Atmospheric Dynamical Cores

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Outline:

- Progress report
- From the pole problem to the wavenumber-5 problem

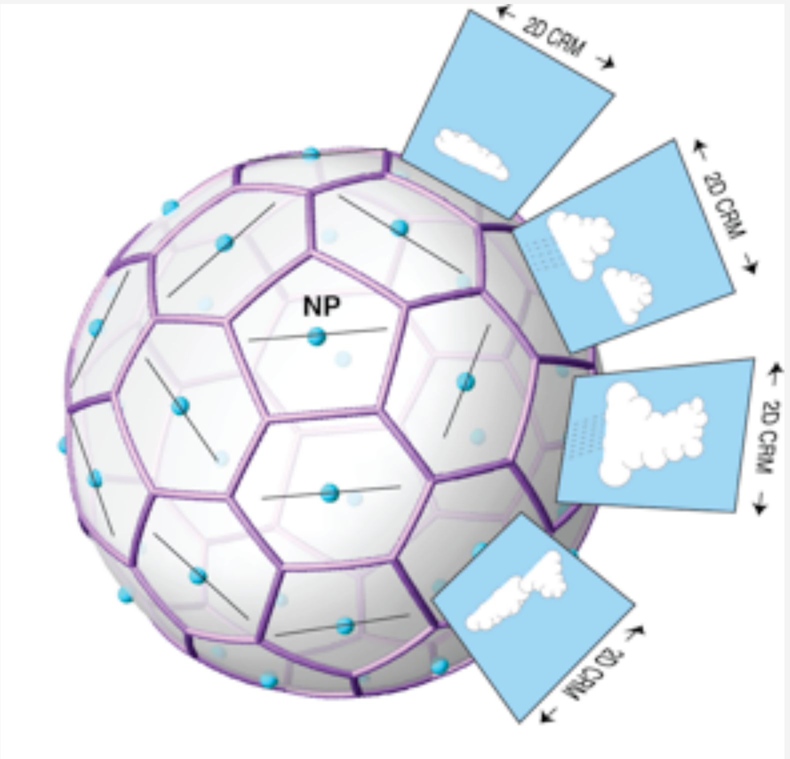
DF working group

17th CMMAP Team Meeting, 5–6 August, 2014, Fort Collins, CO

Progress report:

Development of the Global Cloud Resolving Model

- UZIM (**U**nified **Z**-grid **I**cosahedral **M**odel) development:
  - The *hybrid sigma-pressure* vertical coordinate version of the quasi-hydrostatic UZIM (UZIM-sigma) has been completed.
  - The *hybrid isentropic-sigma* vertical coordinate (gvc) version of the quasi-hydrostatic UZIM (UZIM-gvc) has been completed.
- SUZI (**S**uperparameterized **U**nified **Z**-grid **I**cosahedral Model) has been constructed and tested in an aquaplanet simulation by Don Dazlich.



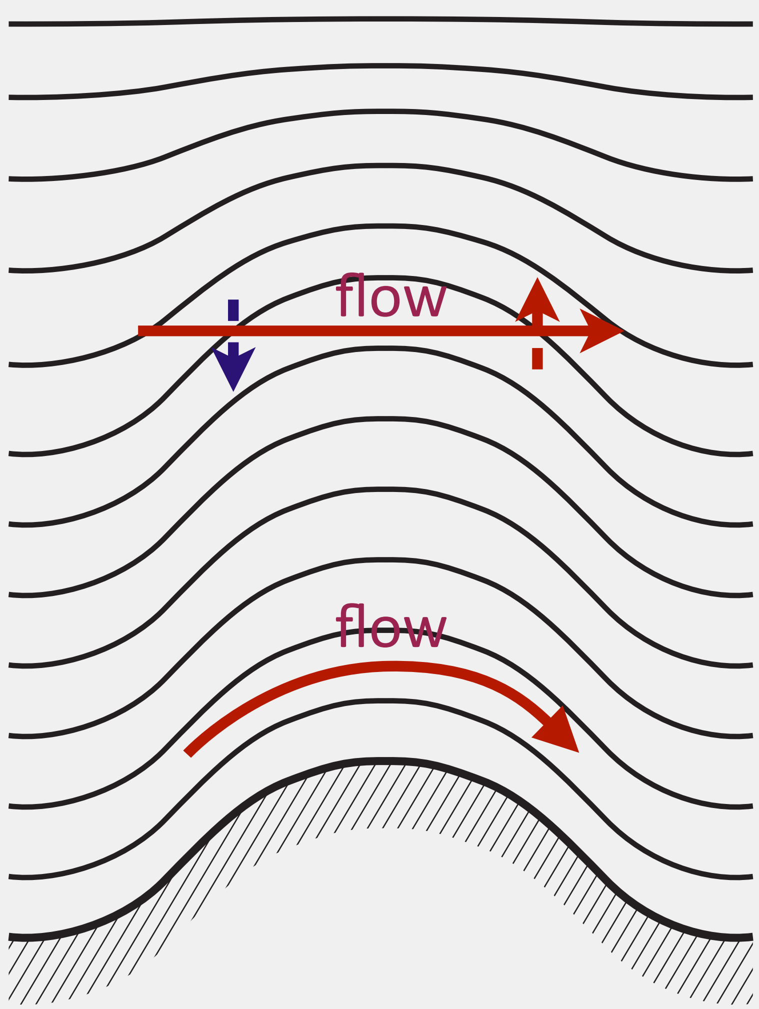
UZIM with the hybrid sigma-pressure and hybrid isentropic-sigma vertical coordinates

