Atmospheric Dynamical Cores

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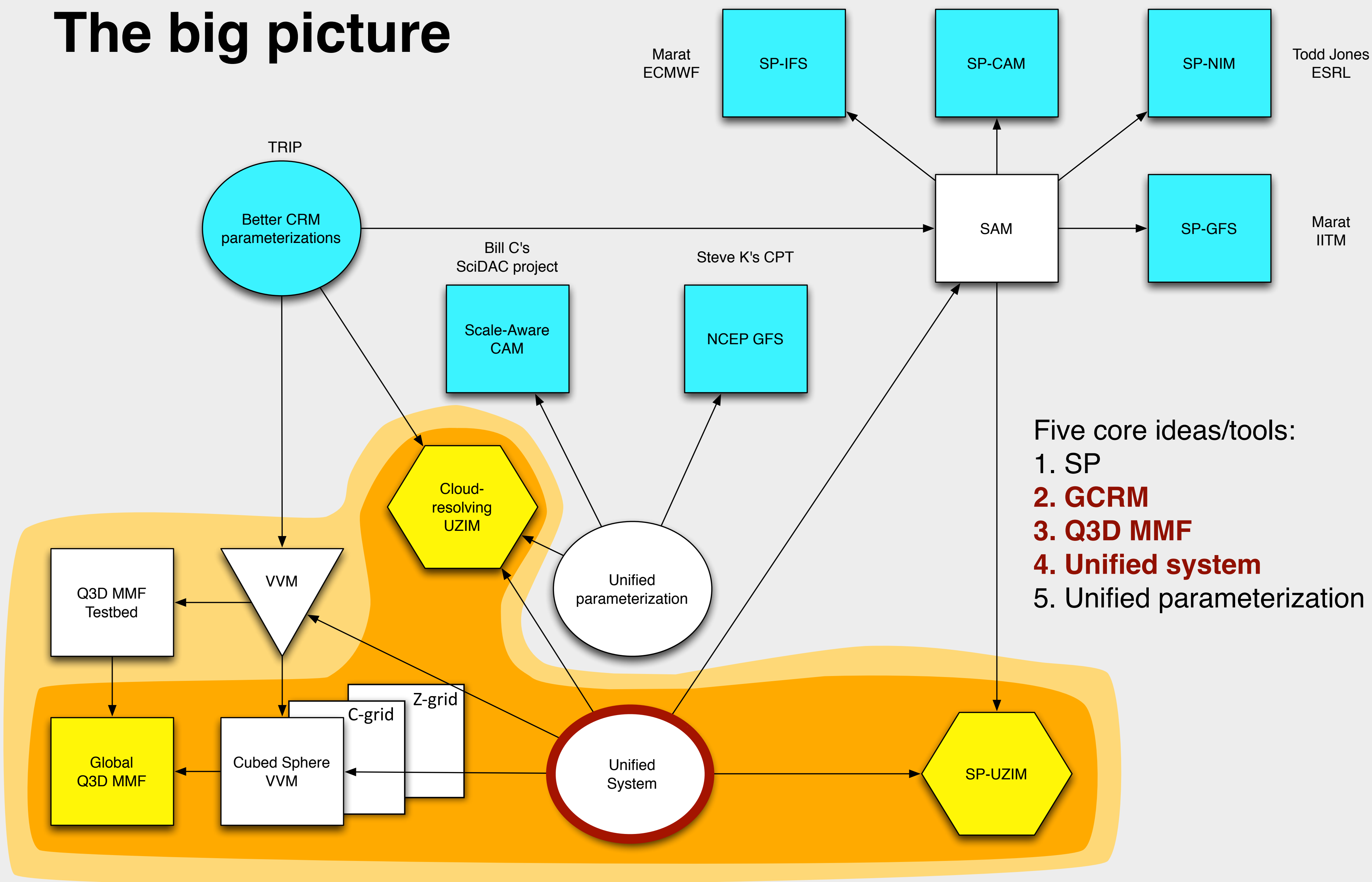
DF working group
15th CMMAP team meeting, 6–8 August, 2013, Fort Collins, CO

The big picture



Five core ideas/tools:
1. SP
2. **GCRM**
3. **Q3D MMF**
4. **Unified system**
5. Unified parameterization

- The unified system described by Arakawa and Konor (*MWR*, 2009) and Konor (*MWR*, 2013).
- The use of the unified system in the Q3D MMF is one of the topics in this presentation.

Progress report:

Developments of the Global Cloud Resolving Model and the cubed-sphere dynamical core

- UZIM (Unified Z-grid Icosahedral Model) development: We think that we have found a proper 3-D elliptic solver for the UZIM.

- Some progress has been made in the development of the sigma version of the UZIM.

- Couple of papers have been accepted for publication (Heikes et al., *MWR* 2013 and Konor, *MWR* 2013).

- Cubed-sphere dynamical core development has started, which will be used as the GCM component and chassis of the global Q3D-MMF.

Features of the unified system of equations

Arakawa and Konor (2009) and Konor (2013)

- Yields elastic solutions for large-scale quasi-hydrostatic motion and anelastic solutions for small-scale nonhydrostatic motion.
- Filters vertically propagating acoustic waves of all scales.
- Does not introduce approximations to the thermodynamic and momentum equations. The continuity equation uses a properly defined quasi-hydrostatic density.
- Does not need a basic (or mean) state.
- Has a total energy equation.
- Covers a wide range of horizontal scales from turbulence to planetary scales so that it is suitable for the use in global cloud resolving models.

Features of the unified system of equations (Cont.)

- Unified equations are available in the height, quasi-hydrostatic pressure, sigma (normalized quasi-hydrostatic pressure), isentropic (quasi-Lagrangian) and hybrid sigma-isentropic vertical coordinates.

- An existing quasi-hydrostatic model can be easily converted to a unified model through add-on modules.

..."The recent "unified" approximation (Arakawa and Konor, 2009) is one of the most accurate"...

Dukowicz (Accepted for publication by MWR, 2013)

The conclusion is from a normal mode analysis of gravity waves

Equations of the unified system

Thermodynamic variables:

θ z (not divided to quasi-hydrostatic and nonhydrostatic components.)

$$\rho \equiv \rho_{qs} + \delta\rho \qquad p \equiv p_{qs} + \delta p \qquad T \equiv T_{qs} + \delta T \qquad \pi \equiv \pi_{qs} + \delta\pi$$

$$\pi \equiv \left(p/p_{00}\right)^{\kappa} \qquad \pi_{qs} \equiv \left(p_{qs}/p_{00}\right)^{\kappa} \qquad T_{qs} \equiv \pi_{qs}\theta$$

where *qs* and δ denote the quasi-hydrostatic and nonhydrostatic components, respectively.

State equations:

$$p = \rho R \pi \theta \qquad p_{qs} = \rho_{qs} R \pi_{qs} \theta$$ "the quasi-hydrostatic state equation"

- The quasi-hydrostatic state in the unified system can be interpreted as a 3D time-dependent balanced state, for which $Dw/Dt = 0$. The 3D time-dependent balanced states are also used by Marshall et al. (1997), Durran (2008) and Smolarkiewicz et al. (2001).
- The quasi-hydrostatic variables are diagnosed from θ.
- The quasi-hydrostatic variables differ from that in the quasi-hydrostatic system because θ has a nonhydrostatic component. For large scales, θ becomes purely quasi-hydrostatic and, then the quasi-hydrostatic variables of the unified system become identical to the true quasi-hydrostatic variables.

Equations of the unified system (Cont.)

