QUASI-3D MMF AND GLOBAL CRM

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in collaboration with Joon-Hee Jung and Celal Konor

ABSTRACT

I. Introduction

Multiscale Modeling Framework is an attempt to link GCM and CRM.

II. Unification of the system of dynamics equations

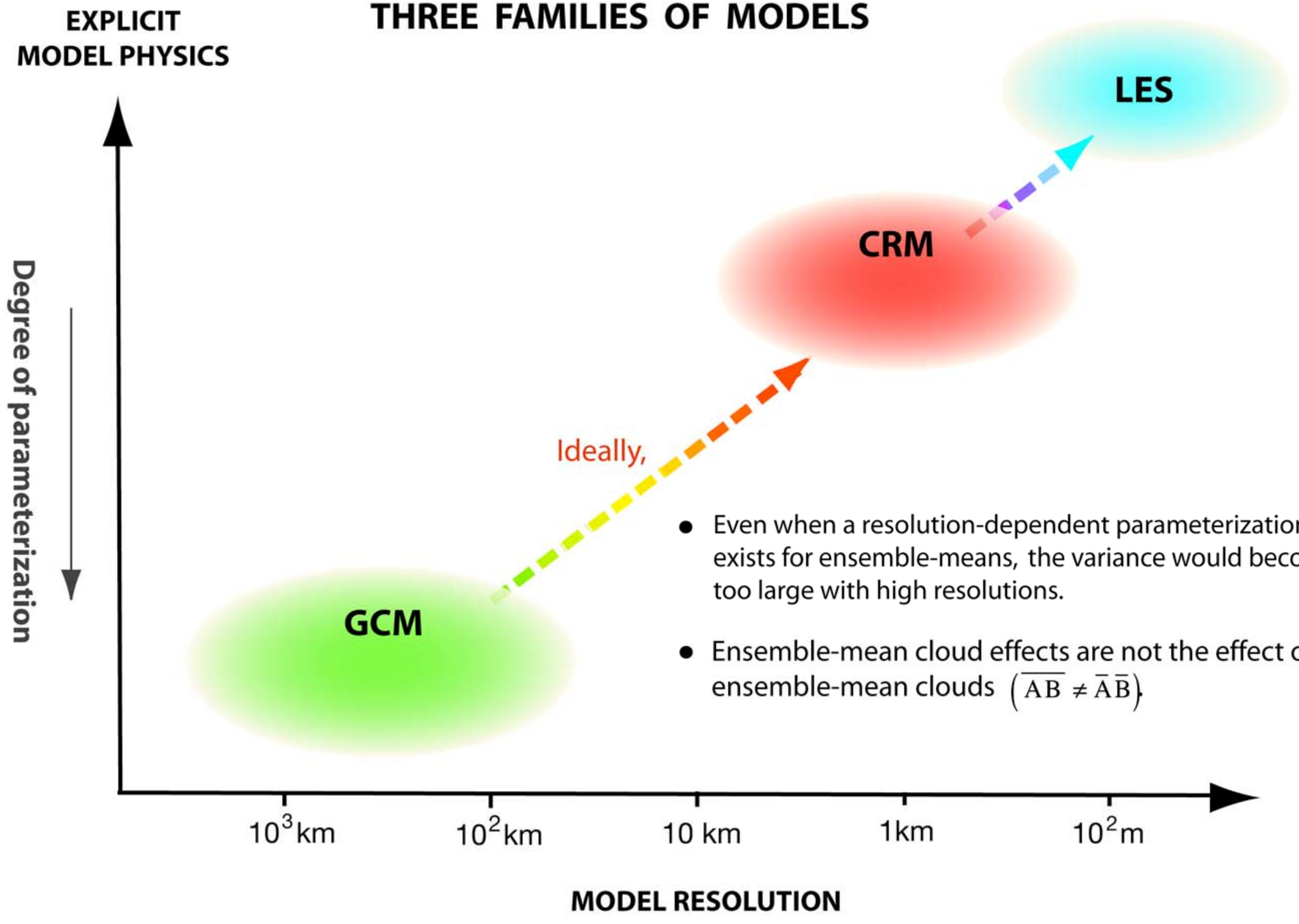
- For such a link, the system of equations must be unified to cover a broad spetrum from turbulence to planetary waves.
- With this objective, a unified system of equations is developed.

III. The quasi-3D multi-scale modeling framework

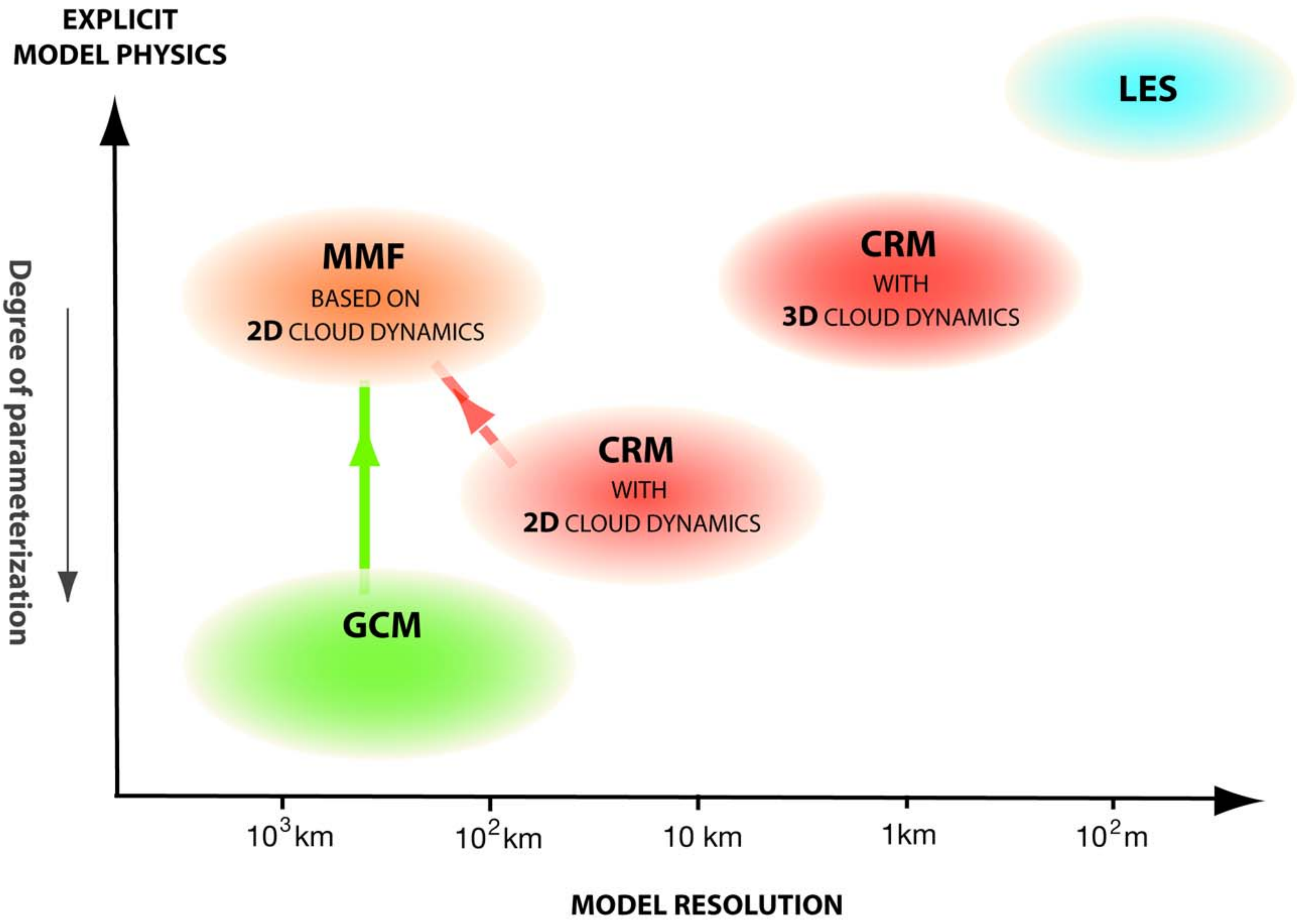
- If we wish to include the dynamical interactions between CRM and GCM, the CRM must be at least quasi-3D.
- Progress has been made in our understanding of the problems involved.

