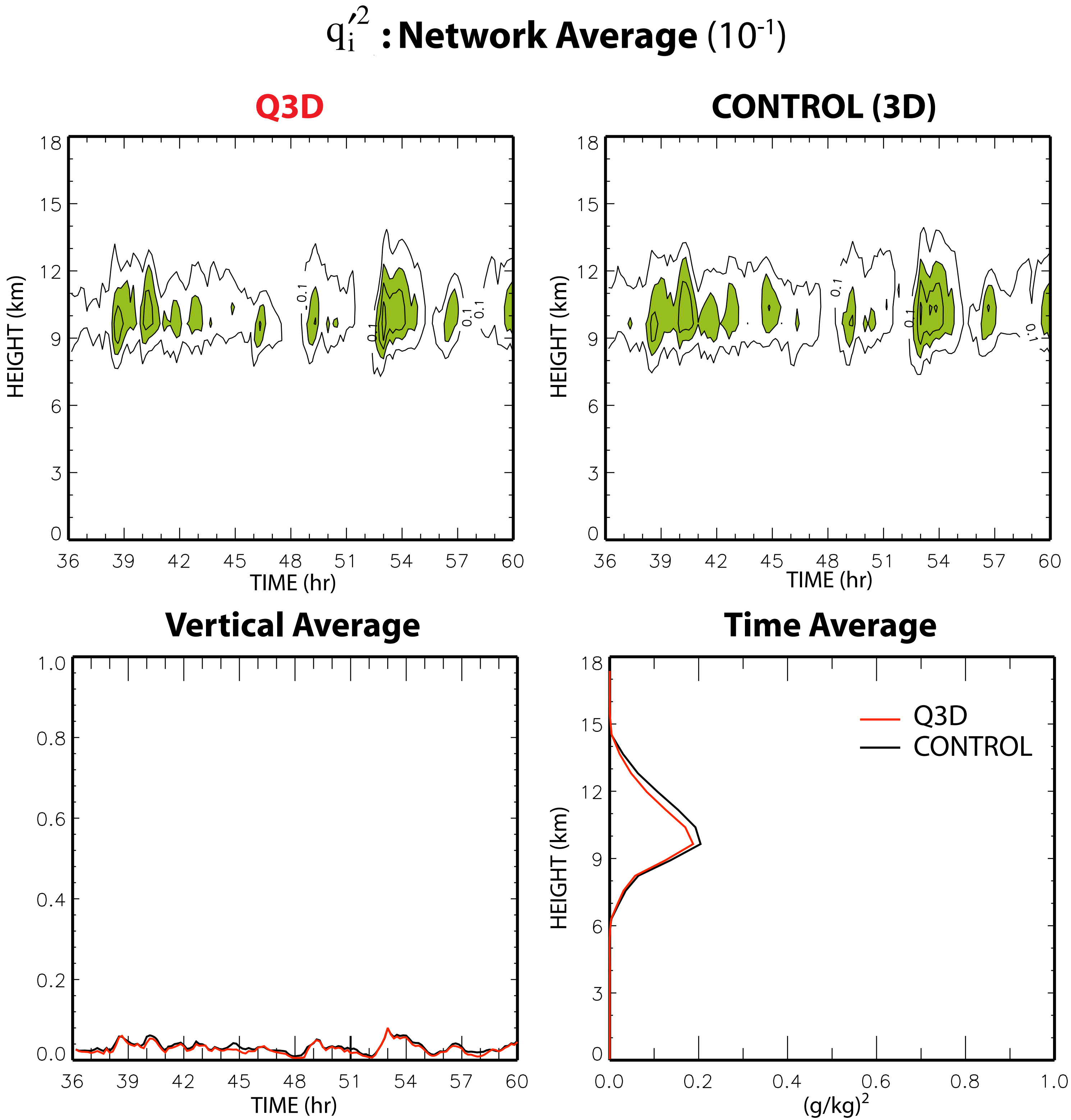




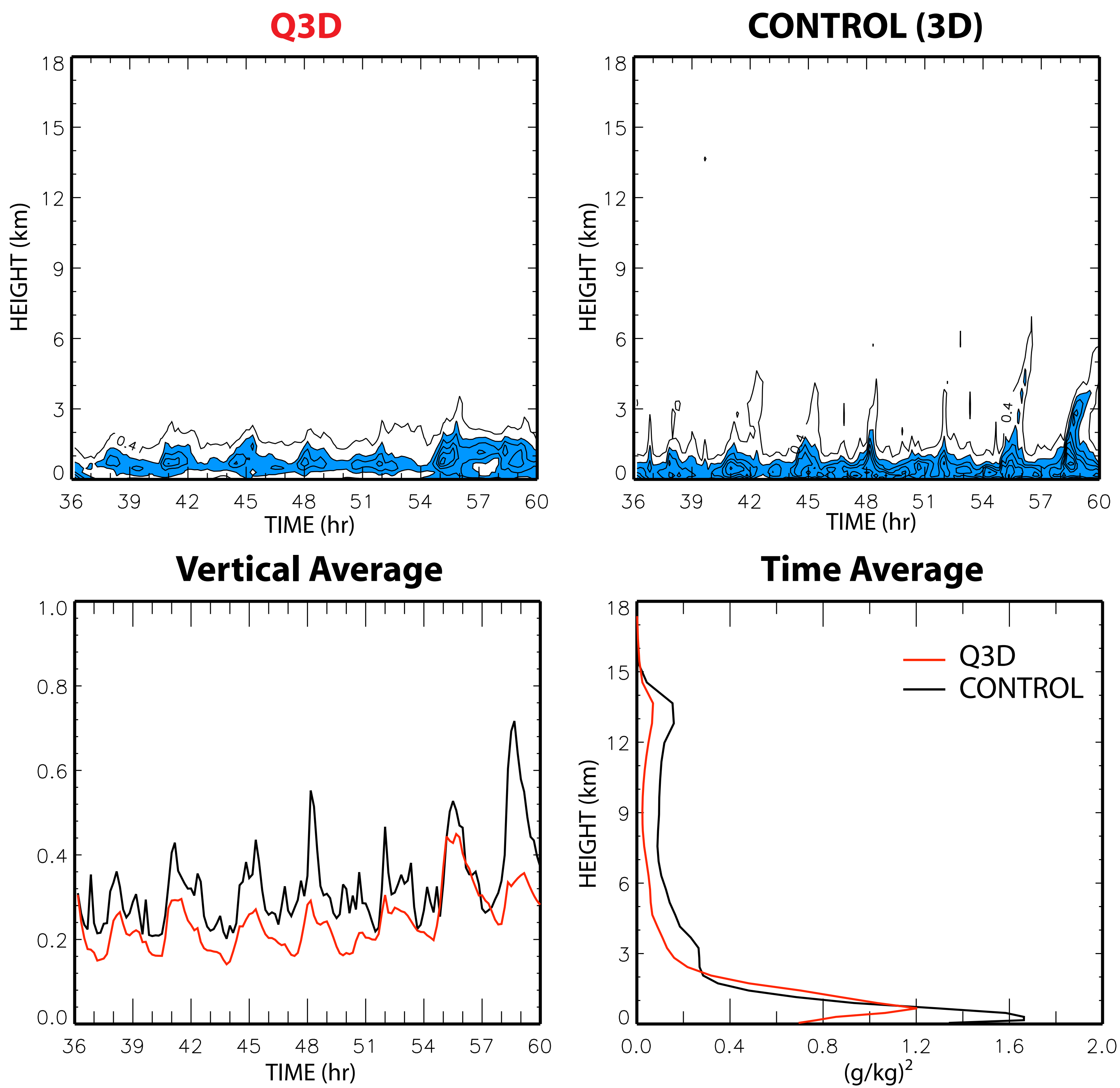
The 3D elliptic equation can be solved with the quasi-3D network, using a statistical method to estimate the 2nd-order derivatives in the direction normal to the grid point arrays.

