Curl Curl

Dynamical core development under CMMAP

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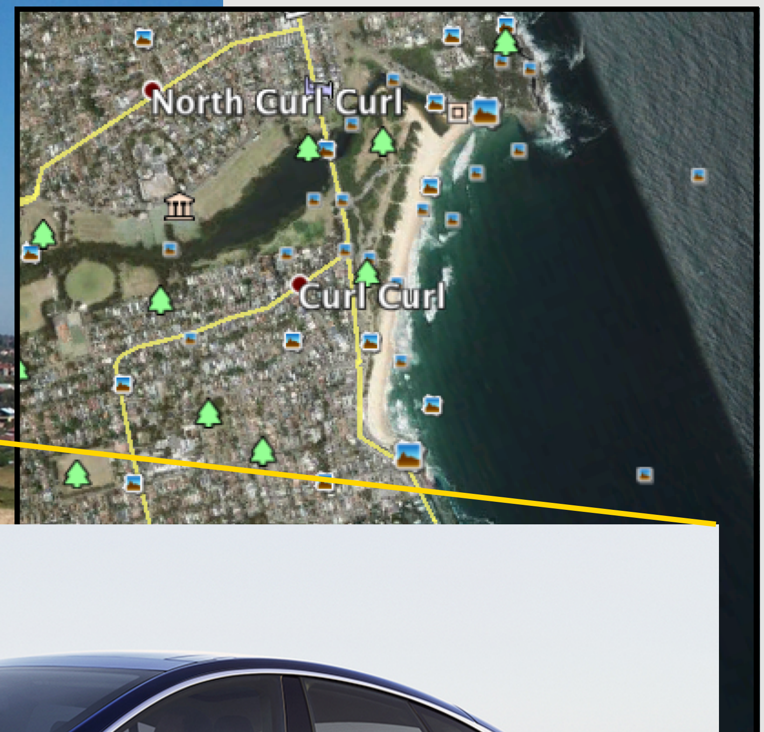
19th CMMAP Team Meeting 4-6 August 2015, Fort Collins, CO

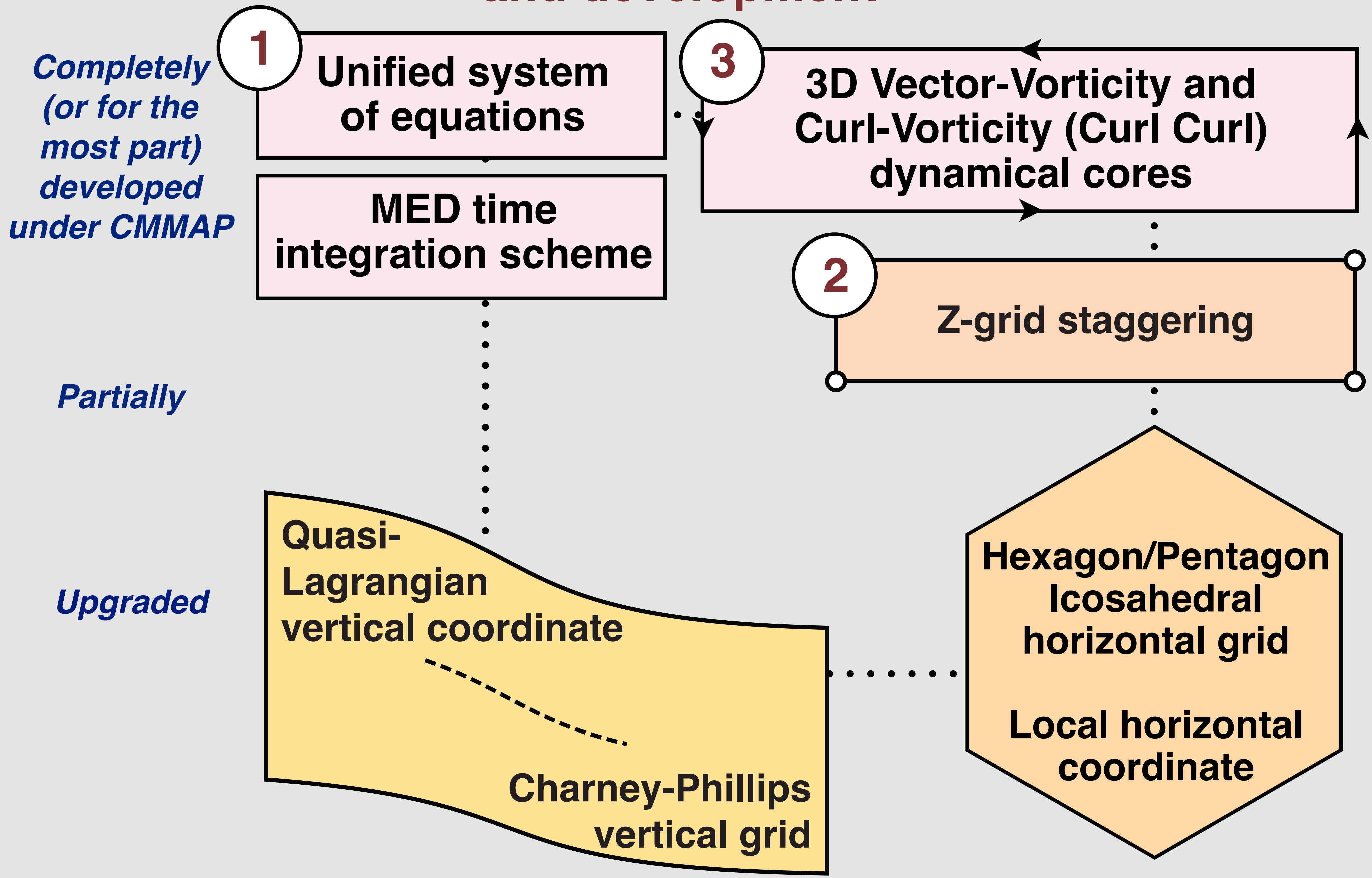
Curl Curl

A suburb of northern Sydney in the state of New South Wales, Australia



The name *Curl Cu*
curial curial, meani

Contributions of CMMAP to the dynamical core design and development



Unified system of equations

A nonhydrostatic system of equations for multiscale global models

Filters vertically propagating acoustic waves of all scales, yet maintains enough elasticity to properly simulate planetary waves

Unifies the quasi-hydrostatic and anelastic systems

Anelastic continuity equation

$$\nabla_{\mathrm{H}} \cdot \left(\rho_0 \mathbf{v} \right) + \frac{\partial}{\partial z} \left(\rho_0 w \right) = 0$$

Unified continuity equation

$$\nabla_{\mathrm{H}} \cdot \left(\rho_{qs} \mathbf{v} \right) + \frac{\partial}{\partial z} \left(\rho_{qs} w \right) = -\frac{\partial \rho_{qs}}{\partial t}$$