(Some of them are related to the basic question of diagnostic parametrizability and the use of "Double-Scale Modeling Framework".)

THREE FAMILIES OF MODELS



- Even when a resolution-dependent parameterization exists for ensemble-means, the variance would become too large with high resolutions.
- Ensemble-mean cloud effects are not the effect of ensemble-mean clouds $\left(\overline{\mathrm{AB}} \neq \bar{\mathrm{A}}\bar{\mathrm{B}}\right)$


EXPLICIT
MODEL PHYSICS
Degree of parameterization
LES
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BASED ON
2D CLOUD DYNAMICS
CRM
WITH
3D CLOUD DYNAMICS
CRM
WITH
2D CLOUD DYNAMICS
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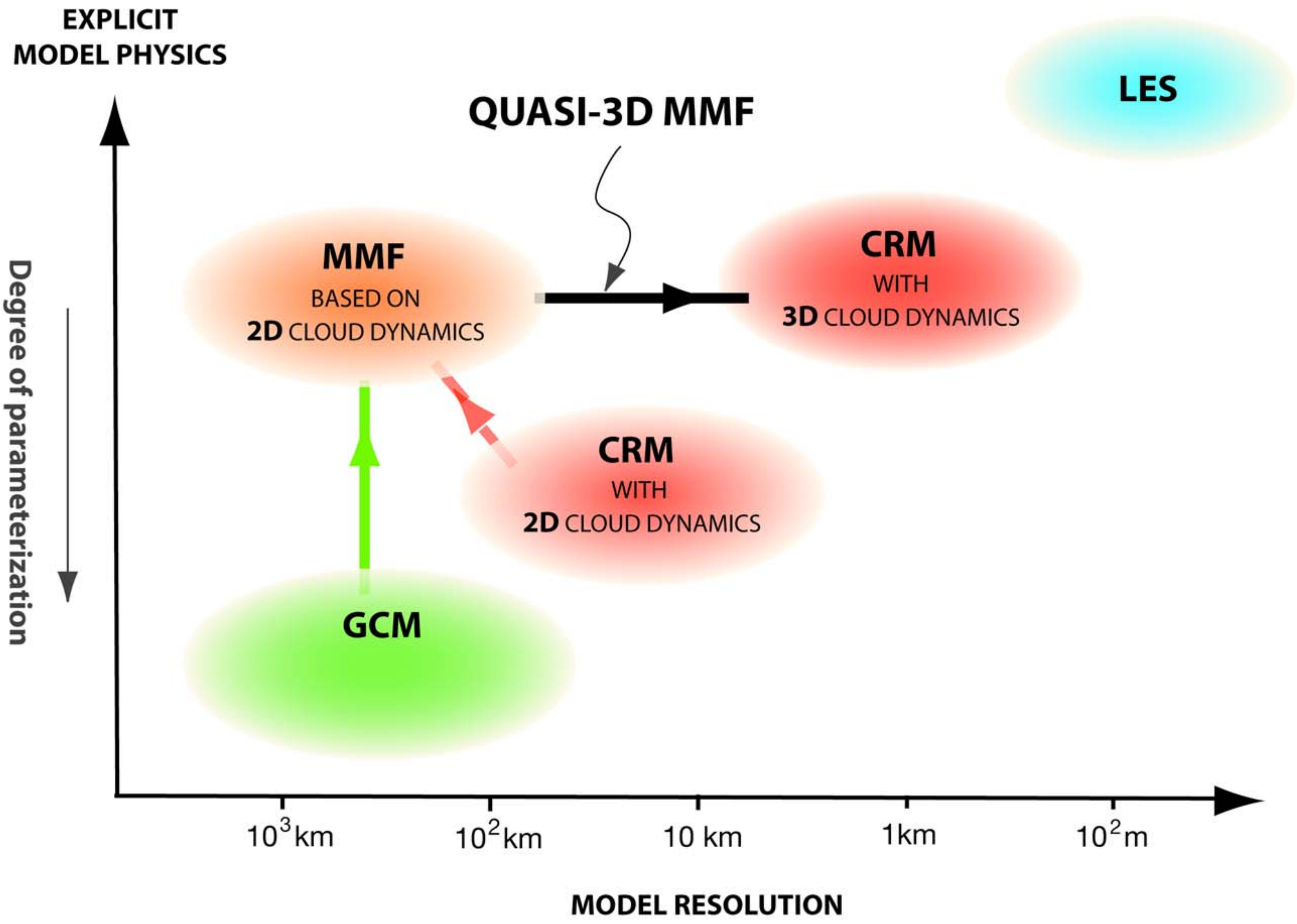
10^3km
10^2km
10 km
1km
10^2m
MODEL RESOLUTION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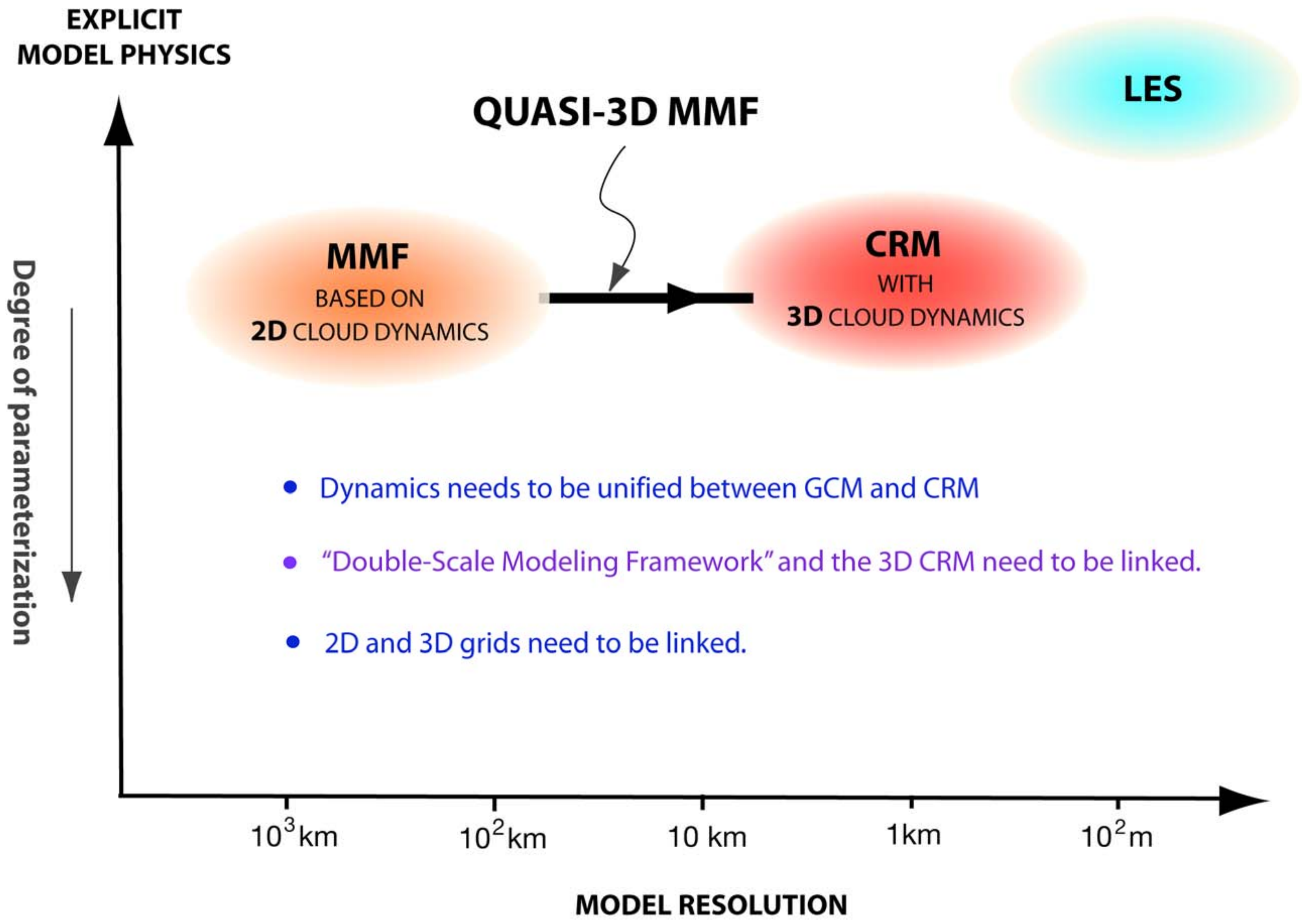


- Dynamics needs to be unified between GCM and CRM
- “Double-Scale Modeling Framework” and the 3D CRM need to be linked.
- 2D and 3D grids need to be linked.

