

A Progress Report and More on the Unified System

Celal Konor

*Department of Atmospheric Science
Colorado State University*

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Progress

- A dynamical core based on the unified system has been developed
- A paper describing the dynamical core and presenting the results has been submitted for publication to JAMES.
- Development of a global dynamical core based on the unified system is nearly completed

Unified System

Arakawa and Konor (2009, *MWR*)

A nonhydrostatic system applicable to wide range of atmospheric scales of motion

- Filters vertically propagating acoustic waves while allowing elasticity due to thermal expansion
- Does not require a basic or mean state
- Does not introduce any approximation to the momentum and thermodynamic equations
- Introduces a minor approximation to the continuity equation
- Conserves energy

Unified System (Cont.)

A normal mode analysis confirms the followings:

- The unified system does not show the errors that appear with the anelastic and pseudo-incompressible systems while filtering vertically propagating acoustic waves
- Ultra-long Rossby waves are compressible and their retrogression speed is realistic

Comparison of the Equations of the Unified System to Those of Some Other Systems

State Equation

Fully Compressible

$$\rho = \frac{p_{00}\pi^{(1-\kappa)/\kappa}}{R\theta}$$

Quasi-Hydrostatic

$$\rho_{qs} = \frac{p_{00}\pi_{qs}^{(1-\kappa)/\kappa}}{R\theta}$$

qs: Quasi-hydrostatic

Anelastic

$$\rho = \frac{p_{00}\pi^{(1-\kappa)/\kappa}}{R\theta} \quad \text{and} \quad \bar{\rho} = \frac{p_{00}\bar{\pi}^{(1-\kappa)/\kappa}}{R\bar{\theta}}$$

$$\pi = \bar{\pi}(z) + \pi' \quad \theta = \bar{\theta}(z) + \theta'$$

$$\rho = \bar{\rho}(z) + \rho'$$

Bar: Basic/Mean state prime: Deviation

Unified

$$\rho = \frac{p_{00}\pi^{(1-\kappa)/\kappa}}{R\theta} \quad \text{and} \quad \rho_{qs} = \frac{p_{00}\pi_{qs}^{(1-\kappa)/\kappa}}{R\theta}$$

$$\pi = \pi_{qs} + \delta\pi \quad \rho = \rho_{qs} + \delta\rho$$

$$T_{qs} = \pi_{qs}\theta \quad T = \pi\theta$$

qs: Quasi-hydrostatic d: Non-hydrostatic

Comparison of Equations

Horizontal Momentum

Fully Compressible

$$\frac{D\mathbf{v}}{Dt} + f\mathbf{k} \times \mathbf{v} + c_p \theta \nabla_H \pi_{qs} + c_p \theta \nabla_H \delta\pi = \mathbf{F}_H$$

$$\pi = \pi_{qs} + \delta\pi$$

Quasi-Hydrostatic

$$\frac{D\mathbf{v}}{Dt} + f\mathbf{k} \times \mathbf{v} + c_p \theta \nabla_H \pi_{qs} = \mathbf{F}_H$$

$$\pi = \pi_{qs}$$

qs: Quasi-hydrostatic

Anelastic

$$\frac{D\mathbf{v}}{Dt} + f\mathbf{k} \times \mathbf{v} + c_p \bar{\theta} \nabla_H \pi' = \mathbf{F}_H$$

$$\pi = \bar{\pi}(z) + \pi' \quad \theta = \bar{\theta}(z) + \theta'$$

$$\pi' \ll \bar{\pi} \quad \theta' \ll \bar{\theta}$$

Bar: Basic/Mean state prime: Deviation

Unified

$$\frac{D\mathbf{v}}{Dt} + f\mathbf{k} \times \mathbf{v} + c_p \theta \nabla_H \pi_{qs} + c_p \theta \nabla_H \delta\pi = \mathbf{F}_H$$

$$\pi = \pi_{qs} + \delta\pi$$

qs: Quasi-hydrostatic d: Non-hydrostatic

Comparison of Equations

Vertical Momentum

Fully Compressible

$$\frac{Dw}{Dt} + c_p \theta \frac{\partial}{\partial z} \delta\pi = F_w$$

$$\pi = \pi_{qs} + \delta\pi$$

Quasi-Hydrostatic

Anelastic

Unified

$$\frac{Dw}{Dt} + c_p \bar{\theta} \frac{\partial}{\partial z} \pi' - g \frac{\theta - \bar{\theta}}{\bar{\theta}} = F_w$$

$$\pi' \ll \bar{\pi} \quad \theta' \ll \bar{\theta}$$

$$\pi = \bar{\pi}(z) + \pi' \quad \theta = \bar{\theta}(z) + \theta'$$

Bar: Basic/Mean state prime: Deviation

$$\frac{Dw}{Dt} + c_p \theta \frac{\partial}{\partial z} \delta\pi = F_w$$

$$\pi = \pi_{qs} + \delta\pi$$

qs: Quasi-hydrostatic d: Non-hydrostatic

Not used to predict w

Comparison of Equations

Hydrostatic Equation

Fully Compressible

$$c_p \theta \frac{\partial}{\partial z} \pi_{qs} = -g$$

Quasi-Hydrostatic

Anelastic

Unified

$$c_p \theta \frac{\partial}{\partial z} \pi_{qs} = -g$$

$$c_p \theta \frac{\partial}{\partial z} \pi_{qs} = -g$$

Comparison of Equations

Thermodynamic Equation

Fully Compressible

$$\frac{D\theta}{Dt} = \frac{Q}{c_p \pi}$$

Quasi-Hydrostatic

$$\frac{D\theta}{Dt} = \frac{Q}{c_p \pi_{qs}}$$

Anelastic

$$\frac{D\theta}{Dt} = \frac{Q}{c_p \pi}$$

Unified

$$\frac{D\theta}{Dt} = \frac{Q}{c_p \pi}$$

Comparison of Equations

Continuity Equation

Fully Compressible

$$\frac{\partial \rho}{\partial t} = -\nabla_H \cdot (\rho \mathbf{v}) - \frac{\partial(\rho w)}{\partial z}$$

Quasi-Hydrostatic

$$\frac{\partial(\rho_{qs} w)}{\partial z} = -\nabla_H \cdot (\rho_{qs} \mathbf{v}) - \frac{\partial \rho_{qs}}{\partial t}$$

