Inclusion of the surface topography into the vector vorticity equation model (VVM)

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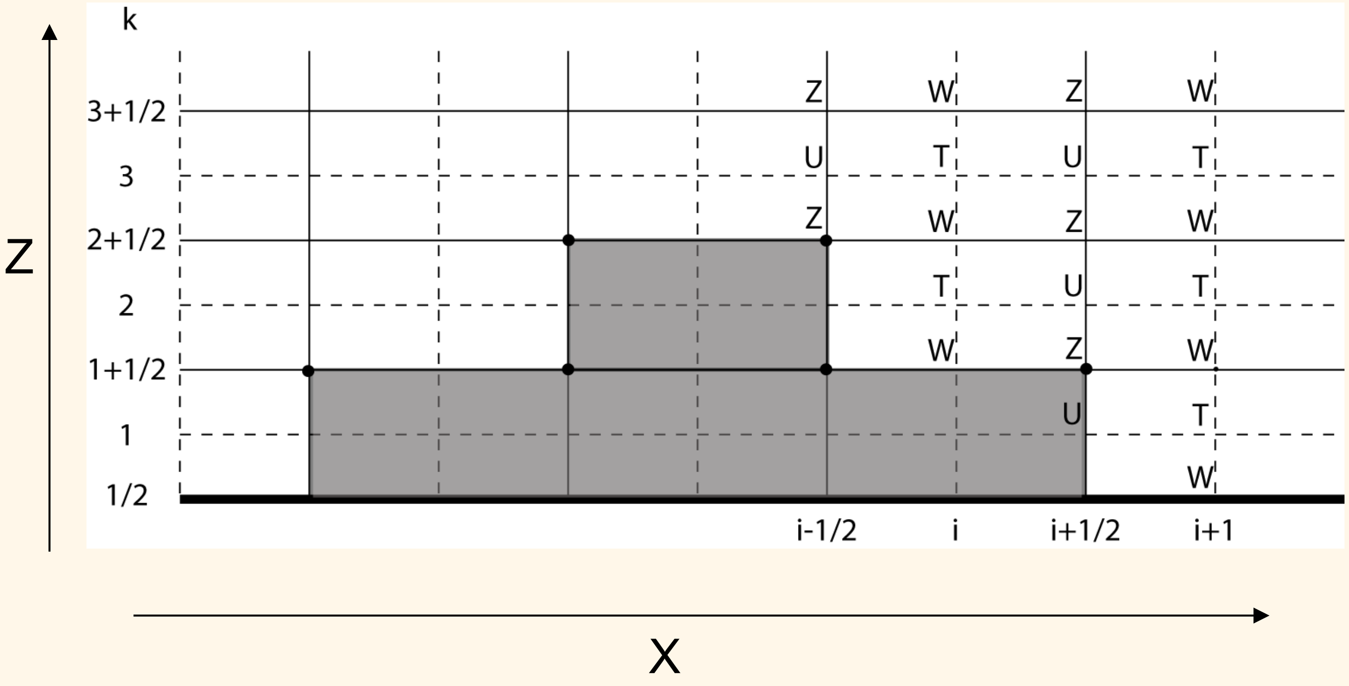
The block mountain apporach

- Rectangular blocks with mountain surface fixed at coordinate surface.

- The kinematic boundary conditions are satisfied with assigning proper computational boundary condition for voritcity.

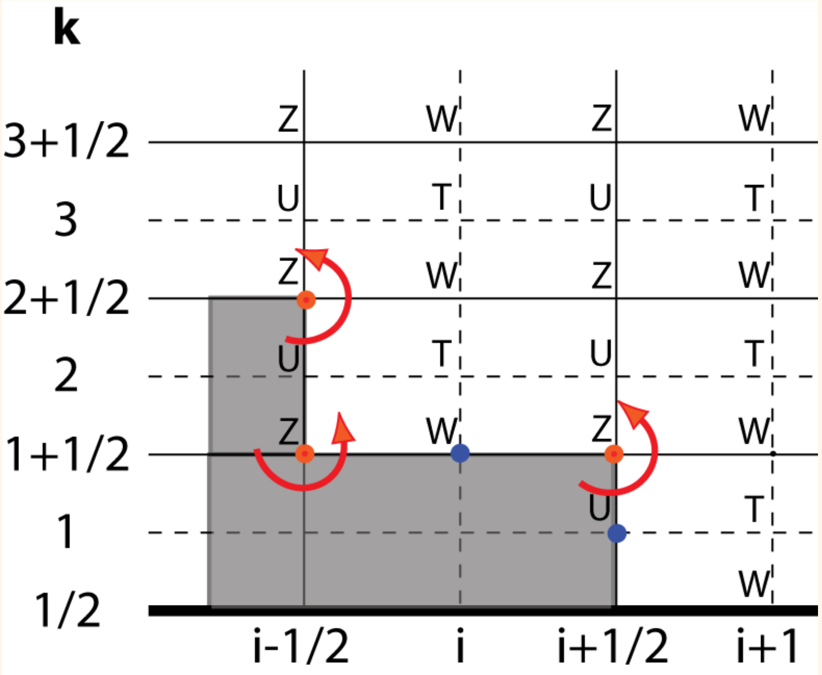


Z : vorticity
T : temperature
U,W: velocity

Determining the vorticity at the corners of the topography

- The strength of the vorticity at the corners is determined through vorticity definition.