Unification of the System of Dynamics Equations between GCM and CRM

This is necessary for the convergence of Quasi-3D MMF to a 3D CRM .

Possibilities :

I. Use of the fully-compressible nonhydrostatic system of equations

Modification of 3D dynamics to Q3D dynamics will be extremely difficult.

II. Use of a system of equations that filters vertically-propagating sound waves

- In the quasi-hydrostatic system of equations (primitive equations), the vertical component of the momentum equation is diagnostic.
- In the anelastic system of equations, the continuity equation is diagnostic.

Our approach unifies these two ways of filtering.

Continuity Equation

$$\begin{array}{c}\dfrac{\partial\rho}{\partial t}\\ \shortparallel \\ \left(\dfrac{\rho}{\gamma p}\right)\dfrac{\partial p}{\partial t}+\left(\dfrac{\rho}{\theta}\right)\dfrac{\partial\theta}{\partial t}\end{array} + \nabla\cdot\left(\rho\mathbf{V}\right)=0$$

isentropic compressibility

adiabatic & diabatic heating

Anelastic (Ogura & Phillips 1962, Lipps & Hemler 1982, Bannon 1996, and many others)

$$\boxed{\nabla\cdot\left(\rho_0\mathbf{V}\right)=0}$$

With modifications of the momentum and/or thermodynamic equations for energetic consistency

Pseudo-incompressible (Durran 1989)

$$\boxed{\left(\frac{\rho_0}{\theta_0}\right)\left(-w\frac{\partial\theta_0}{\partial z}+\frac{\theta_0}{c_p T_0}Q\right)+\nabla\cdot\left(\rho_0\mathbf{V}\right)=0}$$

With no major modification of the momentum and thermodynamic equations

Quasi-hydrostatic

$$\frac{\partial\rho_{qs}}{\partial t}+\nabla\cdot\left(\rho_{qs}\mathbf{V}\right)=0$$

ρ_{qs} : quasi-hydrostatic density

With the hydrostatic equation for the vertical component of the momentum equation

Unified

$$\boxed{\frac{\partial\rho_{qs}}{\partial t}+\nabla\cdot\left(\rho_{qs}\mathbf{V}\right)=0}$$

With no modification of the momentum and thermodynamic equations

Problems with the Anelastic System of Equations

$$\boxed{\nabla \cdot (\rho_0 \mathbf{V}) = 0}$$

I. Too restrictive reference state

- The common way of maintaining energetic consistency is through an assumption that the reference state is neutral (Ogura & Phillips 1962) or approximately neutral (Lipps & Hemler 1982).

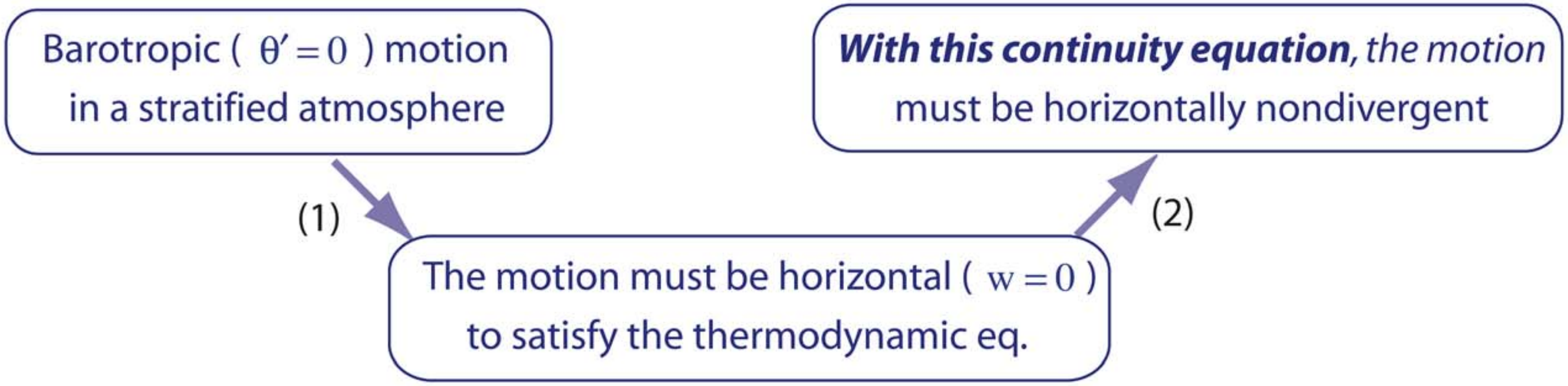
- This assumption introduces a serious error in the vertical structure of the disturbances in a stable atmosphere.

The anelastic system is NOT for a model that includes the stratosphere.

Problems with the Anelastic System of Equations (Continued)

II. Spuriously-fast westward retrogression of barotropic ultra-long waves

$$\boxed{\nabla_{H} \cdot (\rho_0 \mathbf{V}_{H}) + \frac{\partial}{\partial z}(\rho_0 w) = 0}$$



Barotropic ($\theta' = 0$) motion in a stratified atmosphere

(1)

The motion must be horizontal ($w = 0$) to satisfy the thermodynamic eq.

(2)

With this continuity equation*, the motion* must be horizontally nondivergent

Then we are back to the old problem with ultra-long waves recognized during the early years of NWP .

Wolff, P. M., 1958: The error in numerical forecasts due to retrogression of ultra-long waves. *MWR*.

Cressman, G. P., 1958: Barotropic divergence and very long atmospheric waves. *MWR*.

Wiin-Nielsen, A., 1959: On barotropic and baroclinic models, with special emphasis on ultra-long waves. *MWR*.

These papers attempted to bypass the path (1) although the real problem is in (2).

Conserved Dynamics Variable and Retrogression Speed of the Rossby Wave

	Nondivergent motion	Shallow-water motion	Barotropic motion in the atmosphere
Conserved dynamics variable	Absolute vorticity $f+\zeta$	Shallow-water potential vorticity $\frac{f+\zeta}{h}$	Barotropic version of Ertel's potential vorticity $\frac{f+\zeta}{\rho}$
Retrogression speed of Rossby wave	$\frac{\beta}{k^2}$ $\rightarrow \infty$ as $k \rightarrow 0$	$\sim \frac{\beta}{k^2 + f_0^{\ 2} / gH}$ $\rightarrow$ finite as $k \rightarrow 0$	$\sim \frac{\beta}{k^2 + f_0^{\ 2} / c_s^2}$ $\rightarrow$ finite as $k \rightarrow 0$

What makes the atmospheric barotropic motion analogous to the shallow-water motion is *compressibility*.

The anelastic system is NOT for a global model .

- Yet, it is a very good approximation for small-scale convection in the troposphere.
- Also, the quasi-hydrostatic approximation is an excellent approximation for large-scale motions.