Anelastic

$$\frac{\partial(\bar{\rho} w)}{\partial z} = -\nabla_H \cdot (\bar{\rho} \mathbf{v})$$

$$\rho' \ll \bar{\rho} \quad \partial \rho' / \partial t \approx 0$$

$$\rho = \bar{\rho}(z) + \rho'$$

Bar: Basic/Mean state prime: Deviation

Unified

$$\frac{\partial(\rho_{qs} w)}{\partial z} = -\nabla_H \cdot (\rho_{qs} \mathbf{v}) - \frac{\partial \rho_{qs}}{\partial t}$$

$$\delta \rho \ll \rho_{qs} \quad \partial \delta \rho / \partial t \approx 0$$

$$\rho = \rho_{qs} + \delta \rho$$

qs: Quasi-hydrostatic d: Non-hydrostatic

Not used to predict quasi-hydrostatic density, but used to determine w

Comparison of Equations

Elliptic Equation

Fully Compressible

Quasi-Hydrostatic

Anelastic

Unified

$$\bar{\rho} \nabla_H \cdot (c_p \bar{\theta} \nabla_H \pi') + \frac{\partial}{\partial z} \left(\bar{\rho} c_p \bar{\theta} \frac{\partial}{\partial z} \pi' \right) = G_{AN}$$

Generally requires more iterations than the unified counterpart since π' also includes a quasi-hydrostatic component.

$$\rho_{qs} \nabla_H \cdot (c_p \theta \nabla_H \delta\pi) + \frac{\partial}{\partial z} \left(\rho_{qs} c_p \theta \frac{\partial}{\partial z} \delta\pi \right) = -\rho_{qs} \nabla_H \cdot (c_p \theta \nabla_H \pi_{qs}) + G + \frac{\partial^2 \rho_{qs}}{\partial t^2}$$

The Integration Procedure used in the Dynamical Core

1-Predict Q

$$\frac{D\theta}{Dt} = \frac{Q}{c_p \pi}$$

2-Determine quasi-hydrostatic quantities

$$\frac{\partial \pi_{qs}}{\partial z} = -\frac{g}{c_p \theta}$$

$$\rho_{qs} = \frac{p_{00} \pi_{qs}^{(1-\kappa)/\kappa}}{R\theta}$$

$$\underline{\partial \rho_{qs} / \partial t}$$

$$\underline{\partial^2 \rho_{qs} / \partial t^2}$$

3-Determine nonhydrostatic quantities

$$\rho_{qs} \nabla_H \cdot (c_p \theta \nabla_H \delta \pi) + \frac{\partial}{\partial z} \left(\rho_{qs} c_p \theta \frac{\partial}{\partial z} \delta \pi \right)$$

$$= -\rho_{qs} \nabla_H \cdot (c_p \theta \nabla_H \pi_{qs}) + G + \underline{\frac{\partial^2 \rho_{qs}}{\partial t^2}}$$

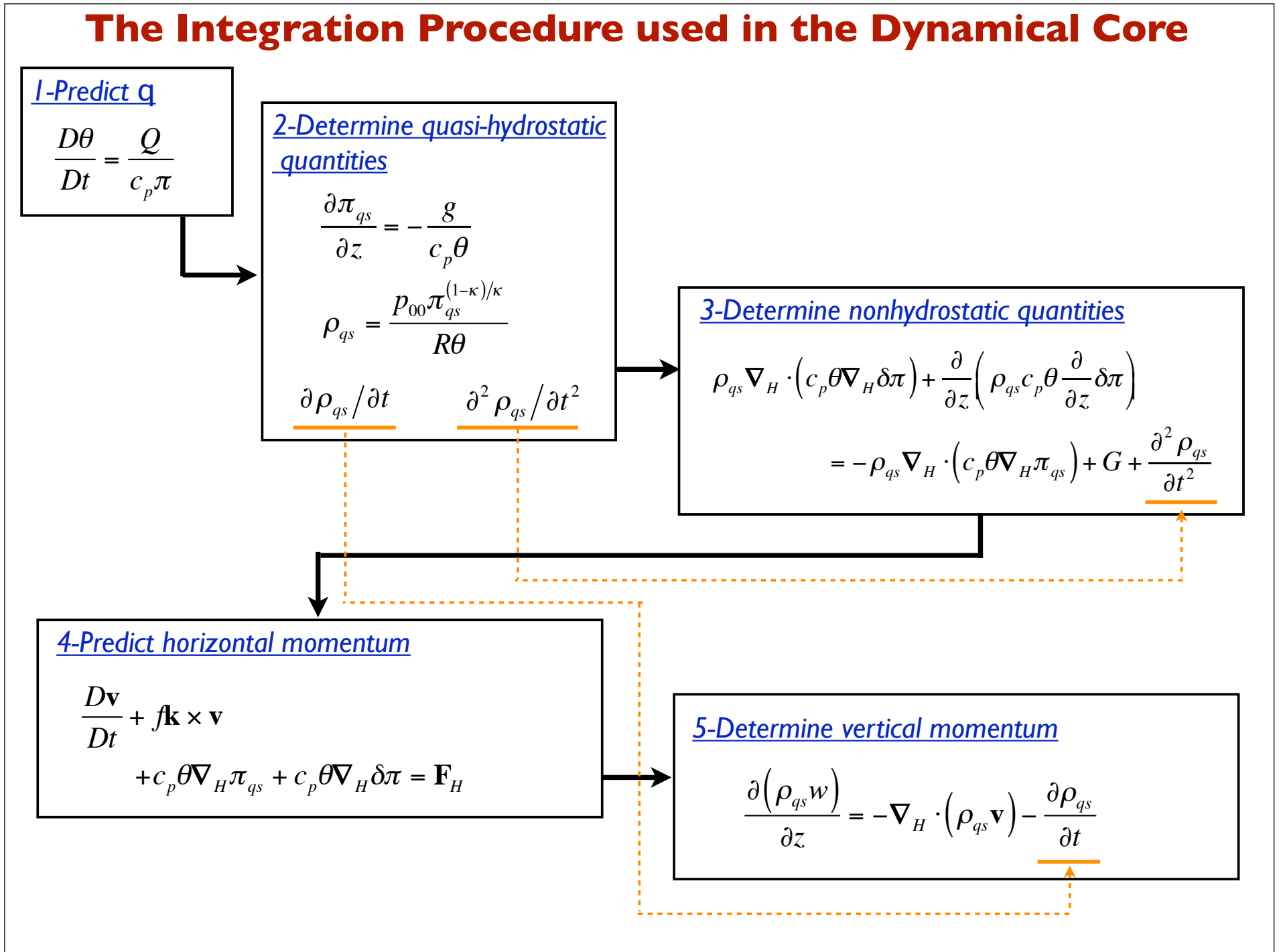
4-Predict horizontal momentum

$$\frac{D\mathbf{v}}{Dt} + f\mathbf{k} \times \mathbf{v}$$

$$+ c_p \theta \nabla_H \pi_{qs} + c_p \theta \nabla_H \delta \pi = \mathbf{F}_H$$