Thermodynamic equation:

$$\frac{D\theta}{Dt} = \frac{Q}{c_p \pi}$$

Unified continuity equation:

$$\nabla_H \cdot \left(\rho_{qs}\mathbf{v}\right) + \frac{\partial}{\partial z}\left(\rho_{qs} w\right) = -\frac{\partial \rho_{qs}}{\partial t}$$

- No approximation is made in the thermodynamic equation.
- The only approximation (or assumption) that distinguishes the unified system from the fully compressible system is made in the continuity equation ($\rho_{qs} >> \delta\rho$ and $\partial\delta\rho/\partial t = 0$).
- The continuity equation is not a prognostic equation in the unified system, which is used to diagnose the vertical mass flux (and vertical velocity).

Equations of the unified system (Cont.)

Determination of $\partial\left(\pi_{qs}\right)_S/\partial t$ **:**

$\partial\left(\pi_{qs}\right)_T/\partial t = 0$ is not a right condition for the *z* coordinate (even for $\left(p_{qs}\right)_T = 0$). Therefore, we need two "hydrostatic" equations.

$$\frac{\partial \pi_{qs}}{\partial z} = -\frac{1}{c_p\theta} \quad \text{and} \quad \frac{\partial p_{qs}}{\partial z} = -g\rho_{qs}$$

From these two equations and the continuity equation, we obtain

$$\frac{\partial\left(\pi_{qs}\right)_S}{\partial t} = \frac{\left(p_{qs}/\kappa\pi_{qs}\right)_T \dfrac{g}{c_p}\displaystyle\int_{z=z_S}^{z_T}\left(1/\theta^2\right)\left(\partial\theta/\partial t\right)dz + g\,\partial M/\partial t}{\left(p_{qs}/\kappa\pi_{qs}\right)_S - \left(p_{qs}/\kappa\pi_{qs}\right)_T}$$

where $\partial M/\partial t = -\nabla_H \cdot \int_{z=z_S}^{z_T}\left(\rho_{qs}\mathbf{v}\right)dz$

Determination of $\partial\rho_{qs}/\partial t$ **:**

$$\frac{\partial\rho_{qs}}{\partial t} = \rho_{qs}\left(\frac{1-\kappa}{\kappa}\frac{1}{\pi_{qs}}\frac{\partial\pi_{qs}}{\partial t} - \frac{1}{\theta}\frac{\partial\theta}{\partial t}\right)$$

Equations of the unified system (Cont.)

Horizontal momentum equation:

$$\frac{\partial \mathbf{v}}{\partial t} = -q\mathbf{k} \times \left(\rho_{qs}\mathbf{v}\right) - w\frac{\partial \mathbf{v}}{\partial z} - \nabla_H K - c_p \theta \nabla_H \left(\pi_{qs} + \delta\pi\right) + \mathbf{F}_H$$

Vertical momentum equation:

$$\frac{Dw}{Dt} + c_p \theta \frac{\partial \delta\pi}{\partial z} = F_z$$

- No approximations are made in the horizontal and vertical momentum equations.
- The vertical momentum equation is not used to predict w. The vertical momentum equation is used to obtain the elliptic equation that determines the nonhydrostatic (Exner) pressure.

Equations of the unified system (Cont.)

Elliptic equation for nonhydrostatic (Exner) pressure:

$$\nabla_H \cdot \left(\rho_{qs} c_p \theta \nabla_H \delta\pi\right) + \frac{\partial}{\partial z}\left(\rho_{qs} c_p \theta \frac{\partial \delta\pi}{\partial z}\right) =$$

$$-\nabla_H \cdot \left(\rho_{qs} c_p \theta \nabla_H \pi_{qs}\right) - \nabla_H \cdot \left(\rho_{qs} \mathbf{G}_H\right) - \frac{\partial\left(\rho_{qs} G_z\right)}{\partial z} + \frac{\partial^2 \rho_{qs}}{\partial t^2}$$

- As the resolution approaches to the quasi-hydrostatic range, the elliptic solver needs to do a smaller job because the nonhydrostatic (Exner) pressure becomes very small.

Algorithm used in the integration of the discrete unified dynamical core

Prediction of θ through thermodynamic equation:

$$\theta^{(n+1)} = \theta^{(n)} + \delta t\, G_\theta\left(\theta^{(n)}, \mathbf{v}^{(n)}, w^{(n)}\right)$$

Prediction of quasi-hydrostatic quantities:

$$\left(\pi_{qs}\right)_S^{(n+1)} = G_{\pi S}\left[\theta^{(n+1)}, \left(\rho_{qs}\right)^{(n)}, \mathbf{v}^{(n)}, w^{(n)}\right]$$

$$\left(\pi_{qs}\right)^{(n+1)} = G_{\pi}\left[\theta^{(n+1)}, \left(\pi_{qs}\right)_S^{(n+1)}\right]$$

$$\left(\rho_{qs}\right)^{(n+1)} = G_{\rho}\left[\theta^{(n+1)}, \left(\pi_{qs}\right)^{(n+1)}\right]$$

Diagnosis of the balanced state

Diagnosis of time derivatives:

$$\left(\partial\rho_{qs}/\partial t\right)^{(n)} = G_{\rho t}\left[\left(\rho_{qs}\right)^{(n+1)}, \left(\rho_{qs}\right)^{(n)}, \left(\rho_{qs}\right)^{(n-1)}\right]$$

$$\left(\partial^2\rho_{qs}/\partial t^2\right)^{(n)} = G_{\rho tt}\left[\left(\partial\rho_{qs}/\partial t\right)^{(n)}, \left(\partial\rho_{qs}/\partial t\right)^{(n-1)}\right]$$

Algorithm used in the integration of the discrete unified dynamical core (cont.)

Diagnosis of nonhydrostatic Exner pressure through elliptic equation:

$$(\delta\pi)^{(n+1)} = G_{\delta\pi}\left[\left(\pi_{qs}\right)^{(n+1)}, \mathbf{v}^{(n)}, w^{(n)}, \left(\partial^2\rho_{qs}/\partial t^2\right)^{(n)}, \cdots\right]$$

Prediction of **v** through the horizontal momentum equation:

$$\mathbf{v}^{(n+1)} = \mathbf{v}^{(n)} + \delta t\, G_H\left[\left(\pi_{qs}\right)^{(n+1)}, (\delta\pi)^{(n+1)}, \mathbf{v}^{(n)}, w^{(n)}, \cdots\right]$$

Diagnosis of *w* through the continuity equation:

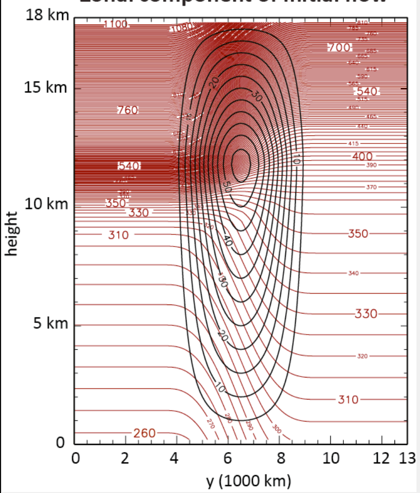
$$\left(\rho_{qs}w\right)_{k+1/2}^{(n+1)} = \left(\rho_{qs}w\right)_{k-1/2}^{(n+1)} + (\delta z)_k\, G_w\left[\left(\rho_{qs}\mathbf{v}\right)^{(n+1)}, \left(\partial\rho_{qs}/\partial t\right)^{(n)}\right]$$

$$w_{k+1/2}^{(n+1)} = \left(\rho_{qs}w\right)_{k+1/2}^{(n+1)} \Big/ \left(\rho_{qs}\right)_{k+1/2}^{(n+1)}$$