The Continuity Equation in the Unified System

— Quasi-Hydrostatic Continuity Equation —

$$\frac{\partial \rho_{qs}}{\partial t} + \nabla \cdot \left(\rho_{qs} \mathbf{V}\right) = 0$$

$$\parallel$$

$$\left(\frac{\rho}{\gamma p}\right)_{qs} \frac{\partial p_{qs}}{\partial t} + \left(\frac{\rho}{\theta}\right)_{qs} \frac{\partial \theta}{\partial t}$$

This is NOT a prognostic equation
because ρ_{qs} *is predicted by*
the surface-pressure tendency and
thermodynamic equations.

- This is exact when the motion is quasi-hydrostatic . The entire system then becomes equivalent to the primitive equations.

- This becomes Durran's pseudo-incompressible equation when
    - the $\partial p_{qs} / \partial t$ term is neglected,
    - the $\partial \theta / \partial t$ term is linearized with respect to the deviations from a reference state.

- This becomes the anelastic continuity equation when the $\partial \theta / \partial t$ term is further neglected.

The Continuity Equation in the Unified System (Continued)

Rewriting,

$$\boxed{\frac{1}{\rho_{qs}}\frac{\partial}{\partial z}(\rho_{qs}w) = -\nabla\cdot\mathbf{V}_H - \left(\frac{D}{Dt}\right)_H \ell n\rho_{qs}}$$

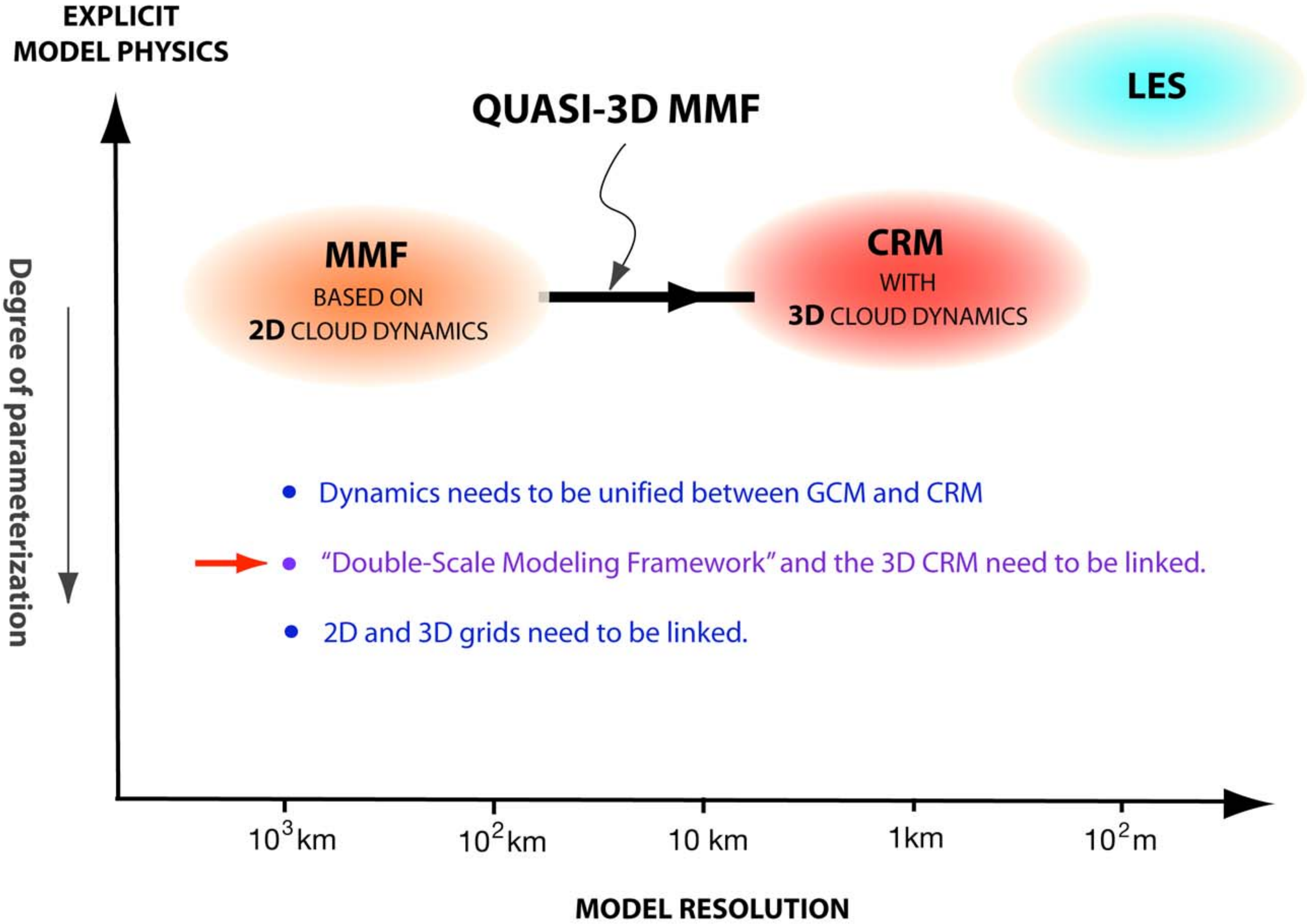
- The last term is evaluated using the surface-pressure tendency and thermodynamic equations.
- In contrast to the Richardson (1922) equation, the unified system treats this term as a generally small correction term.
- When the momentum equation is used,
    - the non-hydrostatic pressure is determined for the predicted 3D velocity to satisfy this continuity equation (parallel to the anelastic system);
- When the horizontal component of vorticity equation is predicted,
    - The unified system solves

$$\boxed{\nabla_H^2 w + \frac{\partial}{\partial z}\left[\frac{1}{\rho_{qs}}\frac{\partial}{\partial z}\left(\rho_{qs}w\right)\right] = -\mathbf{k}\cdot\nabla_H\times\omega_H - \frac{\partial}{\partial z}\left(\frac{D}{Dt}\right)_H \ell n\rho_{qs}}$$

    - the vertical component of the momentum equation can then be used to *diagnose* the non-hydrostatic pressure.

SUMMARY AND CONCLUDING REMARKS

- The anelastic model introduces a serious error in the vetical structure
- The anelastic and pseudo-incompressible approximations introduce a serious error for barotropic ultra-long waves .
- These problems do not exist in the unified system.
- We plan to use (a geodesic-grid version of) the unified model for a global cloud-resolving model.



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MODEL PHYSICS

Degree of parameterization

QUASI-3D MMF

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MMF
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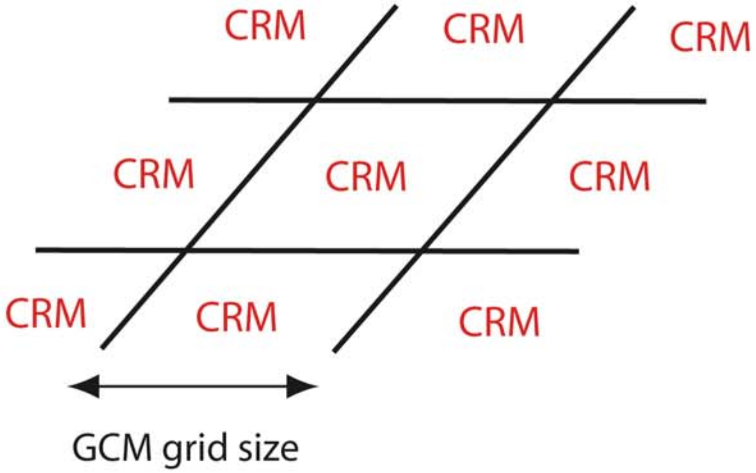
- Dynamics needs to be unified between GCM and CRM
- "Double-Scale Modeling Framework" and the 3D CRM need to be linked.
- 2D and 3D grids need to be linked.

