Atmospheric Dynamical Cores

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Outline:

- Progress report
- From the pole problem to the wavenumber-5 problem

DF working group

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Progress report:

Development of the Global Cloud Resolving Model

- UZIM (**U**nified **Z**-grid **I**cosahedral **M**odel) development:
	- The *hybrid sigma-pressure* vertical coordinate version of the quasi-hydrostatic UZIM (UZIM-sigma) has been completed.
	- The *hybrid isentropic-sigma* vertical coordinate (gvc) version of the quasi-hydrostatic UZIM (UZIM-gvc) has been completed.
- SUZI (**S**uperparameterized **U**nified **Z-grid Icosahedral Model) has been** constructed and tested in an aquaplanet simulation by Don Dazlich.

UZIM with the hybrid sigma-pressure and hybrid isentropic-sigma vertical **coordinates**

Hydrid sigma-pressure vertical coordinate

Hydrid isentropic-sigma vertical coordinate

less isentropic more isentropic

$$
\zeta \equiv \mathcal{F}(\theta, \sigma) \qquad \sigma \equiv \mathcal{G}(p, p_s)
$$

Generalized $\zeta = \mathcal{F}(\theta, p, p_s)$ vertical coordinate

Generalized coordinate definition

$$
\zeta \equiv \mathcal{F}(\theta, \zeta) = \theta_{\min} + (\theta - \theta_{\min}) g(\zeta) - (\partial \theta / \partial \zeta)_{\min} \left\{ (\zeta - \zeta_T) g_0 - \frac{1}{\alpha} [g(\zeta) - 1] \right\}
$$

 $\varsigma \equiv \mathcal{G}(p, p_s)$ (see next page)

$$
g(\zeta) \equiv g_0 \left(1 - e^{-\alpha \zeta} \right) \quad \text{where} \quad g_0 \equiv 1/ \left(1 - e^{-\alpha \zeta_T} \right)
$$

Derivatives

$$
\left(\frac{\partial \mathcal{F}}{\partial \theta}\right)_{\zeta} = g(\zeta) \quad \text{and} \quad \left(\frac{\partial \mathcal{F}}{\partial \zeta}\right)_{\theta} = \alpha \left[\theta + \frac{1}{\alpha} \left(\frac{\partial \theta}{\partial \zeta}\right)_{\text{min}}\right] \left[g_0 - g(\zeta)\right] - \left(\frac{\partial \theta}{\partial \zeta}\right)_{\text{min}} g_0
$$
\n
$$
\left(\frac{\partial \mathcal{F}}{\partial p}\right)_{\theta, p_{\zeta}} = \left(\frac{\partial \mathcal{F}}{\partial \zeta}\right)_{\theta} \left(\frac{\partial \mathcal{G}}{\partial p}\right)_{p_{\zeta}} \quad \text{and} \quad \left(\frac{\partial \mathcal{F}}{\partial \theta}\right)_{p, p_{\zeta}} = \left(\frac{\partial \mathcal{F}}{\partial \theta}\right)_{\zeta} \quad \text{[see next page for } \left(\frac{\partial \mathcal{G}}{\partial p}\right)_{p_{\zeta}}
$$

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1–Traditional sigma $\varsigma \equiv \mathcal{G}(p, p_s)$ vertical coordinate

2–Generalized sigma $\varsigma \equiv \mathcal{G}(p, p_s)$ vertical coordinate

Definition of hybrid *p***–***sigma* **coordinate**

$$
\zeta \equiv \mathcal{G}(p, p_{s}) = \frac{1}{2} \frac{p_{s0} - p_{c}}{p_{s0} - p_{T}} \left[\frac{p_{s} - p}{p_{s} - p_{c}} - \frac{1}{\beta} \ell n \cosh\left(\beta \frac{p_{c} - p}{p_{s} - p_{c}}\right) \right]
$$

+
$$
\frac{1}{2} \frac{p_{s} - p_{c}}{p_{s0} - p_{T}} \left[\frac{p_{s} - p}{p_{s} - p_{c}} + \frac{1}{\beta} \ell n \cosh\left(\beta \frac{p_{c} - p}{p_{s} - p_{c}}\right) \right] + \frac{1}{2} \frac{p_{s0} - p_{s}}{p_{s0} - p_{T}} \left[\frac{1}{\beta} \ell n \cosh(-\beta) \right]
$$