Hydrid sigma-pressure vertical coordinate

Ordinary sigma

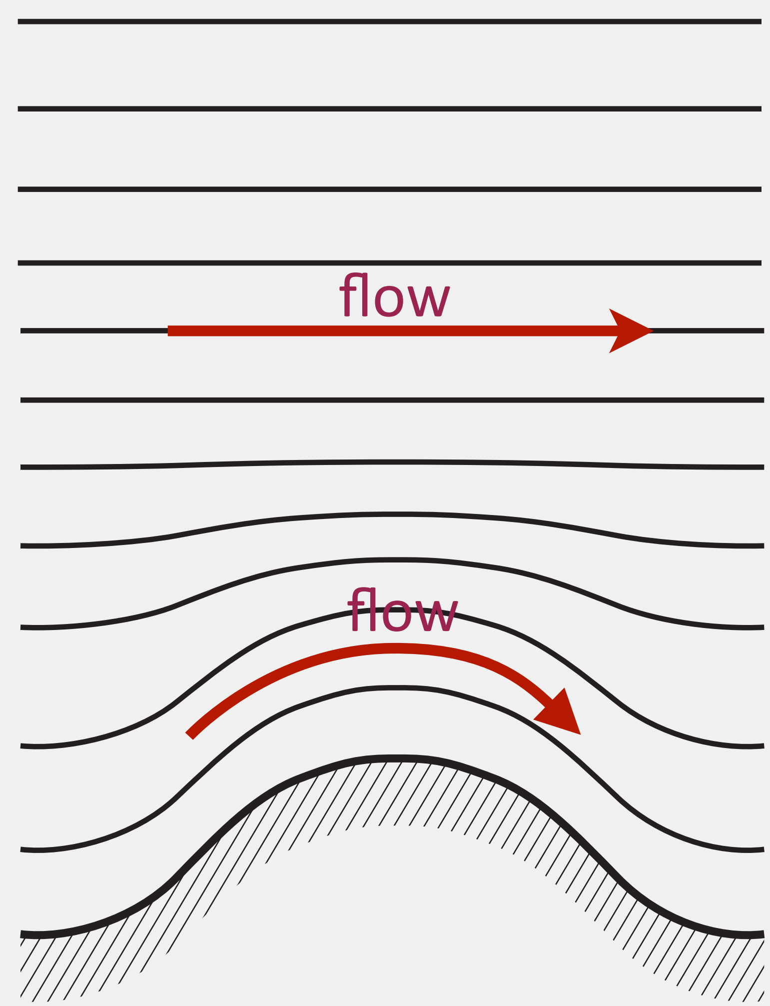
VA errors

PGF errors



$$\sigma \equiv \frac{p - p_T}{p_S - p_T}$$

Hybrid sigma-pressure

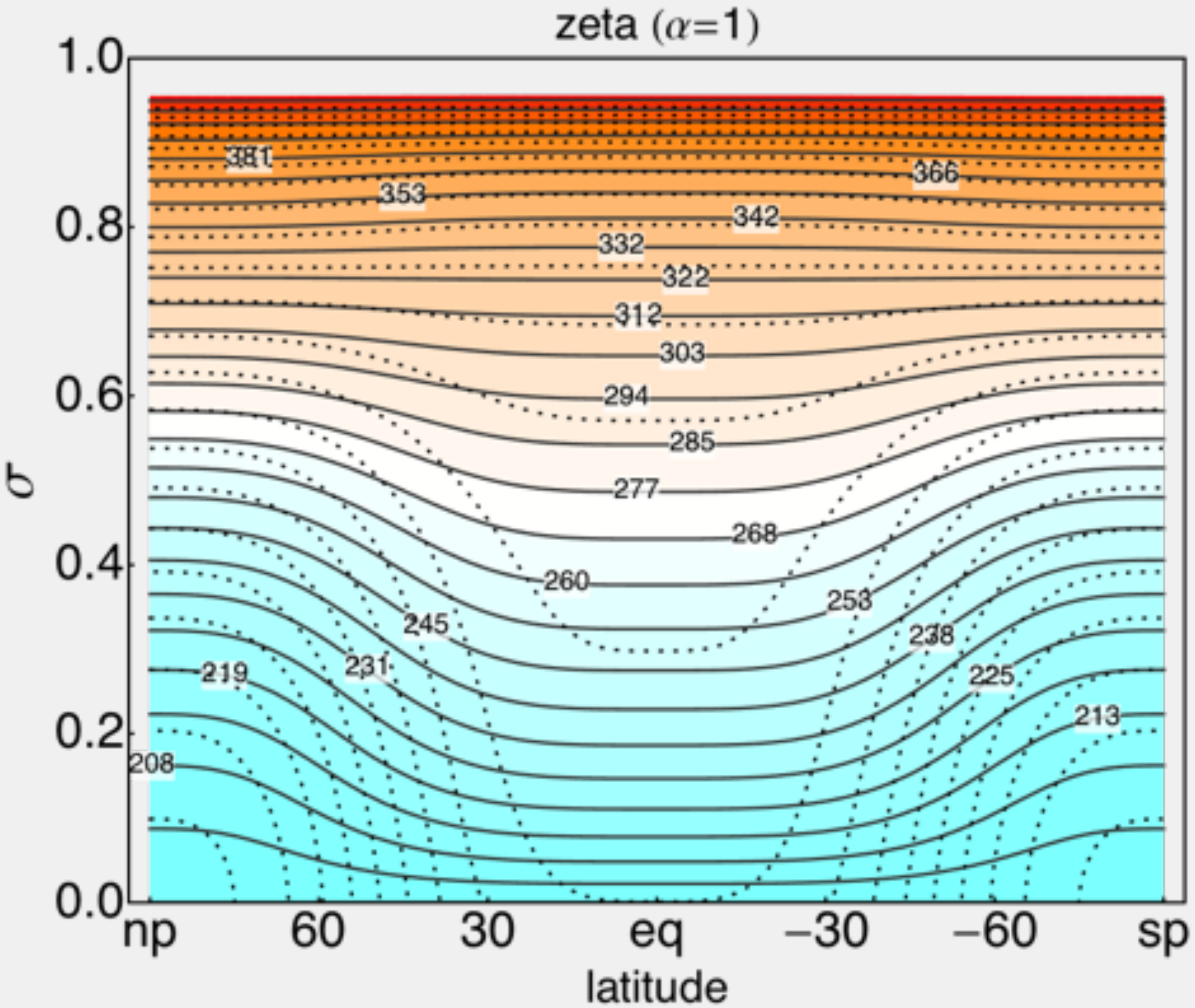


No VA errors

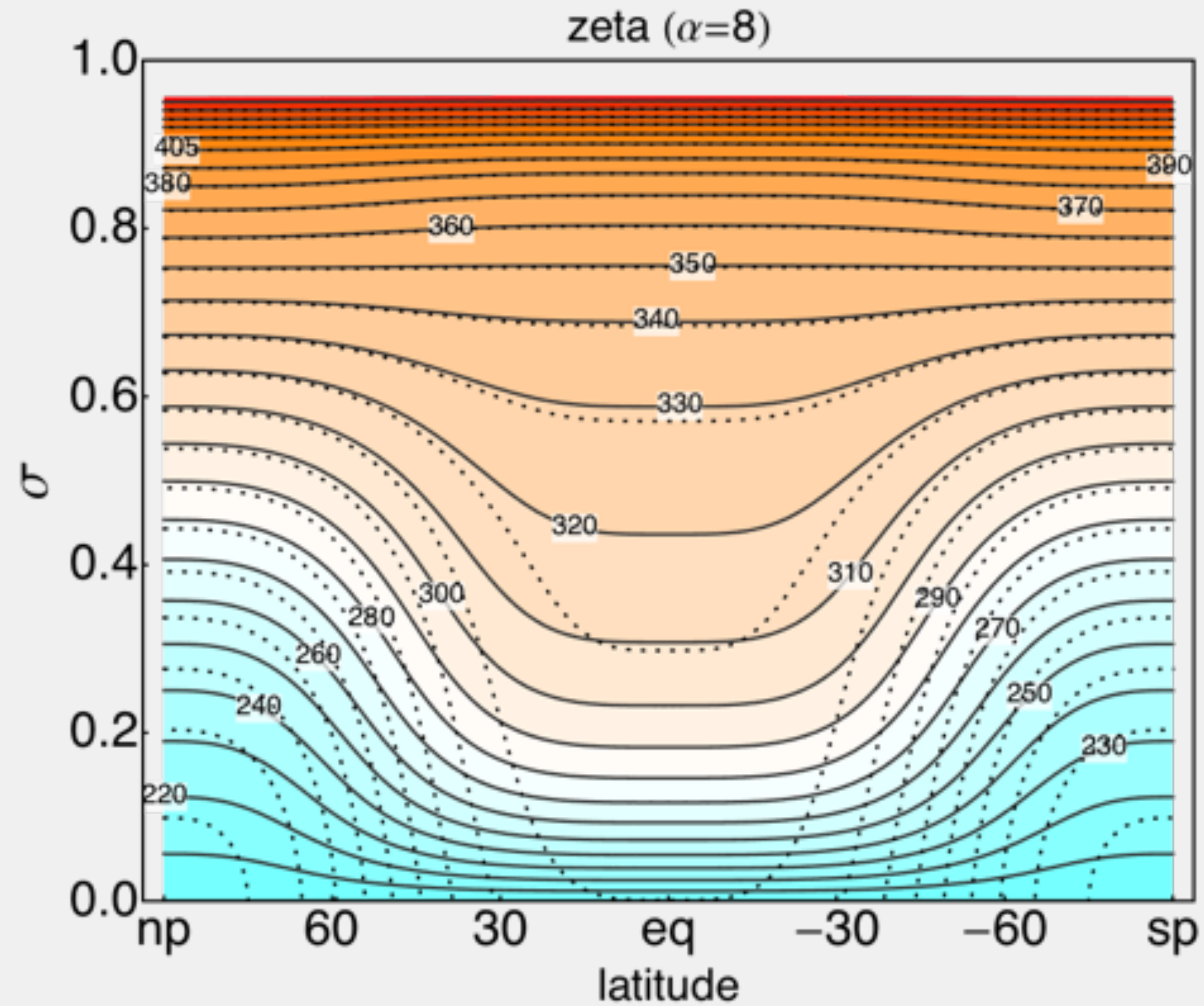
No PGF errors

$$\sigma \equiv \mathcal{G}\left(p, p_S\right)$$

Hydrid isentropic-sigma vertical coordinate



less isentropic



more isentropic

$$\zeta \equiv \mathcal{F}(\theta,\sigma) \qquad \sigma \equiv \mathcal{G}(p, p_S)$$

Generalized $\zeta = \mathcal{F}(\theta, p, p_S)$ vertical coordinate

Generalized coordinate definition

$$\zeta \equiv \mathcal{F}(\theta,\varsigma) = \theta_{\min} + (\theta - \theta_{\min})\, g(\varsigma) - (\partial\theta/\partial\varsigma)_{\min} \left\{ (\varsigma - \varsigma_T)\, g_0 - \frac{1}{\alpha}\left[g(\varsigma) - 1 \right] \right\}$$

$\varsigma \equiv \mathcal{G}(p, p_S)$ (see next page)

$g(\varsigma) \equiv g_0\left(1 - e^{-\alpha\varsigma}\right)$ where $g_0 \equiv 1\big/\left(1 - e^{-\alpha\varsigma_T}\right)$

Derivatives

$$\left(\frac{\partial\mathcal{F}}{\partial\theta}\right)_{\varsigma} = g(\varsigma) \quad \text{and} \quad \left(\frac{\partial\mathcal{F}}{\partial\varsigma}\right)_{\theta} = \alpha\left[\theta + \frac{1}{\alpha}\left(\frac{\partial\theta}{\partial\varsigma}\right)_{\min}\right]\left[g_0 - g(\varsigma)\right] - \left(\frac{\partial\theta}{\partial\varsigma}\right)_{\min} g_0$$

$$\left(\frac{\partial\mathcal{F}}{\partial p}\right)_{\theta,p_S} = \left(\frac{\partial\mathcal{F}}{\partial\varsigma}\right)_{\theta}\left(\frac{\partial\mathcal{G}}{\partial p}\right)_{p_S} \quad \text{and} \quad \left(\frac{\partial\mathcal{F}}{\partial\theta}\right)_{p,p_S} = \left(\frac{\partial\mathcal{F}}{\partial\theta}\right)_{\varsigma}$$

[see next page for $\left(\frac{\partial\mathcal{G}}{\partial p}\right)_{p_S}$]

1–Traditional sigma $\varsigma \equiv \mathcal{G}(p, p_S)$ vertical coordinate

Definition

$$\varsigma \equiv \mathcal{G}(p, p_S) = \frac{p_S - p}{p_S - p_T}$$

Derivatives

$$\left(\frac{\partial \mathcal{G}}{\partial p}\right)_{p_S} = -\frac{1}{p_S - p_T}$$

and

$$\left(\frac{\partial \mathcal{G}}{\partial p_S}\right)_{p} = \frac{p - p_T}{\left(p_S - p_T\right)^2}$$

2–Generalized sigma $\varsigma \equiv \mathcal{G}(p, p_S)$ vertical coordinate

Definition of hybrid *p–sigma* coordinate

$$\varsigma \equiv \mathcal{G}(p, p_S) = \frac{1}{2}\frac{p_{S0} - p_C}{p_{S0} - p_T}\left[\frac{p_S - p}{p_S - p_C} - \frac{1}{\beta}\ell n \cosh\left(\beta\frac{p_C - p}{p_S - p_C}\right)\right]$$

$$+\frac{1}{2}\frac{p_S - p_C}{p_{S0} - p_T}\left[\frac{p_S - p}{p_S - p_C} + \frac{1}{\beta}\ell n \cosh\left(\beta\frac{p_C - p}{p_S - p_C}\right)\right] + \frac{1}{2}\frac{p_{S0} - p_S}{p_{S0} - p_T}\left[\frac{1}{\beta}\ell n \cosh(-\beta)\right]$$

Derivatives

$$\left(\frac{\partial \mathcal{G}}{\partial p}\right)_{p_S} \equiv \frac{1}{2}\frac{p_{S0} - p_C}{p_{S0} - p_T}\left[\frac{-1}{p_S - p_C} + \frac{1}{p_S - p_C}\tanh\left(\beta\frac{p_C - p}{p_S - p_C}\right)\right]$$

$$+\frac{1}{2}\frac{p_S - p_C}{p_{S0} - p_T}\left[\frac{-1}{p_S - p_C} - \frac{1}{p_S - p_C}\tanh\left(\beta\frac{p_C - p}{p_S - p_C}\right)\right]$$

and

$$\left(\frac{\partial \mathcal{G}}{\partial p_S}\right)_{p} \equiv \frac{1}{2}\frac{p_{S0} - p_C}{p_{S0} - p_T}\frac{p - p_C}{(p_S - p_C)^2}\left[1 - \tanh\left(\beta\frac{p_C - p}{p_S - p_C}\right)\right]$$

$$+\frac{1}{2}\frac{1}{p_{S0} - p_T}\left[1 + \frac{1}{\beta}\ell n \cosh\left(\beta\frac{p_C - p}{p_S - p_C}\right) - \frac{p_C - p}{(p_S - p_C)}\tanh\left(\beta\frac{p_C - p}{p_S - p_C}\right)\right] - \frac{1}{2}\frac{1}{p_{S0} - p_T}\left[\frac{1}{\beta}\ell n \cosh(-\beta)\right]$$

Diagnosis of vertical mass flux and predictions of potential temperature, pressure and mass in the generalized $\zeta = \mathcal{F}(\theta, p, p_S)$ vertical coordinate

From thermodynamic equation

$$\theta^{(*)} = \theta^{(n)} - \delta t \times \left(\mathbf{v}\cdot\nabla_\zeta \theta\right)^{(n+1/2)} + \delta t \times \left(Q/c_p \pi\right)^{(n)}$$

$\theta^{(*)}$: Pot. temp. after hor. adv.

From continuity equation

$$p^{(*)} = p^{(n)} + \delta t \times \nabla \cdot \int_{\zeta=\zeta_T}^{\zeta} (m\mathbf{v})^{(n+1/2)}\, d\zeta$$

$$p_S^{(n+1)} = p_S^{(n)} + \delta t \times \nabla \cdot \int_{\zeta=\zeta_T}^{\zeta_S} (m\mathbf{v})^{(n+1/2)}\, d\zeta$$

$p^{(*)}$: Pres. after hor. adv.

$p_S^{(n+1)}$: Surf. pres. for next time step

Diagnosis of vertical mass flux and completion of predictions of pot. temp, pressure and mass

Set before iteration: $\hat{\theta} = \theta^{(*)}$ and $\hat{p} = p^{(*)}$

Iteration:

$$\left(m\dot{\zeta}\right) = \frac{\mathcal{F}_{target} - \mathcal{F}\left(\hat{\theta}, \hat{p}, p_S^{(n+1)}\right)}{\delta t \times \left[\left(\frac{\partial \mathcal{F}}{\partial \hat{\theta}}\right)_{\hat{p}, p_S^{(n+1)}} \left(\frac{\partial \hat{\theta}}{\partial \hat{p}}\right) + \left(\frac{\partial \mathcal{F}}{\partial \hat{p}}\right)_{\hat{\theta}, p_S^{(n+1)}}\right]}$$

$$\hat{\theta} = \hat{\theta} + \delta t \times \left(\frac{\partial \hat{\theta}}{\partial \hat{p}}\right)\left(m\dot{\zeta}\right) \quad \text{and} \quad \hat{p} = \hat{p} + \delta t \times \left(m\dot{\zeta}\right)$$

Values for next time step: $\theta^{(n+1)} = \hat{\theta}$, $p^{(n+1)} = \hat{p}$, $\left(m\dot{\zeta}\right)^{(n+1)} = \left(m\dot{\zeta}\right)$ and $m^{(n+1)} = -\left(\frac{\partial p}{\partial \zeta}\right)^{(n+1)}$

Diagnosis of vertical mass flux in the generalized sigma $\varsigma = \mathcal{G}(p, p_S)$ vertical coordinate

(Coordinate is defined by target sigma)

From continuity equation

$$p^{(*)} = p^{(n)} + \delta t \times \nabla \cdot \int_{\varsigma=\varsigma_T}^{\varsigma} (m\mathbf{v})^{(n+1/2)} \, d\varsigma$$

$p^{(*)}$: Pres. after hor. adv.

$$p_S^{(n+1)} = p_S^{(n)} + \delta t \times \nabla \cdot \int_{\varsigma=\varsigma_T}^{\varsigma_S} (m\mathbf{v})^{(n+1/2)} \, d\varsigma$$

$p_S^{(n+1)}$: Surf. pres. for next time step

Diagnosis of vertical mass flux

$$(m\dot{\varsigma}) = \frac{\mathcal{G}_{target} - \mathcal{G}\left(p^{(*)}, p_S^{(n+1)}\right)}{\delta t \times \left(\frac{\partial \mathcal{G}}{\partial p}\right)^{(*)}_{p_S^{(n+1)}}}$$

Values for next time step: $(m\dot{\varsigma})^{(n+1)} = (m\dot{\varsigma})$

Diagnosis of vertical mass flux in the generalized sigma $\varsigma = \mathcal{G}(p, p_S)$ vertical coordinate

(Coordinate is defined by target pressure)

From continuity equation

$$p^{(*)} = p^{(n)} + \delta t \times \nabla \cdot \int_{\varsigma=\varsigma_T}^{\varsigma} (m\mathbf{v})^{(n+1/2)} d\varsigma$$

$p^{(*)}$: Pres. after hor. adv.

$$p_S^{(n+1)} = p_S^{(n)} + \delta t \times \nabla \cdot \int_{\varsigma=\varsigma_T}^{\varsigma_S} (m\mathbf{v})^{(n+1/2)} d\varsigma$$

$p_S^{(n+1)}$: Surf. pres. for next time step

Diagnosis of target pressure (pressure required by coordinate definition)

$$p_{target} = Ap_{00} + B\left(p_S^{(n+1)} - p_T\right)$$

Diagnosis of vertical mass flux

$$(m\dot{\varsigma}) = \frac{p_{target} - p^{(*)}}{\delta t}$$

Values for next time step: $(m\dot{\varsigma})^{(n+1)} = (m\dot{\varsigma})$

Derivation of $p = Ap_{00} + B\left(p_S - p_T\right)$

Definitions

$$\mathcal{G}(p,p_S) - \mathcal{C} = \left(\frac{\partial \mathcal{G}}{\partial p}\right)_{p_S} p + \left(\frac{\partial \mathcal{G}}{\partial p_S}\right)_p \left(p_S - p_T\right)$$

$$\mathcal{C} \equiv \mathcal{G}\left(p^{(0)}, p_S^{(0)}\right) - \left[\left(\frac{\partial \mathcal{G}}{\partial p}\right)_{p_S} p + \left(\frac{\partial \mathcal{G}}{\partial p_S}\right)_p \left(p_S - p_T\right)\right]^{(0)}$$

