Simulating a Mountain Wave with a Hybrid Vertical Coordinate Model: Experiments with an adaptive vertical grid

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Introduction

We have successfully simulated the growth of a mountain wave with our nonhydrostatic, hybrid vertical coordinate atmospheric model. The coordinate is terrain-following near the surface and transitions to a quasi-Lagrangian (θ) coordinate with height. Almost identical wave characteristics develop as when using a conventional height-based Eulerian (σ) coordinate, yet tracer advection is greatly improved with the quasi-Lagrangian coordinate.

At this point the model is unable to represent a fully developed breaking wave. The challenge for the θ-coordinate is that isentrope overturning cannot be represented because potential temperature always increases monotonically with height for such a coordinate. Despite this shortcoming, we had hoped that the model would give a reasonable solution for the flow associated with wave breaking. However, as the waves amplify, the spatial deformation of the isentropes and thus of the coordinate surfaces becomes large and the resulting gradients apparently become too sharp for the numerics to handle.

Here we present a possible solution to the problem of representing wave breaking using θ-coordinates. We smooth the coordinate surfaces before they become too spatially deformed. In the process, the coordinate becomes locally Eulerian and it is possible to represent isentrope overturning.

Review of the vertical coordinate

Following Konor and Arakawa (1997) we define the vertical coordinate as a prescribed function of height and potential temperature. Since our model is nonhydrostatic we use geometric height instead of pressure as the height metric.

The vertical coordinate (η) is defined as:

$$\eta = F(\theta, \sigma) = f(\sigma) + g(\sigma)\,\theta, \qquad (1)$$

where $\sigma = \dfrac{z - z_S}{z_T - z_S}$, z_S = surface height, and z_T = model top height.

The transition of η from terrain-following to potential temperature coordinates requires:

$$g(\sigma) \to 0 \quad \text{as}\ \sigma \to 0,$$

$$f(\sigma) \to 0 \ \text{and}\ g(\sigma) \to 1 \quad \text{as}\ \sigma \to 1.$$

Model initialization

The test case is the 11 January 1972 Boulder, Colorado windstorm. The setup for this 2D experiment was obtained from Doyle et al. (2000) which presents an intercomparison of various nonhydrostatic model simulations of the windstorm. The initial atmospheric condition is horizontally uniform and is based on the upstream Grand Junction, Colorado sounding shown in Figure 1. The Colorado Front Range profile is represented by a "witch of Agnesi" curve of height 2 km and half-width 10 km. The horizontal grid spacing is 1 km and the horizontal domain is 220 km wide with periodic boundary conditions. The model top is a rigid lid at a height of 25 km and is run with 125 levels. Figure 2 shows the initial vertical profile of the vertical coordinate and potential temperature. Above 10 km the coordinate is basically isentropic.

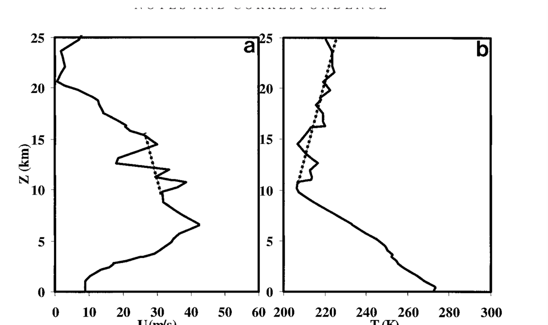

Figure 1: Initial zonal wind and temperature of sounding.

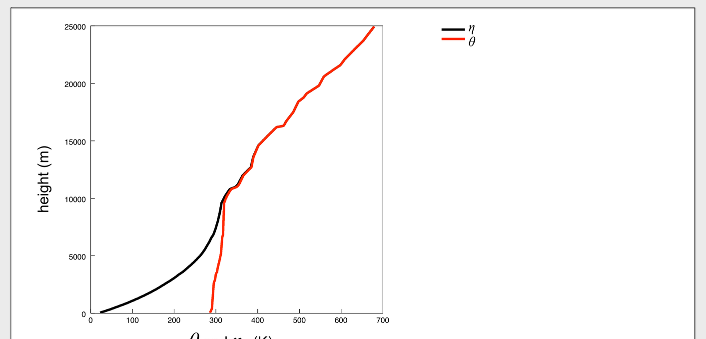

Figure 2: Vertical profile of vertical coordinate and potential temperature at x = 0, t = 0.

