Growth of cloud droplets and raindrops in turbulent clouds

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CMMAP
Reach for the sky.

Grabowski W.W., and L.-P. Wang, 2013: Growth of cloud droplets in a turbulent environment. *Ann. Rev. Fluid Mech*., 45, 293-324.

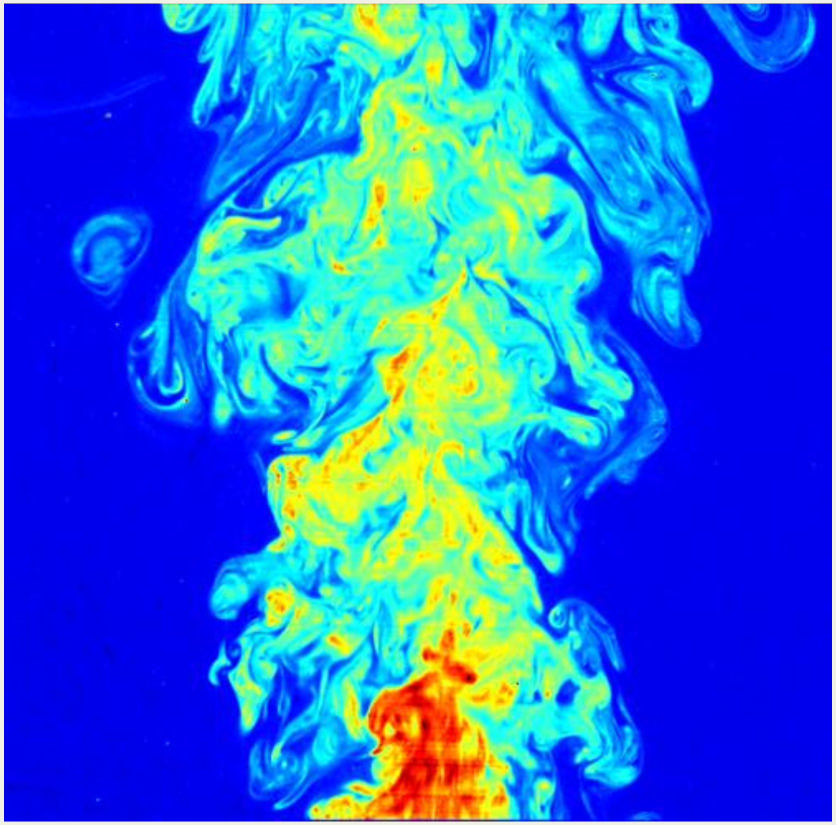
Wyszogrodzki, A. A., W. W. Grabowski, L.-P. Wang, and O. Ayala, 2013: Turbulent collision-coalescence in maritime shallow convection. *Atmos. Chem. Phys.* (in press; available in ACPD).

Cloud droplets grow by the diffusion of water vapor (i.e., by condensation) and by collision/coalescence.

For both cloud turbulence is thought to play a significant role.

Turbulent entrainment---mixing of cloudy air with dry environmental air---significantly affects the spectrum of cloud droplets.

Clouds are turbulent, but what does it mean?



Turbulent jet in the laboratory

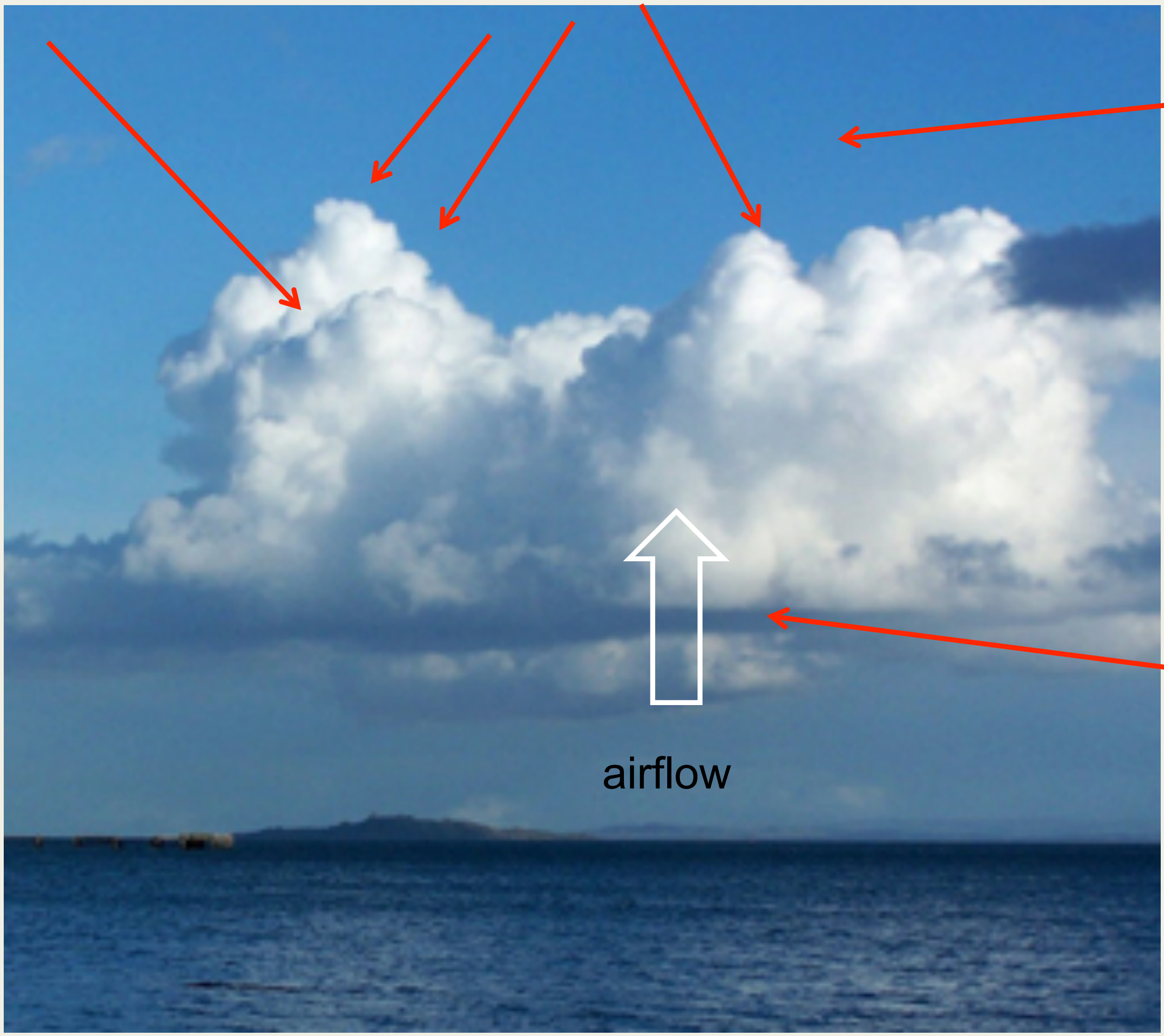
Lewis F. Richardson’s poem:

“Big whirls have little whirls
Which feed on their velocity,
And little whirls have lesser whirls,
And so on to viscosity.”

turbulent cloud

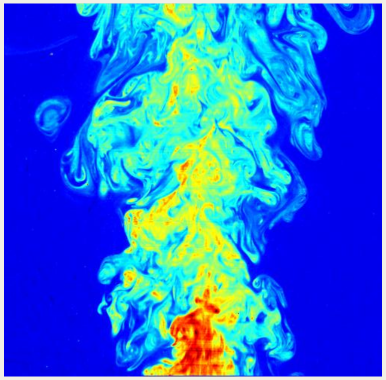
interfacial instabilities

calm (low-turbulence) environment



airflow

cloud base (activation of cloud condensation nuclei)



droplet spectra

vertical and along-track velocity

liquid water content

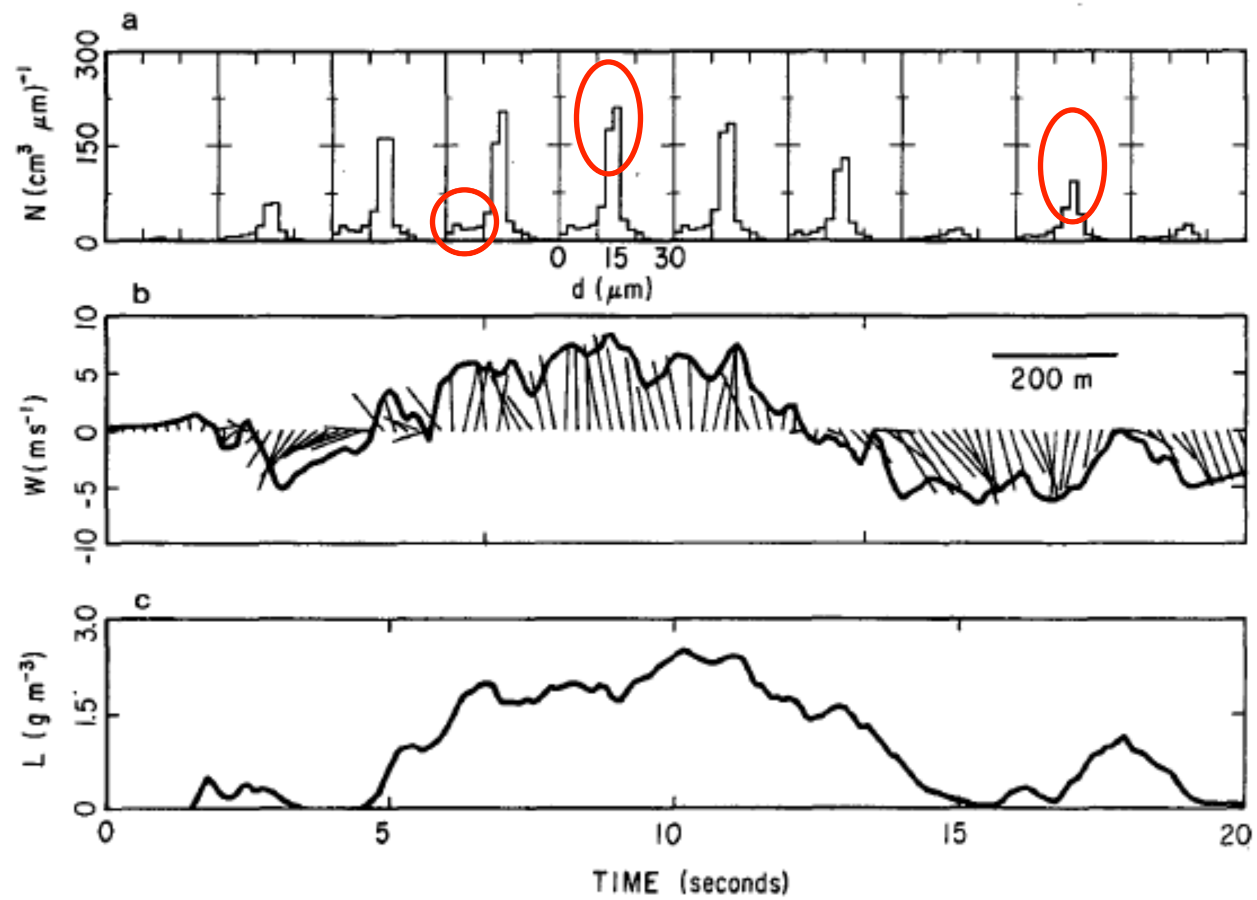
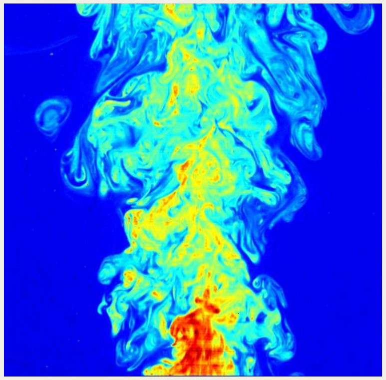


FIG. 3. Penetration at 600 mb, 6 June: (a) two-second averaged droplet spectra (sizes for diameter bins are those given by the manufacturer); (b) wind velocity, the lines represent wind vectors formed from the vertical wind and the wind along the flight path; (c) liquid water density measured by the Johnson-Williams device. All H-2 measurements.



(Austin et al. JAS 1985)

Elementary facts about cloud droplets:

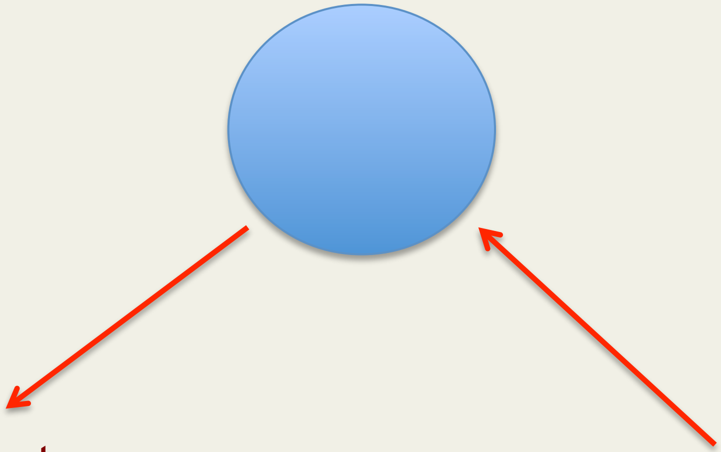
Radius *r* : 5-30 microns (*r* << Kolmogorov length; ~1 mm)

Concentration: 50-2,000 cm^{-3} (mean separation distance >> *r*)

Mass loading: 0.5-5 g kg^{-1} (<< 1; no effects on turbulence)

