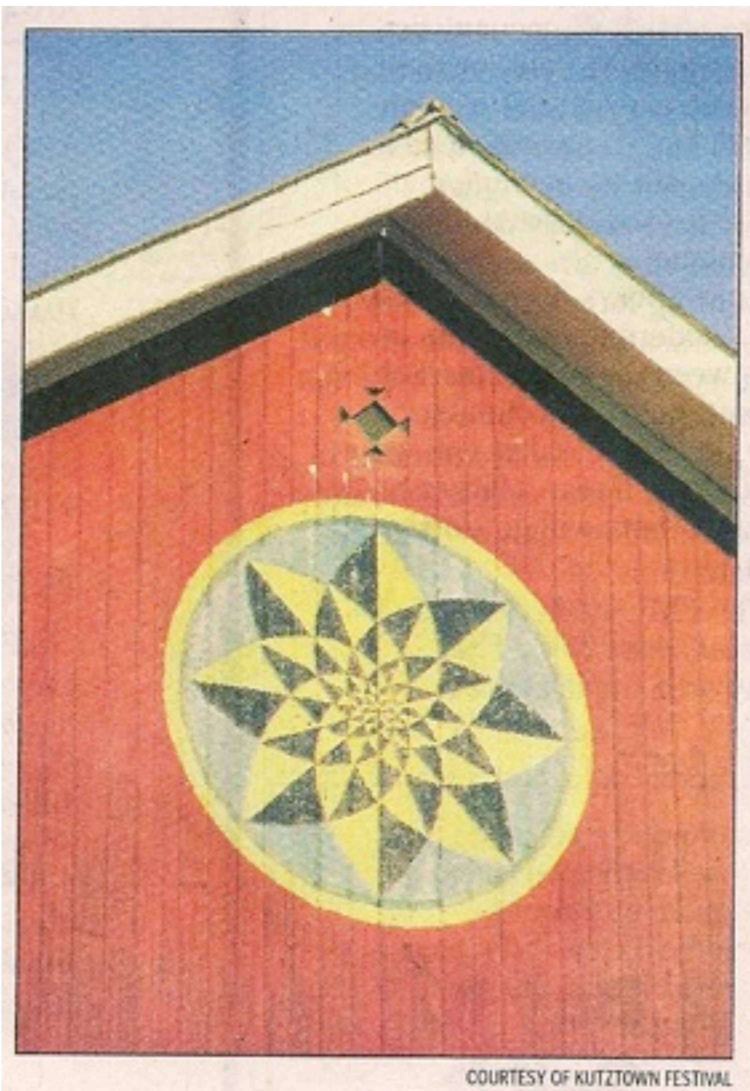
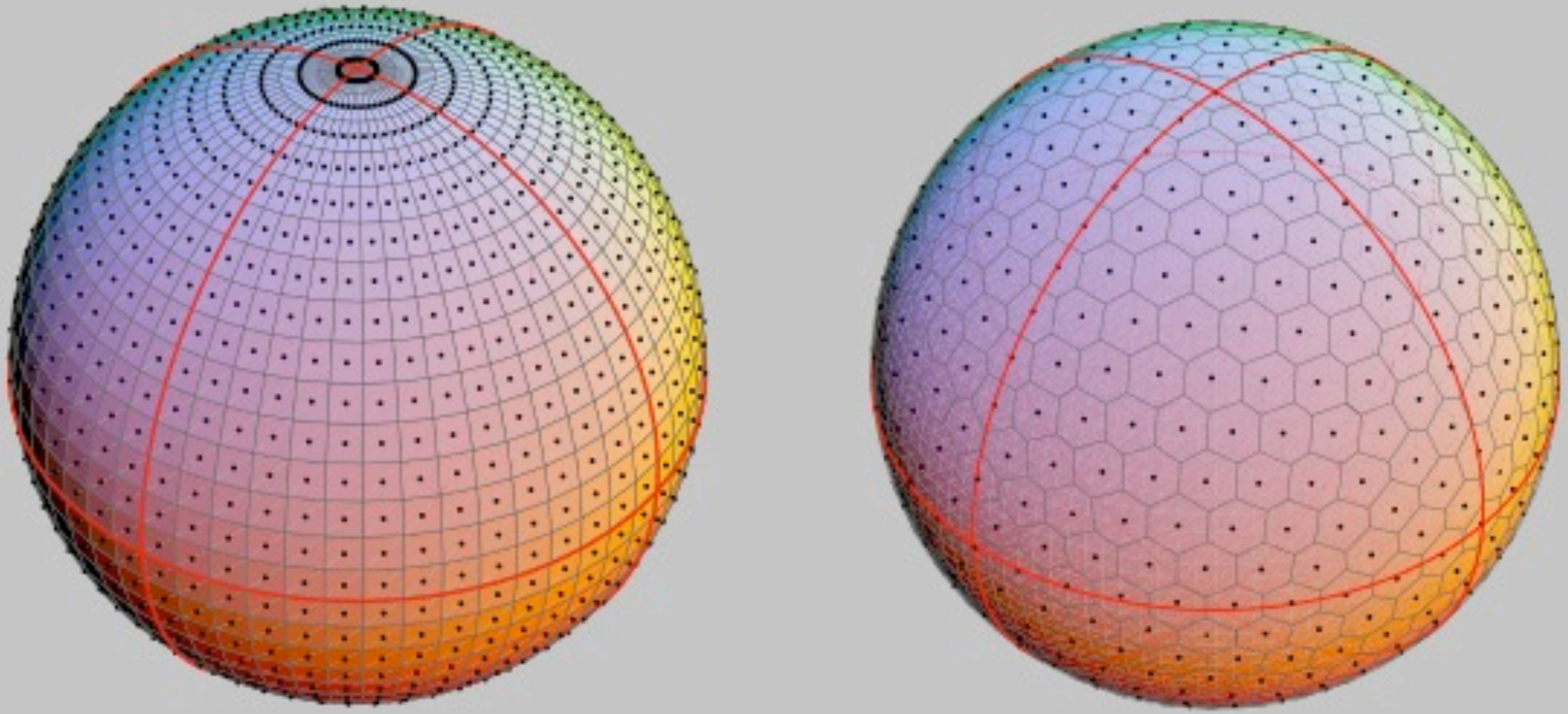