Improved tracer advection with the hybrid-coordinate

Before discussing the adaptive vertical grid we show the advantage of the hybrid-coordinate over the pure σ-coordinate in simulating passive tracer advection. The tracer is initialized with a value of unity within selected bands of potential temperature and zero elsewhere. Since the process is adiabatic, the tracer should remain perfectly correlated with potential temperature. Figure 3 shows that the hybrid-coordinate, in the region above 10 km where the coordinate is potential temperature, does much better at maintaining this correlation. There is no vertical advection in the quasi-Lagrangian coordinate, and therefore no dispersion error as occurs with the σ-coordinate. This is a well known advantage of quasi-Lagrangian coordinates.

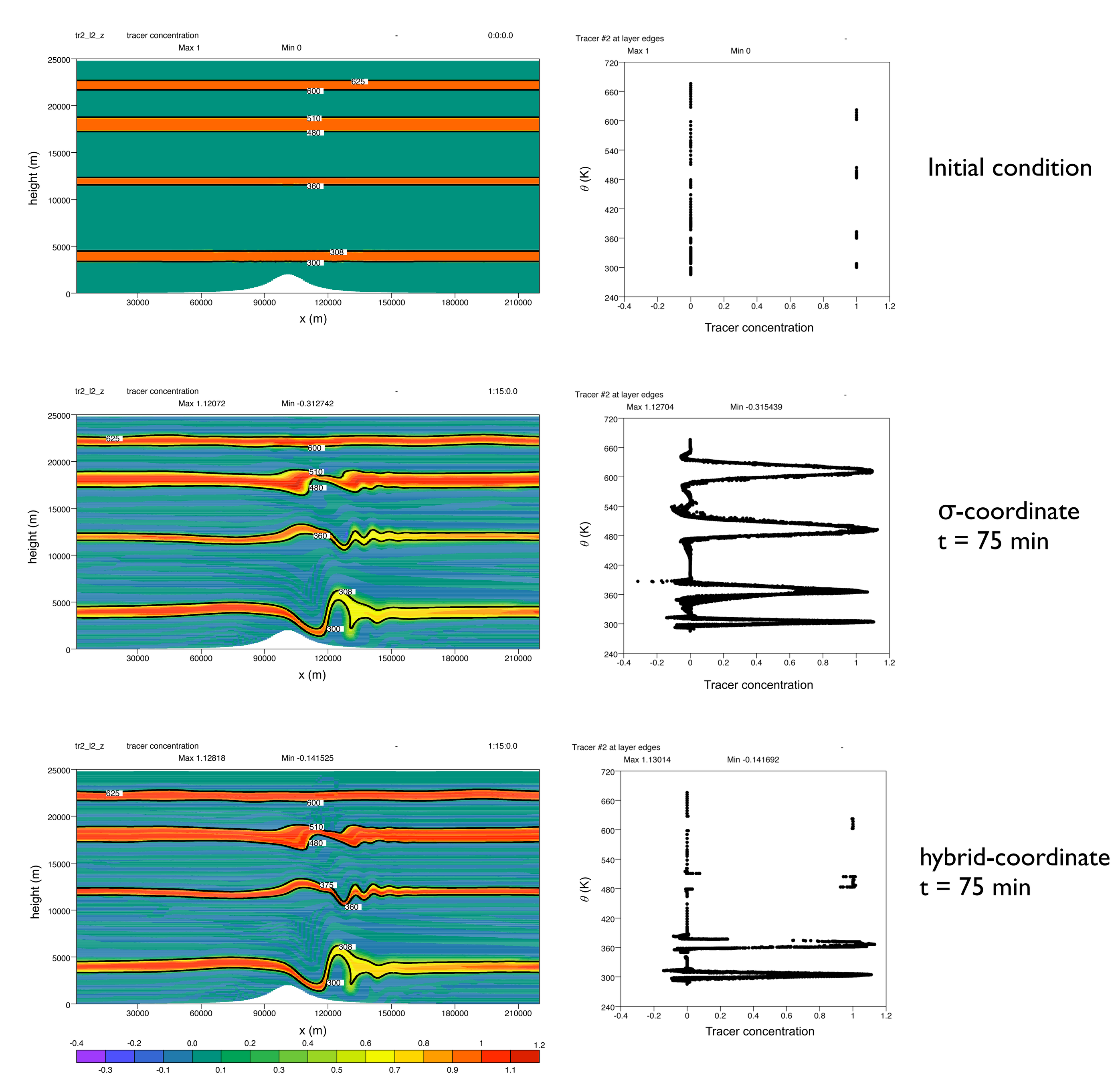

Figure 3: Passive tracer concentration (left column) and scatter plots of potential temperature and tracer concentration (right column). At t = 0 (top row), t = 75 min σ-coordinate (middle row), and t = 75 min hybrid-coordinate (bottom row).
Note: Adaptive grid not used in this simulation.