5-Determine vertical momentum

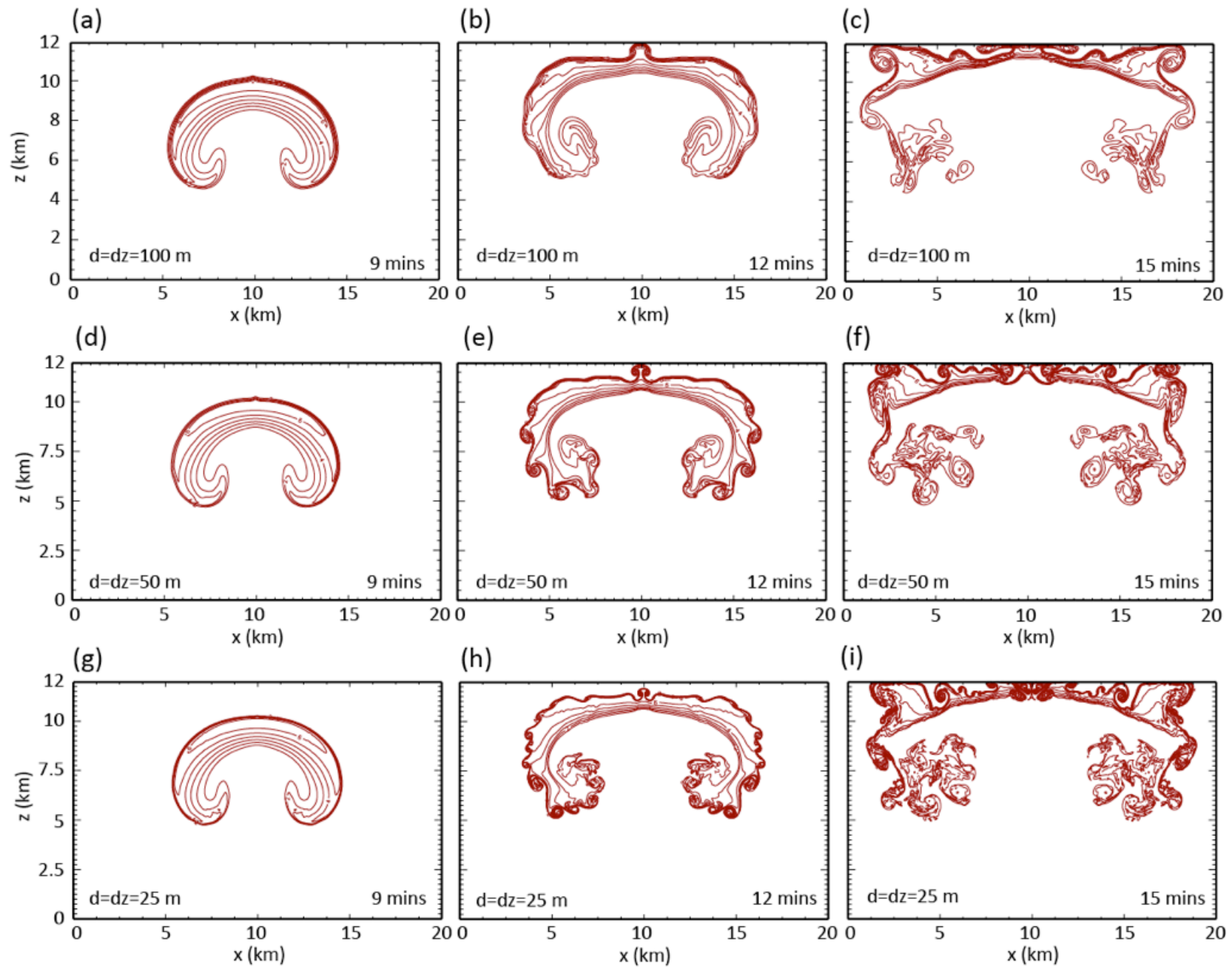
$$\frac{\partial (\rho_{qs} w)}{\partial z} = -\nabla_H \cdot (\rho_{qs} \mathbf{v}) - \underline{\frac{\partial \rho_{qs}}{\partial t}}$$