The superscript (0) denotes a reference state, such as initial

How to obtain $p = Ap_{00} + B\left(p_S - p_T\right)$:

$$\frac{\mathcal{G}(p,p_S) - \mathcal{C}}{\left(\partial \mathcal{G}/\partial p\right)_{p_S}} = p + \frac{\left(\partial \mathcal{G}/\partial p_S\right)_p}{\left(\partial \mathcal{G}/\partial p\right)_{p_S}} \left(p_S - p_T\right)$$

$$p = \underbrace{\left[\frac{1}{p_{00}} \frac{\mathcal{G}(p,p_S) - \mathcal{C}}{\left(\partial \mathcal{G}/\partial p\right)_{p_S}}\right]^{(0)}}_{A} p_{00} + \underbrace{\left[-\frac{\left(\partial \mathcal{G}/\partial p_S\right)_p}{\left(\partial \mathcal{G}/\partial p\right)_{p_S}}\right]^{(0)}}_{B} \left(p_S - p_T\right)$$

$$p = Ap_{00} + B\left(p_S - p_T\right)$$

Derivation of $p = Ap_{00} + B\left(p_S - p_T\right) + C\left(p^{(0)}/\theta^{(0)}\right)\theta$

Definitions

$$\mathcal{F}(\theta,p,p_S)-\mathcal{C}=\left(\frac{\partial\mathcal{F}}{\partial\theta}\right)_{\varsigma}\theta+\left(\frac{\partial\mathcal{F}}{\partial\varsigma}\right)_{\theta}\left(\frac{\partial\mathcal{G}}{\partial p}\right)_{p_S}p+\left(\frac{\partial\mathcal{F}}{\partial\varsigma}\right)_{\theta}\left(\frac{\partial\mathcal{G}}{\partial p_S}\right)_{p}(p_S-p_T)$$

$$\mathcal{C}\equiv\mathcal{F}\left(\theta^{(0)},p^{(0)},p_S^{(0)}\right)-\left[\left(\frac{\partial\mathcal{F}}{\partial\theta}\right)_{\varsigma}\theta+\left(\frac{\partial\mathcal{F}}{\partial\varsigma}\right)_{\theta}\left(\frac{\partial\mathcal{G}}{\partial p}\right)_{p_S}p+\left(\frac{\partial\mathcal{F}}{\partial\varsigma}\right)_{\theta}\left(\frac{\partial\mathcal{G}}{\partial p_S}\right)_{p}(p_S-p_T)\right]^{(0)}$$

The superscript (0) denotes a reference state, such as initial

How to obtain $p = Ap_{00} + B\left(p_S - p_T\right) + C\left(p^{(0)}/\theta^{(0)}\right)\theta$:

$$\frac{\mathcal{F}(\theta,p,p_S)-\mathcal{C}}{(\partial\mathcal{F}/\partial\varsigma)_{\theta}(\partial\mathcal{G}/\partial p)_{p_S}}=\frac{(\partial\mathcal{F}/\partial\theta)_{\varsigma}}{(\partial\mathcal{F}/\partial\varsigma)_{\theta}(\partial\mathcal{G}/\partial p)_{p_S}}\theta+p+\frac{(\partial\mathcal{G}/\partial p_S)_{p}}{(\partial\mathcal{G}/\partial p)_{p_S}}(p_S-p_T)$$

$$p=\underbrace{\left[\frac{1}{p_{00}}\frac{\mathcal{F}(\theta,p,p_S)-\mathcal{C}}{(\partial\mathcal{F}/\partial\varsigma)_{\theta}(\partial\mathcal{G}/\partial p)_{p_S}}\right]^{(0)}}_{A}p_{00}+\underbrace{\left[-\frac{(\partial\mathcal{G}/\partial p_S)_{p}}{(\partial\mathcal{G}/\partial p)_{p_S}}\right]^{(0)}}_{B}(p_S-p_T)+\underbrace{\left[-\frac{\theta}{p}\frac{(\partial\mathcal{F}/\partial\theta)_{\varsigma}}{(\partial\mathcal{F}/\partial\varsigma)_{\theta}(\partial\mathcal{G}/\partial p)_{p_S}}\right]^{(0)}}_{C}\frac{p^{(0)}}{\theta^{(0)}}\theta$$

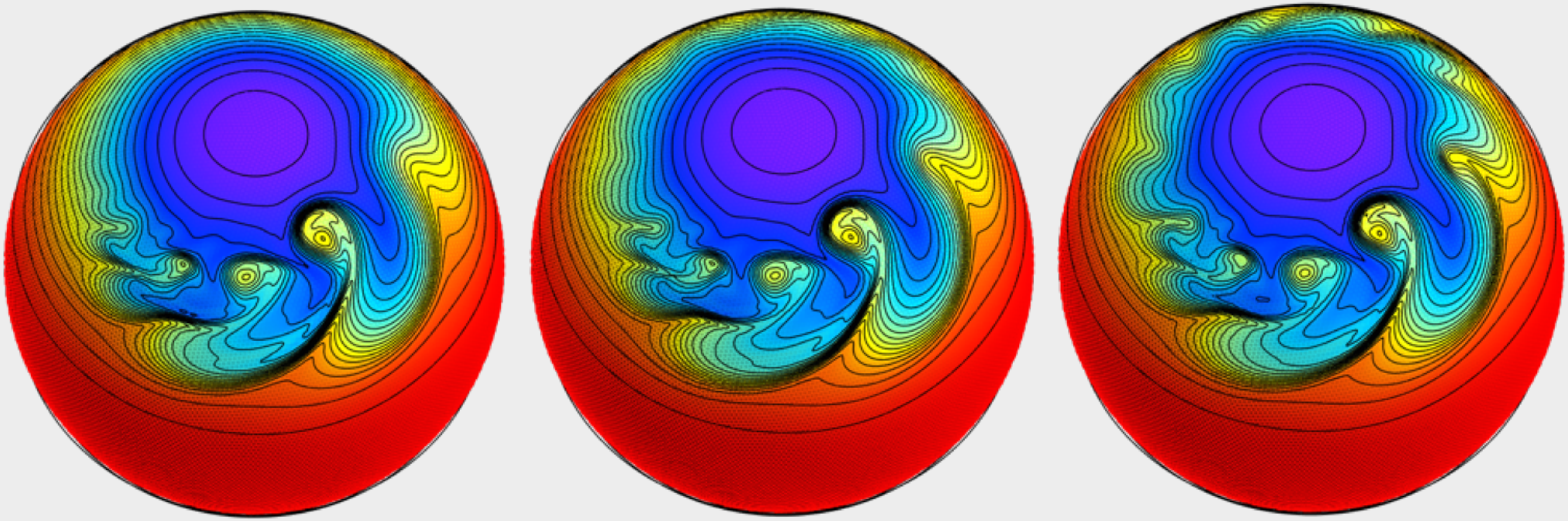
$$p = Ap_{00} + B\left(p_S - p_T\right) + C\left(p^{(0)}/\theta^{(0)}\right)\theta$$

Sigma and hybrid isentropic-sigma model results

Evolution of extratropical disturbances in a broad baroclinic zone

Surface potential temperature at Day 10

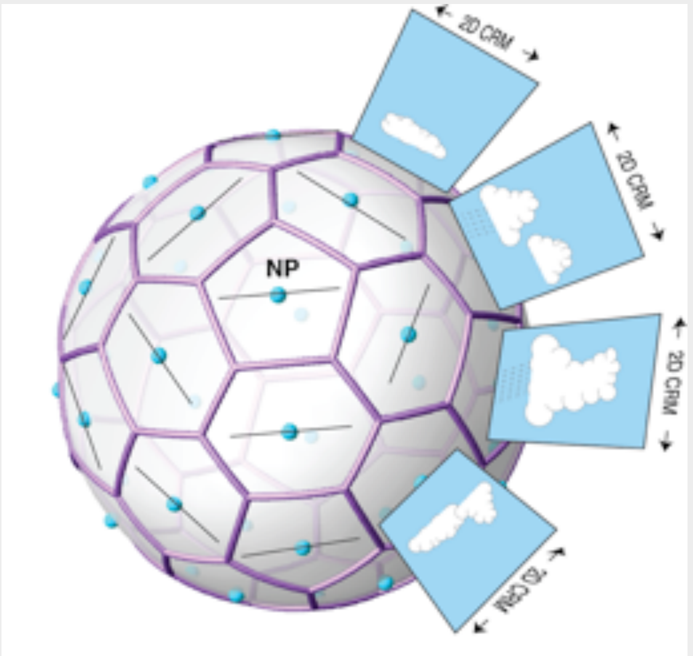
Sigma GVC (alpha=1) GVC (alpha=8)