Fall terminal velocity v_t: $v_t \sim r^2$; $v_t \approx 1$ cm/s for $r = 10$ μm

Growth by the diffusion of water vapor



supersaturated
environment;
S=$e/e_s - 1 > 0$

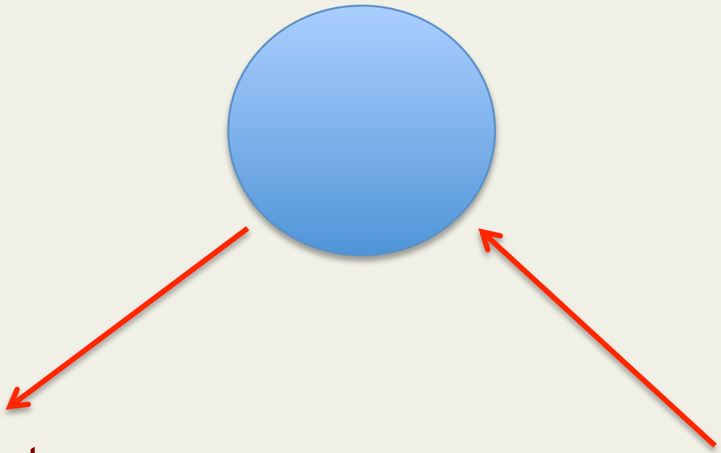
Temperature
flux away from
the droplet

water vapor
flux towards
the droplet

Growth by the diffusion of water vapor

$r\, dr/dt = A\, S$

$A \sim 10^{-10}\ m^2 s^{-1}$



supersaturated
environment;
$S = e/e_s - 1 > 0$

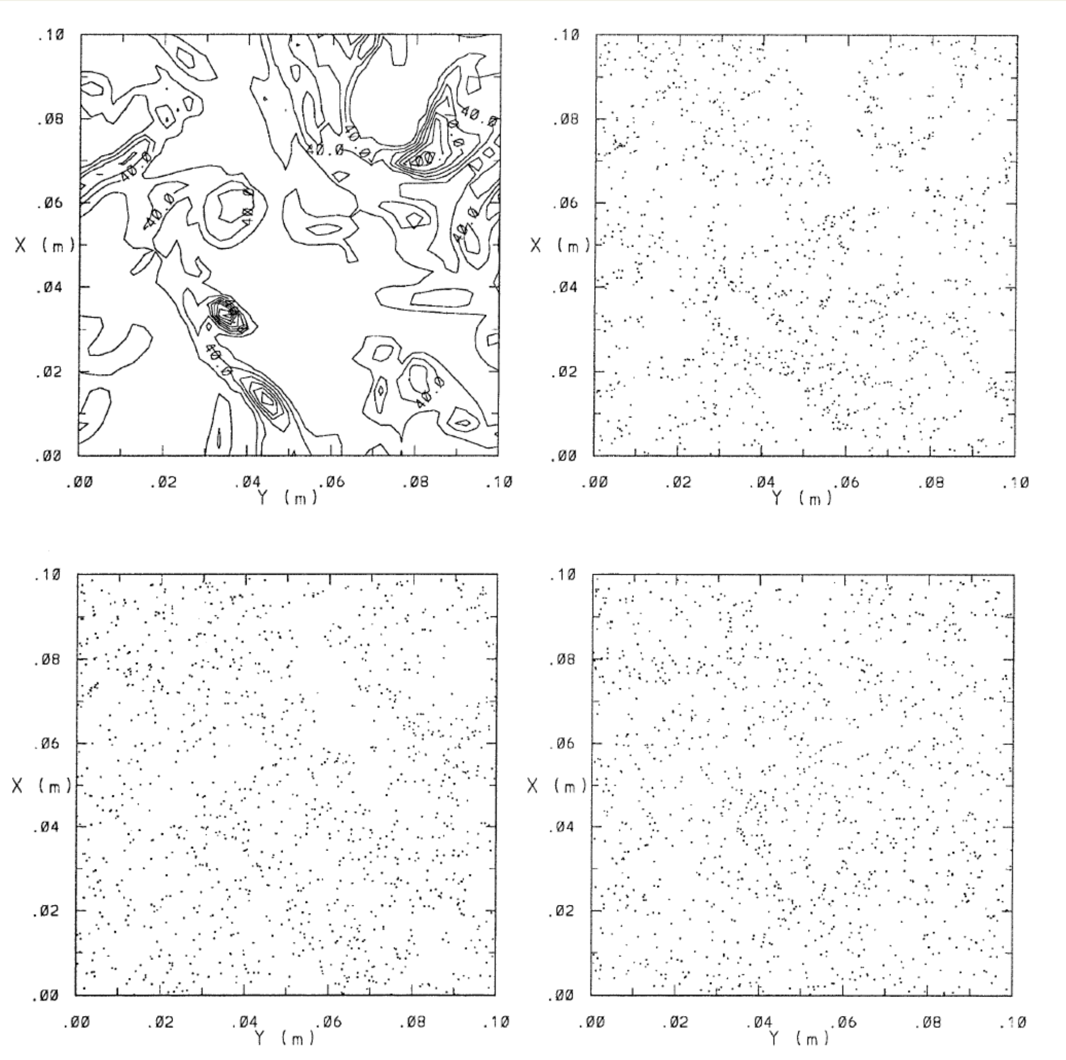
temperature
flux away from
the droplet

water vapor
flux towards
the droplet

Direct Numerical Simulation (DNS) of a turbulent flow with cloud droplets growing by the diffusion of water vapor in conditions relevant to cloud physics (eddy dissipation rate ε=160 cm^2s^{-3})

Vorticity

(contour 15 s^{-1})

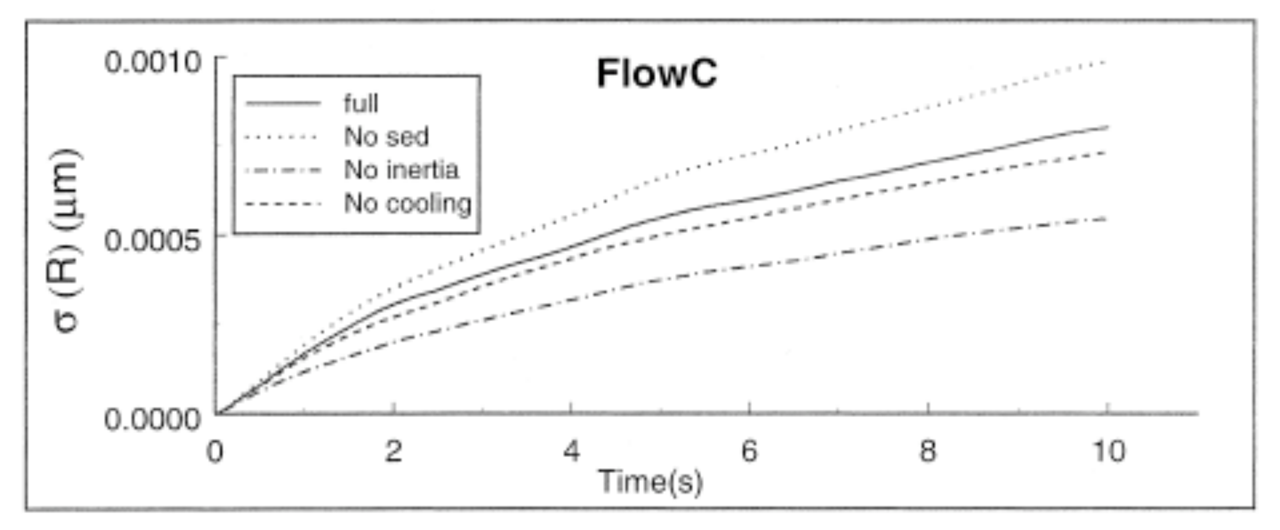


r=20 micron

r=15 micron

r=10 micron

Vaillancourt et al. JAS 2002



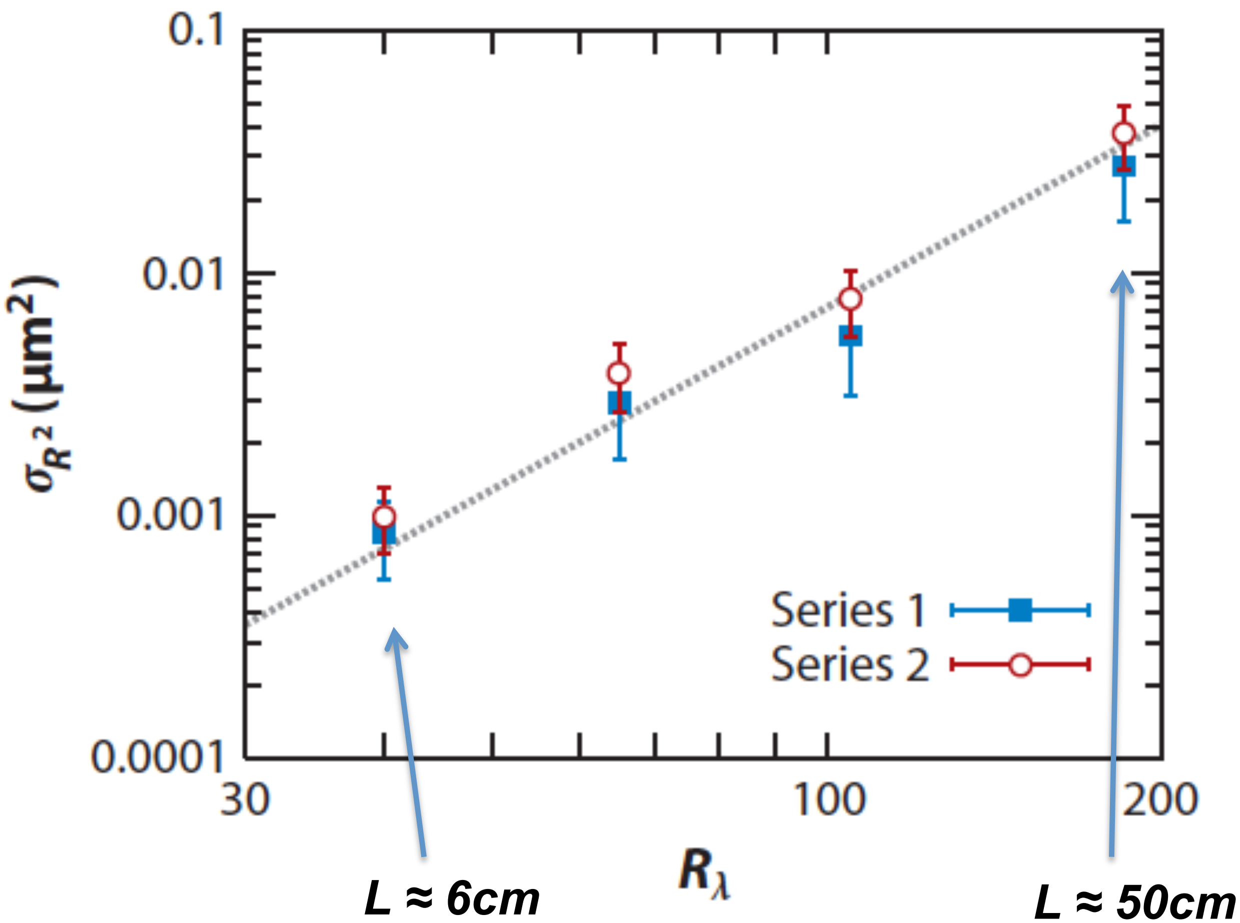
Main conclusion: small-scale turbulence has a negligible effect on the width of the cloud droplet spectrum.

Explanation: droplets rearrange their positions rapidly, growth histories average out…

Vaillancourt et al. JAS 2002

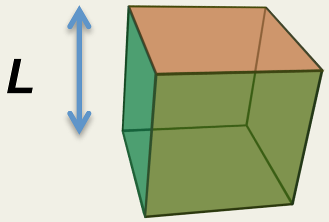
What about those DNS limitations?

Argument: if the Reynolds number increases (i.e., the range of scales involved increases), can small-scale supersaturation fluctuations increase as well?