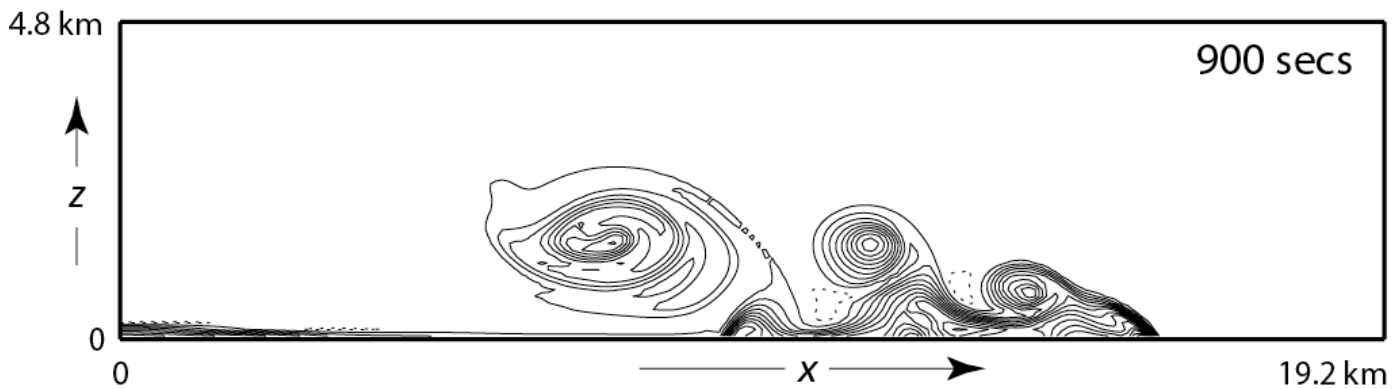
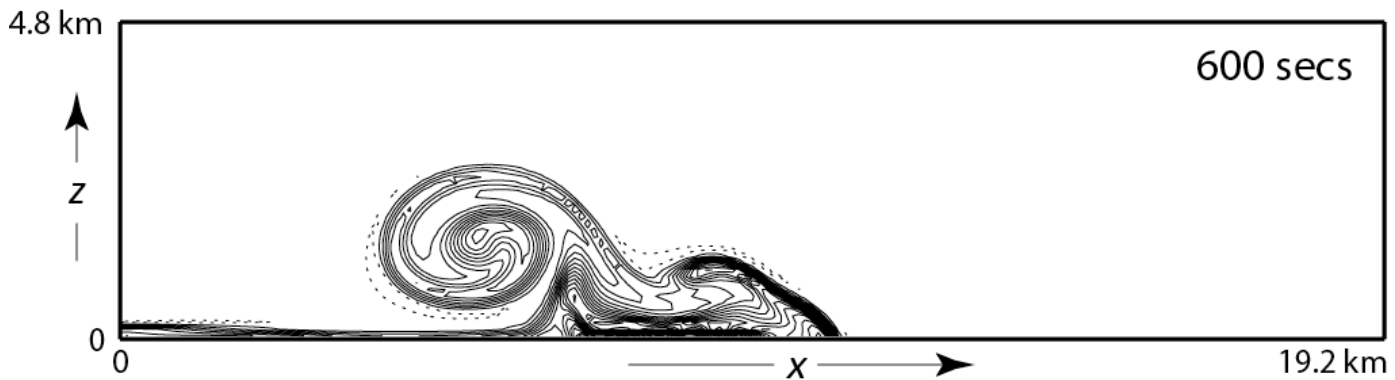
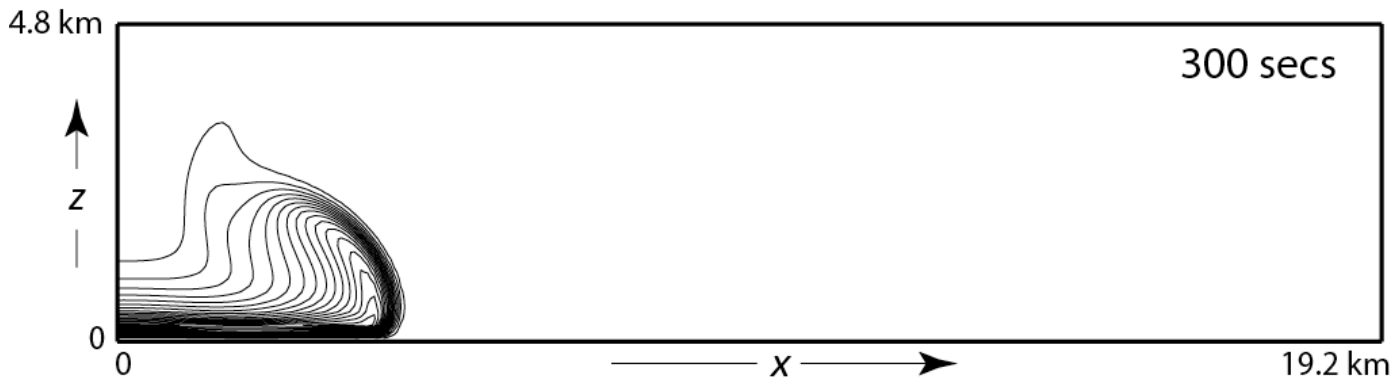
Some results from Konor (2011)

- Warm bubble tests
- Cold bubble tests
- Idealized extratropical cyclogenesis simulations

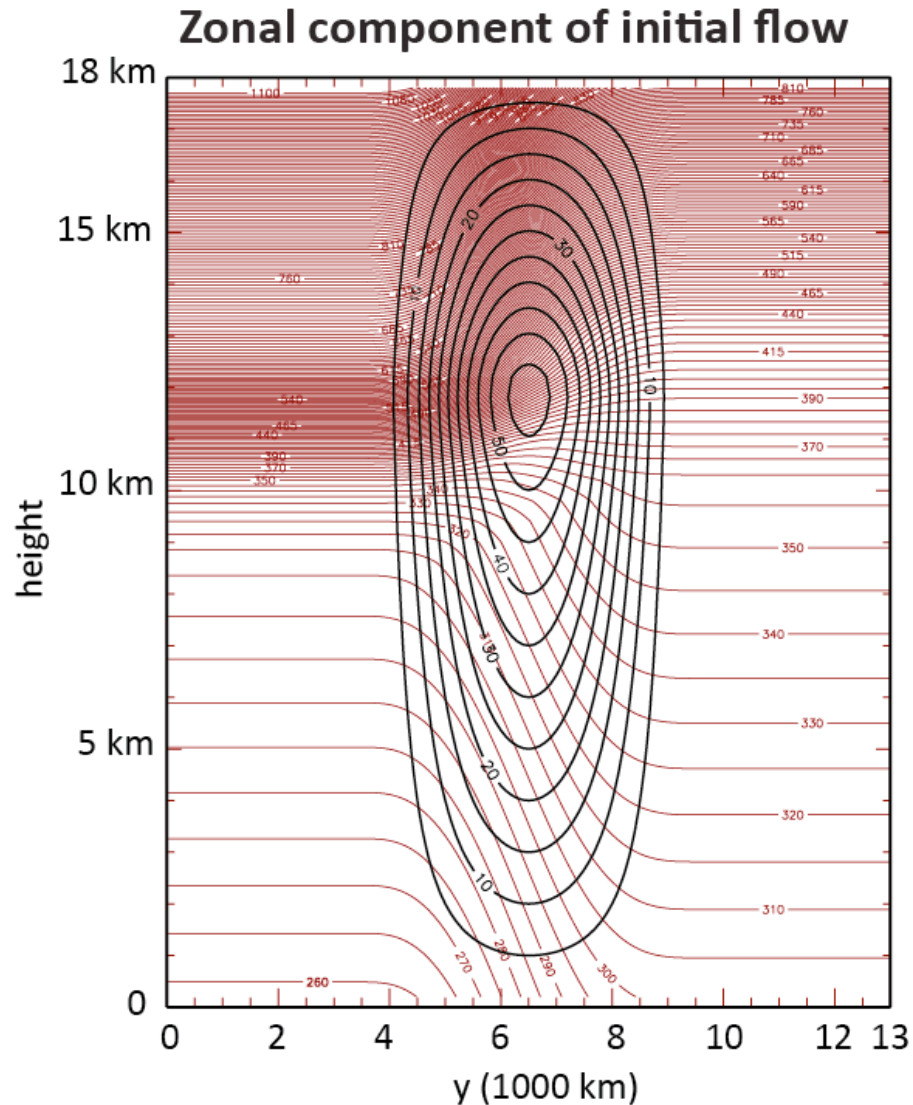
Warm bubble tests [Suggested by Mendez-Nunez and Carroll, 1993]



