DEVELOPMENT OF A QUASI-3D MMF

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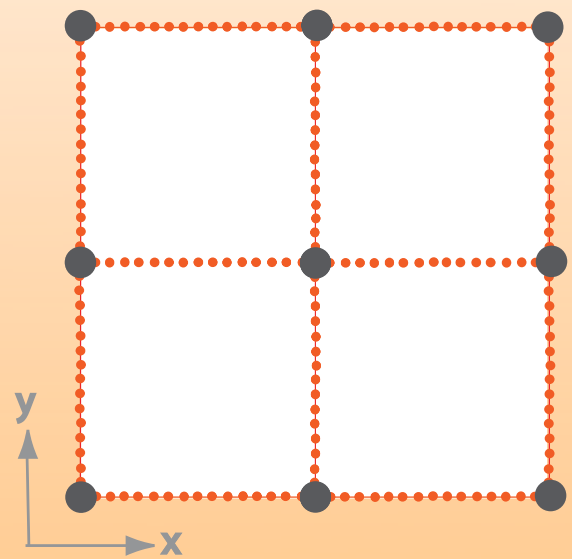
Q3D CRM

Based on the same model dynamics and physics with the 3D CRM, only difference being the use of Q3D grid



CLOUD-RESOLVING MODEL BASED ON 3D VORTICITY EQUATION

Prediction of scalar variables

Water substances (and tracer):

$$\rho_0 \frac{\partial q_x}{\partial t} = -\left[\frac{\partial}{\partial x}(\rho_0 u q_x) + \frac{\partial}{\partial y}(\rho_0 v q_x) + \frac{\partial}{\partial z}(\rho_0 w q_x)\right] + S_{q_x}$$

Potential temperature:

$$\rho_0 \frac{\partial \theta}{\partial t} = -\left[\frac{\partial}{\partial x}(\rho_0 u \theta) + \frac{\partial}{\partial y}(\rho_0 v \theta) + \frac{\partial}{\partial z}(\rho_0 w \theta)\right] + S_{\theta}$$

Prediction of vorticity components

Horizontal components:

$$\frac{\partial \xi}{\partial t} = -\left[\frac{\partial}{\partial x}(u\xi) + \frac{\partial}{\partial y}(v\xi) + \frac{\partial}{\partial z}(w\xi)\right] + \xi\frac{\partial u}{\partial x} + \eta\frac{\partial u}{\partial y} + \zeta\frac{\partial u}{\partial z} + S_{\xi}$$

$$\frac{\partial \eta}{\partial t} = -\left[\frac{\partial}{\partial x}(u\eta) + \frac{\partial}{\partial y}(v\eta) + \frac{\partial}{\partial z}(w\eta)\right] + \eta\frac{\partial v}{\partial y} + \xi\frac{\partial v}{\partial x} + \zeta\frac{\partial v}{\partial z} + S_{\eta}$$

Vertical component:

$$\zeta_z = -\int_{z_T}^{z}\left[\frac{\partial \xi}{\partial x} + \frac{\partial \eta}{\partial y}\right]dz + \zeta_{z_T}$$

Determination of wind components

From anelasticity:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)w + \frac{\partial}{\partial z}\left[\frac{1}{\rho_0}\frac{\partial}{\partial z}(\rho_0 w)\right] = -\frac{\partial \eta}{\partial x} + \frac{\partial \xi}{\partial y}$$

From the definition of vorticity: $u = \int_{z_T}^{z}\left(\frac{\partial w}{\partial x} + \eta\right)dz + u_T(x,y,t)$ $\quad v = \int_{z_T}^{z}\left(\frac{\partial w}{\partial y} - \xi\right)dz + v_T(x,y,t)$

To determine the gradient normal to the grid-point arrays

We first introduce the following multi-scale expression for all variables:

$$q = \overline{q} + q' + q''$$

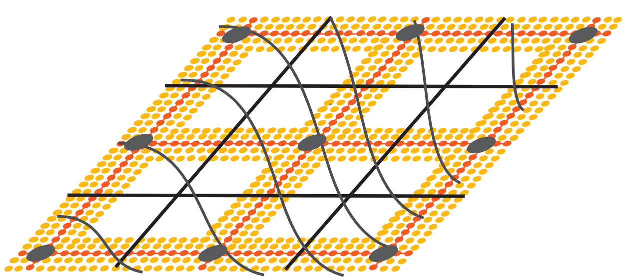
- $\overline{q}$: synoptic-scale (background field)
- q': cloud-system scale
- q'': cloud scale
- q^* = cloud-system scale + cloud scale ($q' + q''$)

DETERMINATION OF 3D STRUCTURES

$\overline{q}$

Determined by interpolation of GCM grid-point values

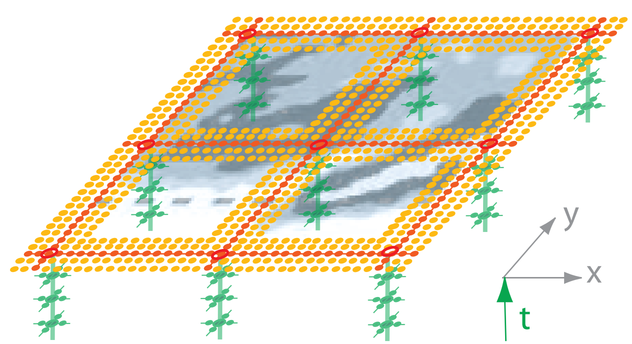
(Currently, this field is prescribed.)



q′

Along grid-point array:
1D Raynolds averaging of q *

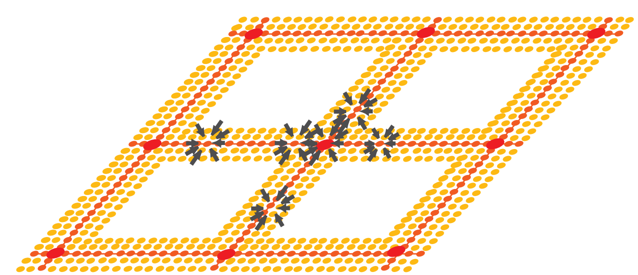
Normal to grid-point array:
By statistical Identification of cloud regime



q″

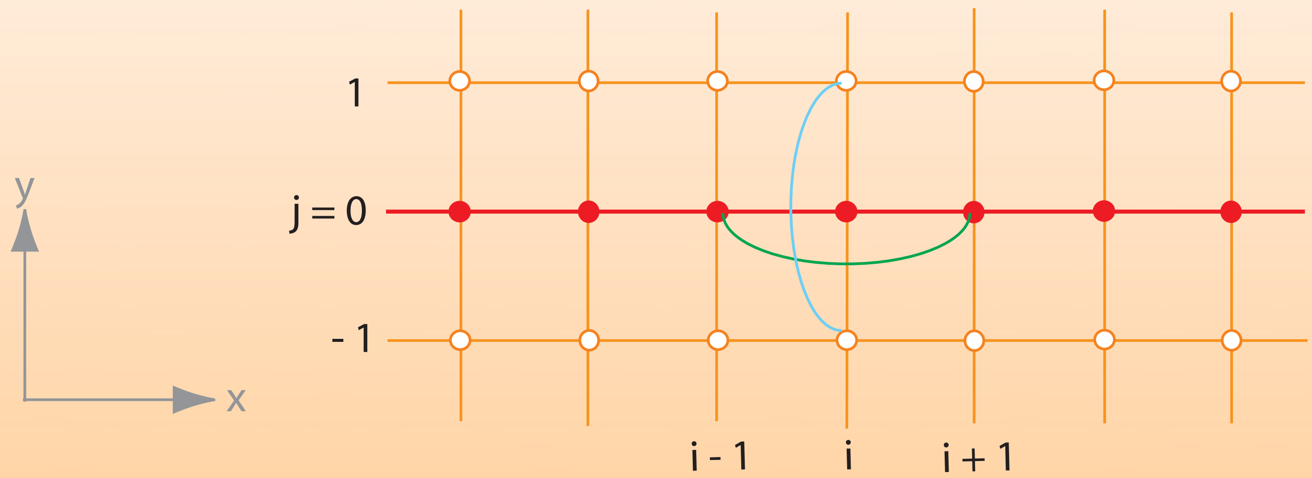
Along grid-point array: q* - q′

Normal to grid-point array:
With a parameterization based on isotropy or inferred anisotropy



Advection of Filtered Variable, q′

GLOBAL STABILITY : Uniform current with $\overline{q'}^{i} = 0$



The array sum of q'^2 is conserved if

$$\Delta_j \widehat{q'} = a_1 + b_1 \Delta_i q'$$

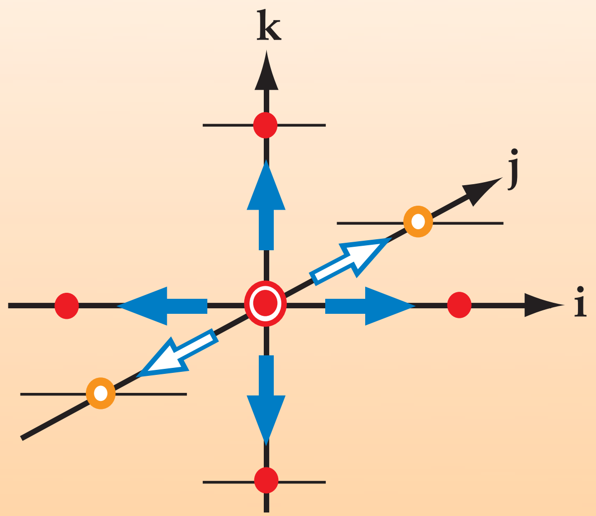
↑ Estimated first-order difference in the normal direction

↑ Predicted first-order difference in the tangential direction

The parameter b_1 represents the dominant orientation of cloud organization.

Advection of Filtered Variable, q′

LOCAL STABILITY : Three-dimensionally variable current



Estimated flux divergence must not produce positive feedback on the perturbation.

$$\delta_j^2 \widehat{q'} = a_2 + b_2 \, \delta_i^2 q'$$

↑ Estimated second-order difference in the normal direction

↑ Predicted second-order difference in the tangential direction

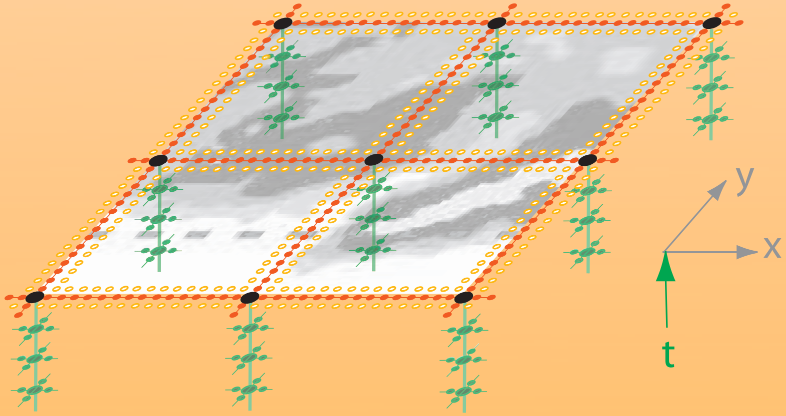
with $b_2 \geq 1$

DETERMINATION OF THE PARAMETERS

$$\Delta_j \hat{q}' = a_1 + b_1 \Delta_i q'$$
$$\delta_j^2 \hat{q}' = a_2 + b_2 \delta_i^2 q'$$
$$\Delta_j \delta_j^2 \hat{q}' = a_3 + b_3 \Delta_i \delta_i^2 q'$$
$$\delta_j^4 \hat{q}' = a_4 + b_4 \delta_i^4 q'$$

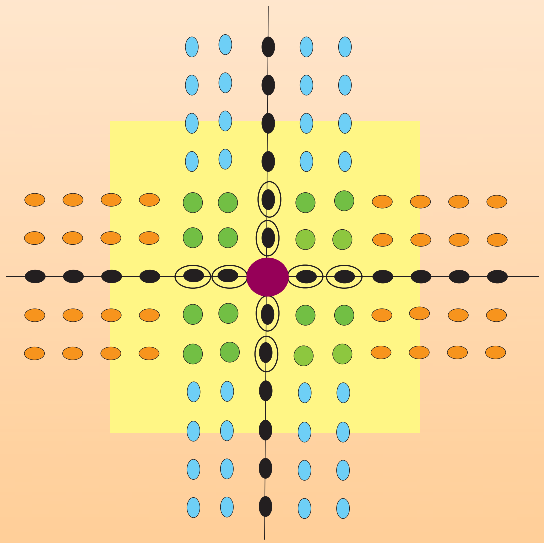
HYPOTHESES:

- These parameters are cloud-regime dependent.
- Cloud regimes have longer spatial and temporal scales than individual clouds.
- These parameters can be statistically estimated through regression analysis of past data at the intersection and neighboring points.



CONTROL OF SINGULARITY AT INTERSECTIONS

Correction of the estimation error near the intersection



$C_{i,j}$: the correction at the data point (i,j) on the net

$c_{i',j'}$: the correction at the ghost point (i',j')

$$c_{i',j'} = \sum_{i,j} C_{i,j} e^{-\left(r_{i,j;i',j'}/r_o\right)^2} \Big/ \sum_{i,j} e^{-\left(r_{i,j;i',j'}/r_o\right)^2}$$

where $r_{i,j;i',j'}$ is the distance between the points and r_o is prescribed.

CONTROL OF SPURIOUS TREND

Rayleigh-type damping

Advection of Non-Filtered Variable, q″: Need for Parameterization

Currently,

For scalar variables: $q'' = 0$ at ghost points (ad hoc).

A Relaxation Method for Solving the Elliptic Equation

The elliptic equation is converted to a parabolic equation whose equilibrium solution is the solution of the elliptic equation.

$$\mu \frac{\partial w}{\partial t} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) w + \frac{\partial}{\partial z}\left[\frac{1}{\rho_0} \frac{\partial}{\partial z}(\rho_0 w) \right] + \frac{\partial \eta}{\partial x} - \frac{\partial \xi}{\partial y}$$

where μ defines the time scale for adjustment toward anelastic balance.

Discretization

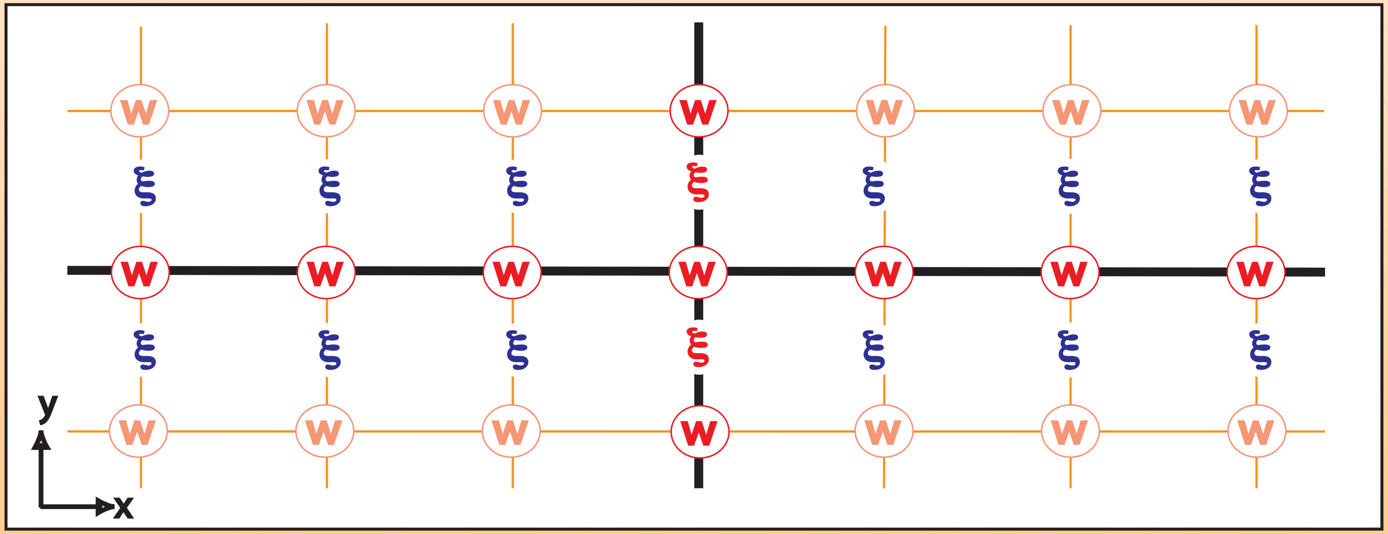
Use a partially backward-implicit scheme for the horizontal derivative term and a fully backward-implicit scheme for the vertical derivative term.

$$\left[\frac{1}{\rho_{k-1/2}} \left(\frac{\mu}{\Delta t} + \frac{2}{\Delta x^2} + \frac{2}{\Delta y^2} \right) + \frac{1}{\Delta z} \left(\frac{1}{\rho_k \Delta z} + \frac{1}{\rho_{k-1} \Delta z} \right) \right] (\rho w)^{n+1}_{i,j,k-1/2} - \frac{1}{\Delta z} \left[\frac{(\rho w)^{n+1}_{i,j,k+1/2}}{\rho_k \Delta z} + \frac{(\rho w)^{n+1}_{i,j,k-3/2}}{\rho_{k-1} \Delta z} \right]$$

$$= \frac{\mu}{\Delta t} w^{n}_{i,j,k-1/2} + \frac{1}{\Delta x^2} \left(w^{n}_{i-1,j,k-1/2} + w^{n}_{i+1,j,k-1/2} \right) + \frac{1}{\Delta y^2} \left(w^{n}_{i,j-1,k-1/2} + w^{n}_{i,j+1,k-1/2} \right) + F^{n+1}_{i,j,k-1/2}$$

A Relaxation Method for Solving the Elliptic Equation in the Q3D Model

$$\mu \frac{\partial w}{\partial t} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) w + \frac{\partial}{\partial z} \left[\frac{1}{\rho_0} \frac{\partial}{\partial z} (\rho_0 w) \right] + \frac{\partial \eta}{\partial x} - \frac{\partial \xi}{\partial y}$$



$\frac{\partial^2 \overline{w}}{\partial y^2}$: prescribed $\quad$ $\frac{\partial^2 w'}{\partial y^2}$: estimated $\quad$ w'' : assumed axi-symmetric

$\frac{\partial \overline{\xi}}{\partial y}$: prescribed $\quad$ $\frac{\partial \xi'}{\partial y}$: estimated $\quad$ $\frac{\partial \xi''}{\partial y} = -\frac{\partial \eta''}{\partial x}$: assumed

VORTICITY AT GHOST POINTS

Obtained with a similar method used for scalar variables.

