

# QUASI-3D MMF and Global CRM

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## I. Unification of Physics between GCM and CRM

1. Introduction - motivation and goal
2. Strategy for development of a quasi-3D algorithm
3. Current major issues

## II. Unification of Dynamics between GCM and CRM

Joon-Hee Jung

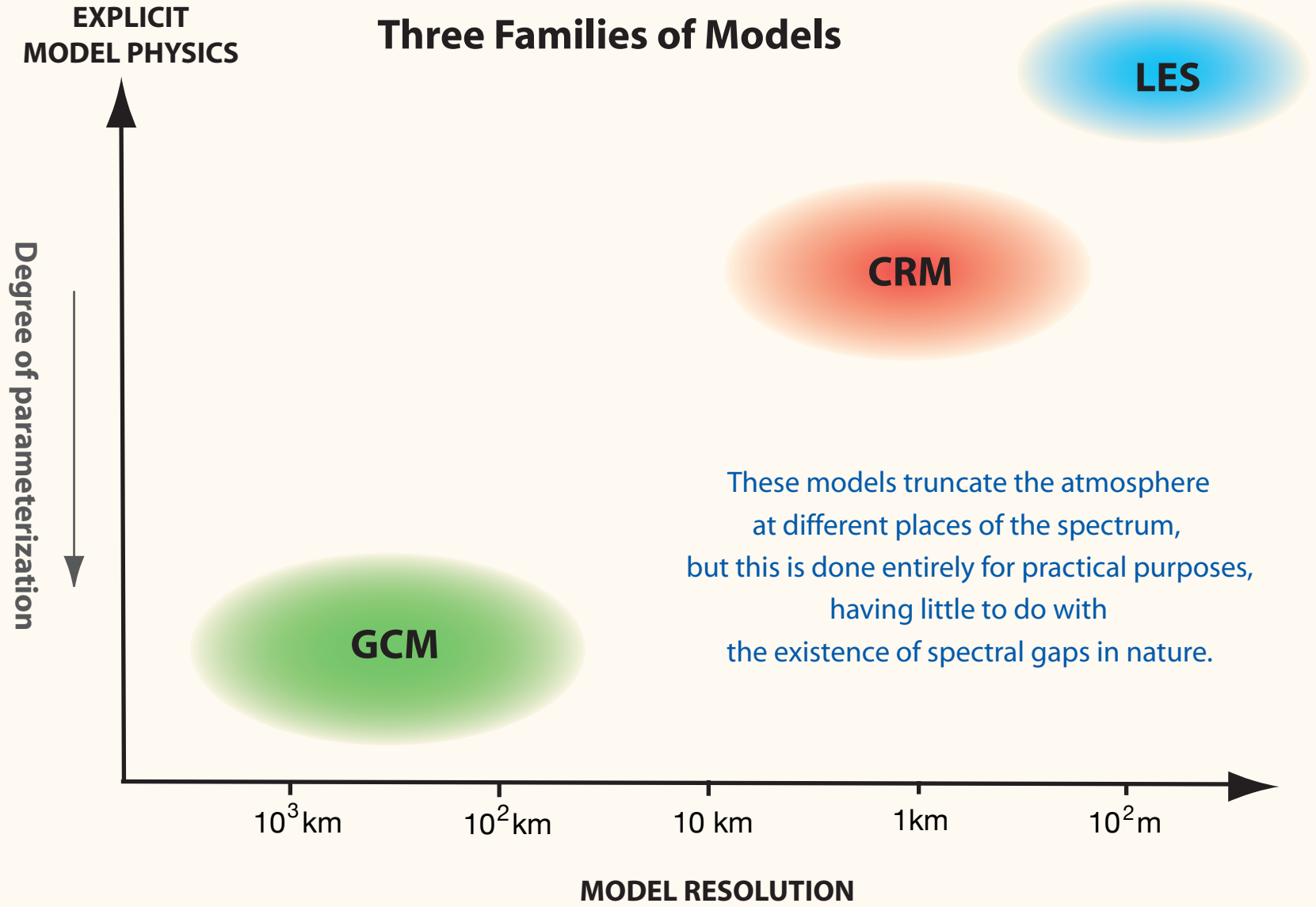
Results of preliminary experiments and future plans