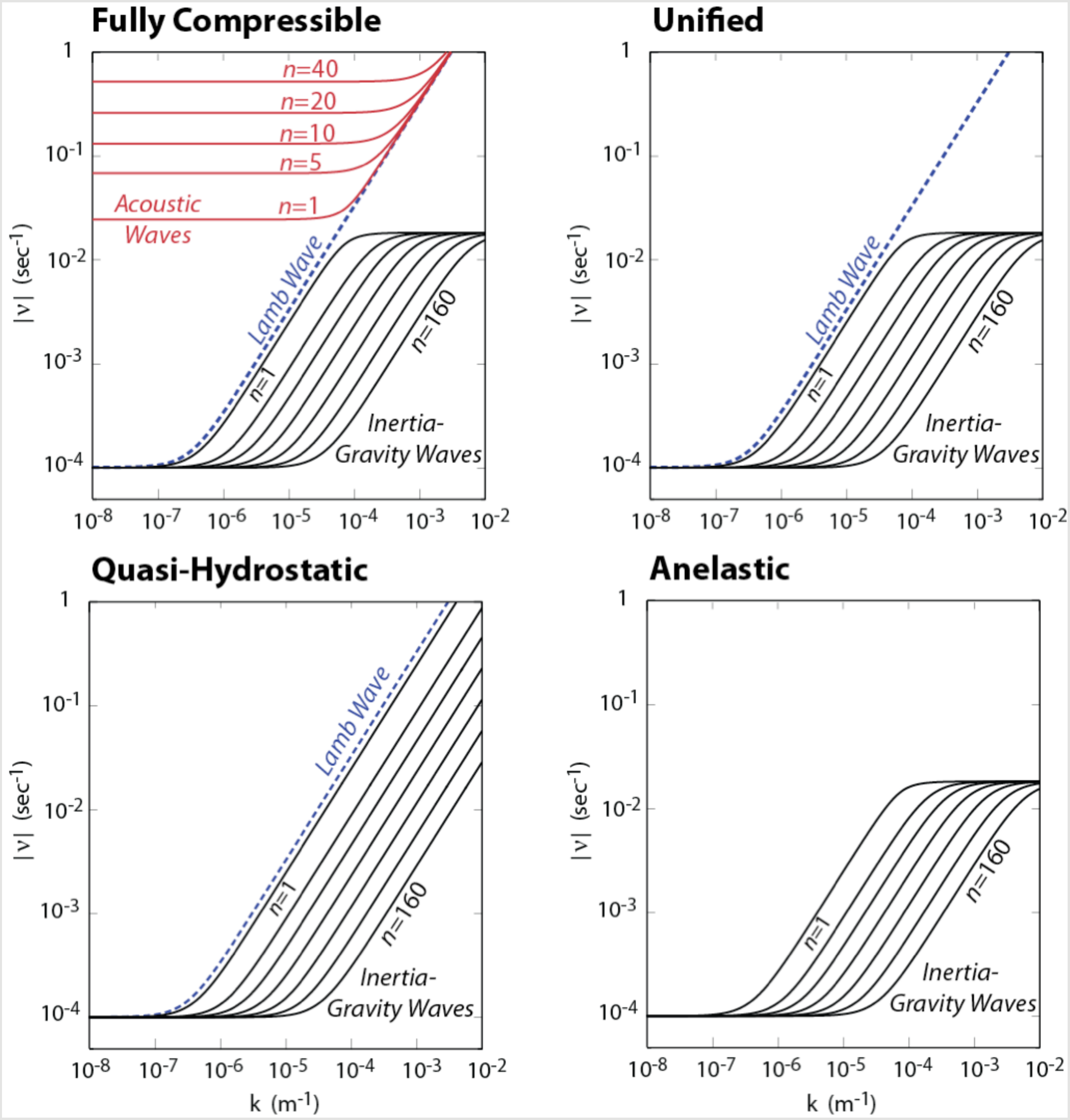
A diagnostic equation

No basic or mean state needed

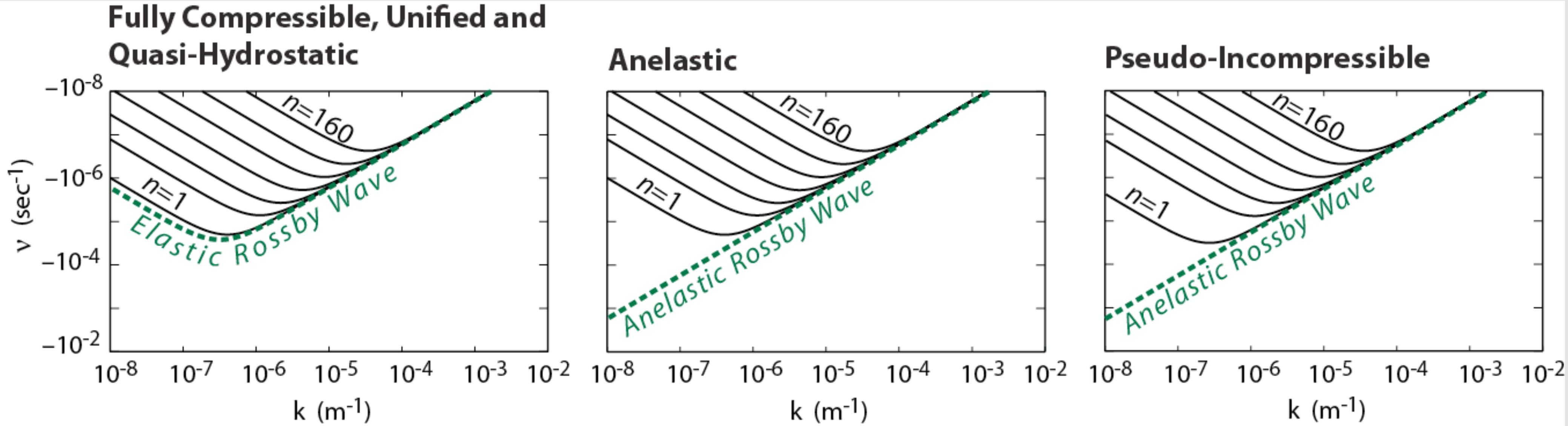
Conserves energy

Can be applied to any quasi-hydrostatic model to add nonhydrostatic effects

Dispersion of Inertia-Gravity Waves



Dispersion of Rossby Waves



Z-grid

Replaces the prediction of vectors by that of (proper dynamical) scalars so that a non-staggered grid can be used

For Vorticity-Divergence dynamical cores

Predicted quantities

$$\omega_z \equiv \mathbf{k} \cdot \nabla_H \times \mathbf{v}$$

$$D \equiv \nabla_H \cdot \mathbf{v}$$

Diagnostic quantities

$$\mathbf{v} \equiv \mathbf{v}_\psi + \mathbf{v}_\chi$$

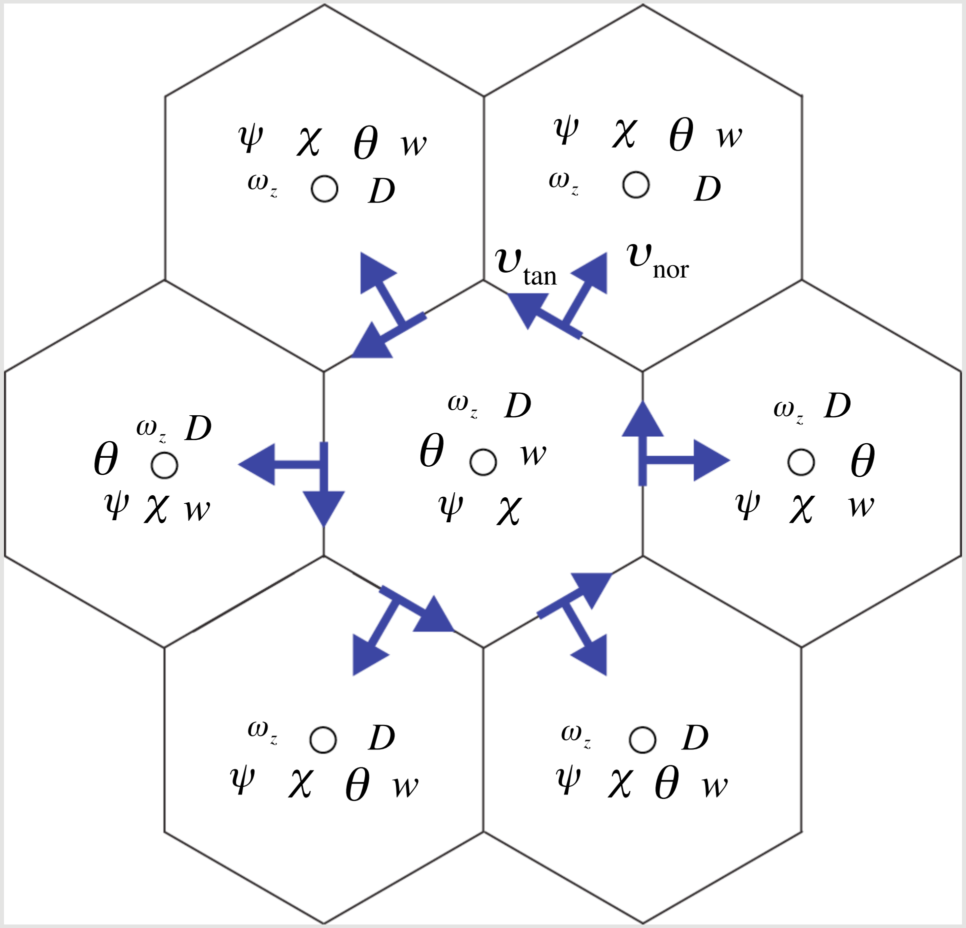
$$\mathbf{v}_\psi \equiv \mathbf{k} \times \nabla_H \psi$$