HORIZONTAL VELOCITY COMPONENTS

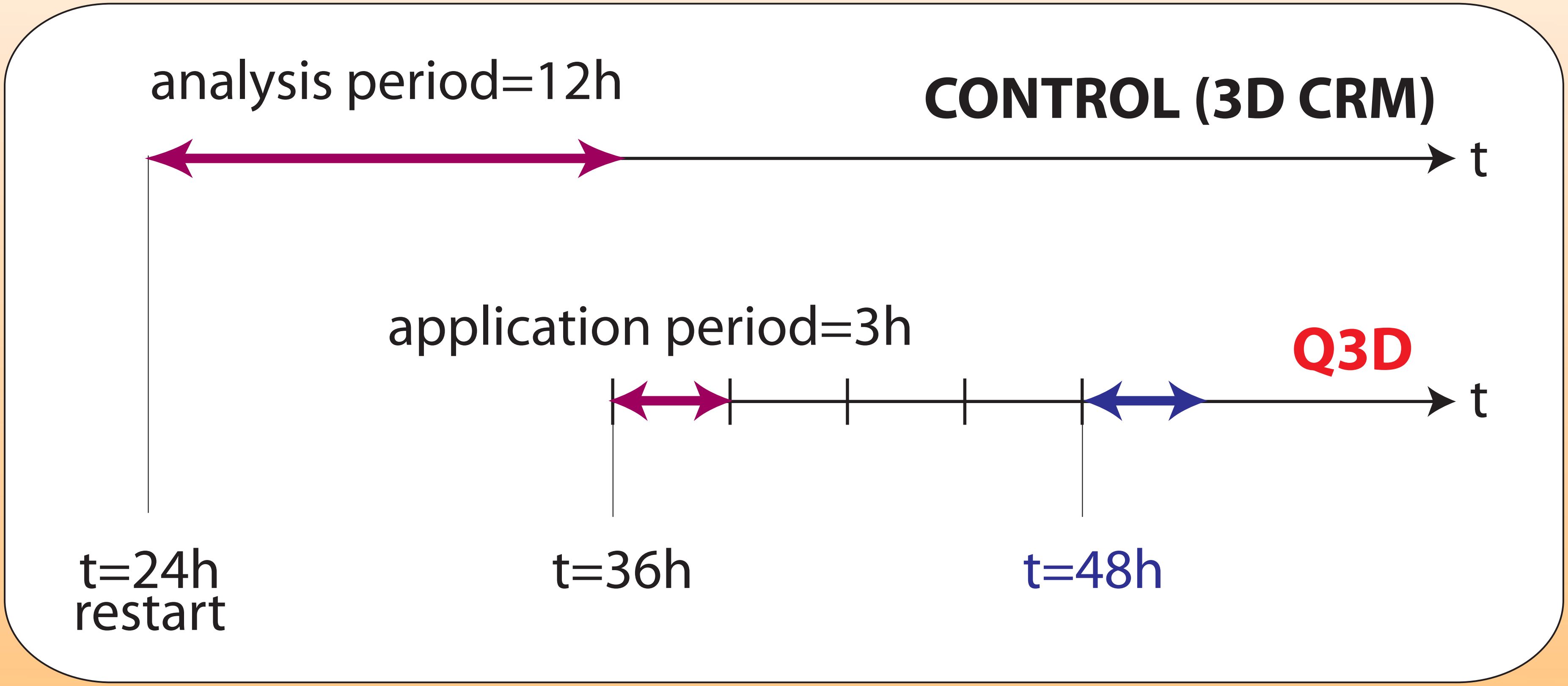
Determined such a way that the continuity equation is satisfied and they are consistent with the predicted vorticity at the network.

EXPERIMENTAL STRATEGY

- Break up the algorithm to pieces, and test one piece at a time.
- Always quantitatively compare with the results of 3D control run.
- Comparison is mainly through the time sequences of spatial variances (and covariances) rather than through spatial/temporal means only.

TESTING PERFORMED

for an idealized, very small domain first



3D CRM

A three-dimensional anelastic model based on the vector vorticity equation

by Joon-Hee Jung and Akio Arakawa (2007), MWR

Control Run

- **Domain size:** 126 km x 126 km x 18 km (height)
- **Horizontal resolution:** 3 km
- **Vertical resolution:** 34 layers with a stretched vertical grid
- **Lower-boundary:** ocean surface with a fixed temperature
- **Idealized tropical condition:** based on the GATE Phase-III mean sounding and wind profile during TOGA COARE
- **Large-scale forcing:** prescribed advective tendency
- **Perturbation:** small, random temperature perturbations into the lowest model layer

SEMI-PROGNOSTIC EXPERIMENT

θ and vorticity components at the network are prescribed.

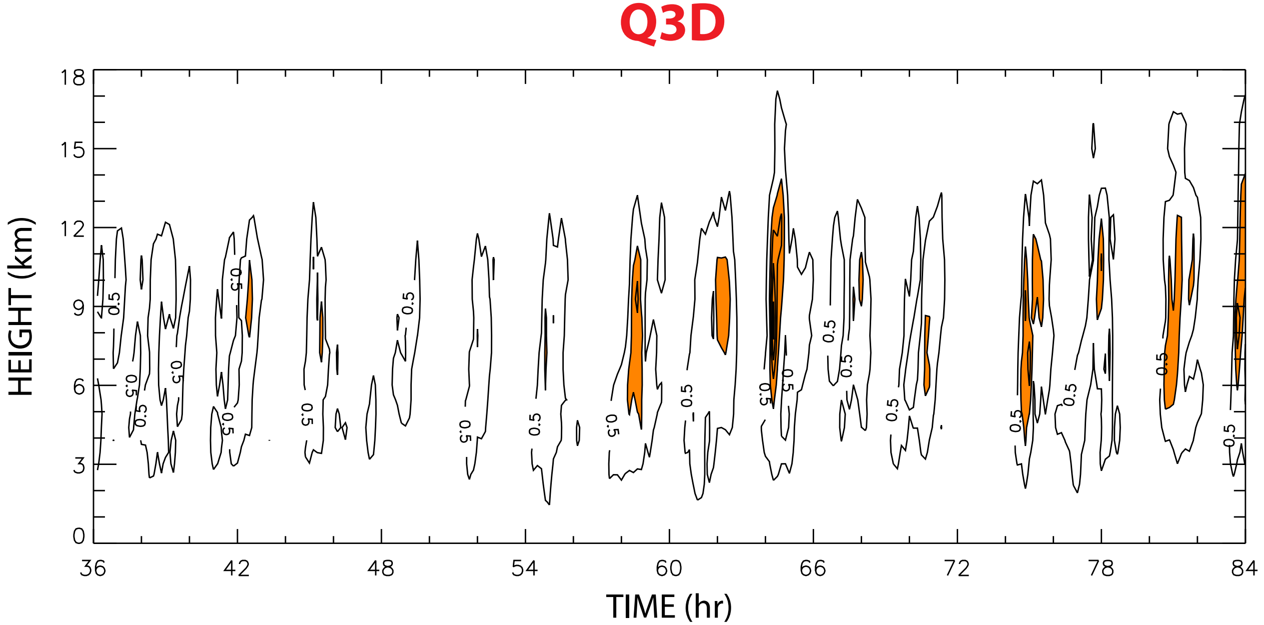
Scalar variables at the network are predicted and those at the ghost points are obtained with the Q3D algorithm.

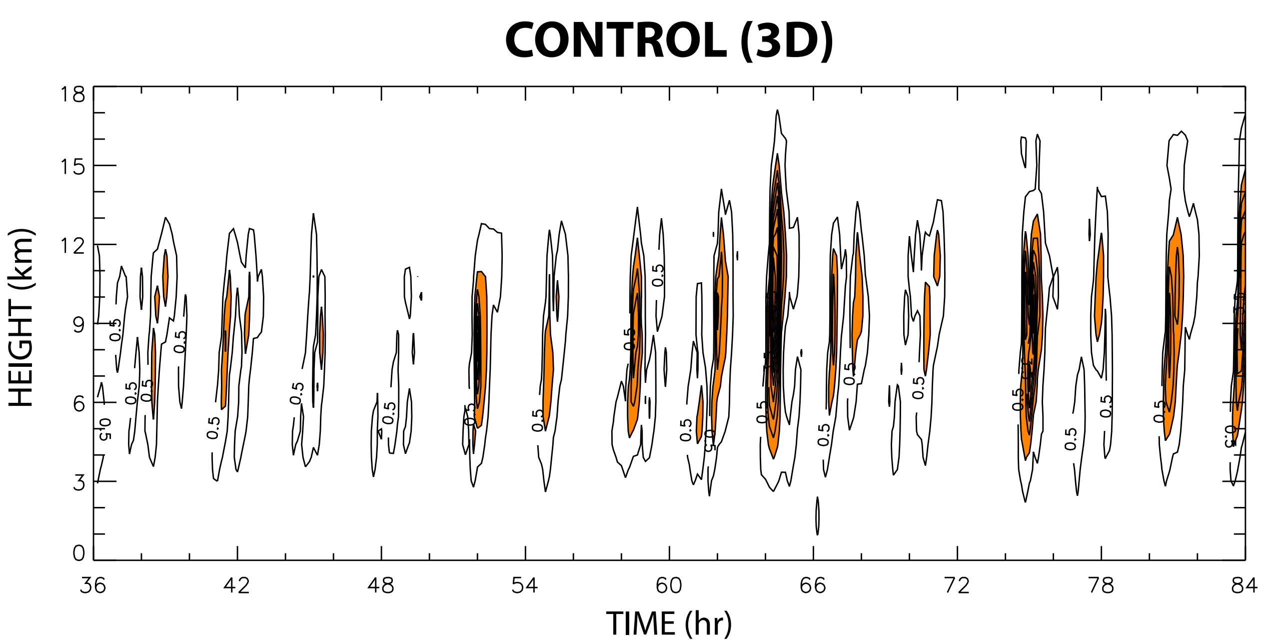
The vorticity gradient in the right hand side of the w-equation is obtained from the Q3D algorithm.

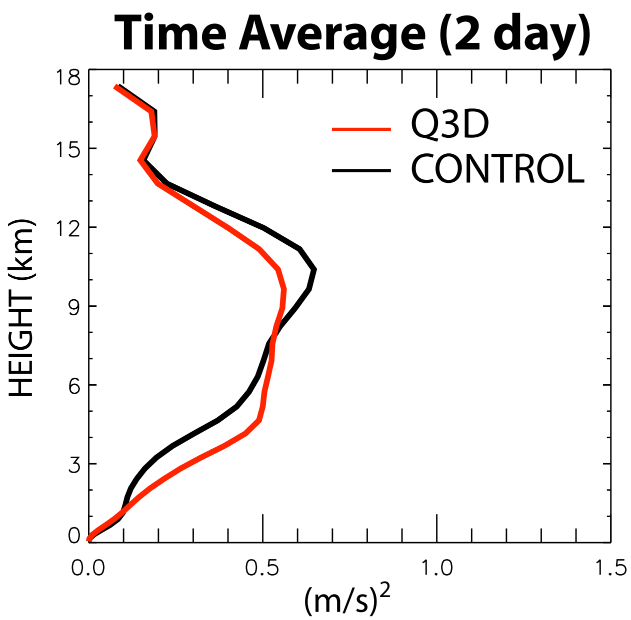
All velocity components and vorticity components at ghost points are obtained from the Q3D algorithm.

