PROGRESS TOWARDS A QUASI-3D MMF: Technical Aspects

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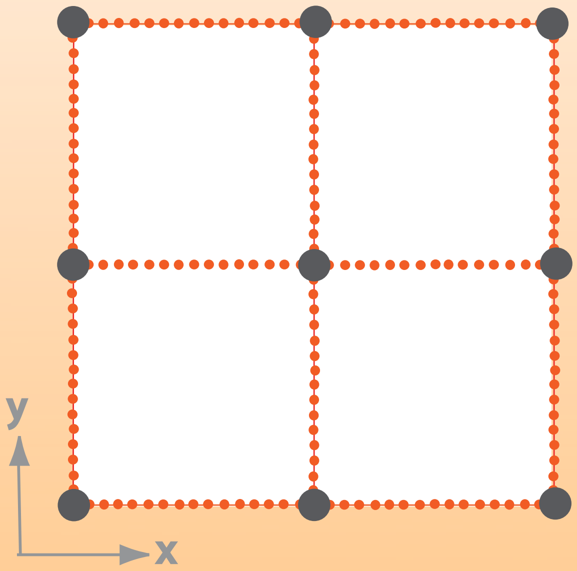
CMMAP Team Meeting
Kauai, Hawaii, February 20-22, 2007

Q3D CRM

Based on the same model dynamics and physics with the 3D CRM, only difference being the use of Q3D grid



Due to the use of a highly-anisotropic horizontal grid,
an algorithm must be developed

1) to determine the gradient normal to the grid-point arrays,
2) to solve the elliptic equation for w, and
3) to determine horizontal velocities.

To determine the gradient normal to the grid-point arrays

We first introduce the following multi-scale expression for all variables:

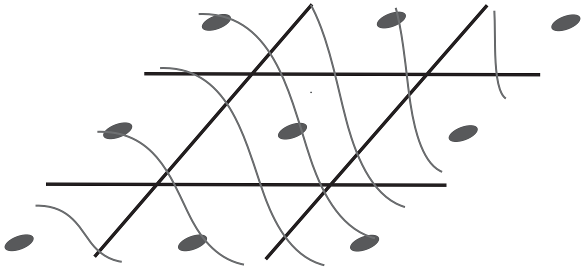
$$q = \underset{\text{synoptic-scale (background field)}}{\bar{q}} + \underbrace{\underset{\text{cloud-system scale}}{q'} + \underset{\text{cloud scale}}{q''}}_{q^*}$$

DETERMINATION OF 3D STRUCTURES

$\bar{q}$

Determined by interpolation of GCM grid-point values

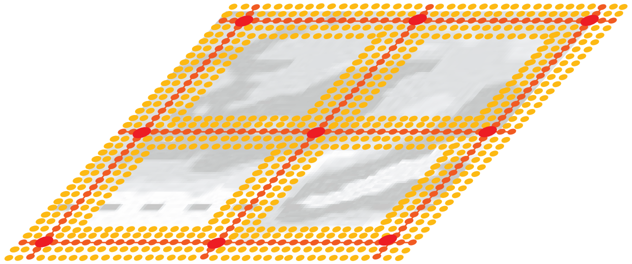
(Currently, this field is prescribed.)



q'

Along grid-point array:
1D Raynolds averaging of q *

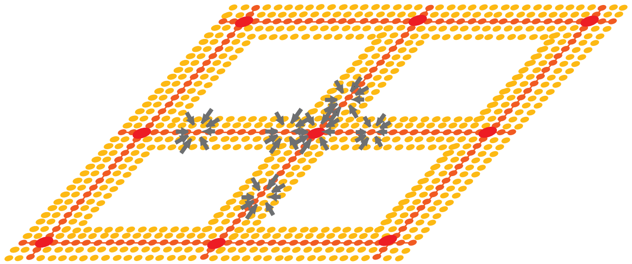
Normal to grid-point array:
By statistical Identification of cloud regime



q''

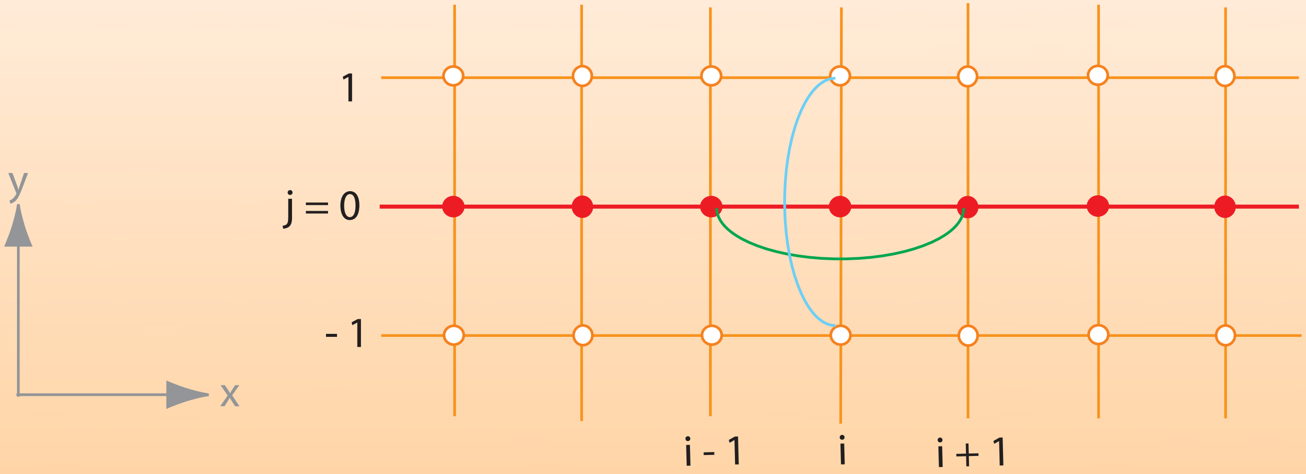
Along grid-point array: q* - q′

Normal to grid-point array:
With a parameterization based on isotropy or inferred anisotropy