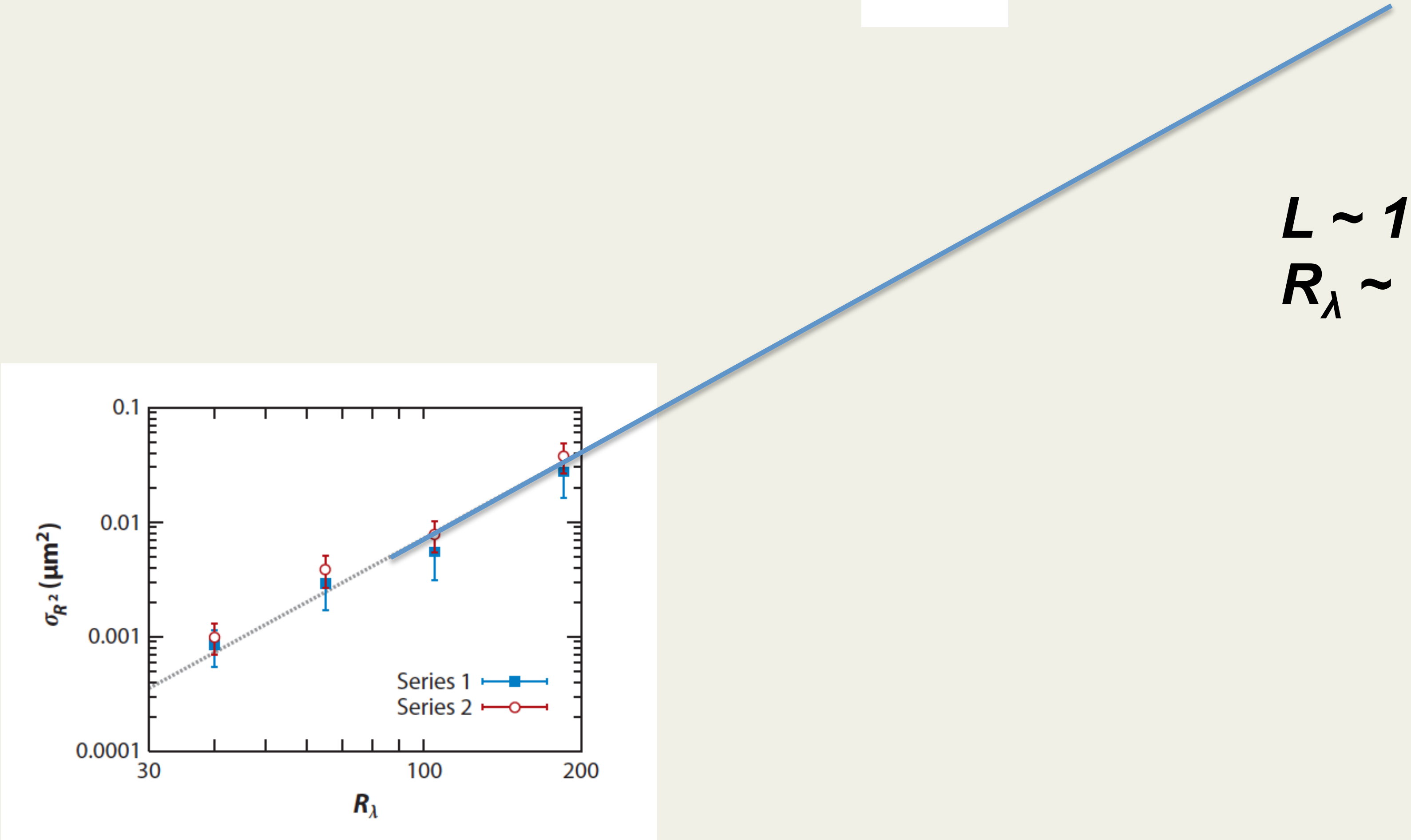
$L \approx 6cm$

$L \approx 50cm$



Lanotte et al. JAS 2009

$\sigma_{R^2} \sim 10^2\ \mu m^2$

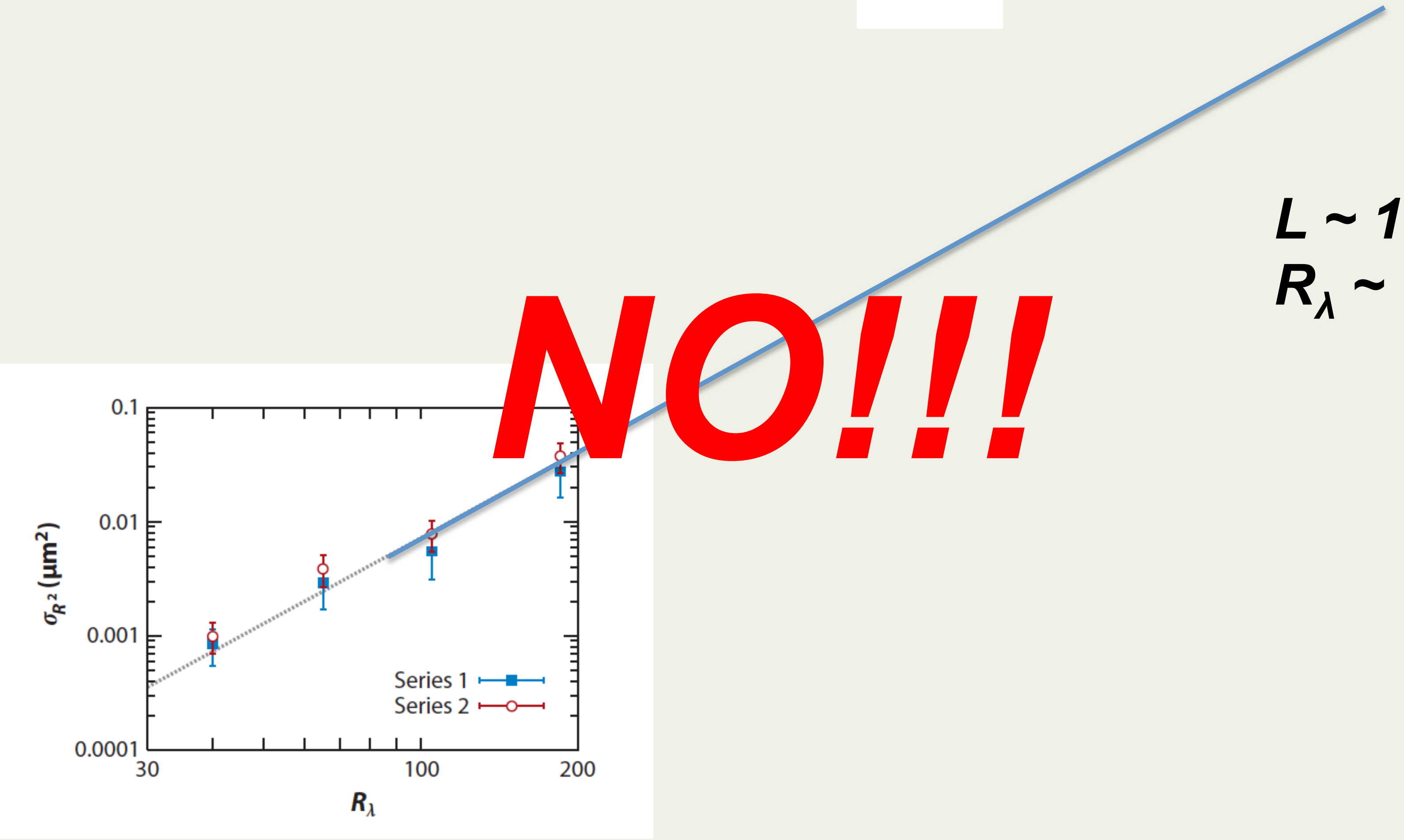


$L \sim 100m$
$R_\lambda \sim 10^4$

Can we extrapolate supersaturation fluctuations into scales relevant to clouds?

σ_{R^2} ~ 10^2 μm²

L ~ 100m
R_λ ~ 10^4



NO!!!

Can we extrapolate supersaturation fluctuations into scales relevant to clouds?

The brake on supersaturation fluctuations:

$$\frac{dS}{dt} = \alpha w - \frac{S}{\tau_{qe}}$$

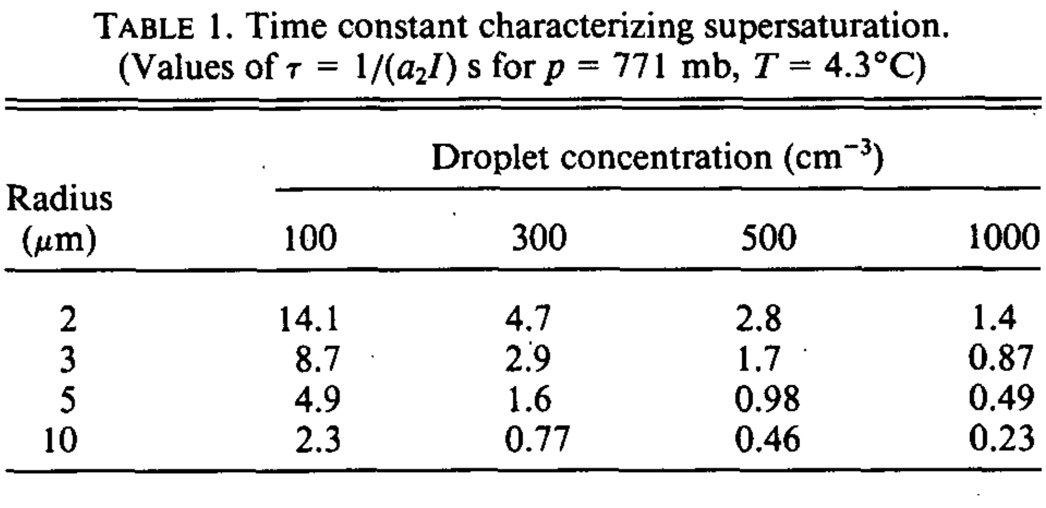


TABLE 1. Time constant characterizing supersaturation. (Values of $\tau = 1/(a_2 I)$ s for p = 771 mb, T = 4.3°C)

Radius (μm)	Droplet concentration (cm^{-3}) 100	300	500	1000
2	14.1	4.7	2.8	1.4
3	8.7	2.9	1.7	0.87
5	4.9	1.6	0.98	0.49
10	2.3	0.77	0.46	0.23

Politovich and Cooper, JAS 1988

$$\tau_{qe} \sim 1\sec$$

$$\frac{dS}{dt} \equiv 0 \rightarrow S_{qe} = \alpha w \tau_{qe}$$

For eddies with time-scale larger than τ_{qe}, fluctuations of S are limited by S_{qe} !!!

So within a uniform cloud (e.g., the adiabatic core), fluctuations of the supersaturation have a small effect.

So within a uniform cloud (e.g., the adiabatic core), fluctuations of the supersaturation have a small effect.

But what about the impact of mixing with the dry air from cloud environment (entrainment)?

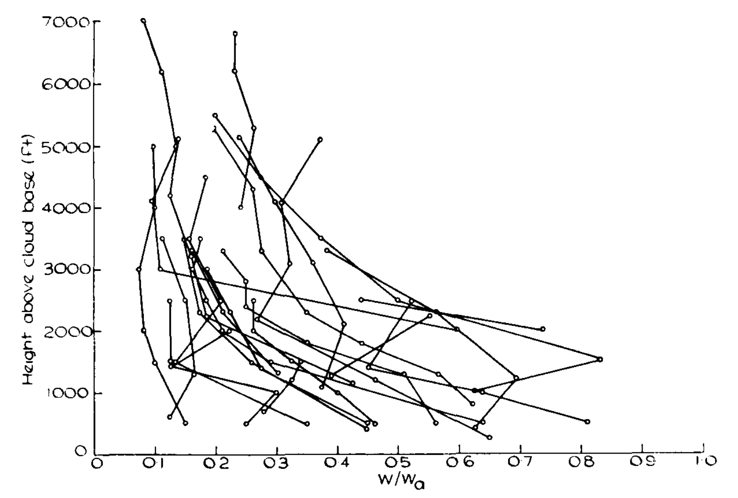
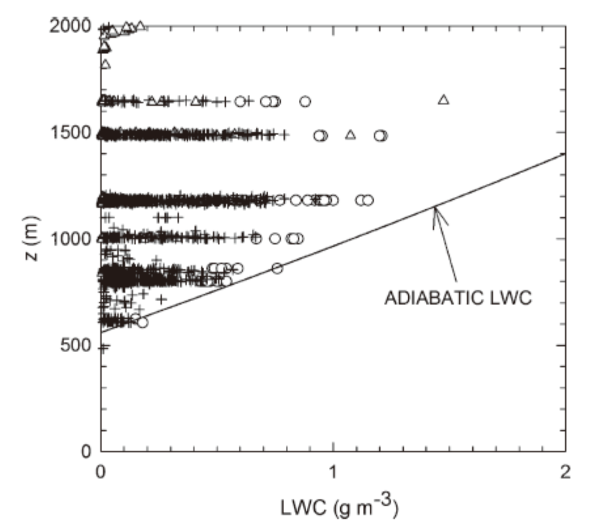


Fig. 7. Ratio of observed liquid water content to adiabatic value versus height above base.

Warner *Tellus* 1955



Gerber *JMSJ* 2008

Entrainment/mixing and cloud droplet spectra:

- Entrainment/mixing leads to partial evaporation of cloud water.

- Entrainment/mixing may lead to activation of cloud droplets above the cloud base.

- Entrainment/mixing allows for different growth histories of cloud droplets arriving at a given location within a cloud.

"Large-eddy hopping" (Grabowski and Wang, ARFM 2013)

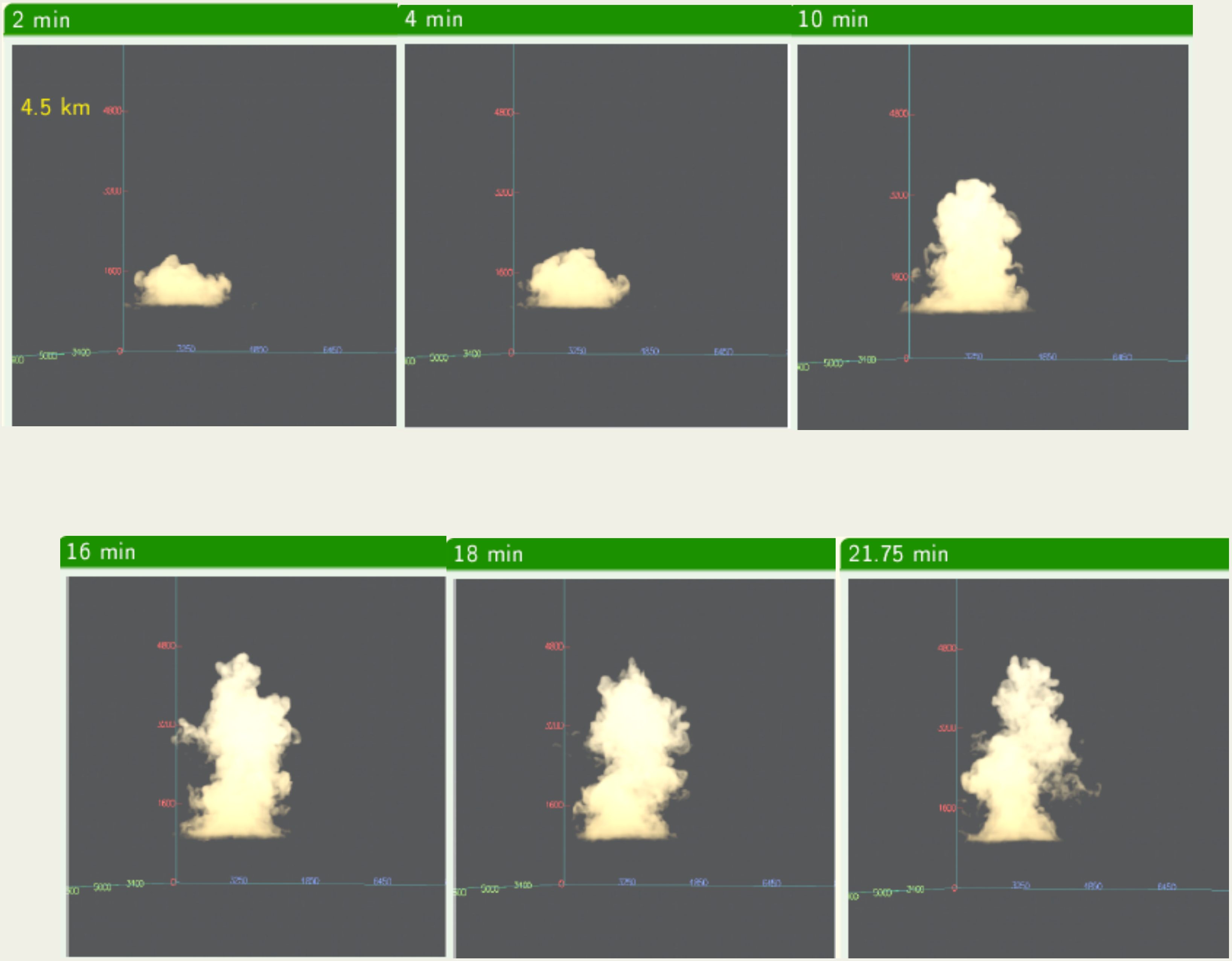
(Al Cooper, NCAR; Sonia Lasher-Trapp, Purdue; Alan Blyth, Leeds):

Droplets observed in a single location within a cloud arrive along a variety of fluid trajectories:

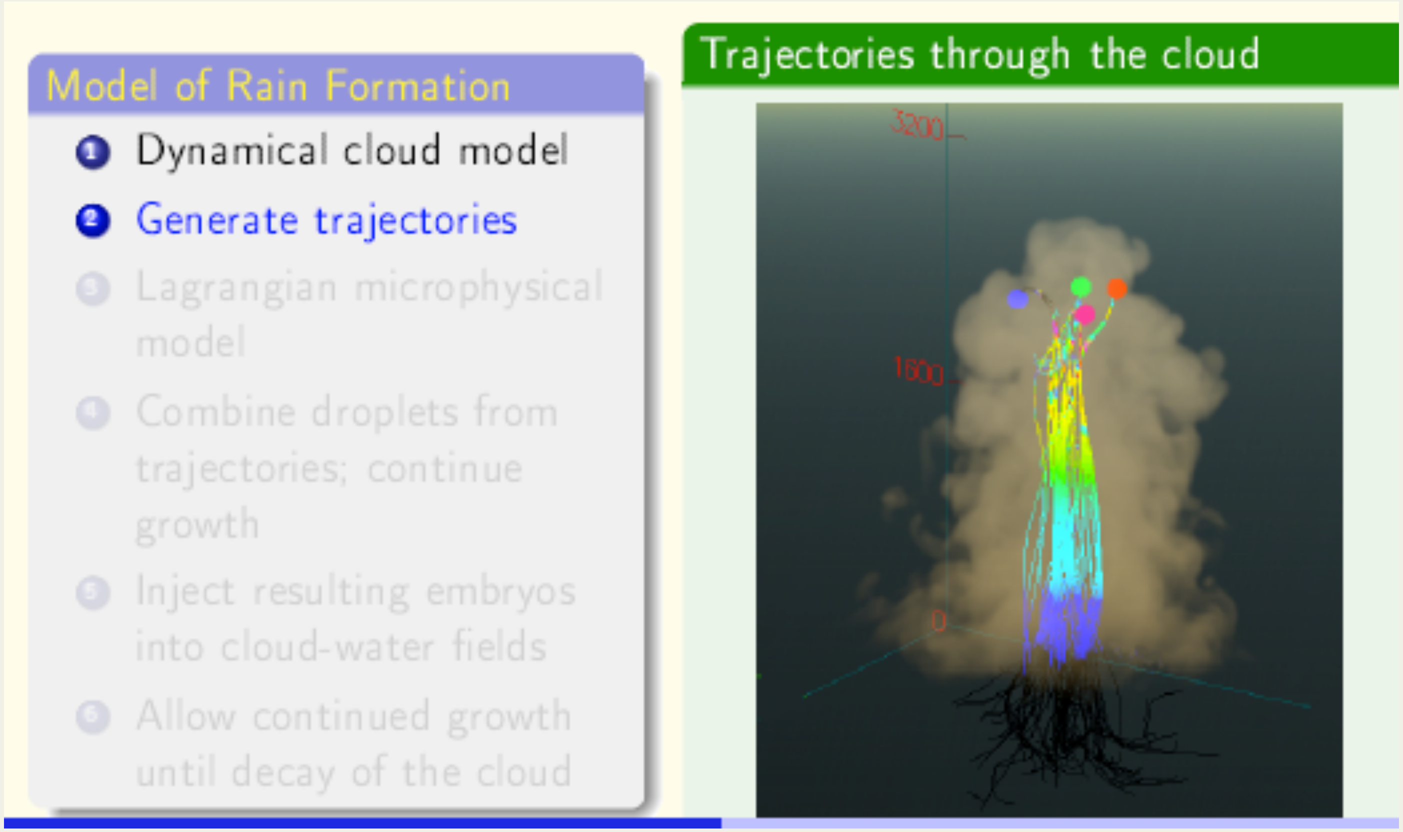
- large scale eddies are needed to provide different droplet activation/growth histories;

- small scale edies needed to allow hopping from one large eddy to another.