Cold bubble tests [Suggested by Straka et al., 1993]



Idealized extratropical cyclogenesis simulations

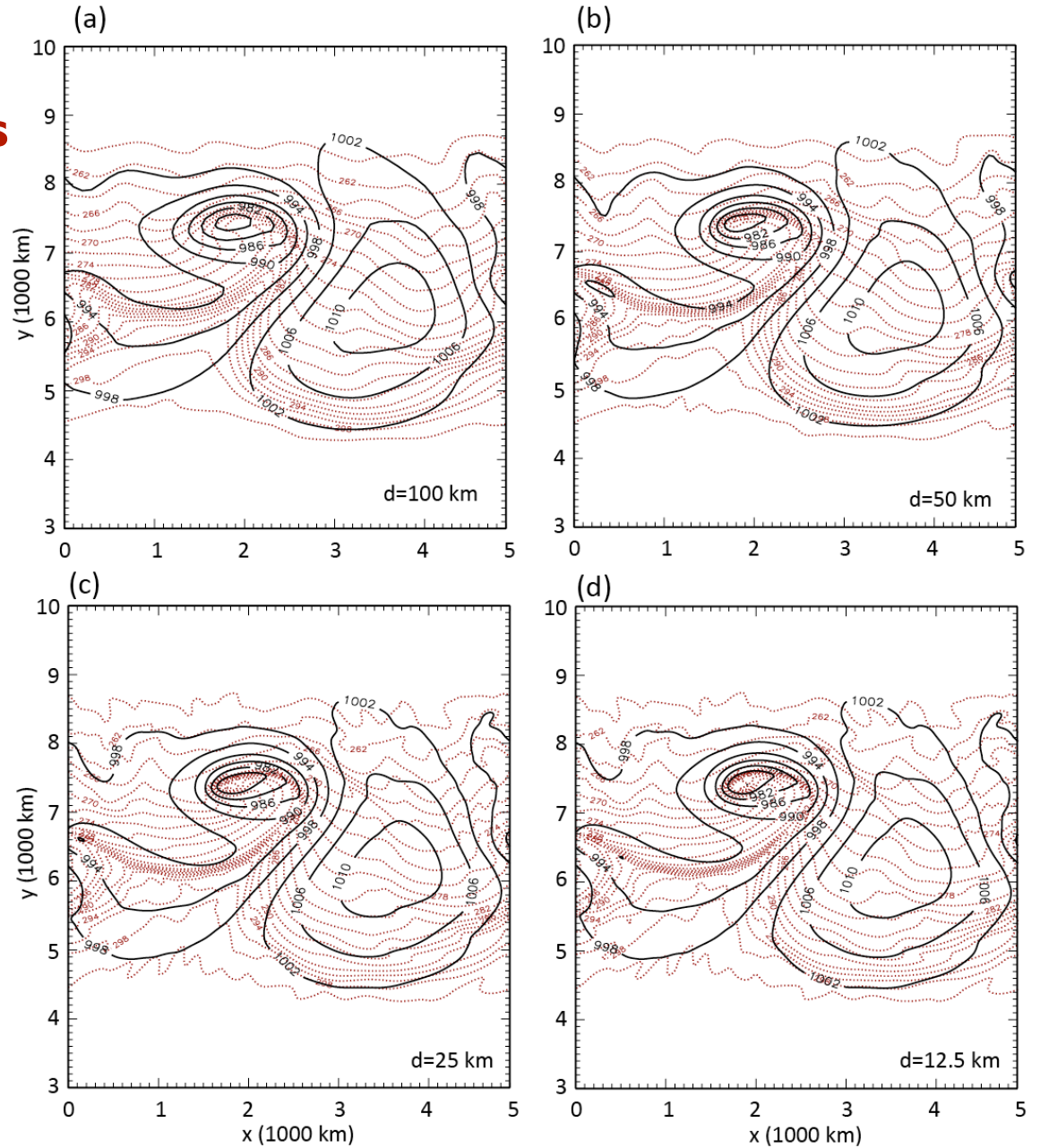


- Domain is a 5000 km long channel on an extratropical b-plane
- Start from random perturbations of potential temperature
- 45 layers (400 m)
- Four different horizontal grid distances: 100 km, 50 km, 25 km and 12.5 km