Advection of Filtered Variable, q’

GLOBAL STABILITY : Uniform current with $\overline{q'}^{i} = 0$



The array sum of q'^{2} is conserved if

$$\Delta_j \widehat{q'} = a_1 + b_1 \Delta_i q'$$

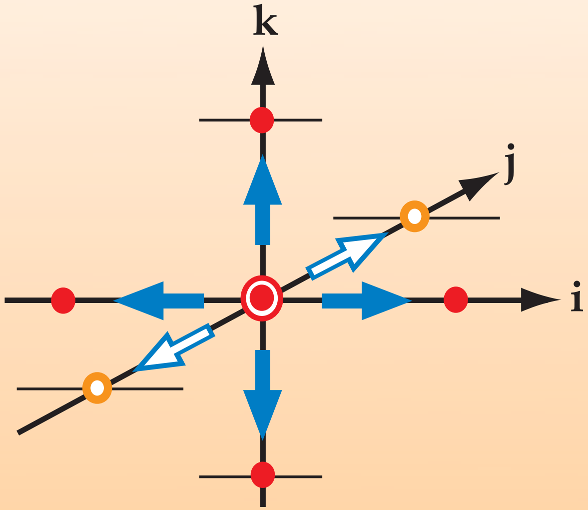
↑
Estimated
first-order difference
in the normal direction

↑
Predicted
first-order difference
in the tangential direction

The parameter b_1 represents the dominant orientation of cloud organization.

Advection of Filtered Variable, q’

LOCAL STABILITY : Three-dimensionally variable current



Estimated flux divergence must not produce positive feedback on the perturbation.

$$\delta_j^2 \widehat{q'} = a_2 + b_2 \, \delta_i^2 q'$$

↑ Estimated second-order difference in the normal direction

↑ Predicted second-order difference in the tangential direction

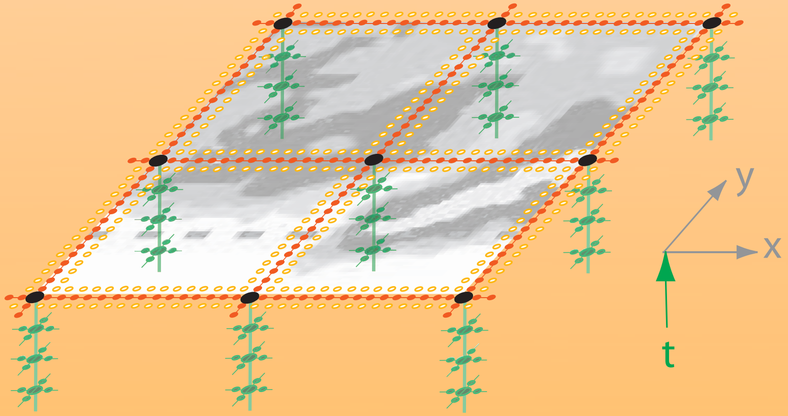
with $b_2 \geq 1$

DETERMINATION OF THE PARAMETERS

$$
\begin{aligned}
\Delta_j \hat{q}' &= a_1 + b_1\, \Delta_i q' \\
\delta_j^2 \hat{q}' &= a_2 + b_2\, \delta_i^2 q' \\
\Delta_j \delta_j^2 \hat{q}' &= a_3 + b_3\, \Delta_i \delta_i^2 q' \\
\delta_j^4 \hat{q}' &= a_4 + b_4\, \delta_i^4 q'
\end{aligned}
$$

HYPOTHESES:

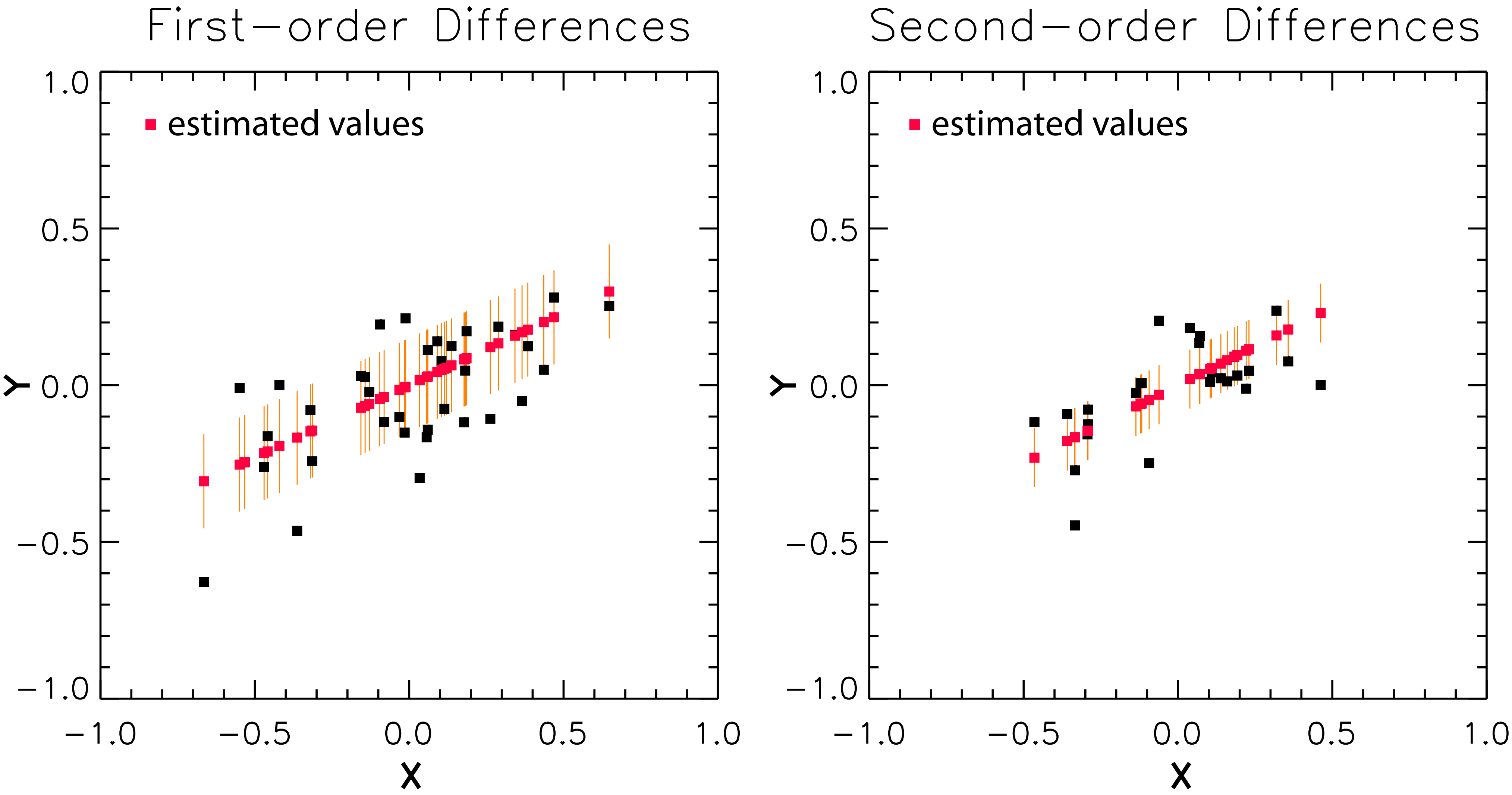
- These parameters are cloud-regime dependent.
- Cloud regimes have longer spatial and temporal scales than individual clouds.
- These parameters can be statistically estimated through regression analysis of past data at the intersection and neighboring points.