$$\mathbf{v}_\chi \equiv \nabla_H \chi$$

$$\nabla_H^2 \psi = \omega_z$$

Elliptic solvers

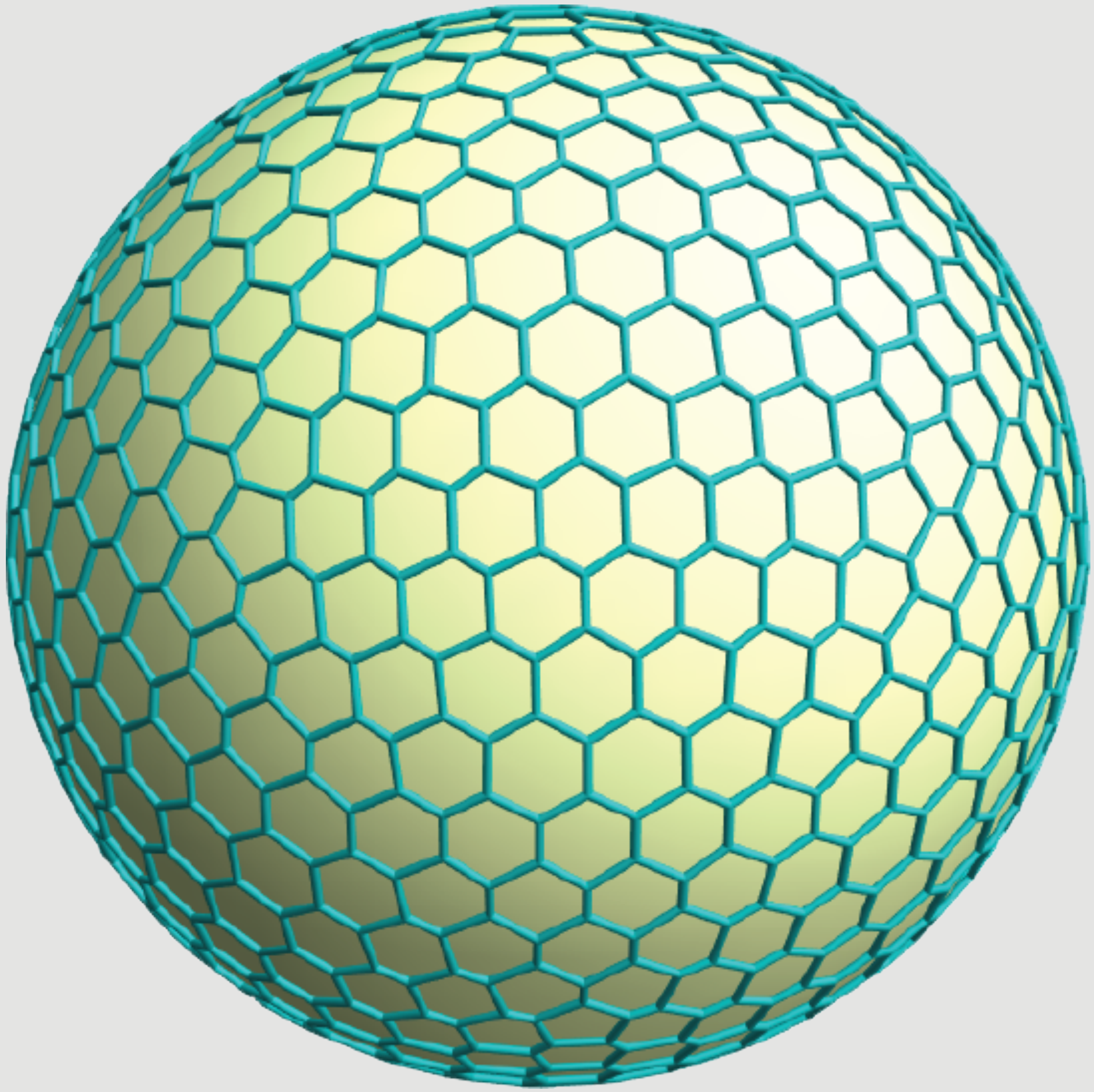
$$\nabla_H^2 \chi = D$$



Computational mode allowed by the C-grid is not allowed by the Z-grid

UZIM (Unified Z-grid Icosahedral Model)

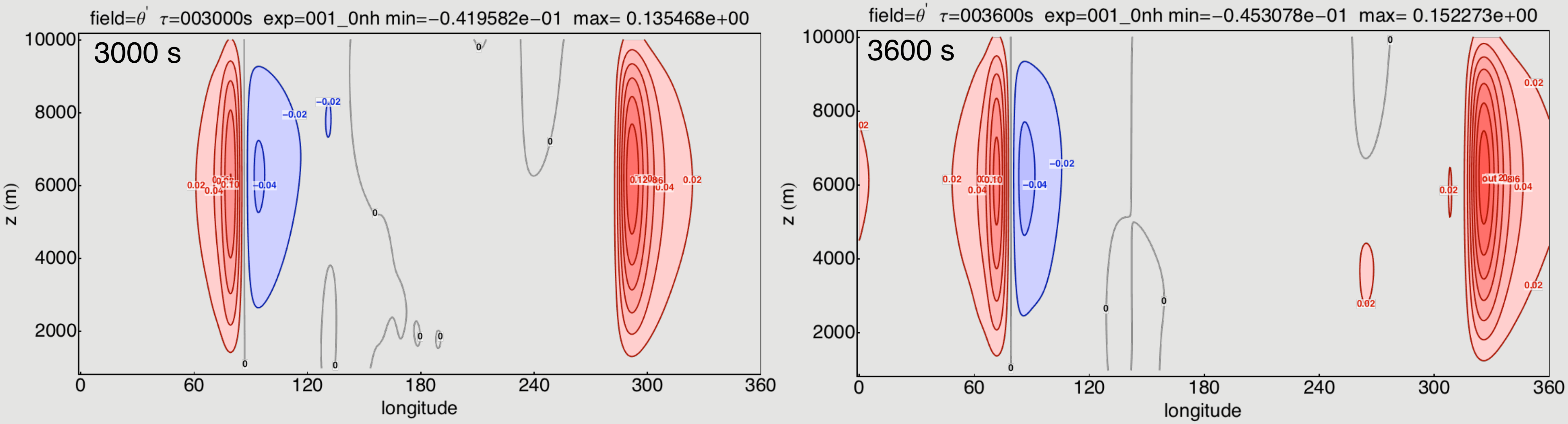
Icosahedral hexagon-pentagon grid



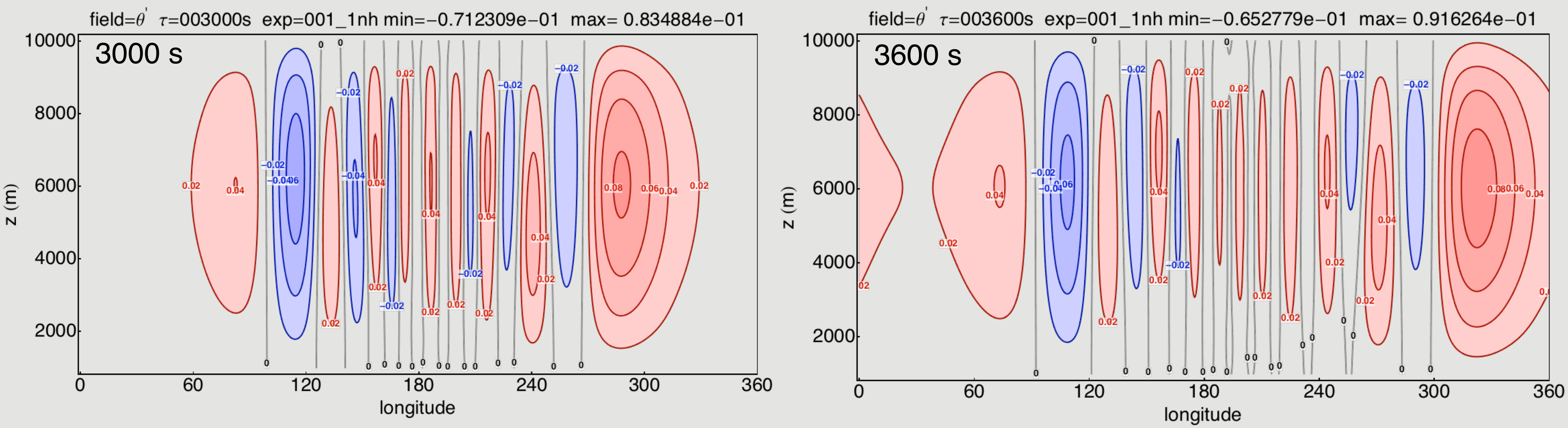
UZIM (Unified Z-grid Icosahedral Model) results