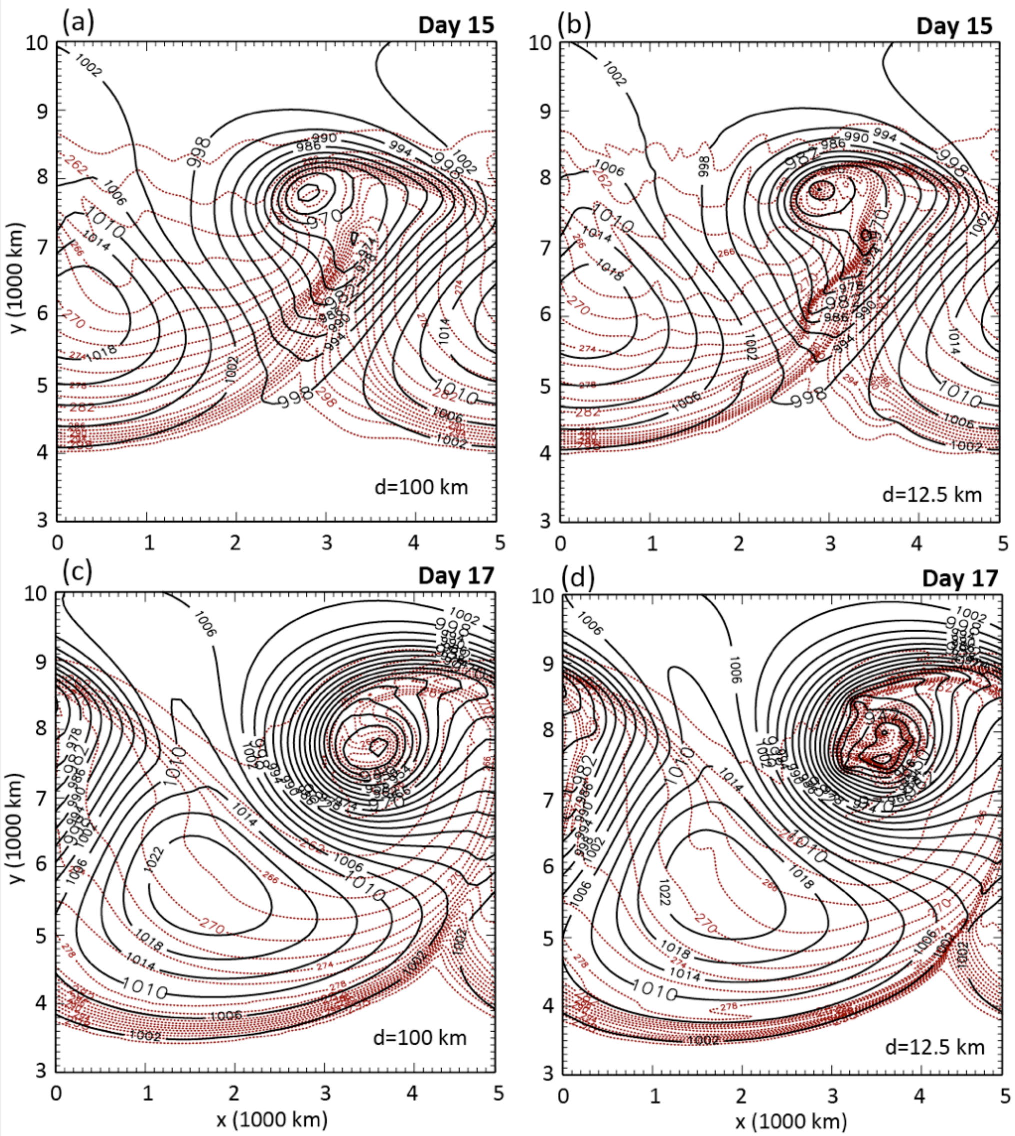
A comparison of the results obtained with the unified system to those obtained with the fully compressible, anelastic and pseudo-incompressible systems

Cyclone-scale simulations with the unified model

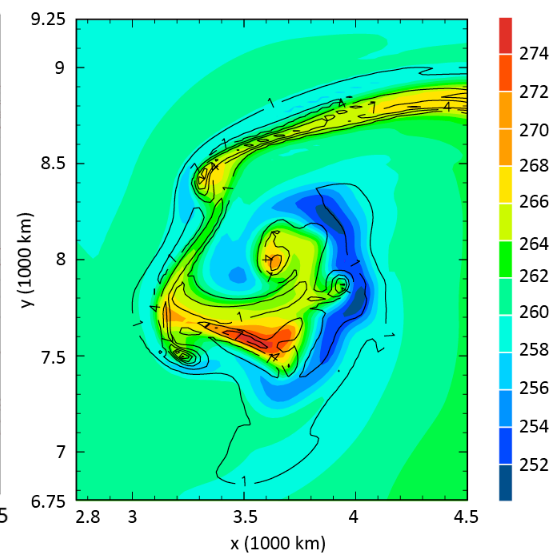
Zonal component of initial flow



Surface pressure ($p=p_{qs}+\delta p$, mb) and surface potential temperature (K)



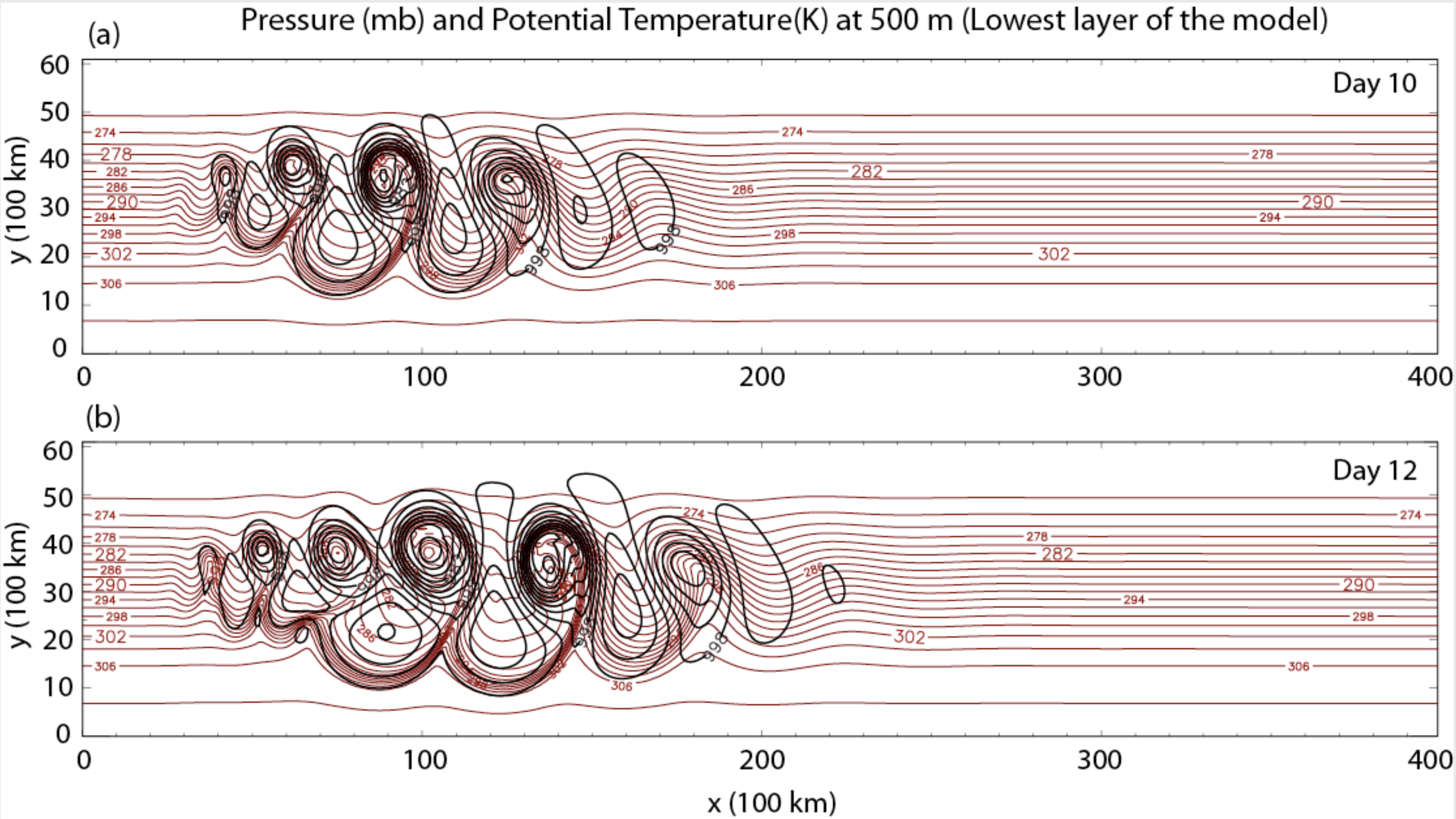
Surface vorticity (10^{-4} 1/s) and surface potential temperature (K) from high-res run (d=12.5 km) at day 17