10^3km 10^2km 10 km 1km 10^2m

MODEL RESOLUTION

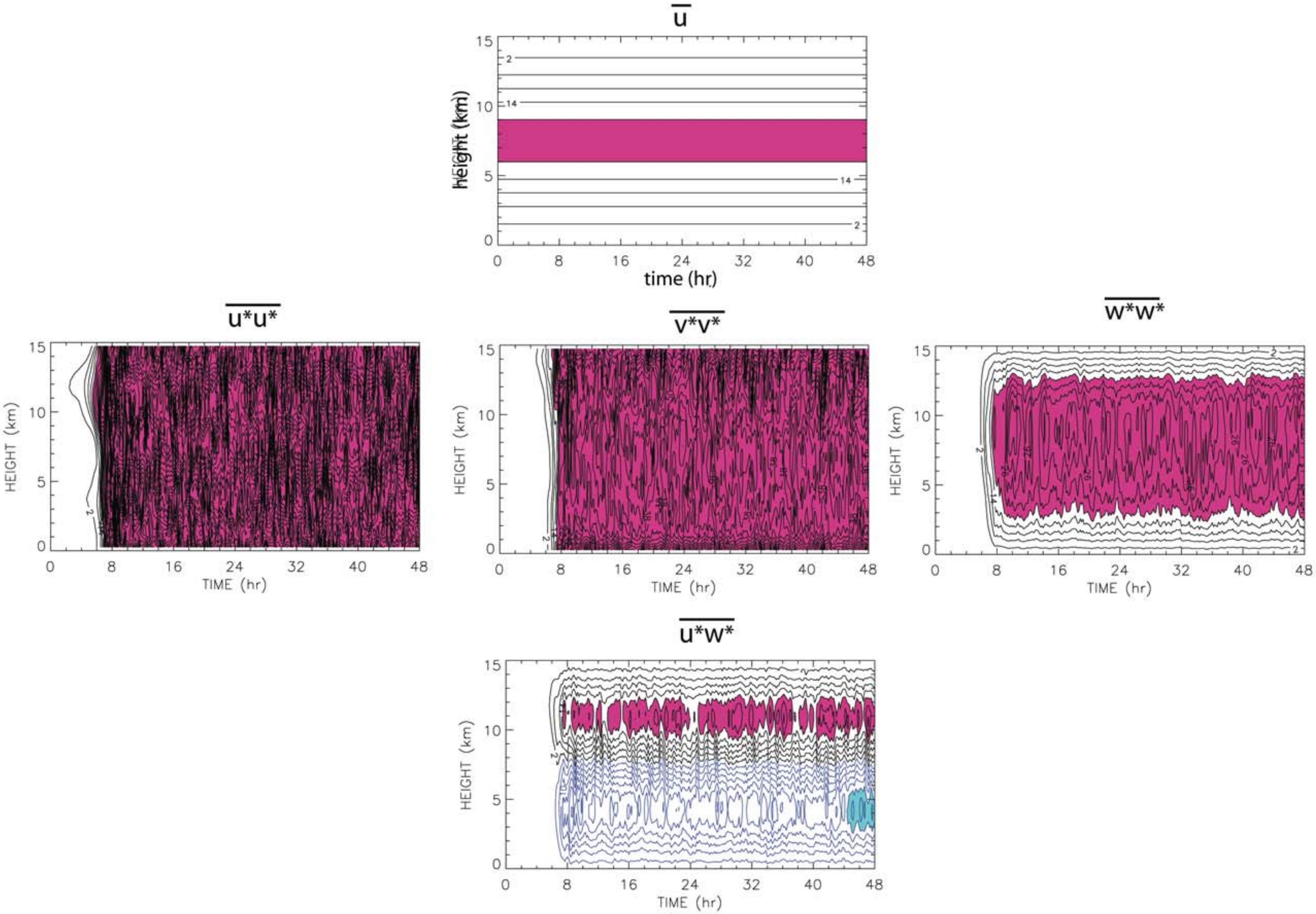
Problems with the "Double-Scale Framework"



- This structure is inherited from the conventional GCMs, which assume "parameterizability".
- The interactive nature of the MMF is an advantage in principle.
- However, if the GCM dynamics is very rigid due to over-dominating large-scale processes, there is not much room for the feedback to operate.
- GCM dynamics without mesoscale dynamics may be too rigid from the point of view of interactions with the cloud scale.

Sensitivity of the Equilibration of Shear Instability to the Rigidity of Mean Flow

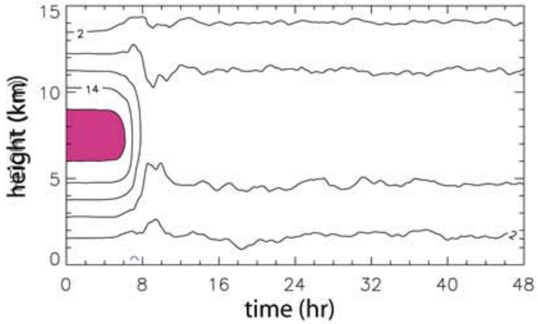
Shear Instability Simulation _Rigid Mean Flow



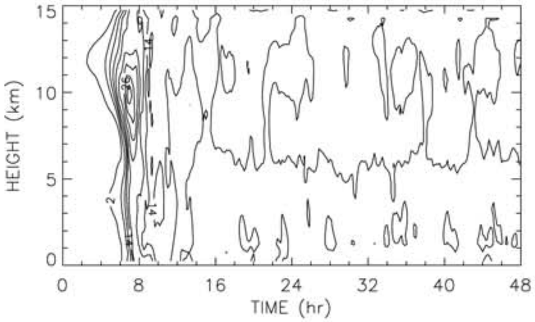
Sensitivity of the Equilibration of Shear Instability to the Rigidity of Mean Flow

Shear Instability Simulation _Relaxed Mean Flow with the Time Scale of 2 hrs

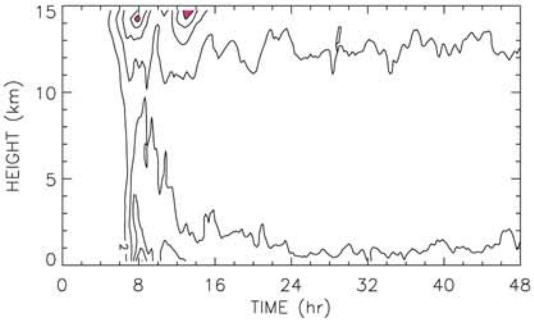
$\overline{u}$



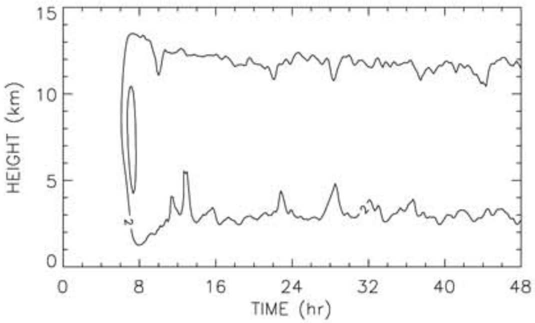
$\overline{u^*u^*}$



$\overline{v^*v^*}$



$\overline{w^*w^*}$



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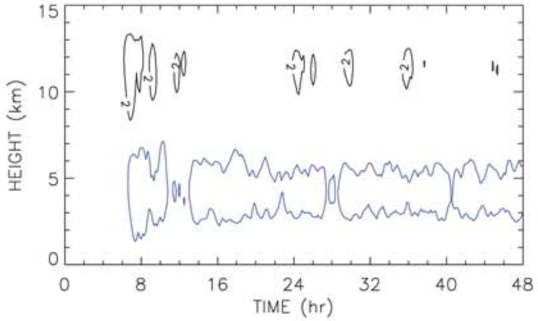

