1 – Validation of the Unified System Through a Normal-Mode Analysis

2 – A Progress Report on

- **a – Parallelization of the VVCM**
- **b – Construction of the Global Cloud Resolving Model**

A Comparison of the Normal Modes of Various Systems of Equations on the Middle-Latitude Beta-Plane

By

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OUTLINE

- Linearized equations of *fully-compressible*, *unified*, *pseudo-incompressible*, *anelastic* and *quasi-hydrostatic* systems
- Dispersion relation for horizontal barotropic and 3-D motions with different systems
- Comparison of vertical phase and amplitude factor with different systems

METHOD

We basically follow Davies et al. (2003 QJRMS, Validity of Anelastic and Other Equation Sets As Inferred from Normal-Mode Analysis)

Exceptions:

- *Unified* system is added to the comparison.
- Middle-latitude beta-plane is assumed to include Rossby waves.
- The variable transformation to obtain constant-coefficient differential equations is different.
- Only the Lipps and Hemler type anelastic system is considered.

Basic State and Definition of Variables

Characteristics of Isothermal (Resting) Basic State:

$T_{00} = 273\ \text{K}$ $\quad H \equiv T_{00} R/g$ $\quad N^2 \equiv g\kappa/H$ $\quad c_s^2 \equiv \gamma R T_{00}$

Perturbation Values:

Hydrostatic pressure: p'_{qs}

Nonhydrostatic pressure: $\delta p'$

Buoyancy: $b' \equiv g\theta'/\theta_{00}$

Transformed and Linearized Equations

<u>Divergence Equation:</u>

$$\frac{\partial}{\partial t}\frac{\partial u'}{\partial x} = f_0 \frac{\partial v'}{\partial x} - \frac{\partial^2}{\partial x^2}\left(p'_{qs} + \delta p'\right)$$

<u>Vorticity Equation:</u>

$$\frac{\partial}{\partial t}\frac{\partial v'}{\partial x} = -\beta v' - f_0 \frac{\partial u'}{\partial x}$$

<u>Boundary Conditions:</u>

$$w'_S = w'_T = 0$$

<u>Vertical Momentum Equation:</u>

$$\delta \frac{\partial w'}{\partial t} = -\left[\frac{\partial}{\partial z} + \underbrace{\frac{1}{2H} - \underline{\underline{\left(\frac{N^2}{g}\right)}}}_{\mu}\right]\delta p'$$

<u>Hydrostatic Equation:</u>

$$\left[\frac{\partial}{\partial z} + \frac{1}{2H} - \underline{\underline{\left(\frac{N^2}{g}\right)}}\right] p'_{qs} = b'$$

<u>Thermodynamic Equation:</u>

$$\frac{\partial b'}{\partial t} = -N^2 w'$$

<u>Continuity Equation:</u>

$$\frac{1}{c_s^2}\frac{\partial}{\partial t}\underline{\left(p'_{qs} + \varepsilon\delta p'\right)} = \left(-\frac{\partial}{\partial z} + \frac{1}{2H} - \underline{\underline{\left(\frac{N^2}{g}\right)}}\right)w' - \left(\frac{\partial u'}{\partial x}\right)$$

Fully-Compressible system: $\varepsilon = 1,\ \delta = 1$ and all terms are retained

Unified: $\varepsilon = 0,\ \delta = 1$ and the rest retained

Pseudo-Incompressible: $\delta = 1$ and terms with single-under bar omitted

Anelastic: $\delta = 1$ and terms with single-and double-under bar omitted

Quasi-Hydrostatic: $\delta = 0$ and the rest retained

Barotropic Motion

Divergence Equation:

$$\frac{\partial}{\partial t}\frac{\partial u'}{\partial x} = f_0 \frac{\partial v'}{\partial x} - \frac{\partial^2 p'_{qs}}{\partial x^2}$$

Vorticity Equation:

$$\frac{\partial}{\partial t}\frac{\partial v'}{\partial x} = -\beta v' - f_0 \frac{\partial u'}{\partial x}$$

Continuity Equation:

$$\frac{1}{c_s^2}\frac{\partial}{\partial t}\left(\underline{p'_{qs}}\right) = -\left(\frac{\partial u'}{\partial x}\right)$$

Fully-Compressible, Unified and Quasi-Hydrostatic systems: All terms are retained

Pseudo-Incompressible and Anelastic: Term with single-under bar omitted

Form of solutions

$$\Phi'(x,z,t) = \mathrm{Re}\left\{\left(\hat{\Phi}_1 e^{imz} + \hat{\Phi}_2 e^{-imz}\right) e^{i(kx-\nu t)}\right\}$$

Solution of w´

To satisfy $w'_S = w'_T = 0$, we obtain

$$w'(x,z,t) = \mathrm{Re}\left\{2i\hat{w}\sin\left(\pi n \frac{z}{z_T}\right) e^{i(kx-\nu t)}\right\}$$

where n=1, 2, 3, . . . is the integer vertical wavenumber $\left[m \equiv \frac{\pi n}{z_T}\right]$

Form of solutions for barotropic modes

$$\Phi'(x,t) = \mathrm{Re}\left\{\hat{\Phi}\, e^{i(kx-\nu t)}\right\}$$

Dispersion Relation

$$\underline{\varepsilon\delta k\nu^5+\varepsilon\delta\beta\nu^4}-\left\{\underline{N^2k}+\delta k^3c_s^2+\underline{\varepsilon\delta k f_0^2}+c_s^2k\left[\left(\pi n/z_T\right)^2+\mu^2\right]\right\}\nu^3$$
$$-\left\{\underline{N^2\beta}+\beta c_s^2\left[\left(\pi n/z_T\right)^2+\mu^2\right]+\delta k^2c_s^2\beta\right\}\nu^2$$
$$+\left\{\underline{N^2kf_0^2}+k^3c_s^2N^2+c_s^2kf_0^2\left[\left(\pi n/z_T\right)^2+\mu^2\right]\right\}\nu+N^2k^2c_s^2\beta=0$$