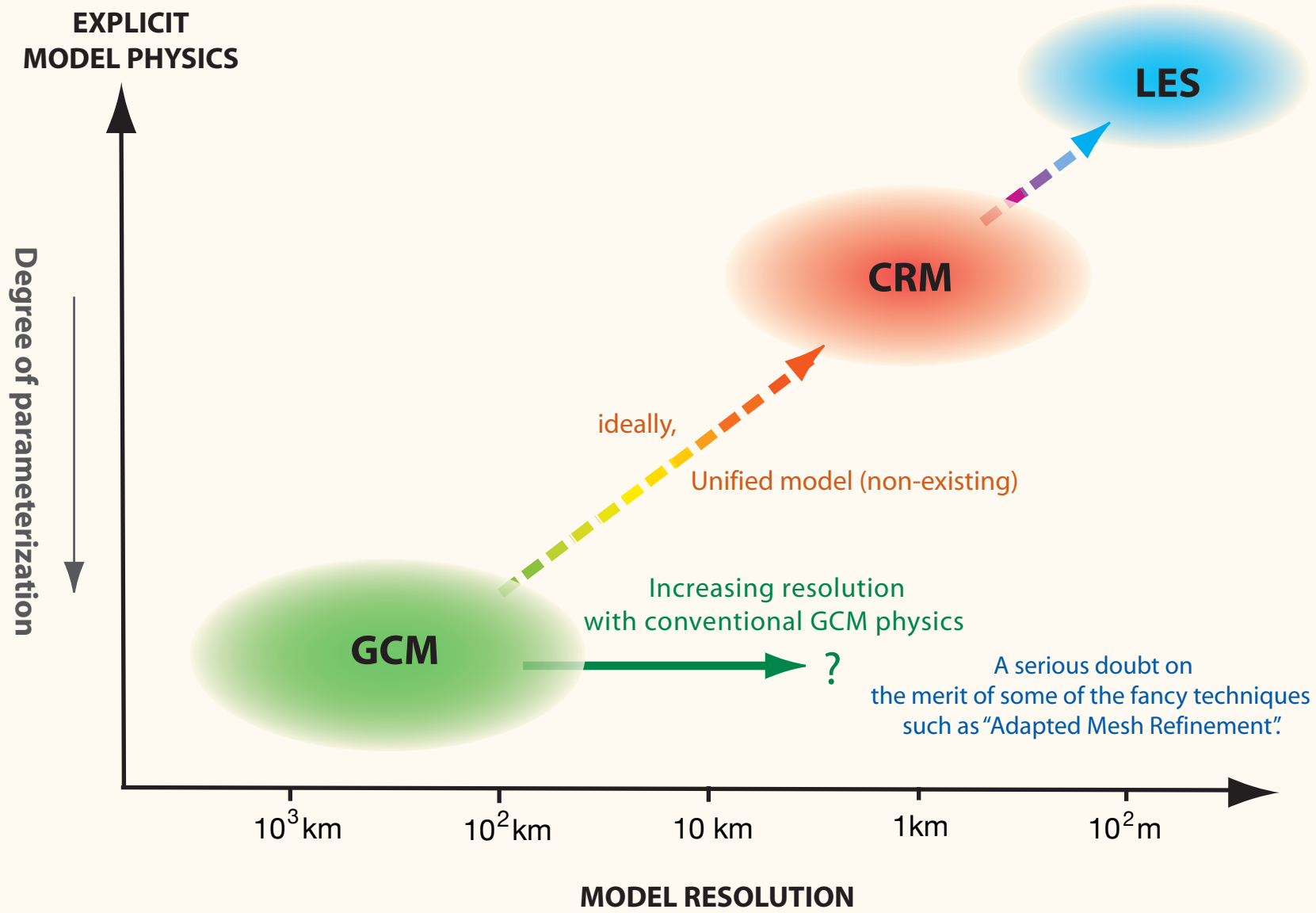
1. Experimental strategy, design and settings
2. Experimental Results
3. Future Plans.

