Shock-like structures in the ITCZ boundary layer

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ABSTRACT

This research focuses on understanding why the Intertropical Convergence Zone (ITCZ) is often thin and zonally elongated over many parts of the globe. We focus on theoretical aspects of the equations used in atmospheric models supported by observations in the central and eastern Pacific ocean. Since the ITCZ boundary layer is a region of significant meridional convergence, $v(\partial v/\partial y)$ should not be neglected. With the inclusion of $v(\partial v/\partial y)$ in the equations of the ITCZ boundary layer, there is an embedded Burgers' equation. Burgers' equation can lead to shocks in the mathematical sense [1], and an embedded Burgers' equation in atmospheric models have been shown to produce shock-like structures in the tropical cyclone boundary layer [3]. Therefore, we believe that shock-like features help organize the ITCZ into a thin, zonally elongated strip of convection. The overarching goal of this research is to improve modeling of the ITCZ, since it is vital in forecasting many atmospheric phenomena, including tropical cyclones, El Niño, and tropical rainfall.

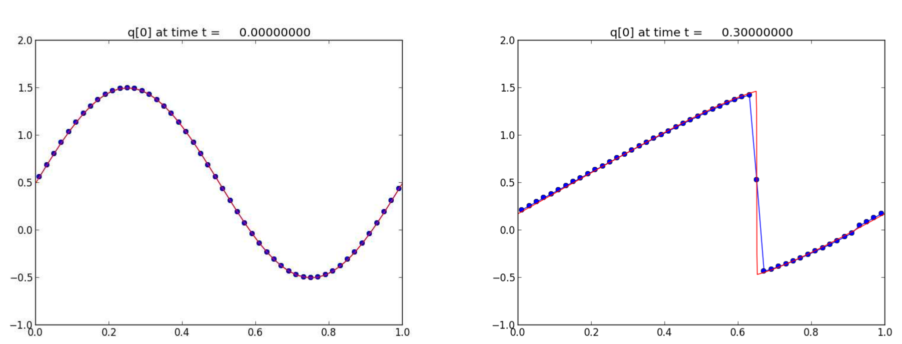
1 Introduction

1.1 What are shocks?

A special form of the 1-D advection equation **is able to produce shocks**,

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial y} = 0, \tag{1}$$

and it is often referred to as Burgers' equation (Burgers 1948). An initial state of convergence of $v(y,t)$ is followed by advection by v that sharpens the convergence. If the $v(\partial v/\partial y)$ is initially large, a discontinuity forms in v and a singularity forms in $\partial v/\partial y$. An example of the time evolution of the **1-D Burgers' equation**, where q is the unknown field rather than v, is shown in the figure below. The initial profile of q is on the left, and q after the shock is on the right. The actual solution is the red line, while the approximation computed using Clawpack software [2] is blue.



1.2 Slab boundary layer equations in the ITCZ

The relevance of (1) is that the meridional advection of the meridional velocity is often very large in the boundary layer of the ITCZ, and therefore cannot be neglected in the equations used to study and forecast weather in the tropical atmosphere. An idealized example of boundary layer equations in the tropics are given by

$$\begin{aligned}\frac{\partial u}{\partial t} + v\frac{\partial u}{a\partial\phi} + w^-\left(\frac{u-u_\mathrm{g}}{h}\right) - \left(f + \frac{u\tan\phi}{a}\right)v \\ = -c_\mathrm{D}U\frac{u}{h} + K\frac{\partial}{a\partial\phi}\left(\frac{\partial(u\cos\phi)}{a\cos\phi\partial\phi}\right),\end{aligned} \tag{2}$$

$$\begin{aligned}\frac{\partial v}{\partial t} + v\frac{\partial v}{a\partial\phi} + w^-\frac{v}{h} + \left(f + \frac{(u+u_g)\tan\phi}{a}\right)(u-u_g) \\ = -c_\mathrm{D}U\frac{v}{h} + K\frac{\partial}{a\partial\phi}\left(\frac{\partial(v\cos\phi)}{a\cos\phi\partial\phi}\right),\end{aligned} \tag{3}$$