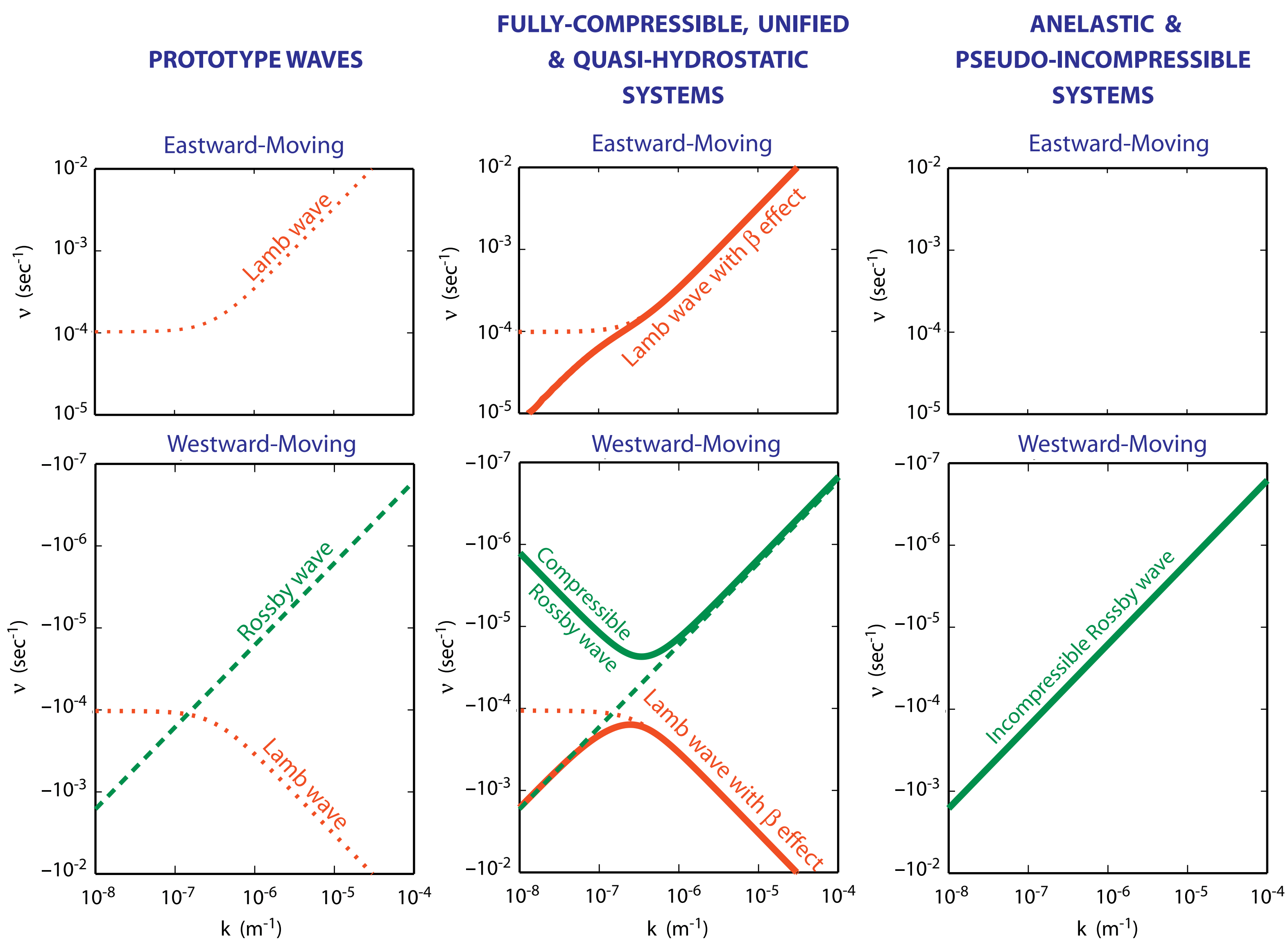
It yields five real roots (modes) for *fully-compressible* system and three modes for the *other* systems.

Dispersion relation for barotropic modes

$$\underline{k\nu^3+\beta\nu^2}-k\left(\underline{f_0^2}+k^2c_s^2\right)\nu-k^2\beta c_s^2=0$$

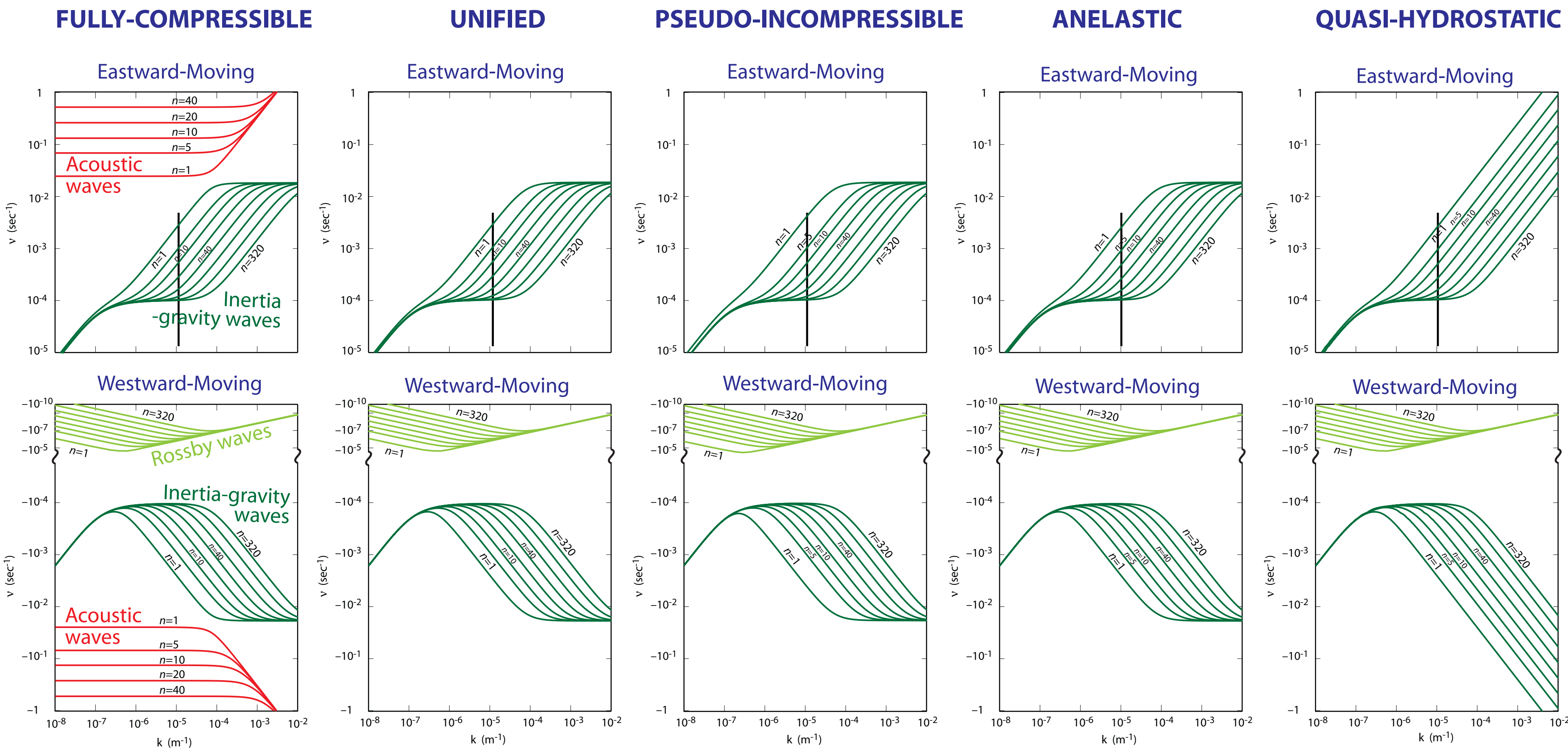
It yields three real roots (modes) for *fully-compressible*, *unified* and *quasi-hydrostatic* systems and one mode for the *pseudo-incompressible* and *anelastic* systems.