Idealized extratropical cyclogenesis simulations

Surface fields at Day 13

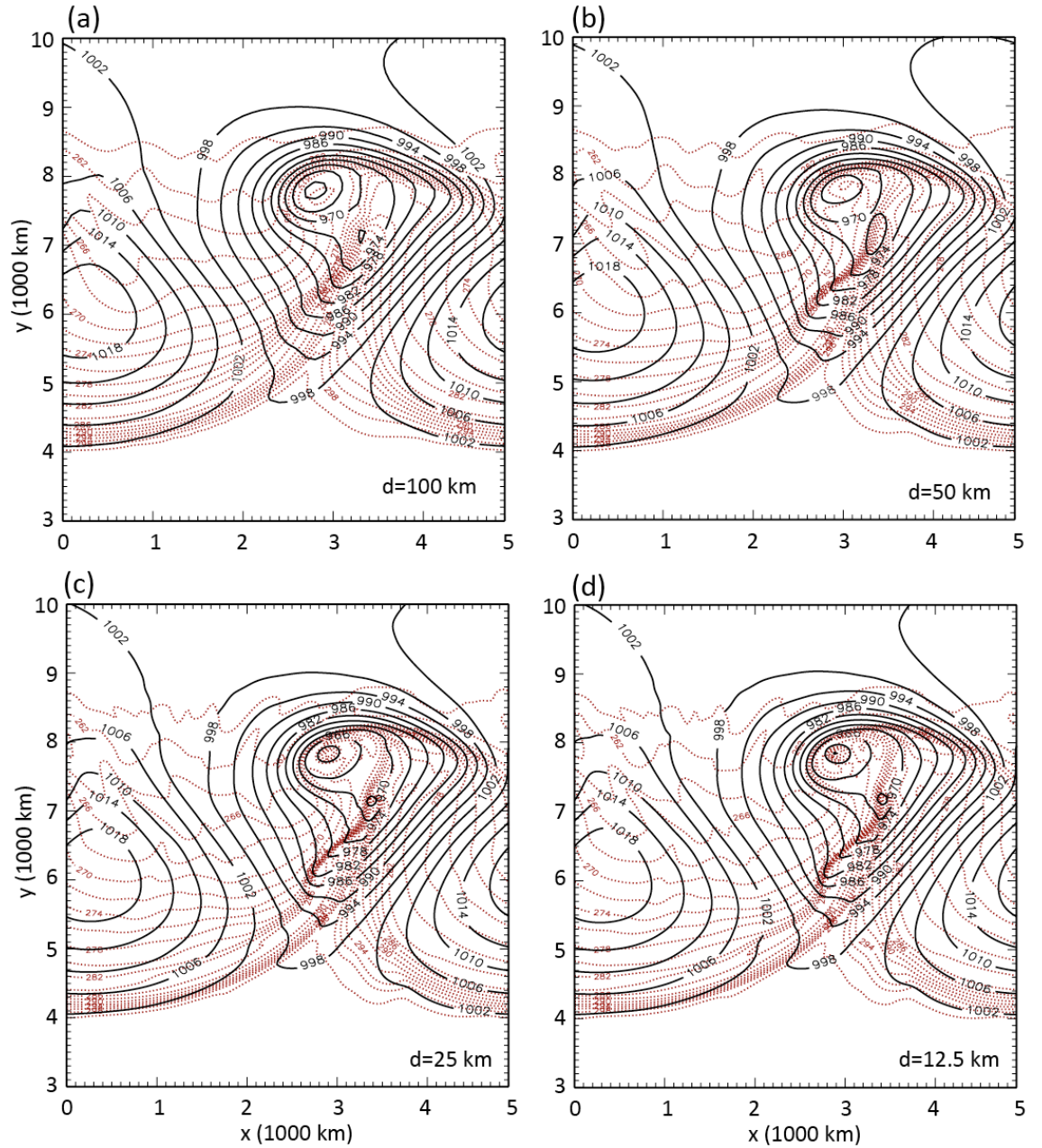
Surface pressure ($p=p_{qs}+\delta p$, mb) and surface potential temperature (K) at day 13



Idealized extratropical cyclogenesis simulations

Surface fields at Day 15

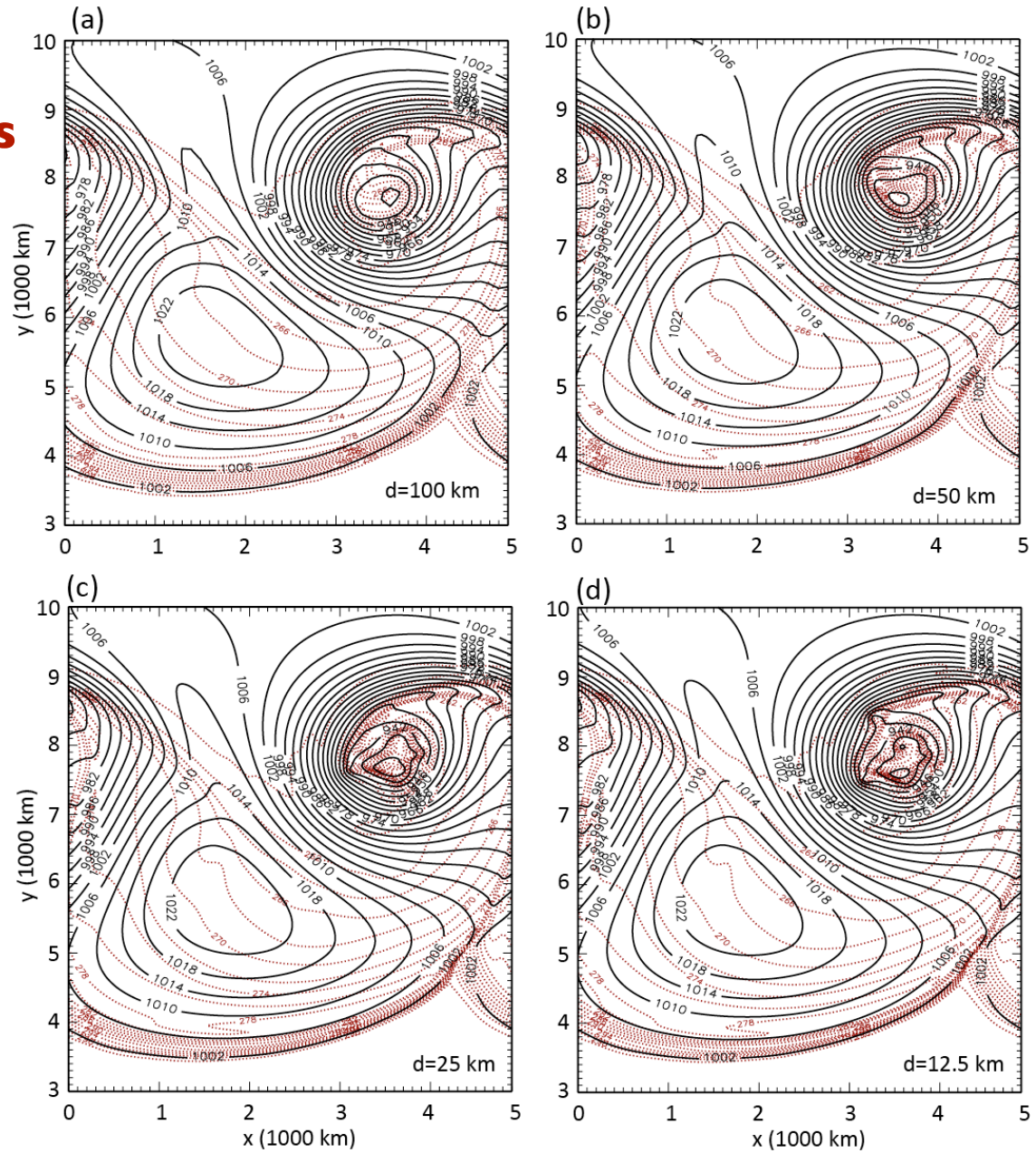
Surface pressure ($p=p_{qs}+\delta p$, mb) and surface potential temperature (K) at day 15