Example of regression analysis

Realized Y vs. Estimated Y

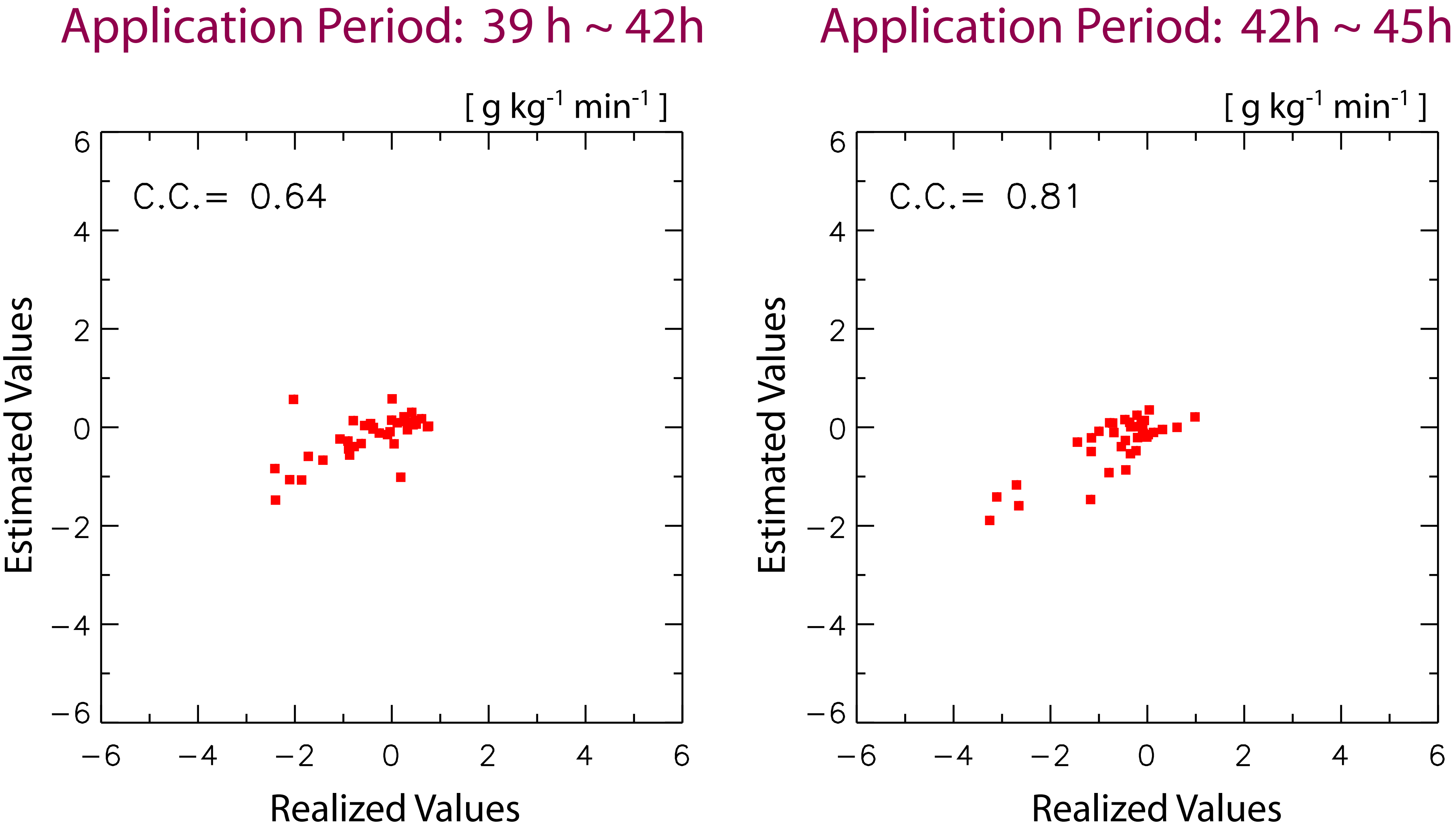
Application Period: 36h ~ 39h



Example of regression analysis

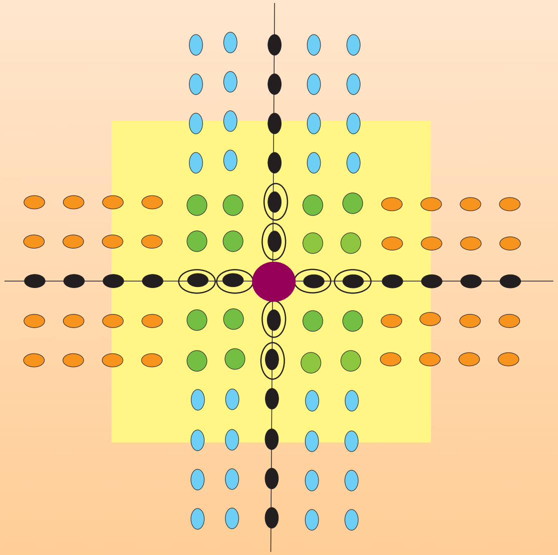
Realized Values vs. Estimated Values

Total Advection



CONTROL OF SINGULARITY AT INTERSECTIONS

1. Correction of the estimation error near the intersection



$C_{i,j}$: the correction at the data point (i,j) on the net

$c_{i',j'}$: the correction at the ghost point (i'j')

$$c_{i',j'} = \sum_{i,j} C_{i,j} e^{-\left(r_{i,j;i',j'}/r_o\right)^2} \Big/ \sum_{i,j} e^{-\left(r_{i,j;i',j'}/r_o\right)^2}$$

where $r_{i,j;i',j'}$ is the distance between the points and r_o is prescribed.

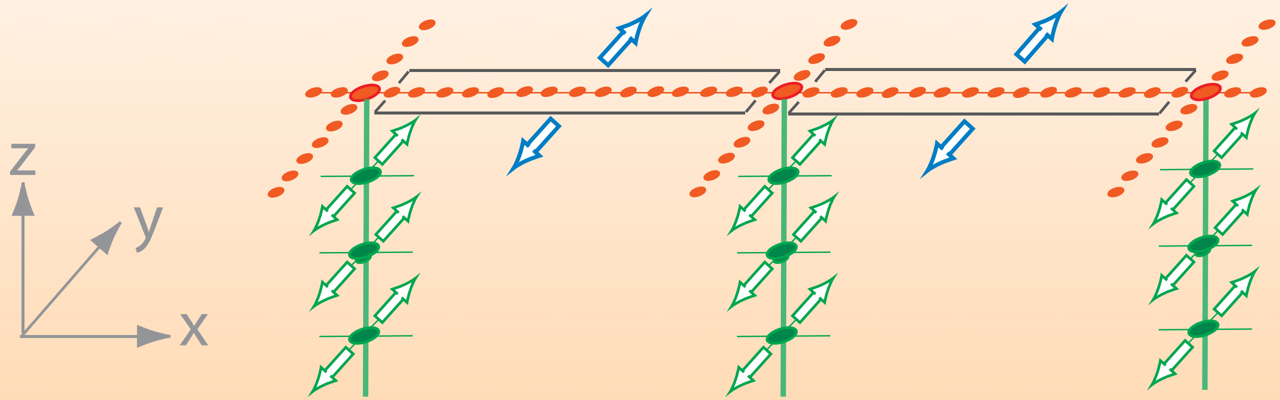
2. Application of semi-local diffusion at and near the intersection

Eddy diffusivity K depends on the distance from the intersection point.

CONTROL OF SPURIOUS TREND

To ensure that $\overline{q^*} = 0$ approximately holds during prediction, the Rayleigh-type damping is applied to q^*.

CONSERVATION OF THE VERTICALLY-INTEGRATED NETWORK MEAN



(Approximate) conservation is achieved by requiring the mean divergence of the flux from/to ghost points is equal to the mean divergence of the flux at the intersection points averaged in time over the analysis period.

$$\underset{\substack{\uparrow\\\text{corrected}\\\text{divergence}}}{\left(\delta_j G\right)_{i,j,k}} \equiv \underset{\substack{\uparrow\\\text{estimated}\\\text{divergence}}}{\left(\delta_j G^*\right)_{i,j,k}} - \left[\underset{\substack{\uparrow\\\text{estimated}\\\text{divergence}\\\text{averaged}\\\text{over the array}}}{\overline{\left(\delta_j G^*\right)_{i,j,k}}^{\,i}} - \underset{\substack{\uparrow\\\text{divergence}\\\text{averaged in time}\\\text{at the intersection}}}{\overline{\left(\delta_j G\right)_{I,J,k}}^{\,\mathrm{INT}}}\right]$$

Advection of Non-Filtered Variable, q” : Need for Parameterization

Currently ,

For scalar variables: $q'' = 0$ at ghost points (ad hoc).