Dispersion Relation for Barotropic Modes on the Middle-Latitude Beta-Plane



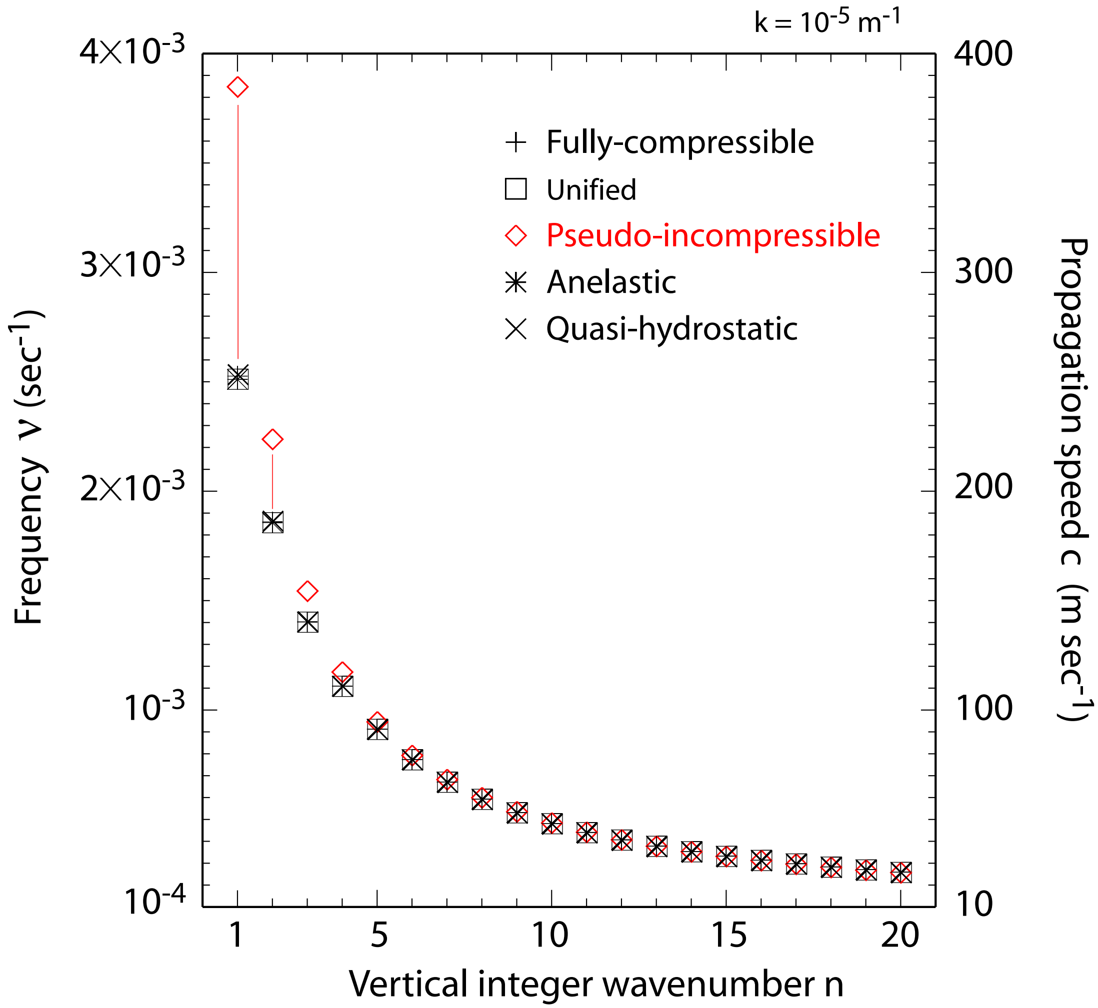
- *Anelastic and pseudo-incompressible systems yield spuriously fast retrogressing ultra-long waves. Thus, they cannot be used for a global model.*

Dispersion Relation for Internal Modes on the Middle-Latitude Beta-Plane



- *Unified, pseudo-incompressible, anelastic and quasi-hydrostatic systems filter acoustic waves.*
- *Among them, quasi-hydrostatic system yields spuriously fast moving short gravity waves.*

Dispersion Relation for Eastward-moving Inertia-Gravity Waves



- *In pseudo-incompressible systems, deep gravity waves (horizontally) move faster than those in the other systems.*

Solutions

$$\delta p'(x,z,t) = -\underbrace{\frac{2\mathcal{A}\delta\nu}{(\pi n/z_T)^2+\mu^2}}_{\text{Amplitude factor}}|\hat{w}|\underbrace{\sin\left(\pi n\frac{z}{z_T}-\varphi\right)}_{\text{Vertical structure}}\mathrm{Re}\left\{e^{i(kx+\phi_w-\nu t)}\right\}$$

Vertical Phase: $\varphi \equiv \arctan\left(\frac{\pi n}{\mu z_T}\right)$ where $\mu \equiv \frac{1}{2H} - \underline{\underline{\left(\frac{N^2}{g}\right)}}$