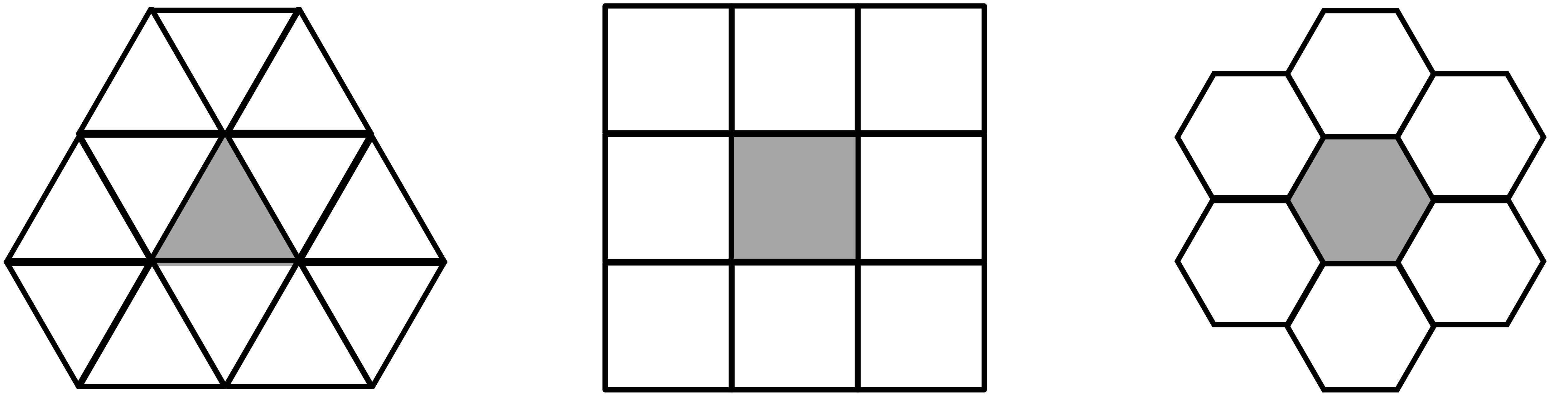
What's up with the hexagons?