A Relaxation Method for Solving the Elliptic Equation

The elliptic equation is converted to a parabolic equation whose equilibrium solution is the solution of the elliptic equation.

$$\mu\frac{\partial w}{\partial t}=\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)w+\frac{\partial}{\partial z}\left[\frac{1}{\rho_0}\frac{\partial}{\partial z}\left(\rho_0 w\right)\right]+\rho_0\frac{\partial \eta}{\partial x}-\rho_0\frac{\partial \xi}{\partial y}$$

where μ defines the time scale for adjustment toward anelastic balance.

Discretization

Use a partially backward-implicit scheme for the horizontal derivative term and a fully backward-implicit scheme for the vertical derivative term.

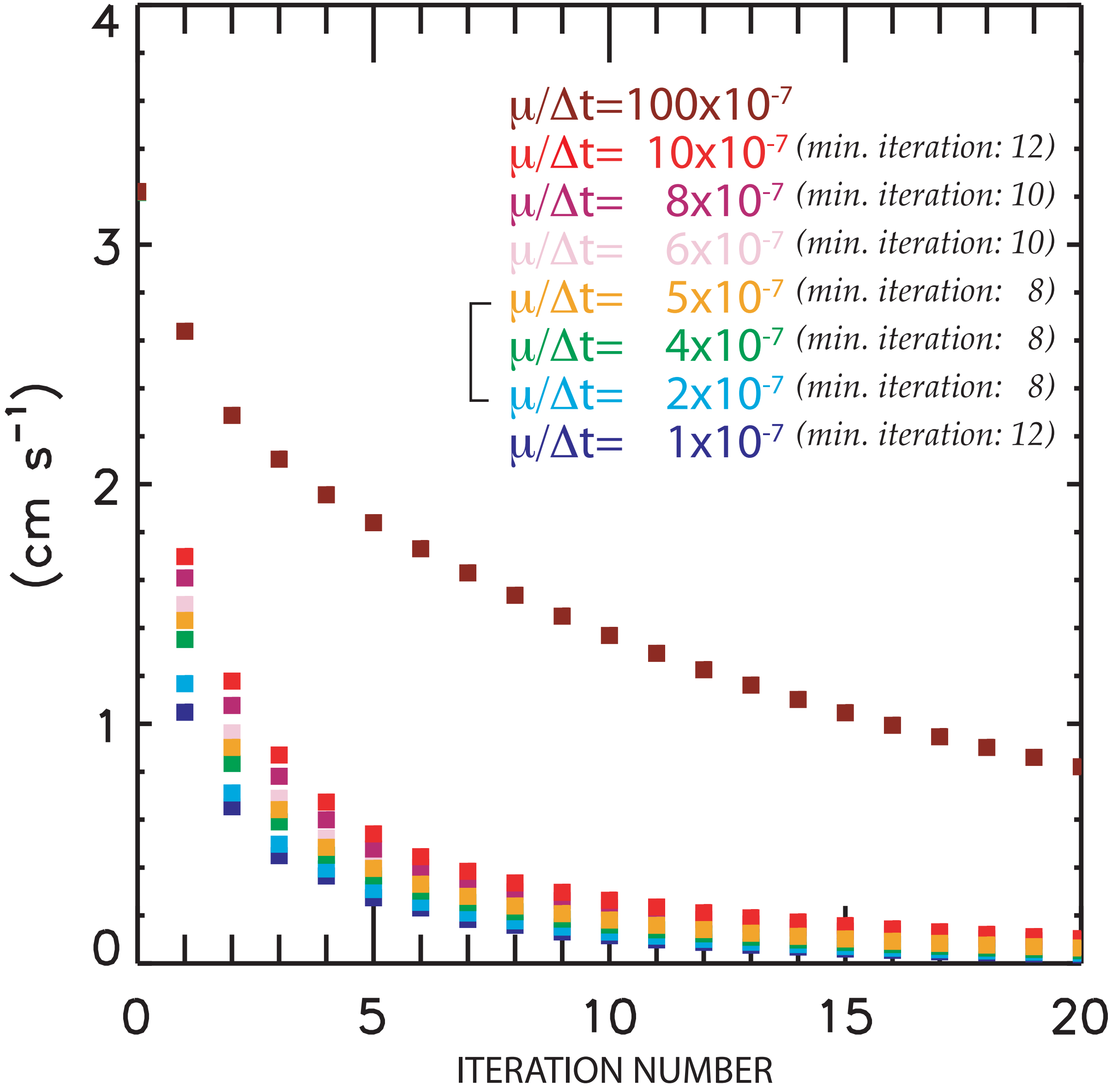
$$\left[\frac{1}{\rho_{k-1/2}}\left(\frac{\mu}{\Delta t}+\frac{2}{\Delta x^2}+\frac{2}{\Delta y^2}\right)+\frac{1}{\Delta z}\left(\frac{1}{\rho_k \Delta z}+\frac{1}{\rho_{k-1}\Delta z}\right)\right](\rho w)^{n+1}_{i,j,k-1/2}-\frac{1}{\Delta z}\left[\frac{(\rho w)^{n+1}_{i,j,k+1/2}}{\rho_k \Delta z}+\frac{(\rho w)^{n+1}_{i,j,k-3/2}}{\rho_{k-1}\Delta z}\right]$$

$$=\frac{\mu}{\Delta t}w^{n}_{i,j,k-1/2}+\frac{1}{\Delta x^2}\left(w^{n}_{i-1,j,k-1/2}+w^{n}_{i+1,j,k-1/2}\right)+\frac{1}{\Delta y^2}\left(w^{n}_{i,j-1,k-1/2}+w^{n}_{i,j+1,k-1/2}\right)+F^{n+1}_{i,j,k-1/2}$$