DCMIP model intercomparison run (test case 3.1)

Quasi-hydrostatic run



Nonhydrostatic Unified run



G7 (~2.5 km) 32 L (dz=500 m) a=300 km

Elliptic equation in UZIM

$$\nabla_H \cdot \left(\rho_{qs} c_p \theta \nabla_H \delta\pi\right) + \frac{\partial}{\partial z}\left(\rho_{qs} c_p \theta \frac{\partial \delta\pi}{\partial z}\right) =$$

$$-\nabla_H \cdot \left(\rho_{qs} c_p \theta \nabla_H \pi_{qs}\right) - \nabla_H \cdot \left(\rho_{qs} \mathbf{G}_H\right) - \frac{\partial\left(\rho_{qs} G_z\right)}{\partial z} + \frac{\partial^2 \rho_{qs}}{\partial t^2}$$

$$\left(\frac{\partial \delta\pi}{\partial z}\right)_S = \left(\frac{\partial \delta\pi}{\partial z}\right)_T = 0$$

Neumann boundary condition

No convergence problem with the Neumann boundary condition in simulations on a shrunken Earth

But, problems in simulations on a true size Earth

No problem with the elliptic solvers using the dirichlet boundary condition on all sizes

Vector-Vorticity and Curl-Vorticity (Curl-Curl) dynamical cores

VVM

Predicts horizontal vorticity : $\left(\omega_{\mathrm{H}}\right)_{\tan}$

Diagnoses the vertical vorticity : ω_z

Horizontal staggering : **C-grid**

CVM (Curl-Curl)

Predicts the curl of horizontal vorticity vector :

$$\Gamma \ \left(\equiv \mathbf{k}\cdot\nabla_{\mathrm{H}}\times\boldsymbol{\omega}_{\mathrm{H}}\right)$$

Predicts the vertical vorticity : ω_z

Horizontal staggering : **Z-grid**

Important common features

Three-dimensional vorticity vector is nondivergent : $\nabla_{\mathrm{H}}\cdot\boldsymbol{\omega}_{\mathrm{H}}+\dfrac{\partial\omega_z}{\partial z}=0$

Three-dimensional elliptic equation solves vertical velocity w instead of pressure perturbation (or nonhydrostatic Exner pressure):

$$\nabla_{\mathrm{H}}^2 w+\frac{\partial}{\partial z}\left[\frac{1}{\rho_{qs}}\frac{\partial}{\partial z}\left(\rho_{qs}w\right)\right]=-\Gamma-\frac{\partial}{\partial z}\left[\frac{1}{\rho_{qs}}\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\nabla_{\mathrm{H}}\right)\rho_{qs}\right]$$

$w_S = w_T = 0$ ***Dirichlet boundary condition***

Z-grid of CVM

Replaces the prediction of vectors by that of scalars so that a non-staggered grid can be used

For Vorticity-Divergence dynamical cores	For Curl-Vorticity dynamical cores
Predicted quantities	
$\omega_z \equiv \mathbf{k} \cdot \nabla_{\mathrm{H}} \times \mathbf{v}$	$\Gamma \equiv \mathbf{k} \cdot \nabla_{\mathrm{H}} \times \left(\boldsymbol{\omega}_{\mathrm{H}} \right)$
$D \equiv \nabla_{\mathrm{H}} \cdot \mathbf{v}$	$D_{\omega} \equiv \nabla_{\mathrm{H}} \cdot \left(\boldsymbol{\omega}_{\mathrm{H}} \right)$
Diagnostic quantities	
$\mathbf{v} \equiv \mathbf{v}_{\psi} + \mathbf{v}_{\chi}$	$\boldsymbol{\omega}_{\mathrm{H}} \equiv \left(\boldsymbol{\omega}_{\mathrm{H}} \right)_{\psi} + \left(\boldsymbol{\omega}_{\mathrm{H}} \right)_{\chi}$
$\mathbf{v}_{\psi} \equiv \mathbf{k} \times \nabla_{\mathrm{H}} \psi$	$\left(\boldsymbol{\omega}_{\mathrm{H}} \right)_{\psi} \equiv \mathbf{k} \times \nabla_{\mathrm{H}} \psi_{\omega}$
$\mathbf{v}_{\chi} \equiv \nabla_{\mathrm{H}} \chi$	$\left(\boldsymbol{\omega}_{\mathrm{H}} \right)_{\chi} \equiv \nabla_{\mathrm{H}} \chi_{\omega}$
$\nabla_{\mathrm{H}}^2 \psi = \omega_z$	$\nabla_{\mathrm{H}}^2 \psi_{\omega} = \Gamma$
$\nabla_{\mathrm{H}}^2 \chi = D$	$\nabla_{\mathrm{H}}^2 \chi_{\omega} = D_{\omega}$

Equations of Curl-Vorticity (Curl Curl) Dynamical core

Prediction equation of the curl of horizontal vorticity–curl curl equation:

$$\frac{\partial \Gamma}{\partial t} = -\nabla_{\mathrm{H}} \cdot (\Gamma \mathbf{v}) - \frac{\partial}{\partial z}(\Gamma w) + \nabla_{\mathrm{H}} \cdot \left[\left(\mathbf{k} \cdot \boldsymbol{\omega}_{\mathrm{H}} \times \nabla_{\mathrm{H}} \right) \mathbf{v} \right] + \frac{\partial}{\partial z}\left(\mathbf{k} \cdot \boldsymbol{\omega}_{\mathrm{H}} \times \nabla_{\mathrm{H}} w \right) + \mathbf{k} \cdot \nabla_{\mathrm{H}} \times \mathbf{G}_{\mathrm{H}}$$

$$\mathbf{G}_{\mathrm{H}} \equiv \left(\boldsymbol{\omega}_{\mathrm{H}} \cdot \boldsymbol{\nabla}_{\mathrm{H}} \right) \mathbf{v} + \left(\omega_z + f \right) \frac{\partial \mathbf{v}}{\partial z} - \left[\boldsymbol{\nabla}_{\mathrm{H}} \times \left(P_z \mathbf{k} \right) + \frac{\partial}{\partial z} \left(\mathbf{k} \times \mathbf{P}_{\mathrm{H}} \right) \right]$$

Prediction equation of the vertical vorticity:

$$\frac{\partial \omega_z}{\partial t} = -\boldsymbol{\nabla}_{\mathrm{H}} \cdot \left[\left(\omega_z + f \right) \mathbf{v} \right] - \frac{\partial}{\partial z} \left[\left(\omega_z + f \right) w \right] + G_z$$

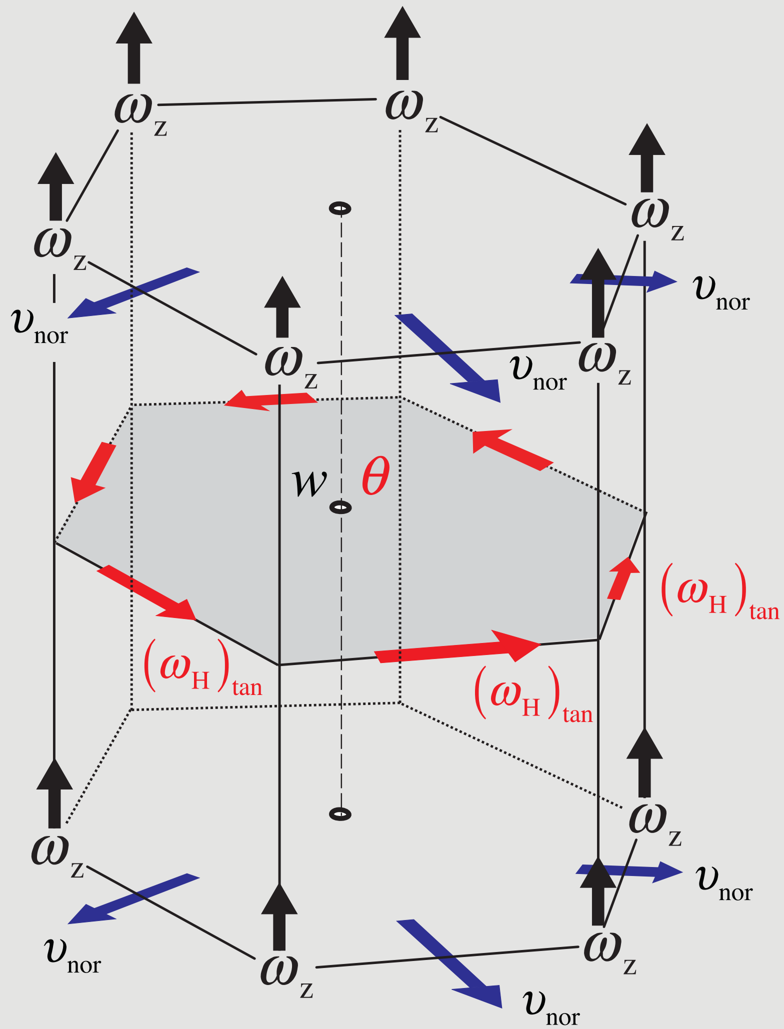
$$G_z \equiv \boldsymbol{\omega}_{\mathrm{H}} \cdot \boldsymbol{\nabla}_{\mathrm{H}} w + \left(\omega_z + f \right) \frac{\partial w}{\partial z} - \mathbf{k} \cdot \boldsymbol{\nabla}_{\mathrm{H}} \times \mathbf{P}_{\mathrm{H}}$$