COURTESY OF KUTZTOWN FESTIVAL

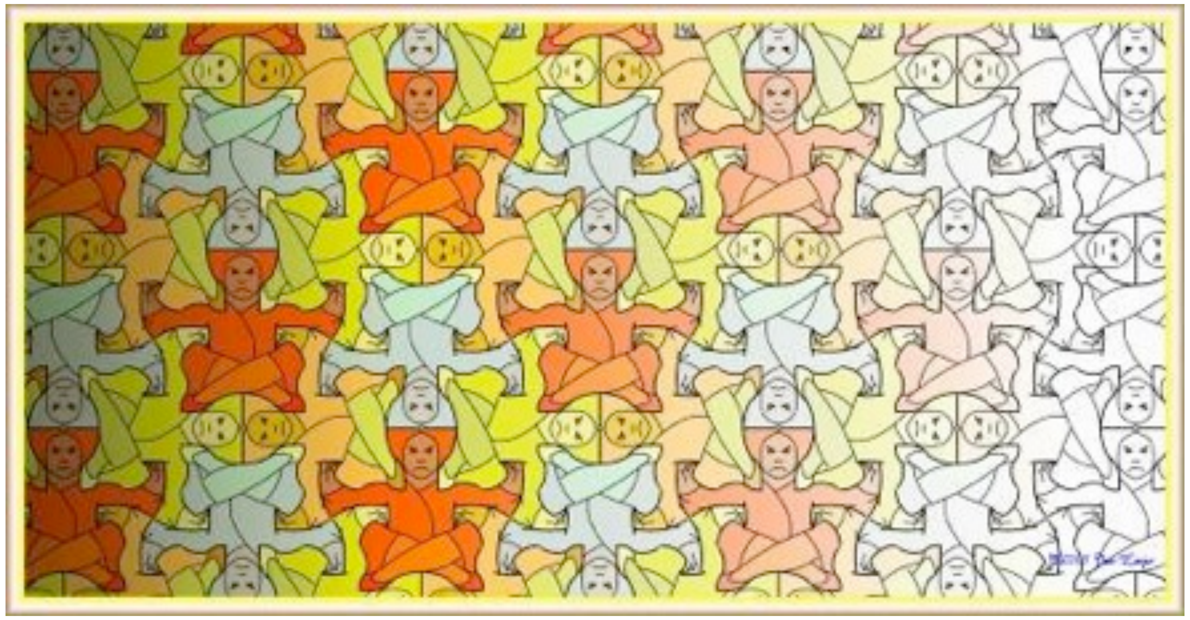
Lat-Lon Versus Geodesic Grids



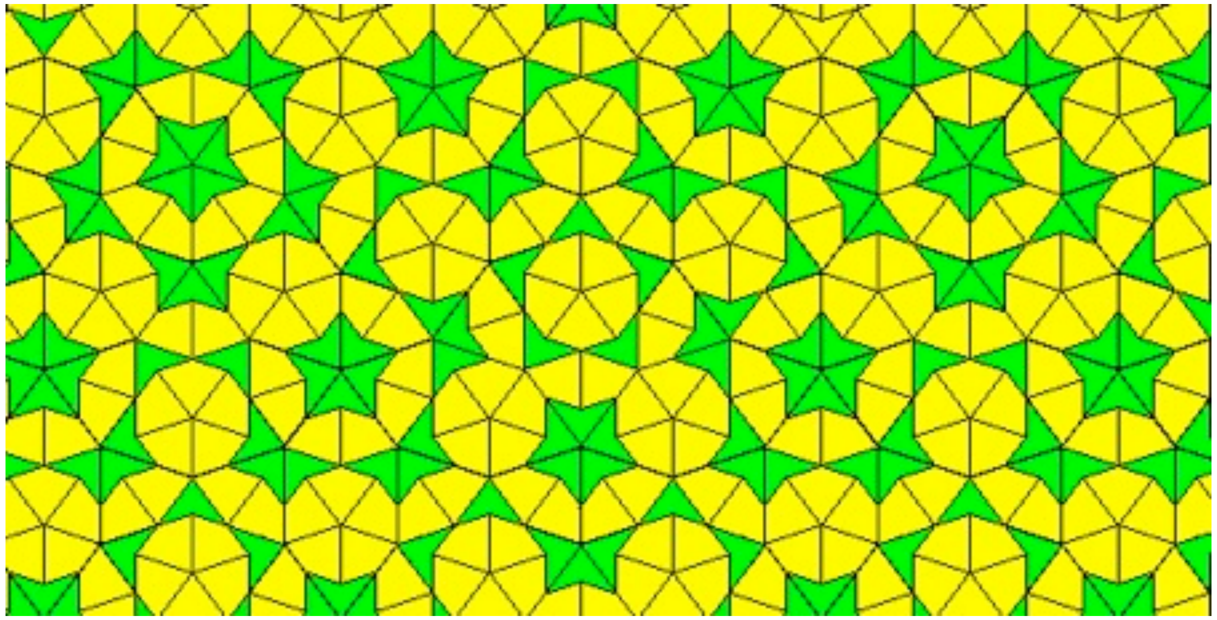
Triangles, Squares, and Hexagons



Other ways to tile the plane

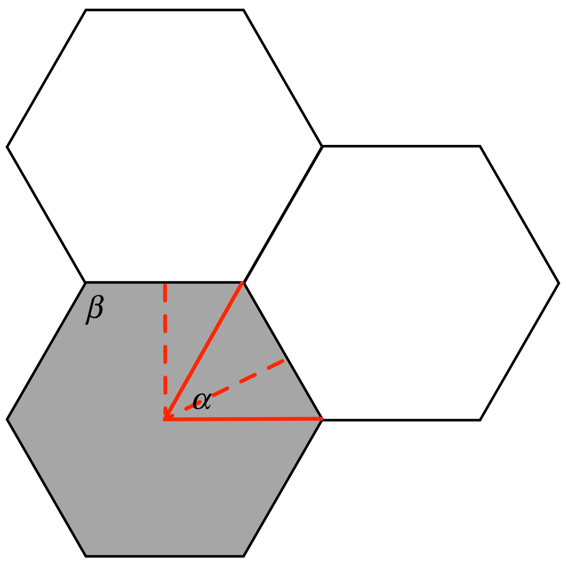


Escher



Penrose

Tiling the plane



Suppose that the polygon has N sides. The interior angles have to divide evenly into 360 degrees, so

$$N\alpha = 360^{\circ}.$$

Supposes that M polygons come together at each vertex. The exterior angles, β, have to add up to 360 degrees, so