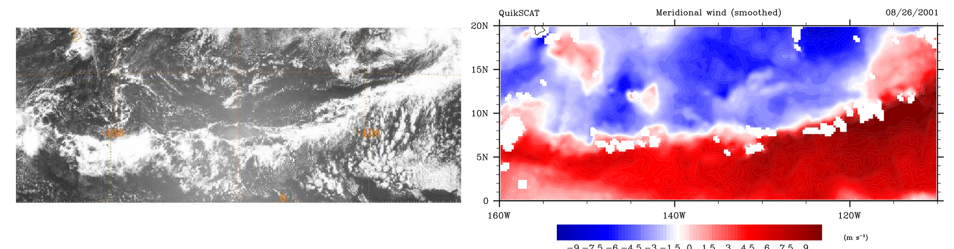
$$w = w^+ - w^- = -h\frac{\partial(v\cos\phi)}{a\cos\phi\partial\phi}, \tag{4}$$

where (2) is the boundary layer averaged zonal momentum equation, (3) is the boundary layer averaged meridional momentum equation, and (4) is the continuity equation where w is the vertical velocity at the top of the boundary layer. These equations assume zonal symmetry and a slab boundary layer. **Notice that the first two terms of (3) make up the 1-D Burgers' equation!** The initial condition is a prescribed geostrophic zonal velocity $u_\mathrm{g}(y,0)$ above the boundary layer, and the boundary conditions are $\partial(u\cos\phi)/\partial\phi = \partial(v\cos\phi)/\partial\phi = 0$ at $\phi = \pm\pi/6$.

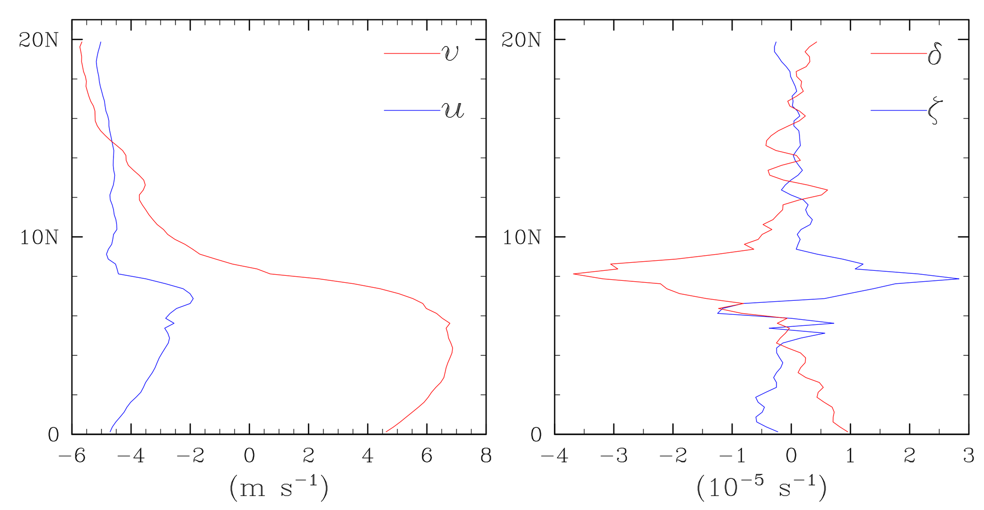
2 Observations

2.1 Satellites: GOES and QuikSCAT

Visible GOES satellite imagery over the central and eastern Pacific ocean show that the ITCZ often has thin cloud structures that correspond with large meridional wind convergence retrieved by QuikSCAT (below).