[see also Sidin et al. (*Phys. Fluids* 2009) for idealized 2D synthetic turbulence simulations]



courtesy of Al Cooper, NCAR

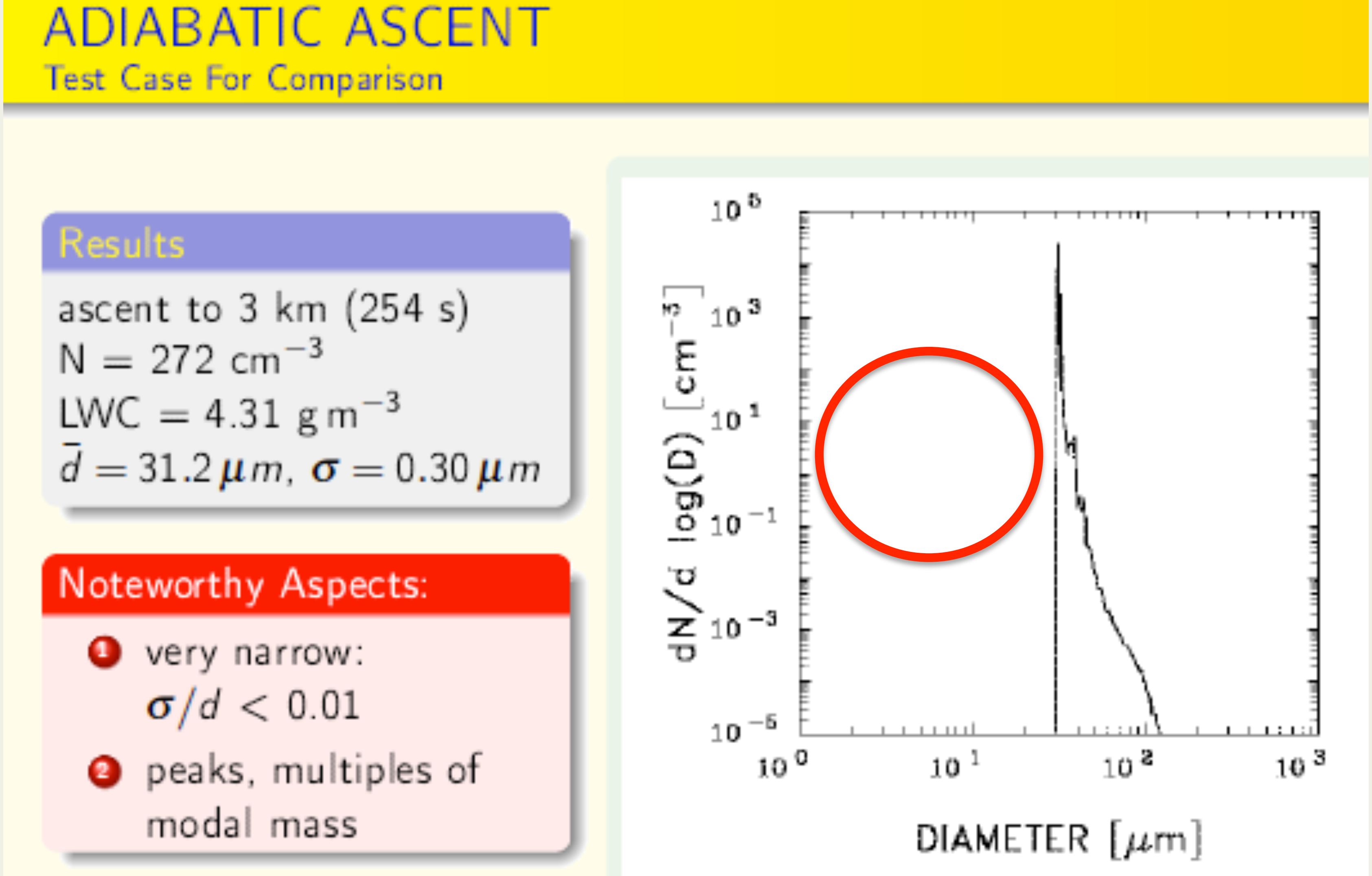


Model of Rain Formation

1. Dynamical cloud model
2. Generate trajectories
3. Lagrangian microphysical model
4. Combine droplets from trajectories; continue growth
5. Inject resulting embryos into cloud-water fields
6. Allow continued growth until decay of the cloud

Trajectories through the cloud

courtesy of Al Cooper, NCAR



ADIABATIC ASCENT

Test Case For Comparison

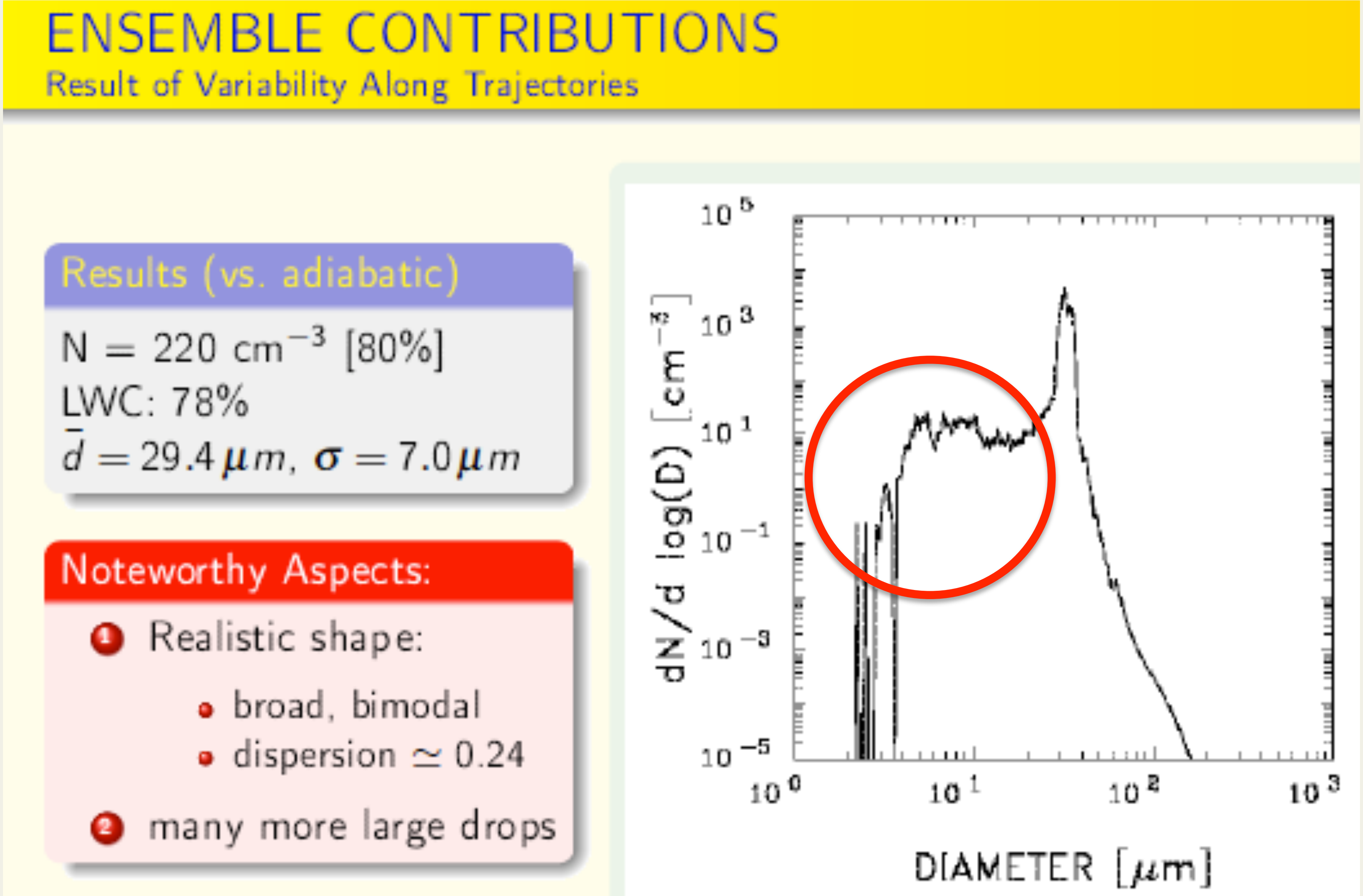
Results

ascent to 3 km (254 s)
$N = 272\ \mathrm{cm}^{-3}$
$\mathrm{LWC} = 4.31\ \mathrm{g\,m}^{-3}$
$\bar{d} = 31.2\,\mu m,\ \sigma = 0.30\,\mu m$

Noteworthy Aspects:

① very narrow: $\sigma/d < 0.01$
② peaks, multiples of modal mass

courtesy of Al Cooper, NCAR



ENSEMBLE CONTRIBUTIONS

Result of Variability Along Trajectories

Results (vs. adiabatic)

N = 220 cm^{-3} [80%]
LWC: 78%
$\bar{d} = 29.4\,\mu m$, $\sigma = 7.0\,\mu m$

Noteworthy Aspects:

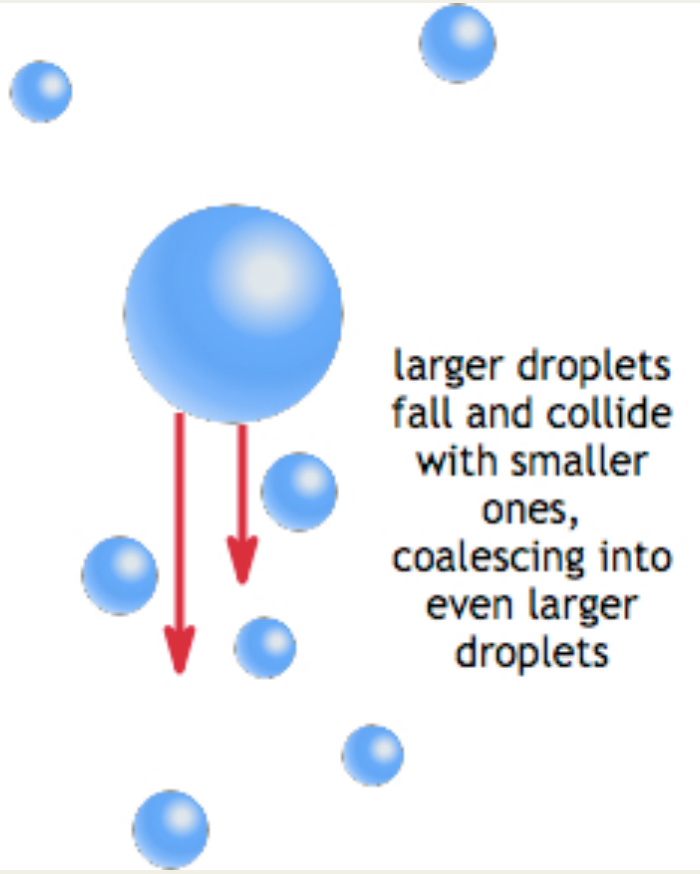
1. Realistic shape:
   - broad, bimodal
   - dispersion $\simeq$ 0.24
2. many more large drops

courtesy of Al Cooper, NCAR

Small impact of small-scale turbulence: because condensational growth is reversible, droplets grow more in higher S, and then less in lower S, and the two situations change rapidly.

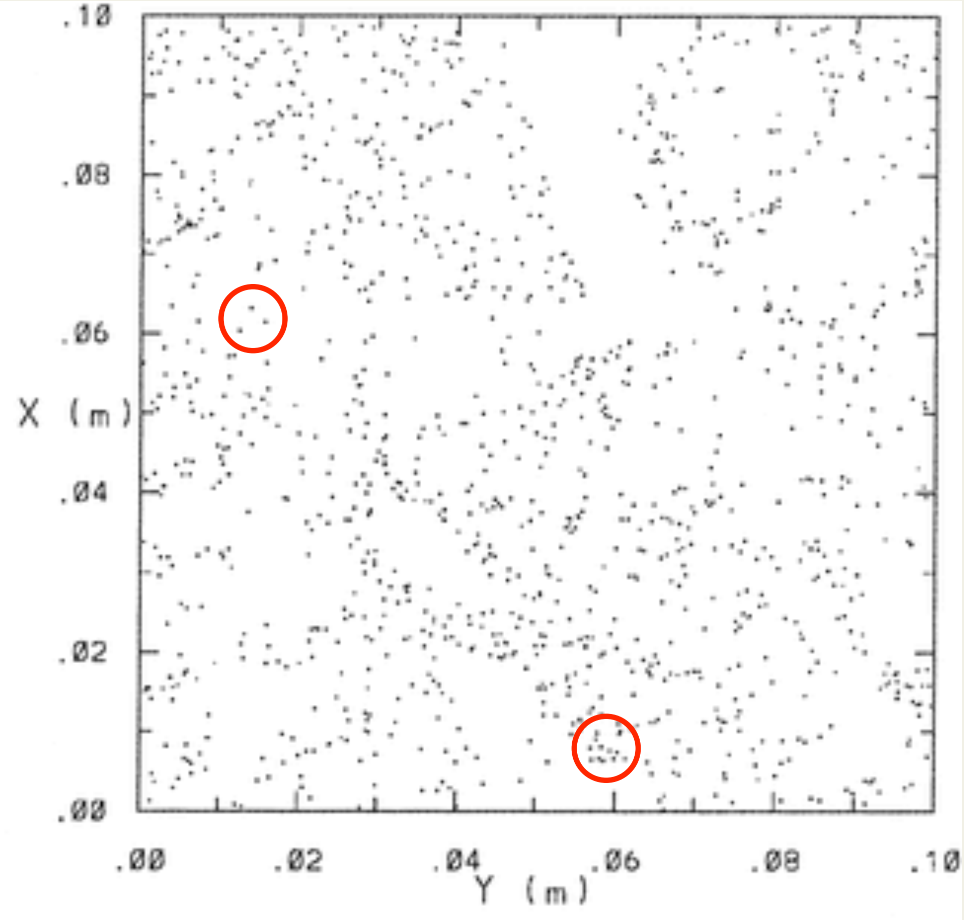
Entrainment/mixing and "large-eddy hopping" provide additional effects contributing to the spectral broadening.