$$M\beta = 360^{\circ}.$$

The exterior angles satisfy

$$\beta / 2 = 180^{\circ} - 90^{\circ} - \alpha / 2,$$

or

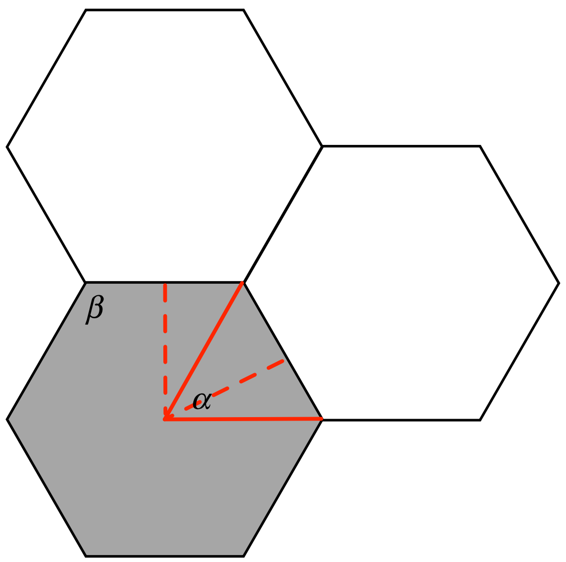
$$\beta = 180^{\circ} - \alpha.$$

By combining the formulas above, we can show that

$$M = \frac{2N}{N-2}.$$

This only makes sense for $N \geq 3$. That's OK, because there are no regular polygons with less than three sides.

Tiling the plane



N	M
3	6
4	4
5	10/3
6	3
7	14/5
∞	2

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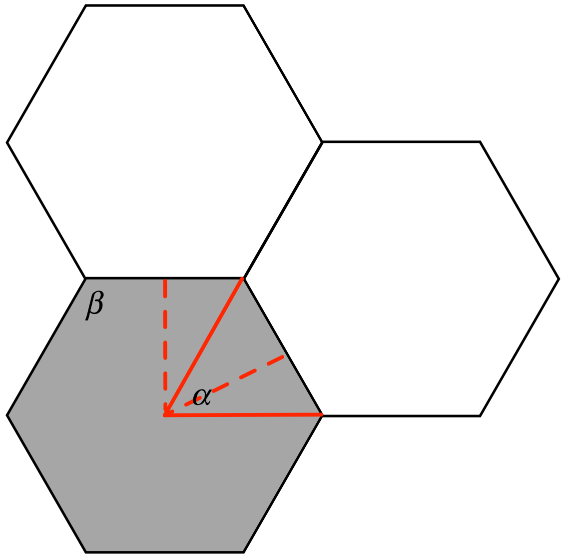