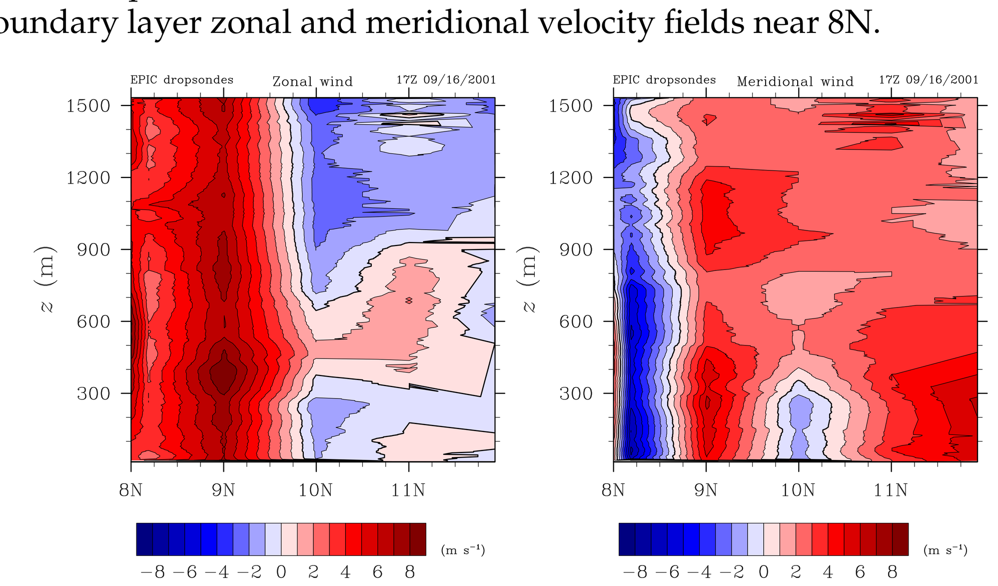


The 120-140W average of the QuikSCAT zonal (u) and meridional (v) velocities, as well as derived relative vorticity (ζ) and divergence (δ), demonstrate the **validity to use a zonally symmetric model** (figure below). Note the spike in both ζ and δ near 9N.

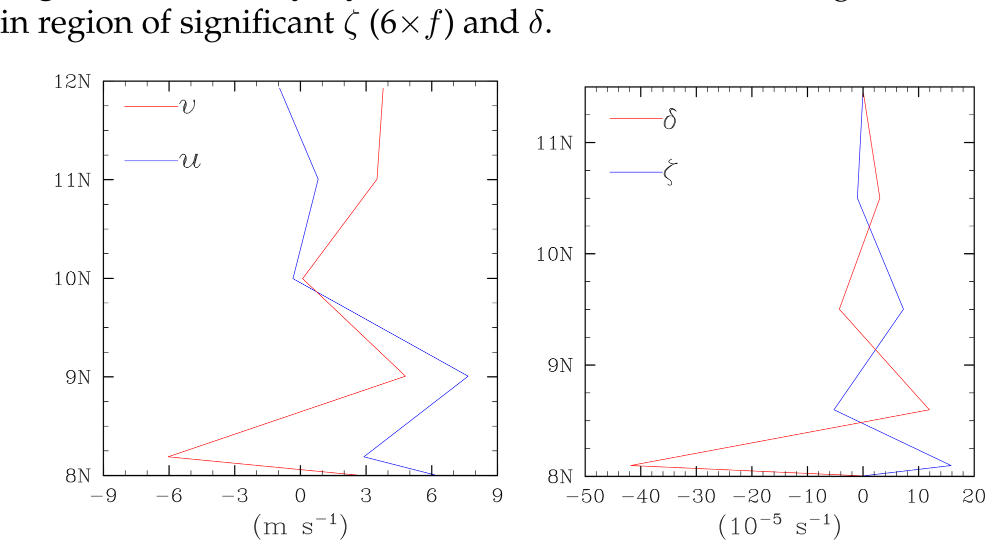


2.2 Dropsondes: EPIC (2001)

During the East Pacific Investigations of Climate (EPIC) field experiment, dropsondes were released from both C-130 and P-3 aircraft. In the figure below, dropsonde data from the P-3 illustrate shock-like structures in the boundary layer zonal and meridional velocity fields near 8N.



The 0–700 m vertical average of the zonal and meridional velocities, as well as the relative vorticity and divergence demonstrate the **validity of using a slab boundary layer model**, shown below. Once again, there is a thin region of significant ζ ($6\times f$) and δ.



Now that we have some observations of zonally elongated, shock-like features in the boundary layer of the central and eastern Pacific ocean, we simplify the equations introduced in section 2 to produce some analytical solutions to slab boundary layer equations appropriate in the tropics.

3 Analytical slab boundary layer model

3.1 Model equations

Consider the zonally symmetric slab boundary layer equations on the equatorial β-plane

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} - \beta yv = \frac{u}{\tau}, \quad \Longleftrightarrow \quad \frac{d}{dt}\left(ue^{t/\tau}\right) = \beta yve^{t/\tau}, \tag{5}$$

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial y} = \frac{v}{\tau}, \quad \Longleftrightarrow \quad \frac{d}{dt}\left(ve^{t/\tau}\right) = 0, \tag{6}$$

$$w = w^+ - w^- = -h\frac{\partial v}{\partial y}, \tag{7}$$

where $\tau = 40$ hours is a constant surface drag e-folding time scale and the boundary layer depth $h = 1$ km. Note that **there is an embedded Burgers' equation in (6)!** This set of equations is a simplified version of (2)–(4), in that vertical exchange between the boundary layer and the air above it terms involving w^-, the $u - u_\mathrm{g}$ term in the meridional momentum equation, and horizontal diffusion are all neglected. Also, surface drag is linearized. **After integration along v, (5)–(6) simplify to**