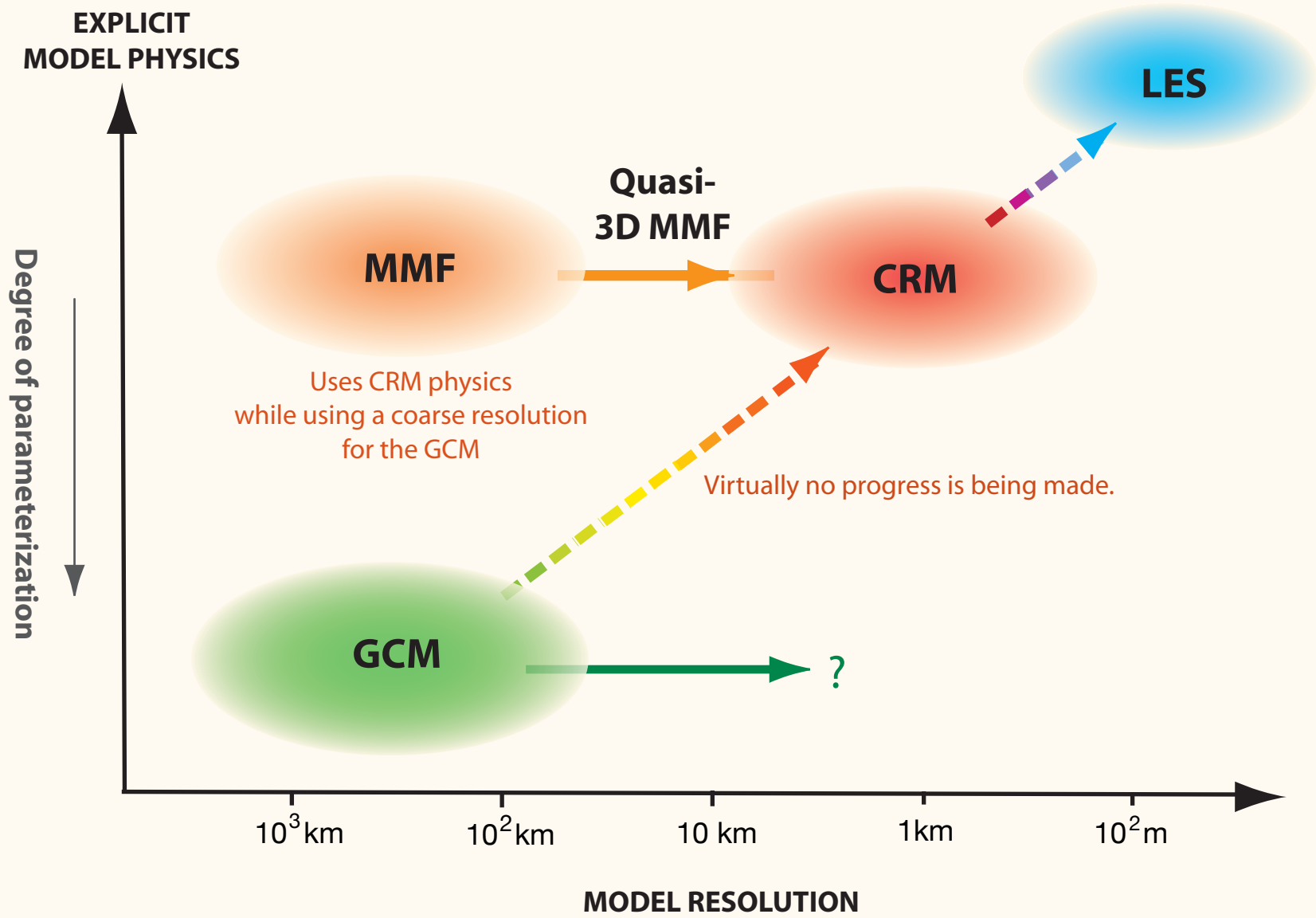
## MOTIVATION AND GOAL

- Use of a discrete model can be justified only when its solution converges to the solution of the original system as the resolution is refined.
- When truncation is made in or near an energetically-active range of the spectrum, model physics must also be changed as the resolution changes.
- At present, there is no unified formulation of model physics that automatically provides such changes.
- Since the original system is 3D, a converging framework must be at least quasi-3D.