$$\boxed{\eta_b = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \qquad u_b = w_b = 0}$$



$$(\eta_b)_{i,1+1/2} =$$

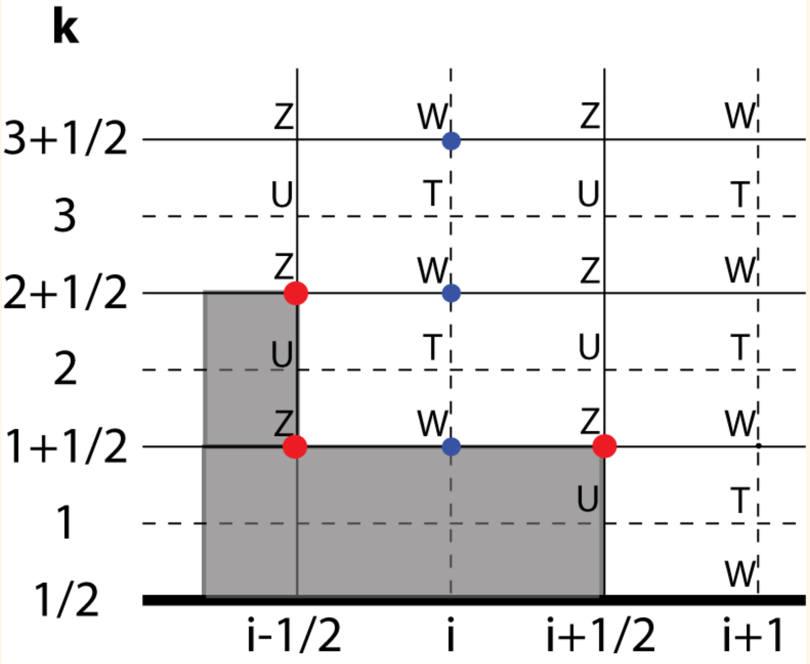
$$\frac{u_{i,2} - \underline{(u_b)_{i,1}}}{\Delta z} - \frac{w_{i+1/2,1+1/2} - \underline{(w_b)_{i-1/2,1+1/2}}}{\Delta x}$$

$$(u_b)_{i,1} = (w_b)_{i-1/2,1+1/2} = 0$$

Solving the relaxed w-equation

- Solving the relaxed w-equation with the addition of vorticies at the corners

$$\mu\frac{\partial w}{\partial t}+\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)w+\frac{\partial}{\partial z}\left[\frac{1}{\rho_0}\left(\frac{\partial}{\partial z}\rho_0 w\right)\right]=-\frac{\partial\eta}{\partial x}+\frac{\partial\xi}{\partial y}$$



First term of RHS=

$$-\left\{\frac{\eta_{i,k+1/2}-\eta_{i-1,k+1/2}}{\Delta x}\right\}$$

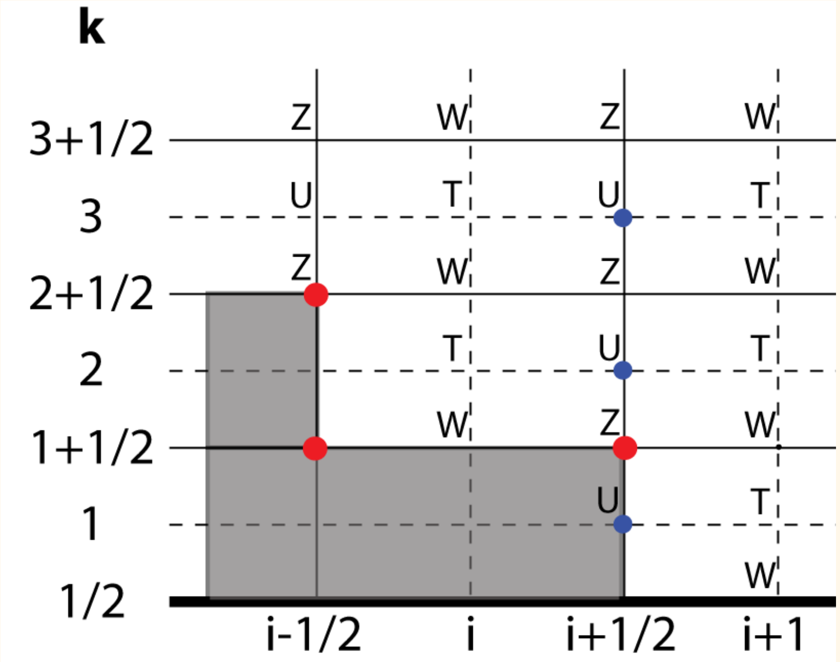
$$-\left\{\frac{\eta_{i,2+1/2}-\underline{(\eta_b)_{i-1,2+1/2}}}{\Delta x}\right\}$$

$$-\left\{\frac{\underline{(\eta_b)_{i,1+1/2}}-\underline{(\eta_b)_{i-1,1+1/2}}}{\Delta x}\right\}$$

Update horizontal velocity

- Update horizontal velocity with the addition of vorticity and the updated w.

$$u = \int_{z_t}^{z} \left(\frac{\partial w}{\partial x} + \eta \right) dz + u_T(x,y,t)$$



$$u_{i,k} =$$

$$u_{i,k+1} - \left[\frac{w_{i+1/2,k+1/2} - w_{i-1/2,k+1/2}}{\Delta x} + (\rho_0 \eta)_{i,k+1/2} \right] \cdot \Delta z$$

$$u_{i,2} - \left[\frac{w_{i+1/2,1+1/2} - w_{i-1/2,1+1/2}}{\Delta x} + \underline{(\rho_0 \eta_b)_{i,1+1/2}} \right] \cdot \Delta z$$