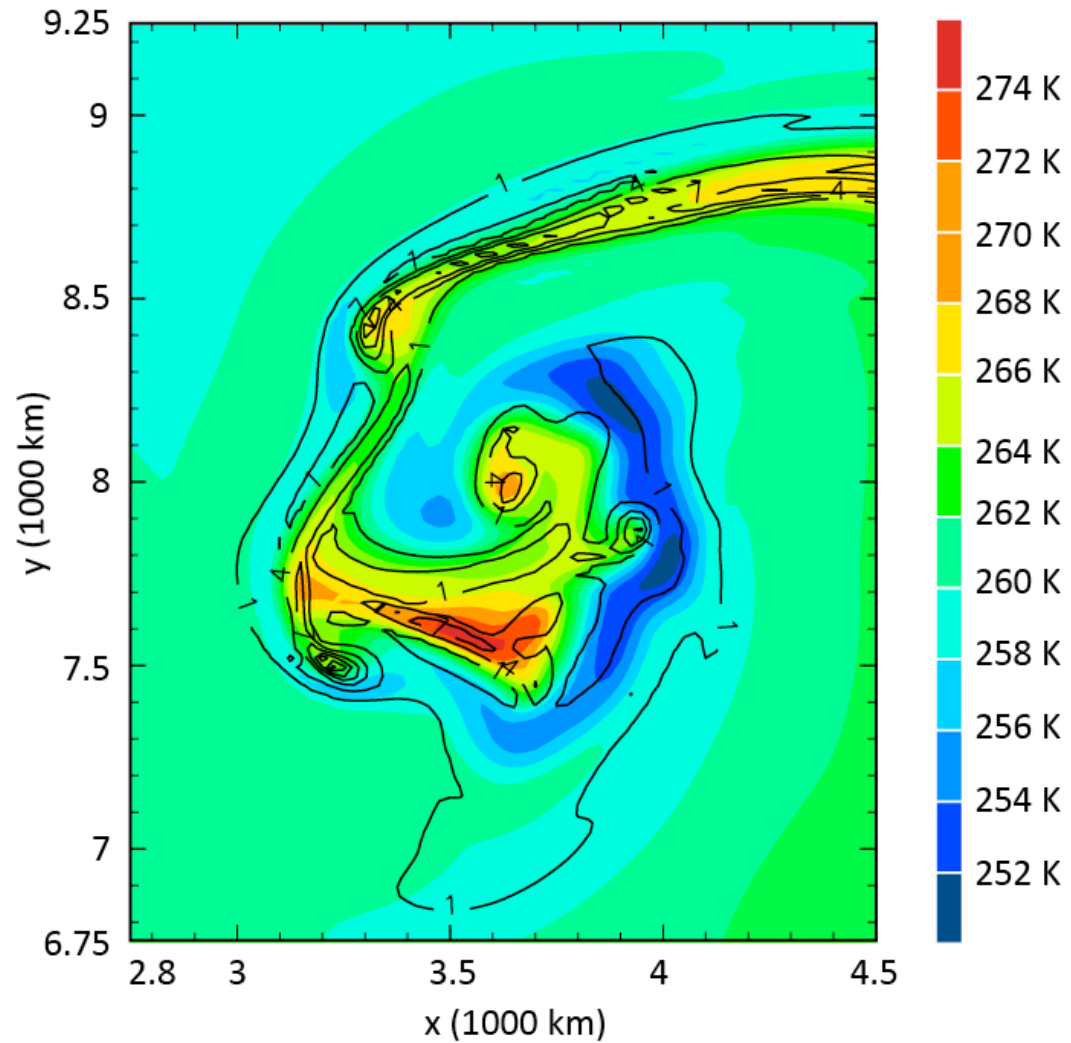
Idealized extratropical cyclogenesis simulations

Surface fields at Day 17

Surface pressure ($p=p_{qs}+\delta p$, mb) and surface potential temperature (K) at day 17