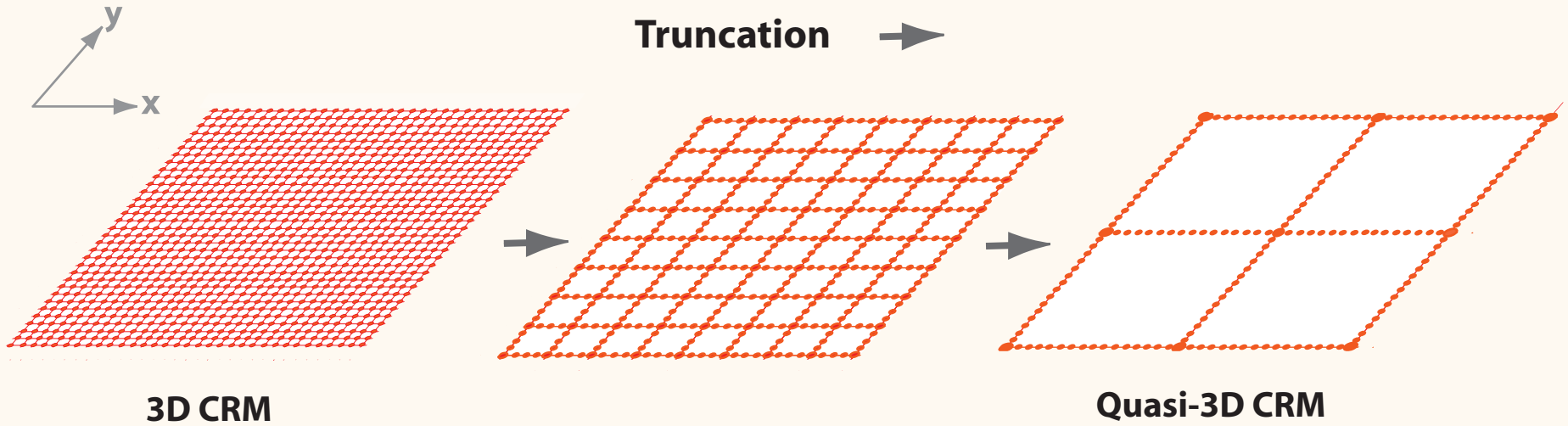
# Three Families of Models



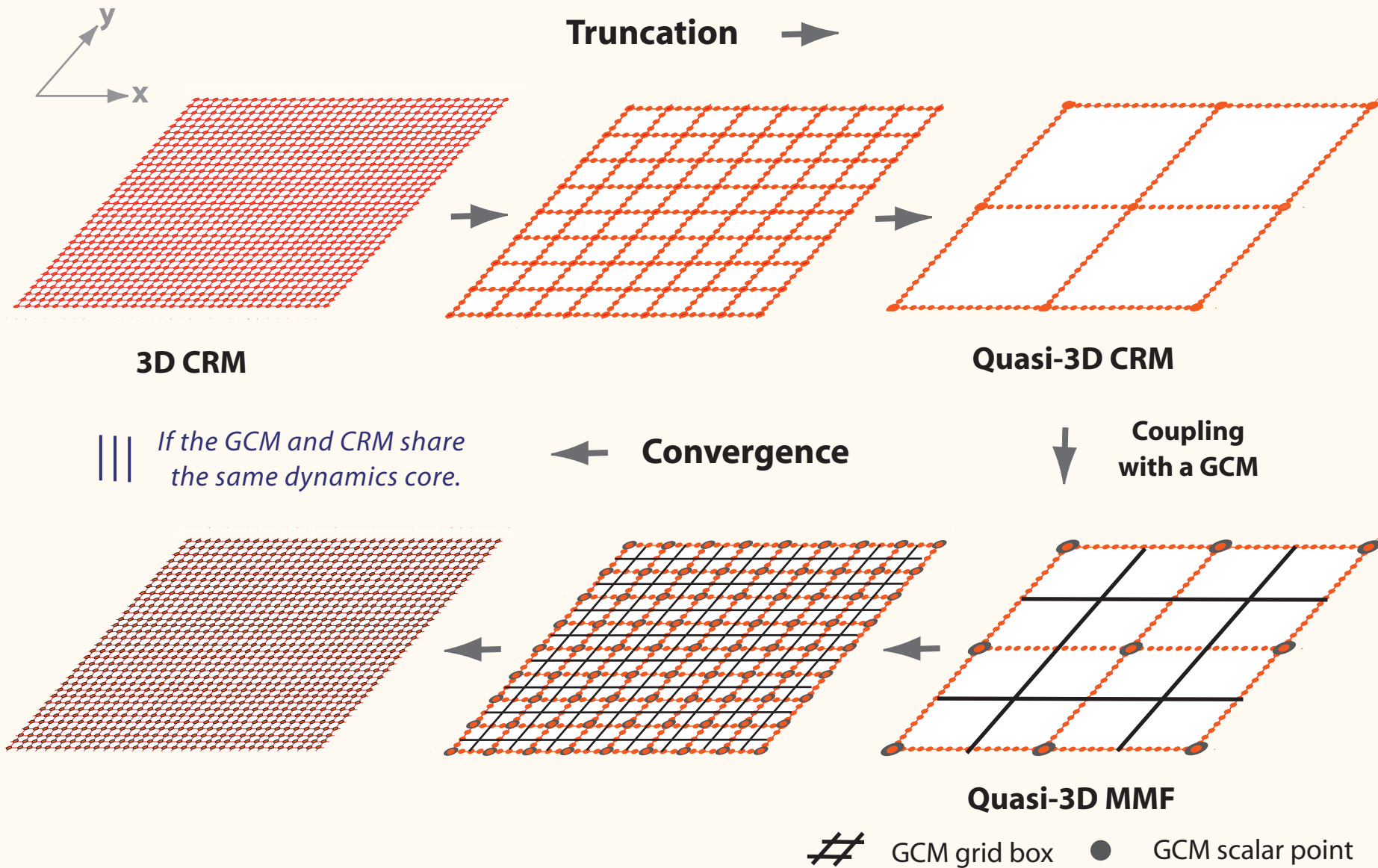




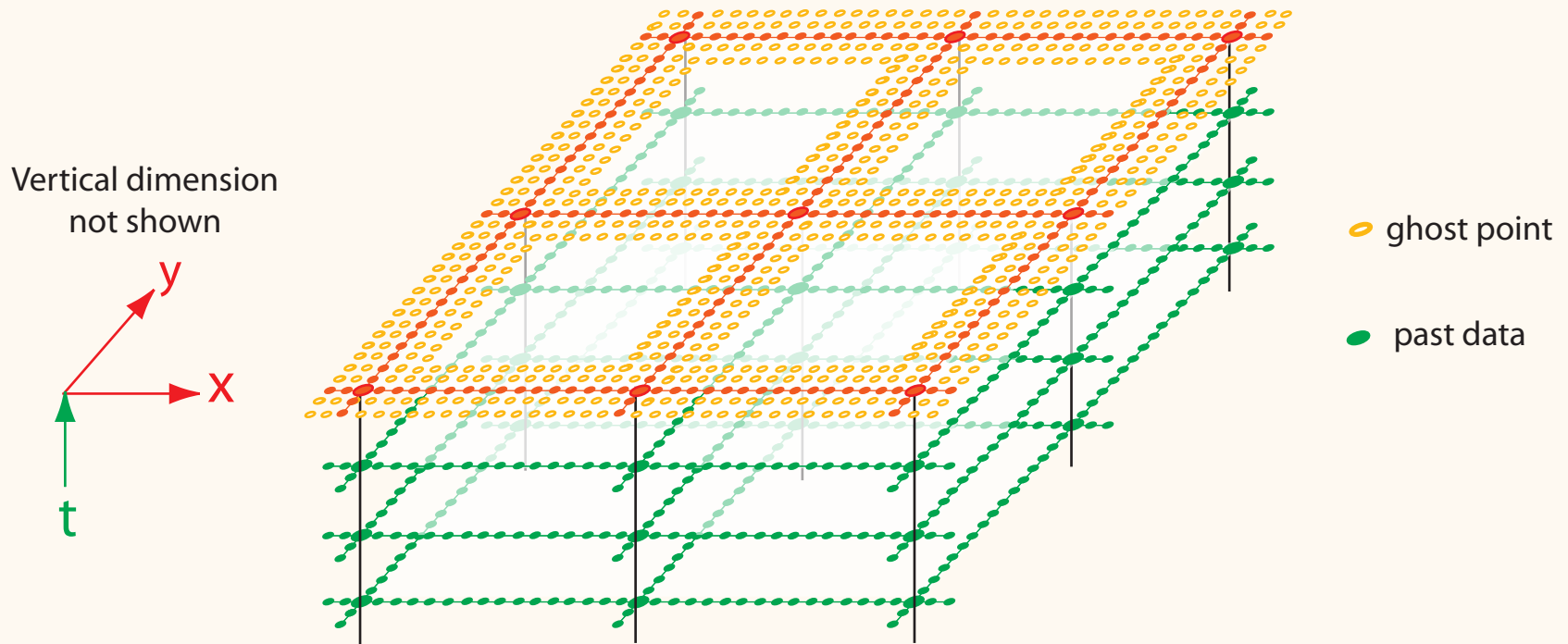
# HIERARCHY of QUASI-3D CRM AND MMF



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## Quasi-3D CRM : A prerequisite to a quasi-3D MMF



A 4D estimation/prediction problem

with a highly-anisotropic multi-resolution horizontal grid with singular points ...