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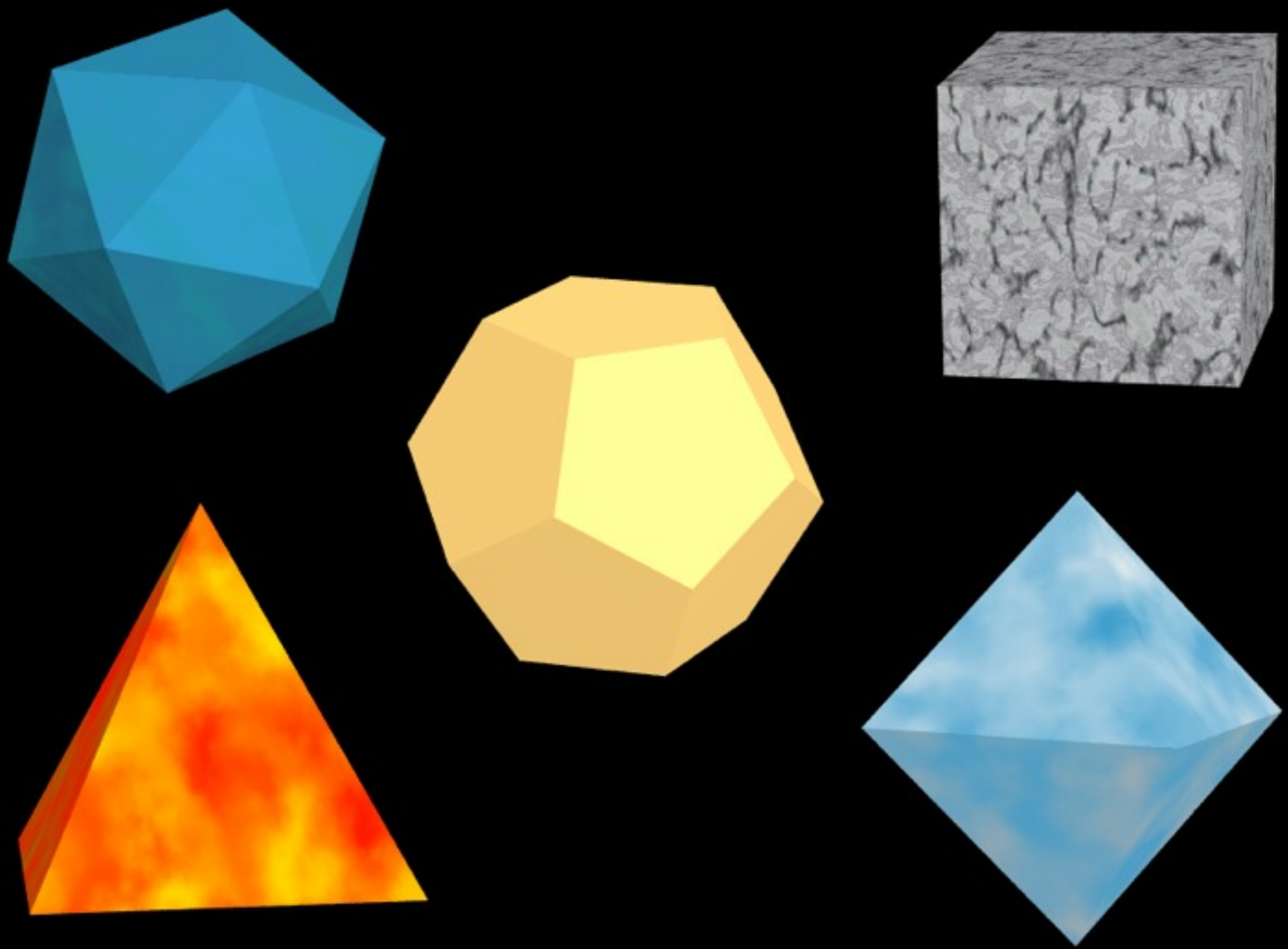
Conclusion:

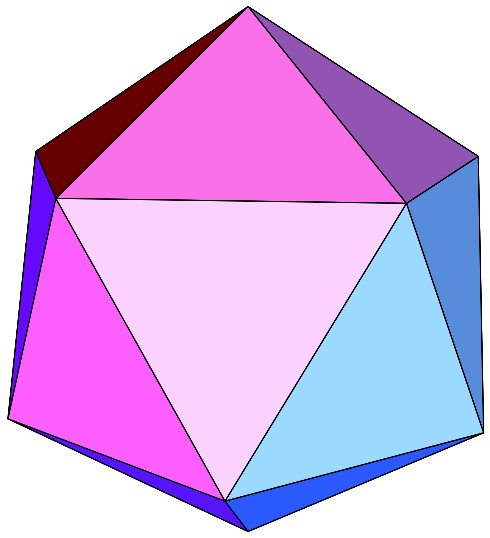
Triangles, squares, and hexagons tile the plane.
No other regular polygons do.

Platonic Solids



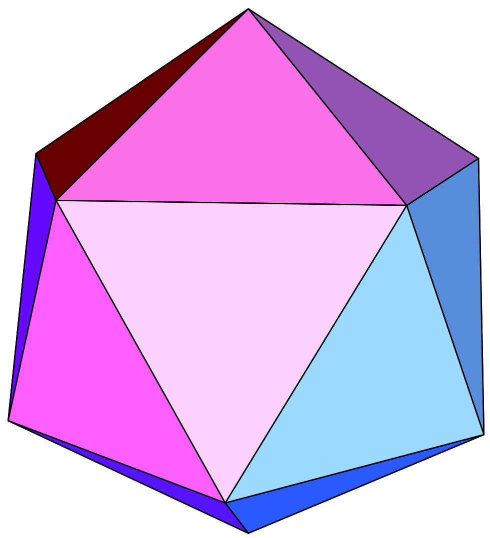
These are the only regular polyhedrons.