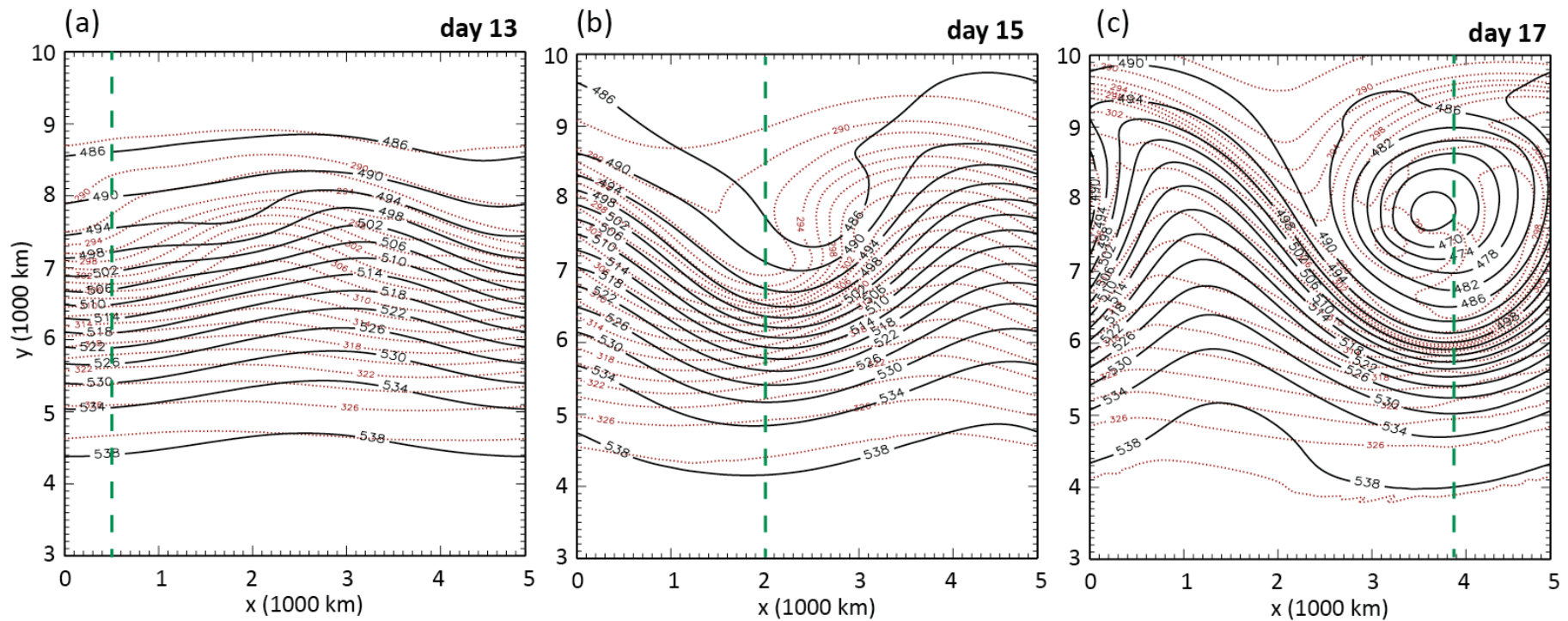
Surface vorticity (10^{-4} 1/s)
and surface potential temperature (K)
at day 17 from high-res run (d=12.5 km)



Idealized extratropical cyclogenesis simulations

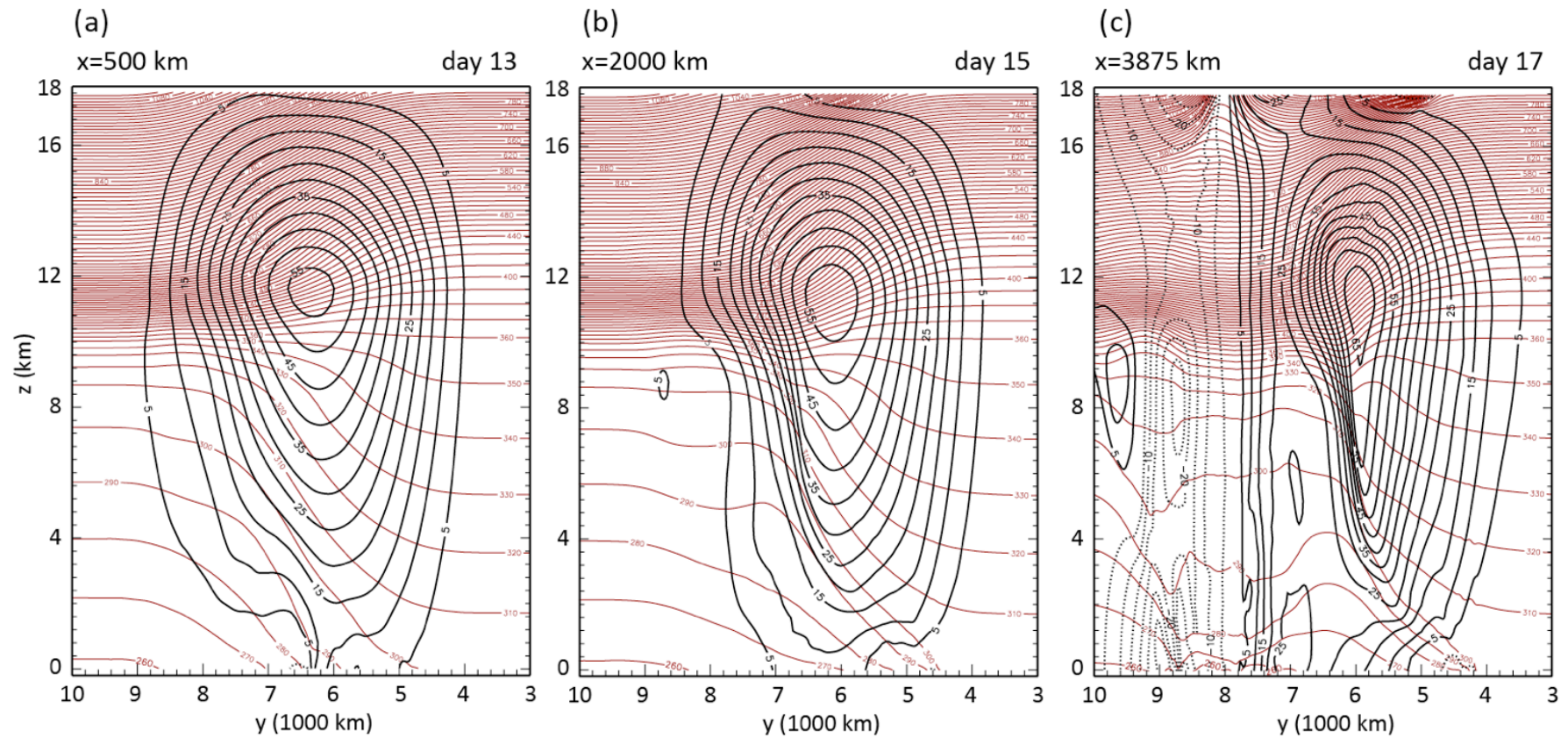
Middle troposphere fields from 12.5-km run at Days 13, 15 and 17

Pressure ($p=p_{qs}+\delta p$, mb) and potential temperature (K) for 5200 m height from high-resolution run ($d=12.5$ km)



Idealized extratropical cyclogenesis simulations

y-z cross-sections of zonal velocity (m/s) and potential temperature (K) from high-resolution run (d=12.5 km)



Summary

- The dynamical core based on the unified system performs well in the warm and cold bubble tests and in simulating idealized extratropical cyclogenesis
- A paper describing the dynamical core and presenting the results has been submitted for publication to JAMES.

Remaining tasks

- Completion of a global dynamical core based on the unified system
- Inclusion of physics into the global dynamical core
- Construction of a new global MMF