**A treasure house of computational problems !**

**PATIENCE !**

*"If I have ever made valuable discoveries, it has been owing to patient attention, than to any other talent."*

Sir Isaac Newton



# CLOUD-RESOLVING ANELASTIC MODEL BASED ON THE 3D VORTICITY EQUATION

## Prediction of scalar variables

Water substances (and tracer):

$$\rho_0 \frac{\partial q_x}{\partial t} = - \left[ \frac{\partial}{\partial x} (\rho_0 u q_x) + \frac{\partial}{\partial y} (\rho_0 v q_x) + \frac{\partial}{\partial z} (\rho_0 w q_x) \right] + S_{q_x}$$

Potential temperature:

$$\rho_0 \frac{\partial \theta}{\partial t} = - \left[ \frac{\partial}{\partial x} (\rho_0 u \theta) + \frac{\partial}{\partial y} (\rho_0 v \theta) + \frac{\partial}{\partial z} (\rho_0 w \theta) \right] + S_\theta$$

## Prediction of vorticity components

Horizontal components:

$$\rho_0 \frac{\partial \xi}{\partial t} = - \left[ \frac{\partial}{\partial x} (\rho_0 u \xi) + \frac{\partial}{\partial y} (\rho_0 v \xi) + \frac{\partial}{\partial z} (\rho_0 w \xi) \right] + \rho_0 \xi \frac{\partial u}{\partial x} + \rho_0 \eta \frac{\partial u}{\partial y} + \rho_0 \zeta \frac{\partial u}{\partial z} + S_\xi$$

$$\rho_0 \frac{\partial \eta}{\partial t} = - \left[ \frac{\partial}{\partial x} (\rho_0 u \eta) + \frac{\partial}{\partial y} (\rho_0 v \eta) + \frac{\partial}{\partial z} (\rho_0 w \eta) \right] + \rho_0 \eta \frac{\partial v}{\partial y} + \rho_0 \xi \frac{\partial v}{\partial x} + \rho_0 \zeta \frac{\partial v}{\partial z} + S_\eta$$

Vertical component:

$$(\rho_0 \zeta)_z = - \int_{z_T}^z \left[ \frac{\partial (\rho_0 \xi)}{\partial x} + \frac{\partial (\rho_0 \eta)}{\partial y} \right] dz + (\rho_0 \zeta)_{z_T}$$

## Determination of wind components

From anelasticity:

$$\left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) w + \frac{\partial}{\partial z} \left[ \frac{1}{\rho_0} \frac{\partial}{\partial z} (\rho_0 w) \right] = - \rho_0 \frac{\partial \eta}{\partial x} + \rho_0 \frac{\partial \xi}{\partial y}$$

From the definition of vorticity:

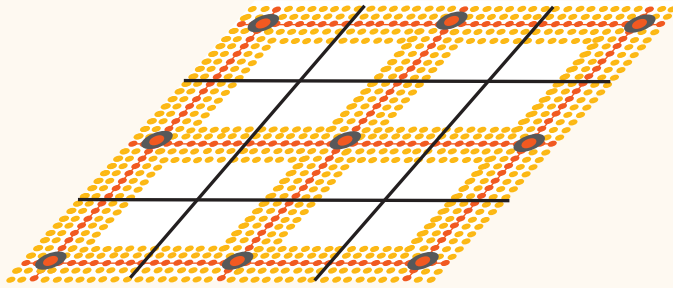
$$u = \int_{z_T}^z \left( \frac{\partial w}{\partial x} + \rho_0 \eta \right) dz + u_T(x, y, t) \quad v = \int_{z_T}^z \left( \frac{\partial w}{\partial y} - \rho_0 \xi \right) dz + v_T(x, y, t)$$

## **A Problem with the Highly Anisotropic Grid :**

Error imbalance between  
explicitly -resolved tangential derivative and  
statistically-estimated normal derivative.

*The degree of imbalance is scale-dependent.*

# MULTI-SCALE REPRESENTATION OF VARIABLES



$$q = \bar{q} + \overbrace{q' + q''}^{q^*}$$

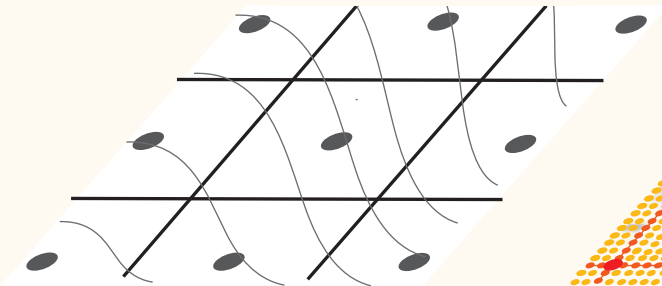
Typically, BACKGROUND FIELD COHERENT PART OF DEVIATION NON-COHERENT PART OF DEVIATION  
synoptic-scale cloud-system scale cloud scale

## DETERMINATION OF 3D STRUCTURES

$\bar{q}$

Determined by interpolatin of GCM grid-point values

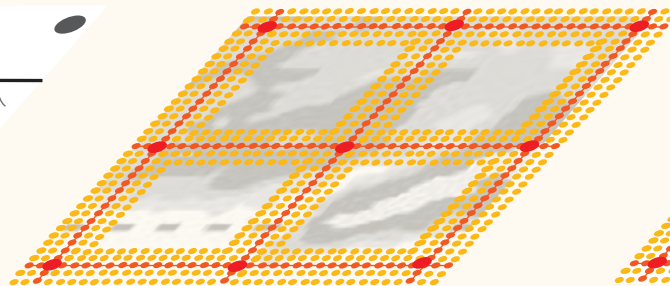
(Currently, this field is prescribed.)