Diagnostic equation for the divergence of the horizontal vorticity:

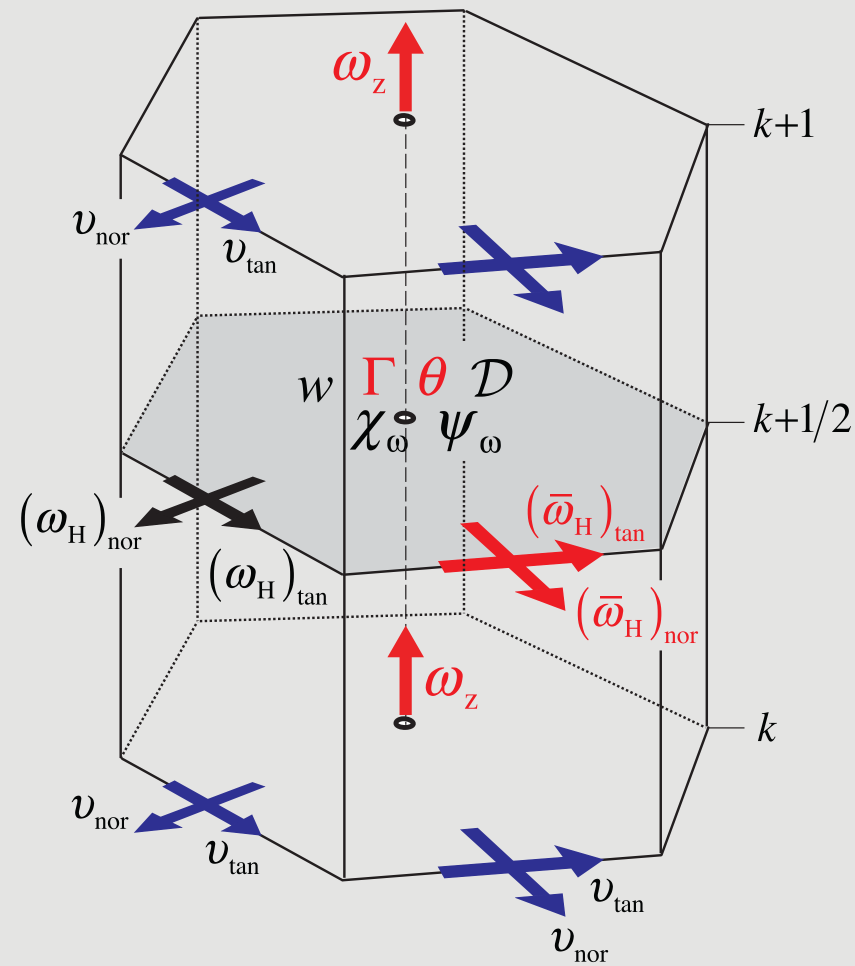
$$D_{\boldsymbol{\omega}} \equiv \boldsymbol{\nabla}_{\mathrm{H}} \cdot \boldsymbol{\omega}_{\mathrm{H}} = -\frac{\partial \omega_z}{\partial z}$$

Vector-Vorticity and Curl-Vorticity (Curl-Curl) dynamical cores

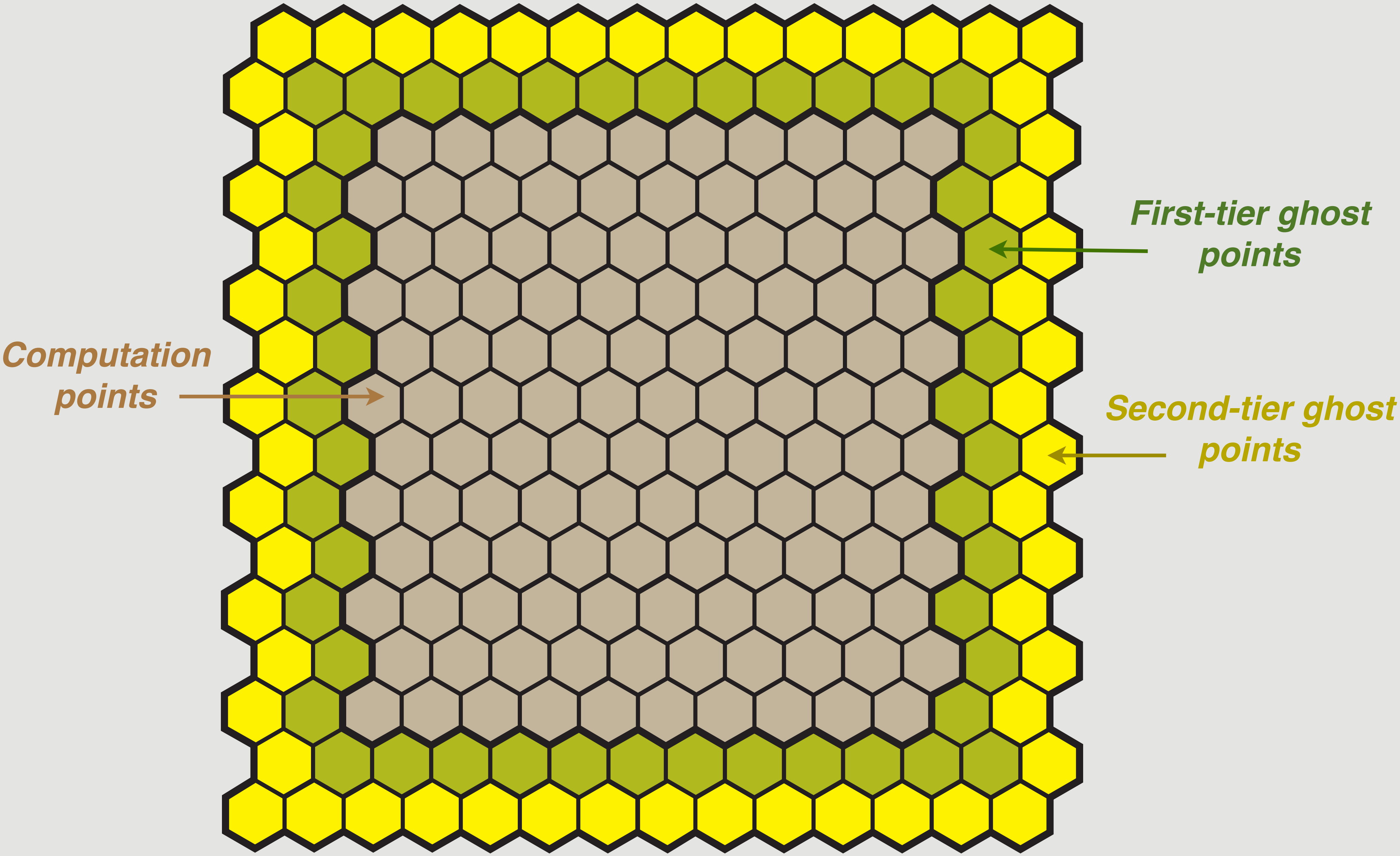
"C- horizontal grid" and CP-vertical grid of VVM