Derivatives

$$
\left(\frac{\partial \mathcal{G}}{\partial p}\right)_{p_{S}} \equiv \frac{1}{2} \frac{p_{S0} - p_{C}}{p_{S0} - p_{T}} \left[\frac{-1}{p_{S} - p_{C}} + \frac{1}{p_{S} - p_{C}} \tanh\left(\beta \frac{p_{C} - p}{p_{S} - p_{C}}\right) \right] + \frac{1}{2} \frac{p_{S} - p_{C}}{p_{S0} - p_{T}} \left[\frac{-1}{p_{S} - p_{C}} - \frac{1}{p_{S} - p_{C}} \tanh\left(\beta \frac{p_{C} - p}{p_{S} - p_{C}}\right) \right]
$$

anu

$$
\left(\frac{\partial \mathcal{G}}{\partial p_{s}}\right)_{p} = \frac{1}{2} \frac{p_{s0} - p_{c}}{p_{s0} - p_{T}} \frac{p - p_{c}}{(p_{s} - p_{c})^{2}} \left[1 - \tanh\left(\beta \frac{p_{c} - p}{p_{s} - p_{c}}\right)\right]
$$

+
$$
\frac{1}{2} \frac{1}{p_{s0} - p_{T}} \left[1 + \frac{1}{\beta} \ln \cosh\left(\beta \frac{p_{c} - p}{p_{s} - p_{c}}\right) - \frac{p_{c} - p}{(p_{s} - p_{c})} \tanh\left(\beta \frac{p_{c} - p}{p_{s} - p_{c}}\right)\right] - \frac{1}{2} \frac{1}{p_{s0} - p_{T}} \left[\frac{1}{\beta} \ln \cosh(-\beta)\right]
$$

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Diagnosis of vertical mass flux and predictions of potential temperature, pressure and mass in the generalized $\zeta = \mathcal{F}(\theta, p, p_s)$ vertical coordinate

Diagnosis of vertical mass flux in the generalized sigma $\varsigma = \mathcal{G}(p, p_s)$ vertical coordinate

(Coordinate is defined by target sigma)

Diagnosis of vertical mass flux in the generalized sigma $\varsigma = \mathcal{G}(p, p_{s})$ vertical coordinate

(Coordinate is defined by target pressure)

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Derivation of $p = Ap_{00} + B(p_S - p_T)$

Definitions

$$
\mathcal{G}(p, p_{s}) - \mathcal{C} = \left(\frac{\partial \mathcal{G}}{\partial p}\right)_{p_{s}} p + \left(\frac{\partial \mathcal{G}}{\partial p_{s}}\right)_{p} \left(p_{s} - p_{T}\right)
$$

$$
\mathcal{C} \equiv \mathcal{G}\left(p^{(0)}, p_{s}^{(0)}\right) - \left[\left(\frac{\partial \mathcal{G}}{\partial p}\right)_{p_{s}} p + \left(\frac{\partial \mathcal{G}}{\partial p_{s}}\right)_{p} \left(p_{s} - p_{T}\right)\right]^{(0)}
$$

The superscript (0) denotes a reference state, such as initial

How to obtain $p = Ap_{00} + B(p_S - p_T)$:

$$
\frac{\mathcal{G}(p,p_{\scriptscriptstyle S})\!-\!\mathcal{C}}{\big(\partial\mathcal{G}/\partial p\big)_{p_{\scriptscriptstyle S}}} = p + \frac{\big(\partial\mathcal{G}/\partial p_{\scriptscriptstyle S}\big)_{p}}{\big(\partial\mathcal{G}/\partial p\big)_{p_{\scriptscriptstyle S}}} \big(p_{\scriptscriptstyle S}-p_{\scriptscriptstyle T}\big)
$$

$$
p = \left[\frac{1}{p_{00}} \frac{\mathcal{G}(p, p_{s}) - \mathcal{C}}{(\partial \mathcal{G}/\partial p)_{p_{s}}}\right]^{(0)} p_{00} + \left[\frac{(\partial \mathcal{G}/\partial p_{s})_{p}}{(\partial \mathcal{G}/\partial p)_{p_{s}}}\right]^{(0)} (p_{s} - p_{T})
$$

$$
p = Ap_{00} + B(p_{s} - p_{T})
$$

Derivation of
$$
p = Ap_{00} + B(p_s - p_T) + C(p^{(0)}/\theta^{(0)})\theta
$$

Definitions

$$
\mathcal{F}(\theta, p, p_{s}) - \mathcal{C} = \left(\frac{\partial \mathcal{F}}{\partial \theta}\right)_{\zeta} \theta + \left(\frac{\partial \mathcal{F}}{\partial \zeta}\right)_{\theta} \left(\frac{\partial \mathcal{G}}{\partial p}\right)_{p_{s}} p + \left(\frac{\partial \mathcal{F}}{\partial \zeta}\right)_{\theta} \left(\frac{\partial \mathcal{G}}{\partial p_{s}}\right)_{p} \left(p_{s} - p_{T}\right)
$$

$$
\mathcal{C} \equiv \mathcal{F}\left(\theta^{(0)}, p^{(0)}, p_{s}^{(0)}\right) - \left[\left(\frac{\partial \mathcal{F}}{\partial \theta}\right)_{\zeta} \theta + \left(\frac{\partial \mathcal{F}}{\partial \zeta}\right)_{\theta} \left(\frac{\partial \mathcal{G}}{\partial p}\right)_{p_{s}} p + \left(\frac{\partial \mathcal{F}}{\partial \zeta}\right)_{\theta} \left(\frac{\partial \mathcal{G}}{\partial p_{s}}\right)_{p} \left(p_{s} - p_{T}\right)\right]^{(0)}
$$