Konor (*MWR*, 2013)

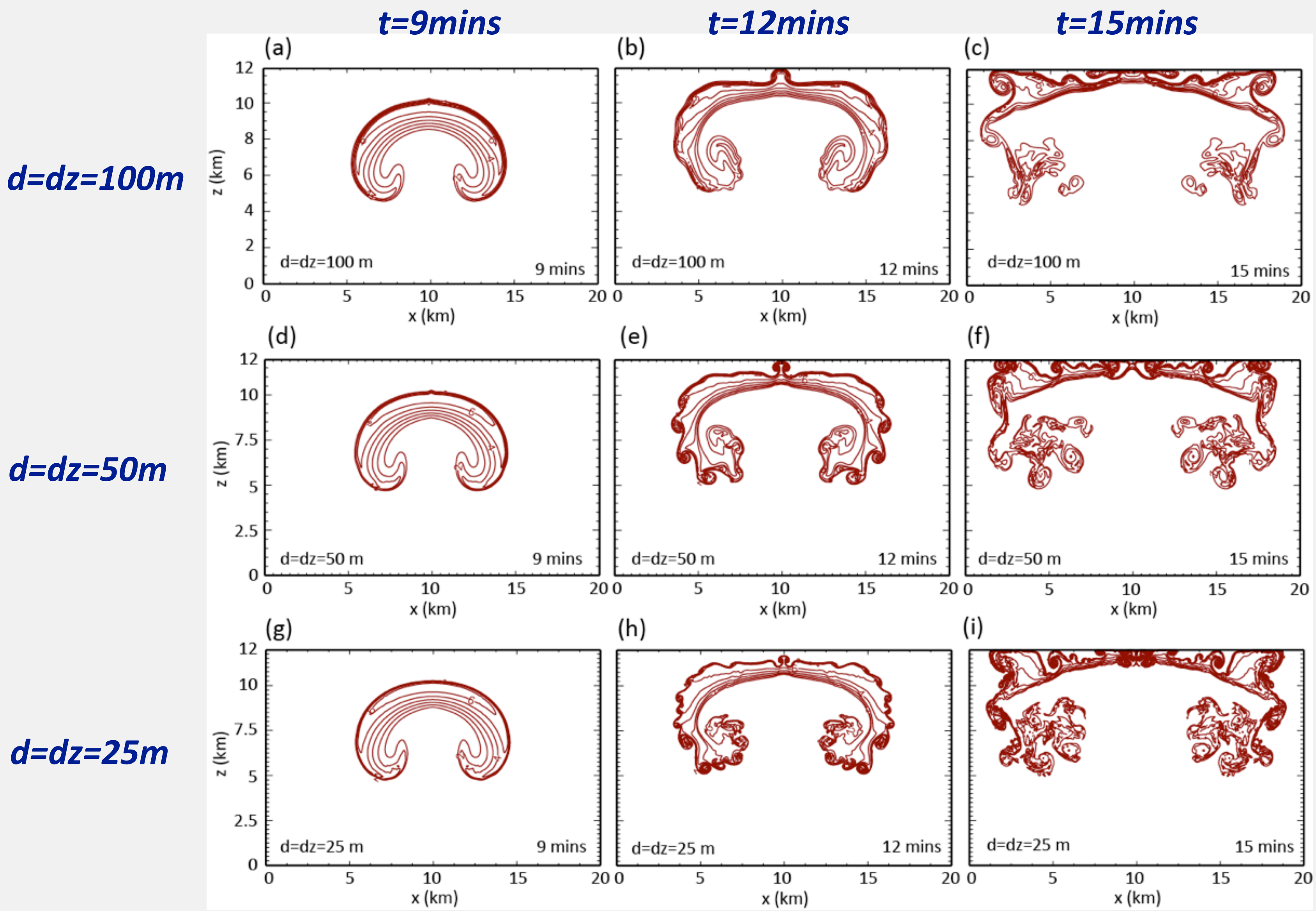
Cyclone-scale simulations with the unified model

Experiment setup from Ullrich and Jablonowski (2011)