Growth by collision/coalescence



Textbook explanation of rain formation in ice-free clouds: gravitational collision-coalescence…

Growth by collision/coalescence: nonuniform distribution of droplets in space (because of inertial clustering) affects droplet collisions…



Number of collisions: $N_i \, N_j \, K_{ij}$

N_i , N_j - concentrations

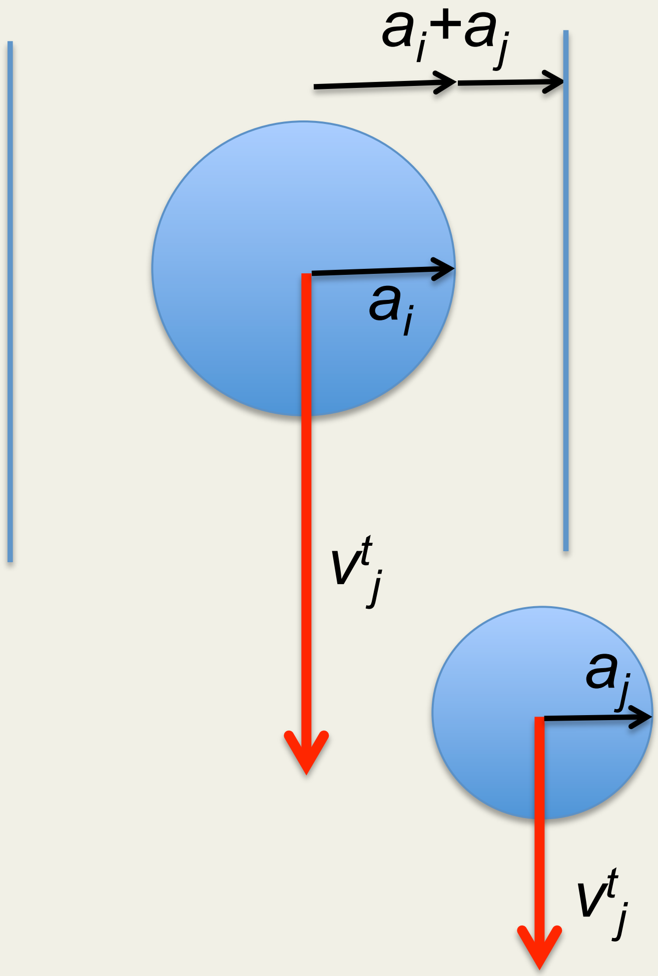
K_{ij} - collision kernel
(~probability of a collision between two droplets)

gravitational kernel:

$$K_{ij} = \pi \, (a_i + a_j)^2 \, |v_i^t - v_j^t| E_{ij}^g$$

gravitational kernel:

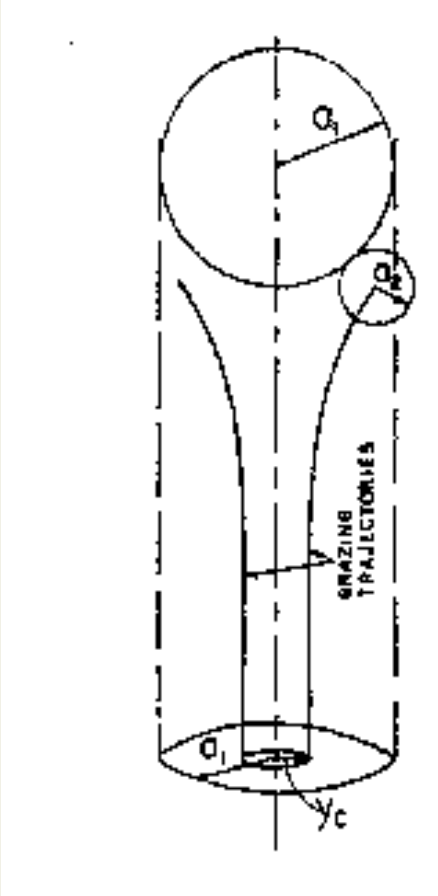
$$K_{ij} = \boxed{\pi\ (a_i + a_j)^2\ |v_i^t - v_j^t|} E_{ij}^g$$



collisional cylinder –
“geometric collision”

gravitational kernel:

$$K_{ij} = \pi \, (a_i + a_j)^2 \, |v_i^t - v_j^t| \boxed{E_{ij}^g}$$



collision efficiency

gravitational kernel:

$$K_{ij} = \pi\,(a_i + a_j)^2\,|v_i^t - v_j^t|E_{ij}^g$$

generalized kernel (gravity plus turbulence):

$$K_{ij} = K_{ij}^{tg}\,E_{ij}^g\,\eta_E$$

Saffman and Turner
(*JFM* 1956)
Wang et al.
(*JAS* 2005)
Grabowski and Wang
(*ARFM* 2013)

$$K_{ij}^{tg} = 2\pi R^2\,\langle|w_r(r = R)|\rangle\,g_{ij}(r = R)$$

$$R = a_i + a_j$$

$$w_r = \mathbf{r}\cdot(V_i - V_j)/r$$

radial relative velocity

g_{ij}

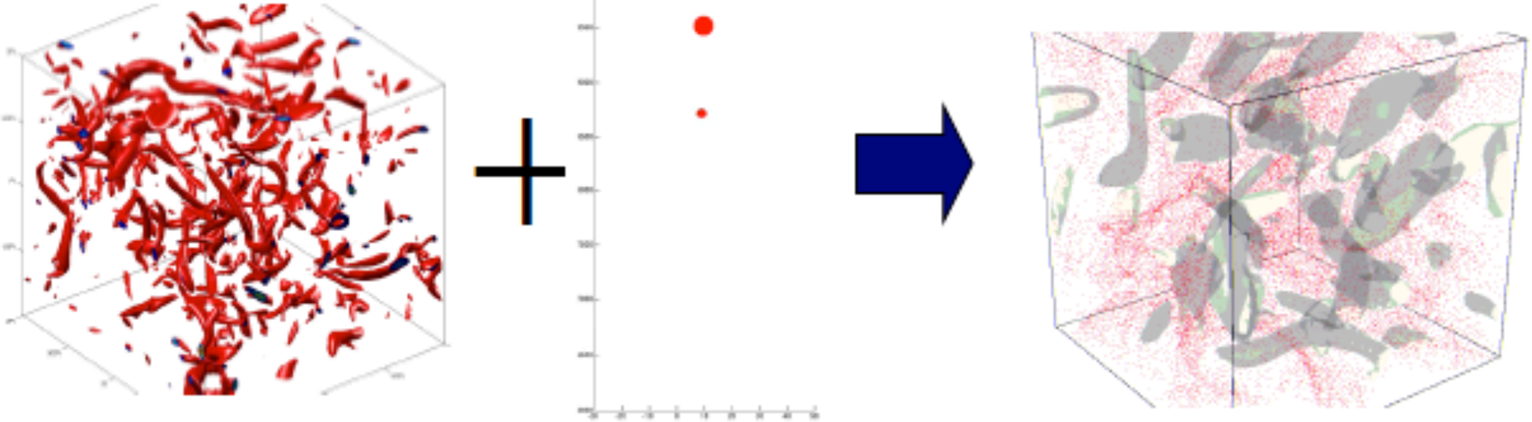
radial distribution function

The hybrid DNS approach: including disturbance flows due to droplets

$$\vec{U}(\vec{x},t)+\sum_{k=1}^{N_p}\vec{u}_s(\vec{r}_k;a_k,\vec{V}_k-\vec{U}(\vec{Y}_k,t)-\vec{u}_k)$$

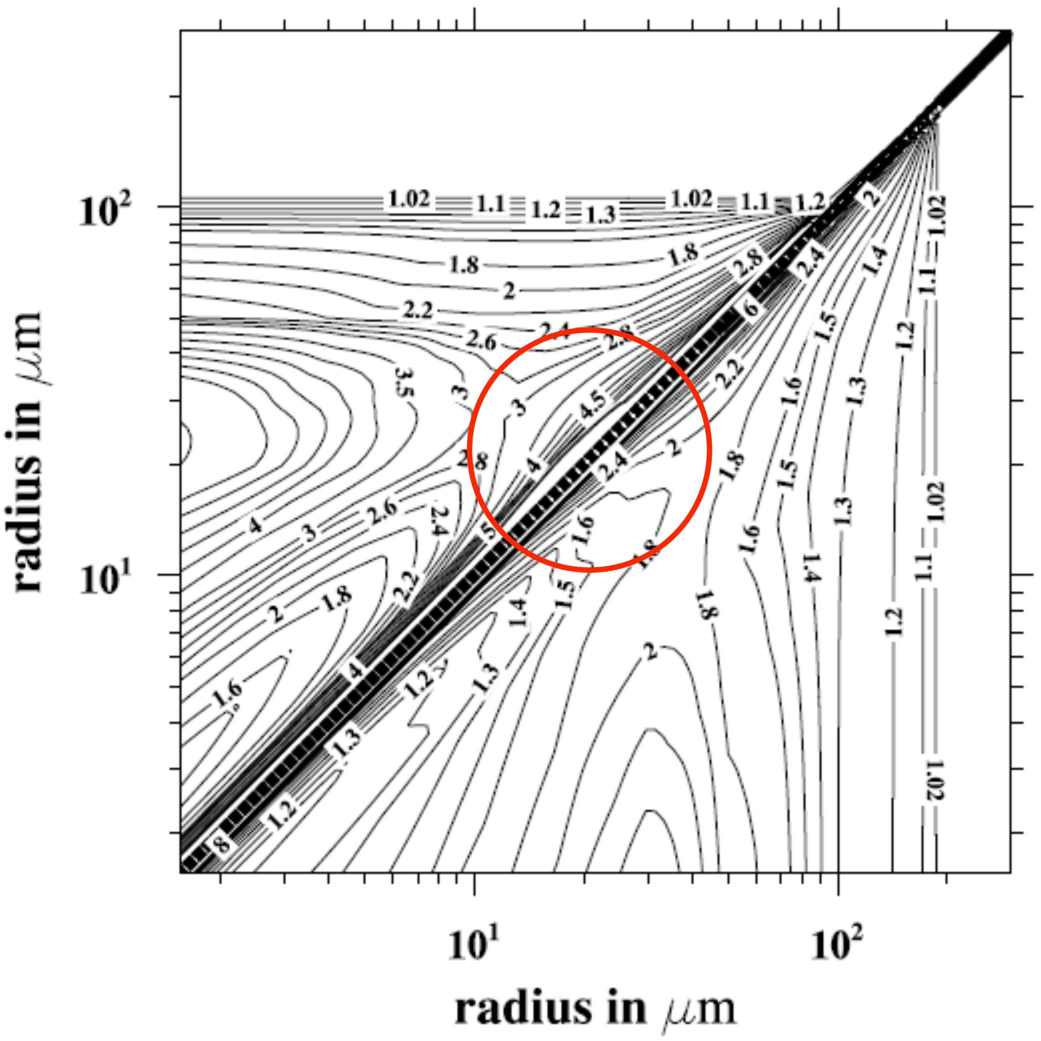
Background turbulent flow

Disturbance flows due to droplets



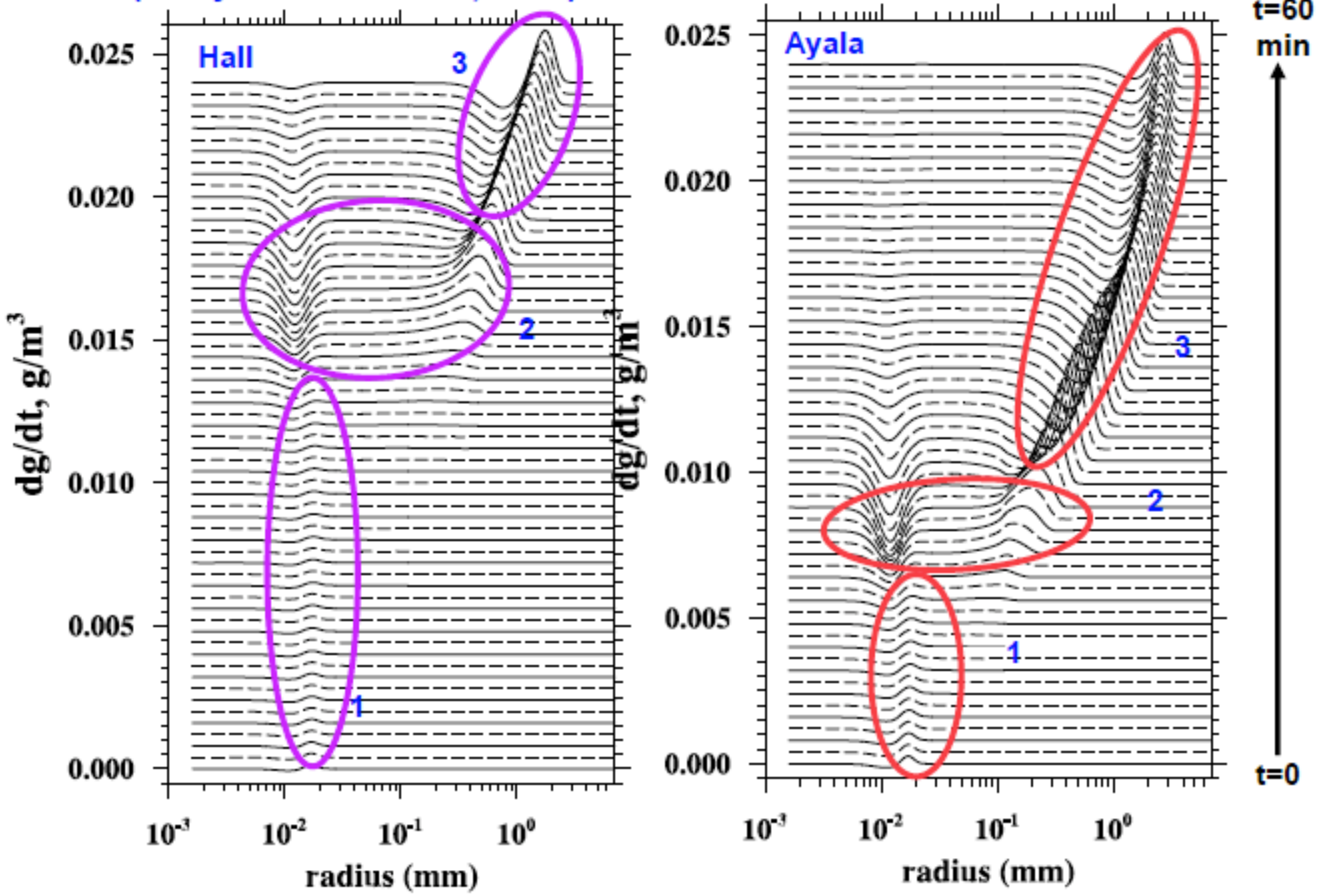
Features: Background turbulent flow can affect the disturbance flows;
No-slip condition on the surface of each droplet is satisfied on average;
Both near-field and far-field interactions are considered.

Wang, Ayala, and Grabowski, J. Atmos. Sci. **62**: 1255-1266 (2005).
Ayala, Wang, and Grabowski, J. Comp. Phys. **225**: 51-73 (2007).