Determination of μ

Max. Value of $|w^{relaxed} - w^{true}|$

(Starting from $w^{guess} = w^{linear\ extrapolation}$)



$\Delta x = \Delta y = 2\text{km}$

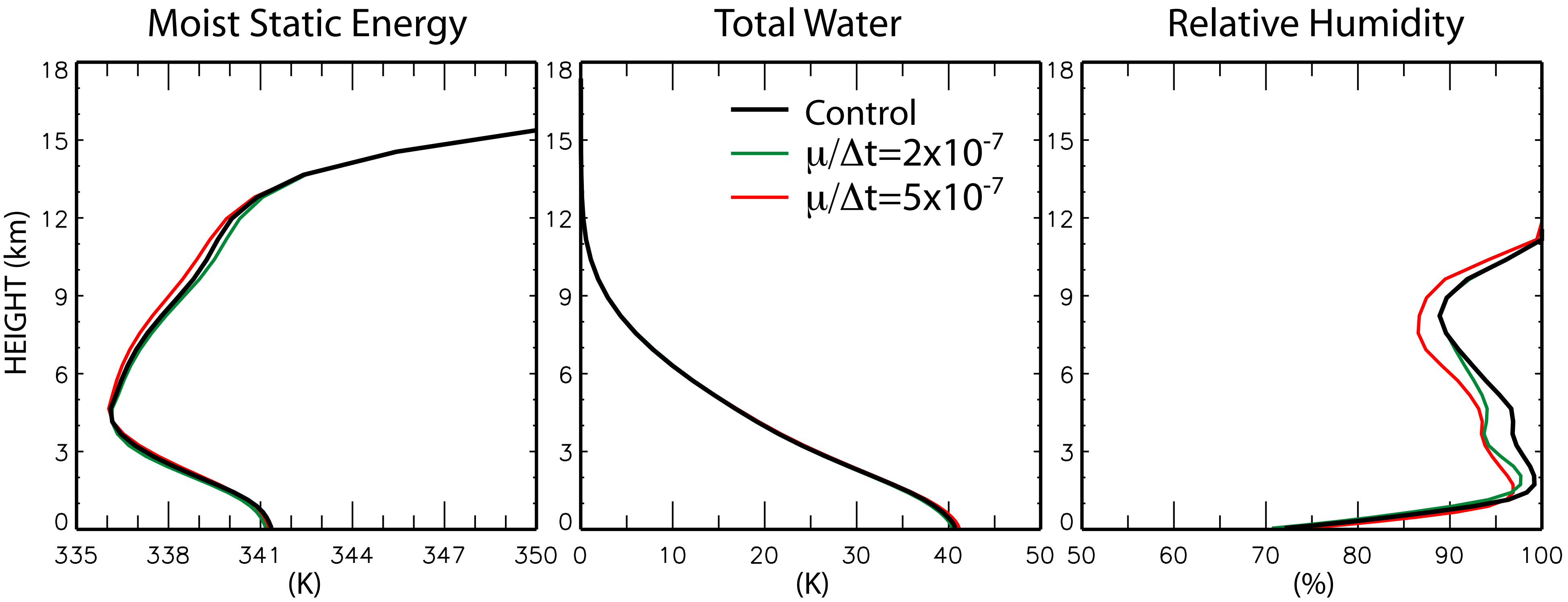
$$\frac{\mu}{\Delta t} \sim \frac{2}{\Delta x^2}, \frac{2}{\Delta y^2} = \quad 5 \times 10^{-7}$$

$$\frac{\mu}{\Delta t} \sim \frac{1}{\Delta x^2}, \frac{1}{\Delta y^2} = 2.5 \times 10^{-7}$$

Relaxaion Method vs. Direct Method

Domain and Time Averaged Variables

(last 12 hour-average)



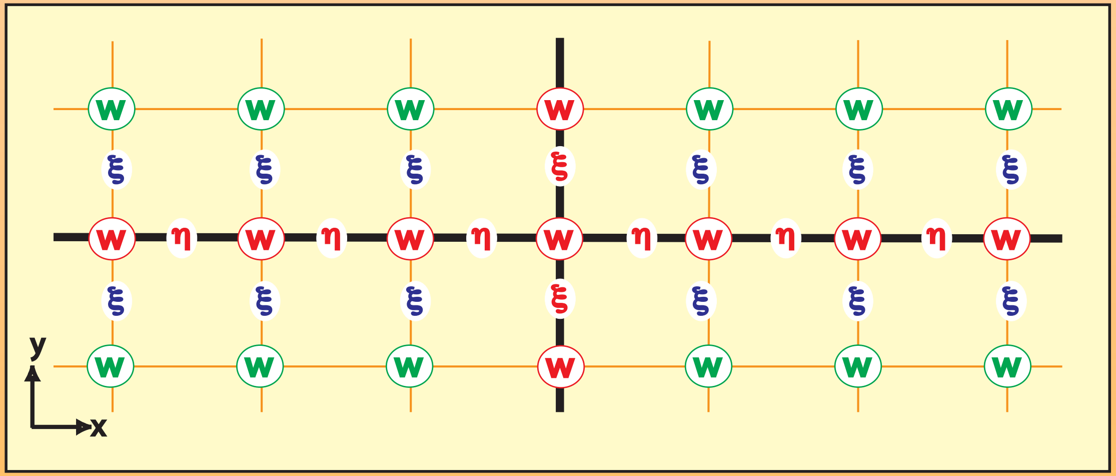
The test results show that the error due to the relaxation is comparable to the error due to the non-deterministic nature of the simulation.