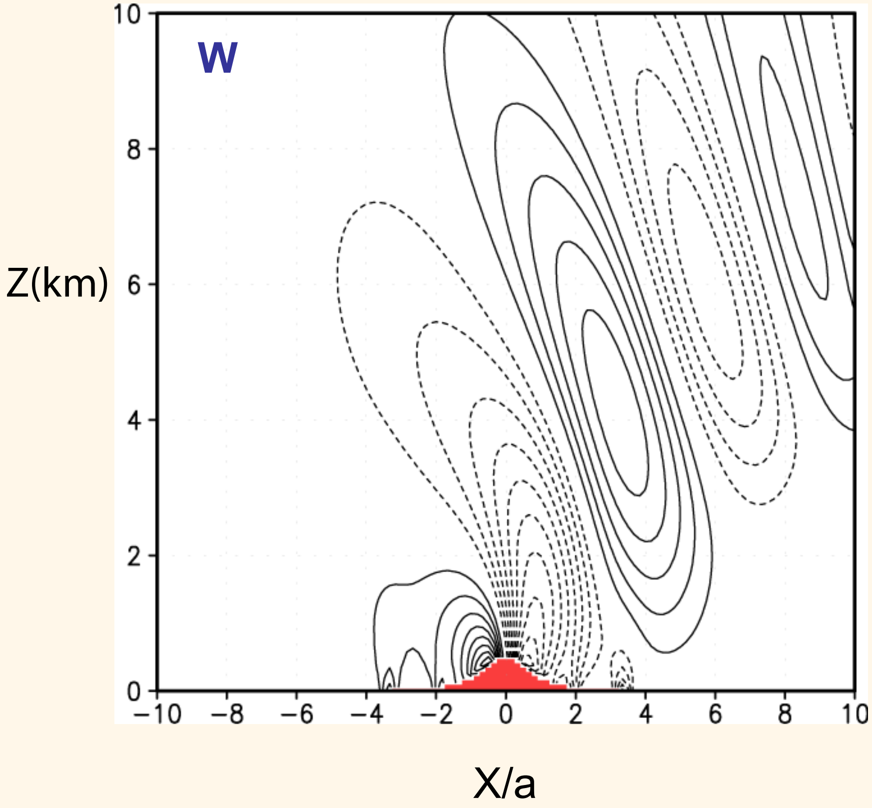
- ***The kinematic boundary conditions are satisfied with the addition of the voritcity.***

Validation of model performance

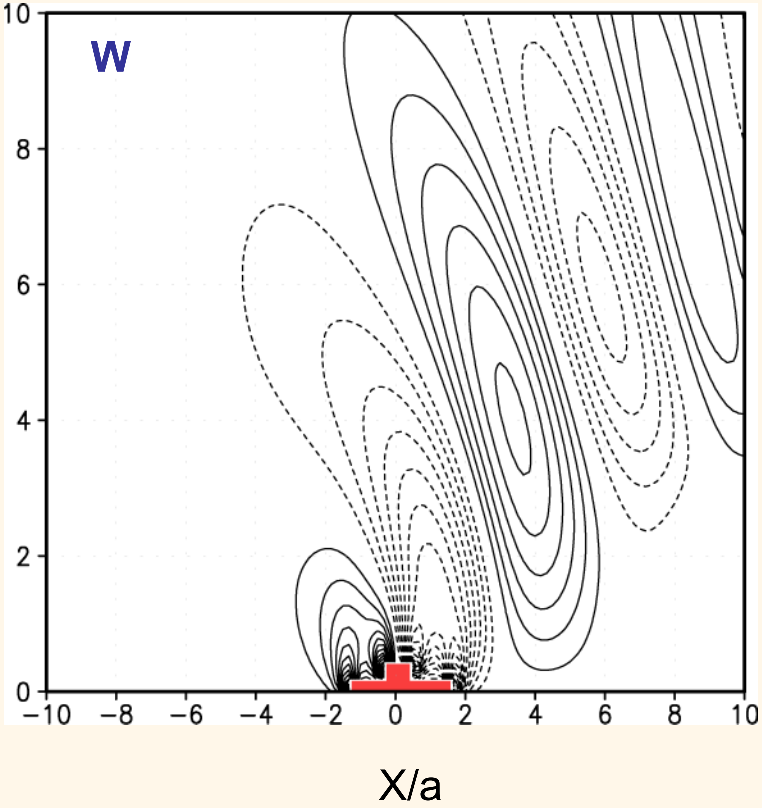
1. Idealized 2D mountain wave.
2. Boulder downslope windstorm.
3. Orographic precipitation over a ridge.

Idealized 2D mountain wave

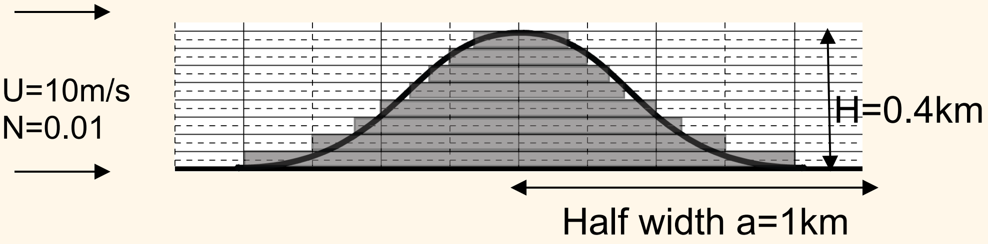
- A block representation of bell-shaped mountain (Na/U=1) is introduced.
- The mountain wave generated by the block mountain is reasonable.