"Z- horizontal grid" and CP-vertical grid of CVM



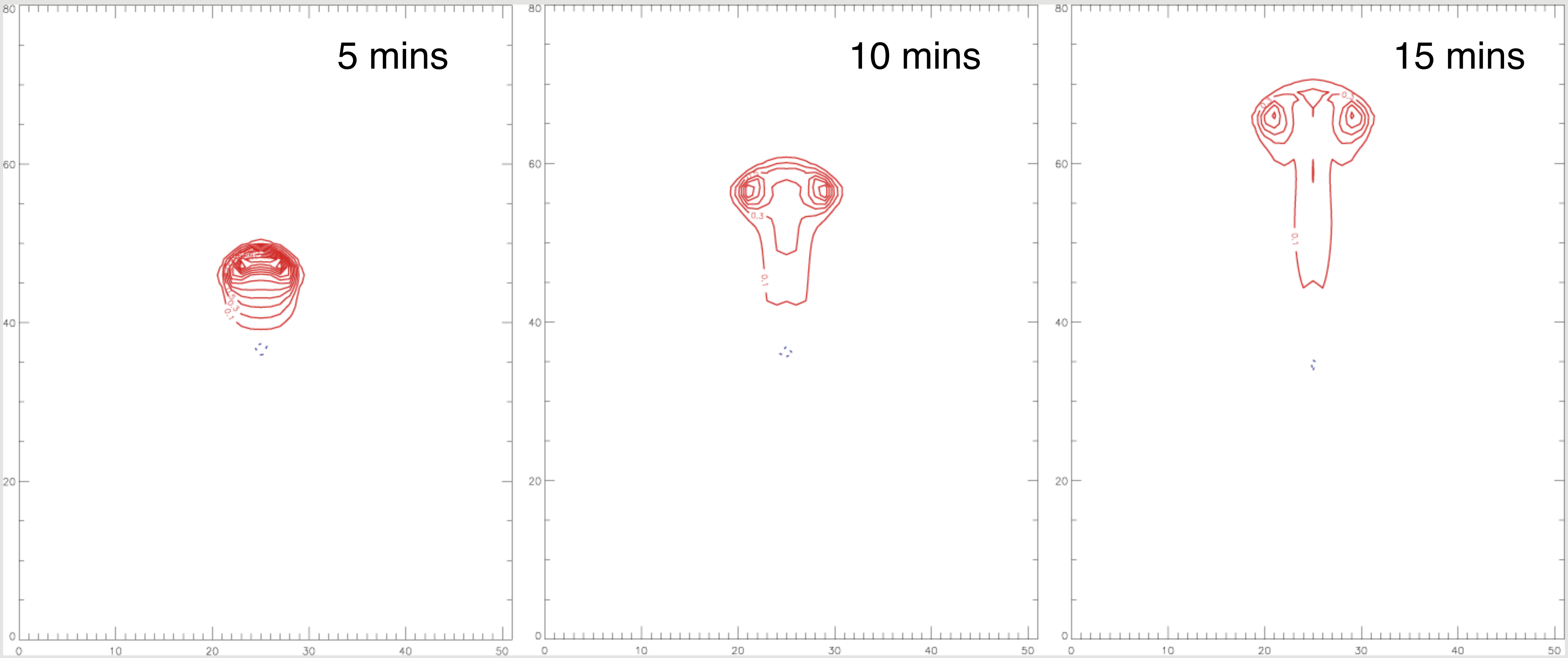
Hexagonal grid and domain of Planar CVM and VVM



Doubly periodic boundary conditions

Curl-Vorticity (Curl-Curl) dynamical core

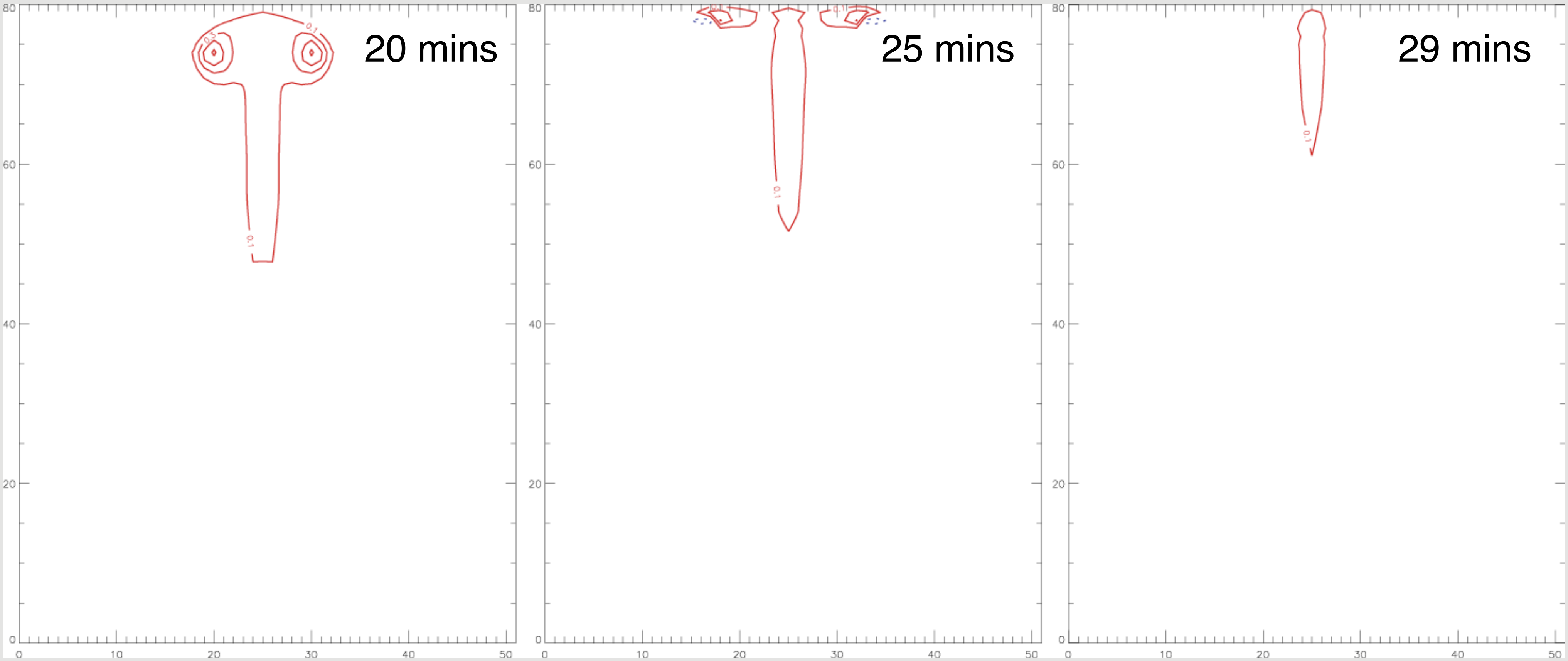
Potential temperature perturbation in warm bubble test