SEMI-PROGNOSTIC TEST

X-array *variance* of w



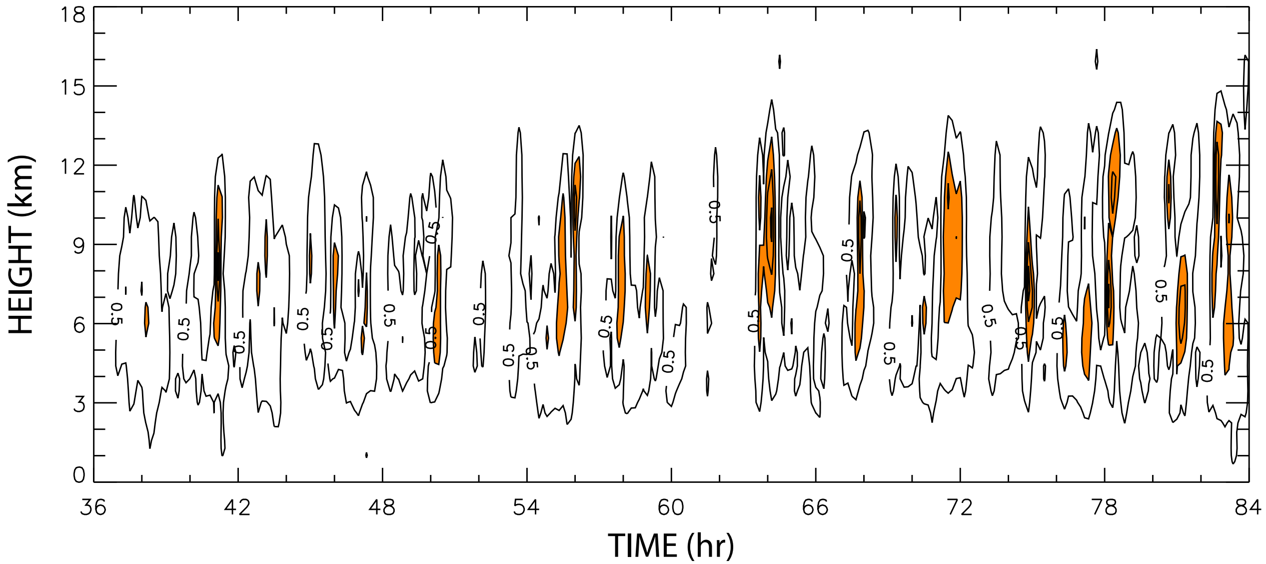




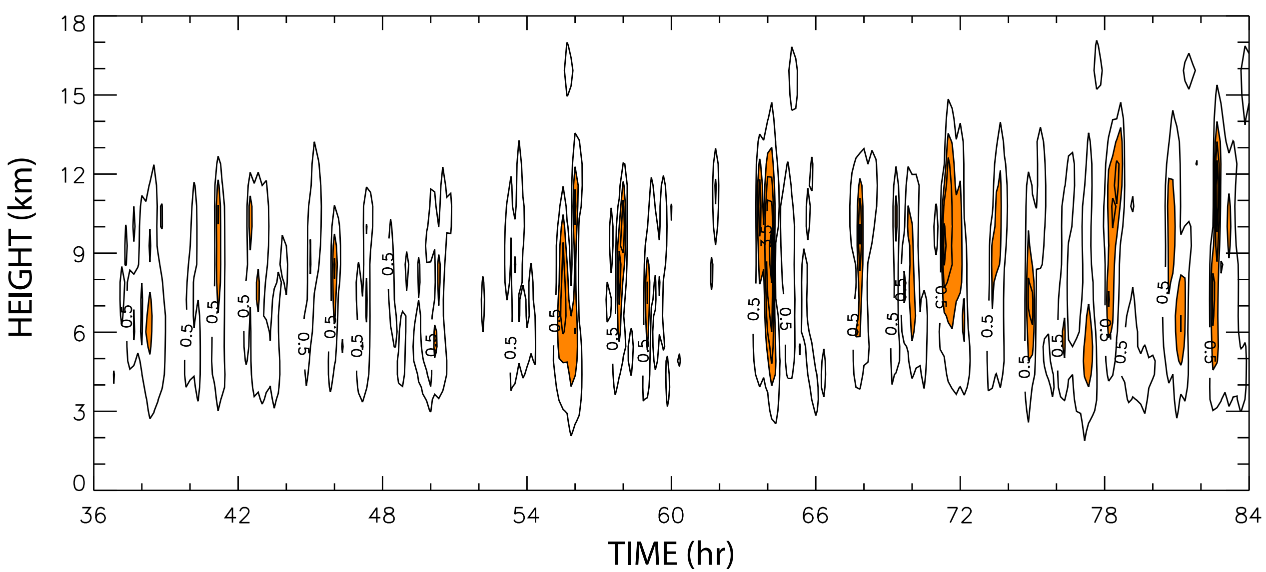
SEMI-PROGNOSTIC TEST

Y-array ***variance*** of w

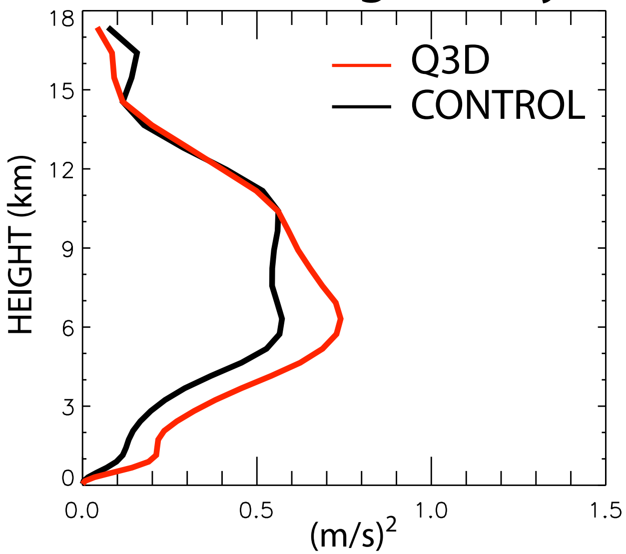
Q3D

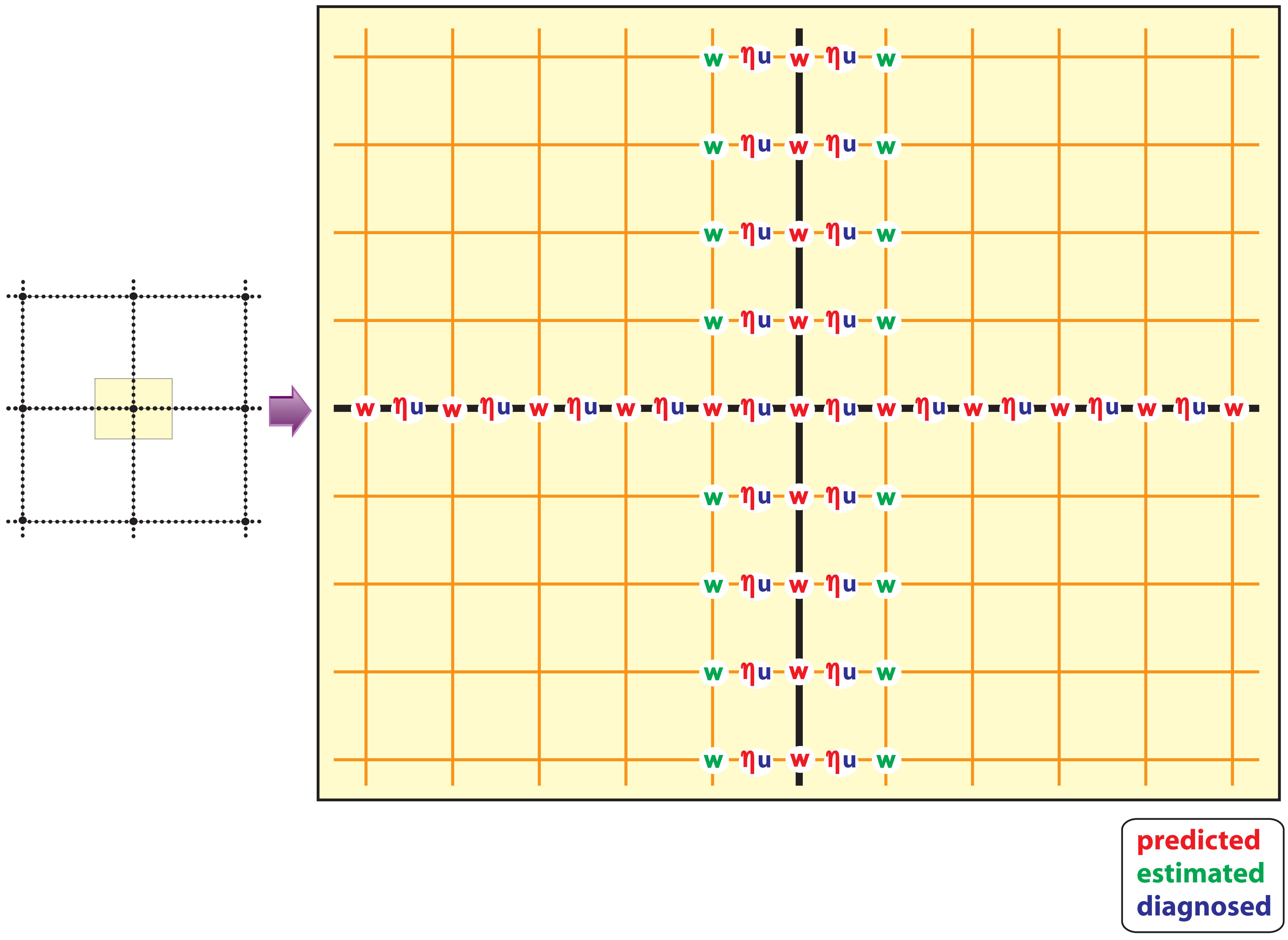


CONTROL (3D)



Time Average (2 day)

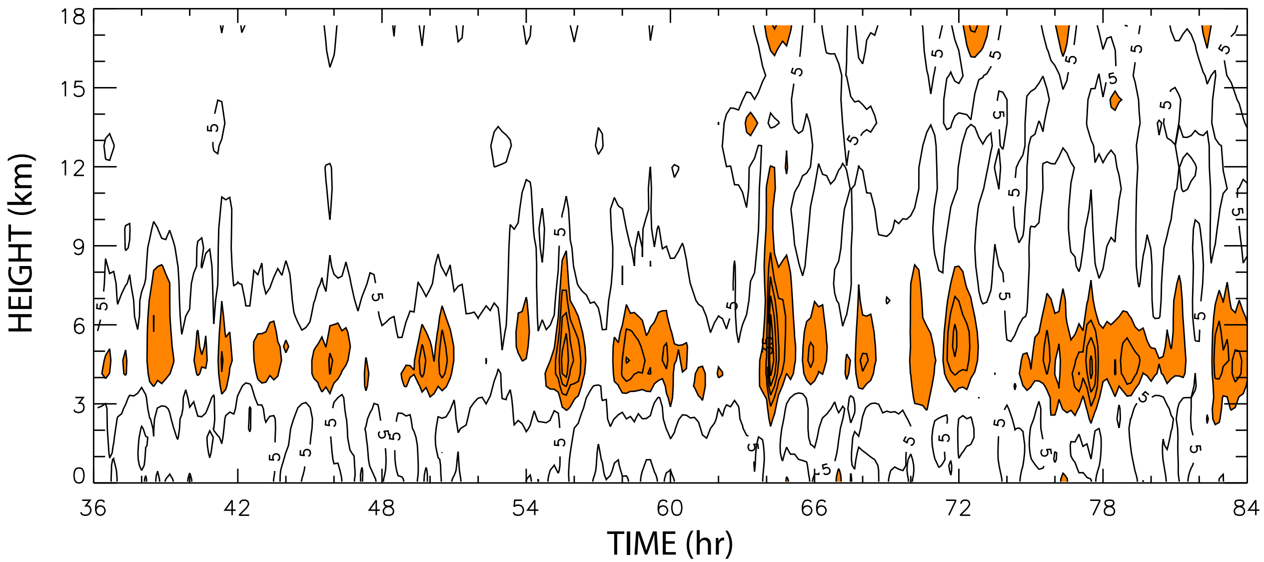



predicted
estimated
diagnosed

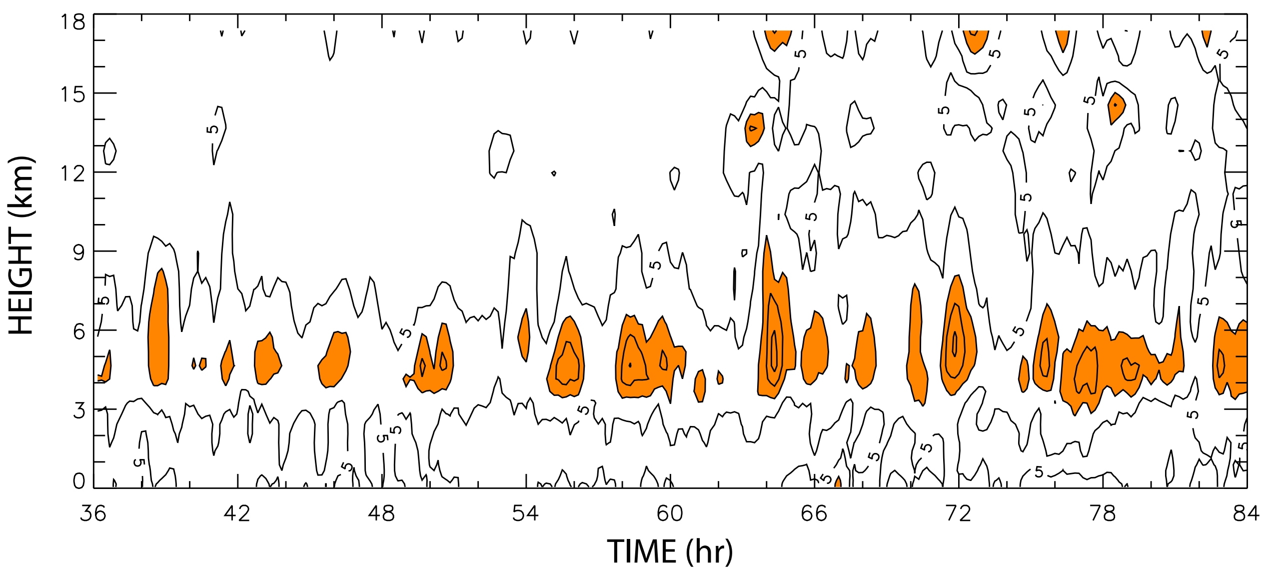
SEMI-PROGNOSTIC TEST

Y-array ***variance*** of u

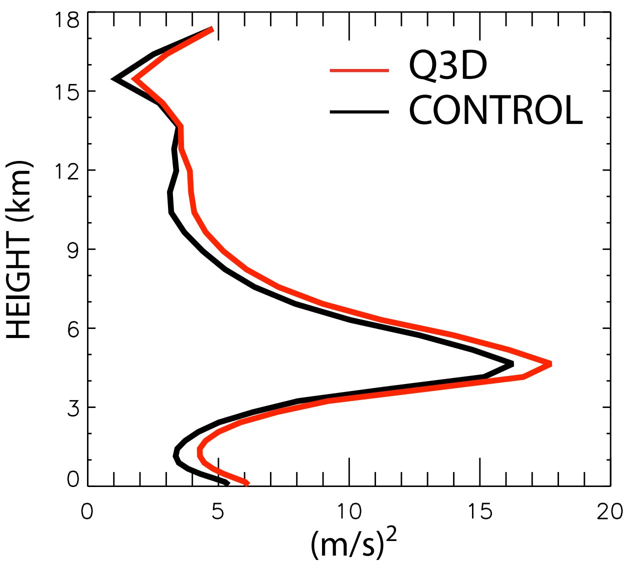
Q3D



CONTROL (3D)



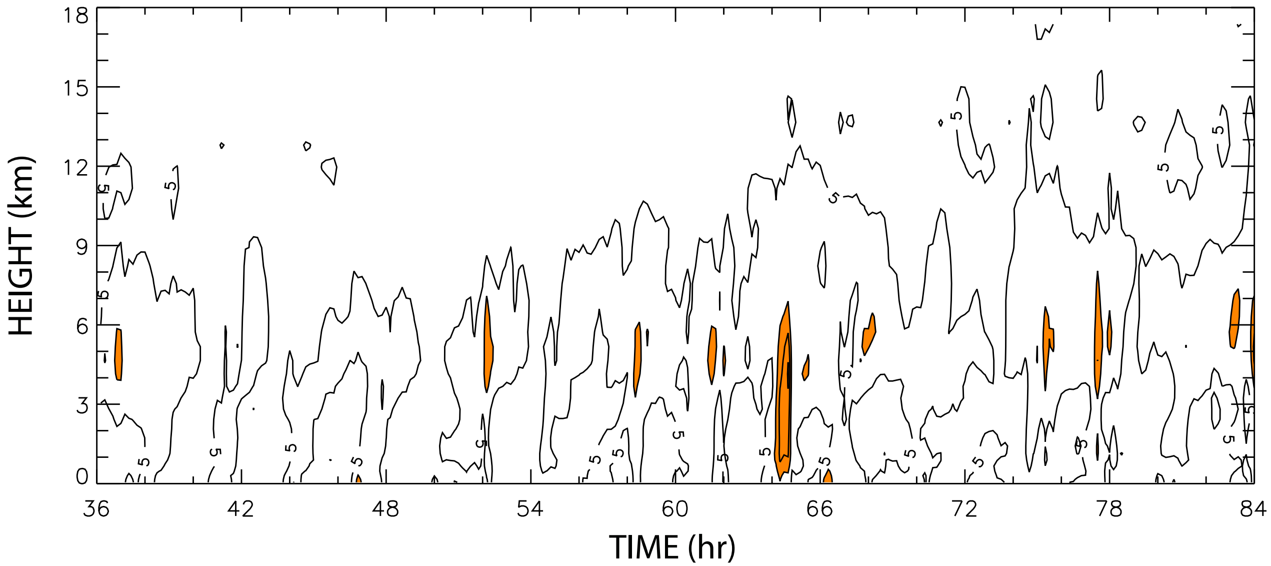
Time Average (2 day)