$q_c'^2$ **: Network Average** (10^{-1})

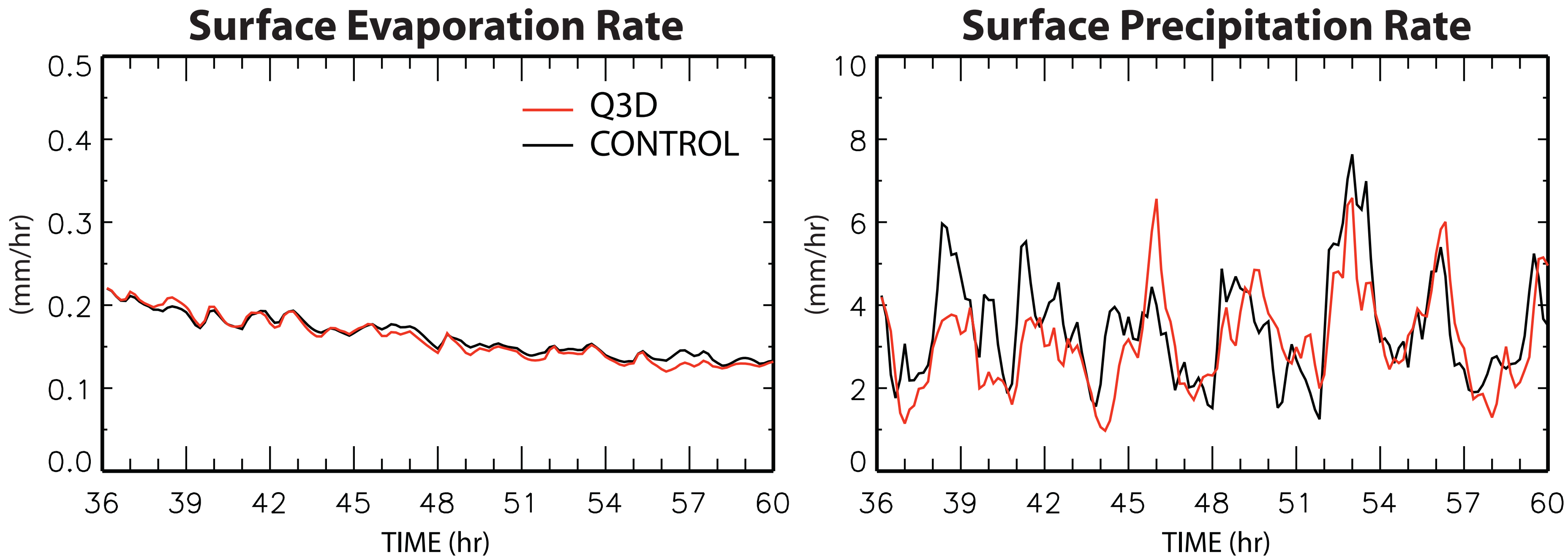



$q_i'^2$: Network Average (10^{-1})
Q3D
CONTROL (3D)
HEIGHT (km)
TIME (hr)
0.1
Vertical Average
Time Average
Q3D
CONTROL
$(g/kg)^2$