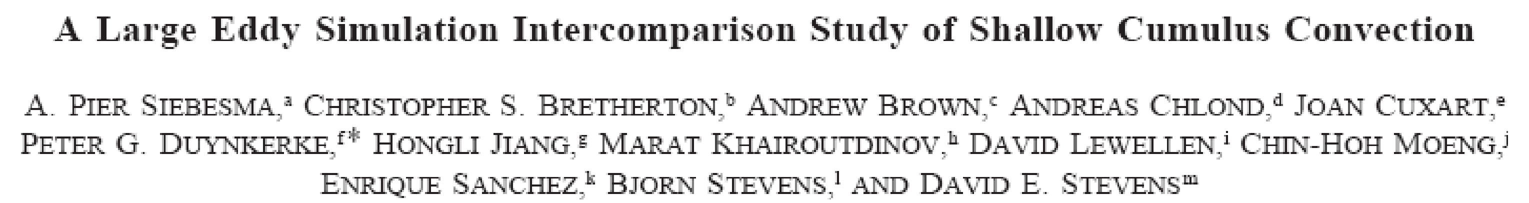
Enhancement factor for the collision kernel (the ratio between turbulent and gravitation collision kernel in still air) including turbulent collision efficiency; $\varepsilon =$ 100 and 400 cm^2 s^{-3}.

1. Autoconversion; 2. Accretion; 3. Hydrometeor self-collection (Berry and Reinhardt, 1974)



without turbulence

with turbulence, ε = 400 cm^{2}s^{-3}



A Large Eddy Simulation Intercomparison Study of Shallow Cumulus Convection

A. PIER SIEBESMA,[a] CHRISTOPHER S. BRETHERTON,[b] ANDREW BROWN,[c] ANDREAS CHLOND,[d] JOAN CUXART,[e]
PETER G. DUYNKERKE,[f]* HONGLI JIANG,[g] MARAT KHAIROUTDINOV,[h] DAVID LEWELLEN,[i] CHIN-HOH MOENG,[j]
ENRIQUE SANCHEZ,[k] BJORN STEVENS,[l] AND DAVID E. STEVENS[m]

JAS
2003

Δx=Δy=50m; Δz=20m

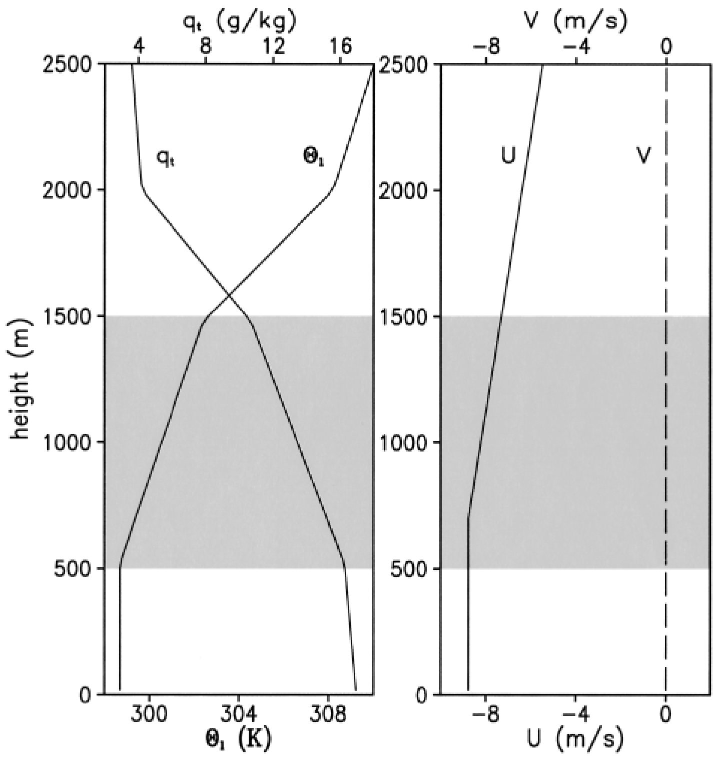
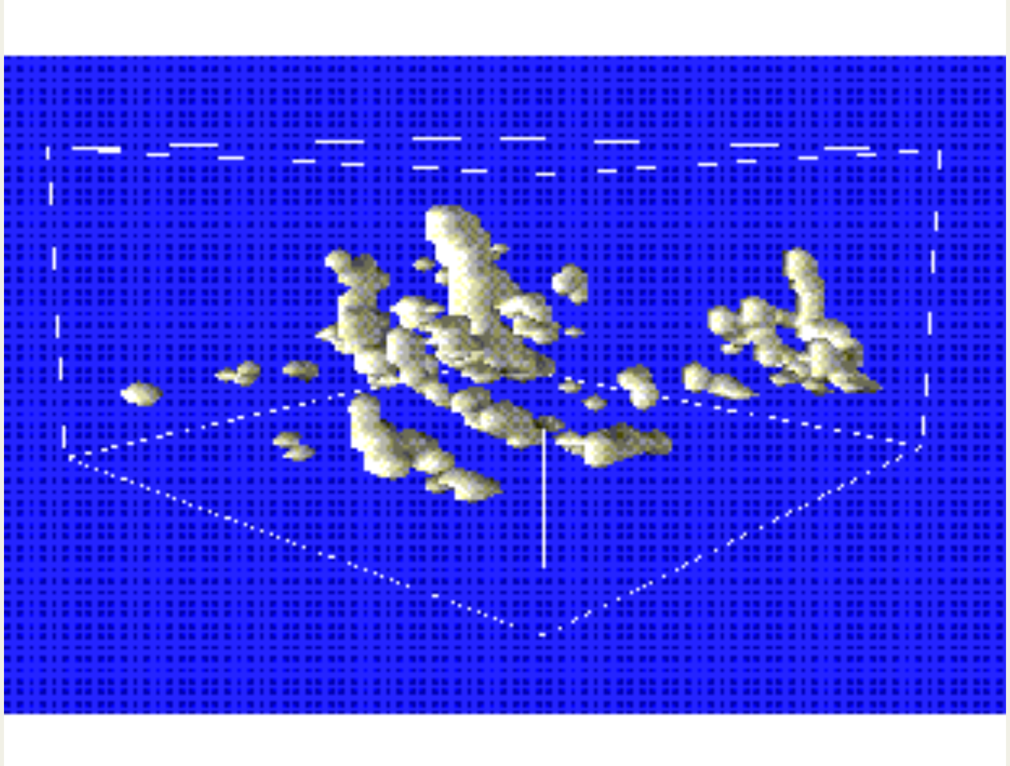


FIG. 1. Initial profiles of the total water specific humidity q_t, the liquid water potential temperature θ_ℓ, and the horizontal wind components u and v. The shaded area denotes the conditionally unstable cloud layer.



The Barbados Oceanographic and Meteorological Experiment (BOMEX) case (Holland and Rasmusson 1973)

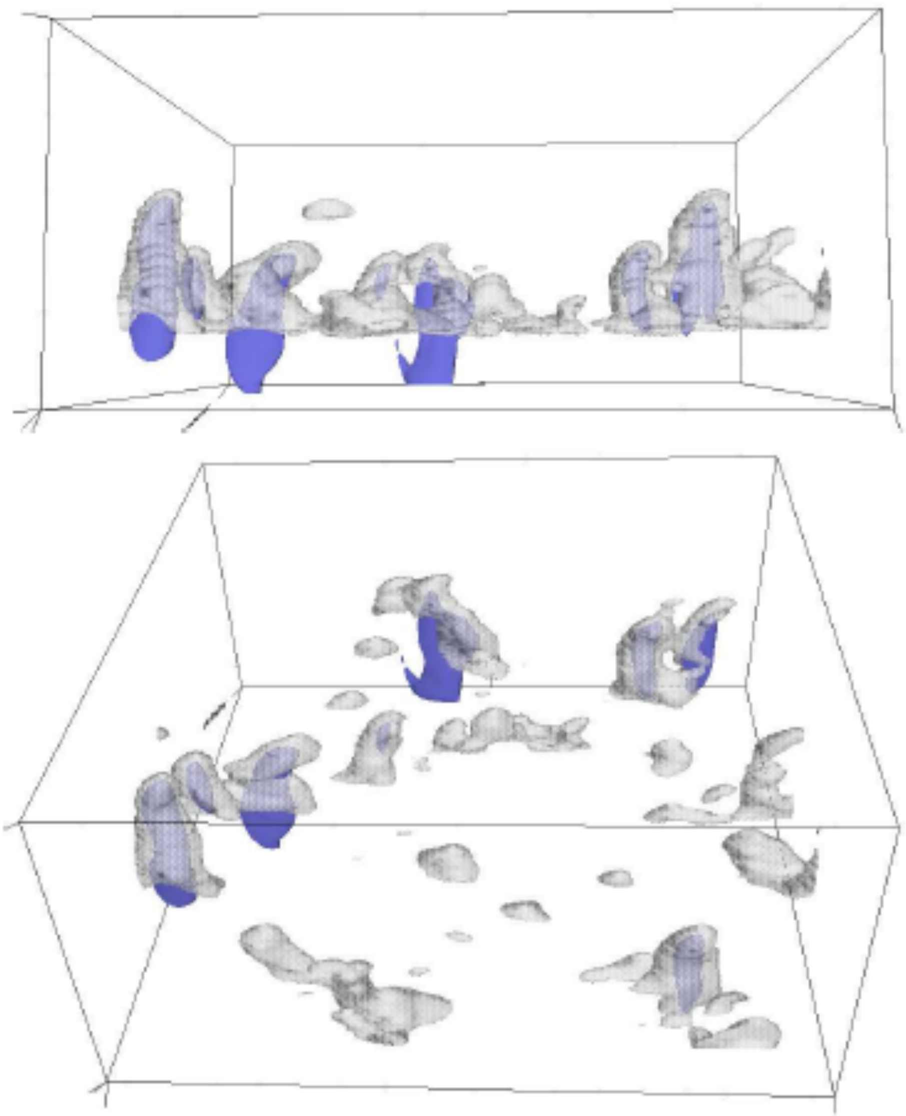
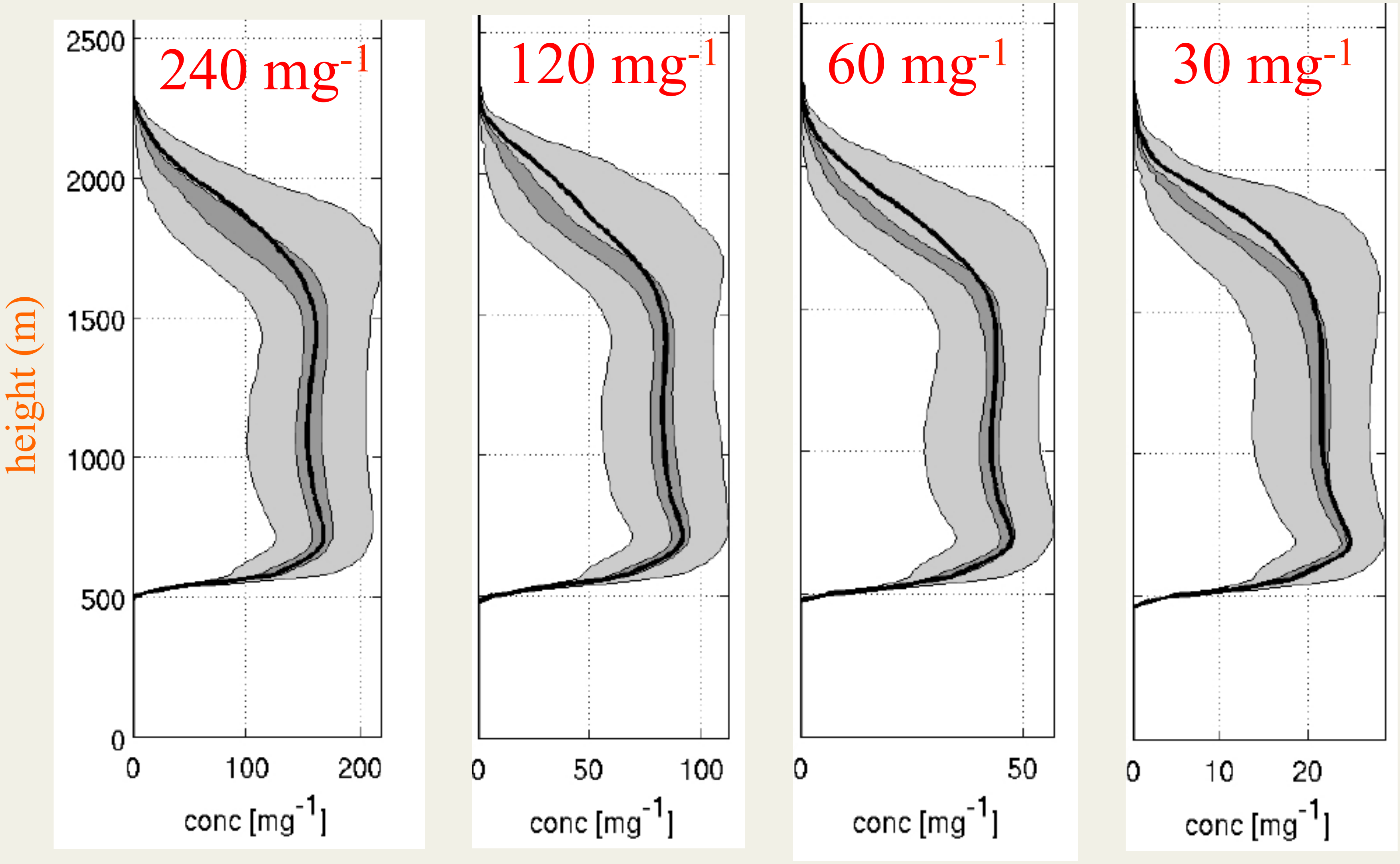
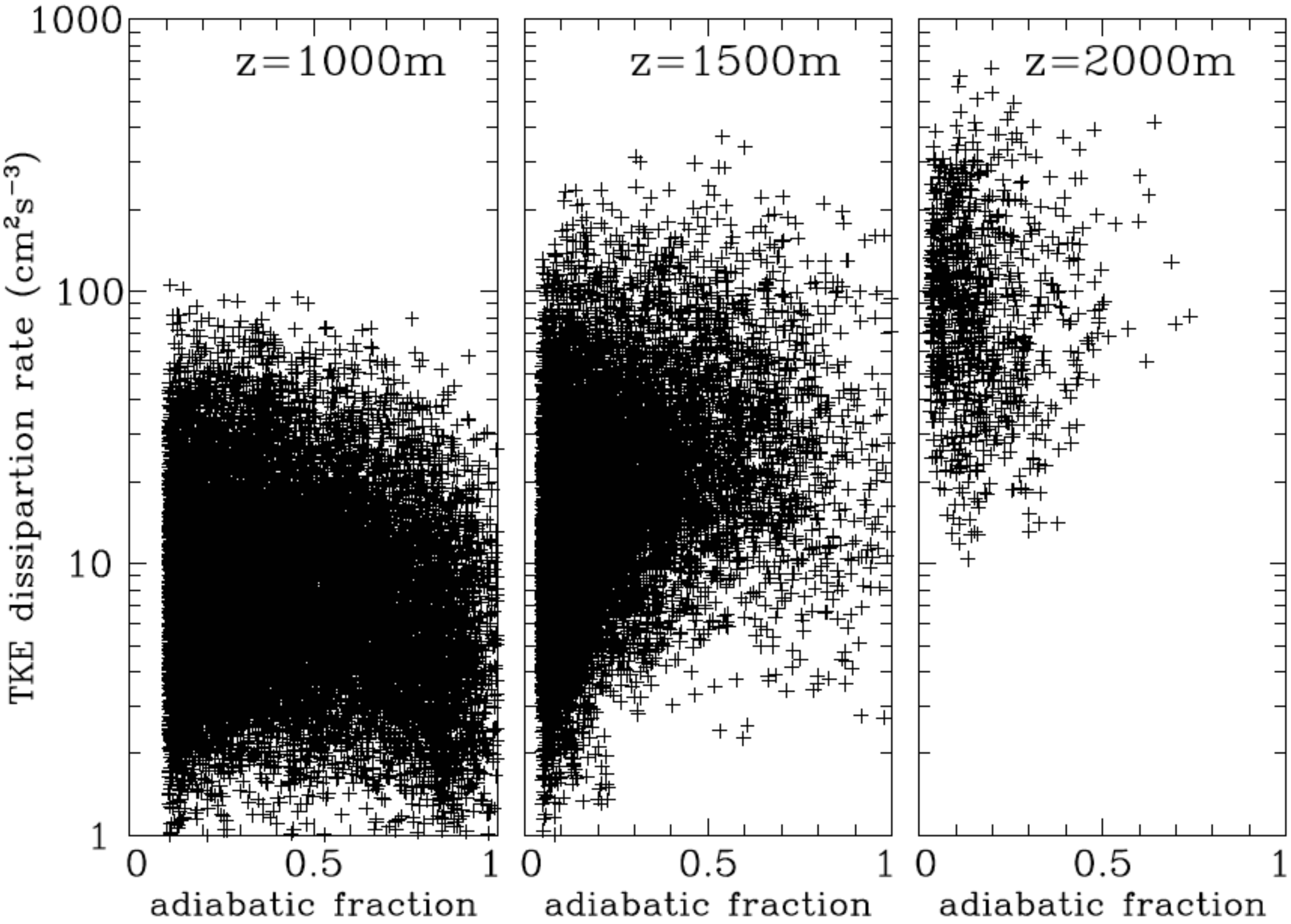