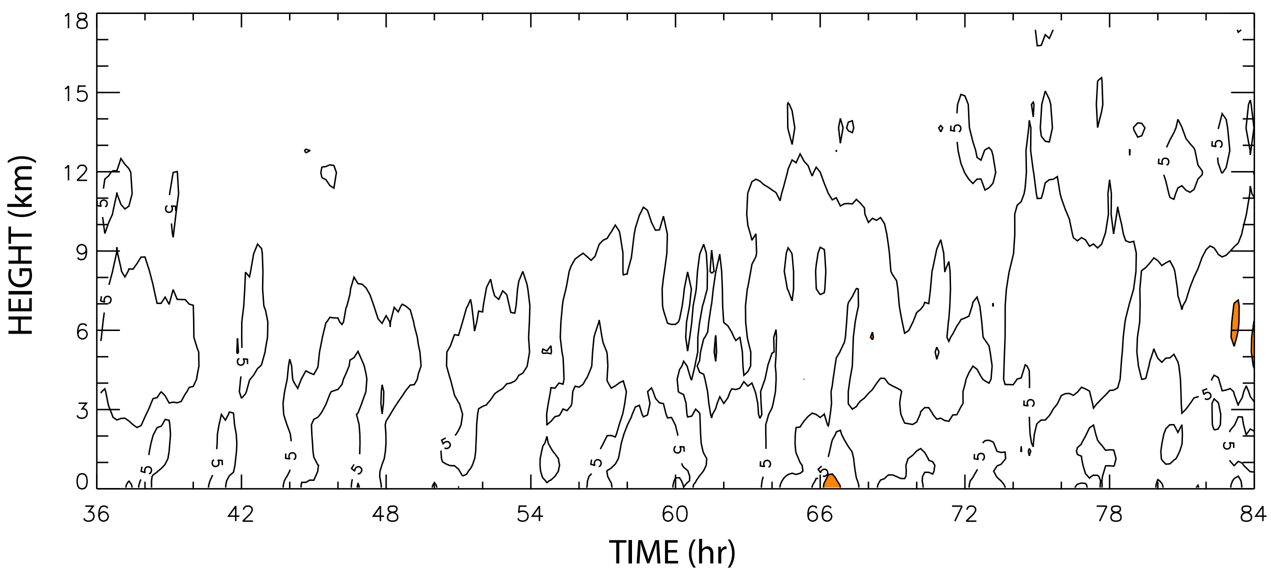
SEMI-PROGNOSTIC TEST

X-array ***variance*** of v

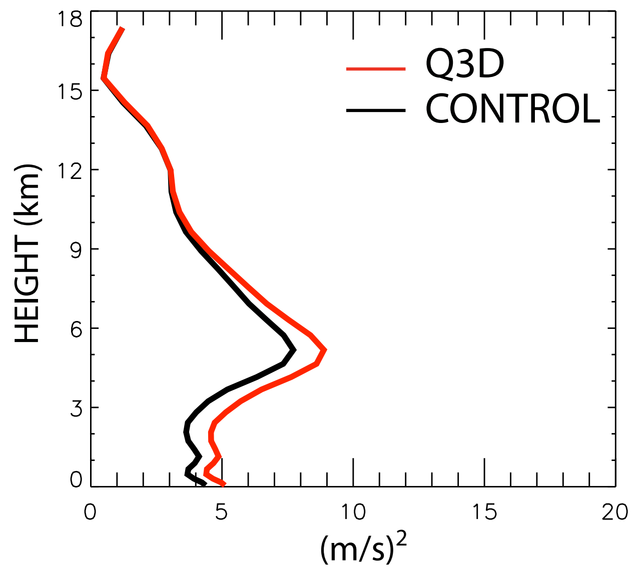
Q3D



CONTROL (3D)



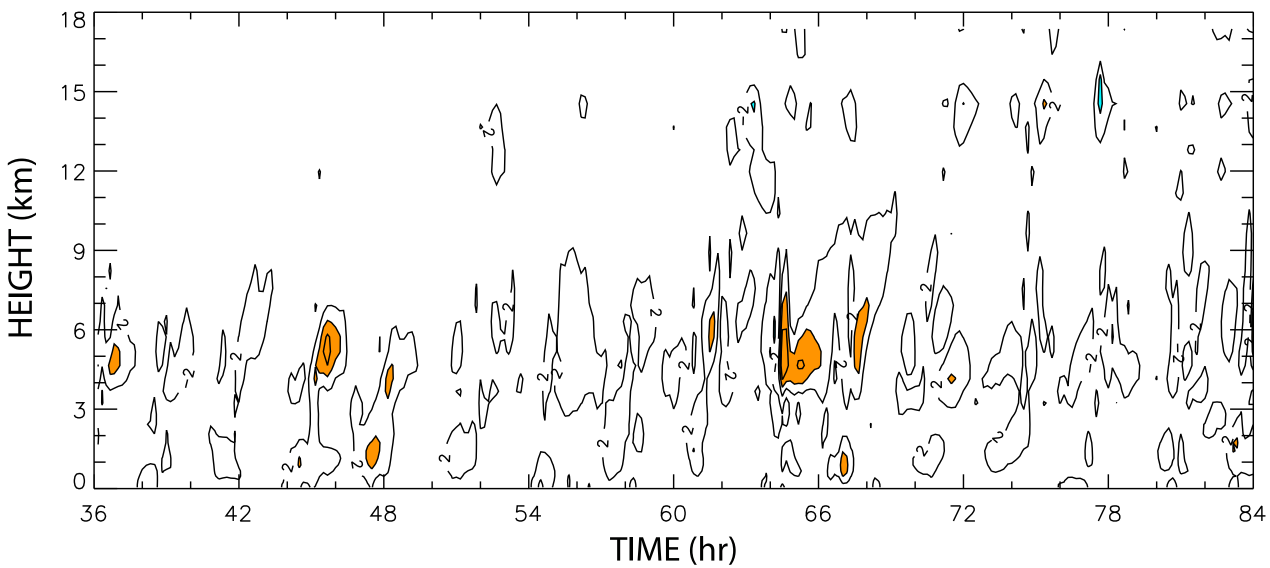
Time Average (2 day)



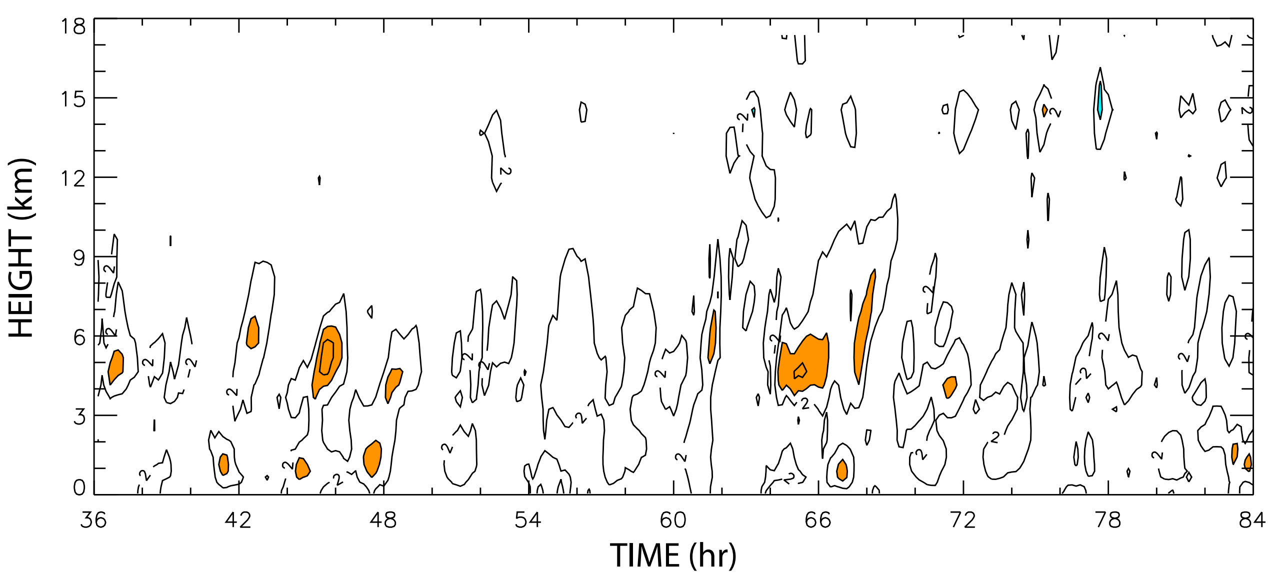
SEMI-PROGNOSTIC TEST

X-array ***covariance*** of uv

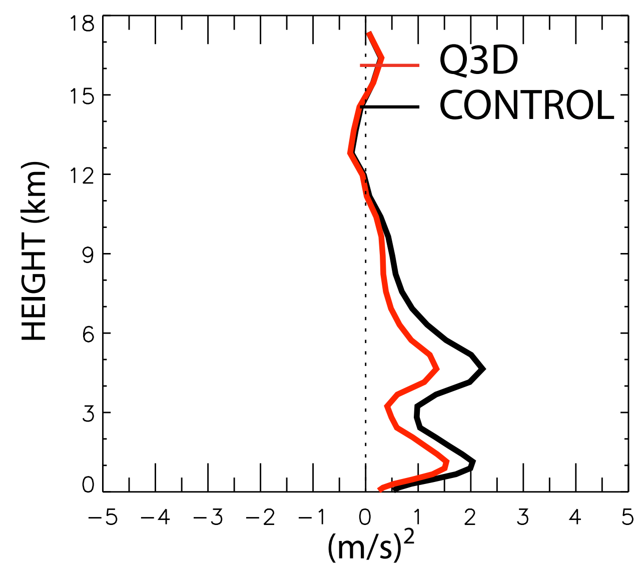
Q3D



CONTROL (3D)



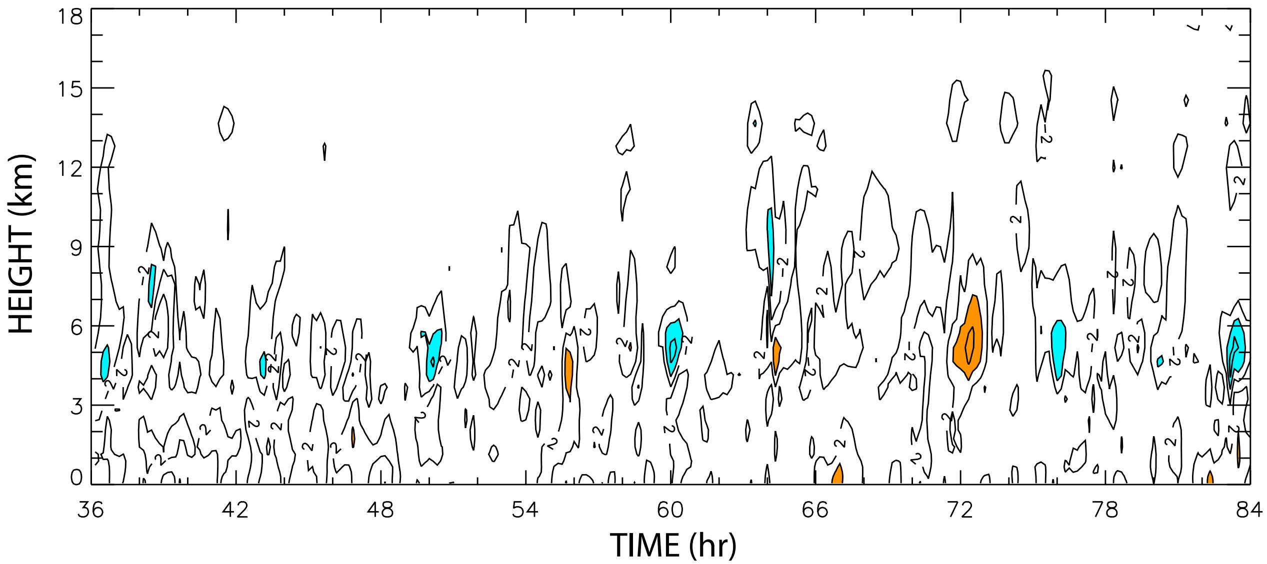
Time Average (2 day)



SEMI-PROGNOSTIC TEST

Y-array ***covariance*** of uv

Q3D



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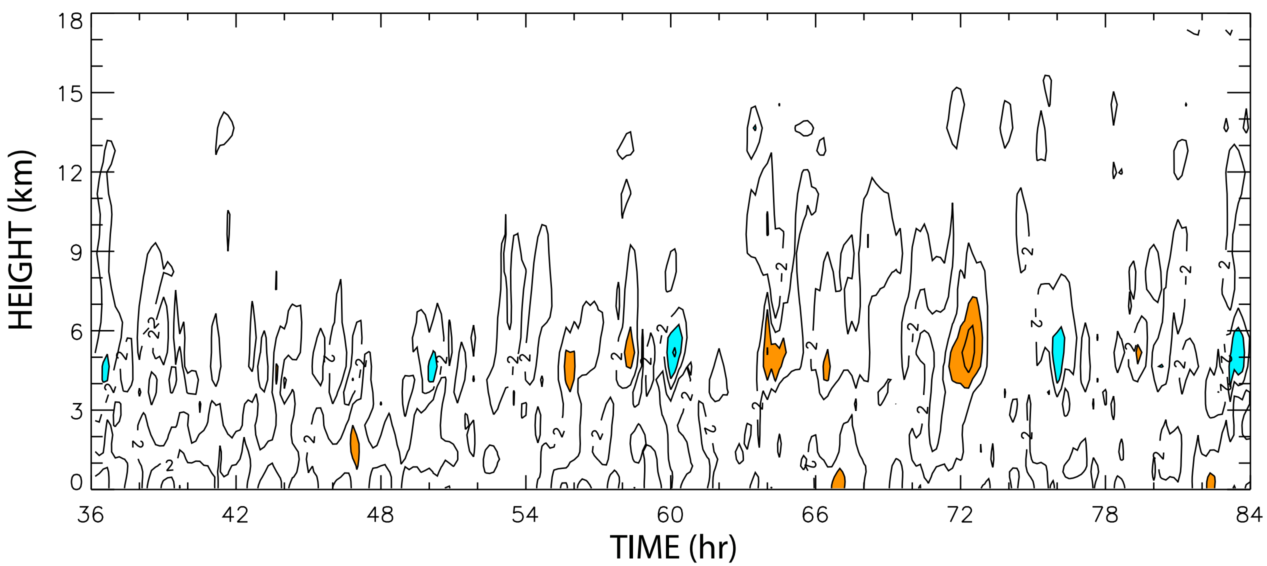
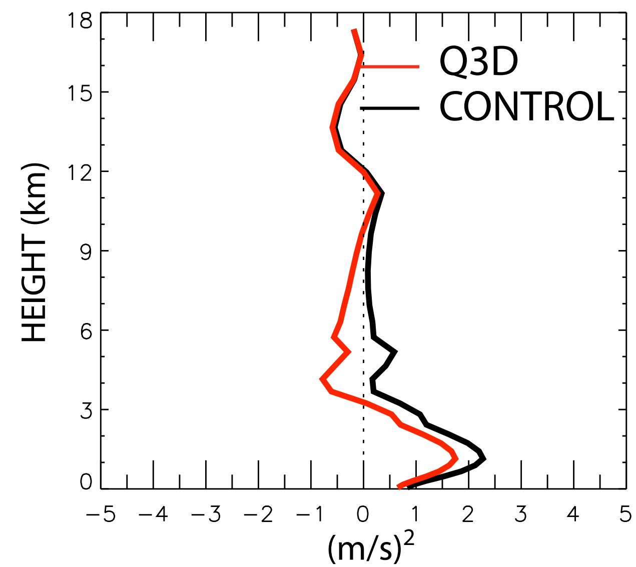


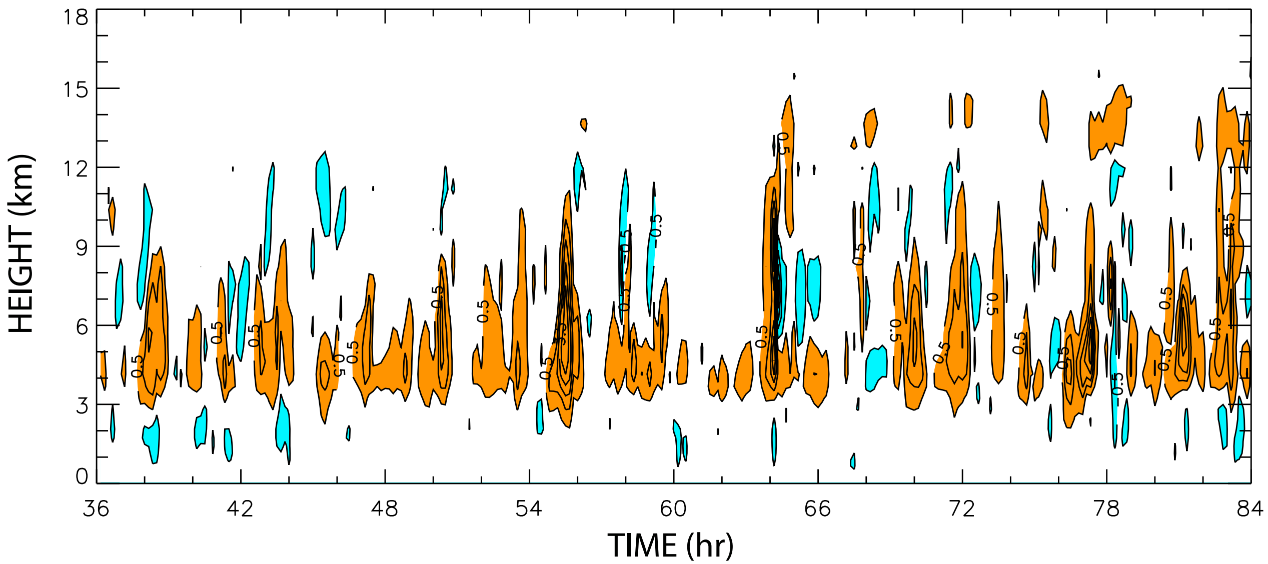
Time Average (2 day)