A Relaxation Method for Solving the Elliptic Equation in the Q3D Model

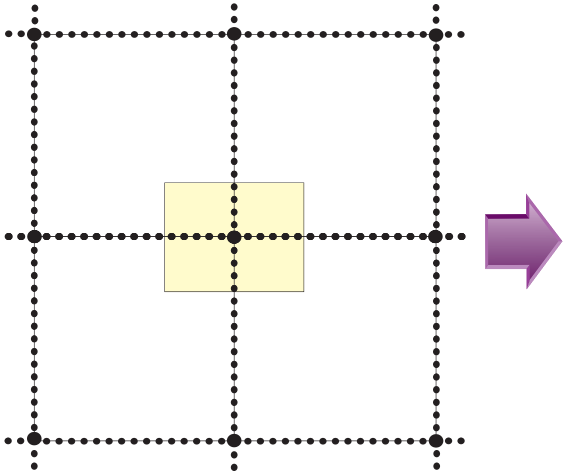
$$\mu \frac{\partial w}{\partial t} = \left(\frac{\partial^2}{\partial x^2} + \underline{\frac{\partial^2}{\partial y^2}} \right) w + \frac{\partial}{\partial z} \left[\frac{1}{\rho_0} \frac{\partial}{\partial z} \left(\rho_0 w \right) \right] + \rho_0 \frac{\partial \eta}{\partial x} - \rho_0 \underline{\frac{\partial \xi}{\partial y}}$$

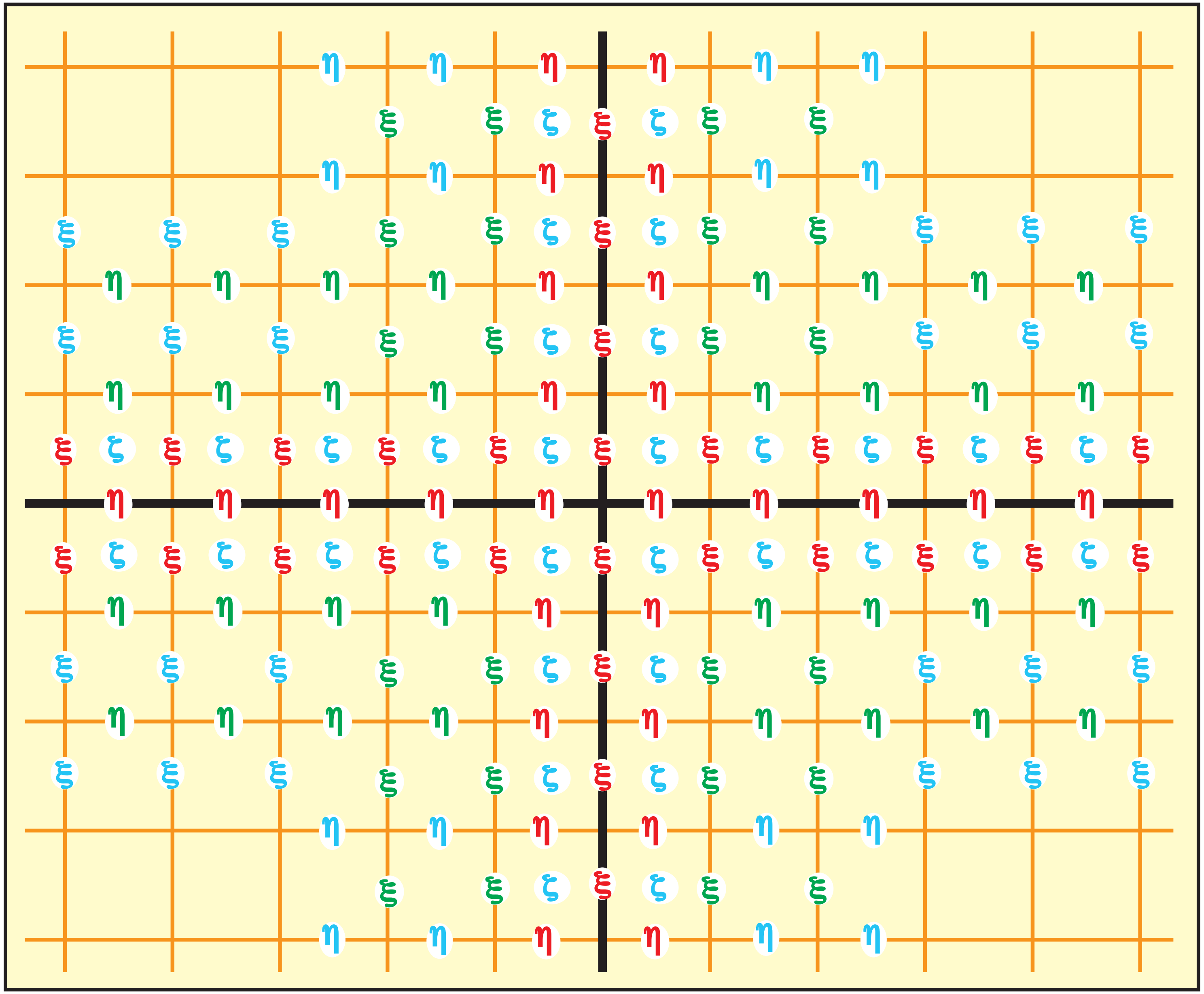
$\frac{\partial^2 \overline{w}}{\partial y^2}$: prescribed $\frac{\partial^2 w'}{\partial y^2}$: estimated $w'' = 0$: assumed at ghost points