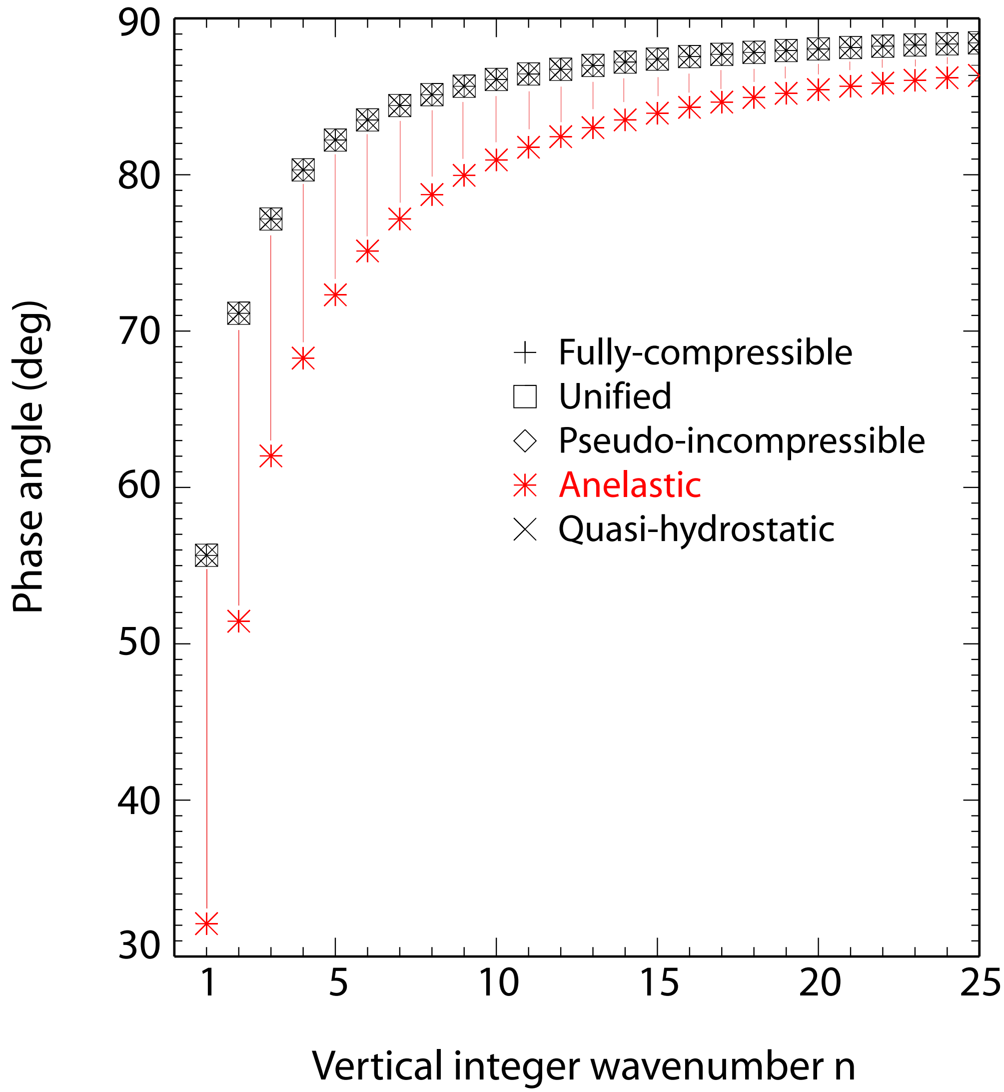
Amplitude factor: Varies for variables

$$p'_{qs}(x,z,t) = \frac{2\mathcal{A}N^2}{\nu\left[(\pi n/z_T)^2+\mu^2\right]}|\hat{w}|\sin\left(\pi n\frac{z}{z_T}-\varphi\right)\mathrm{Re}\left\{e^{i(kx+\phi_w-\nu t)}\right\}$$

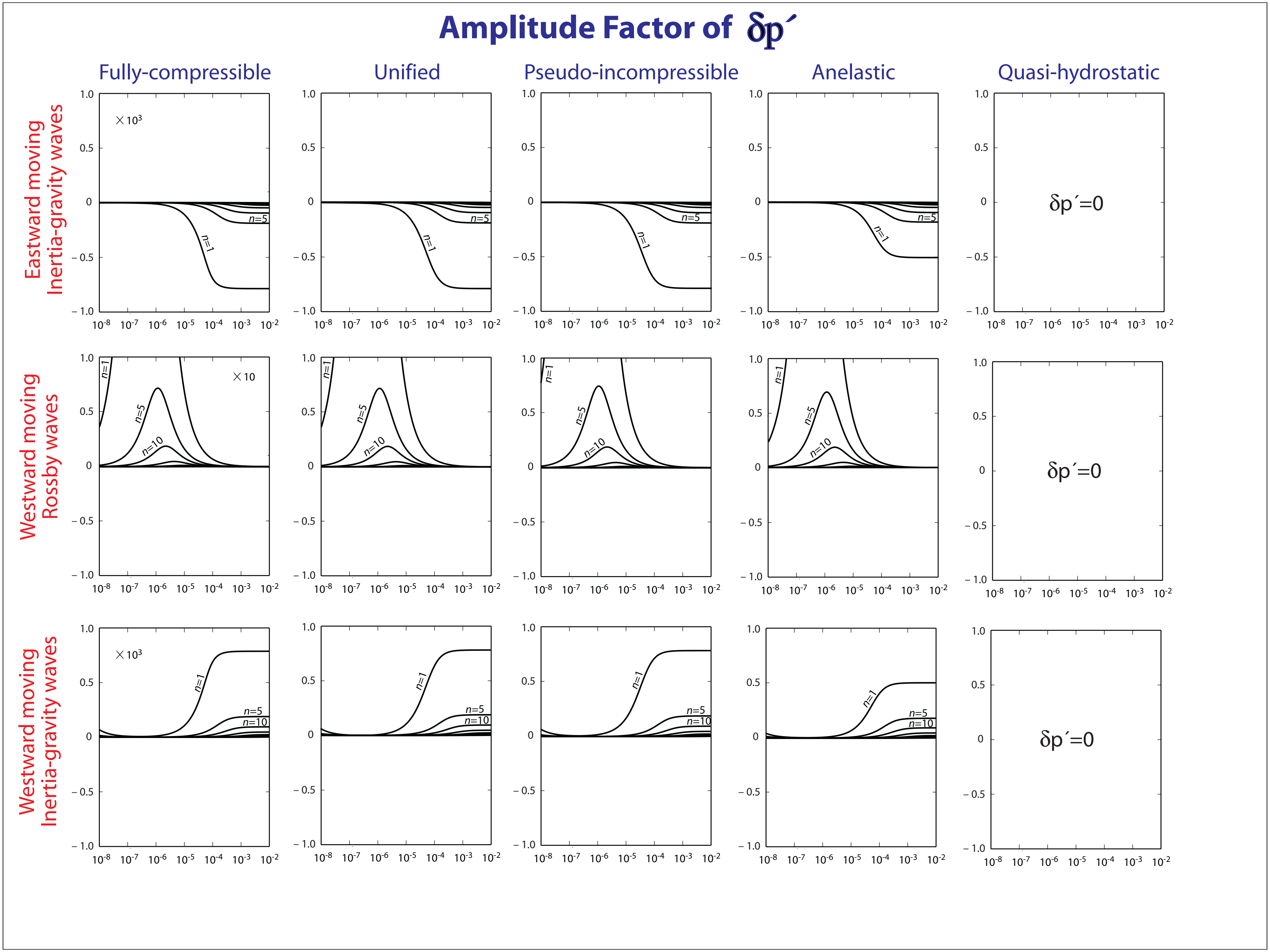
$$u'(x,z,t) = -\frac{2\mathcal{A}}{k}\left[\frac{\underline{-N^2+\varepsilon\delta\nu^2}}{c_s^2\left[(\pi n/z_T)^2+\mu^2\right]}-1\right]|\hat{w}|\sin\left(\pi n\frac{z}{z_T}-\varphi\right)\mathrm{Re}\left\{e^{i(kx+\phi_w-\nu t)}\right\}$$

$$v'(x,z,t) = \frac{2\mathcal{A}f_0}{k\nu+\beta}\left[\frac{\underline{-N^2+\varepsilon\delta\nu^2}}{c_s^2\left[(\pi n/z_T)^2+\mu^2\right]}-1\right]|\hat{w}|\sin\left(\pi n\frac{z}{z_T}-\varphi\right)\mathrm{Re}\left\{e^{i(kx+\phi_v-\nu t)}\right\}$$