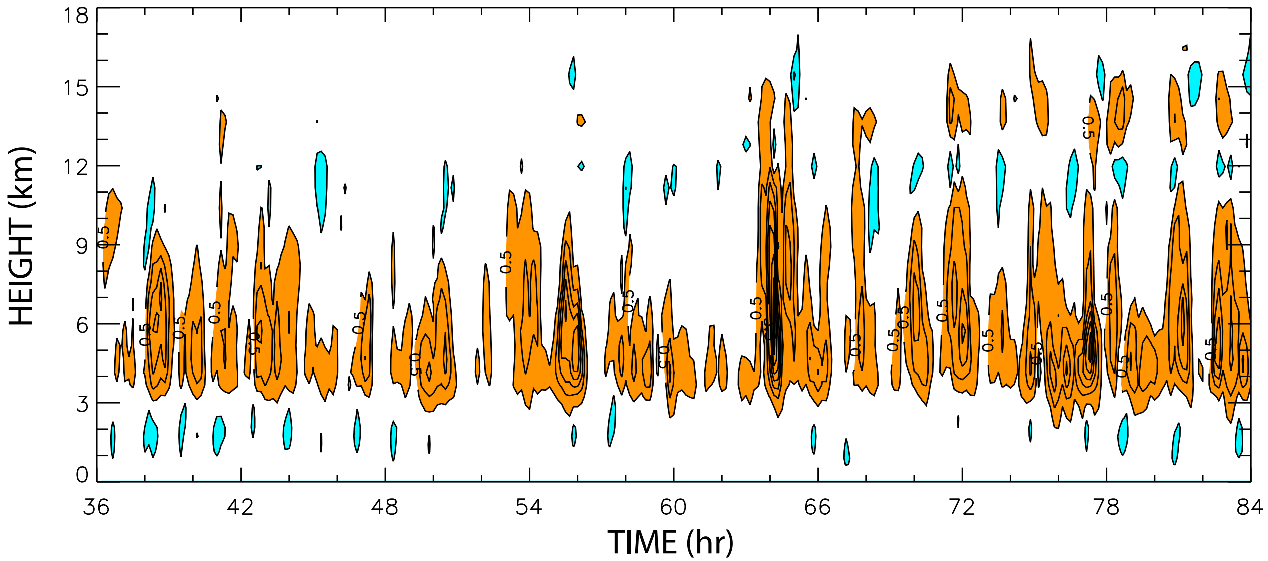
SEMI-PROGNOSTIC TEST

Y-array ***covariance*** of uw

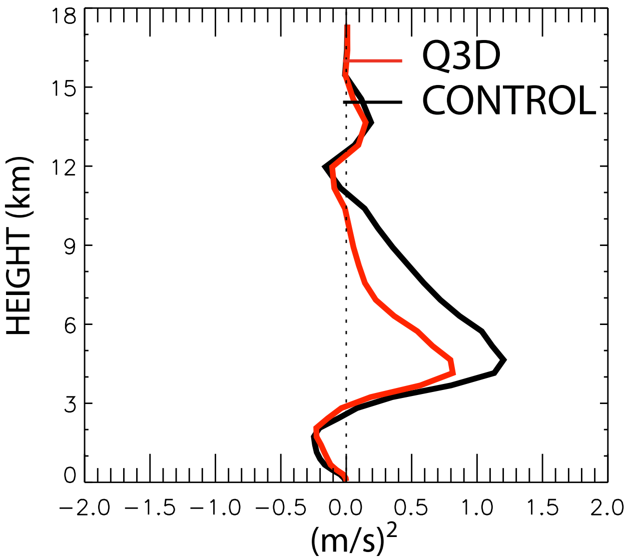
Q3D



CONTROL (3D)



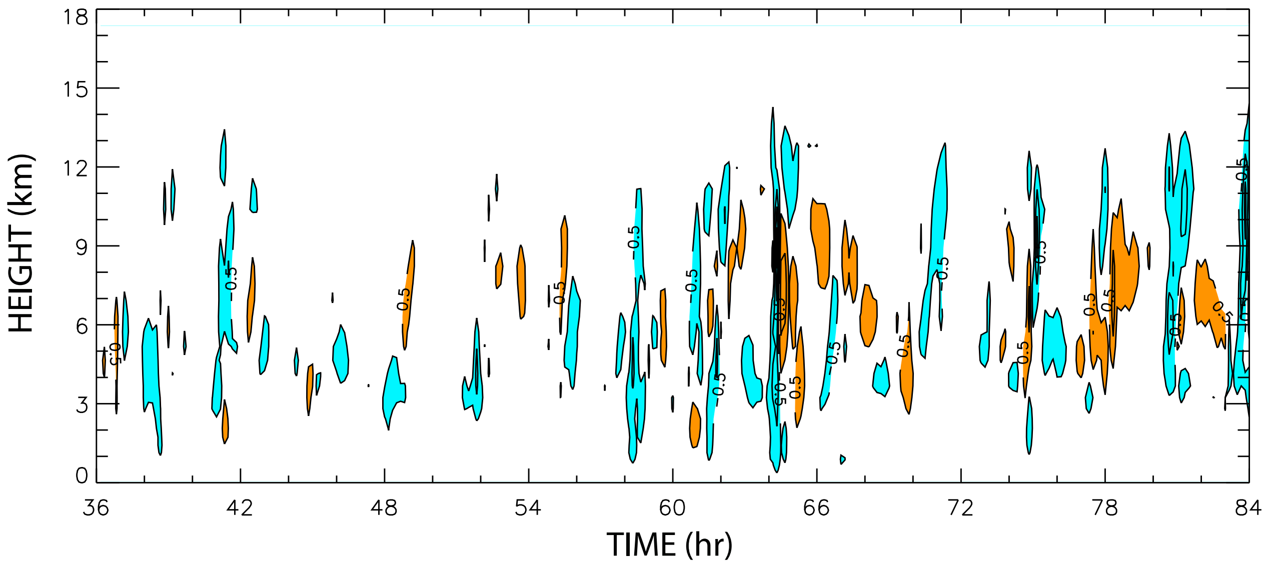
Time Average (2 day)



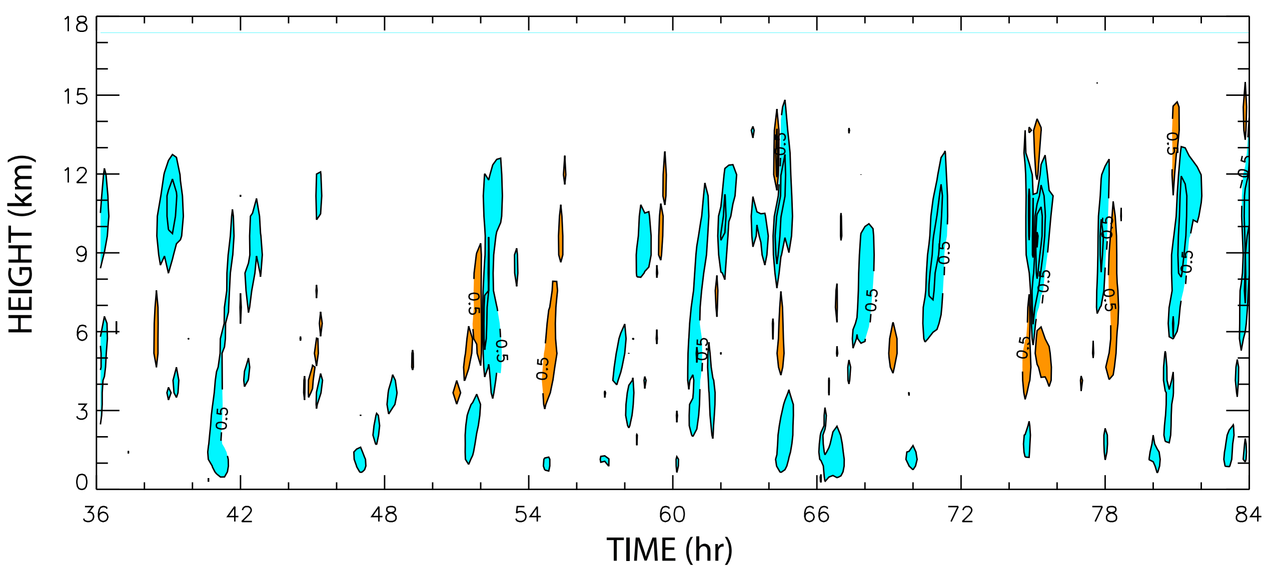
SEMI-PROGNOSTIC TEST

X-array ***covariance*** of vw

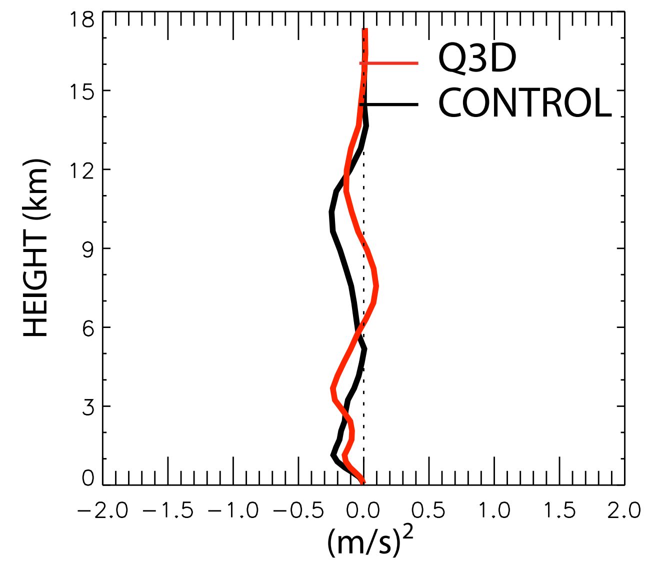
Q3D



CONTROL (3D)



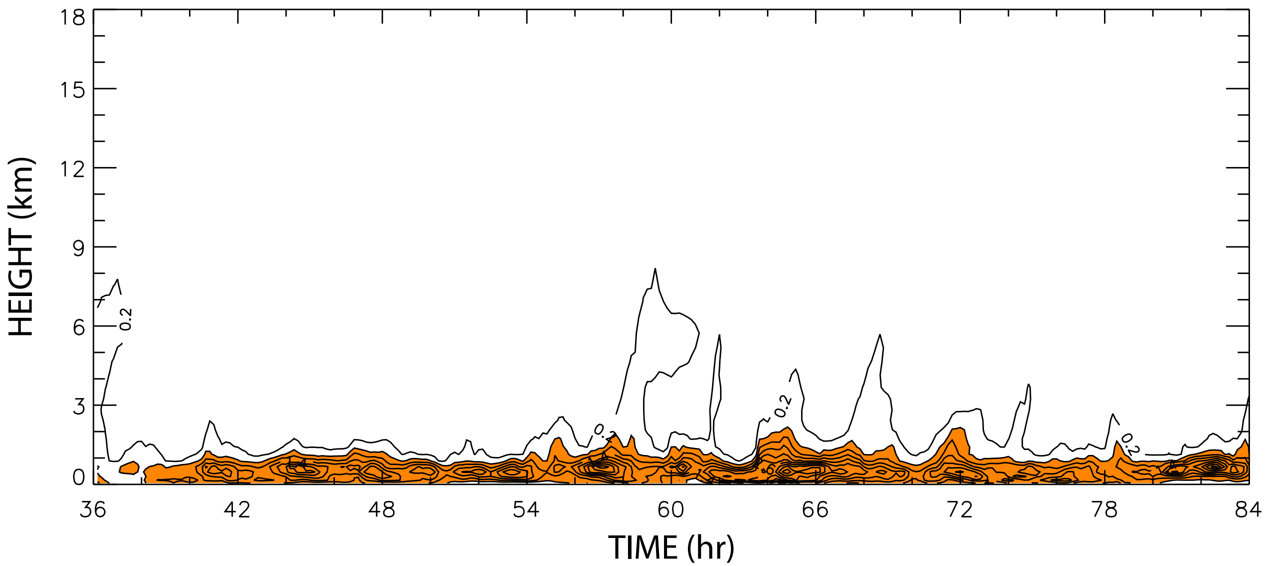
Time Average (2 day)



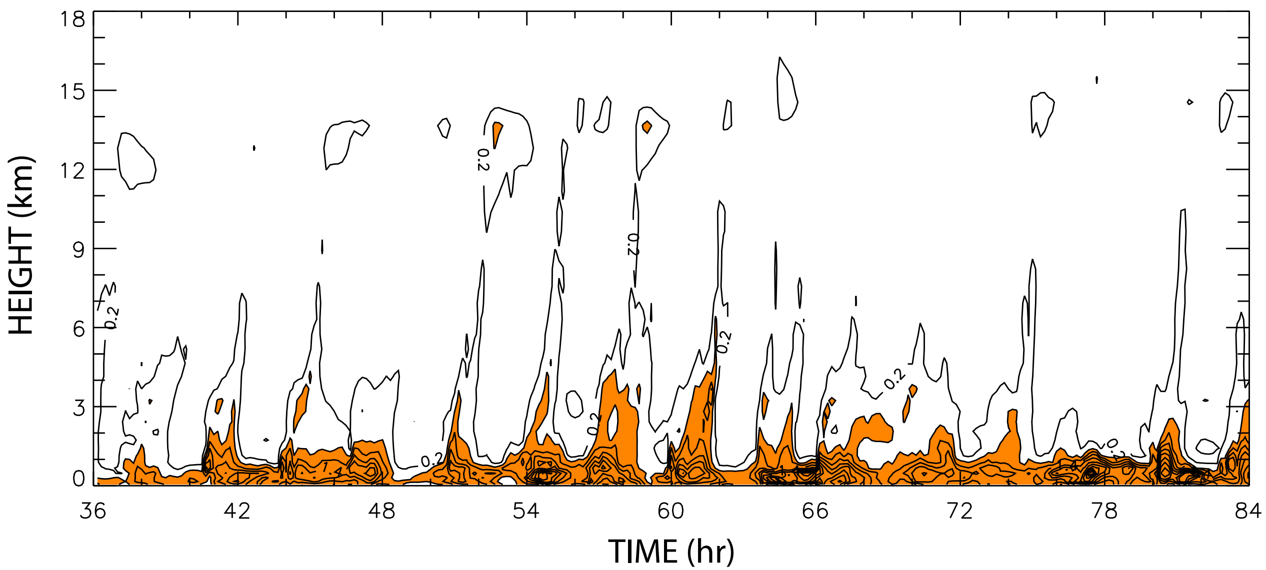
SEMI-PROGNOSTIC TEST

X-array ***variance*** of tracer mixing ratio

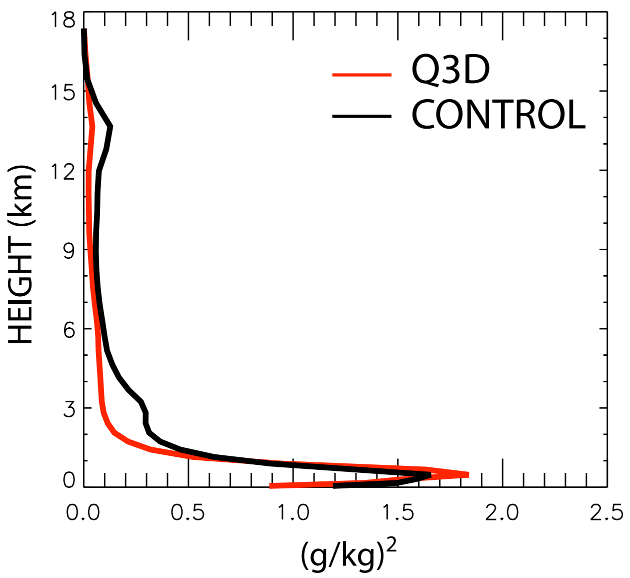
Q3D



CONTROL (3D)