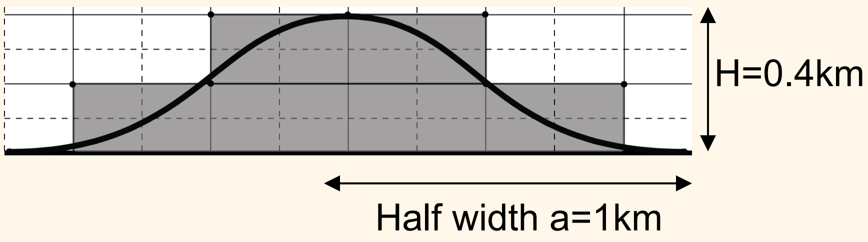


Smooth mountain shaped (H=8dz)



Rough mountain shape (H=2dz)

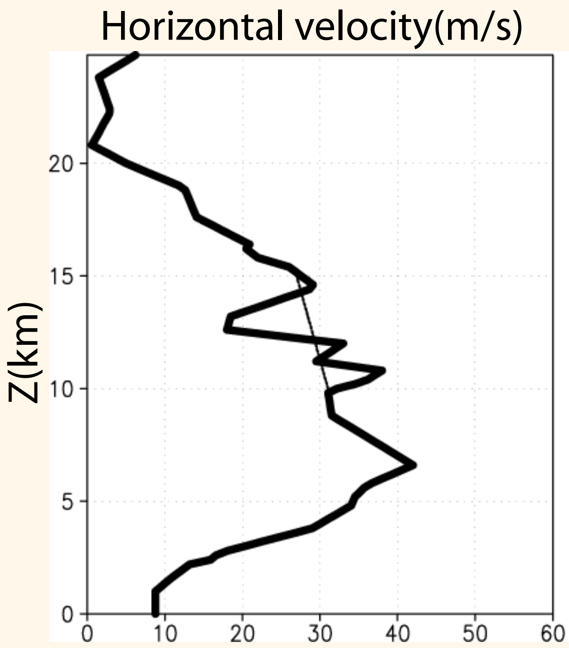


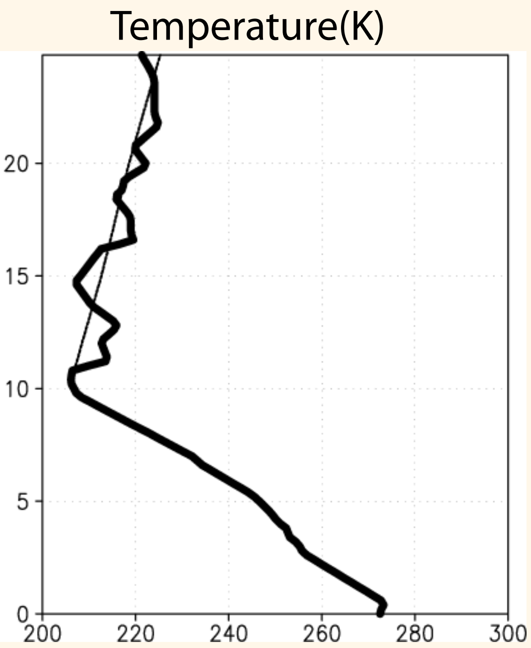


Boulder wind storm case

Initial conditions

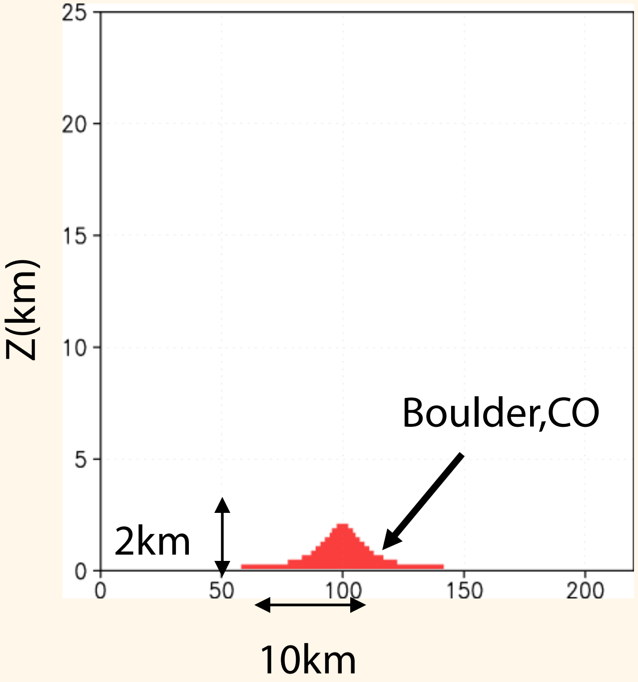
- Based on upstream Grand Junction, CO sounding for 1200 UTC 11 January, 1972