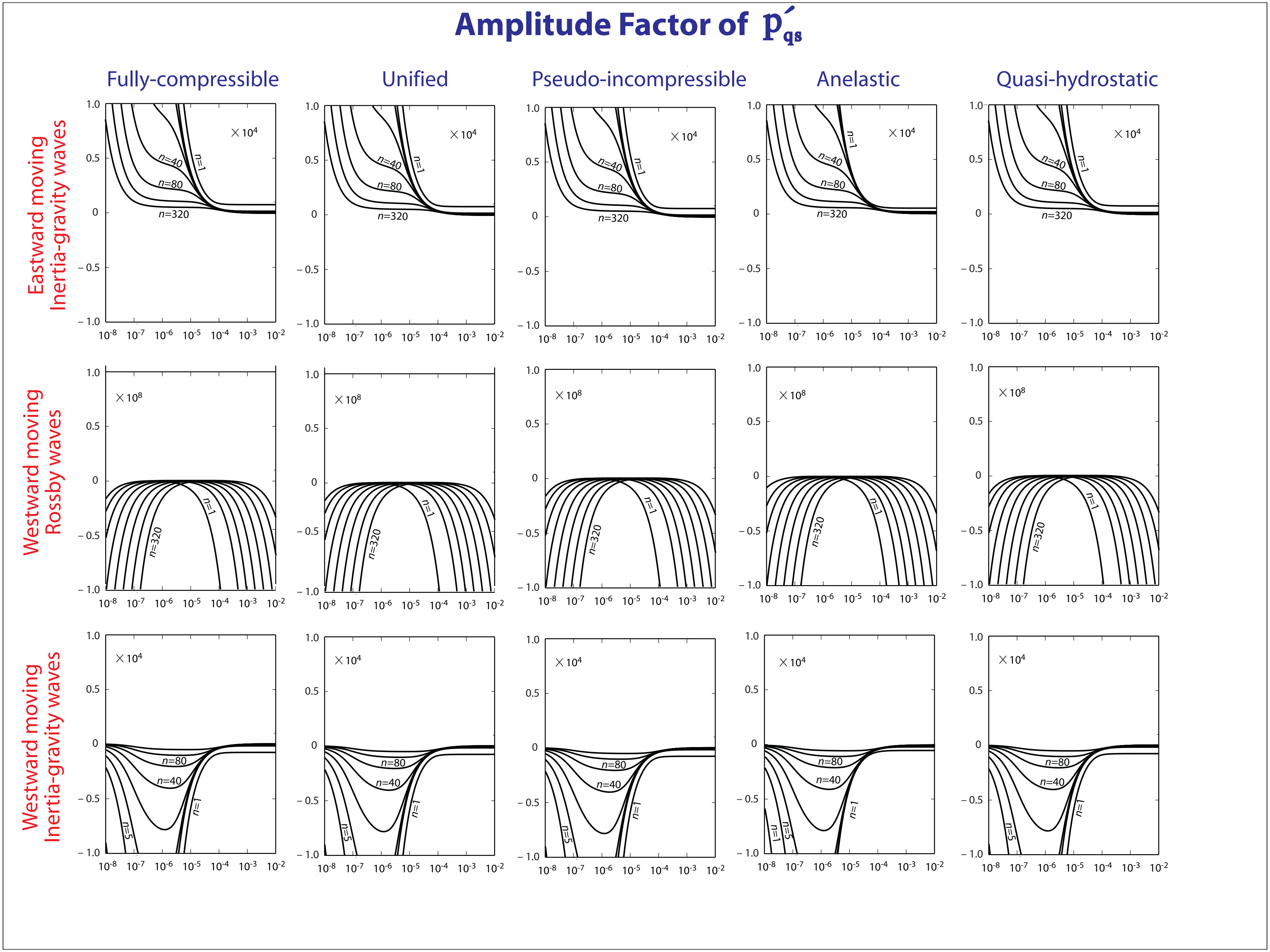
Vertical Phase

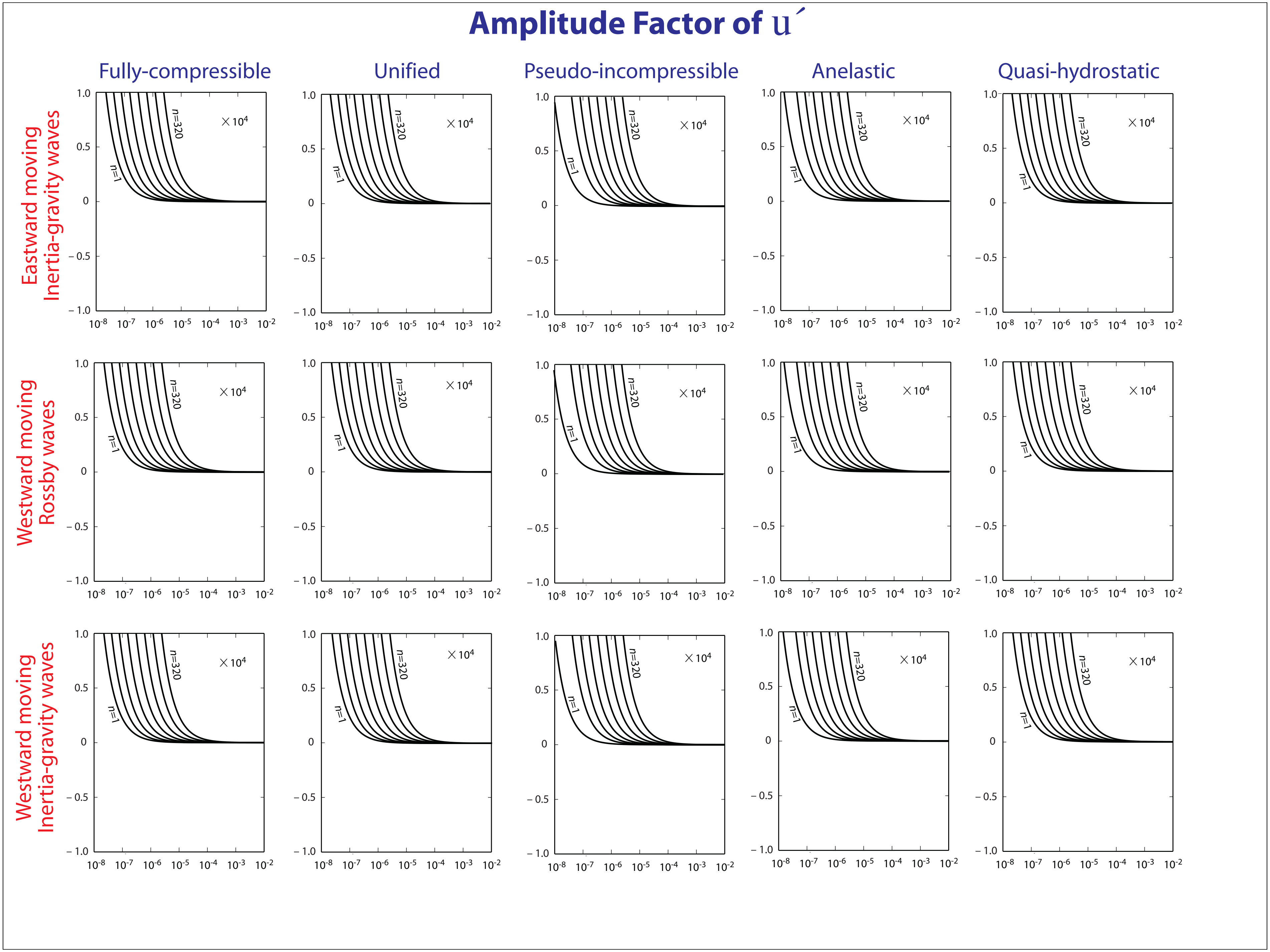


- *Anelastic system distorts the vertical structure of gravity waves.*

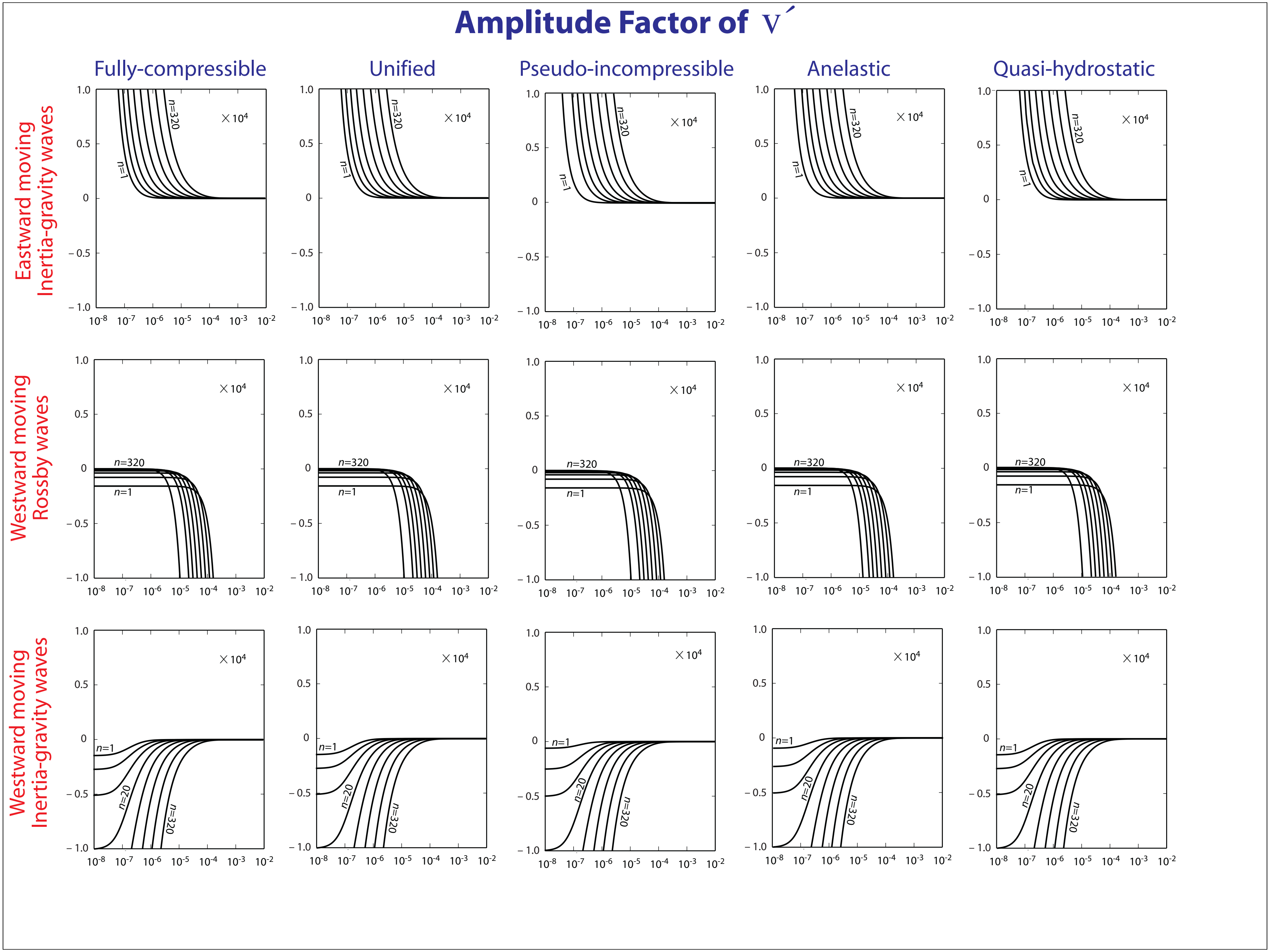


Amplitude Factor of δp′
Fully-compressible
Unified
Pseudo-incompressible
Anelastic
Quasi-hydrostatic
Eastward moving Inertia-gravity waves
Westward moving Rossby waves
Westward moving Inertia-gravity waves
×10³
×10
n=1
n=5
n=10
δp′=0


Amplitude Factor of p'_{qs}
Fully-compressible
Unified
Pseudo-incompressible
Anelastic