$q'$

Along grid-point array:  
1D Raynolds averaging of  $q^*$

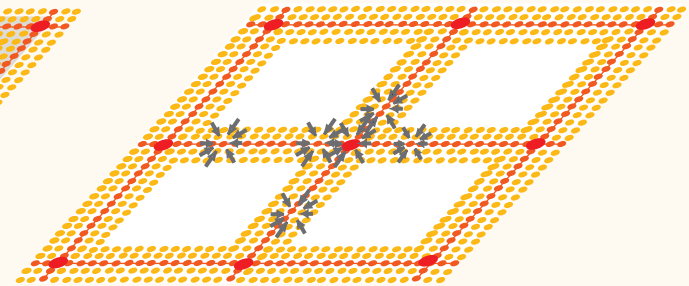
Normal to grid-point array:  
By statistical Identification of cloud regime



$q''$

Along grid-point array:  
 $q^* - q'$

Normal to grid-point array:  
With a parameterization based on isotropy or inferred anisotropy



In CRMs with a horizontal resolutions of  $\geq 1$  km,  
parameterization problems still exist ***even for deep convection***

(due to the existence of the internal structure and adjacent small-scale processes  
such as entrainment, detrainment, convection in anvils, convective downdrafts,  
triggering due to surface inhomogeneities, etc.)

*Additional parameterization problem exists in MMFs,  
which have cloud-scale resolution only in limited directions*

## WAVENUMBER SPACE

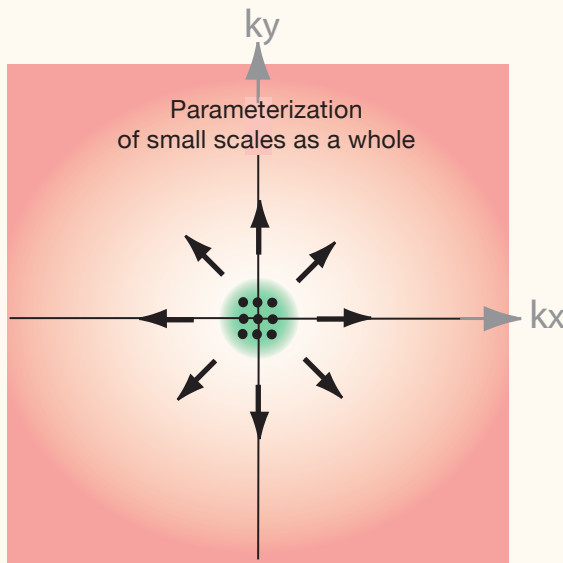


Synoptic scale

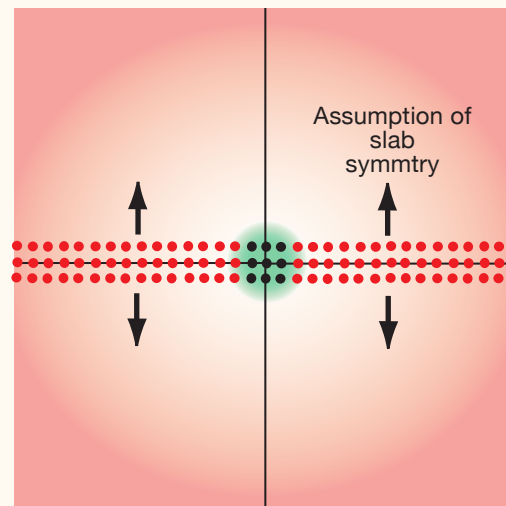


Meso & cloud scales

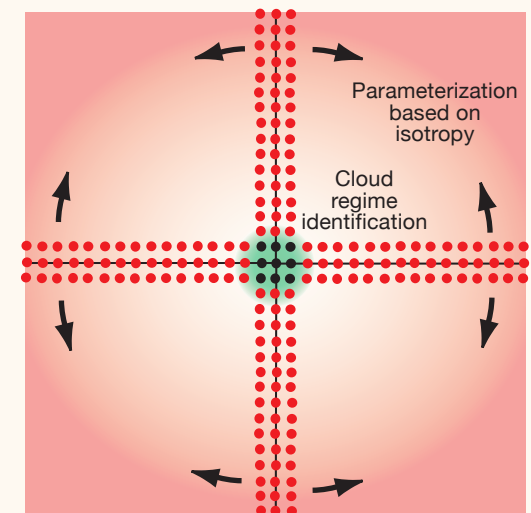
### Conventional model



### Prototype MMF



### Quasi-3D MMF



# Emphasis of the Development Work up to Now

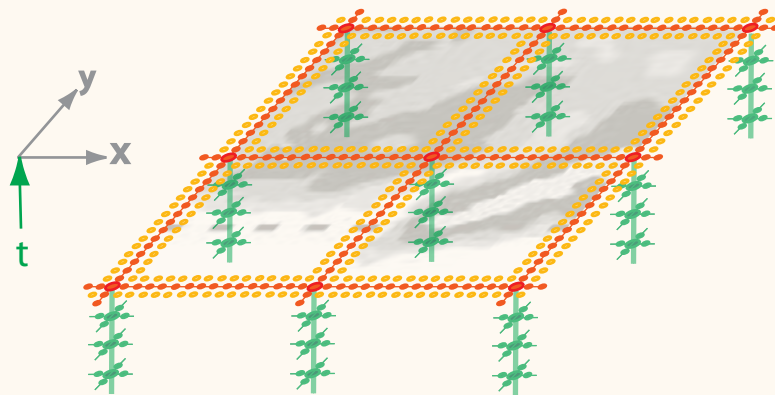
$$q = \bar{q} + \overbrace{q' + q''}^{q^*},$$