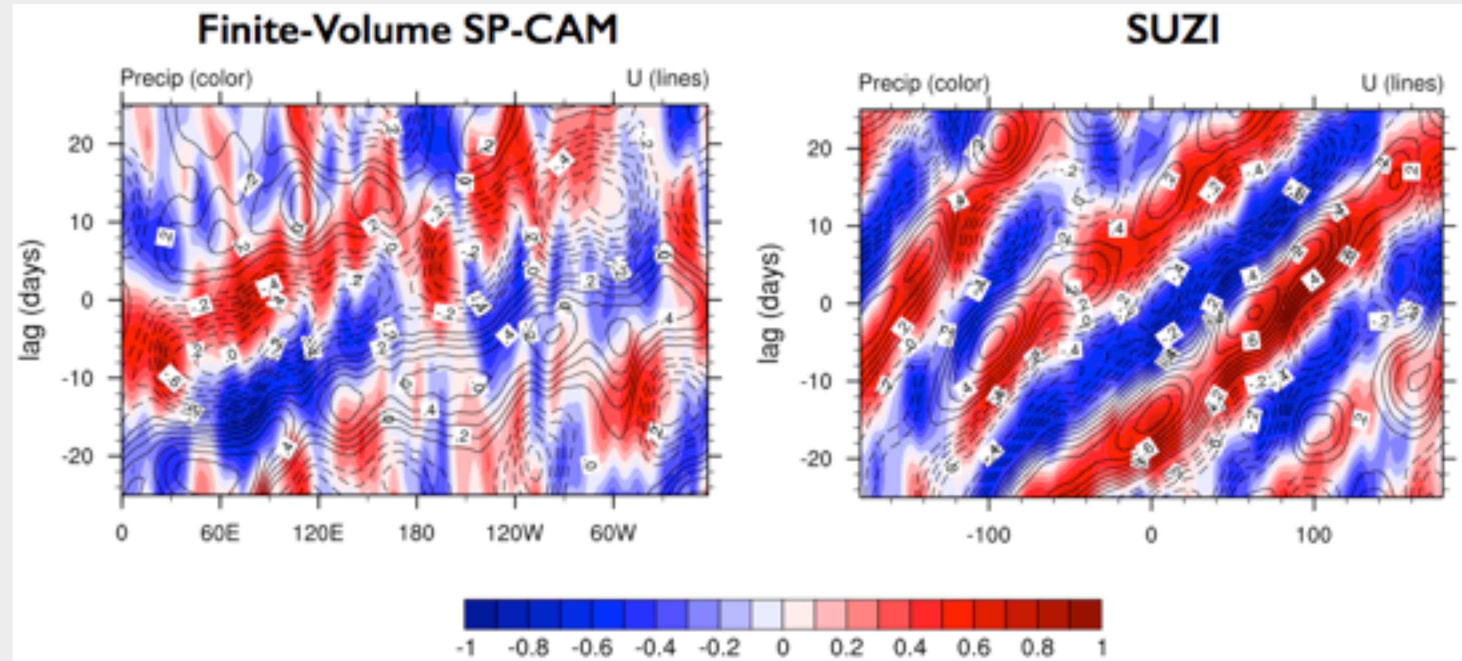
less isentropic more isentropic

SUZI aquaplanet simulation

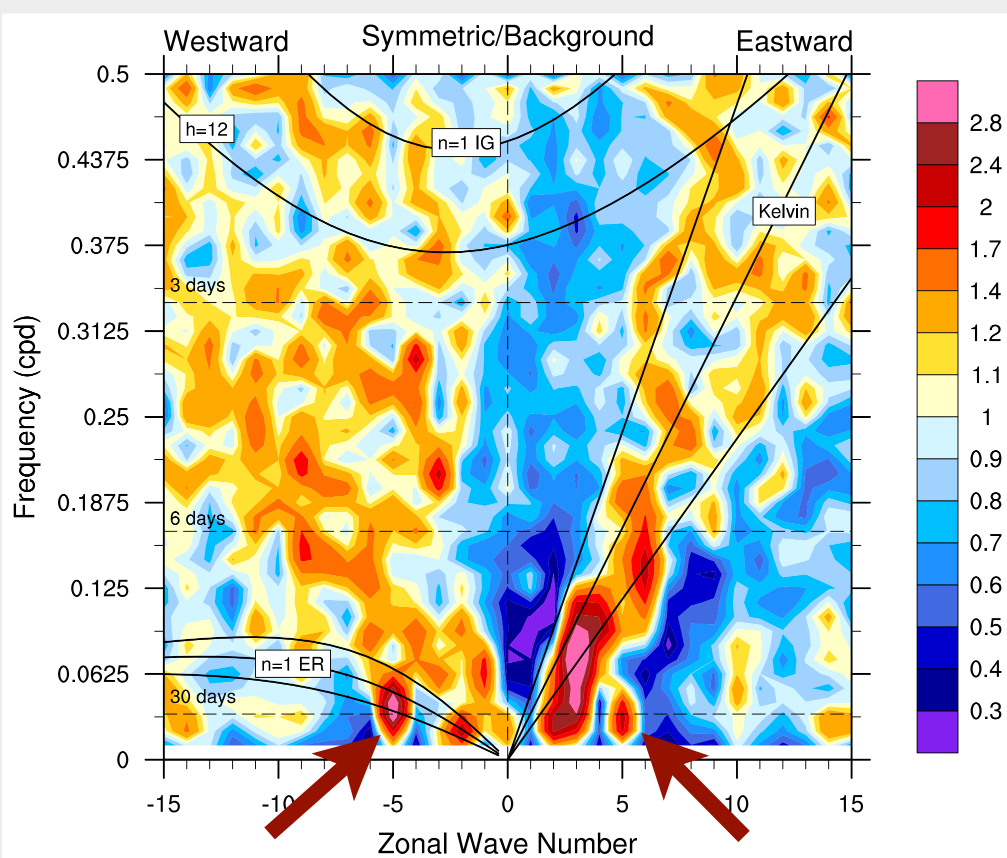
GCM and CRMs in SUZI

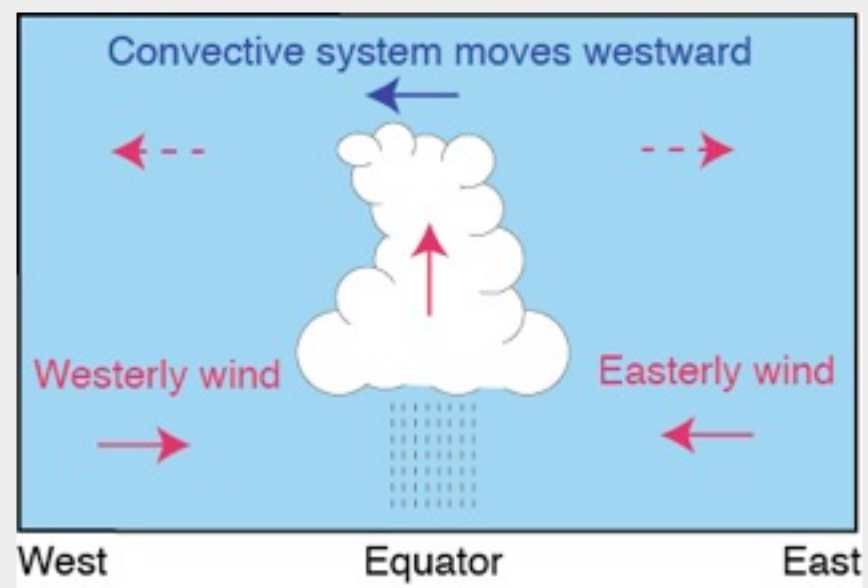


Lagged correlations of precipitation (color) and zonal wind (contour) along Equator



Dispersion of EQ waves

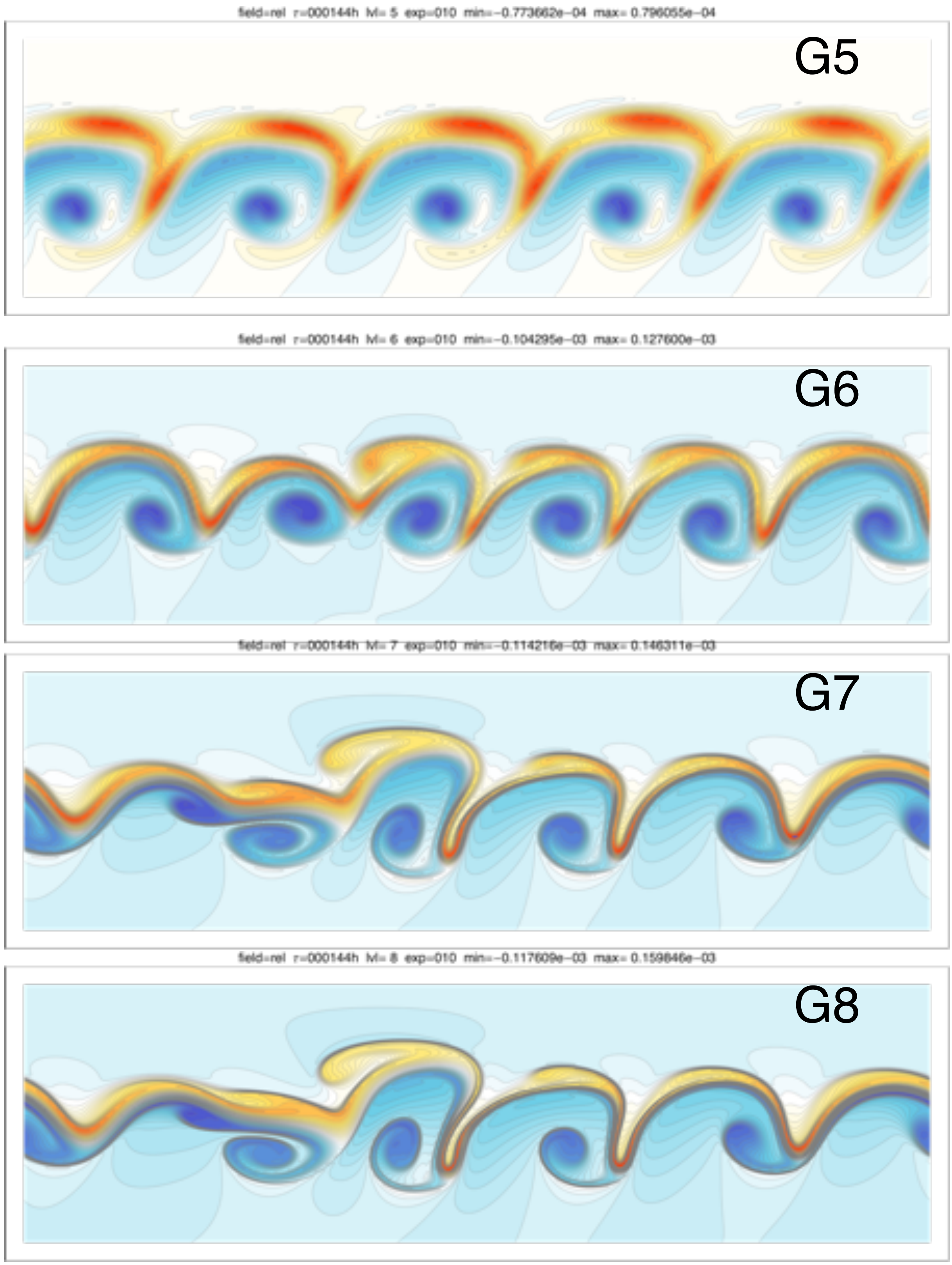




Shallow-water test case

Galewsky et al.

(Wavenumber-6 perturbation)

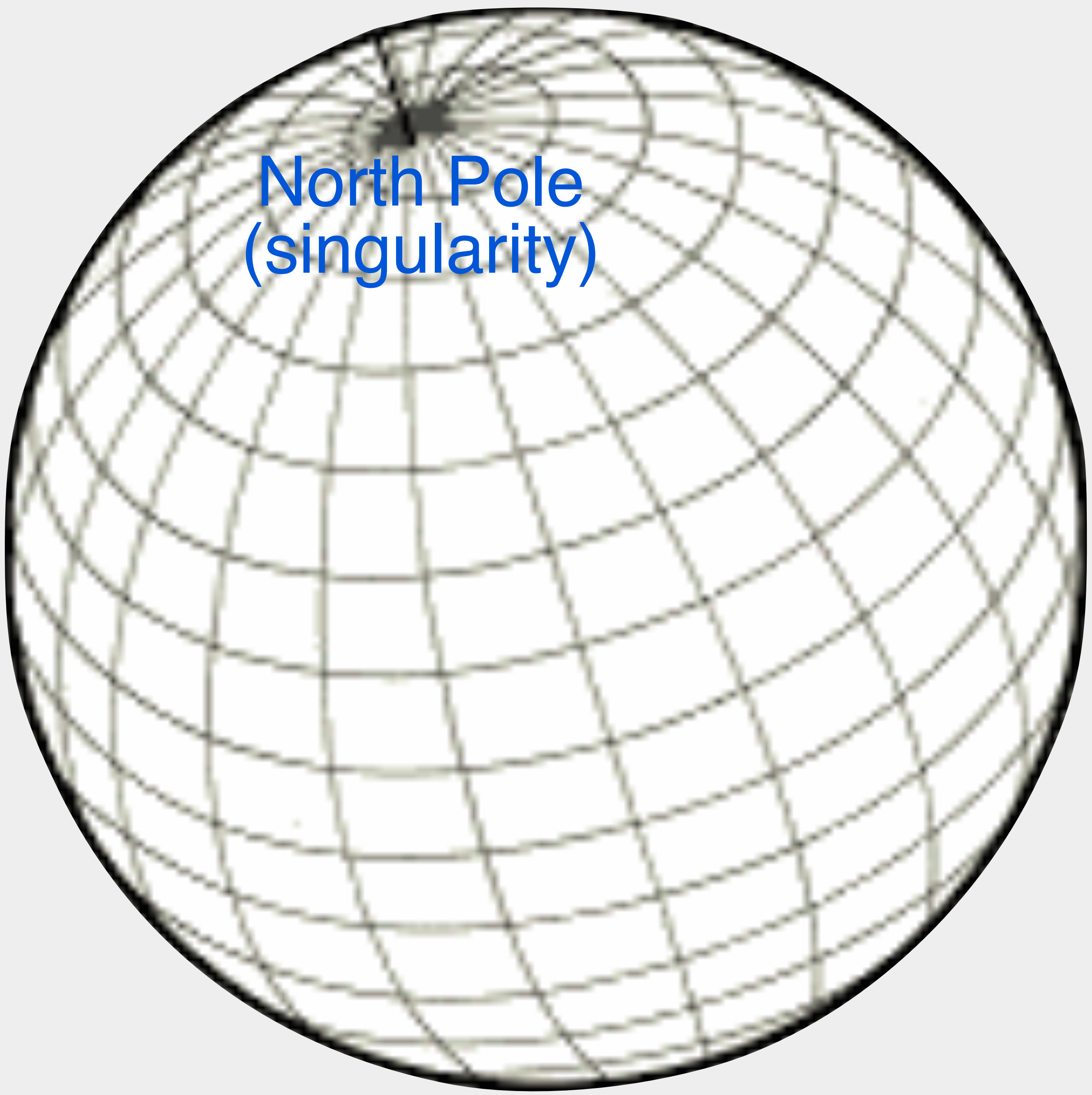


Grid	Number of grid points	Grid distance (km)
G5	10 242	240.9
G6	40 962	120.4
G7	163 842	60.2
G8	655 362	30.1