Model configuration

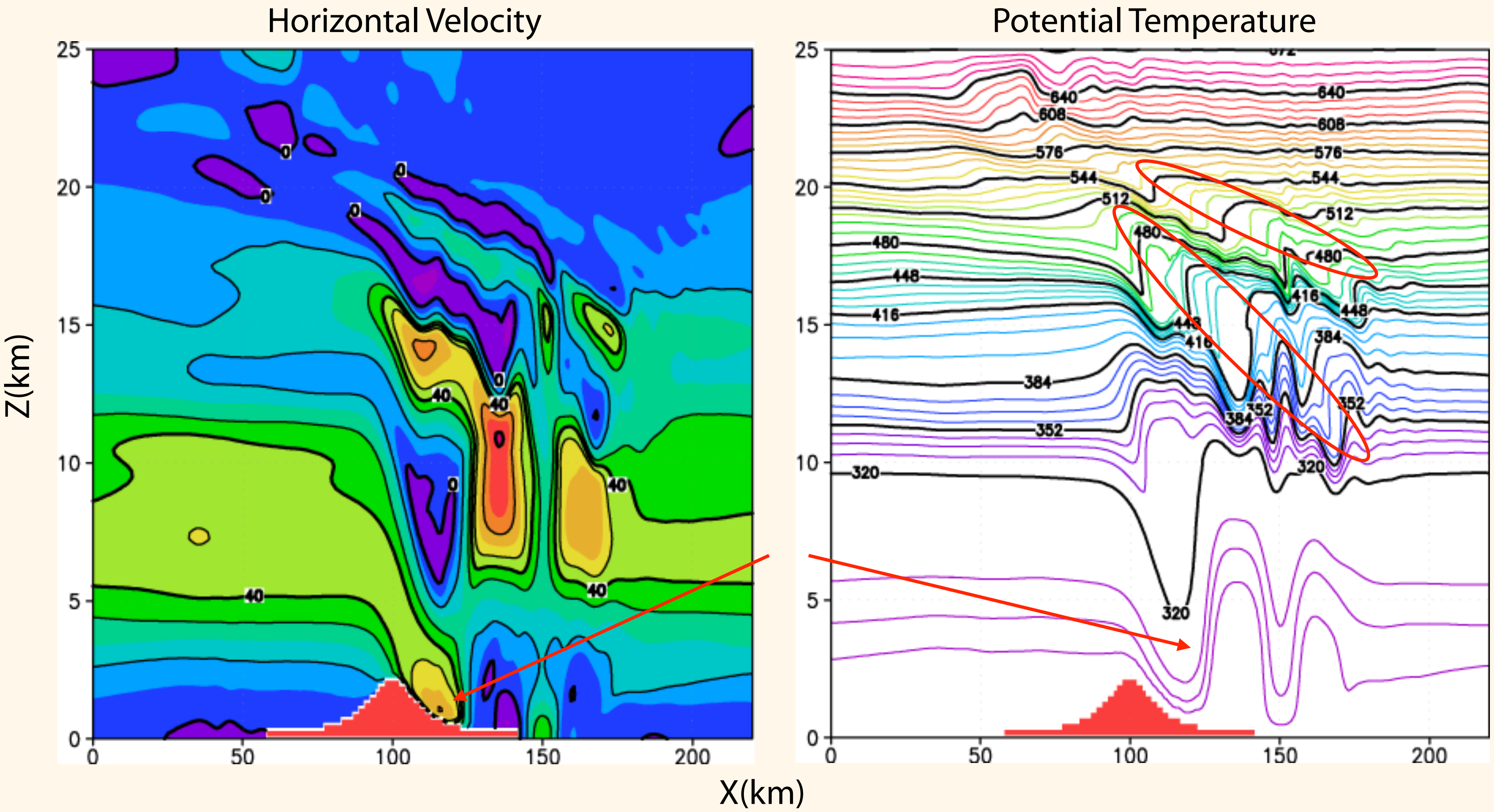
- 2D with no physics
- Periodic boundary
- Bell shaped mountain
- Dx=1000m,dz=200m



The experiment setup follows Doyle et al. (2000).

Boulder windstorm case VVM results

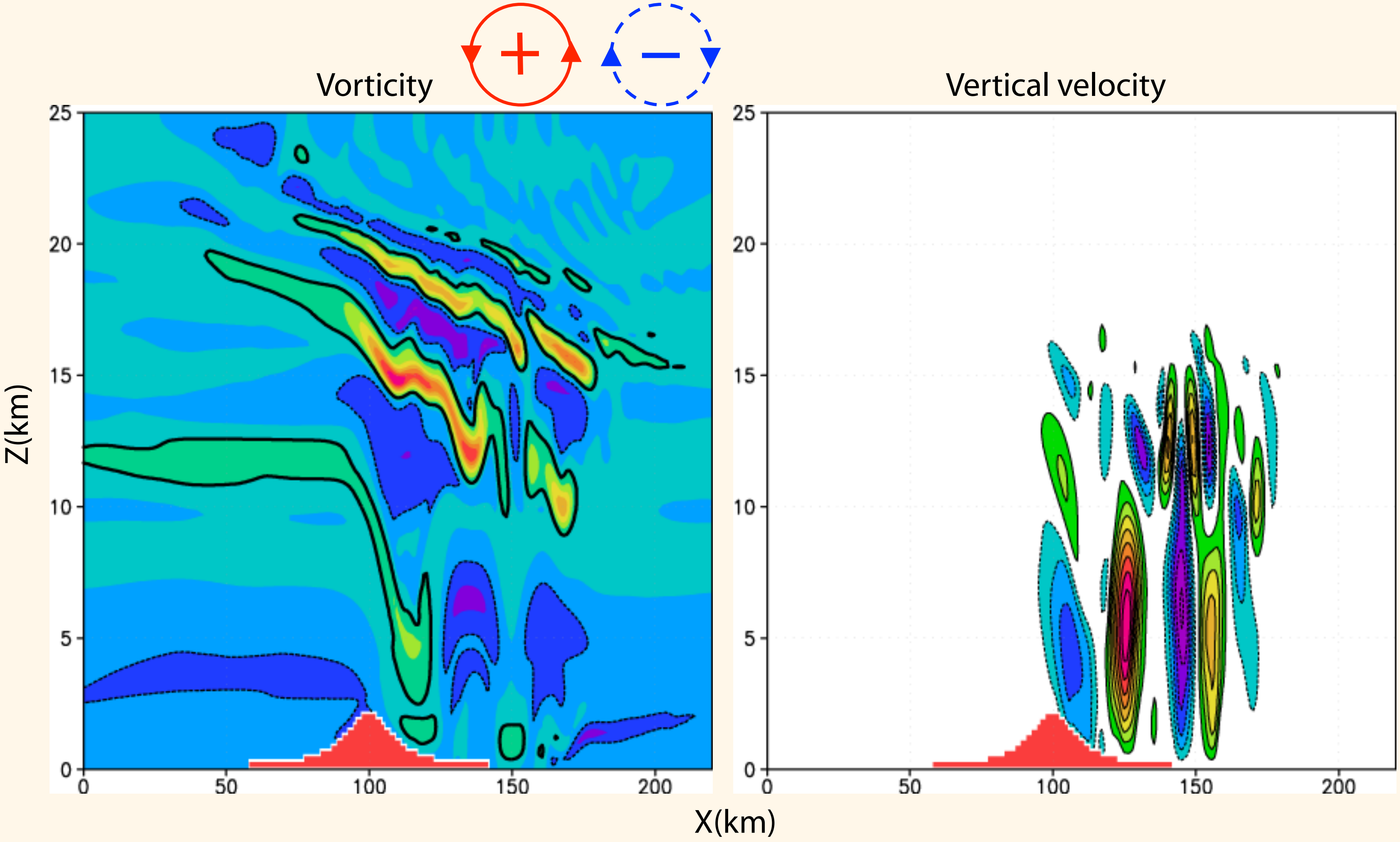
- VVM captures the upper level wave breaking and hydraulic at downstream region very well.
- The downslope wind exceeds 64m/s.