Quasi-hydrostatic
Eastward moving Inertia-gravity waves
Westward moving Rossby waves
Westward moving Inertia-gravity waves
× 10⁴
× 10⁸
n=1
n=5
n=40
n=80
n=320
1.0
0.5
0
− 0.5
− 1.0
10⁻⁸ 10⁻⁷ 10⁻⁶ 10⁻⁵ 10⁻⁴ 10⁻³ 10⁻²



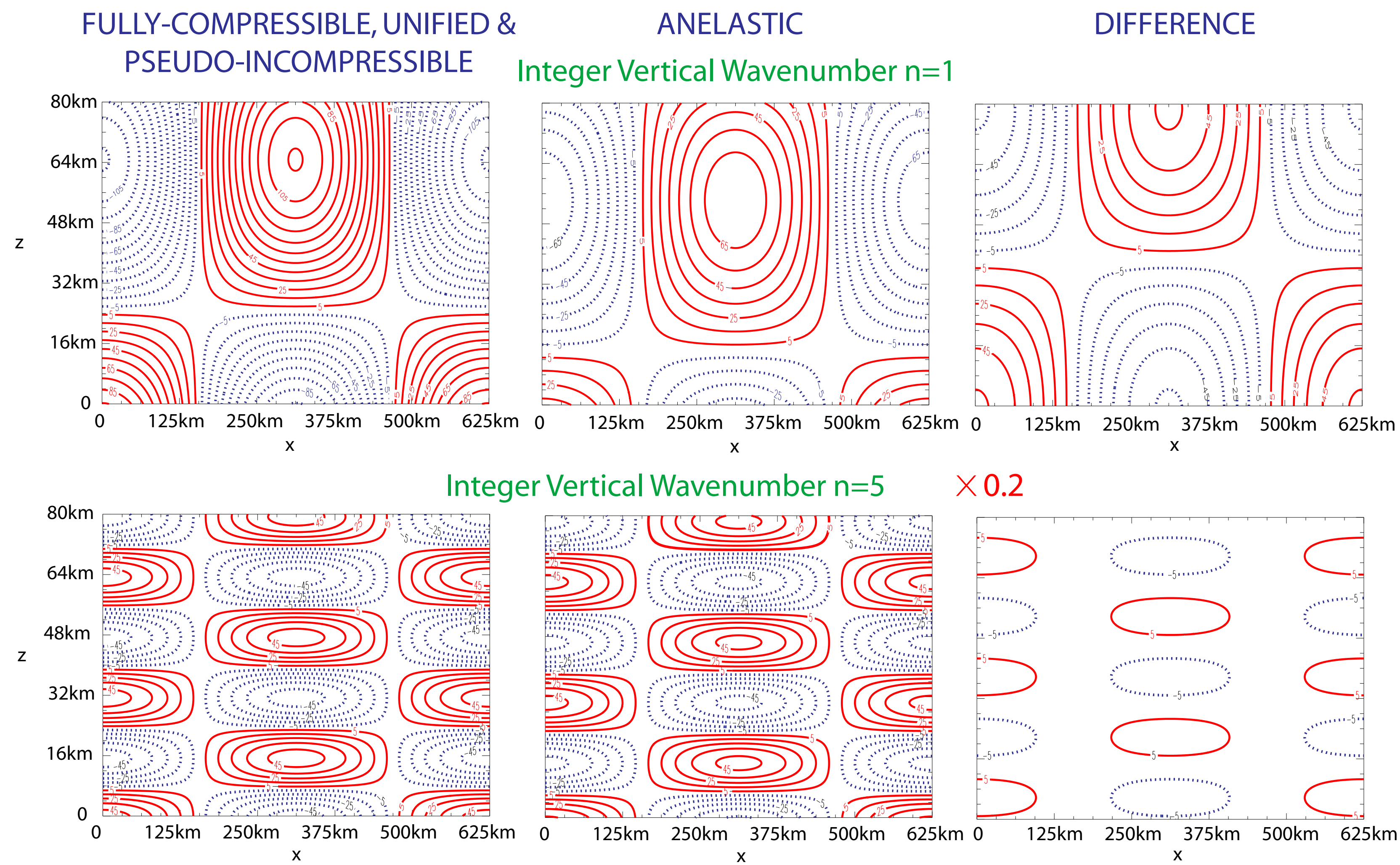
Amplitude Factor of u´
Fully-compressible
Unified
Pseudo-incompressible
Anelastic
Quasi-hydrostatic
Eastward moving Inertia-gravity waves
Westward moving Rossby waves
Westward moving Inertia-gravity waves
n=320
n=1
×10⁴
1.0
0.5
0
−0.5
−1.0
10⁻⁸ 10⁻⁷ 10⁻⁶ 10⁻⁵ 10⁻⁴ 10⁻³ 10⁻²