From the pole problem to the wavenumber-5 problem

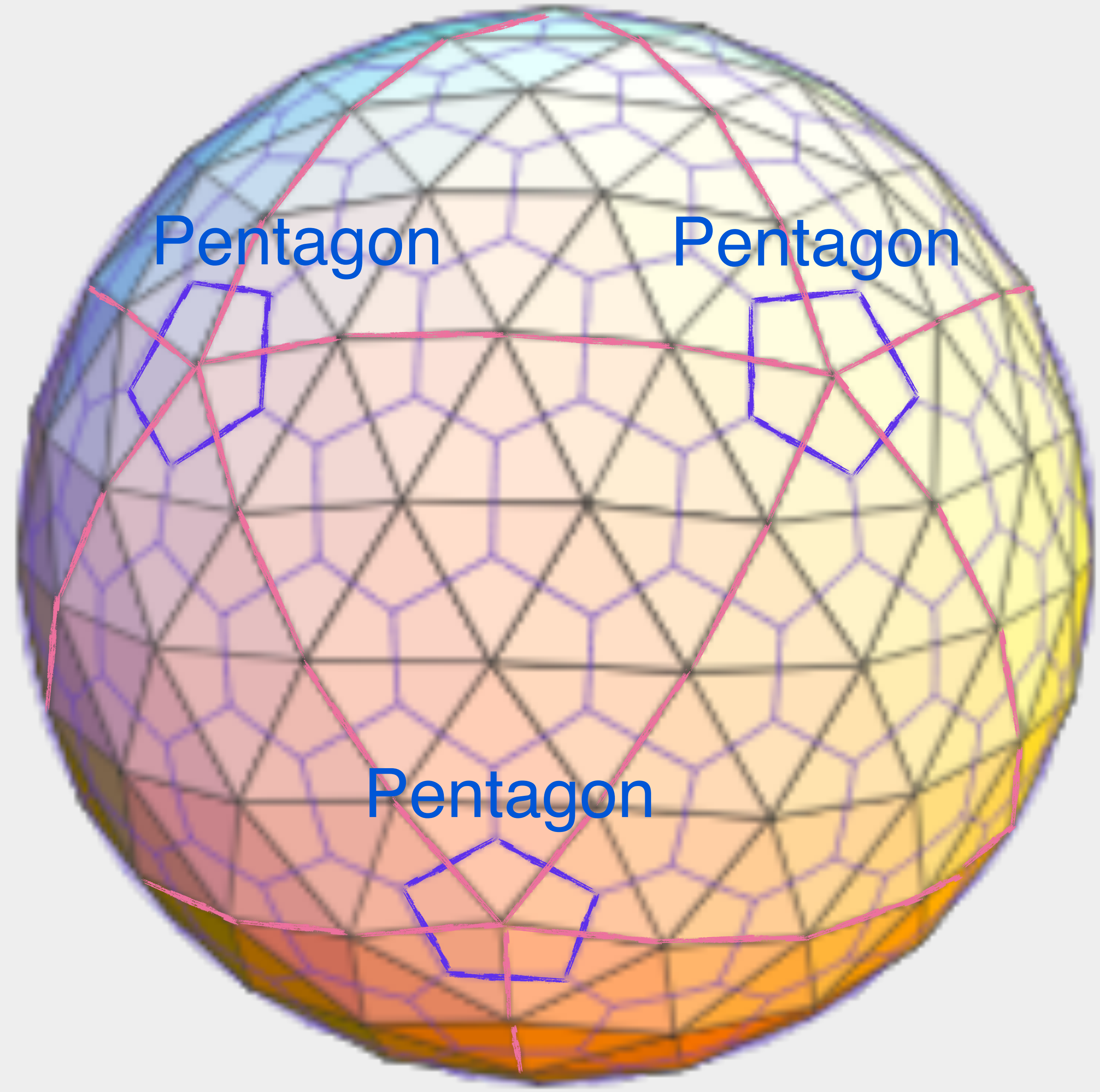
Grids

Longitude-Latitude grid



The grid of CAM-FV, etc.

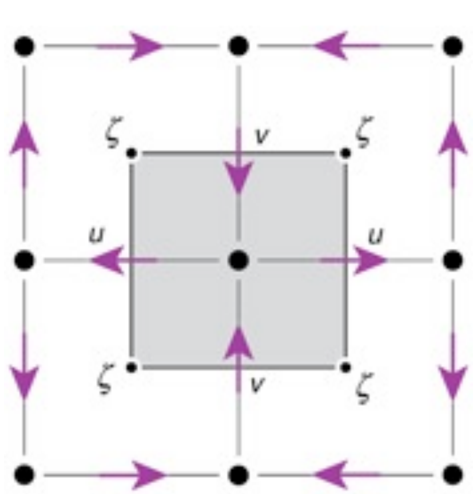
Icosahedral hexagon-pentagon grid



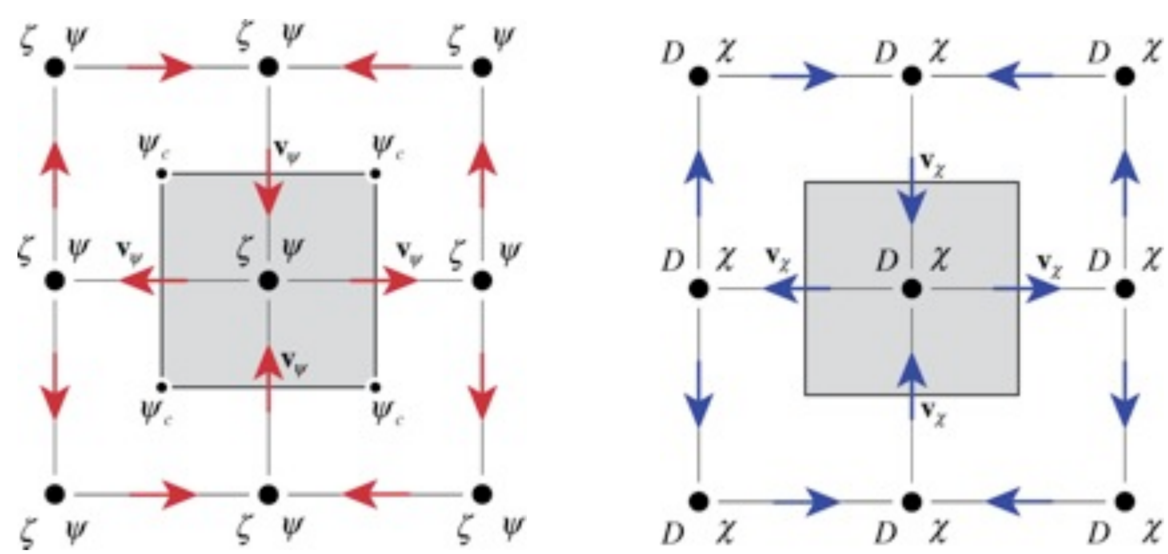
The grid of UZIM, NICAM, etc.

Momentum equation	Vorticity and Divergence equations	Barotropic vorticity equation
$\frac{\partial \mathbf{v}}{\partial t} = -\mathbf{k} \times \eta \mathbf{v} + \cdots$ $\eta \equiv \zeta + f$ $\zeta \equiv \mathbf{k} \cdot \nabla_H \times \mathbf{v}$	$\frac{\partial \eta}{\partial t} = -\nabla_H \cdot (\eta \mathbf{v}_\psi) - \nabla_H \cdot (\eta \mathbf{v}_\chi) + \cdots$ $= J(\eta, \psi) - \nabla_H \cdot (\eta \mathbf{v}_\chi) + \cdots$ $\eta \equiv \zeta + f \qquad \zeta \equiv \mathbf{k} \cdot \nabla_H \times \mathbf{v}_\psi$ $\frac{\partial D}{\partial t} = -\nabla_H \cdot (\mathbf{k} \times \eta \mathbf{v}_\psi) - \nabla_H \cdot (\mathbf{k} \times \eta \mathbf{v}_\chi) + \cdots$ $= \nabla_H \cdot (\eta \nabla_H \psi) + J(\eta, \chi) + \cdots$ $\mathbf{v}_\psi \equiv \mathbf{k} \times \nabla_H \psi \qquad \mathbf{v}_\chi \equiv \nabla_H \chi$ $\nabla_H^2 \psi = \zeta \qquad \nabla_H^2 \chi = D$	$\frac{\partial \eta}{\partial t} = -\nabla_H \cdot (\eta \mathbf{v}_\psi) = J(\eta, \psi)$ $\eta \equiv \zeta + f$ $\mathbf{v}_\psi \equiv \mathbf{k} \times \nabla_H \psi$ $\nabla_H^2 \psi = \zeta$