Fig. 4. Snapshots of cloud water mixing ratio (transparent gray) and rain water mixing ratio (solid blue) at the 6th hour of the simulation. The isosurfaces show values $q_c = 0.05\ \mathrm{g\,kg^{-1}}$ and $q_r = 0.02\ \mathrm{g\,kg^{-1}}$.

8 simulations: 4 CCN concentrations (extra clean to weakly polluted), contrasting gravitational and turbulent collision kernels

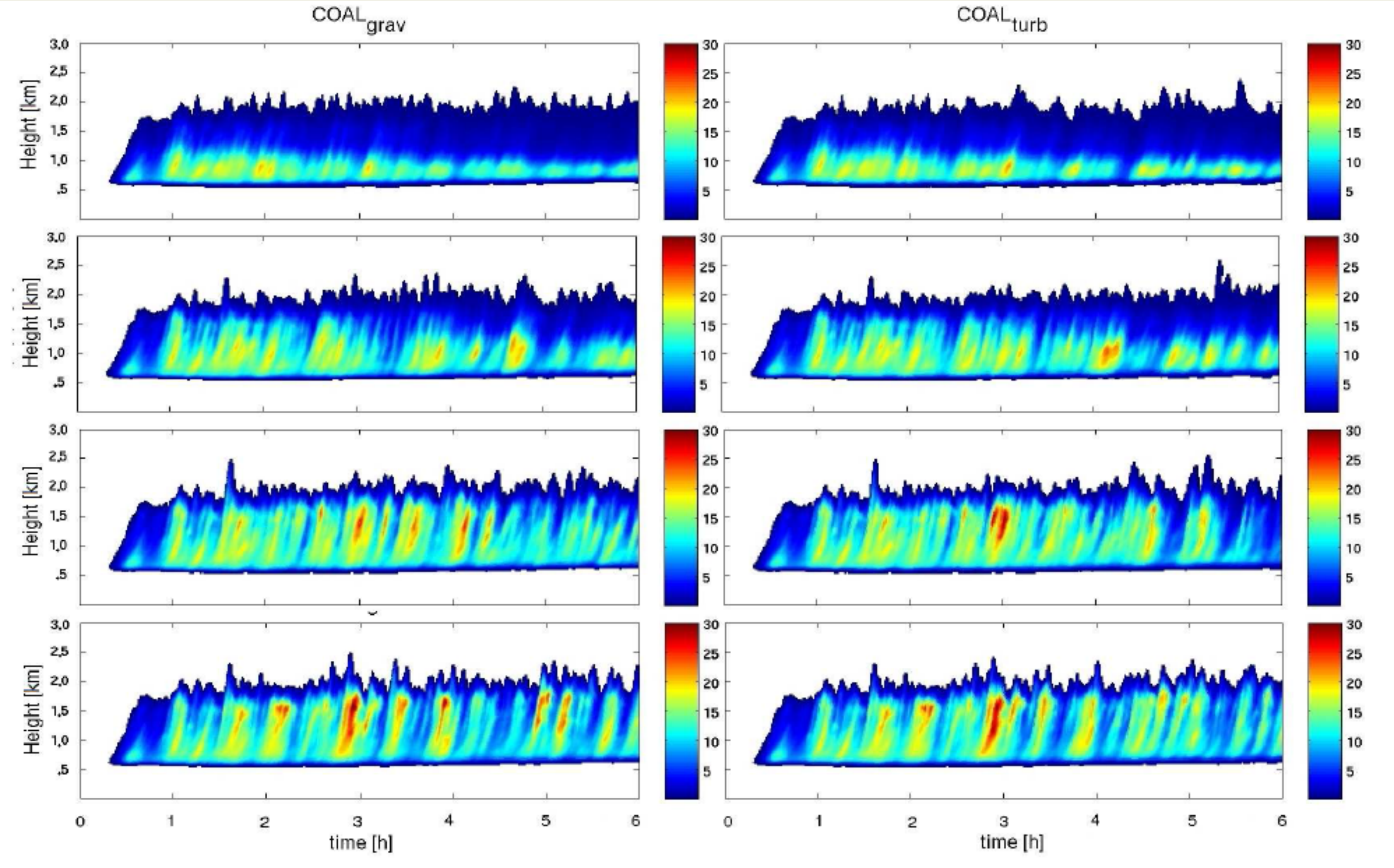


droplet concentration (mg^{-1})



N120 simulation; all cloudy points (q_c> 0.1g/kg) with ε > 1cm^2/s^3; hours 3-6

Domain-averaged cloud water mixing ratio (r < 25 μm)



30

60

120

240

Gravitational kernel

Turbulent kernel

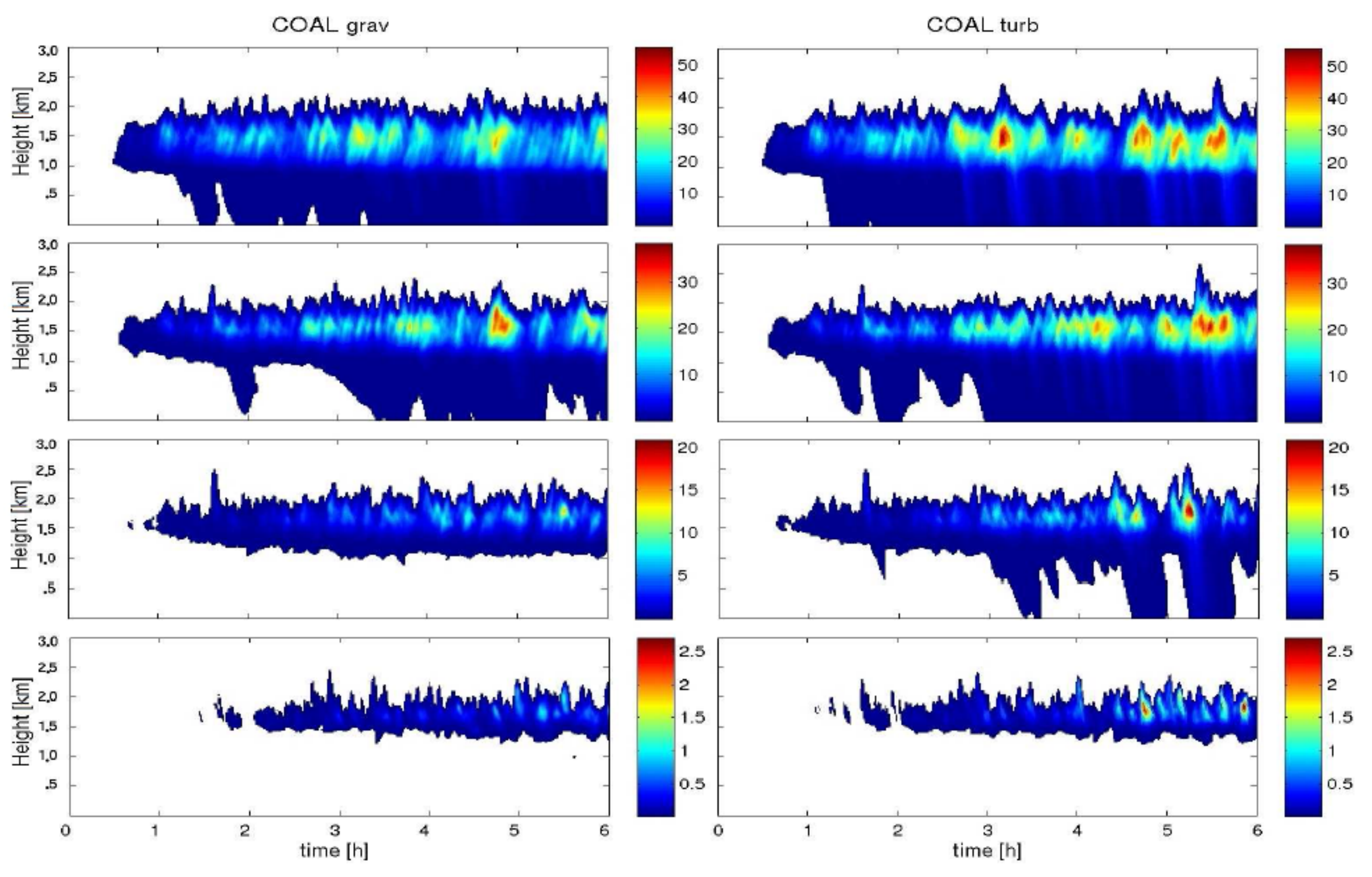
Domain-averaged drizzle/rain water mixing ratio (r > 25 μm)