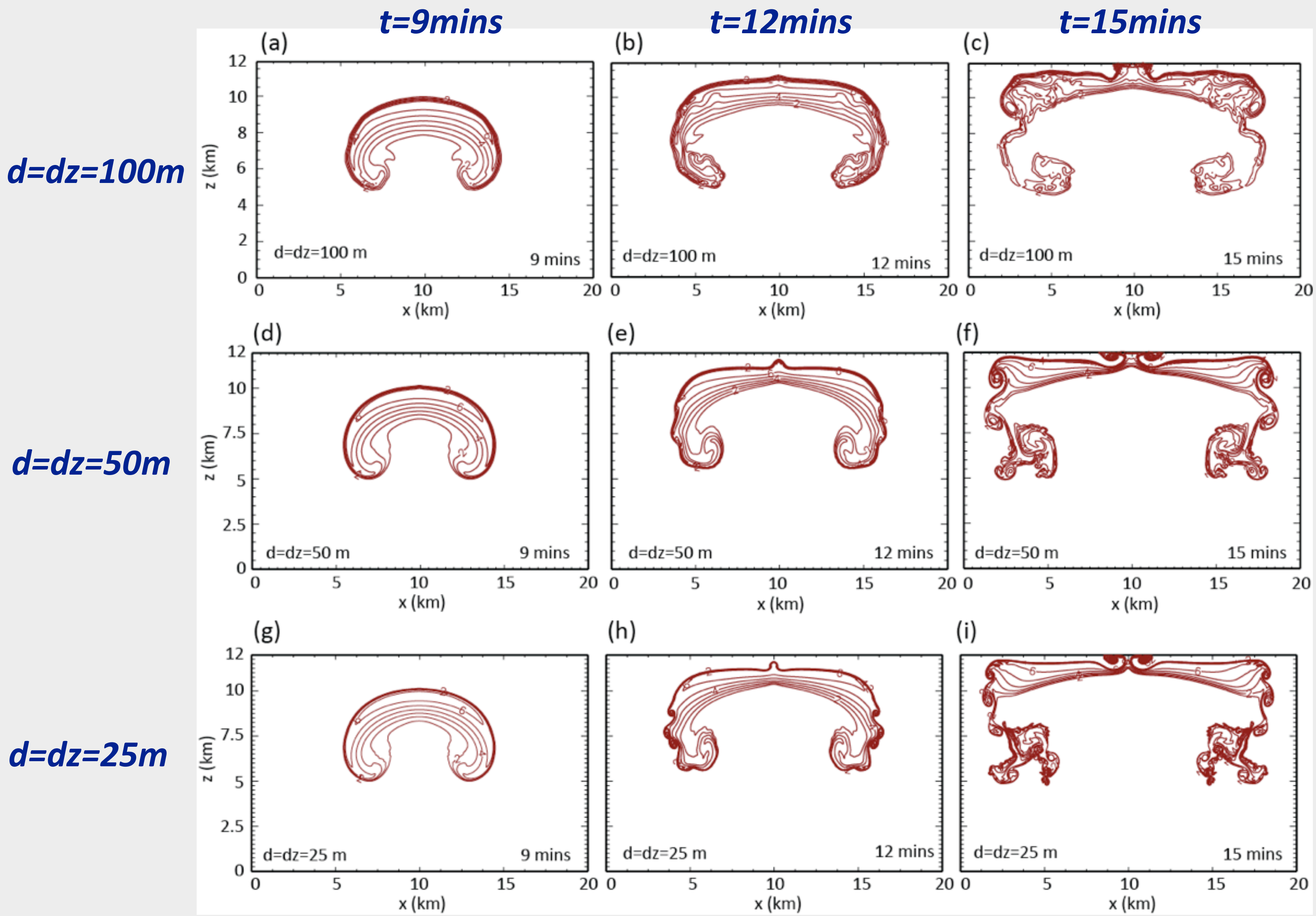
Konor (*MWR*, 2013)

Cloud-scale simulations with the *unified* model (Warm bubble)



Konor (*MWR*, 2013)

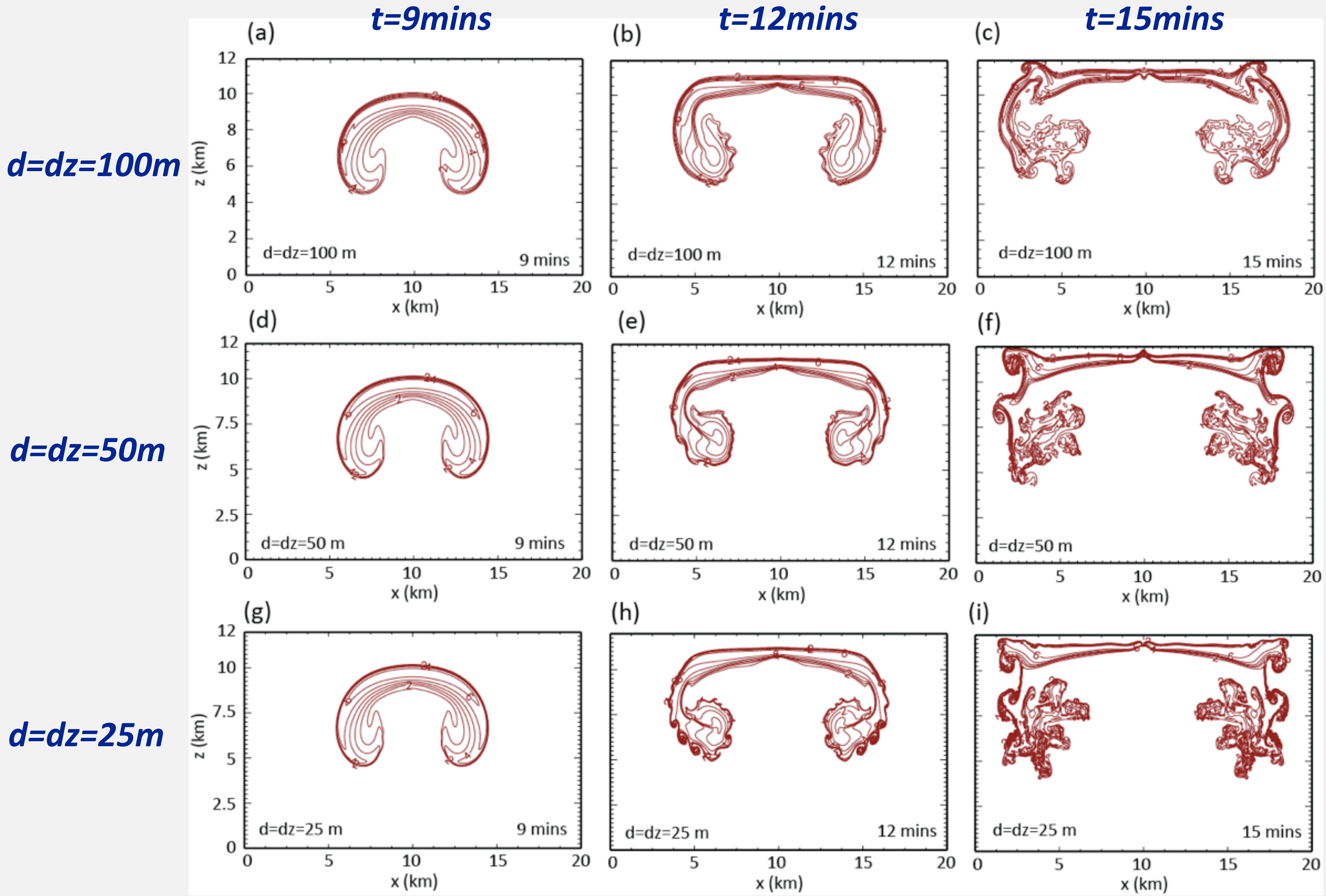
With the ***fully compressible*** model (Warm bubble)



- The fully compressible dynamical core has been constructed by using the schemes of the unified dynamical core as much as possible. A very small time step is used in the fully compressible dynamical core.

Konor (*MWR*, 2013)

With the *fully compressible* model using *higher order schemes* (Warm bubble)