Horizontal Velocity

Potential Temperature

Boulder windstorm case VVM results

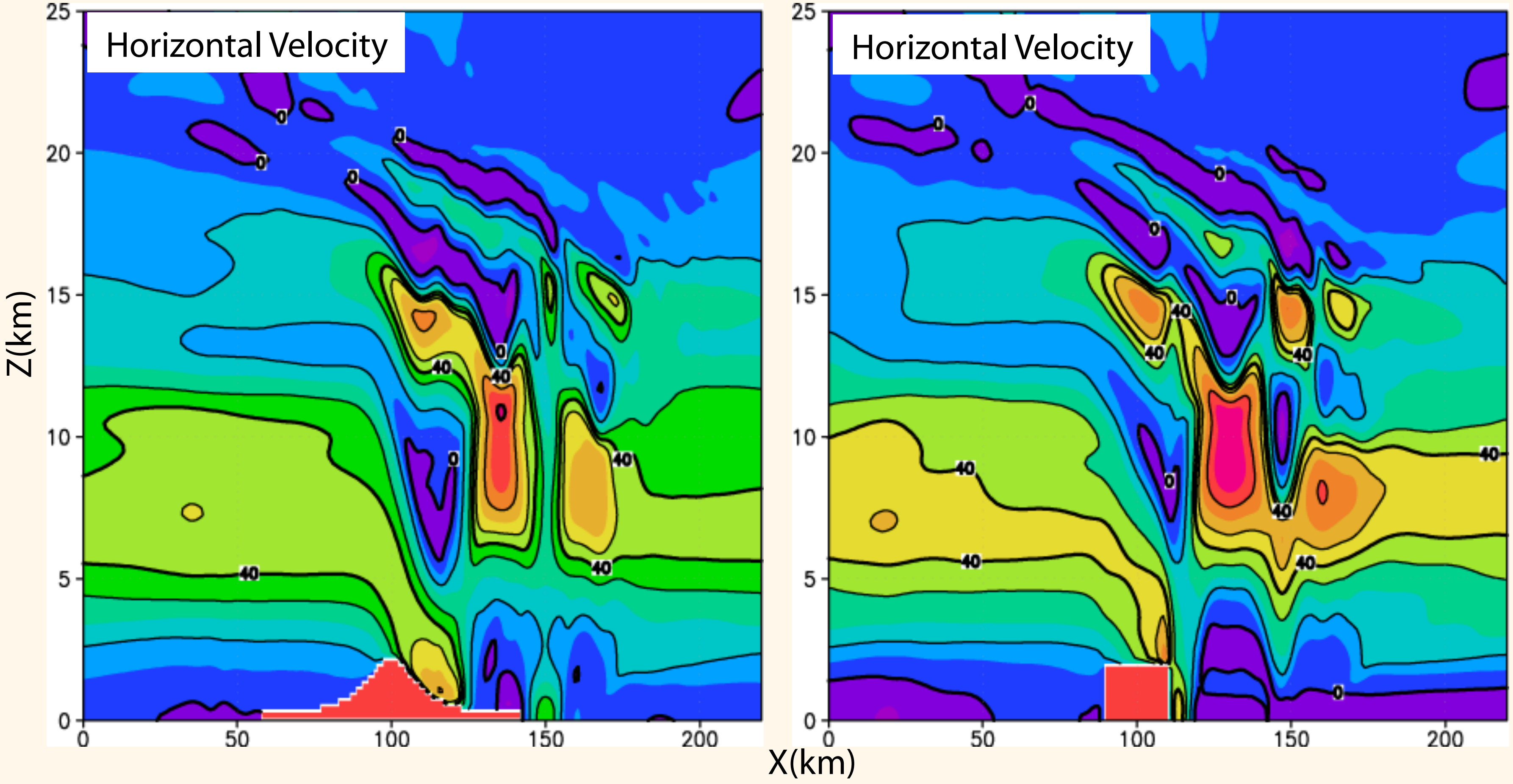
- Vorticity fields in VVM suggest the upstream coherent structure for the enhancement of downslope wind.



Boulder windstorm case sensitivity experiments

Stress test for steep mountain shape

- VVM handles steep topography without problem.
- Mountain shape determines the distribution of surface wind.