References

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An adaptive vertical grid

We are experimenting with horizontally smoothing the vertical coordinate surfaces when the curvature, due to wave amplification, reaches a specified amount. For now, the degree of curvature is determined by the second spatial derivative of geopotential height. The smoothing process is equivalent to "perforating" the coordinate surfaces to allow mass to flow from one layer to the next, resulting in a vertical mass flux. Therefore where smoothing occurs, the benefit of the quasi-Lagrangian coordinate is partially lost, but in a localized and intermittent manner. Other models in which coordinate smoothing is used include He (2002) and Zangl (2007).

The amplification of the mountain wave in the lower stratosphere is shown in Figure 4. Red curves indicate the position of model coordinate surfaces, and black curves are isolines of constant $F(\theta,\sigma)$ defined in Equation (1). These curves are collocated when the vertical coordinate is exactly defined as in Equation (1), and they deviate only when smoothing takes place which begins at about t = 40 min.

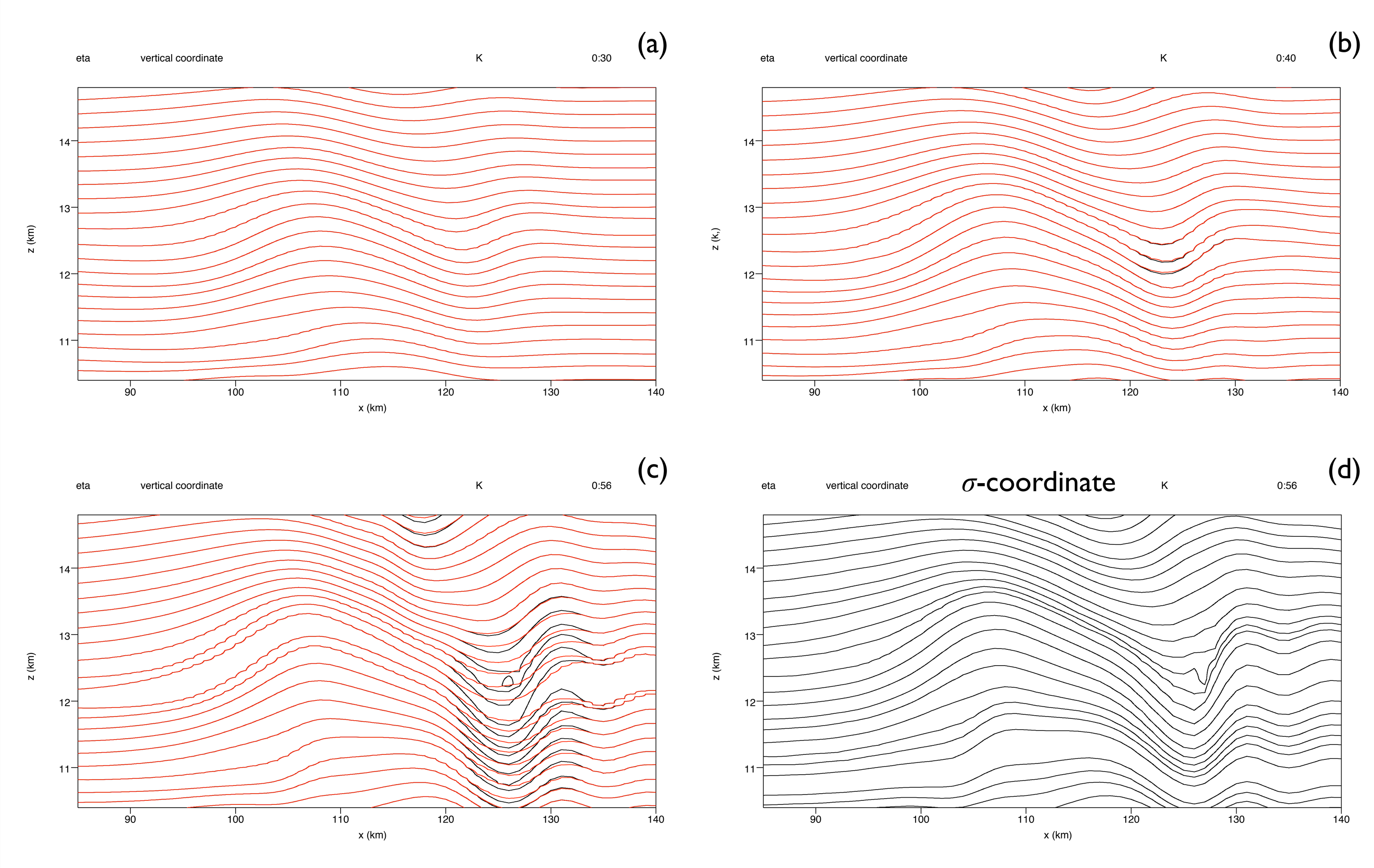