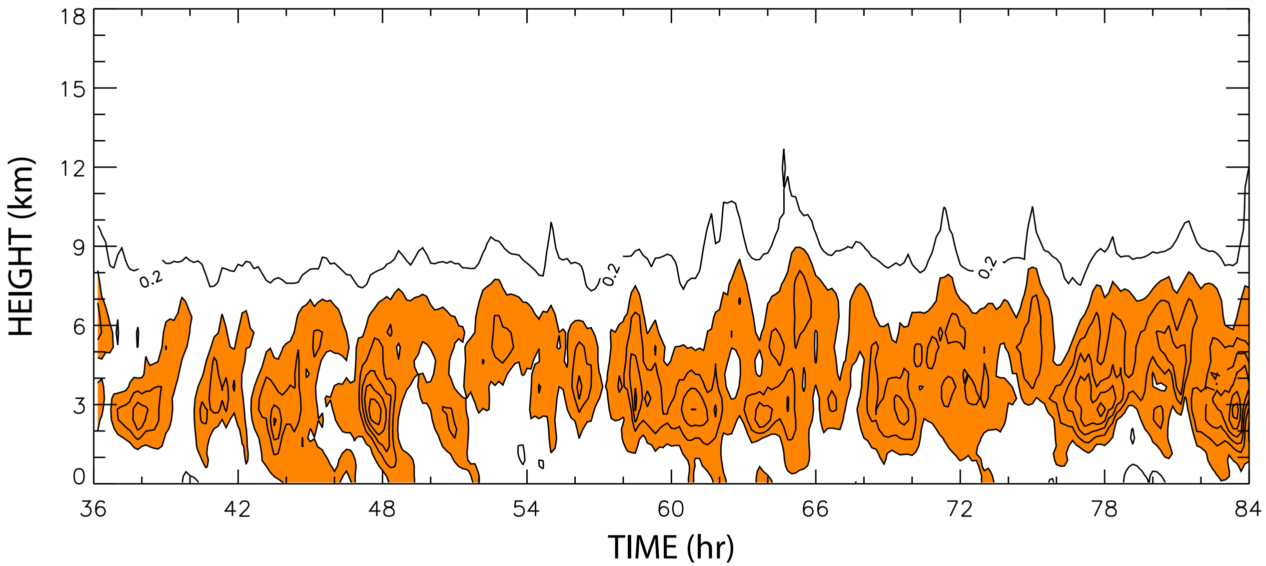
Time Average (2 day)



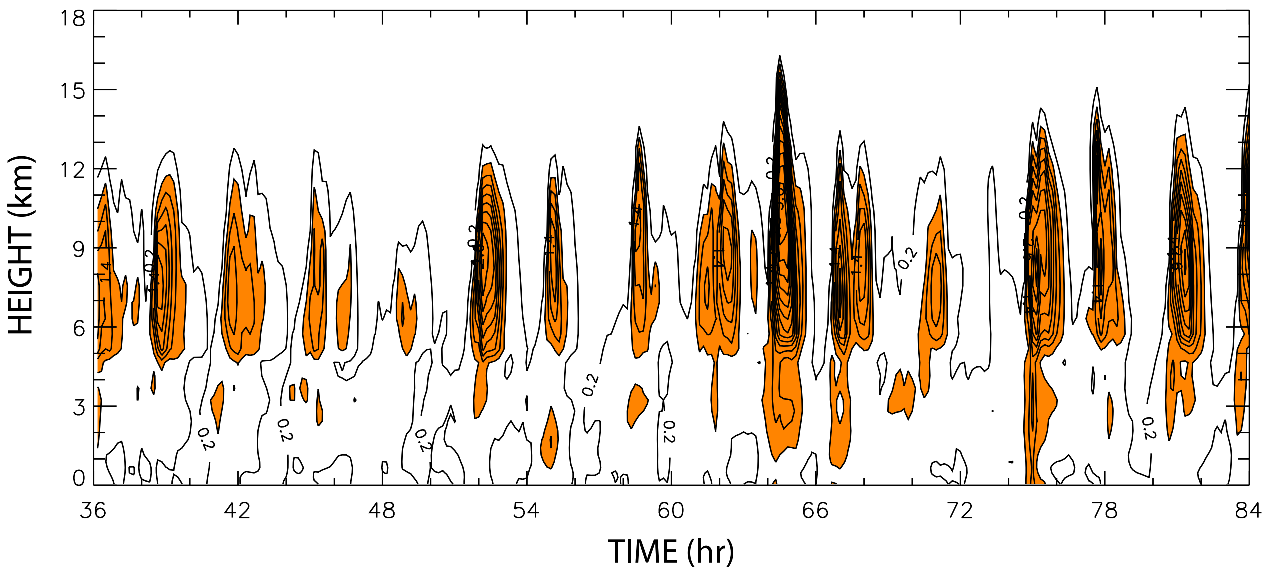
SEMI-PROGNOSTIC TEST

X-array ***variance*** of total water mixing ratio

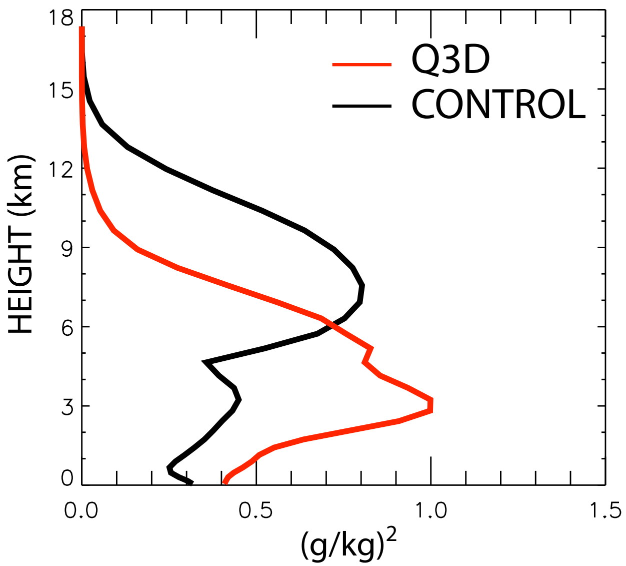
Q3D



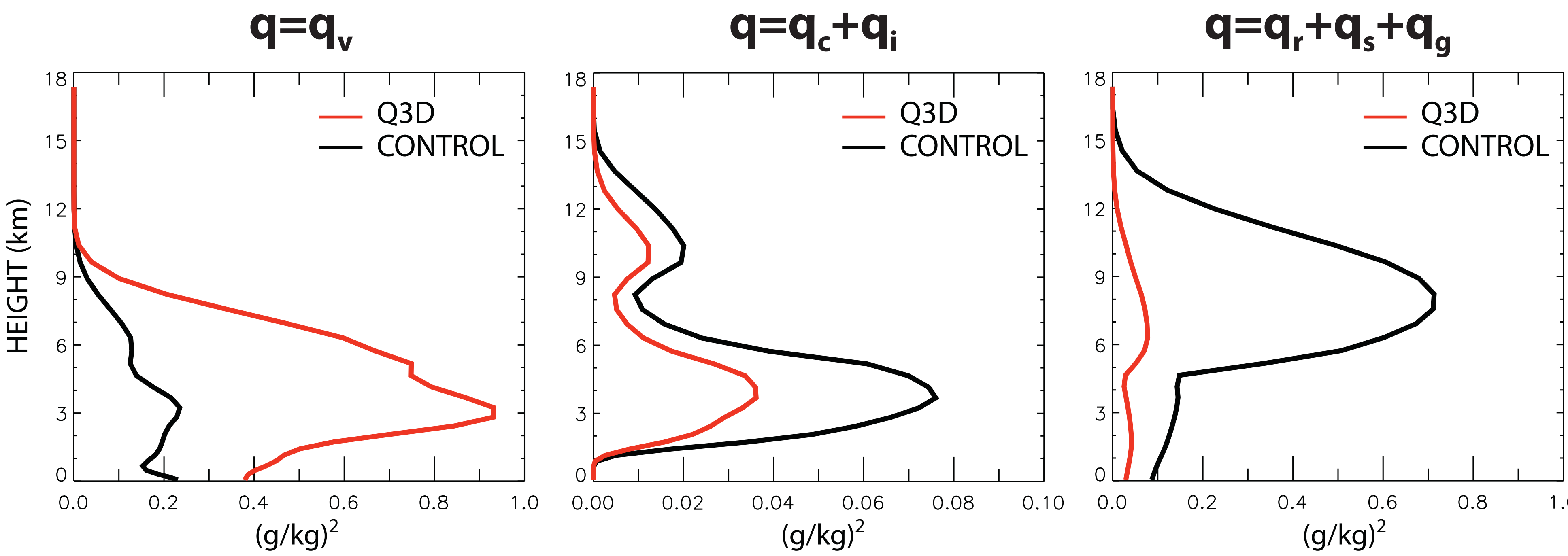
CONTROL (3D)



Time Average (2 day)



X-array Variance of q



FULL-PROGNOSTIC EXPERIMENT

θ and vorticity components at the network are predicted.

Scalar variables at the network are predicted and those at the ghost points are obtained with the Q3D algorithm.

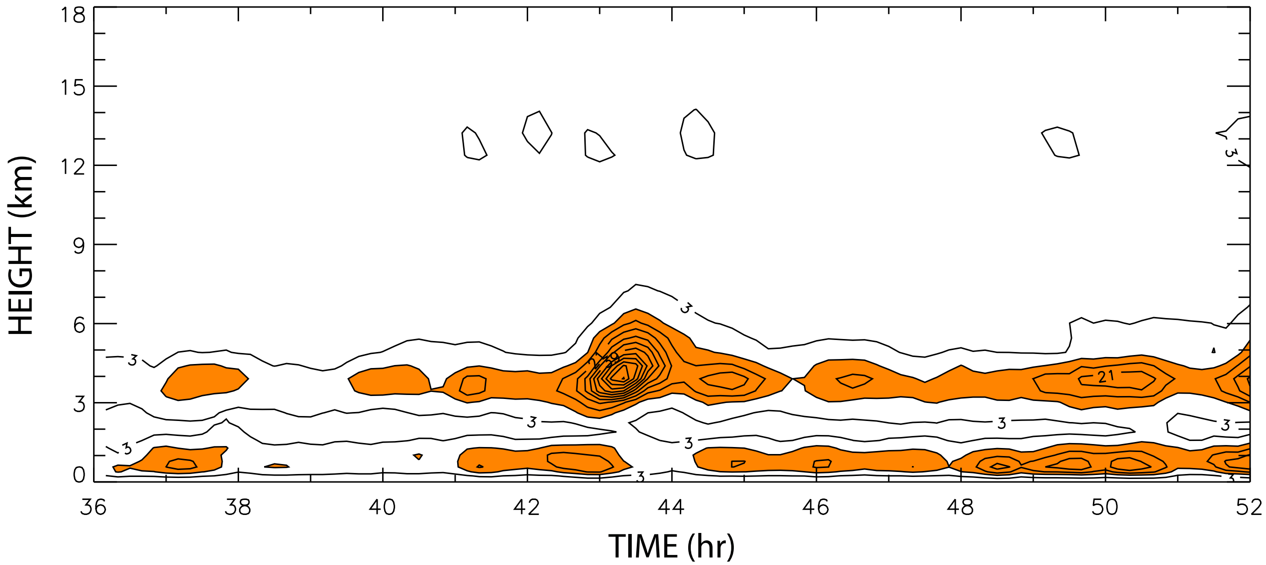
The vorticity gradient in the right hand side of the w-equation is obtained from the Q3D algorithm.

All velocity components and vorticity components at ghost points are obtained from the Q3D algorithm.

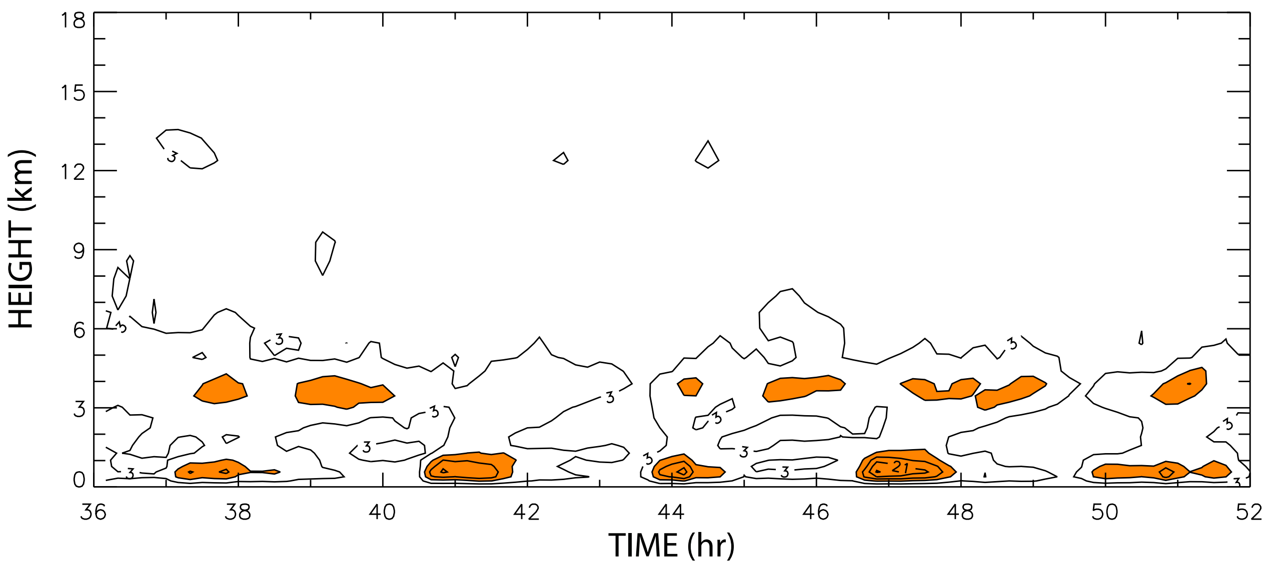
FULL PRONOSTIC TEST

X-array ***variance*** of $\xi\ (10^{-6})$

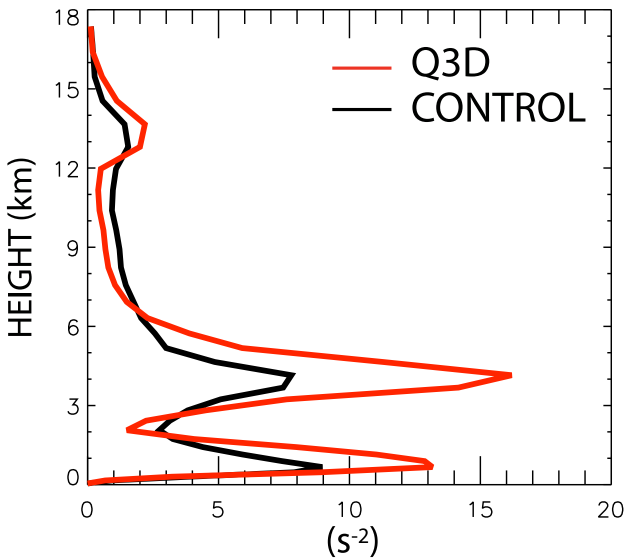
Q3D



CONTROL (3D)