"FILTERED"      "NON-FILTERED"

$q'$

Along grid-point array:  
1D Reynolds averaging of  $q^*$

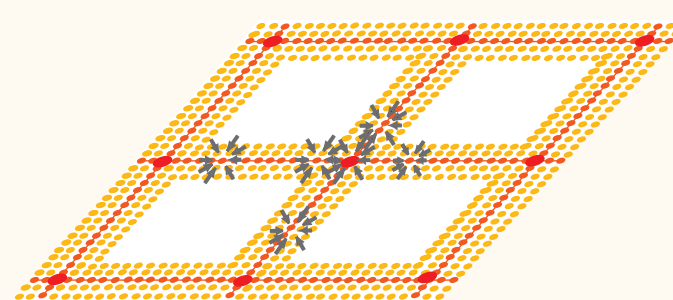
**Normal to grid-point array:**  
**By statistical Identification of cloud regime**



$q''$

Along grid-point array:  
 $q^* - q'$

**Normal to grid-point array:**  
**With a parameterization based on isotropy or inferred anisotropy**



- *Cloud regimes have longer spatial and temporal scales than individual clouds.*
- *Cloud regimes can be statistically inferred through regression analysis of past data at the intersection and neighboring points.*

# Technical Problems in Quasi3D Advection

## Computational problems

1. Global stability with 2-dimensional uniform current
2. Local stability with 3-dimensional non-uniform current
3. Control of singularity at intersections
4. Control of spurious trend
5. Control of noise
- ( 6. Conservation )

**Regression analysis of past data to identify cloud regimes**

# Solving 3D Elliptic Equation using the Quasi-3D Network

In anelastic models, 3D elliptic equation must be solved.

In momentum-equation models : for pressure

In vorticity-equation models : for vertical velocity

- *In our model*, the elliptic equation for  $w$  is converted to a parabolic equation whose equilibrium solution is the solution of the elliptic equation (mimicing the relaxation method).

$$\mu \frac{\partial w}{\partial t} = \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) w + \frac{\partial}{\partial z} \left[ \frac{1}{\rho_0} \frac{\partial}{\partial z} (\rho_0 w) \right] + \rho_0 \left( \frac{\partial \eta}{\partial x} - \frac{\partial \xi}{\partial y} \right)$$

- The second-order finite difference term is estimated based on ghost point values given by
  - $w'$ : Statistically estimated as in advection
  - $w''$ : Currently assumed to be zero
- No local deformation of the horizontal vorticity vector for cloud scale.

## Problem with the Stretching and Twisting Terms in the Vorticity Equation

$$\rho_0 \frac{\partial \xi}{\partial t} = - \left[ \frac{\partial}{\partial x} (\rho_0 u \xi) + \frac{\partial}{\partial y} (\rho_0 v \xi) + \frac{\partial}{\partial z} (\rho_0 w \xi) \right] + \underbrace{\rho_0 \xi \frac{\partial u}{\partial x}}_{\text{stretching}} + \underbrace{\rho_0 \eta \frac{\partial u}{\partial y} + \rho_0 \zeta \frac{\partial u}{\partial z}}_{\text{twisting}} + S_\xi$$

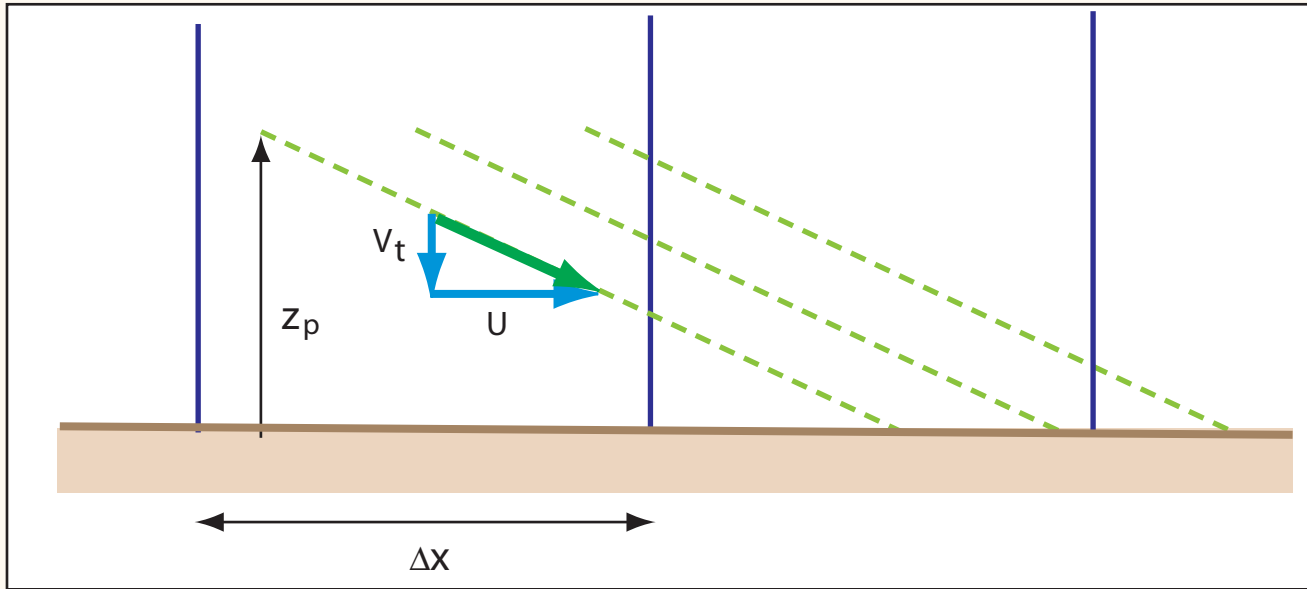
$$\rho_0 \frac{\partial \eta}{\partial t} = - \left[ \frac{\partial}{\partial x} (\rho_0 u \eta) + \frac{\partial}{\partial y} (\rho_0 v \eta) + \frac{\partial}{\partial z} (\rho_0 w \eta) \right] + \underbrace{\rho_0 \eta \frac{\partial v}{\partial y}}_{\text{stretching}} + \underbrace{\rho_0 \xi \frac{\partial v}{\partial x} + \rho_0 \zeta \frac{\partial v}{\partial z}}_{\text{twisting}} + S_\eta$$