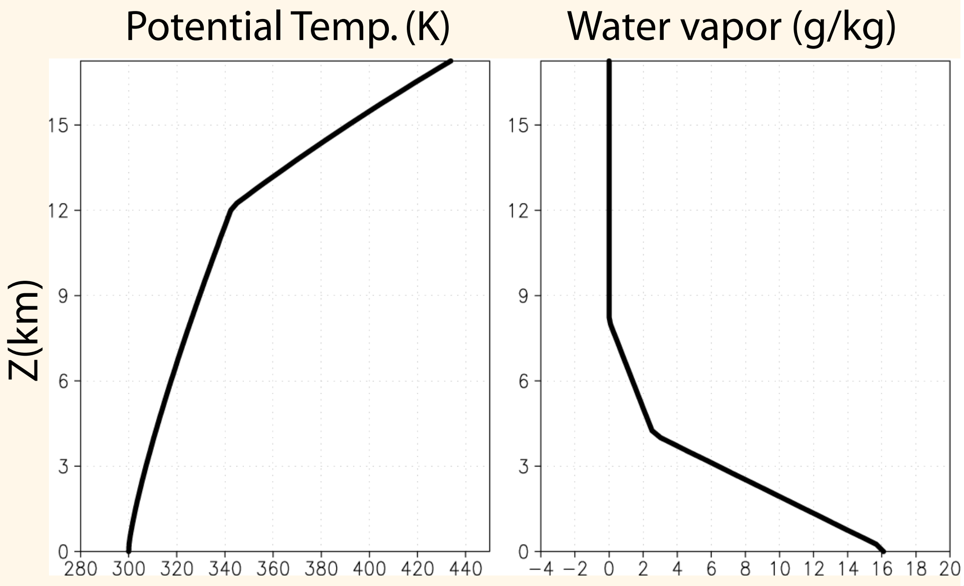
Control

Block shape mountain

Orographic precipitation over a ridge

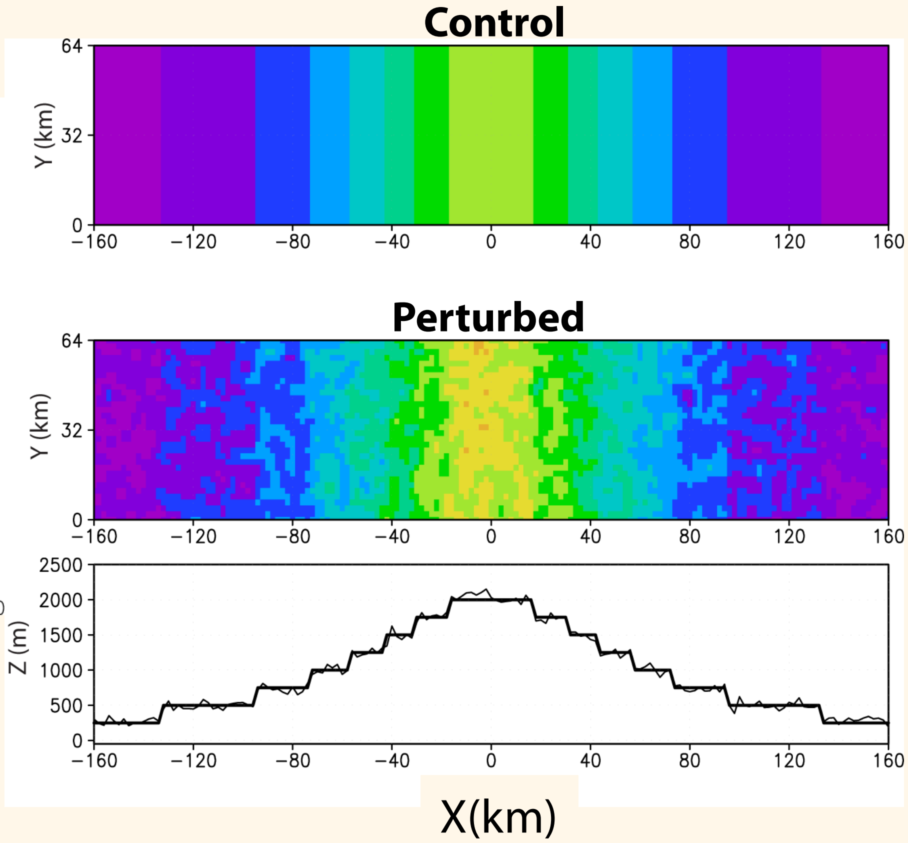
Initial conditions

- Typical conditions in the Midwestern United States during the spring (Wiseman and Klemp 1982)



Model configuration

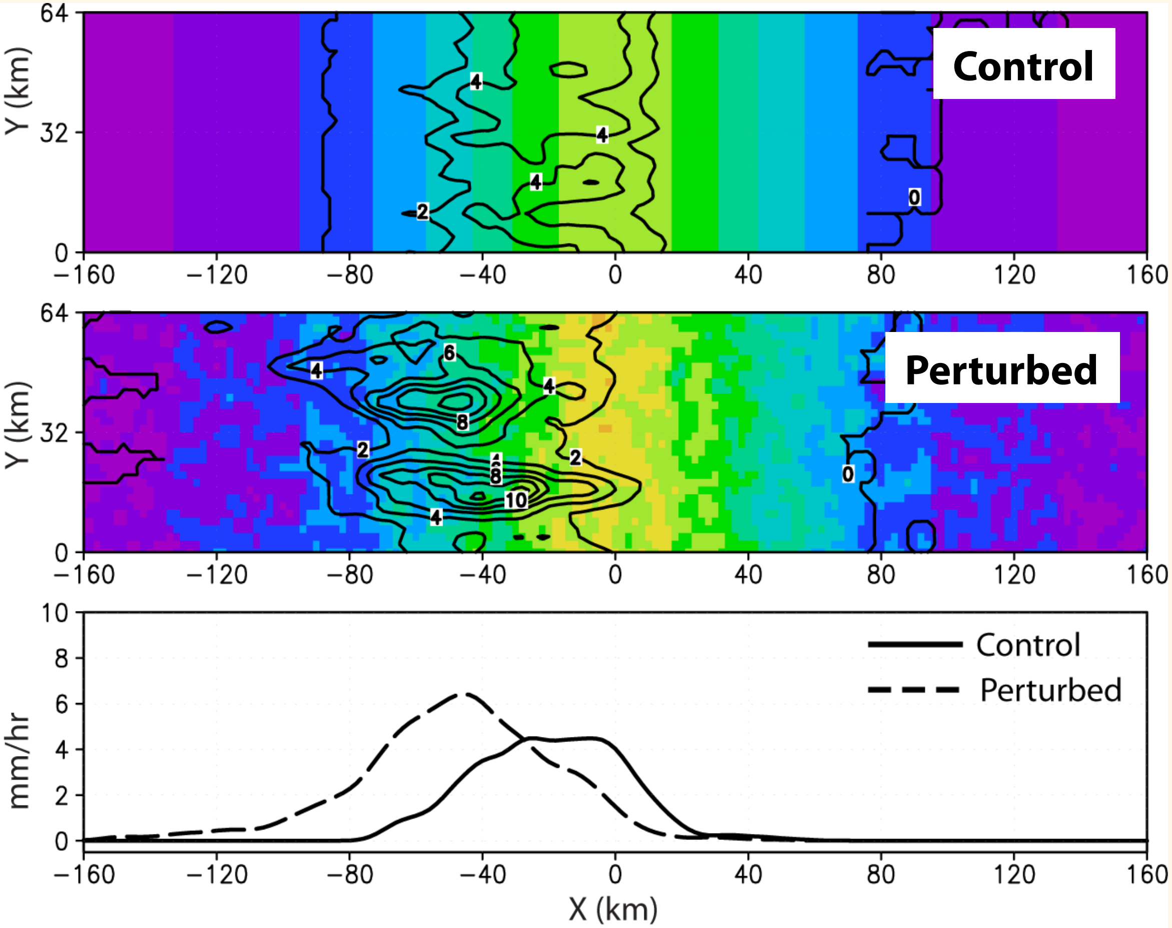
- 3D with physics (no radiation).
- Bell shaped mountain uniform in y direction.
- Randomly perturbed mountain height (2dz).
- Dx=2000m,dz=250m.