Amplitude Factor of v´
Fully-compressible
Unified
Pseudo-incompressible
Anelastic
Quasi-hydrostatic
Eastward moving Inertia-gravity waves
Westward moving Rossby waves
Westward moving Inertia-gravity waves
×10^4
n=1
n=20
n=320
1.0
0.5
0
− 0.5
− 1.0
10^-8
10^-7
10^-6
10^-5
10^-4
10^-3
10^-2

x-z Structure of an Eastward moving Gravity Wave

Nonhydrostatic Pressure $\delta p'$ $|\hat{w}|=1$, $\phi_w = 0$ and $t = 0$ **$k = 10^{-5}$ (m^{-1})**



$$\delta p'(x,z,t) = -\frac{2\mathcal{A}\delta\nu}{(\pi n/z_T)^2 + \mu^2}|\hat{w}|\sin\left(\pi n\frac{z}{z_T} - \varphi\right)\mathrm{Re}\left\{e^{i(kx+\phi_w-\nu t)}\right\}$$

Summary of Comparisons

	ACOUSTIC WAVES	INTERTIA-LAMB WAVES	INTERTIA-GRAVITY WAVES	ROSSBY WAVES	VERTICAL PHASE	AMPLITUDE FACTOR dp	p_{qs}	u	v
UNIFIED	**FILTERED**	NOT MODIFIED	NOT MODIFIED	NOT MODIFIED (COMPRES-SIBLE)	NOT MODIFIED	NOT MODIFIED	NOT MODIFIED	NOT MODIFIED	NOT MODIFIED
PSEUDO-INCOMPRESSIBLE	**FILTERED**	FILTERED	SLIGHTLY MODIFIED FOR LONG WAVES	INCOMP-RESSIBLE ULTRA-LONG WAVE CATASTROPHE	NOT MODIFIED	SLIGHTLY MODIFIED	NOT MODIFIED	SLIGHTLY MODIFIED	SLIGHTLY MODIFIED
ANELASTIC	**FILTERED**	FILTERED	NOT MODIFIED	INCOMP-RESSIBLE ULTRA-LONG WAVE CATASTROPHE	MODIFIED UNABLE TO RECOGNIZE HIGH STATIC-STABILITY	SLIGHTLY MODIFIED	SLIGHTLY MODIFIED	SLIGHTLY MODIFIED	SLIGHTLY MODIFIED
QUASI-HYDROSTATIC	**FILTERED**	NOT MODIFIED	GREATLY MODIFIED SHORT WAVES	NOT MODIFIED (COMPRES-SIBLE)	NOT MODIFIED	ZERO NO NON-HYDROSTATIC PRESSURE	NOT MODIFIED	NOT MODIFIED	NOT MODIFIED

Indicates major deficiency

Indicates minor deficiency

Conclusions

- As far as the normal-mode analysis is concerned, the *unified* and *fully-compressible* systems are virtually identical. *Except, the unified system filters acoustic waves*.

  - Unlike the *pseudo-incompressible* and *anelastic* systems, the *unified* system is *free* from the spuriously fast retrogression of ultra-long waves.

  - *Unified* system distorts neither the structure of the inertia-gravity waves nor modifies the dispersion relation.

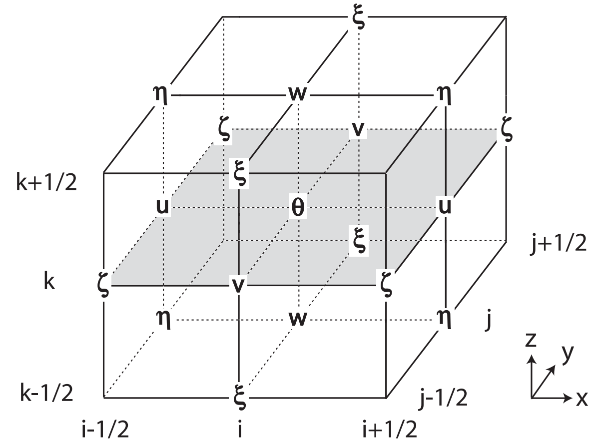
Testing the Unified equations in a Model

- Parallelized Jung-Arakawa model (VVCM) can be modified to test the unified equations on a middle-Latitude b-plane.

- Global cloud resolving model with the unified equations on a hexagon-pentagon geodesic grid is the ultimate target.

Parallelization of the VVCM

Parallelization done by
Shao-Ching Huang of UCLA Academic Technology Services



- Completed Tasks (Feb 2007 – Aug. 3, 2007)

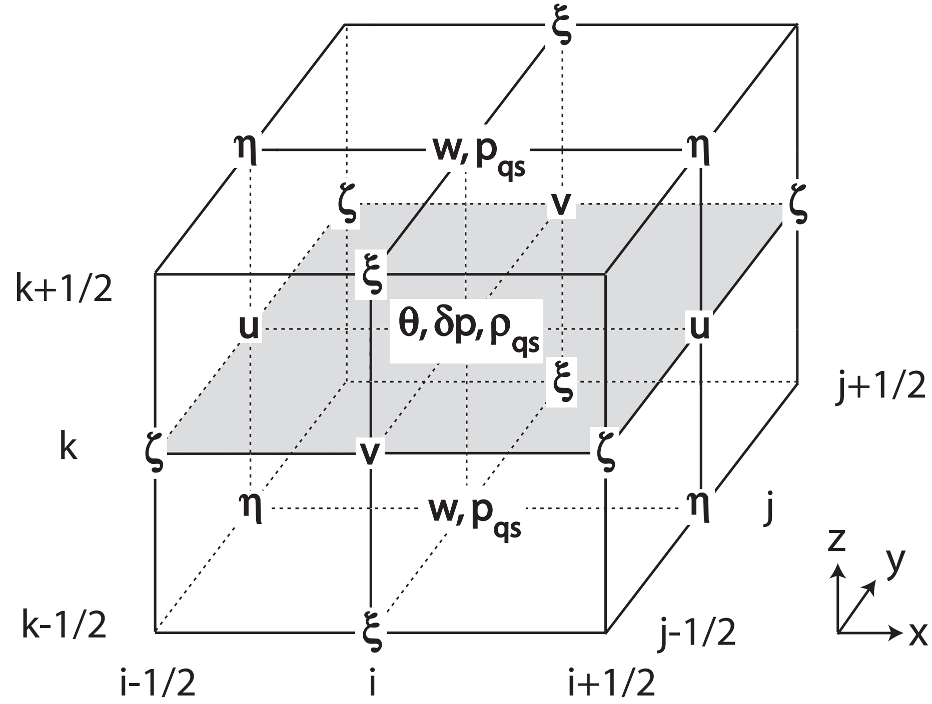
  – Data structure following horizontal domain decomposition (Variable number of ghost points and one wrap per time step)
  – Data structure following horizontal 3D Poisson equation solver for w (Based on 1D FFTs)
  – 3D vorticity advection*, and stretching and twisting
  – I/O and restart procedure
  – Advection of potential temperature and water species