$$u(y,t) = u_0(\hat{y})e^{-t/\tau} + \beta v_0(\hat{y})\left\{\hat{y}t + \tau v_0(\hat{y})\hat{t}\right\}e^{-t/\tau}, \tag{8}$$

$$v(y,t) = v_0(\hat{y})e^{-t/\tau}, \tag{9}$$

where $\hat{t} = \tau(1 - e^{-t/\tau})$, and the characteristic lines $\hat{y}(y,t)$ are given implicitly via integration of $dy/dt = v$ by

$$y = \hat{y} + \hat{t}v_0(y). \tag{10}$$

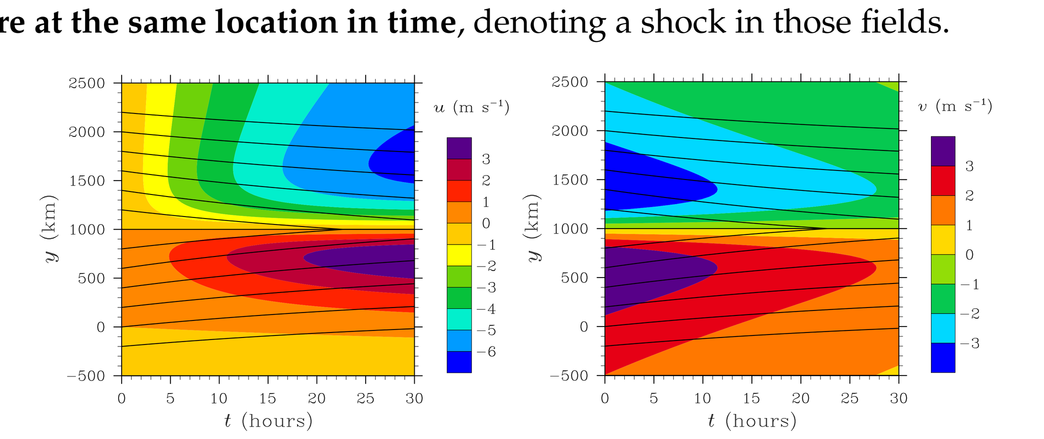
The initial condition is prescribed via the meridional velocity field as a broad region of convergence

$$v_0(y) = v_\mathrm{max}\left[\frac{2b(y_0 - y)}{b^2 + (y - y_0)^2}\right], \tag{11}$$

centered at $y_0 = 1000$ km, $v_\mathrm{max} = 4$ m/s is the maximum initial meridional velocity, and $b = 400$ km is the meridional half-width of the convergence region. All other fields are zero initially.

3.2 Model results

The contoured $u(y,t)$ and $v(y,t)$ fields are displayed below along with selected characteristic lines ($\hat{y} = -200, 0, 200, ..., 2200$ km). Each characteristic line represents a line of constant $\hat{y}$, but variable $u(y,t)$ and $v(y,t)$. **When two characteristic lines intersect, two different values of u and v are at the same location in time**, denoting a shock in those fields.