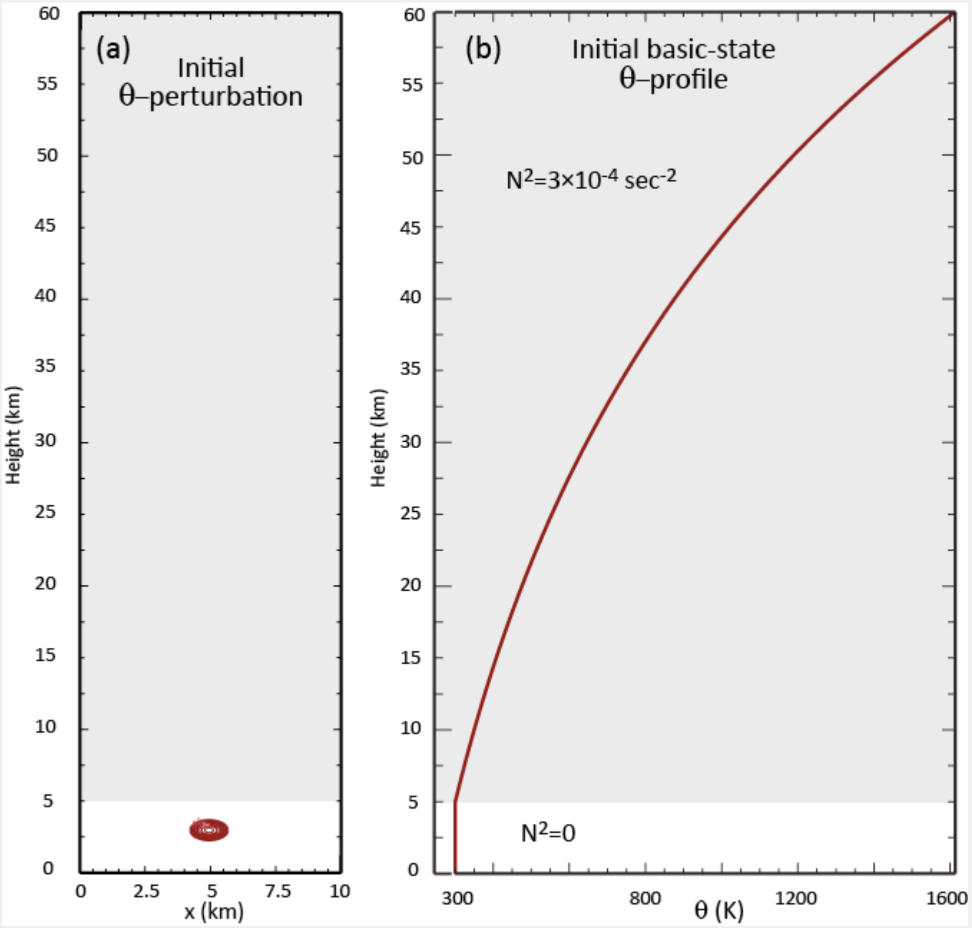


- The schemes used in the spatial discretization of the original fully compressible dynamical core are increased two orders.

Konor (*MWR*, 2013)

Cloud-scale simulations including a stratosphere with the unified model (Warm bubble)

Initial warm bubble and environment

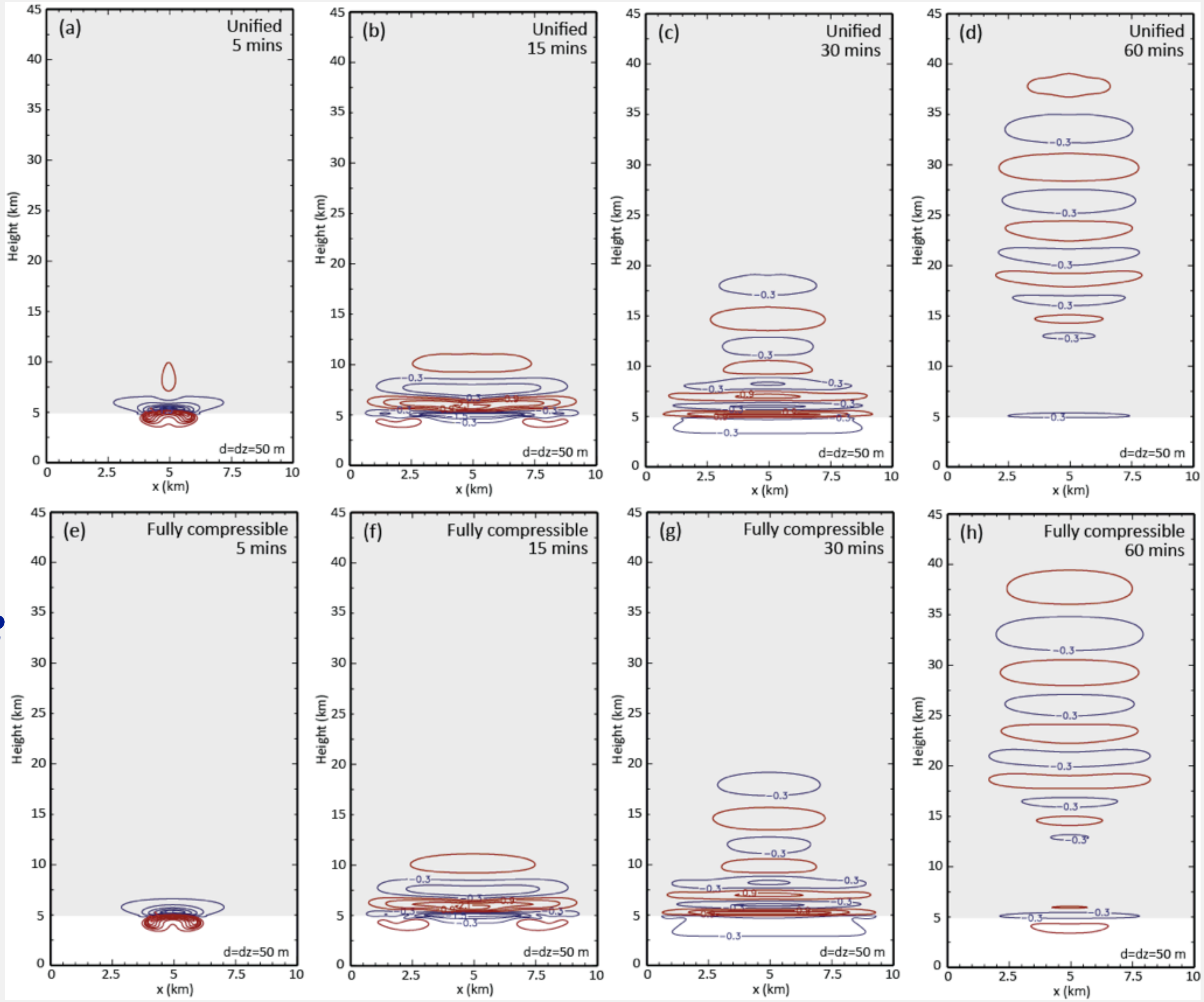


Konor (*MWR*, 2013)

Cloud-scale simulations including a stratosphere with the unified, fully compressible, anelastic and pseudo-incompressible models (Warm bubble and stratosphere)

Unified

Fully compressible



Konor (*MWR*, 2013)

