A Nonhydrostatic Hybrid Vertical Coordinate Dynamical Core

Michael D. Toy and David A. Randall -- *Department of Atmospheric Science, Colorado State University*

Introduction

We have developed a nonhydrostatic atmospheric dynamical core which takes advantage of an isentropic vertical coordinate in the free atmosphere and a terrain-following, height-based coordinate near the surface. Use of such a hybrid vertical coordinate has previously been developed for hydrostatic models. Our goal here is to apply similar methods to a high-resolution cloud resolving model in which nonhydrostatic effects are important.

We note here that other work in nonhydrostatic modeling with a hybrid vertical coordinate has been performed using a regridding method for handling the vertical coordinate, e.g., He (2002). In our model we follow the approach of Konor and Arakawa (1997) to define the vertical coordinate.

The vertical finite difference scheme is based on the fully elastic system of Euler equations. It conserves mass and total energy is conserved with a second-order, centered advection scheme. An integral constraint on the circulation developed around a contour of surface topography is also satisfied. These features are important as we intend to use the dynamical core for long term global climate simulations.

Vertical coordinate

Following Konor and Arakawa (1997) we define the vertical coordinate as a prescribed function of height and potential temperature. Since our model is nonhydrostatic we use geometric height instead of pressure as the height metric.

The vertical coordinate (η) is defined as:

$$\eta = F(\theta, \sigma) = f(\sigma) + g(\sigma)\,\theta, \qquad (1)$$

where $\sigma = \dfrac{z - z_S}{z_T - z_S}$, z_S = surface height, and z_T = model top height.

The transition of η from terrain-following to potential temperature coordinates requires:

$$g(\sigma) \to 0 \quad \text{as } \sigma \to 0,$$

$$f(\sigma) \to 0 \text{ and } g(\sigma) \to 1 \quad \text{as } \sigma \to 1.$$

First, we choose the form of $g(\sigma)$ as shown below, where r is a constant which controls how quickly the transition to θ-coordinates occurs:

$$g(\sigma) = 1 - (1 - \sigma)^r.$$

In order to guarantee that the vertical coordinate be monotonic, $f(\sigma)$ is determined from the following equation where $\theta_{\min}$ and $\left(\frac{\partial\theta}{\partial\sigma}\right)_{\min}$ are suitably chosen minimum values of potential temperature and static stability respectively:

$$\frac{df}{d\sigma} + \frac{dg}{d\sigma}\theta_{\min} + g\left(\frac{\partial\theta}{\partial\sigma}\right)_{\min} = 0.$$

Vertical grid

We use the Charney-Phillips grid in which potential temperature and mass are staggered. The staggering of the prognostic variables is shown in Figure 1.

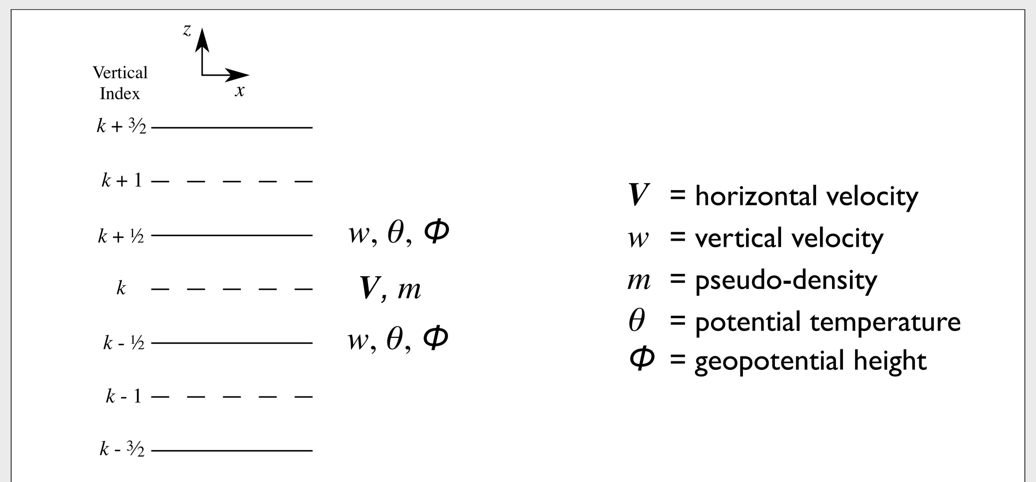

Figure 1: Vertical staggering of the prognostic variables.