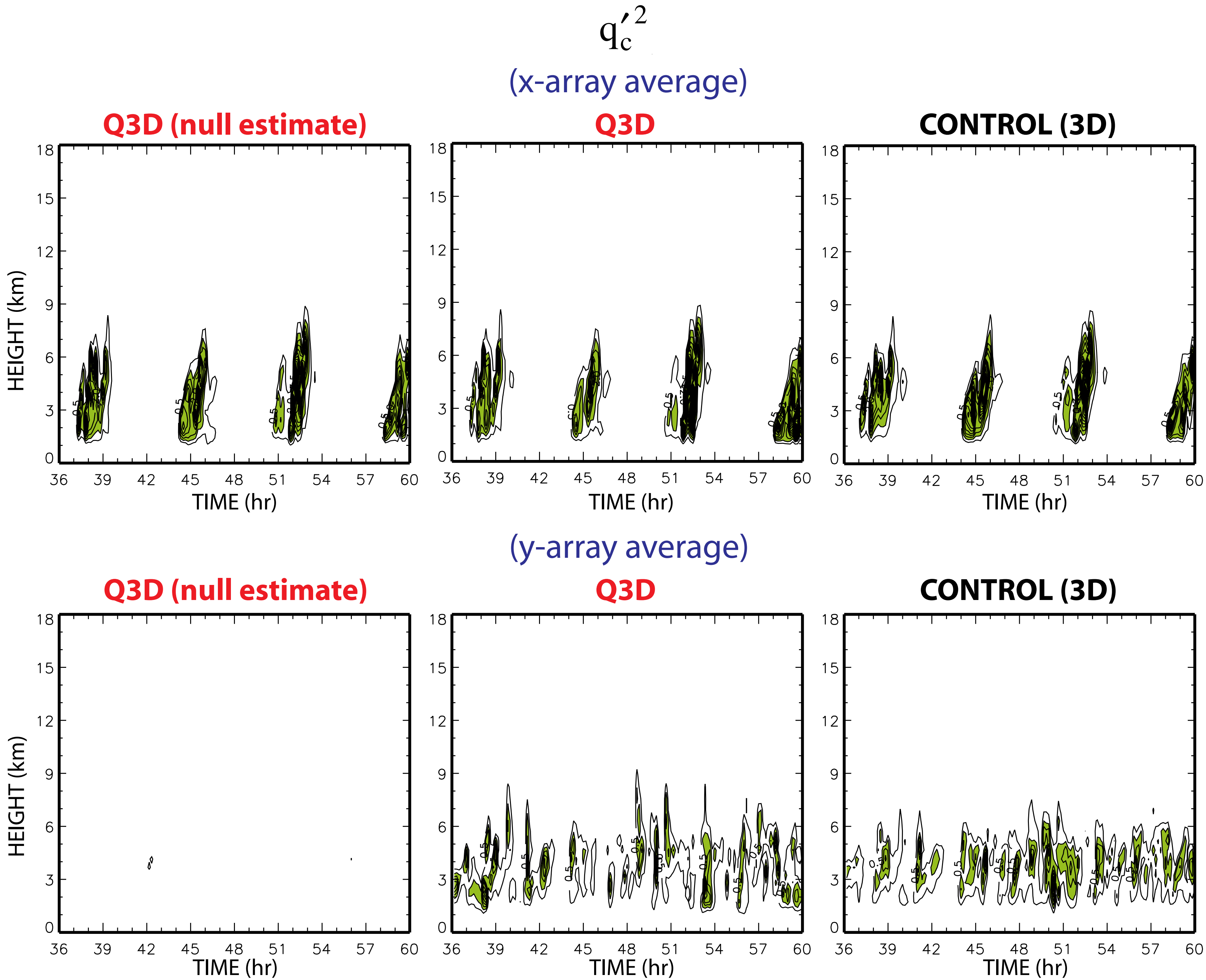
$q_t'^2$: Network Average

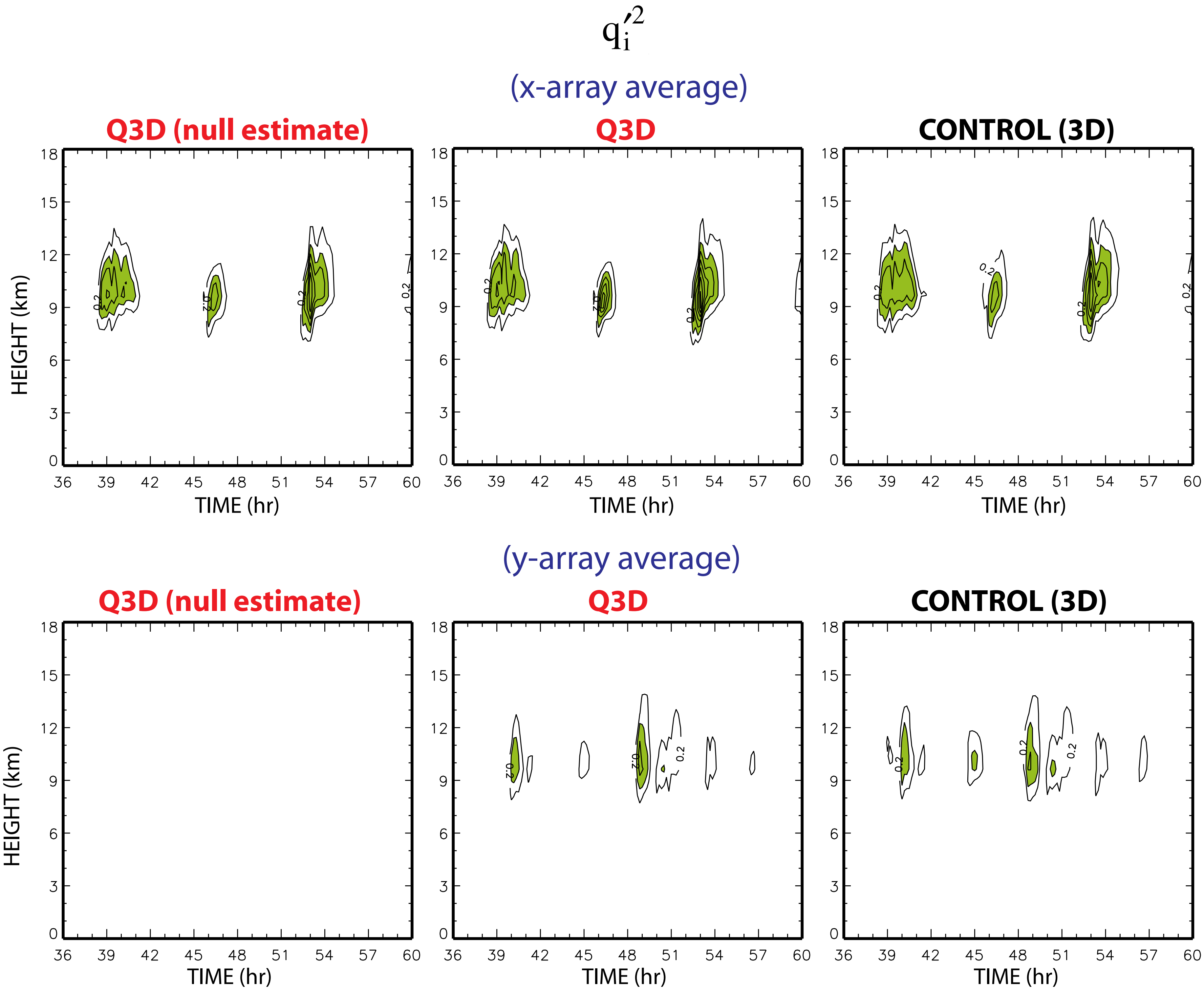