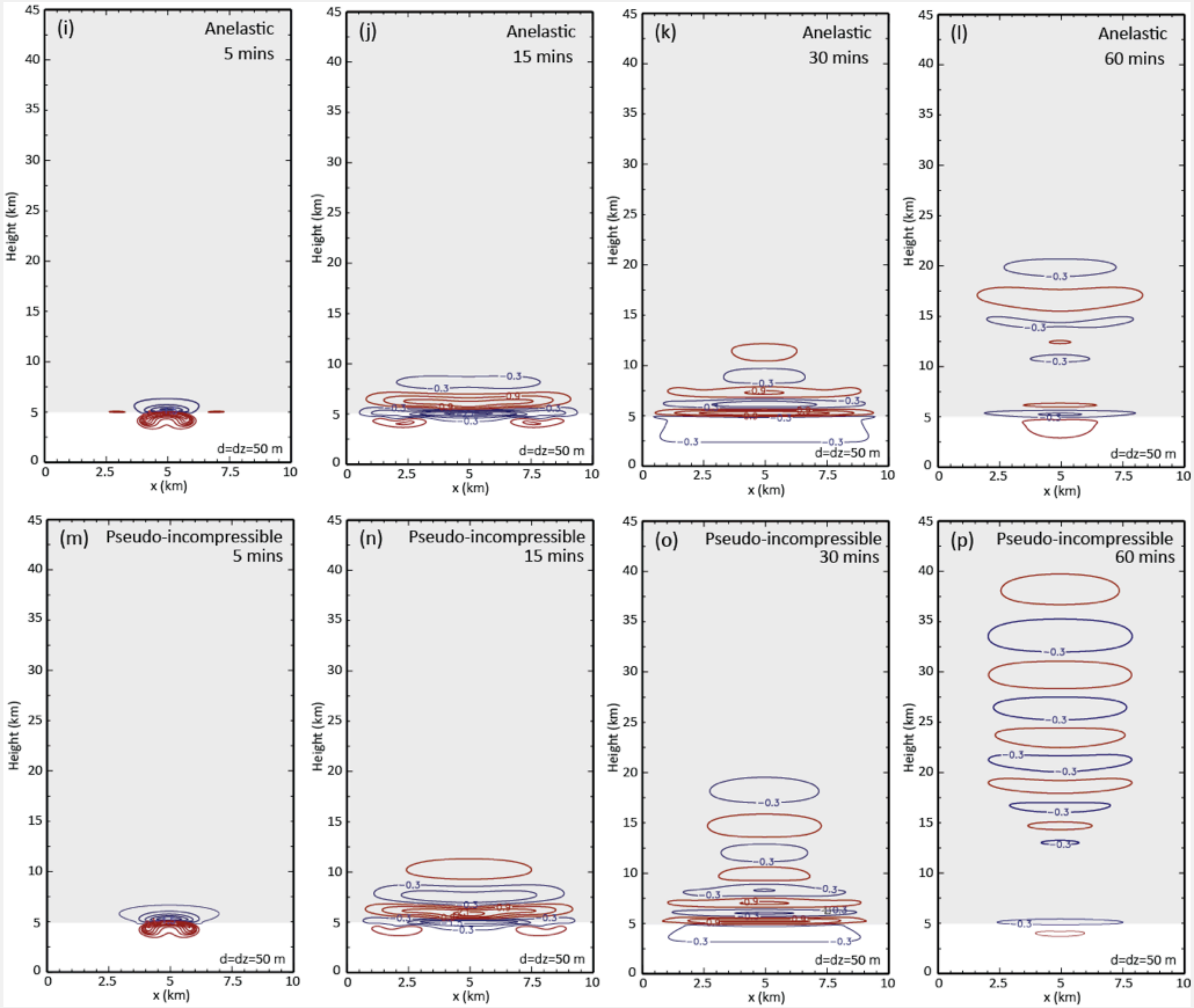
Cloud-scale simulations including a stratosphere with the unified, fully compressible, anelastic and pseudo-incompressible models (Warm bubble and stratosphere)

Anelastic (Lipps and Hemler)

Lipps and Hemler (1983)

Pseudo-incompressible

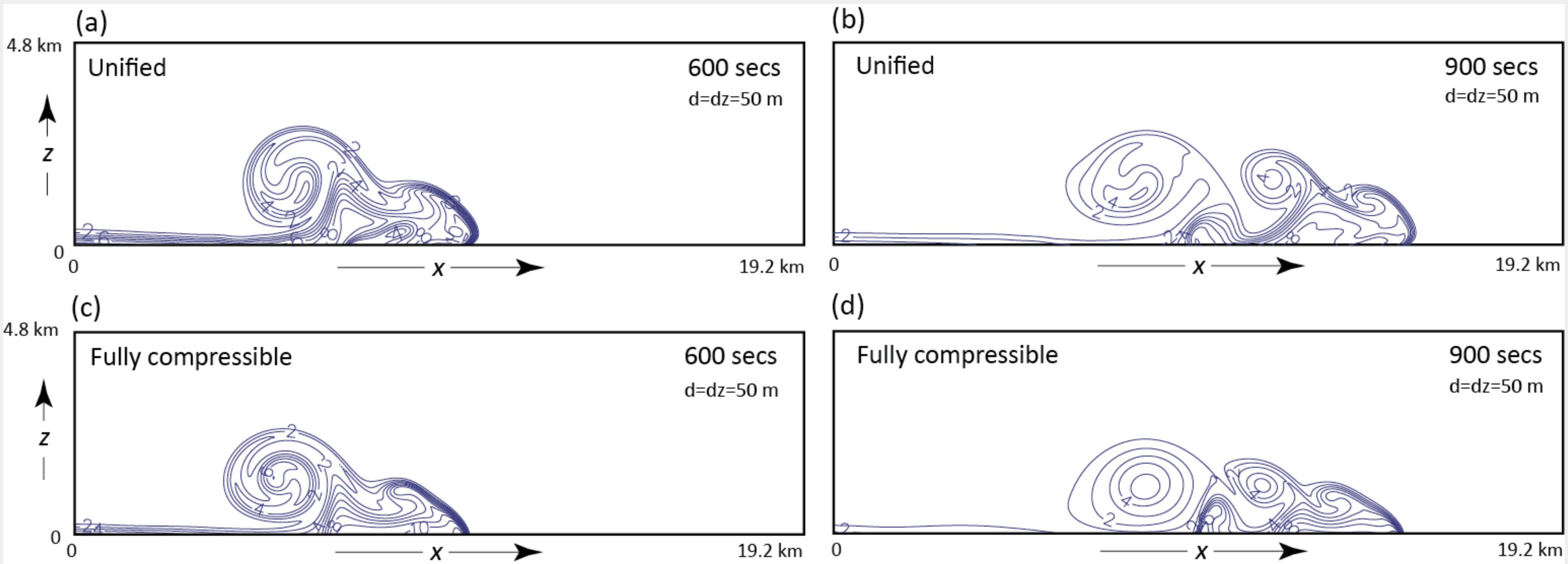
Durran (1989 and 2008)



Konor (*MWR*, 2013)

Cloud-scale simulations with the unified and fully compressible models (Cold bubble)

Experiment setup from Straka et al. (1993)