Vertical mass flux diagnosis

With the hybrid coordinate, both potential temperature and geopotential must be predicted in such a way as to maintain the relationship given by Equation 1 on coordinate surfaces. As in Konor and Arakawa (1997) we achieve this by diagnosing the vertical velocity $\dot{\eta} = D\eta/Dt$ from the vertical advection terms of the θ and Φ tendency equations shown below:

$$\frac{\partial\theta_{k+1/2}}{\partial t} + (\boldsymbol{V}\cdot\nabla\theta)_{k+1/2} + \left(\dot{\eta}\,\frac{\partial\theta}{\partial\eta}\right)_{k+1/2} = \left(\frac{Q}{\Pi}\right)_{k+1/2}, \qquad (2)$$

$$\frac{\partial\phi_{k+1/2}}{\partial t} + (\boldsymbol{V}\cdot\nabla\phi)_{k+1/2} + \left(\dot{\eta}\,\frac{\partial\phi}{\partial\eta}\right)_{k+1/2} = g\,(m\,w)_{k+1/2}. \qquad (3)$$

Mountain wave simulation

We tested the hybrid-coordinate model for the case of the 11 January 1972 Boulder, Colorado windstorm. The setup for this 2D experiment was obtained from Doyle et al. (2000) which presents an intercomparison of various nonhydrostatic model simulations of the windstorm. The initial atmospheric condition is horizontally uniform and is based on the upstream Grand Junction, Colorado sounding shown in Figure 2. The Colorado Front Range profile is represented by a "witch of Agnesi" curve of height 2 km and half-width 10 km. The horizontal grid spacing is 1 km and the horizontal domain is 220 km wide with periodic boundary conditions. The model top is a rigid lid at a height of 25 km.

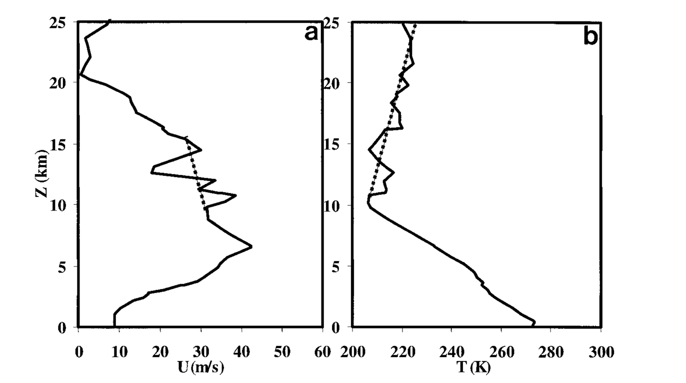

Figure 2: Initial zonal wind and temperature of sounding.

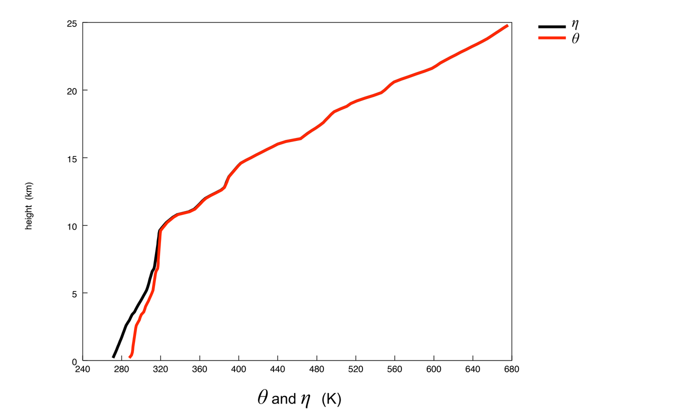

Figure 3: Vertical profile of vertical coordinate and potential temperature at x = 0, t = 0.

We achieved a 1-hour hybrid coordinate run with 125 levels. Figure 3 shows the initial vertical profile of the vertical coordinate and potential temperature. Note that we used $\theta_{\min}$ = 270 K and $\left(\frac{\partial\theta}{\partial\sigma}\right)_{\min}$ = 0. Above 10 km the coordinate is basically isentropic. Figure 4a shows the initial position of the coordinate surfaces which are equally spaced in z away from the mountain. In Figure 4b the position of the coordinate surfaces at t = 1 hour is shown.