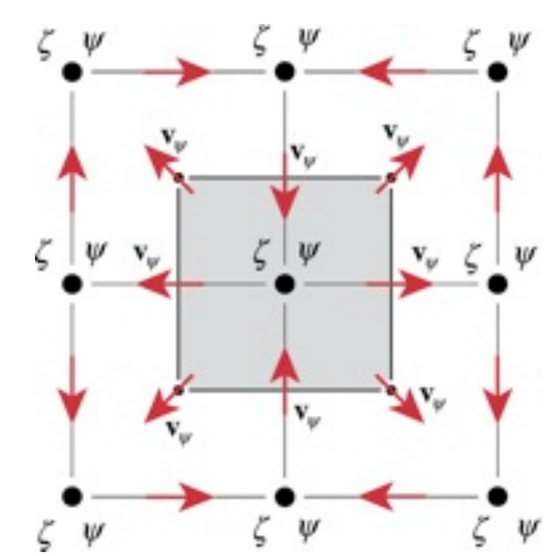
Arakawa C-grid



Randall Z-grid

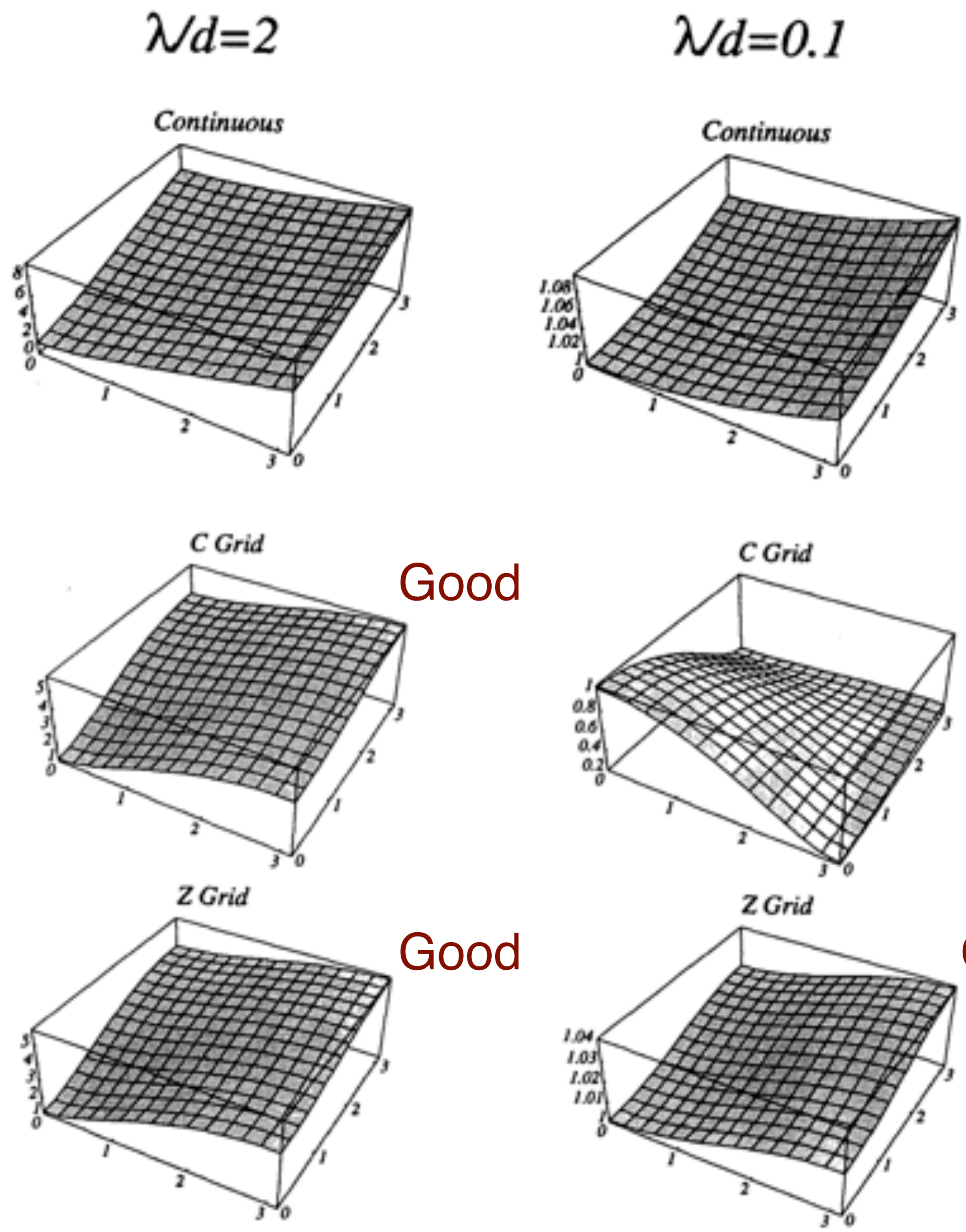


Grid of Arakawa Jacobian



C- and Z-grids in linearized systems

Thick layer solution | Thin layer solution



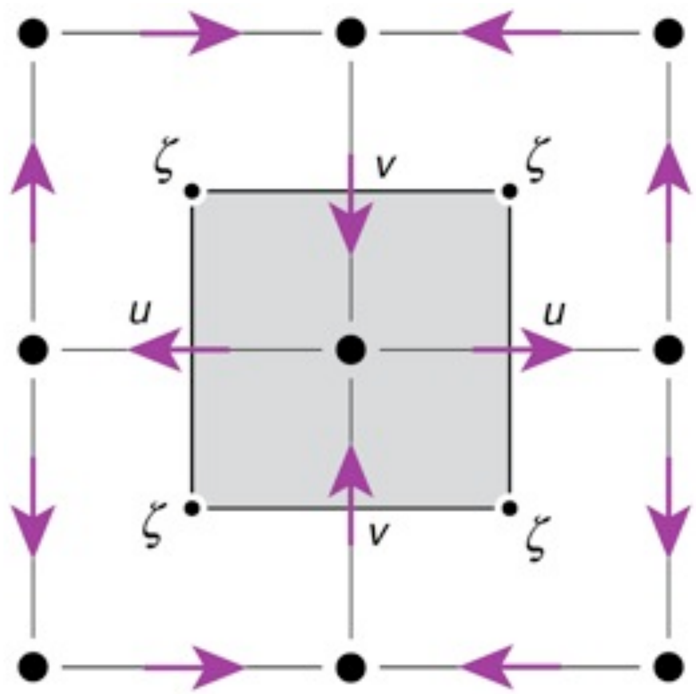
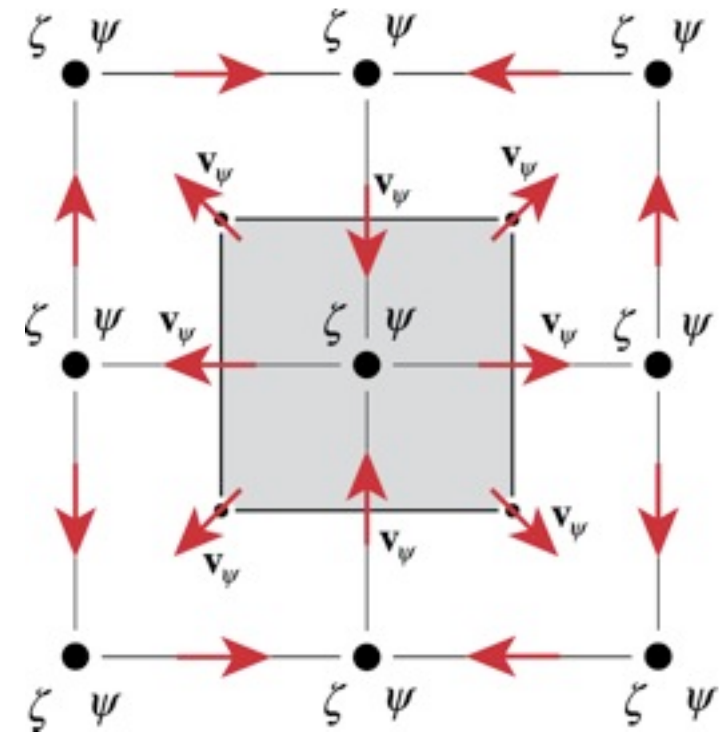
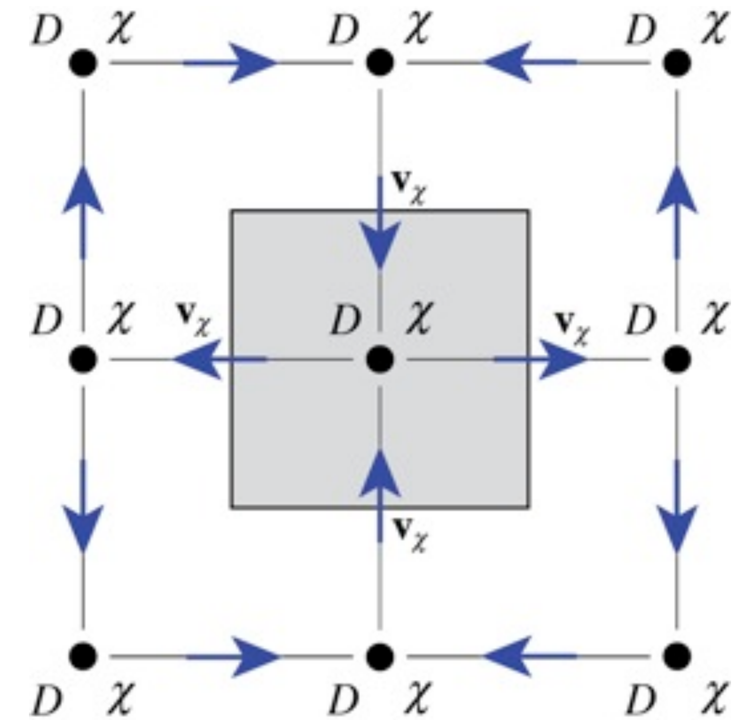
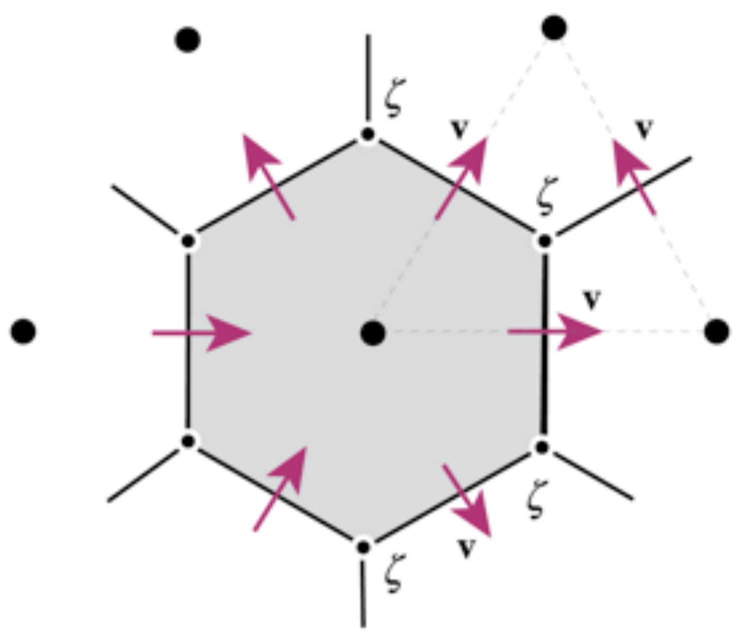
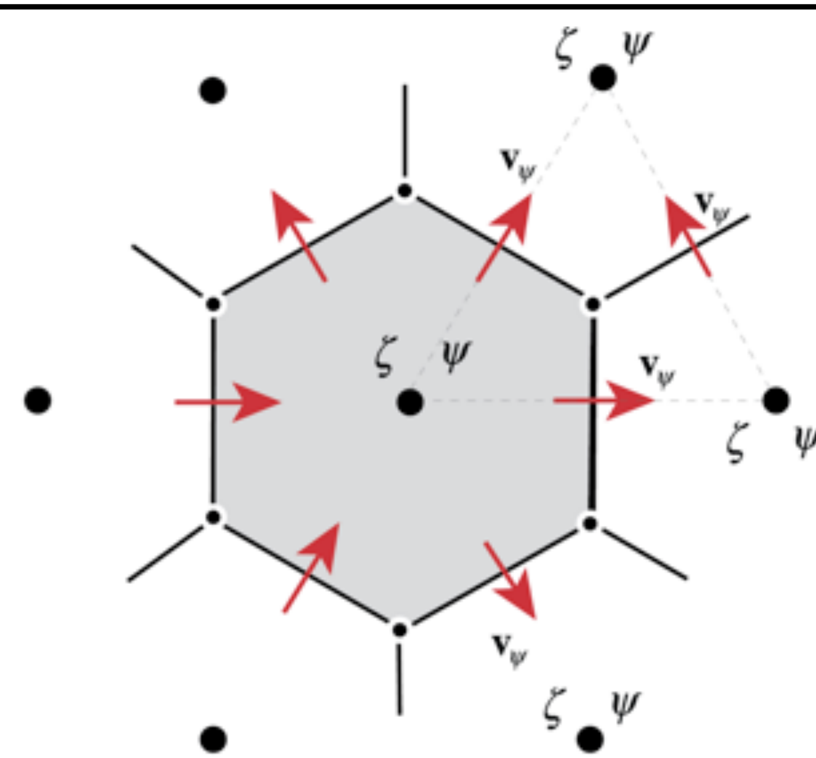
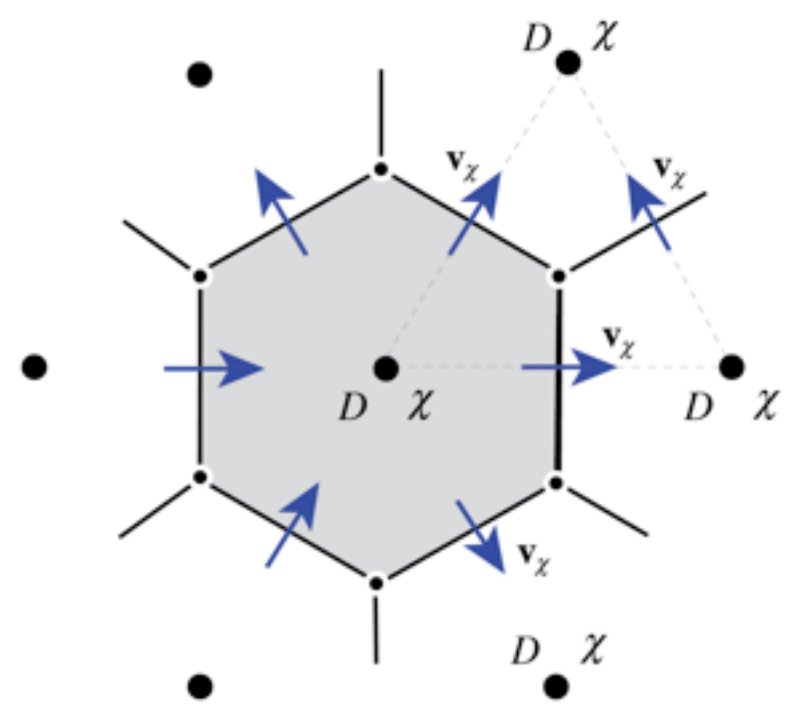
True dispersion