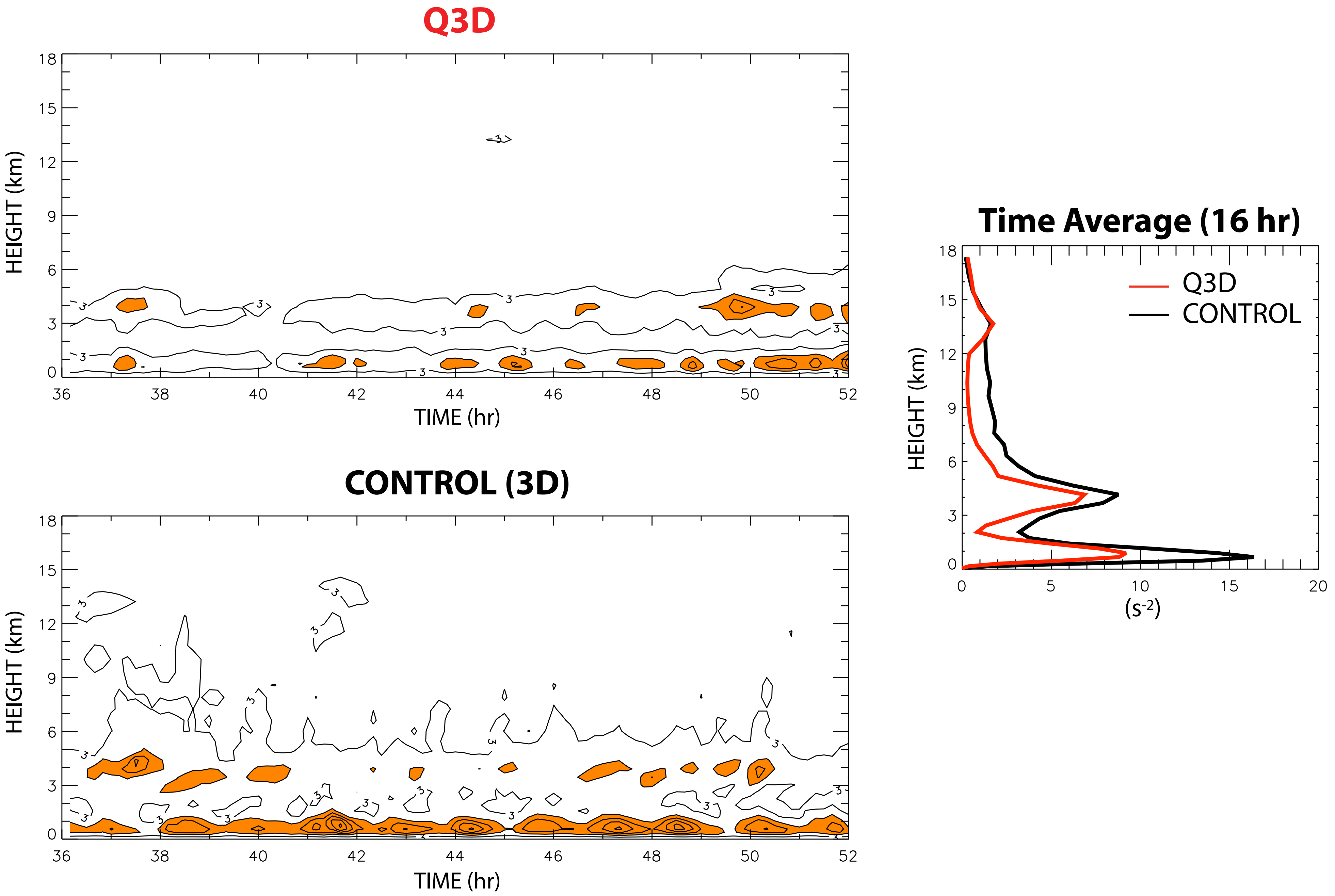


Time Average (16 hr)



FULL PRONOSTIC TEST

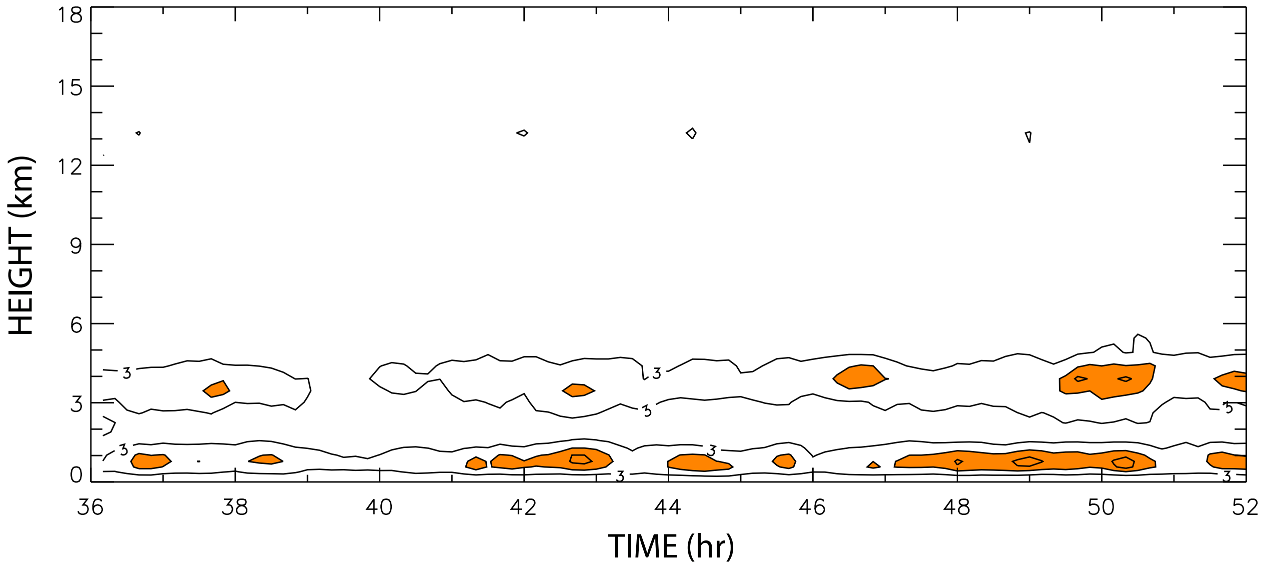
Y-array ***variance*** of ξ (10^{-6})



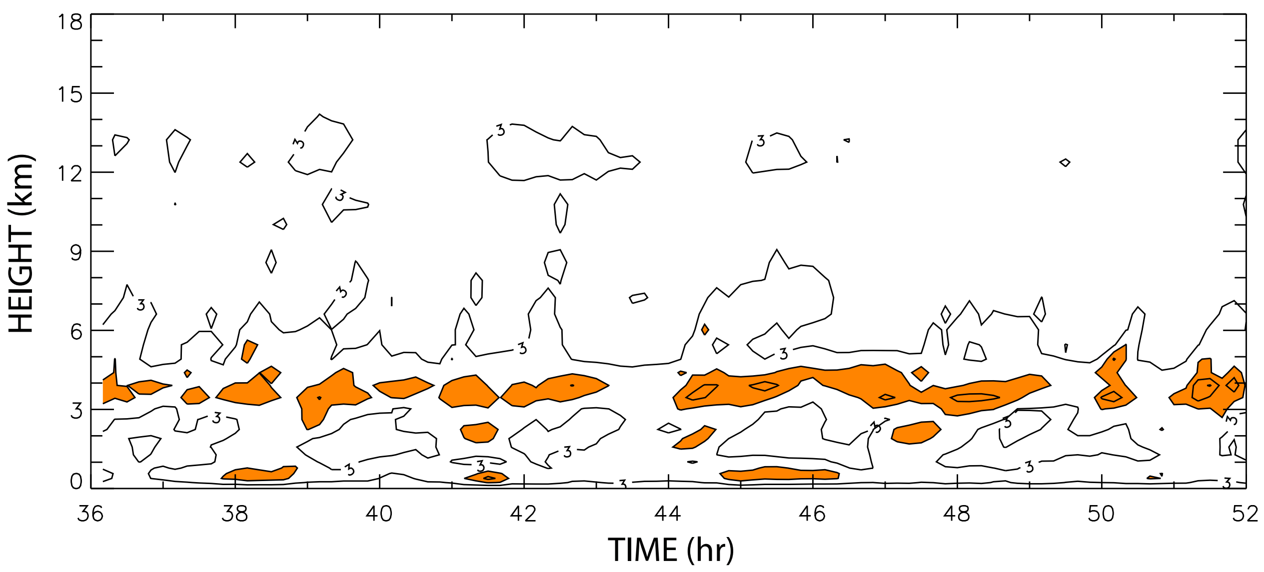
FULL PRONOSTIC TEST

X-array ***variance*** of η $\left(10^{-6}\right)$

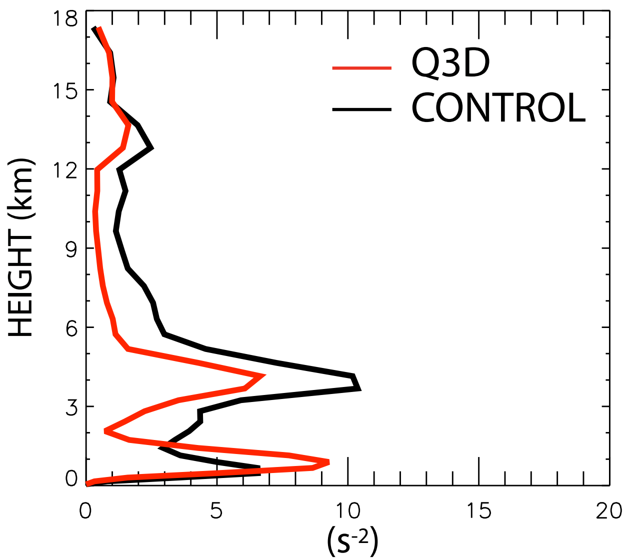
Q3D



CONTROL (3D)



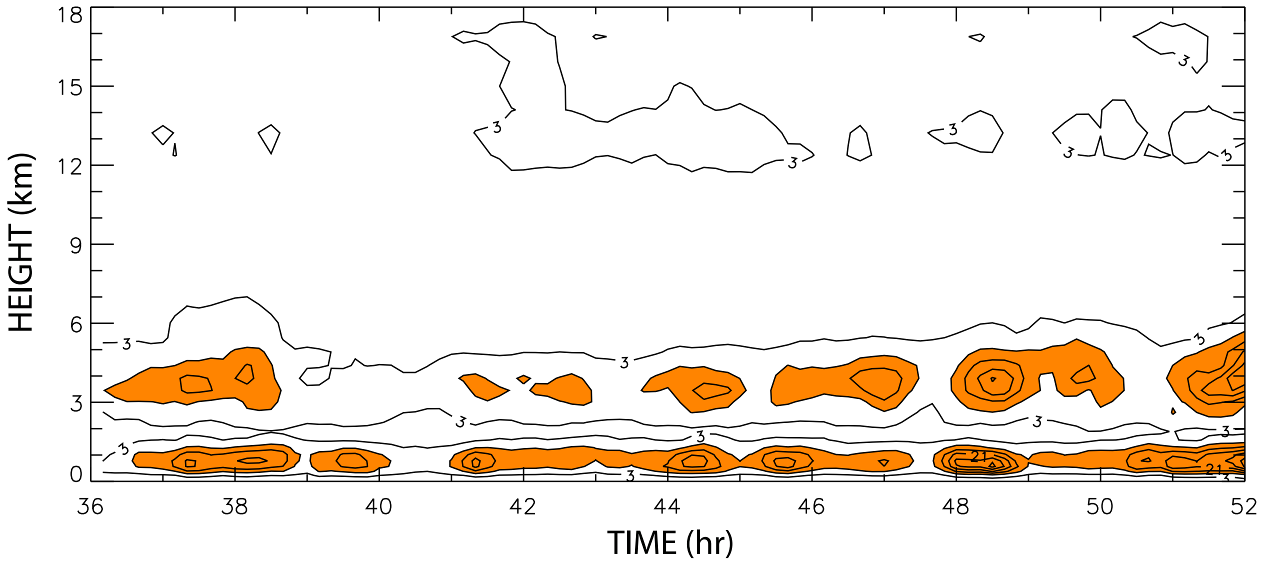
Time Average (16 hr)



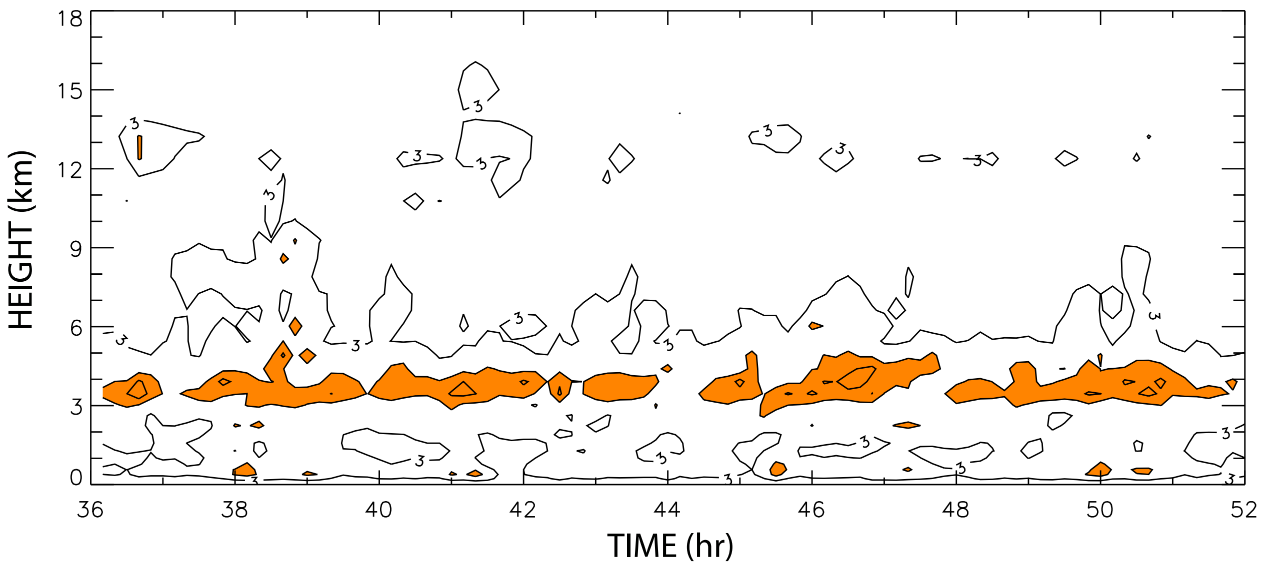
FULL PRONOSTIC TEST

Y-array ***variance*** of $\eta\ (10^{-6})$

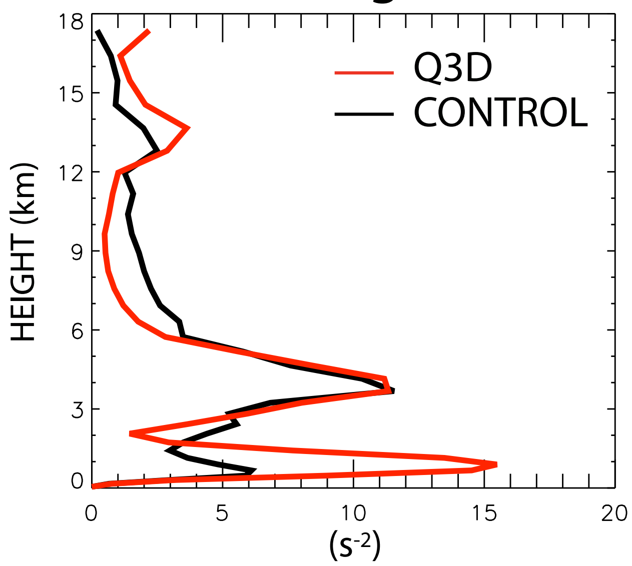
Q3D



CONTROL (3D)



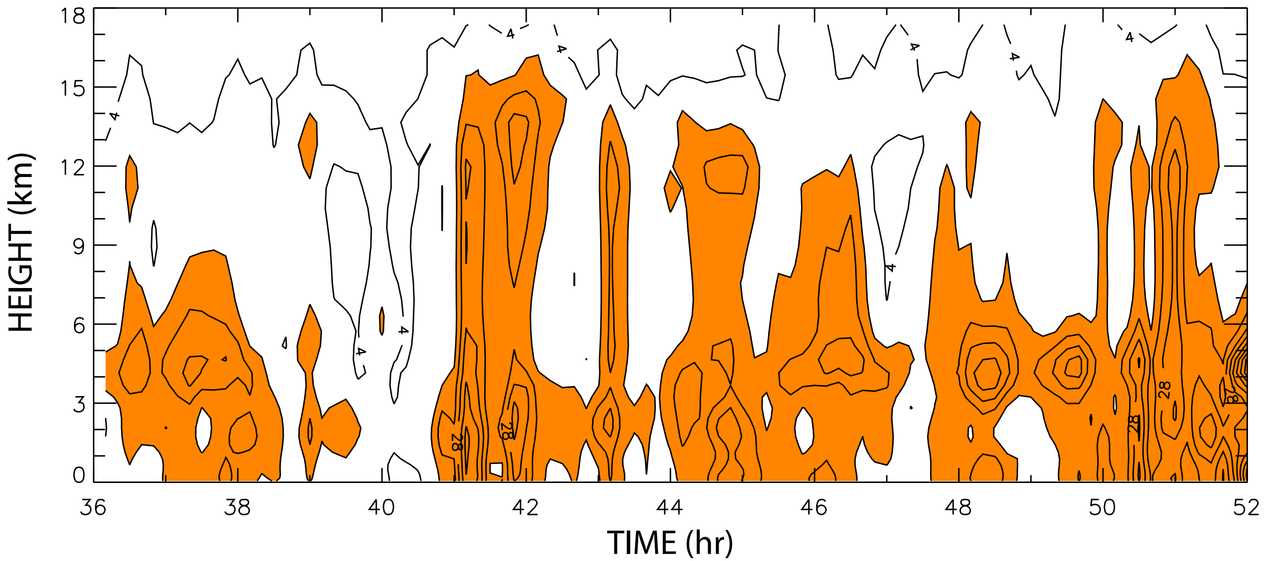
Time Average (16 hr)