30

60

120

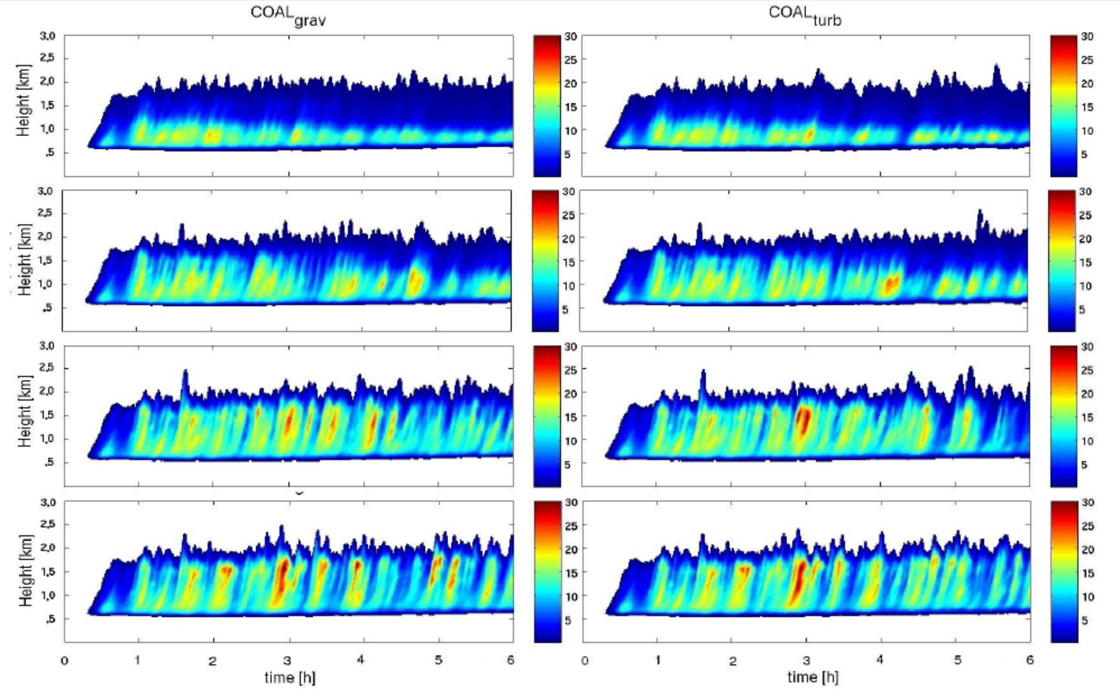
240



Gravitational kernel

Turbulent kernel

Cloud water



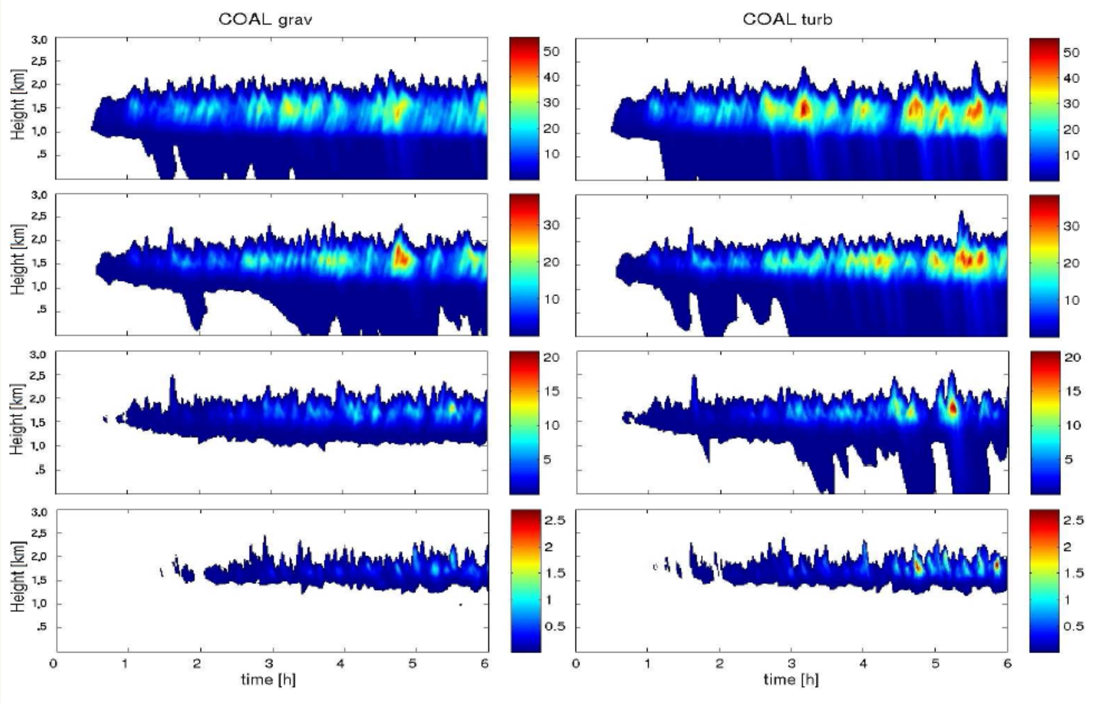
30

60

120

240

Drizzle/rain water



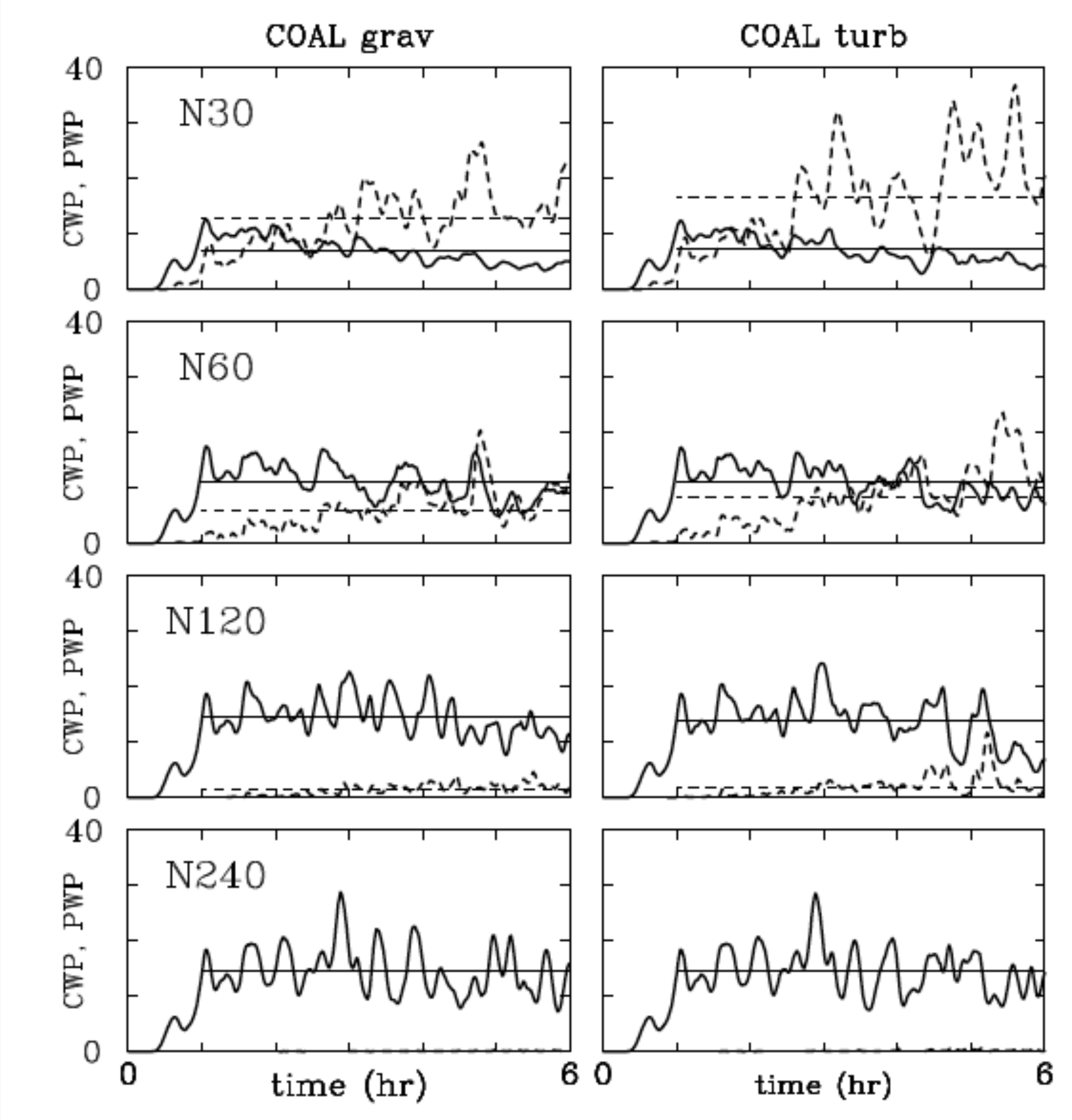
30

60

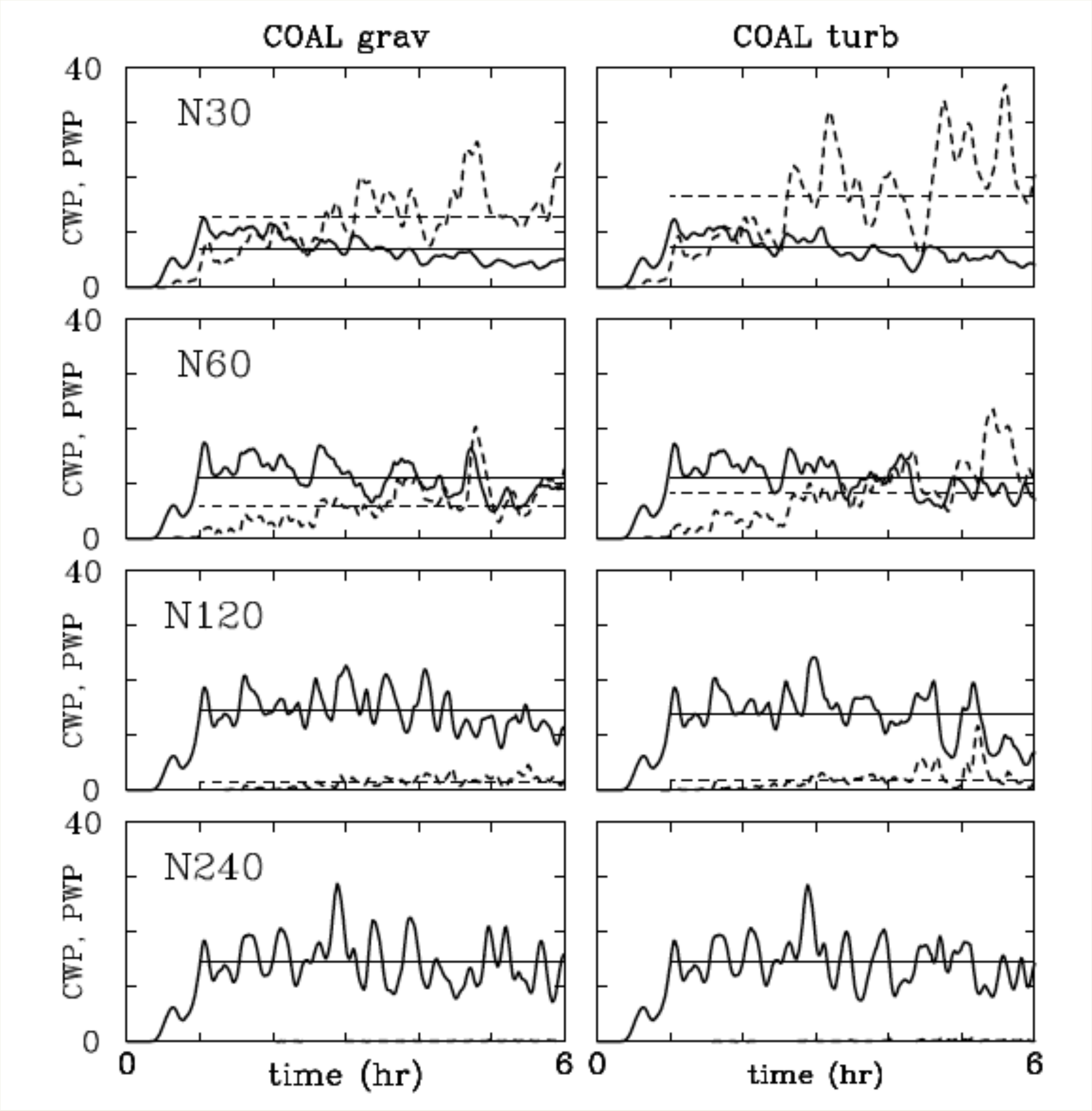
120

240

Domain-averaged cloud water path (CWP, solid line) and precip water path (PWP, dashed line)

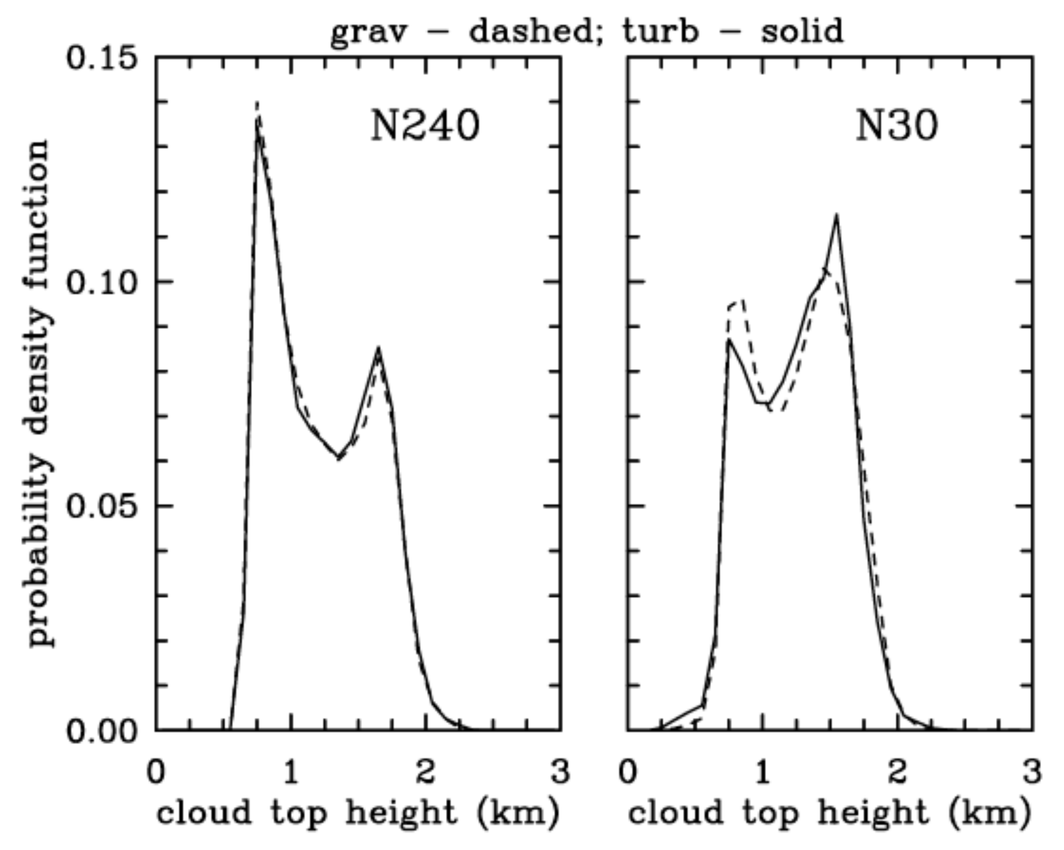


Domain-averaged cloud water path (CWP, solid line) and precip water path (PWP, dashed line)



Microphysical + dynamical enhancement

Microphysical enhancement

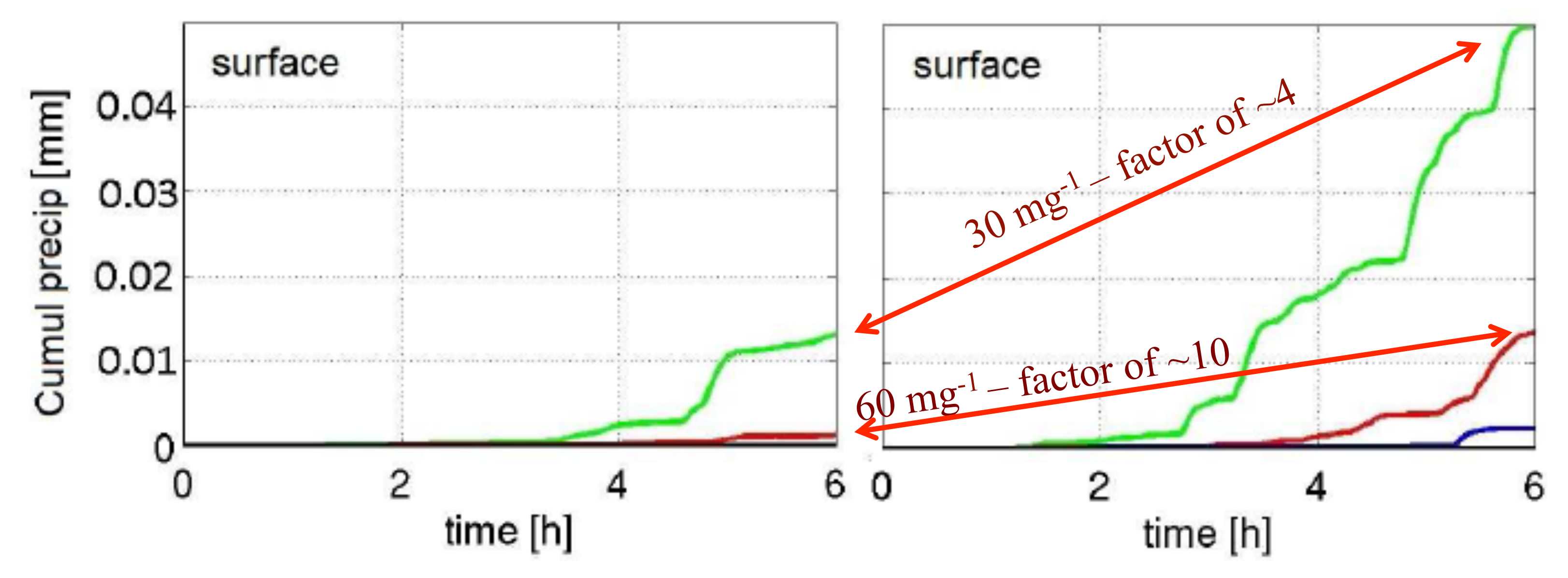


Dynamical enhancement: turbulent precipitating clouds seem to reach higher levels…

Surface rain accumulation from the cloud field:

Gravitational kernel

Turbulent kernel



30 mg^{-1} – factor of ~4

60 mg^{-1} – factor of ~10

Summary:

Small-scale turbulence seems to have an insignificant effect on diffusional growth of cloud droplets.

Turbulence seems to plays a significant role when entrainment and mixing is considered through the “large-eddy hopping” mechanism, local heterogeneity of mixing, and in-cloud activation.

Small-scale turbulence appears to have a significant effect on collisional growth. Rain tends to form earlier in a single cloud, and turbulent clouds rain more. This appears to be a combinations of two effects:

i) the microphysical enhancement: more cloud water available to be converted to rain when rain forms earlier in the cloud lifecycle;

ii) the dynamical enhancement: off-loading cloud condensate increases cloud buoyance allowing clouds to reach higher levels.