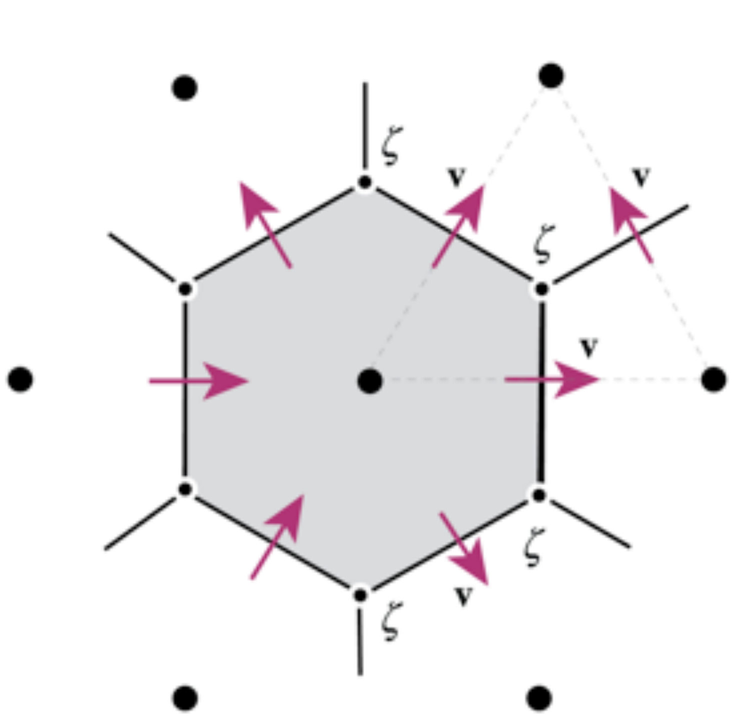
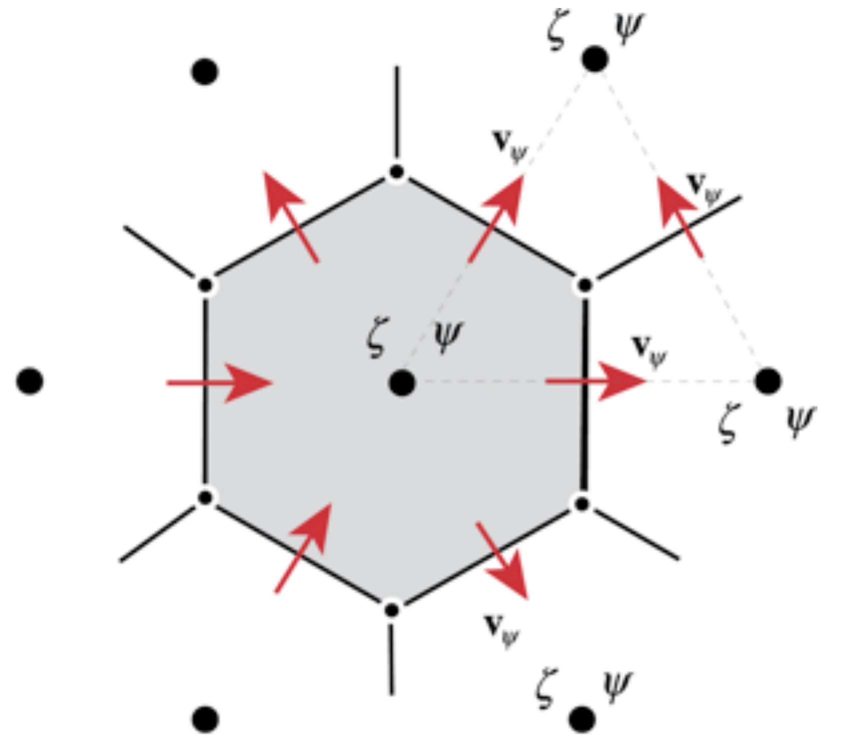
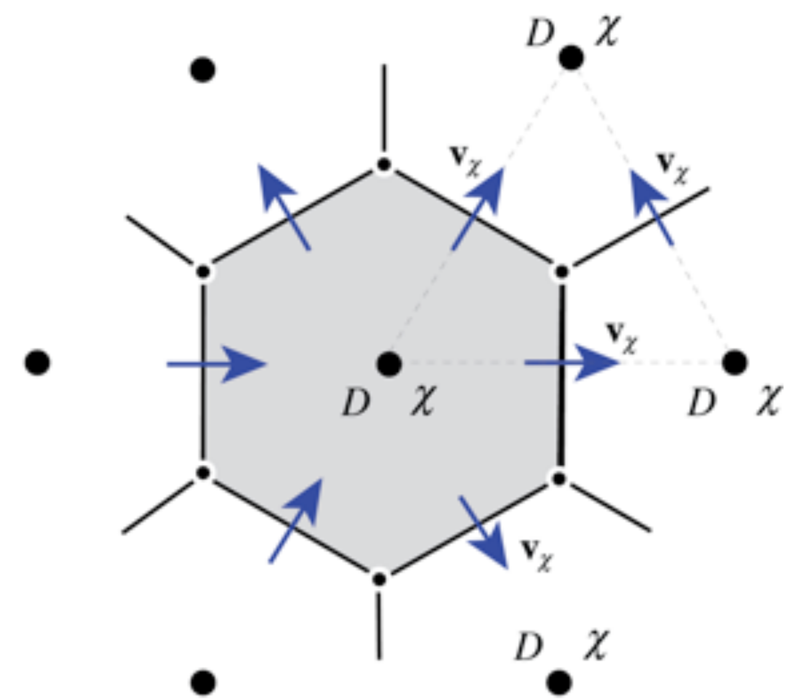
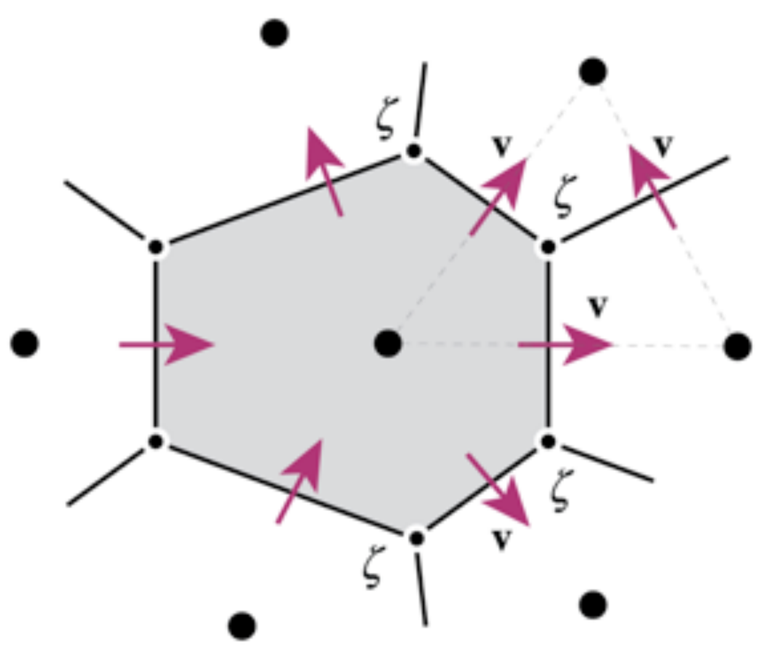
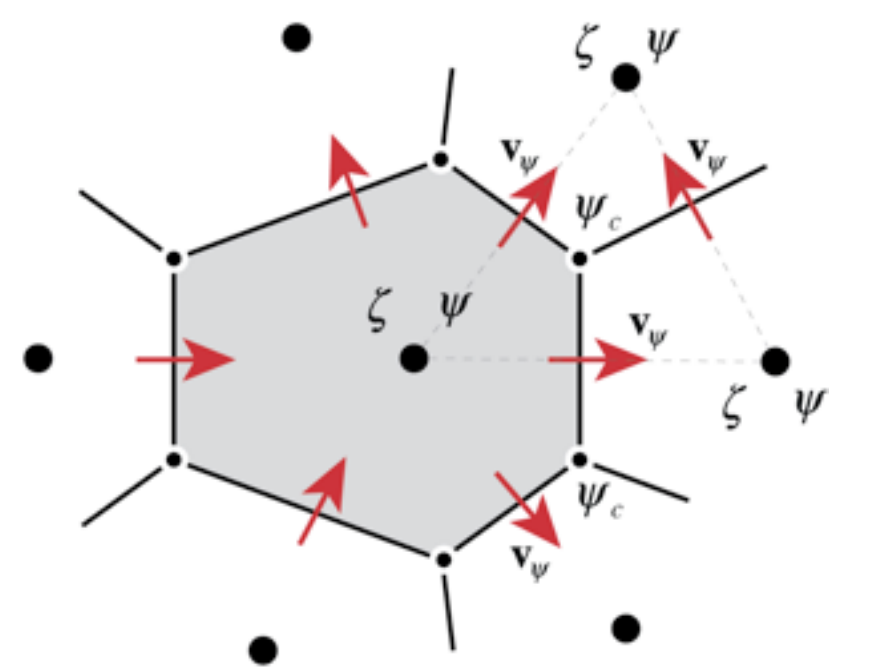
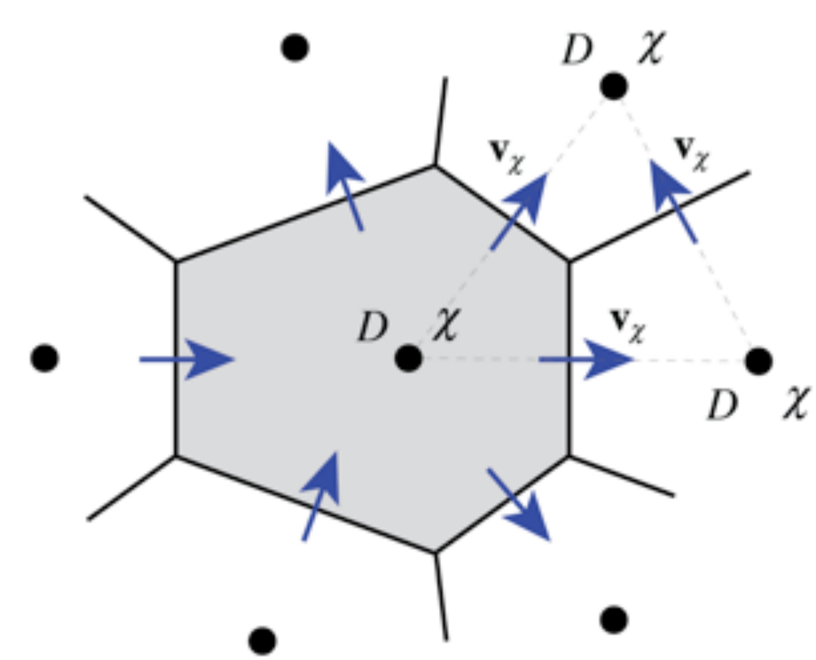
C-grid dispersion — Good (thick layer), Bad (thin layer) — Mixed results

Z-grid dispersion — Good (thick layer), Good (thin layer) — Overall good results

- Linearized system tests the surface wave propagation, and thus only the divergent component of velocity plays a role.

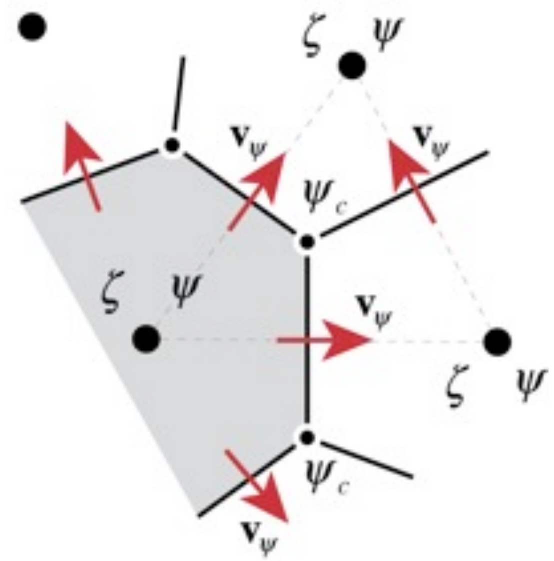
Taken from Randall 1994

C-grid	Z-grid	
 Vorticity is a product	 Vorticity is *unverifiable* Small chance for computational mode	 Divergence is *verifiable*
 Vorticity is a product	 Vorticity is *unverifiable* Small chance for computational mode	 Divergence is *verifiable*

C-grid	Z-grid	
 Vorticity is a product	 Vorticity is *unverifiable* Small chance for computational mode	 Divergence is *verifiable*
 Vorticity is a product	 Vorticity is *unverifiable* *High* chance for computational mode	 Divergence is *verifiable*