250m horizontal (cell center to cell center) and vertical grid distances

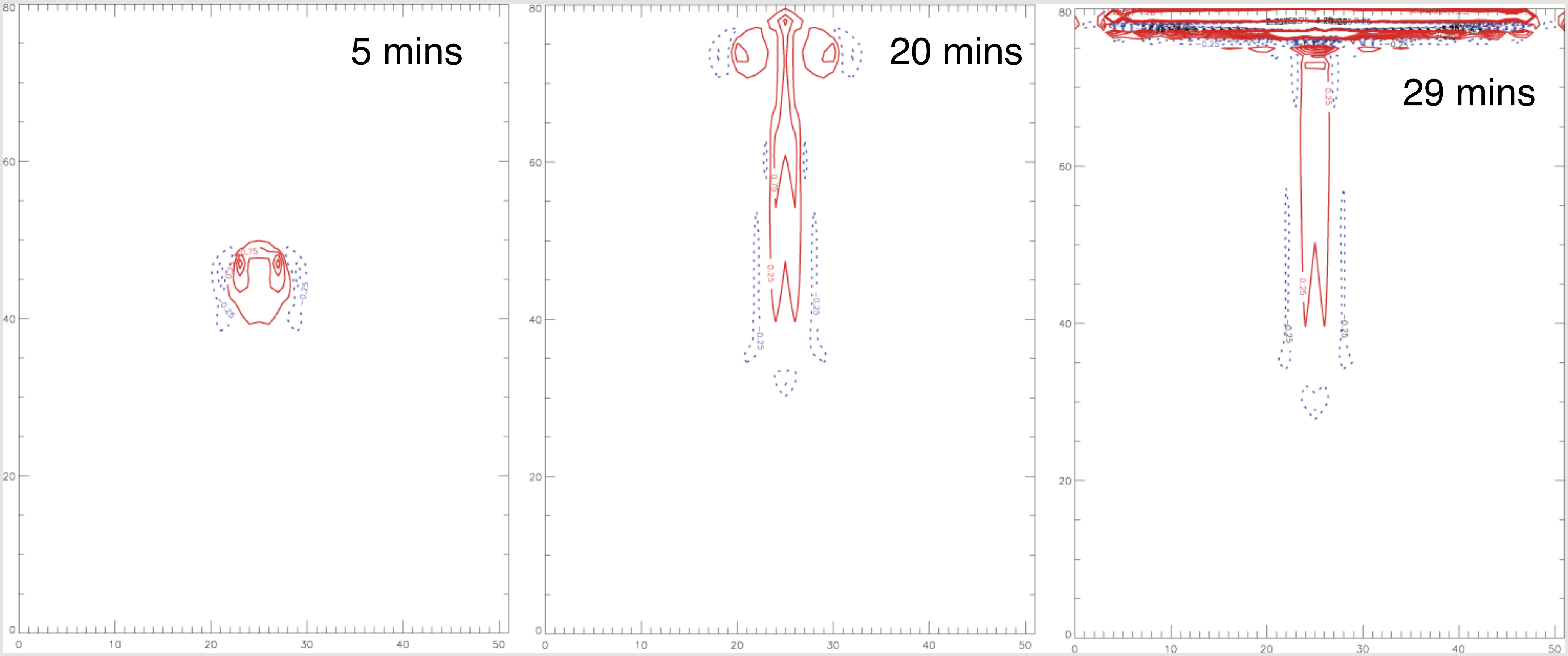
Curl-Vorticity (Curl-Curl) dynamical core

Potential temperature perturbation in warm bubble test



Curl-Vorticity (Curl-Curl) dynamical core

Gamma in warm bubble test

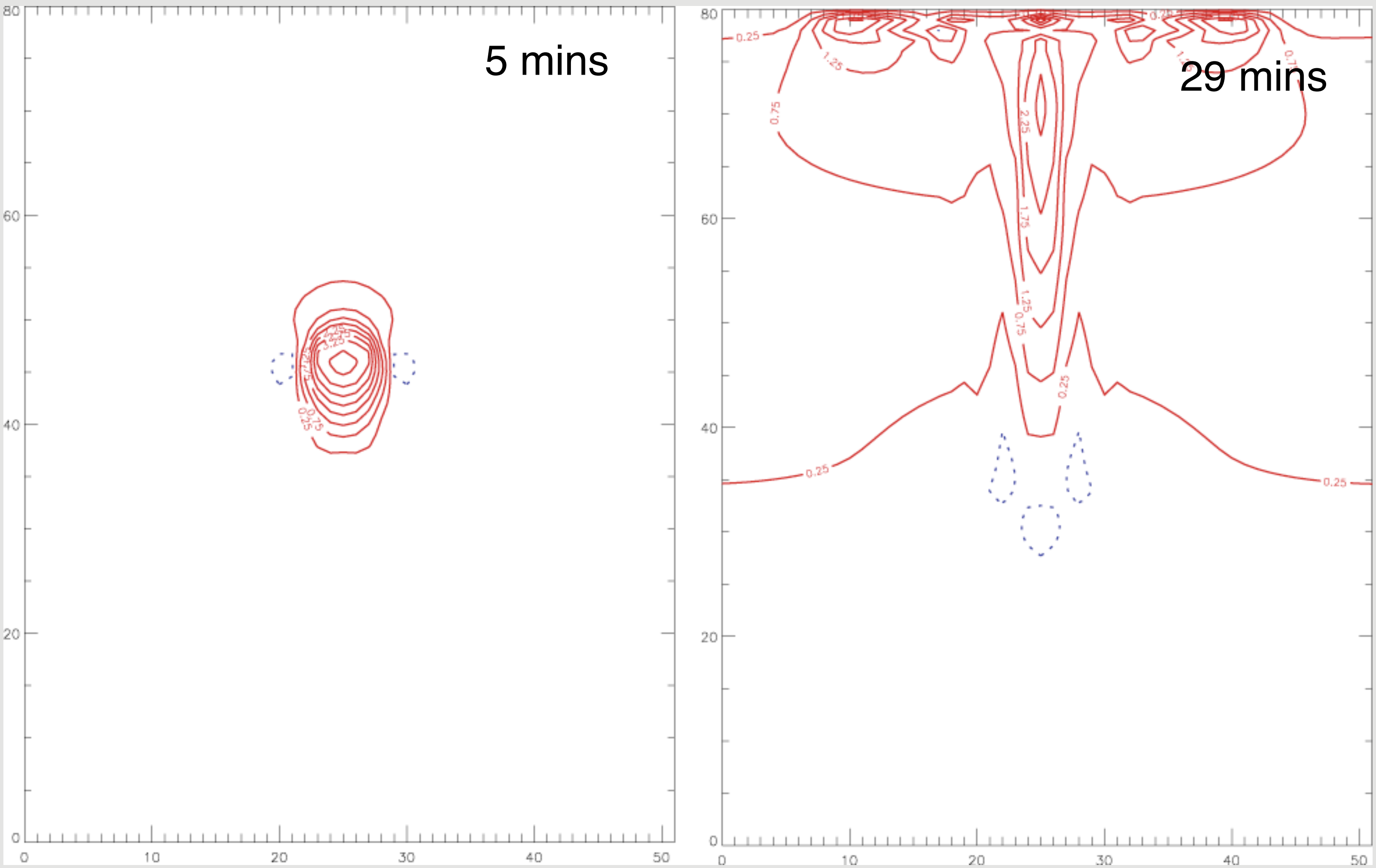


Palinistrophy increases

$$\overline{\Gamma^2} \equiv \overline{\left(\mathbf{k}\cdot\nabla_H \times \boldsymbol{\omega}_H\right)^2}$$

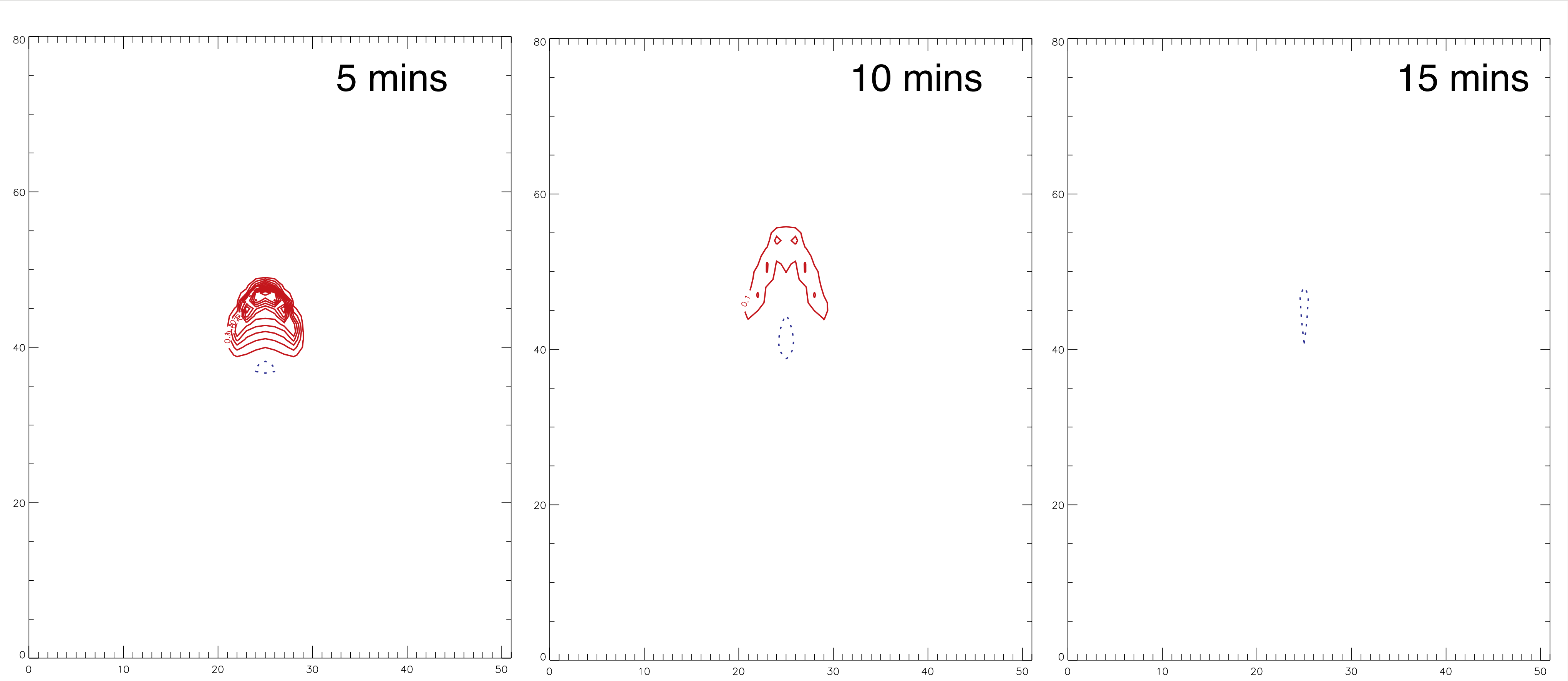
Curl-Vorticity (Curl-Curl) dynamical core