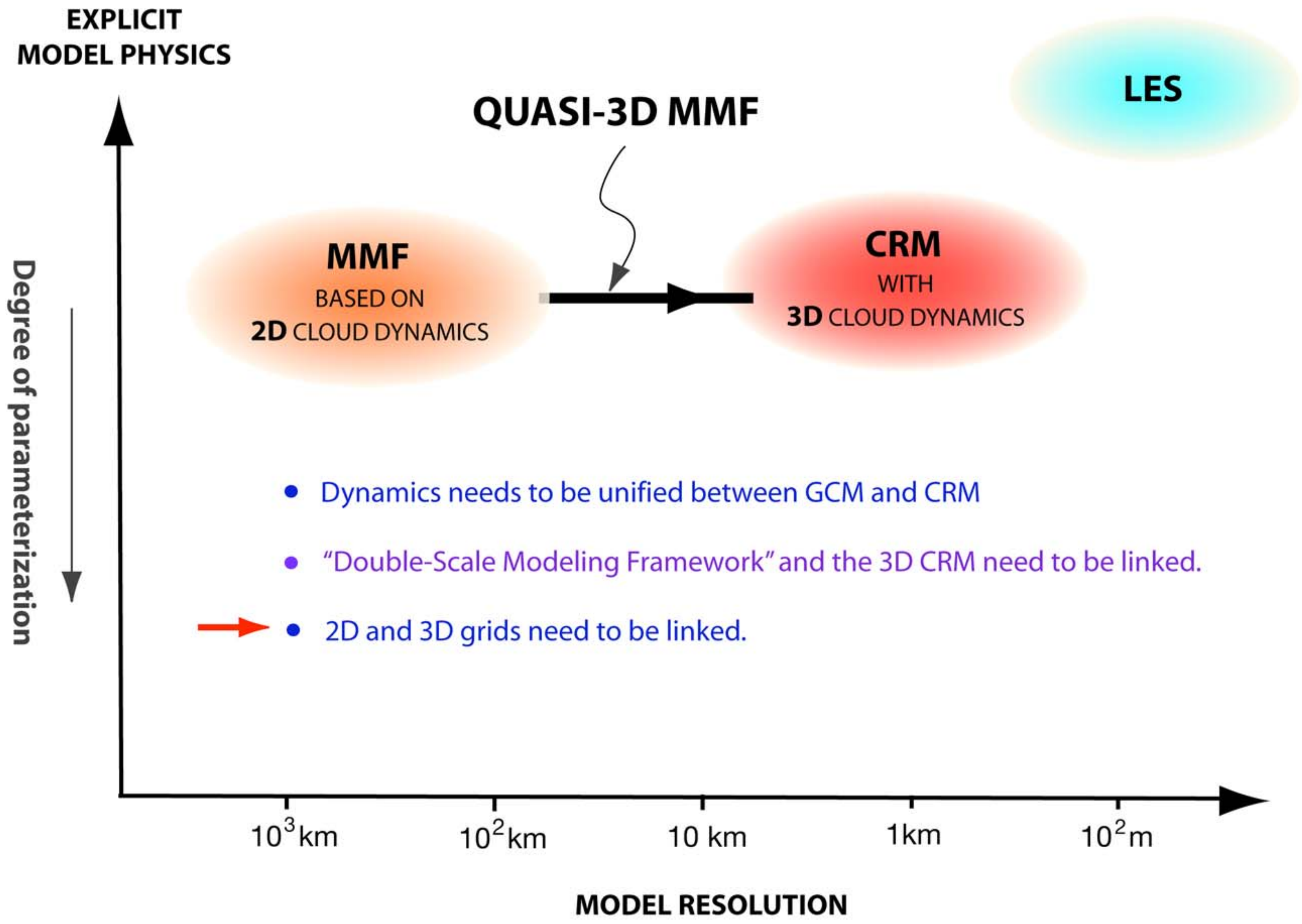

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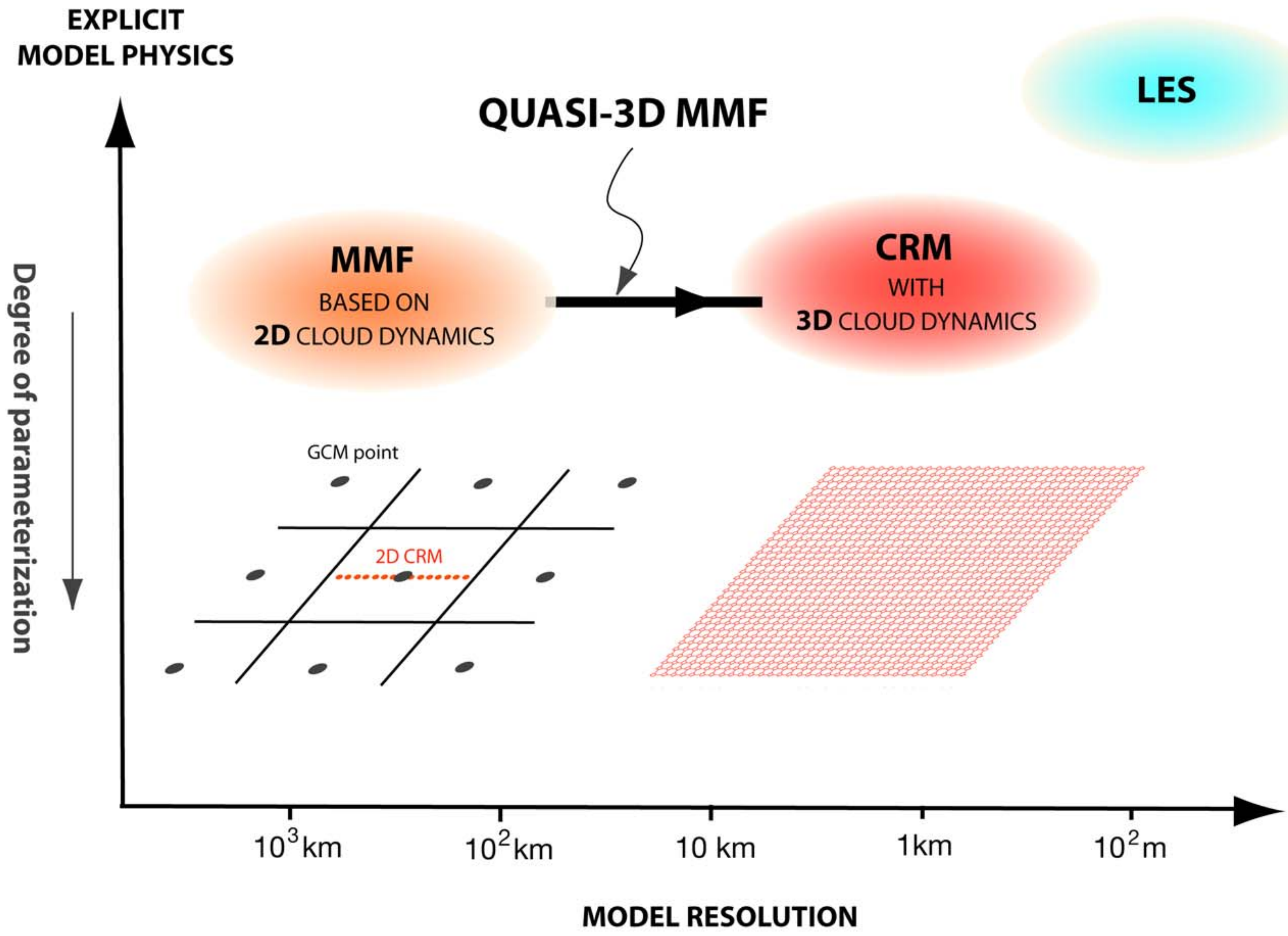
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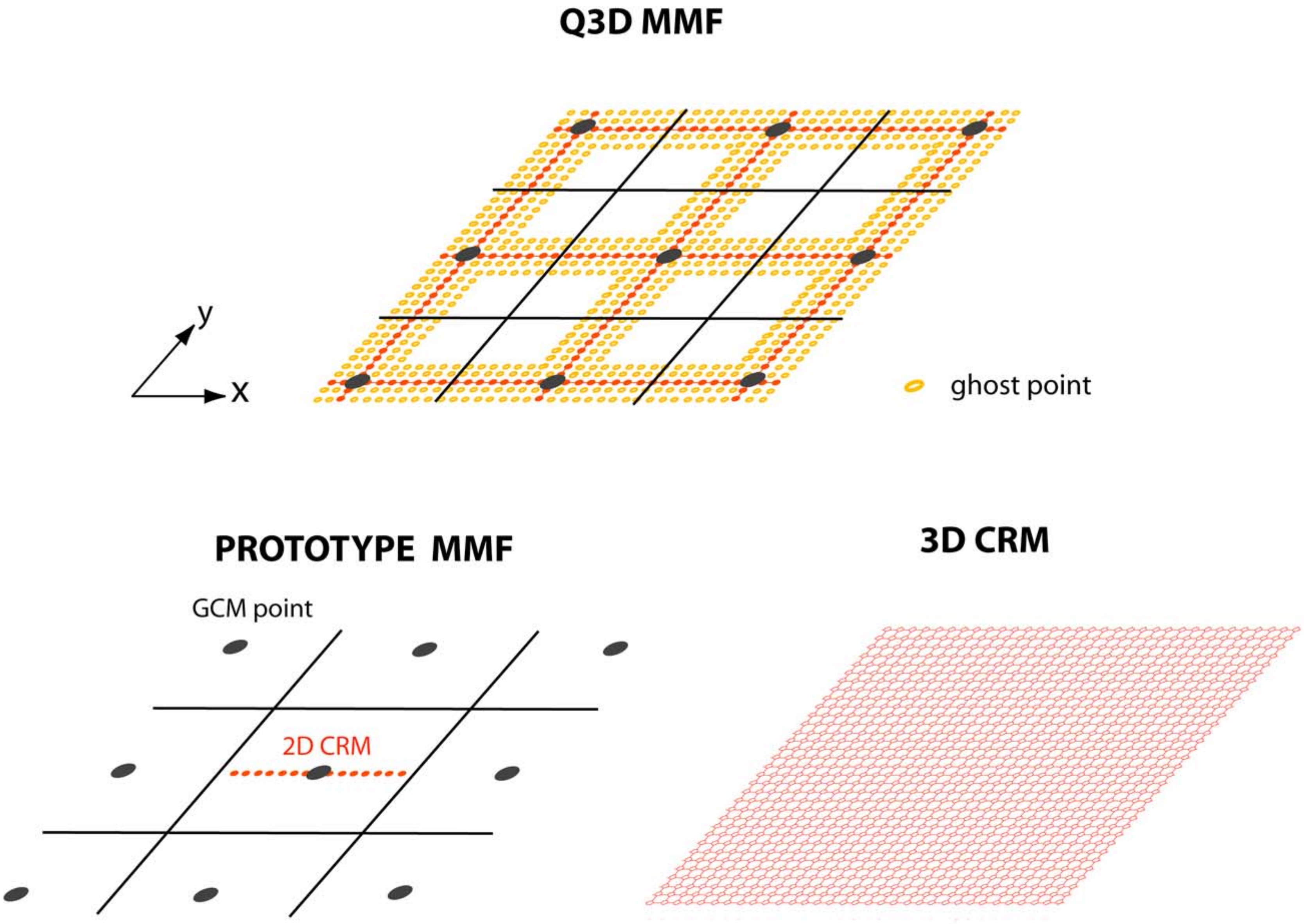