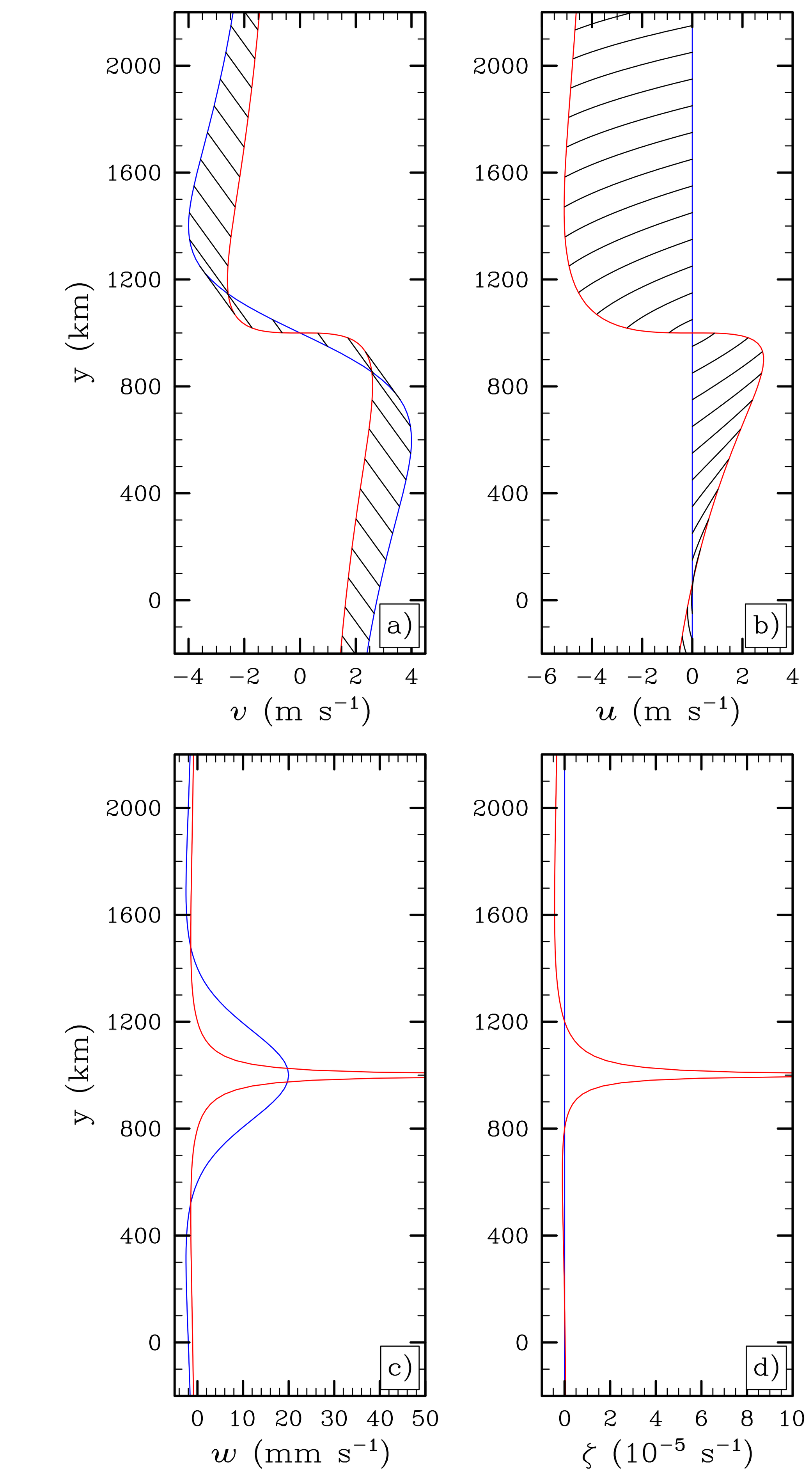


The time of shock formation is calculated as the time when $\partial v/\partial t \to \infty$

$$t_\mathrm{s} = -\tau\ln\left(1 - \frac{b}{2v_\mathrm{max}\tau}\right) \approx 17\,\text{hours}. \tag{12}$$

Once the shock has occurred, the model becomes multivalued and oscillates wildly, therefore is it no longer valid. In the figure on the next column to the right, we illustrate the boundary layer zonal (a) and meridional velocity (b), as well as the vertical velocity at the top of the boundary layer (c), and the boundary layer relative vorticity (d) at $t = 0$ in blue and at the time step before $t = t_\mathrm{s}$ in red. Black lines indicate the change in horizontal velocity in time at selected meridional locations. Notice the symmetry of v and w and the asymmetry of u and ζ about y_0. This is due to the βy term in (5). The initial field of broad convergence thins out over time due to $v(\partial v/\partial y)$, until the horizontal velocities are discontinuous and the derived fields $\to \infty$. **If the initial meridional velocity field is large enough and the initial meridional convergence is thin enough, surface drag cannot overcome the shock.**

3.2 Model results



4 Conclusions and Future Work

- Observations illustrate that the ITCZ in regions such as the central and eastern Pacific are often zonal elongated and thin with shock-like features.
- An idealized zonally symmetric slab boundary layer model on the equatorial β-plane with an embedded Burgers' equation in (6) shocks in the zonal and meridional velocity fields after 17 hours, and derived fields $\to \infty$. The model is invalid when $t \geq t_\mathrm{s}$.
- The next step is to solve a more complete set of slab boundary layer equations [(2)–(4)], which also has an embedded Burgers' equation in (3), and use both horizontal diffusion and shock capturing methods so that the solution is valid after the shock forms.

References

[1] Burgers, J. M. (1948), A mathematical model illustrating the theory of turbulence, *Adv. Appl. Mech.*, *1*, 171–199.

[2] R. J. LeVeque, M. J. Berger, et. al., Clawpack Software 4.6.3, www.clawpack.org, May 2013.

[3] Williams, G. J., R. K. Taft, B. D. McNoldy, and W. H. Schubert (2013), Shock-like structures in the tropical cyclone boundary layer. *J. Adv. Model. Earth Syst.*, *accepted*.