Vertical velocity in warm bubble test



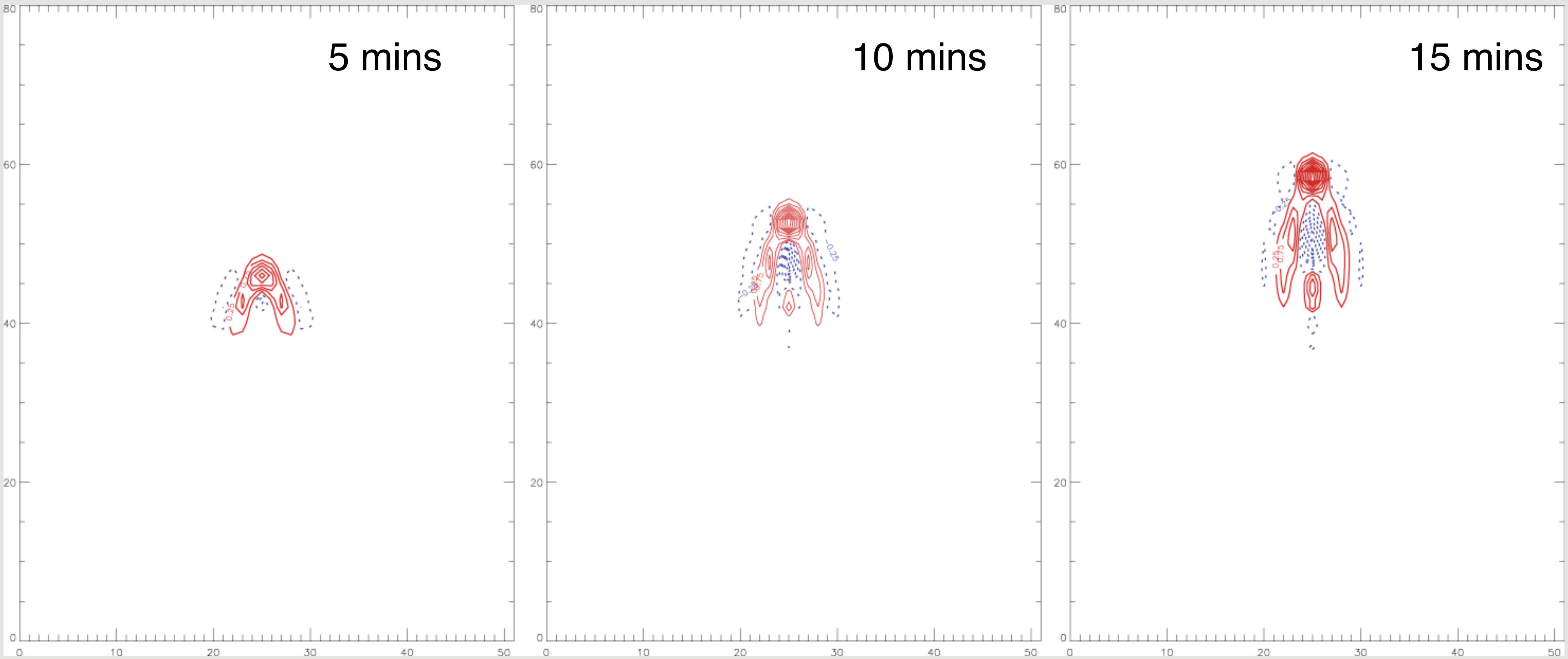
Vector-Vorticity (VVM) dynamical core

Potential temperature perturbation in warm bubble test



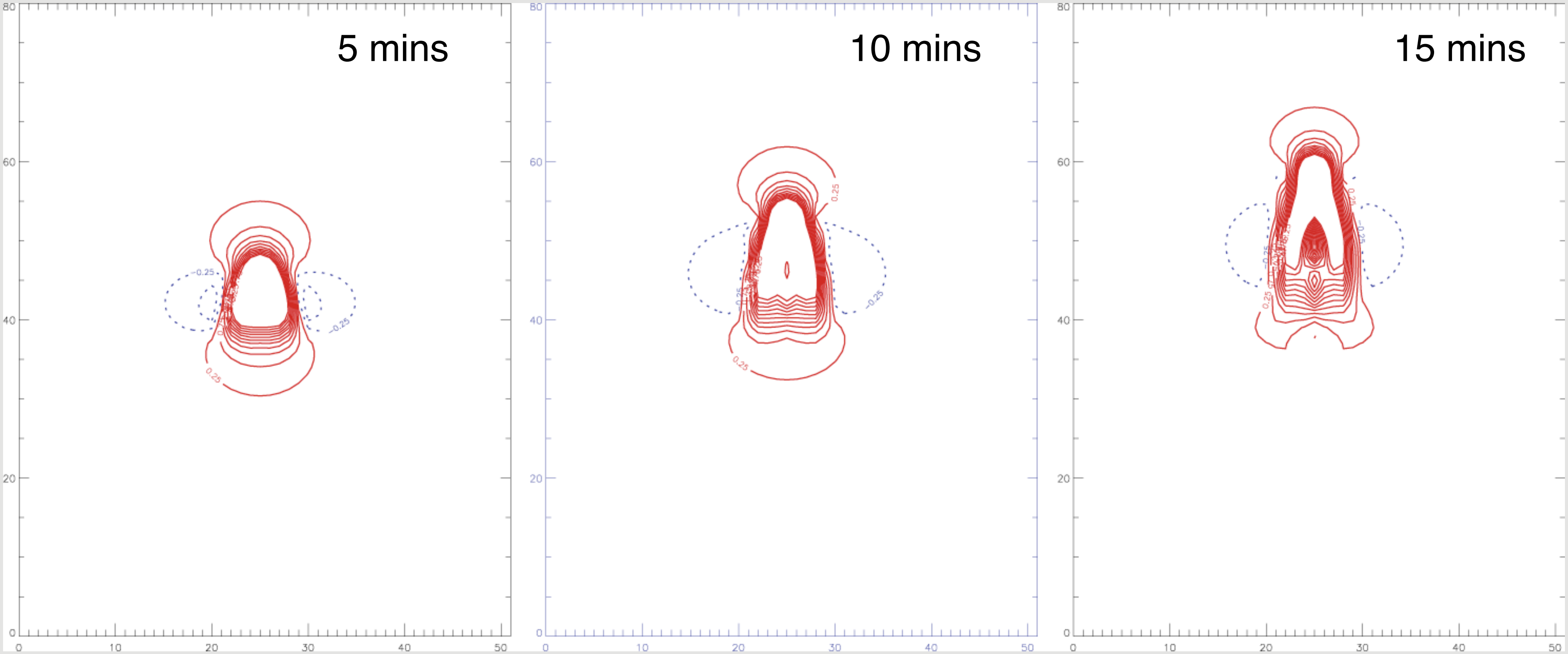
Vector-Vorticity (VVM) dynamical core

Gamma in warm bubble test



Vector-Vorticity (VVM) dynamical core

Vertical velocity in warm bubble test



2D (x-z) Cartesian Grid comparison

Potential temperature perturbation in warm bubble test