Q3D MMF

PROTOTYPE MMF

3D CRM

MULTI-SCALE REPRESENTATION OF VARIABLES

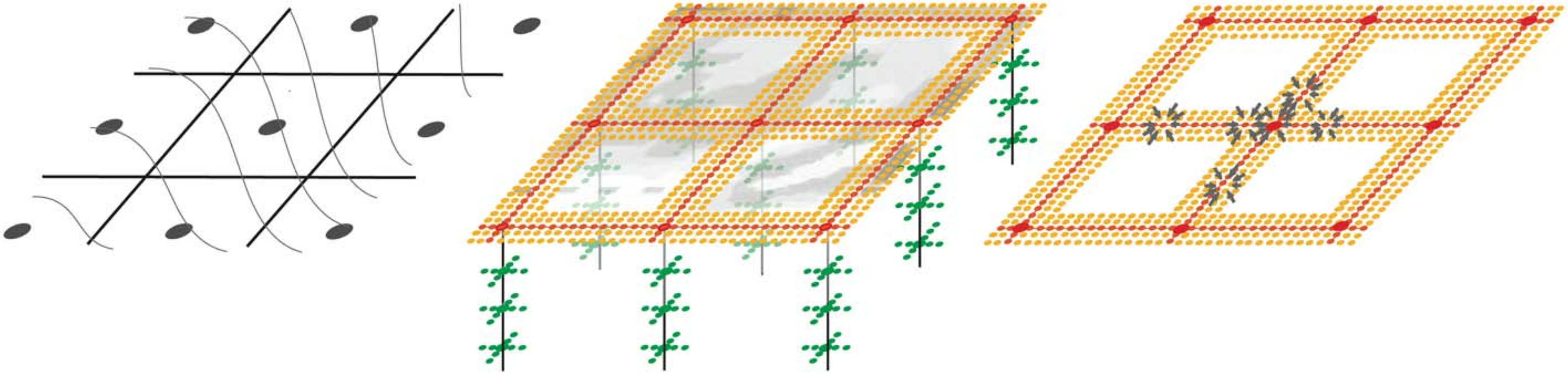
$$q = \bar{q} + q' + q''$$

BACKGROUND

CLOUD-SYSTEM SCALE

CLOUD SCALE

$\bar{q}$		q'	q''
Determined by interpolation of GCM grid-point values (Currently, this field is prescribed.)	Along the array	Raynolds averaging of $q - \bar{q}$	$q - \bar{q} - q'$
	Normal to the array	Statistical identification of cloud regime	Parameterization based on isotropy



Problems in Quasi-3D Advection of Scalar Variables

1. Global stability with 2-dimensional uniform current
2. Local stability with 3-dimensional non-uniform current
3. Control of singularity at intersections
4. Control of spurious trend
5. Identifying the orientation of cloud organization by regression analysis of past data at the intersections

The major remaining problem:

Advection of cloud water causes "computational detrainment"
due to computational dispersion/dissipation
and incompatibility with the movement of updraft.

Solving a 3D Elliptic Equation using the Quasi-3D Network

In an anelastic model, a 3D elliptic equation must be solved.

(The same is true in the unified system.)

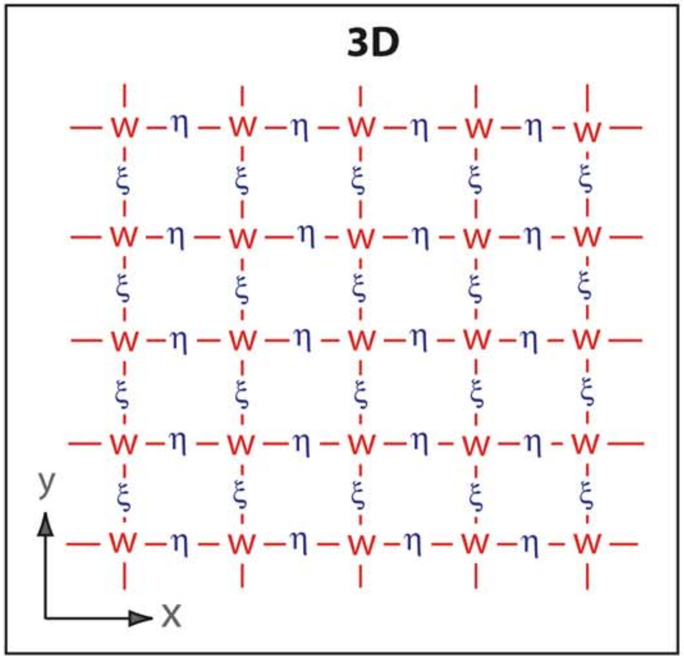
In our model,

$$\left(\frac{\partial^2}{\partial x^2}+\underline{\frac{\partial^2}{\partial y^2}}\right)w+\frac{\partial}{\partial z}\left[\frac{1}{\rho_0}\frac{\partial}{\partial z}(\rho_0 w)\right]=-\left(\frac{\partial \eta}{\partial x}-\underline{\frac{\partial \xi}{\partial y}}\right),$$