Network Average

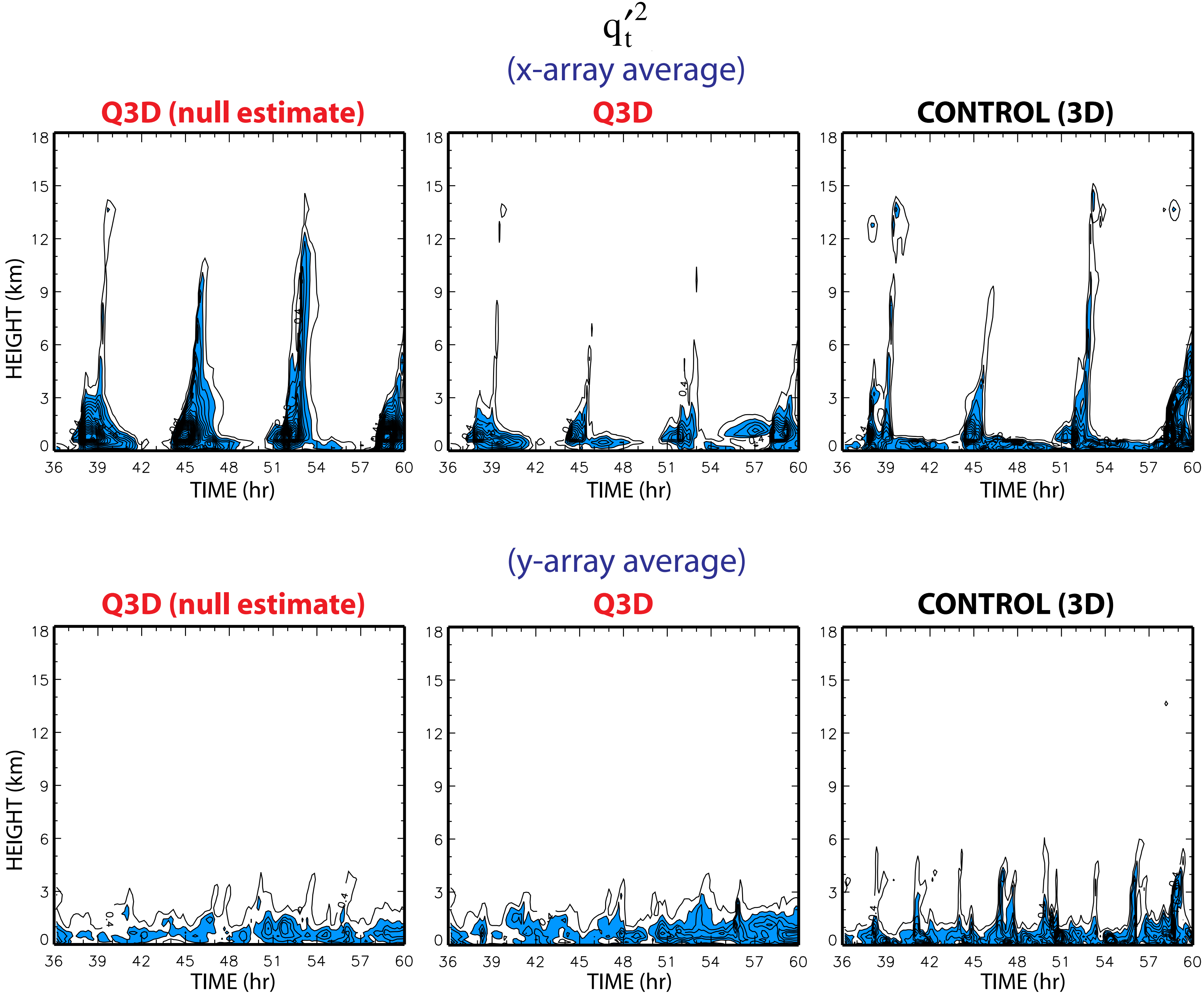


The results from the quasi-3D run show no apparent signs of violating the requirements of stability and conservation property.