- These terms, which are responsible for enstrophy increase and energy cascade in 3D flow, are missing in 2D models.
- The Q3D CRM includes these terms.
  - Fully for filtered scale (i.e., for cloud-system scale) vorticity;
  - Partially for non-filtered scale (i.e., individual cloud scale) vorticity by neglecting the effect of local deformation on this scale.
- The enstrophy increase and energy cascade must be balanced by dissipation on average. So far we have not found a satisfactory formulation of the dissipation for the quasi-3D network.

**Dilemma**      Too strong dissipation : Inactive dynamics  
 Too weak dissipation:      Blowing up



# Horizontal Advection of Precipitates in 3D CRM



If  $\Delta X < \frac{U}{V_t} z_p \sim 10 \text{ km}$ , horizontal advection of precipitates must be explicitly formulated.

Lagrangian view: All precipitates in the column move into the next column before reaching ground.

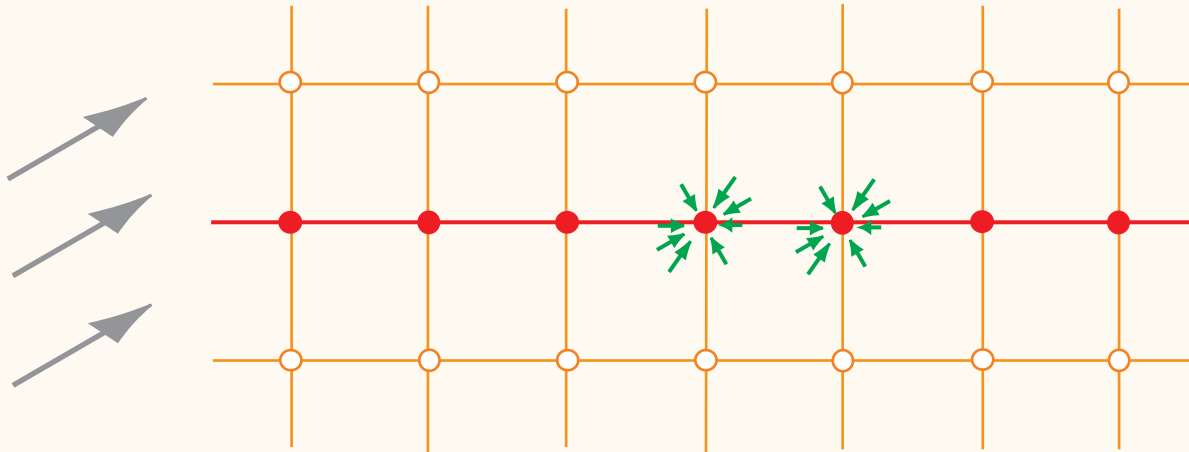
Eulerian view: Air of the column is entirely replaced by the upwind-side precipitate-free air.

- If the precipitating system is poorly resolved ( e.g., if  $\Delta X \gtrsim 1 \text{ km}$ ) by an Eulerian grid, serious dispersion error may appear.
- Consequently, a significant part of the precipitates can be left in the original column, which tends to evaporate as a result of drier-air inflow.

*Thus in a CRM with  $1 \text{ km} \lesssim \Delta X < \frac{U}{V_t} Z_p \sim 10 \text{ km}$ , the area average of precipitate due to convection tends to be under-predicted, and the humidity of the convective column tends to be over-predicted, eventually producing stratiform rain.*

**These tendencies can be exaggerated in a Q3D CRM** through

- Under-estimation of the precipitate mixing ratio at the up-wind side due to the use of statistics including non-precipitating cases.
- Overestimation of the normal component of velocity.



In our tests, generally the variance of water-vapor mixing ratio over the grid points tends to be over-predicted and those of rain/graupel mixing ratios tend to be under-predicted.

# Unification of Dynamics between GCM and CRM

Two possibilities :

- (1) Use of the fully-compressible nonhydrostatic system of equations for both.
- (2) Use of a generalized anelastic system of equations for both, with compressibility included only for quasi-static motions.

*neglected in  
the anelastic system*

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0$$

$$\left( \frac{\partial \rho}{\partial p} \right)_\theta \frac{\partial p}{\partial t} + \left( \frac{\partial \rho}{\partial \theta} \right)_p \frac{\partial \theta}{\partial t}$$

compressibility

thermal  
expansion

*Included with  
hydrostatic  
approximation*

*fully  
included*

This system automatically becomes  
the primitive equation model  
for large scales.