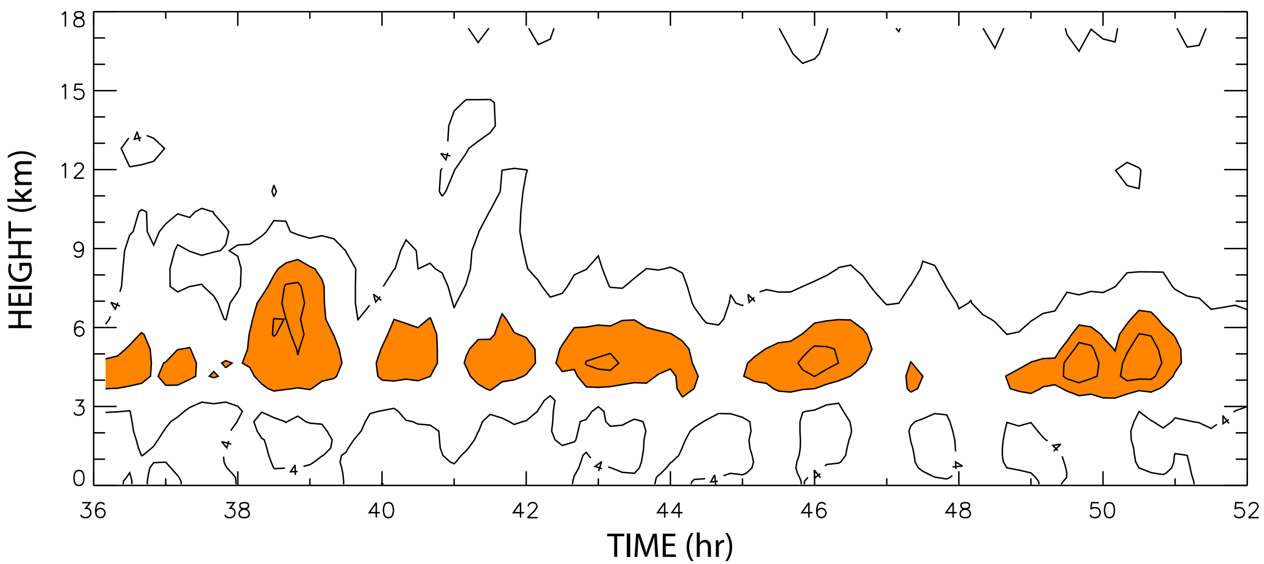
FULL PRONOSTIC TEST

Y-array ***variance*** of u

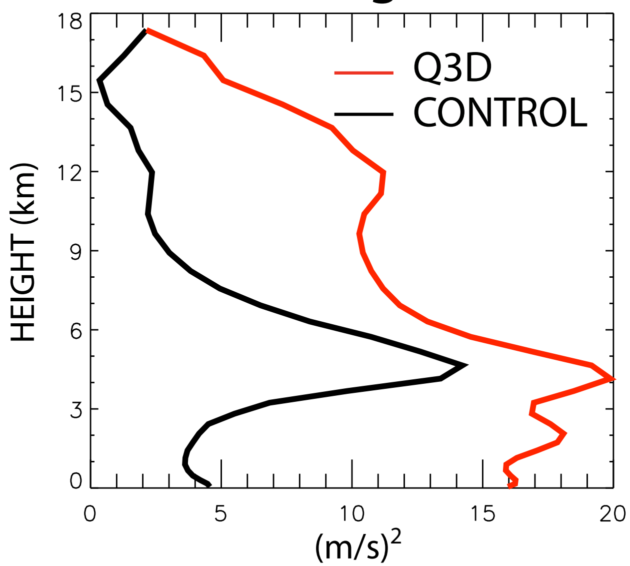
Q3D



CONTROL (3D)



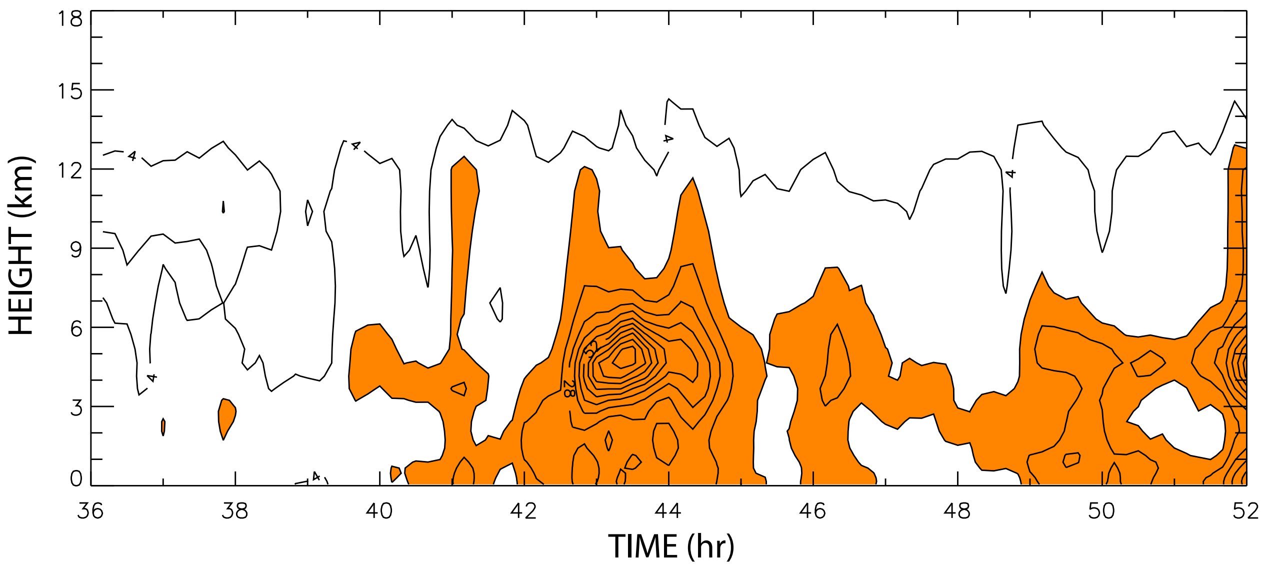
Time Average (16 hr)



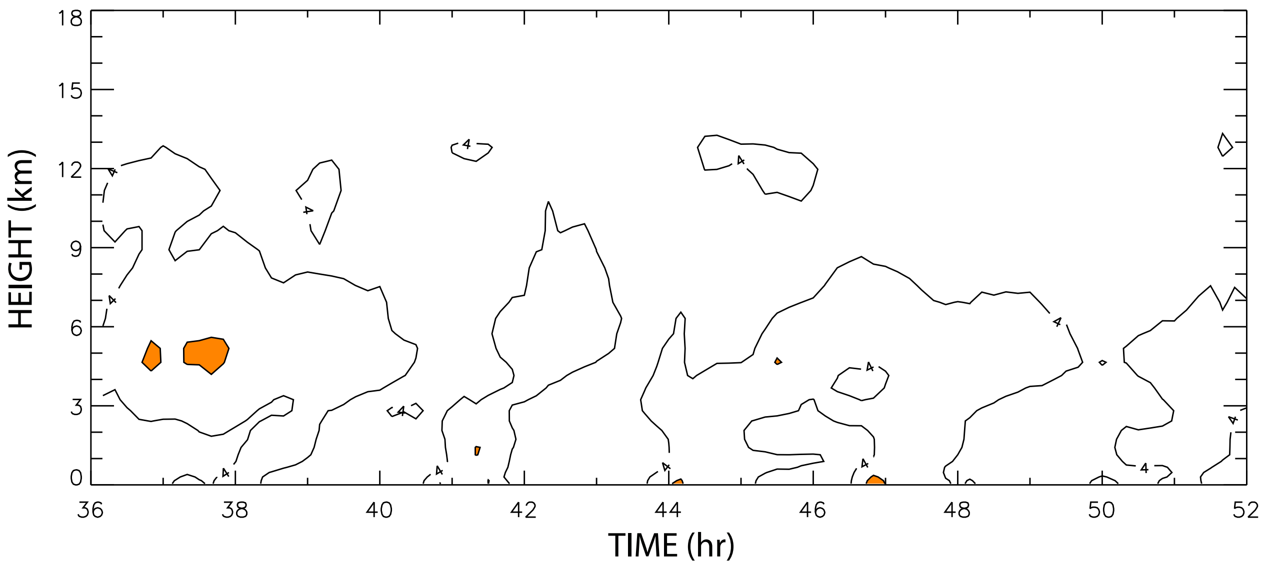
FULL PRONOSTIC TEST

X-array ***variance*** of v

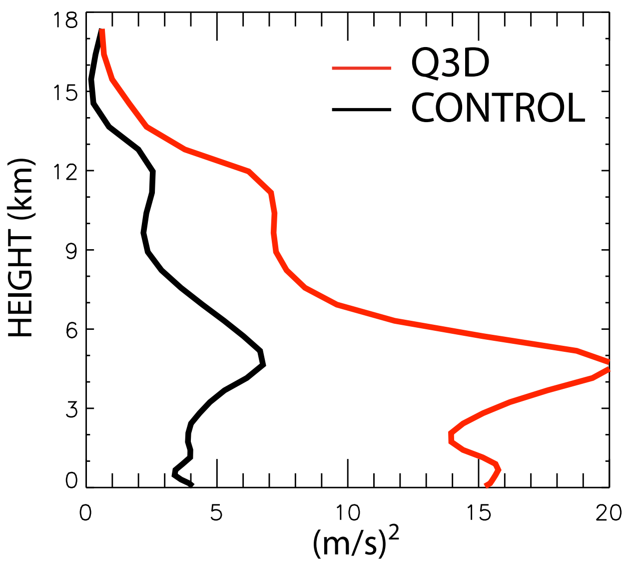
Q3D



CONTROL (3D)



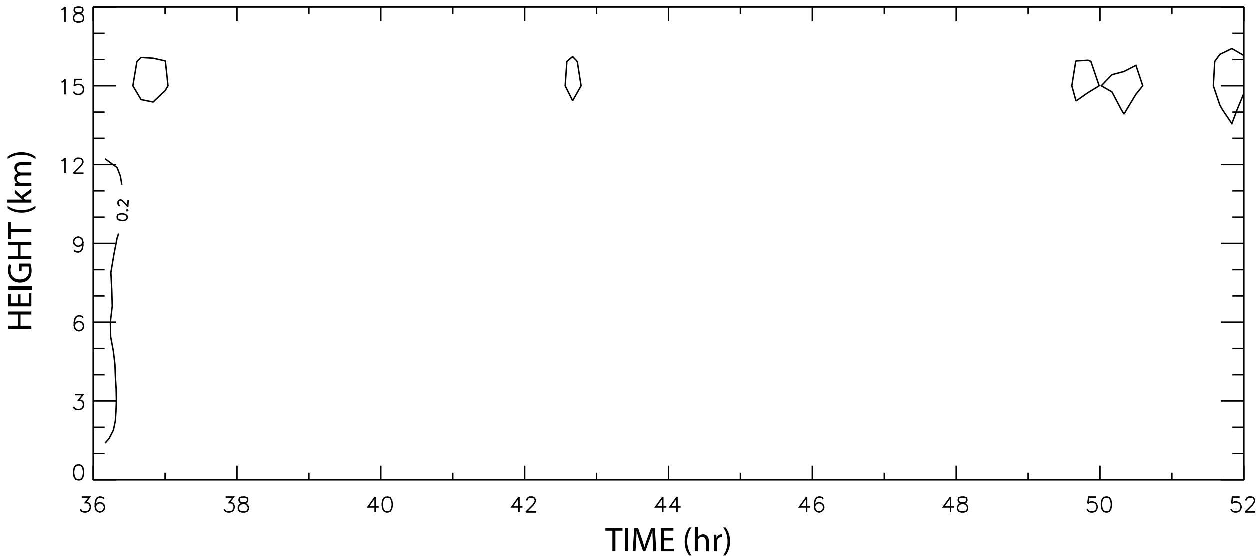
Time Average (16 hr)



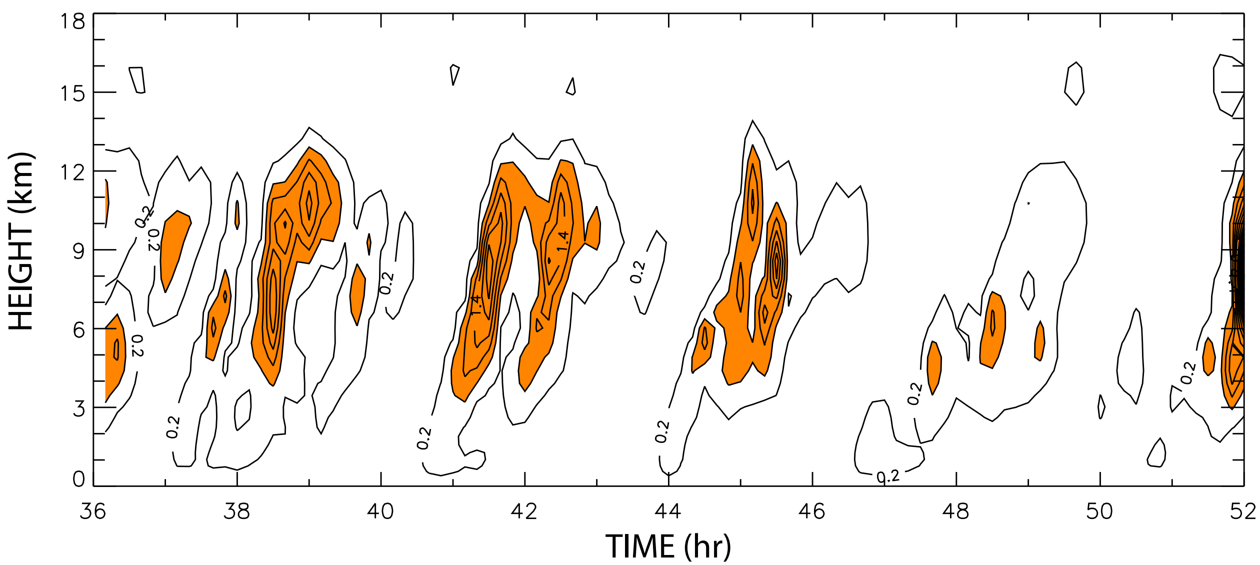
FULL PRONOSTIC TEST

X-array ***variance*** of w

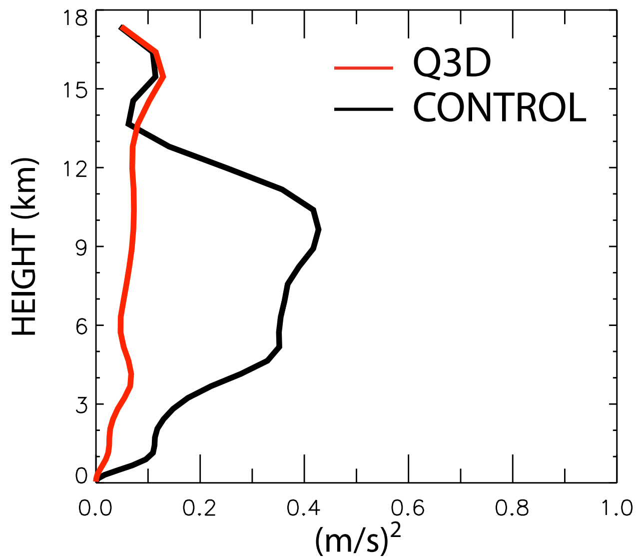
Q3D



CONTROL (3D)



Time Average (16 hr)



Sensitivity to the Horizontal Spectrum of Forcing

- Solution of the elliptic equation has the important role of determining the partition between the vertical and horizontal components of velocity.
- The partition crucially depends on the horizontal spectrum of the forcing (see below).

The 2D Boussinesq version of the w-equation: $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right) w = F\left(\equiv -\frac{\partial \eta}{\partial x}\right)$ (1)

where $\eta = \partial u / \partial z - \partial w / \partial x$ is the y-component of vorticity.

For a fixed vertical wavenumber m, let

$F = \int F(k,x)\,dk$, where k is the horizontal wavenumber.

From (1) and $\partial u / \partial x + \partial w / \partial z = 0$, we can show

$$\frac{\partial w}{\partial x} = -\int \frac{1}{k^2 + m^2} \frac{\partial}{\partial x} F(k,x)\,dk \qquad \frac{\partial u}{\partial z} = \int \left(\frac{m}{k}\right)^2 \frac{1}{k^2 + m^2} \frac{\partial}{\partial x} F(k,x)\,dk$$