Geodesic Grid

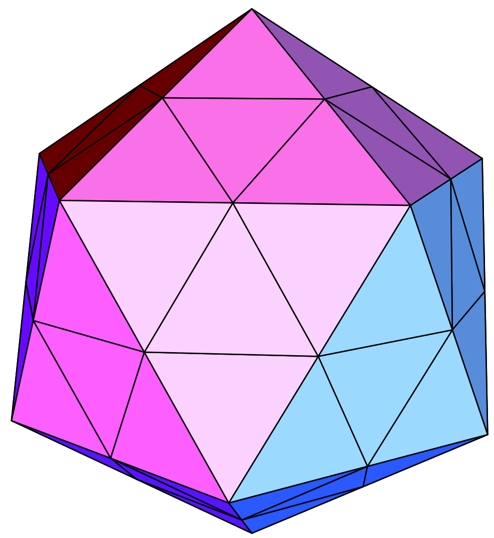


Icosahedron

Geodesic Grid

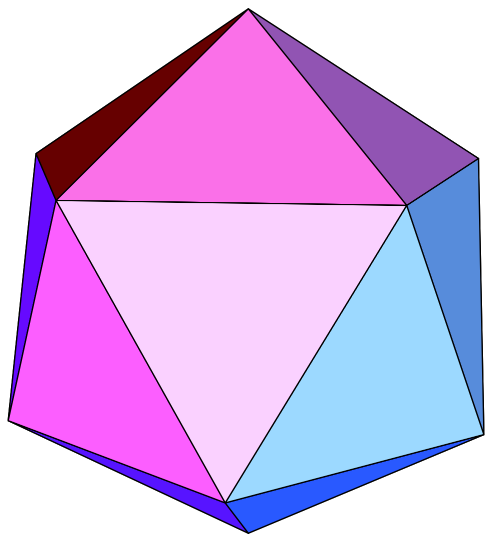


Icosahedron

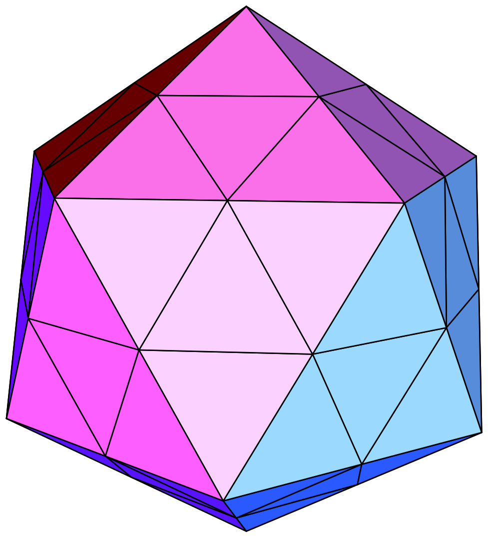


Bisect each edge
and connect the dots

Geodesic Grid



Icosahedron



Bisect each edge
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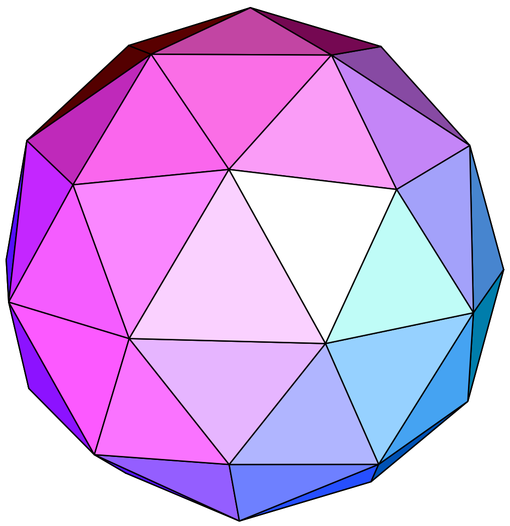
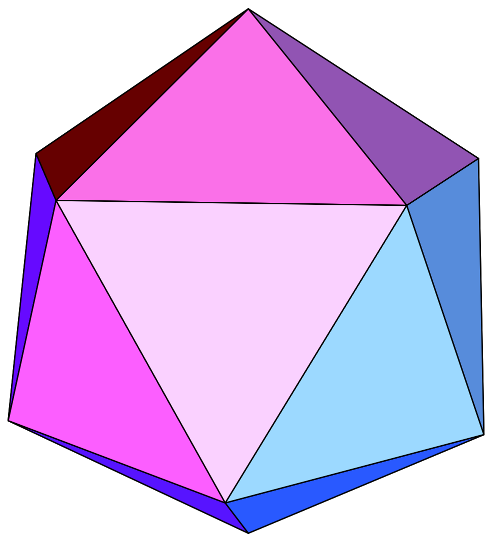
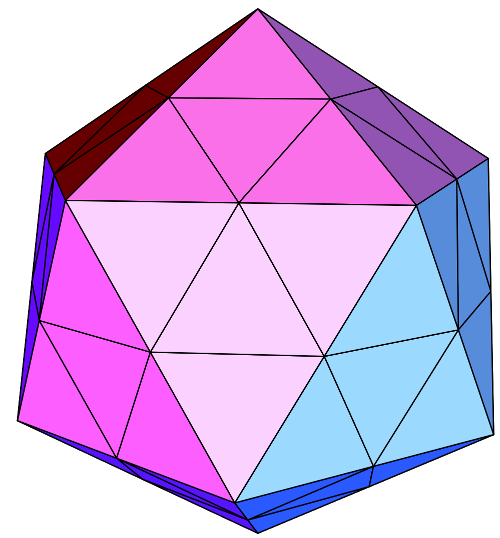