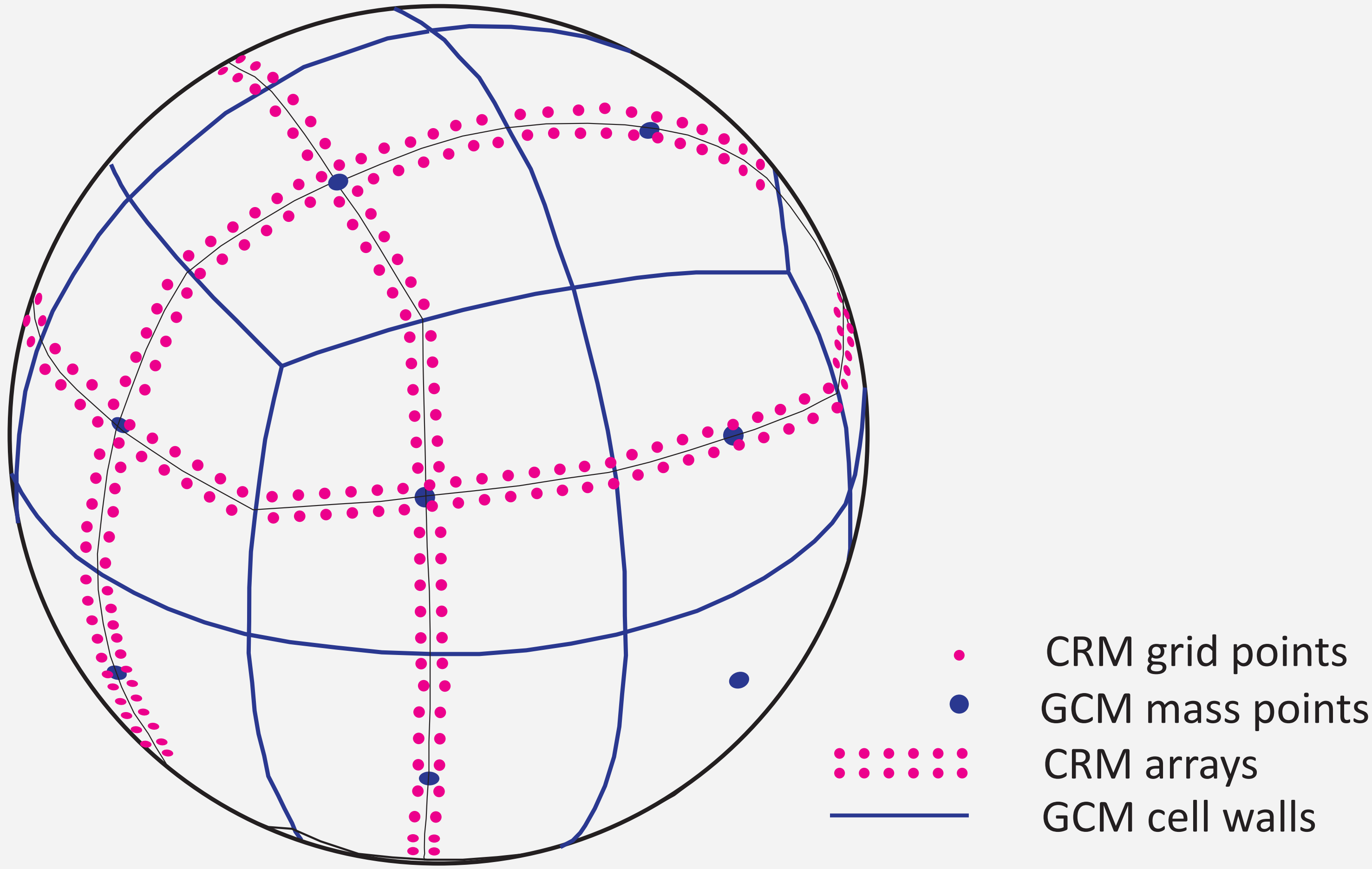
Konor (*MWR*, 2013)

Cubed-sphere dynamical core

Place of the cubed-sphere grid in the big picture

- The GCRM under development uses (and will always use) the icosahedral grid.
- There are compelling reasons to use a cubed-sphere grid for the implementation of the Q3D-MMF to the globe.

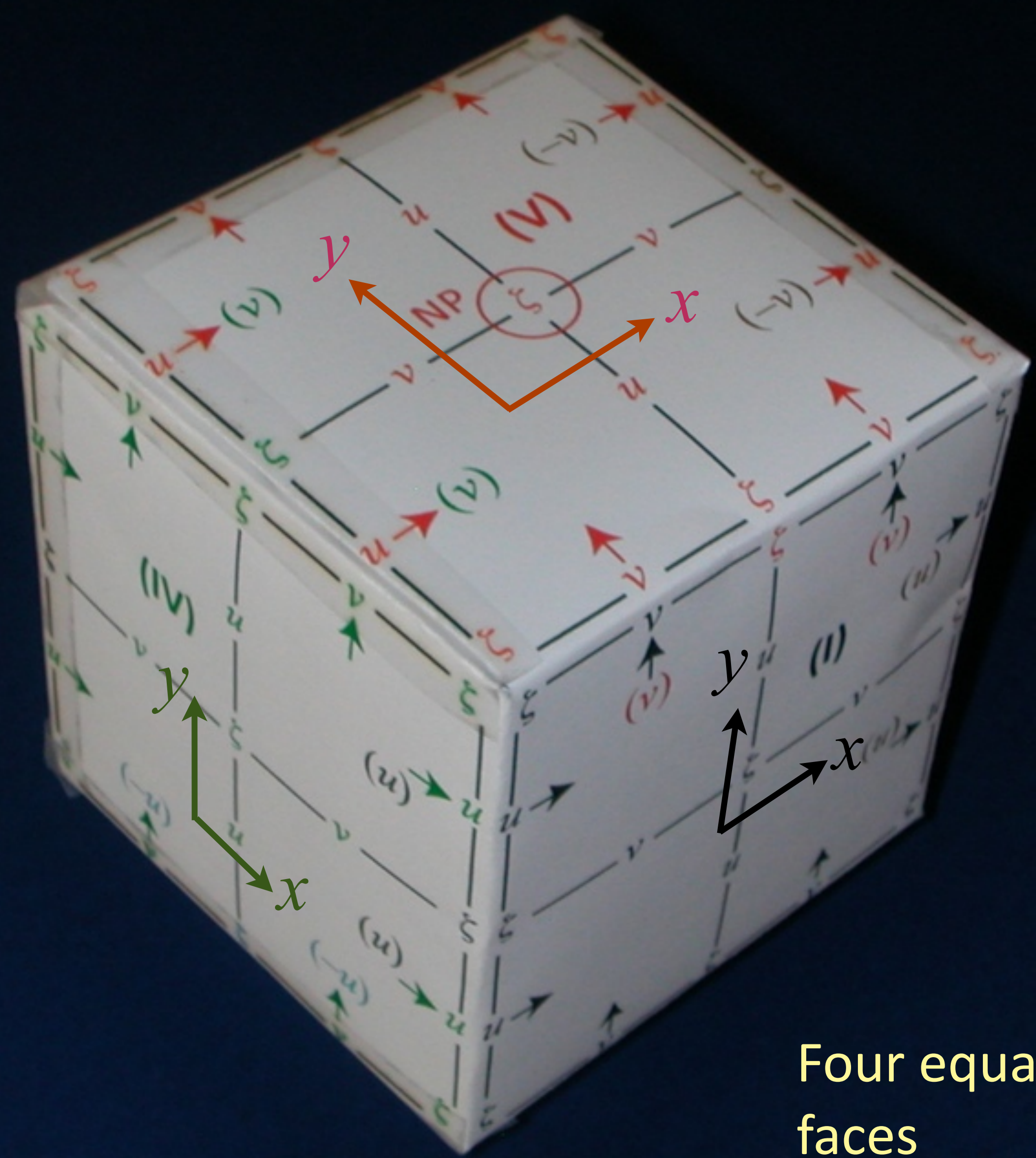
Global Q3D-MMF on a cubed-sphere



Cubed-sphere dynamical core

- GCM component of the Q3D-MMF
- Cubed-sphere VVM dynamical core. First version: C-grid momentum dynamical core
- Curvilinear coordinate
- Unified equations in a sigma vertical coordinate. First version: Quasi-hydrostatic equations
- First version: Momentum prediction with the potential enstrophy conserving (or dissipating) fourth-order scheme (Takano and Wurtele, 1983; Konor, 2013)

C-grid on a cubed sphere



Four equatorial and two polar faces

Unified system in the global Q3D MMF

Unified system provides a conceptual link to unify the components of the Q3D Multiscale Modeling Framework

Unified CRM dynamics

$$p_{CRM} \equiv p_{qs} + \delta p$$

Unified GCM dynamics

$$p_{GCM} \equiv \underline{p_{qs}} + \delta p$$

$\delta p = 0$ for quasi-hydrostatic dynamics

- Quasi-hydrostatic variables of the GCM can be used as the quasi-static balanced state of the unified CRM

Plans for the next period:

- Conclude the search for a 3D elliptic solver for the UZIM.
- Continue the work on the sigma version of the UZIM.
- Submit two+one papers related to the development of the UZIM.
- Accelerate work on the cubed-sphere dynamical core.
- Seek funding for the development of the Global Q3D-MMF.