The superscript (0) denotes a reference state, such as initial

How to obtain $p = Ap_{00} + B(p_s - p_T) + C(p^{(0)}/\theta^{(0)})\theta$ *:*

$$
\frac{\mathcal{F}(\theta, p, p_{S}) - \mathcal{C}}{(\partial \mathcal{F}/\partial \varsigma)_{\theta} (\partial \mathcal{G}/\partial p)_{p_{S}}} = \frac{(\partial \mathcal{F}/\partial \theta)_{\varsigma}}{(\partial \mathcal{F}/\partial \varsigma)_{\theta} (\partial \mathcal{G}/\partial p)_{p_{S}}} \theta + p + \frac{(\partial \mathcal{G}/\partial p_{S})_{p}}{(\partial \mathcal{G}/\partial p)_{p_{S}}} (p_{S} - p_{T})
$$

$$
p = \left[\frac{1}{p_{00}}\frac{\mathcal{F}(\theta, p, p_{s}) - \mathcal{C}}{(\partial \mathcal{F}/\partial \zeta)_{\theta}(\partial \mathcal{G}/\partial p)_{p_{s}}}\right]^{(0)} p_{00} + \left[\frac{(\partial \mathcal{G}/\partial p_{s})_{p}}{(\partial \mathcal{G}/\partial p)_{p_{s}}}\right]^{(0)} (p_{s} - p_{T}) + \left[\frac{\theta}{p} \frac{(\partial \mathcal{F}/\partial \theta)_{\zeta}}{(\partial \mathcal{F}/\partial \zeta)_{\theta}(\partial \mathcal{G}/\partial p)_{p_{s}}}\right]^{(0)} \frac{p^{(0)}}{\theta^{(0)}} \theta
$$
\n
$$
p = Ap_{00} + B(p_{s} - p_{T}) + C\left(p^{(0)}/\theta^{(0)}\right)\theta
$$

Sigma and hybrid isentropic-sigma model results Evolution of extratropical disturbances in a broad baroclinic zone

Surface potential temperature at Day 10

less isentropic more isentropic

SUZI aquaplanet simulation

Lagged correlations of precipitation (color) and zonal wind (contour) along Equator

Dispersion of EQ waves

GCM and CRMs in SUZI

Shallow-water test case Galewsky et al. G0 12 6699.1 100 (100) 100 (100) 0.0 (0.0) 0.0 (0.0) $\text{GUT } \text{GUT}$ dalewsky et di.

G9 2 621 442 15.0 78.6 (83.6) 95.3 (73.4) 0.0522 (9.6715) 0.0344 (0.0377)

 G_{10} 10 μ 10 $\$

 $G_{1,2}$ $G_{2,3}$ $G_{3,4}$ $G_{4,3}$ ($G_{5,4}$) $G_{6,4}$ ($G_{7,5}$) $G_{8,4}$ ($G_{9,5}$) $G_{9,6}$ ($G_{9,6}$) $G_{1,6}$ ($G_{1,6}$) $G_{2,6}$ ($G_{3,6}$) $G_{4,6}$

G12 167 772 162 1.88 78.6 (83.6) 95.3 (73.4) 0.0065 (9.6715) 0.0043 (0.0047)

 G_{13} 671 G_{13} G_{14} G_{15} G_{16} G_{17} G_{18} G_{19} (G_{19}

Fiurbation) 91.6 (84.8) 91.6 (84.8) 5.8020 (9.9718) 3.6172 (3.6700) 3.6172 (3.6700) 3.6172 (3.6700) 3.6172 (3. (Wavenumber-6 perturbation)

field=rel r=000144h M= 6 exp=010 min=-0.104295e-03 max= 0.127600e-03

field-rel r=000144h M= 8 exp=010 min=-0.117609e-03 max= 0.159846e-03

From the pole problem to the wavenumber-5 problem

Grids

Longitude-Latitude grid Icosahedral hexagon-pentagon grid

The grid of CAM-FV, etc. The grid of UZIM, NICAM, etc.

C- and Z-grids in linearized systems

Linearized system tests the surface wave propagation, and thus only the divergent component of velocity plays a role.