Pop out onto
the unit sphere

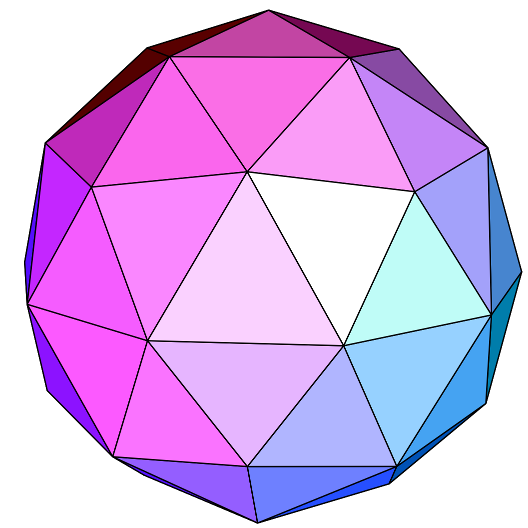
Geodesic Grid



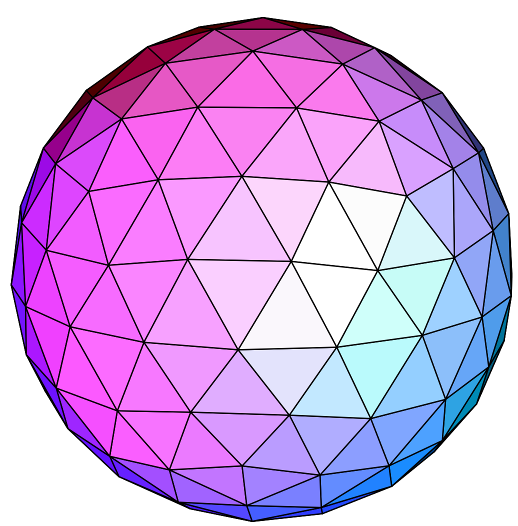
Icosahedron

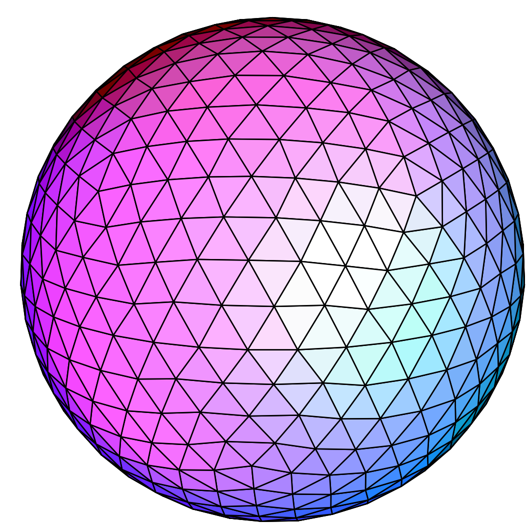


Bisect each edge
and connect the dots



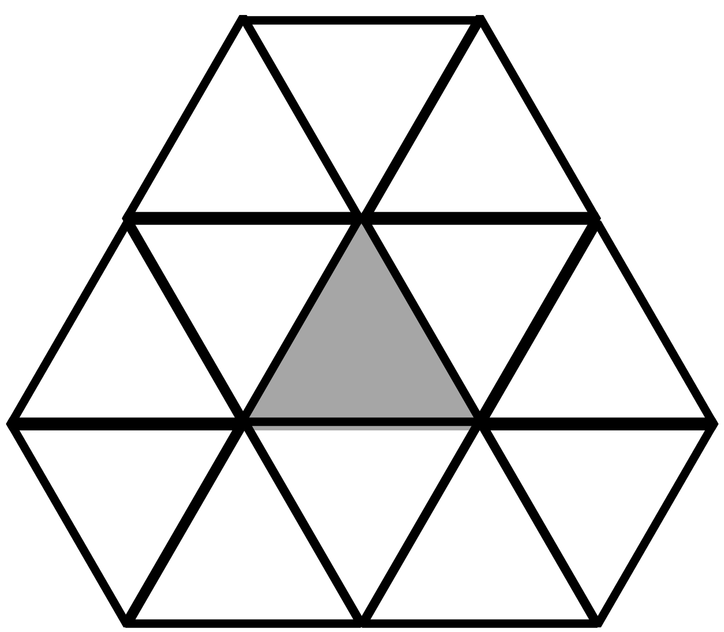
Pop out onto
the unit sphere



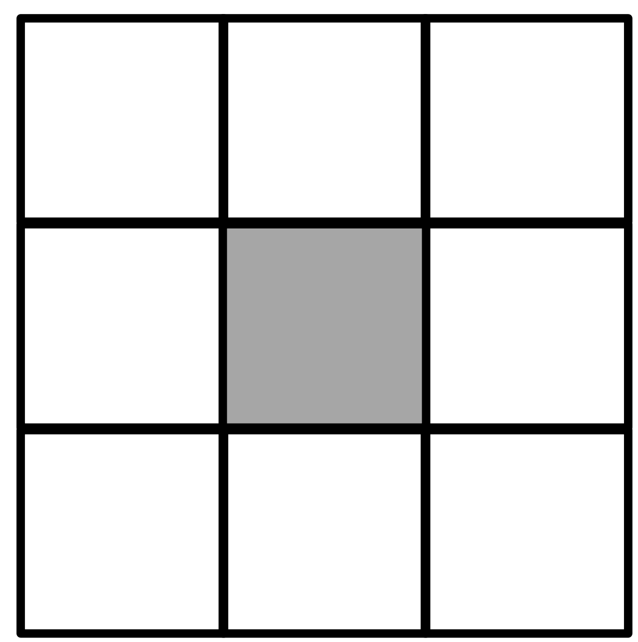


And so on, until we reach our target resolution...

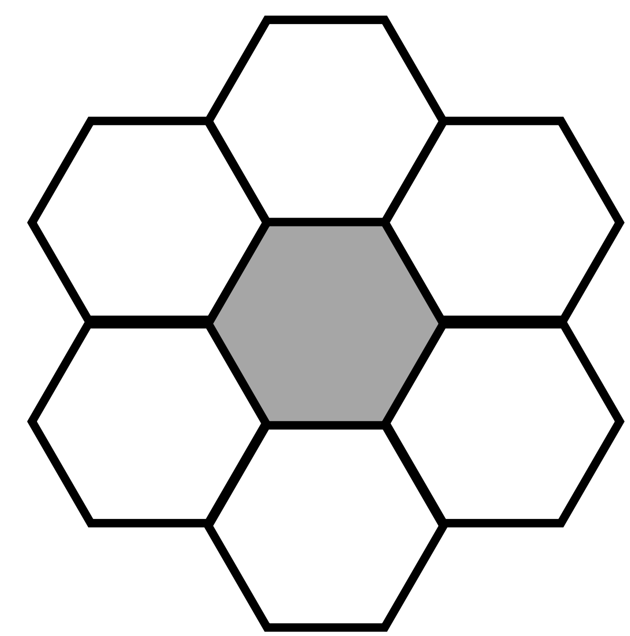
All places the same.
All directions the same.
All neighbors the same.



**12 neighbors,
3 wall neighbors**



**8 neighbors,
4 wall neighbors**



**6 neighbors,
6 wall neighbors**

“Tweaked” geodesic grids

“Tweaked” geodesic grids

Number of cells
12
42
162
642
2562
10242
40962

“Tweaked” geodesic grids

Number of cells	Average distance between cell centers, km
12	7054
42	3717
162	1909
642	961
2562	481
10242	240
40962	120

“Tweaked” geodesic grids

Number of cells	Average distance between cell centers, km	Area ratio (smallest to largest)
12	7054	1
42	3717	0.885
162	1909	0.916
642	961	0.942
2562	481	0.948
10242	240	0.951
40962	120	0.952

Some grids of interest

Level of recursion	Number of grid columns	Distance between grid columns, km
9	2,621,442	15.64
10	10,485,762	7.819
11	41,943,042	3.909
12	167,772,162	1.955
13	671,088,642	0.977

And then ... do the math