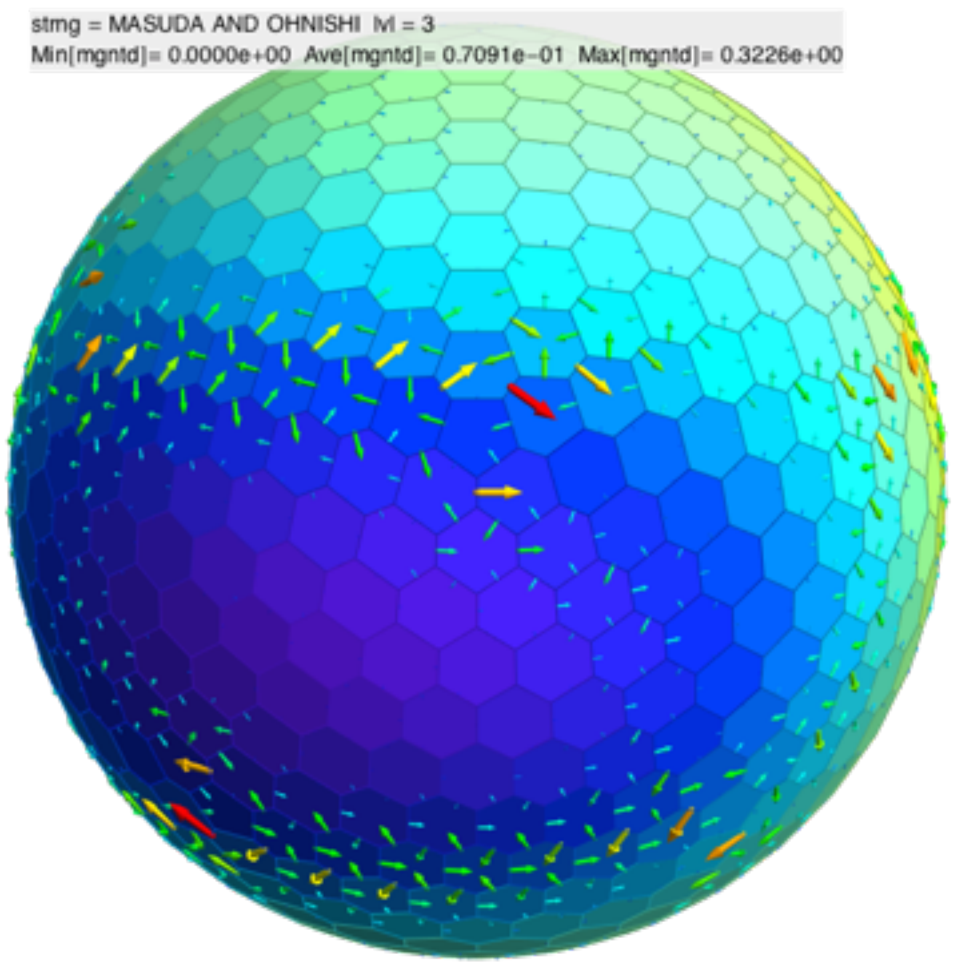
Rotational wind error

Interpolation of streamfunction to the corners from centers



Enstrophy and kinetic energy conserving interpolation

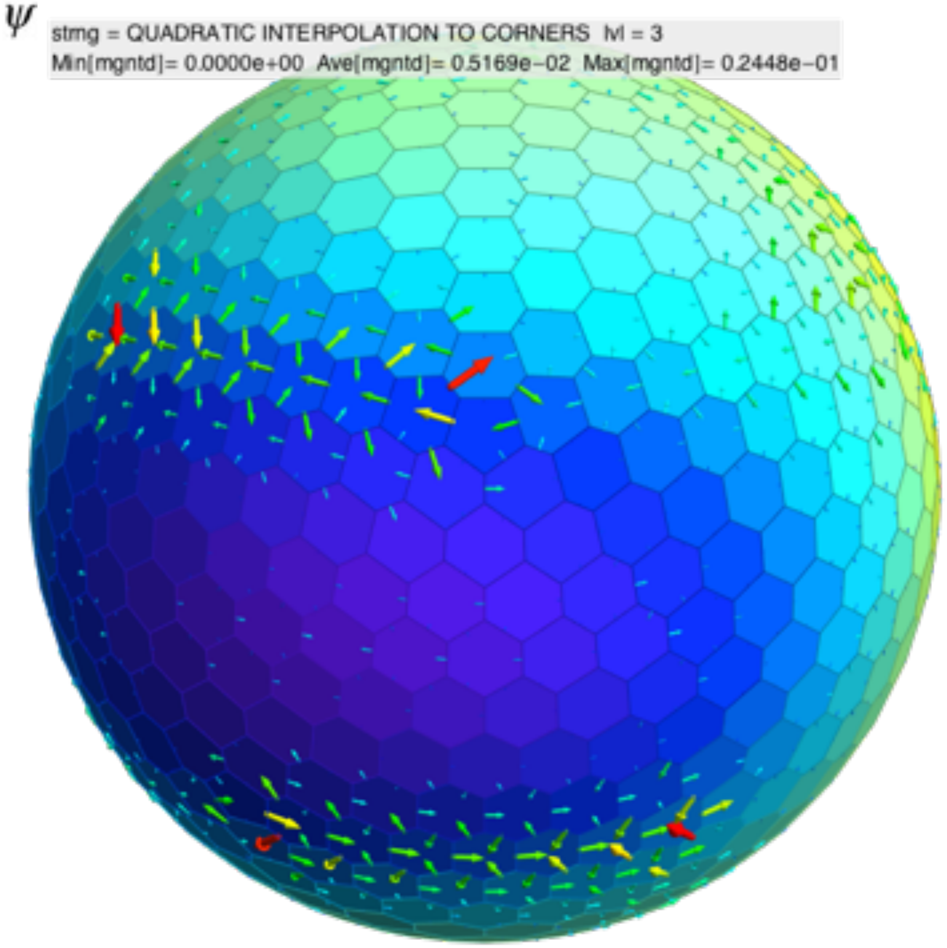
(Arithmetic average)



Max error is 28% of true wind

Quadratic interpolation

(Fitting a curvy surface)

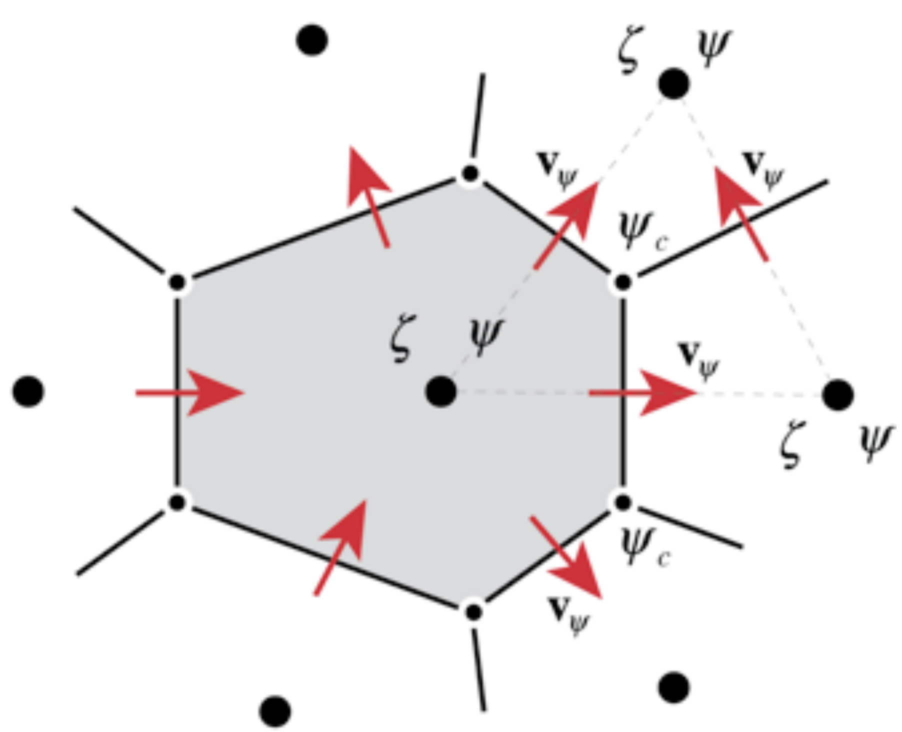


Max error is 2.1% of true wind

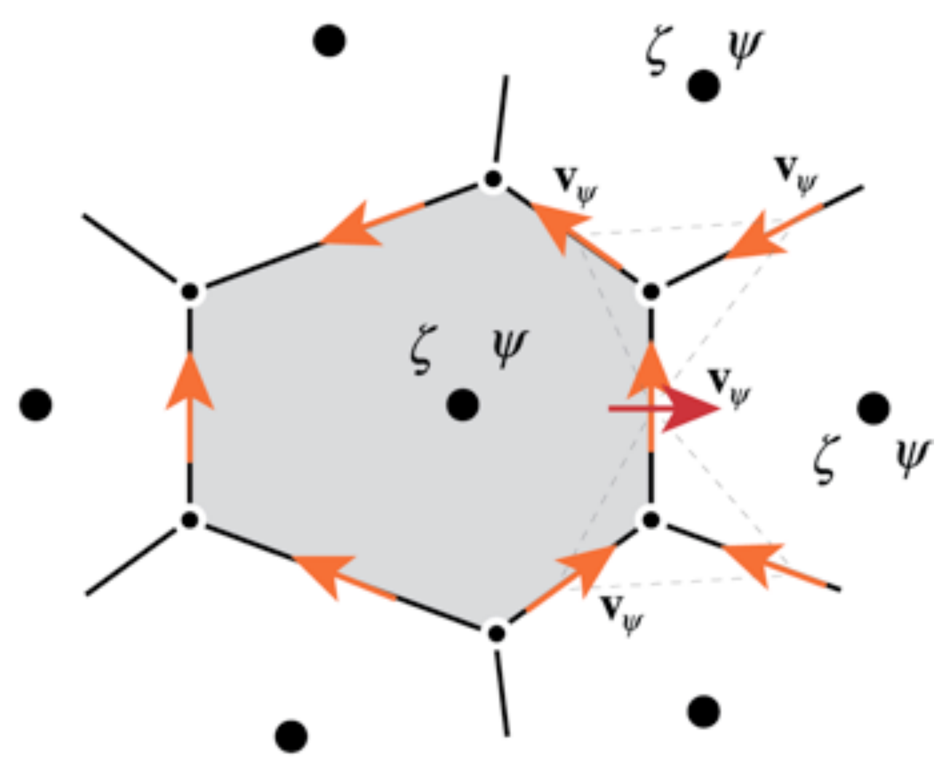
Mitigation

Improve the accuracy of the normal component of rotational velocity

- Improved interpolation of the streamfunction to the corners



- First compute the tangential component of rotational velocity. Then interpolate to obtain normal component



2D-least-square based interpolation scheme

Conservative Transport Schemes for Spherical Geodesic Grids: High-Order Reconstructions for Forward-in-Time Schemes

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$$\psi - \psi_0 = c_x x + c_y y + c_{xx} x^2 + c_{xy} xy + c_{yy} y^2 + c_{xxx} x^3 + c_{xxy} x^2 y + c_{xyy} xy^2 + c_{yyy} y^3 + c_{xxxx} x^4 + c_{xxxy} x^3 y + c_{xxyy} x^2 y^2 + c_{xyyy} xy^3 + c_{yyyy} y^4, \quad (6)$$

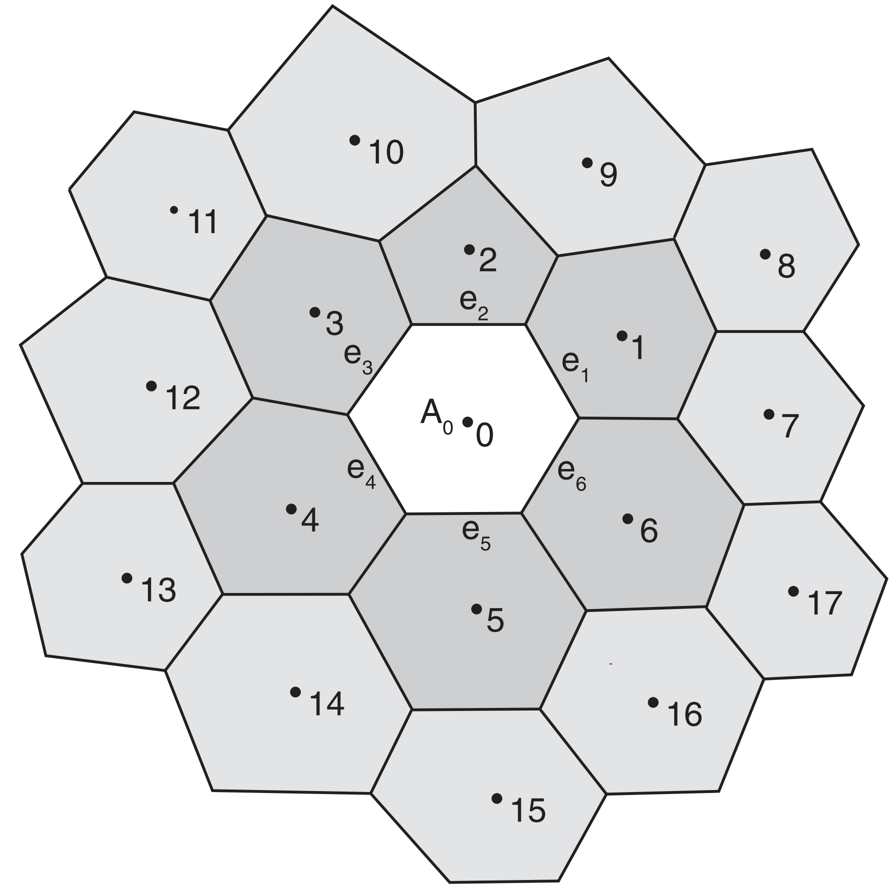
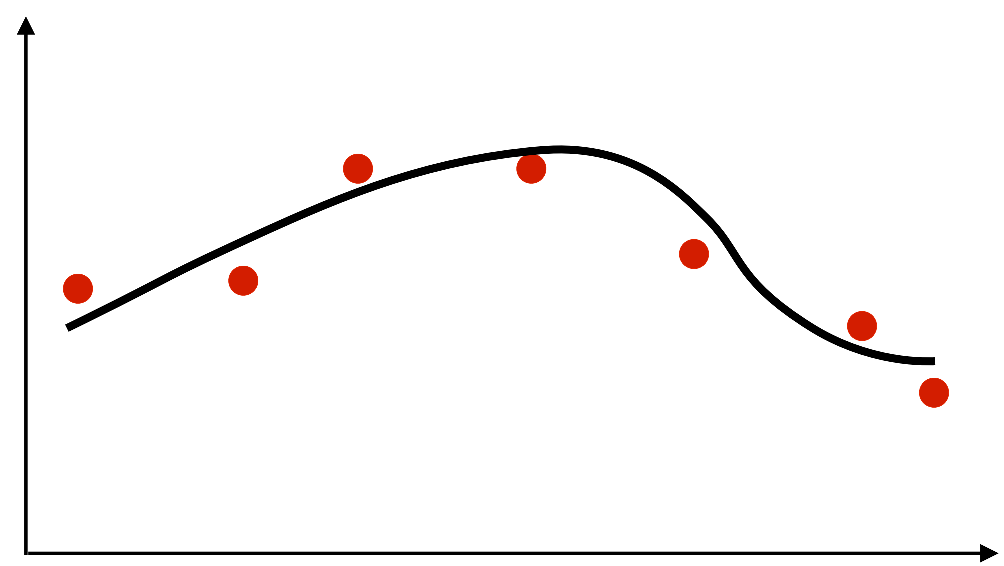


FIG. 2. Schematic showing a grid centered about cell 0. The dark-shaded cells (1–6) are used in the reconstruction of polynomials less than or equal to order 2. The lighter-shaded cells (7–17) are used in the reconstruction of polynomials on the order of 3 and 4.



Summary

Development of the Global Cloud Resolving Model

- UZIM (**U**nified **Z**-grid **I**cosahedral **M**odel) development:
  - The *hybrid sigma-pressure* vertical coordinate and the gvc versions of the quasi-hydrostatic UZIM have been completed.
- SUZI (**S**uperparameterized **U**nified **Z**-grid **I**cosahedral Model) has been constructed and tested in an aquaplanet simulation by Don Dazlich.
- We face a wavenumber-5 problem. We will work on the mitigation of the problem.
- We are working on a new Global VVM which predicts the curl of horizontal vorticity. The 3D elliptic equation has more convenient boundary conditions with the VVM.