Taken from Randall 1994

Rotational wind error

Interpolation of streamfunction to the corners from centers

Mitigation

Improve the accuracy of the normal component of rotational velocity

- Improved interpolation of the streamfunction to the corners
- First compute the tangential component of rotational velocity. Then interpolate to obtain normal component

2D-least-square based **Interpolation scheme** Z DECEMBER 2010 S K A 10 S K A 40 S K

Conservative Transport Schemes for Spherical Geodesic Grids: High-Order Reconstructions for Forward-in-Time Schemes

practice we observe no discernible difference using (5) WILLIAM C. SKAMAROCK

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FIG. 1. Schematic showing the 2D scalar mass flux regions for (left) the M07 scheme and (right) the LR05 scheme. The shaded areas show the mass fluxed through the cell edge e¹² over a time step Dt.

$$
\psi - \psi_0 = c_x x + c_y y + c_{xx} x^2 + c_{xy} xy + c_{yy} y^2 + c_{xxx} x^3
$$

+
$$
c_{xxy} x^2 y + c_{xyy} xy^2 + c_{yyy} y^3 + c_{xxxx} x^4
$$

+
$$
c_{xxxy} x^3 y + c_{xxyy} x^2 y^2 + c_{xyyy} xy^3 + c_{yyyy} y^4, \quad (6)
$$

reconstructions of the scalar mass field and using the same

In L02, M07, LR05, Yeh (2007), and herein, the areaaveraged value is computed by integrating polynomials representing the spatial distribution of c over the mass

flux area (Am in Fig. 1). For a first-order polynomial

M07 uses a least squares method to determine the coefficients of the polynomial following Stuhne and Peltier (1996). A least squares fit to a quadratic polynomial is used to obtain the coefficients for the linear polynomial in (3) by M07 and for the results presented herein. Other approaches are possible, including using the Stokes method (Tomita et al. 2001) or by fitting planes to the cell-averaged values lying at the vertices of the dual grid (triangles) whose centers are the vertices of the hexagons, and producing cell-averaged values of cx and cy by averaging these vertex values (LR05; Yeh 2007). These approaches produce identical results for perfect hexagons on a plane, but may produce different results on the imperfect hexagonal grid on the sphere. We have found very little difference in the absolute errors or error convergence rates in our tests using the different approaches, and we have chosen to use a least squares fit to determine the coefficients because it is easily extended to higher-order

For finite-volume transport schemes that compute mass

of the higher-order terms over the cell area will not equal zero. In our approach we compute the polynomial coefficients using a least squares fit after which we adjust the constant term c⁰ such that the constraint is satisfied. L02 also constructs the polynomial using the least squares fit but there the constraint is satisfied by including it directly,

polynomials.

FIG. 2. Schematic showing a grid centered about cell 0. The $\frac{1}{10}$. 2. Schematic showing a grid centered about cell 0. The dark-shaded cells $(1-6)$ are used in the reconstruction of polynomials δ less than or equal to order 2. The lighter-shaded cells (7–17) are used \sum in the reconstruction of polynomials on the order of 3 and 4. approaches are possible, including using the Stokes are possible, including using $\mathcal{L}(\mathcal{A})$

projected onto the tangent plane using (preserving) the

where c is the area-averaged mixing ratio. Our procedure α

where α is the indicates a volume-averaged α cell boundary, nⁱ is the unit vector normal to the cell boundary, and Ai is the cell area. Applied to a discrete

our case the edges of the icosahedral mesh cell ⁱ) and dei is the length of edge e for cell i. No approximations have been made developing (2) because we have not yet spec-

The second term on the right-hand side of (2) is the sum of the fluxes through the control volume edge. M07

schematically described in the left-hand panel of Fig. 1. The mass flux is approximated by the mass contained with the shaded area Am. The area Am. swept out by the edge moving at a constant velocity 2V

ified how we evaluate the term in the summation.

refers to the edges of the control volume i (in

mesh, (1) can be written as

 $\overline{}$

approximates the scalar mass flux dei

Summary

Development of the Global Cloud Resolving Model

- UZIM (**U**nified **Z**-grid **I**cosahedral **M**odel) development:
	- The *hybrid sigma-pressure* vertical coordinate and the gvc versions of the quasi-hydrostatic UZIM have been completed.
- SUZI (**S**uperparameterized **U**nified **Z**-grid **I**cosahedral Model) has been constructed and tested in an aquaplanet simulation by Don Dazlich.
- We face a wavenumber-5 problem. We will work on the mitigation of the problem.
- We are working on a new Global VVM which predicts the curl of horizontal vorticity. The 3D elliptic equation has more convenient boundary conditions with the VVM.