Orographic precipitation over a ridge

- Precipitation falls over the windward slope and the mountain crest.
- Precipitation tends to be stronger with localized band structure in perturbed exp.

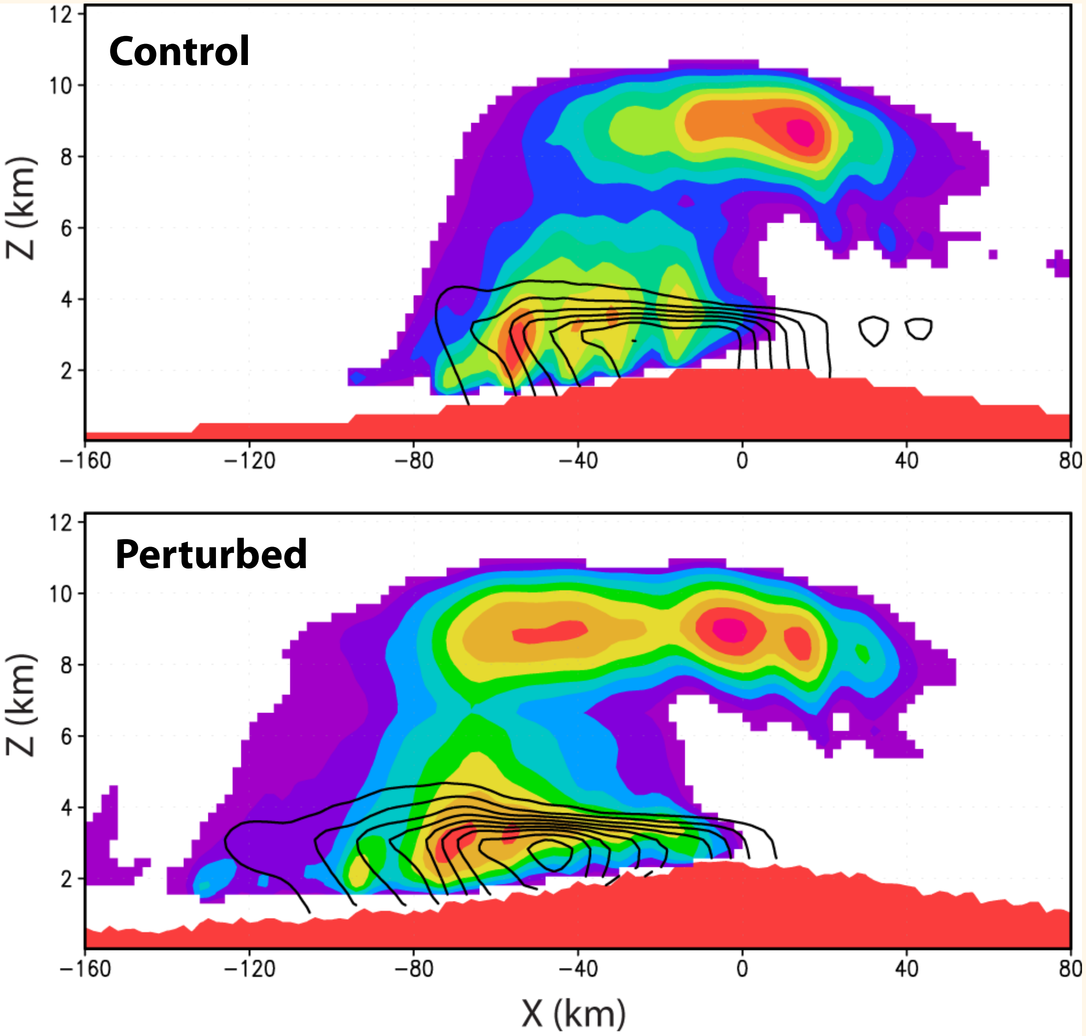
Mountain height (shaded) and Precipitation (contour)



Orographic precipitation over a ridge

- The flow recognizes the mountain shape relatively upstream due to the irregular topography and the mechanical lifting of the mountain is stronger with steep slope.

Cloud condensate (shaded) and rain water (contour)



Summary

- Surface topography is implemented into the vector vorticity equation model (VVM) with a block representation of mountains .

- The kinematic boundary conditions at the surface are satisfied with assigning a proper computational boundary condition for vorticity.

- The VVM with implemented topography performs reasonably well in the idealized 2D mountain waves, Boulder downslope windstorm and orographic precipitation cases.