Figure 4: Position of coordinate surfaces (red curves) and isolines of $F(\theta,\sigma)$ defined in Equation (1) (black curves) at: a) t = 30 min, b) t = 40 min, c) t = 56 min, and d) t = 56 min (σ-coordinate results shown for comparison).

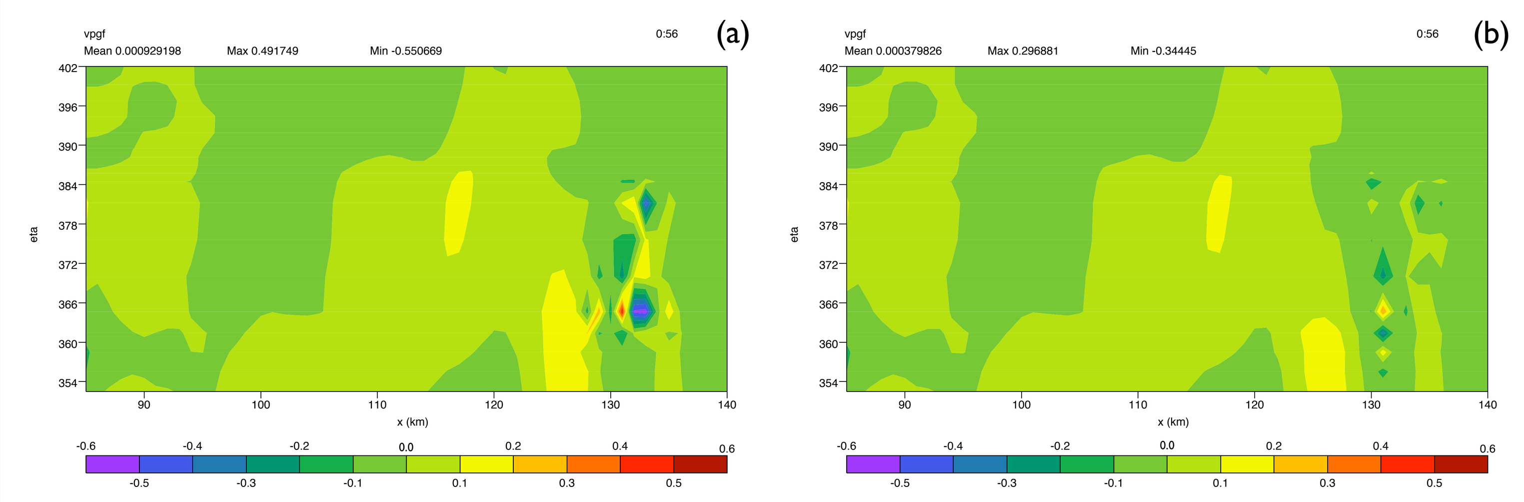

Figure 5: Departure from hydrostatic balance given by discrete form of $\left(-\frac{1}{\rho}\frac{\partial p}{\partial z}-g\right)$ at t = 56 min: a) without smoothing, and b) with smoothing (adaptive grid).

The first sign of trouble in the model run appears in the "departure from hydrostatics" term of the vertical momentum equation given by $\left(-\frac{1}{\rho}\frac{\partial p}{\partial z}-g\right)$. This term is plotted above in Figure 5. It is seen in the plot on the right that when smoothing is applied, the severity of the noise is reduced, but at this point not eliminated. Further work is required to obtain better results and to capture the entire wave breaking process.

Conclusion

The hybrid-coordinate has demonstrated its advantage over pure σ-coordinates in terms of the vertical transport of a passive tracer. This is a desirable feature for the planned development of a cloud model where accurate water transport is important.

The adaptive grid technique shows promise as a method to allow localized isentrope overturning while retaining the quasi-Lagrangian nature of the coordinate throughout most of the domain.