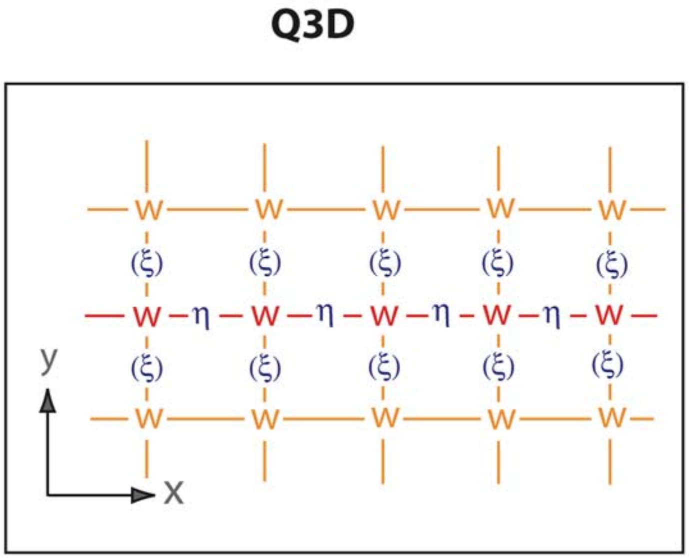
where

$$\xi \equiv (\nabla \times \mathbf{V})_x$$

$$\eta \equiv (\nabla \times \mathbf{V})_y$$



"Rotated C grid"

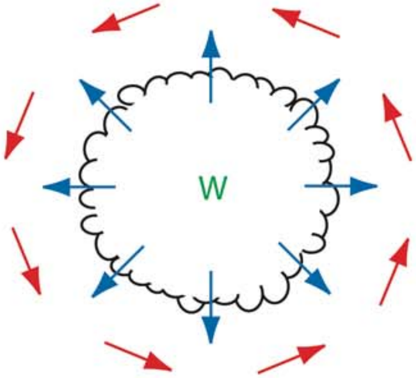


- There is an extra degree of freedom for ξ.
- *The extra degree of freedom turns out to be unstable.*

Estimation of the y-derivatives in the w-equation

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\underline{\partial y^2}}\right)w+\frac{\partial}{\partial z}\left[\frac{1}{\rho_0}\frac{\partial}{\partial z}(\rho_0 w)\right]=-\left(\frac{\partial\eta}{\partial x}-\frac{\partial\xi}{\underline{\partial y}}\right)$$

For Cloud Scale :



→ Horizontal component of velocity

→ Horizontal component of vorticity

We assume that cloud-scale w axi-symmetric and horizontal flow is radial. This means that the horizontal component of vorticity is circular (deformation-free).

For Cloud-System Scale :

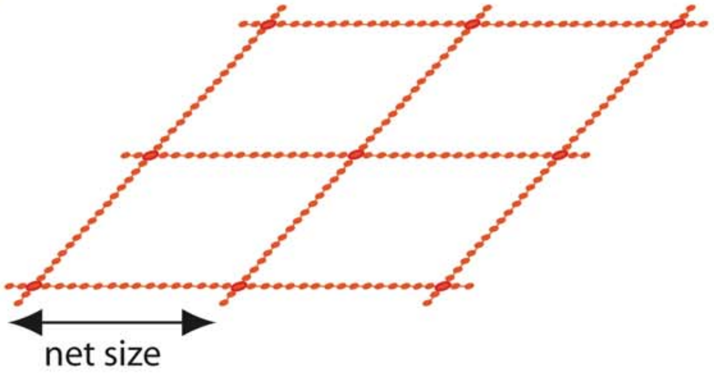
Statistical estimation as in the advection equation.

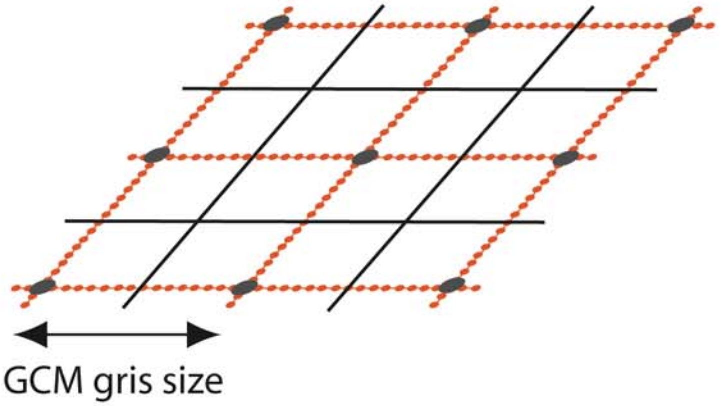
Semi-prognostic tests using prescribed vorticity on the network points are highly successful.

Problems in Vorticity Prediction

- In spite of the purely 3D nature, the effects of twisting followed by stretching are handled reasonably well.
- The local singularity at the intersections is relatively well controlled (no noise)
- Due to the inhomogeneous structure of the grid, however, there is a tendency toward development of large-scale circulations that have scales comparable to the net size.
- Not only this influences the overall partition between vertical and horizontal wind components, it suppresses smaller-scale convection by subsidence.
- Since the net size is the GCM grid size in MMF, the net-size circulation should be controlled also by the GCM dynamics.

We need to couple with a GCM
for real evaluation of the Q3D MMF





EXPERIMENTAL STRATEGY FOLLOWED SO FAR

- Break up the algorithm to pieces, and test one piece at a time.
- Always quantitatively compare with the results of 3D control run.
- Comparison is mainly through the time sequences of spatial variances (and covariances) rather than through spatial/temporal means.

We should start to test coupling the Q3D CRM with a toy GCM soon.

CONCLUDING REMARKS

- The semi-prognostic test with prescribed vorticity at the net points are
  - very successful in predicting velocity components,
  - but not in predicting the individual phases of water, very likely due to "computational detrainment".
- Fully-prognostic tests produces a "red" spectrum of vorticity, very likely due to the lack of interactions with the GCM.
- It seems that we are approaching the limit of the "peace by peace" test strategy.
- We also seem to have mixed up the problems (2) and (3).

(1) Dynamics needs to be unified between GCM and CRM

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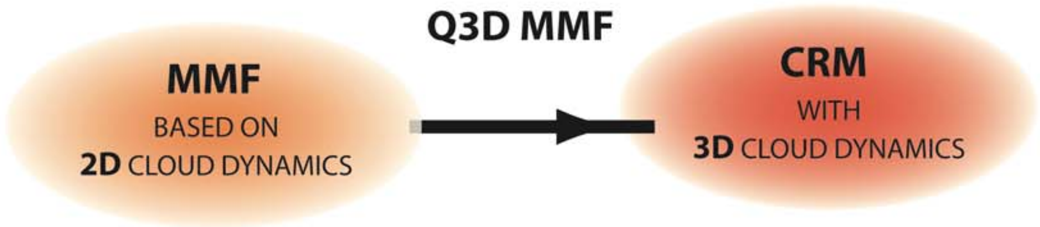
(3) 2D and 3D grids need to be linked.



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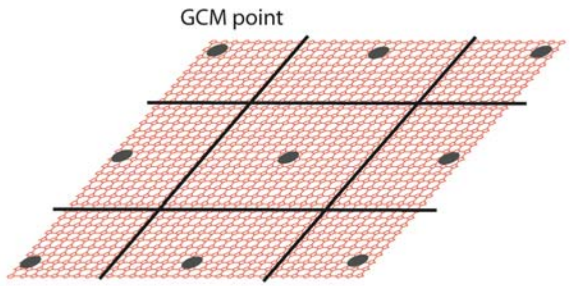

We need to test a "3D MMF" as a benchmark.