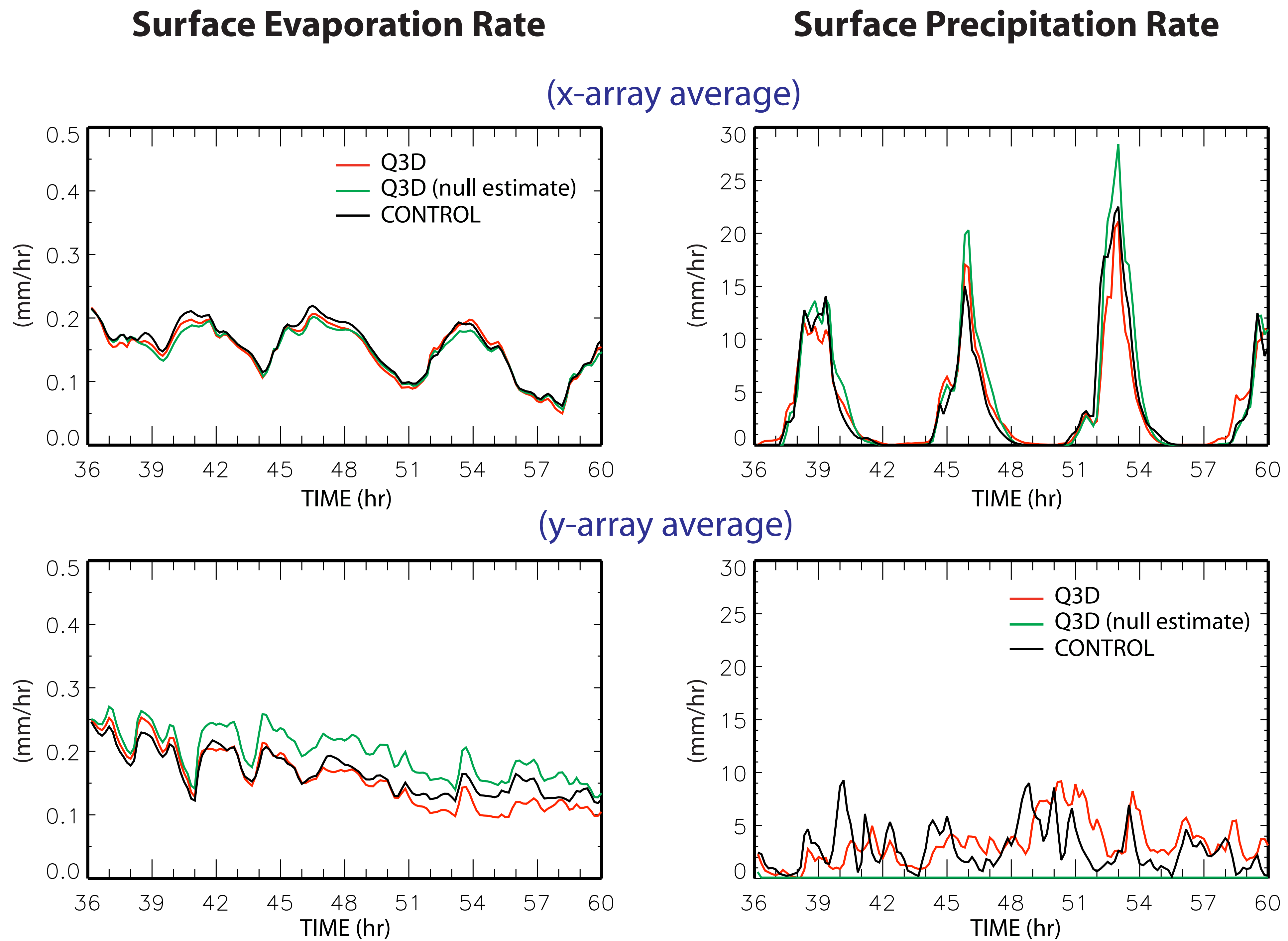

$q_c'^2$
(x-array average)
Q3D (null estimate)
Q3D
CONTROL (3D)
(y-array average)
Q3D (null estimate)
Q3D
CONTROL (3D)
HEIGHT (km)
TIME (hr)
0
3
6
9
12
15
18
36
39
42
45
48
51
54
57
60
0.5



$q_i'^2$
(x-array average)
Q3D (null estimate)
Q3D
CONTROL (3D)
(y-array average)
Q3D (null estimate)
Q3D
CONTROL (3D)
HEIGHT (km)
TIME (hr)
0.2
0 3 6 9 12 15 18
36 39 42 45 48 51 54 57 60



$q_t'^2$
(x-array average)
Q3D (null estimate)
Q3D
CONTROL (3D)
HEIGHT (km)
TIME (hr)
0.4
(y-array average)
Q3D (null estimate)
Q3D
CONTROL (3D)
HEIGHT (km)
TIME (hr)
0.4



Surface Evaporation Rate
Surface Precipitation Rate
(x-array average)
(y-array average)
Q3D
Q3D (null estimate)
CONTROL
(mm/hr)
TIME (hr)
0.0
0.1
0.2
0.3
0.4
0.5
0
5
10
15
20
25
30
36
39
42
45
48
51
54
57
60

Conclusions

- The 3D elliptic equation can be solved with the quasi-3D network, using a statistical method to estimate the second-order derivatives in the direction normal to the grid point arrays.

- The results from the quasi-3D run show no apparent signs of violating the requirements of stability and conservation property.

- The 2D and quasi-3D algorithms do not make much difference for tracer. The same is true for water substance where convection is active. In such a case, the horizontal advection can be negligible compared to vertical advection, and vertical velocity is prescribed from the control in this test.

The results of the partially prognostic tests are encouraging and we are ready to challenge fully prognostic tests.