- Ongoing Work (as of Aug. 5, 2007)

  – Debugging for a problem with the indexing in the advection of vertical vorticity

- Remaining Tasks

  - Replacing the Poisson solver with the parobolic solver
  - Adding molecular viscosity
  - Adding "hooks" and necessary data structure to the parallel code for the unified system

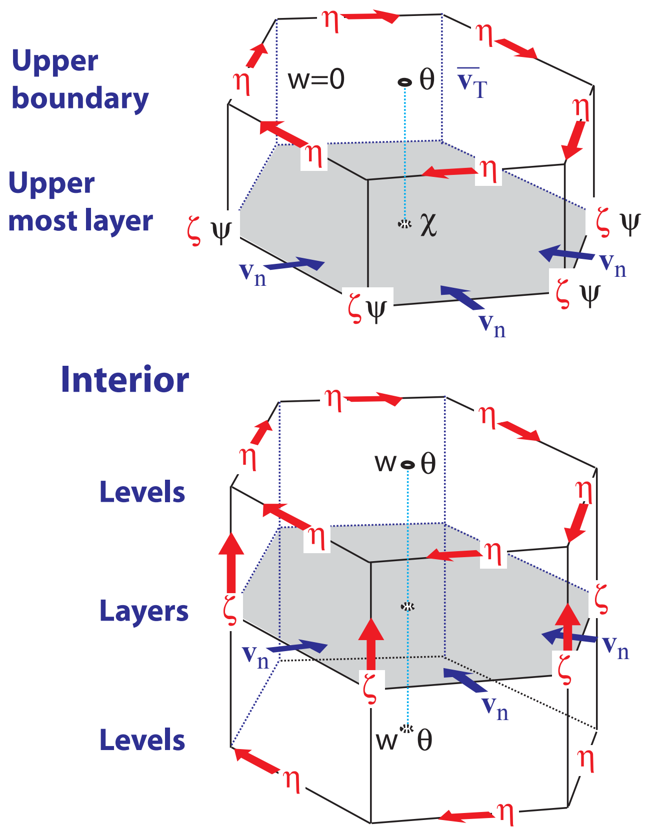


- Inclusion of an Improved Turbulence Paramaterization

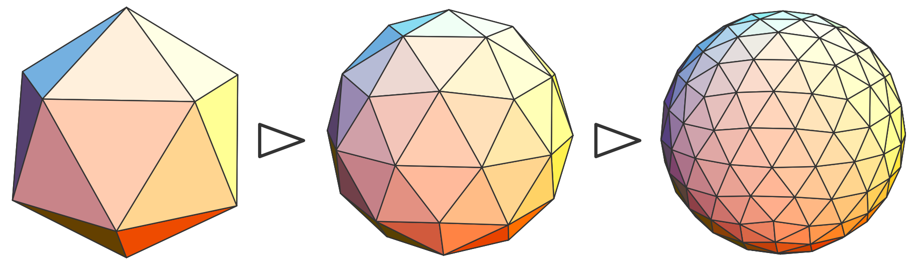
- Inclusion of Microphysics

Construction of the Global Cloud Resolving Model

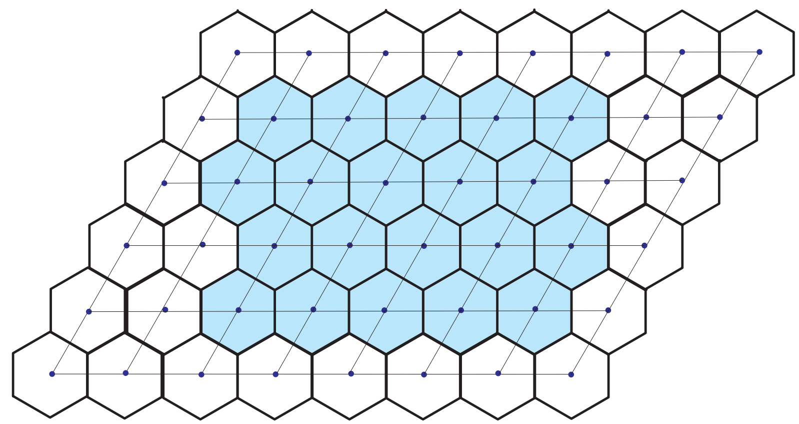
3-D View of the Grid (Pseudo-incompressible and Anelastic)



Hexagon-Pentagon Geodesic Grid



Grid of the Planer Version



- Completed Tasks for the Planer Version of the Model
  - Horizontal advection of vorticity components, potential temperature and moisture species
  - Elliptic solvers for streamfunction and velocity potential* at the upper boundary
- Ongoing Work
  - Debugging of the elliptic solver applied the cell centers (problems near boundaries).
  - Planning for the unified equations

Vertical Grid for the GCRM Based on the Unified Equations

Charney-Phillips Type

η $\overline{\mathbf{v}}_T$ θ $w=0$ p_{qs} ρ_{qs}	z_T
ζ $\mathbf{v}_n$ δp	z_1
η θ w p_{qs} ρ_{qs}	$z_{\ell+1/2}$
ζ $\mathbf{v}_n$ δp	z_ℓ
η θ w p_{qs} ρ_{qs}	$z_{\ell-1/2}$
ζ $\mathbf{v}_n$ δp	z_L
η θ $w=0$ p_{qs} ρ_{qs}	z_S

Lorenz Type

η $\overline{\mathbf{v}}_T$ $w=0$ p_{qs}	z_T
ζ θ $\mathbf{v}_n$ δp p_{qs} ρ_{qs}	z_1
η w	$z_{\ell+1/2}$
ζ θ $\mathbf{v}_n$ δp p_{qs} ρ_{qs}	z_ℓ
η w	$z_{\ell-1/2}$
ζ θ $\mathbf{v}_n$ δp p_{qs} ρ_{qs}	z_L
η $w=0$	z_S

- Remaining Tasks

  - Understand and solve the problems with the elliptic solver
  - Code the parabolic w-equation
  - Transport the model to the global hexagon-pentagon domain