$\frac{\partial \overline{\xi}}{\partial y}$: prescribed $\frac{\partial \xi'}{\partial y}$: estimated $\frac{\partial \xi''}{\partial y} = -\frac{\partial \eta''}{\partial x}$: assumed



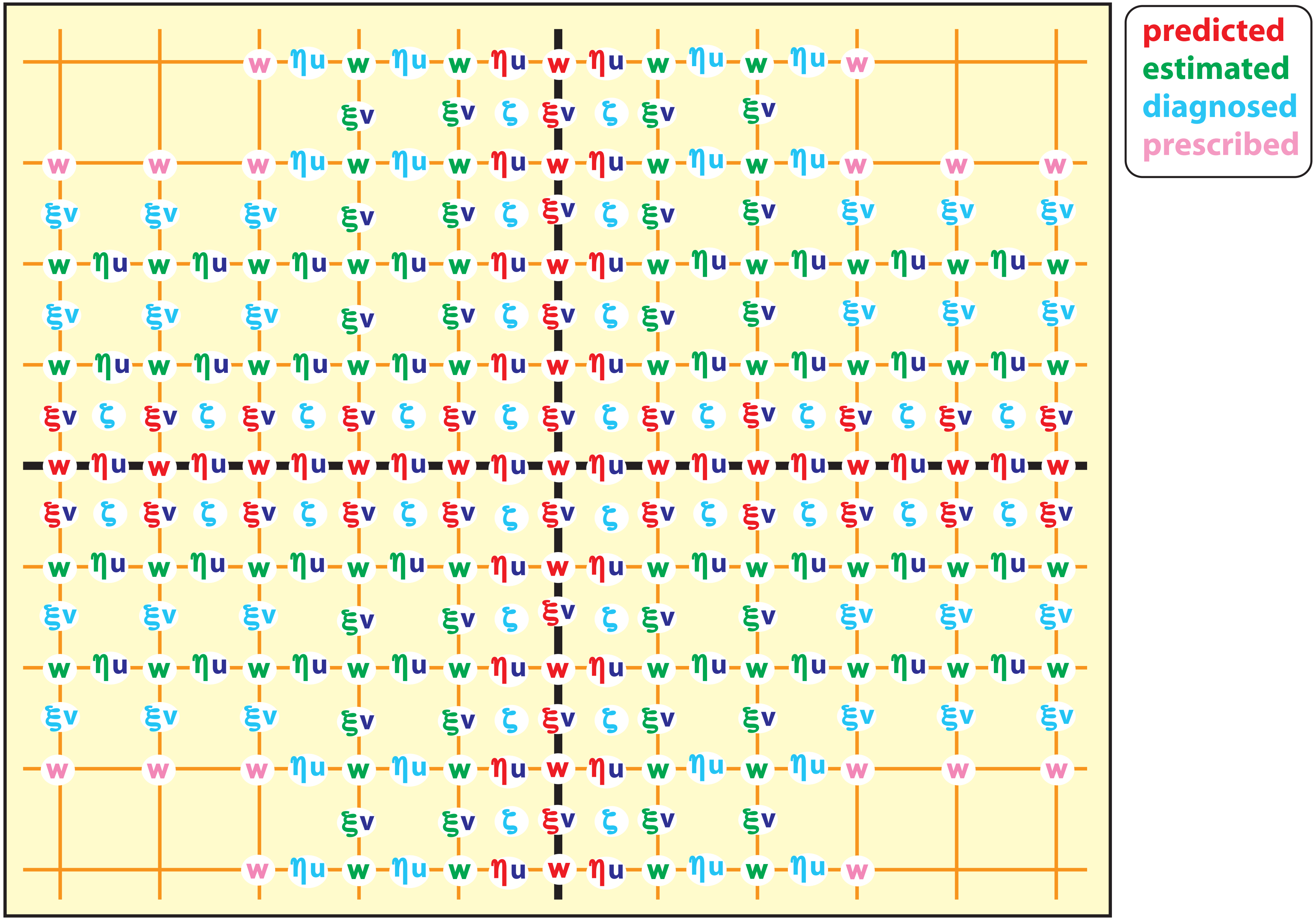
Vorticity Components in Q3D Dynamics





predicted variables
estimated variables
diagnosed variables

Variables Needed in Q3D Dynamics



The velocity components are determined such a way that
the continuity equation is satisfied and
they are consistent with the predicted vorticity on the network.

The main structure of quasi-3D cloud-resolving model has been developed.

But, there are still many things to be done……

- We plan to replace the ad hoc formulations on the individual cloud-scale vorticity by less arbitrary formulations.
- We plan to improve the quasi-3D vorticity dynamics:
  problems with the stretching and twisting terms
  in the vorticity equation.