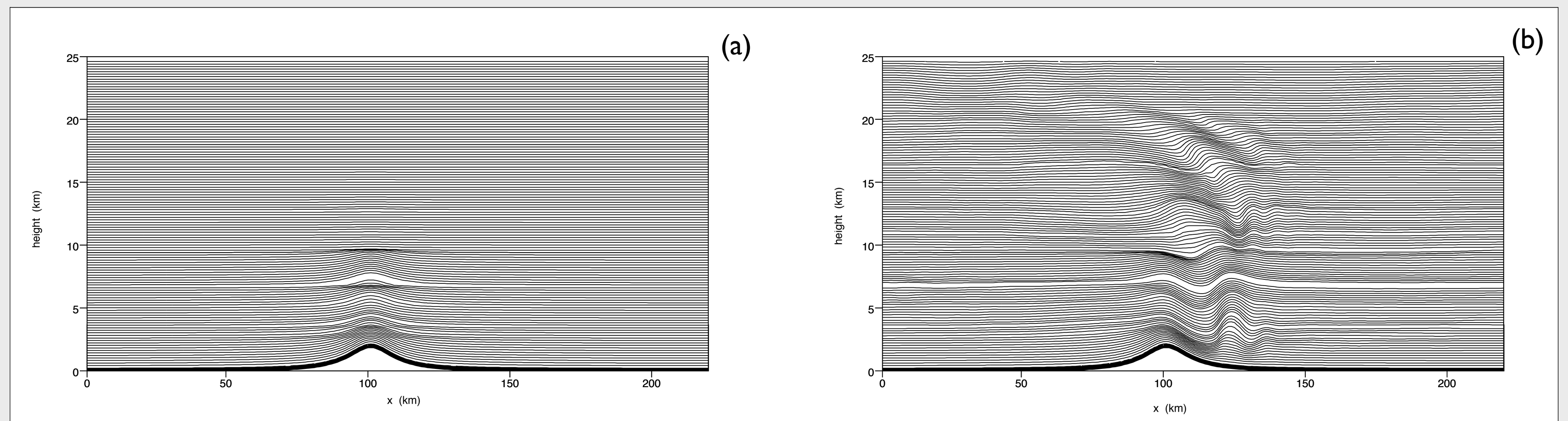

Figure 4: Profile of hybrid vertical coordinate: a) at t = 0, and b) at t = 1 hour.

Figure 5 shows the potential temperature field after one hour for two different runs: a) with a terrain-following, height-based coordinate, and b) with the hybrid coordinate. The simulations produced similar results with a "hydraulic jump" appearing in the lower troposphere downstream of the mountain, and wave development with an upstream vertical tilt in the stratosphere. These features were observed by NCAR research aircraft during the windstorm.

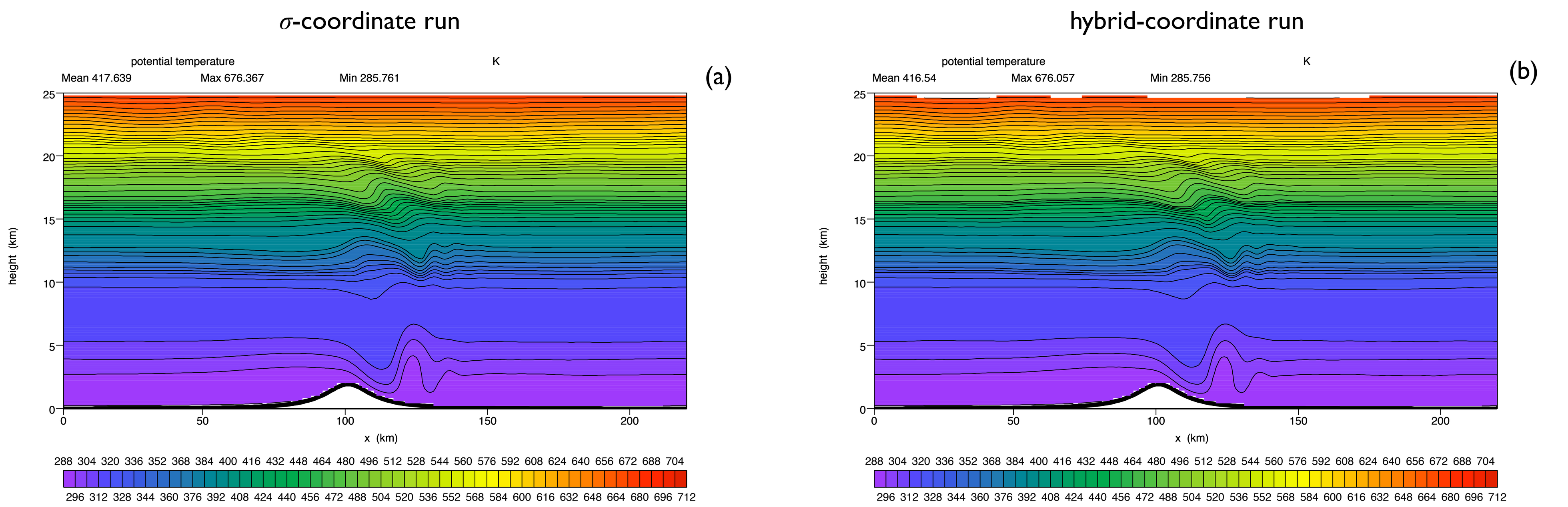

Figure 5: Potential temperature field at t = 1 hour: a) Terrain-following height-based coordinate throughout domain, and b) hybrid coordinate.

Challenges and Goals

One of the main challenges with isentropic coordinates is handling massless layers and preventing coordinate surfaces from crossing each other. We have experienced these issues in our model and are working to solve them. So far we have made progress by experimenting with an upstream advection scheme in the continuity equation. Our goal is to obtain longer runs with the hybrid coordinate, as we have done with the model run in pure σ-coordinate mode as shown in Figure 6.

We have had success with a previous σ-coordinate version of our nonhydrostatic model in simulating phenomena at various scales -- from a sheared rising thermal with a capping stable layer to the development of a synoptic scale, baroclinic disturbance on a β-plane. These results are shown in Figure 7.

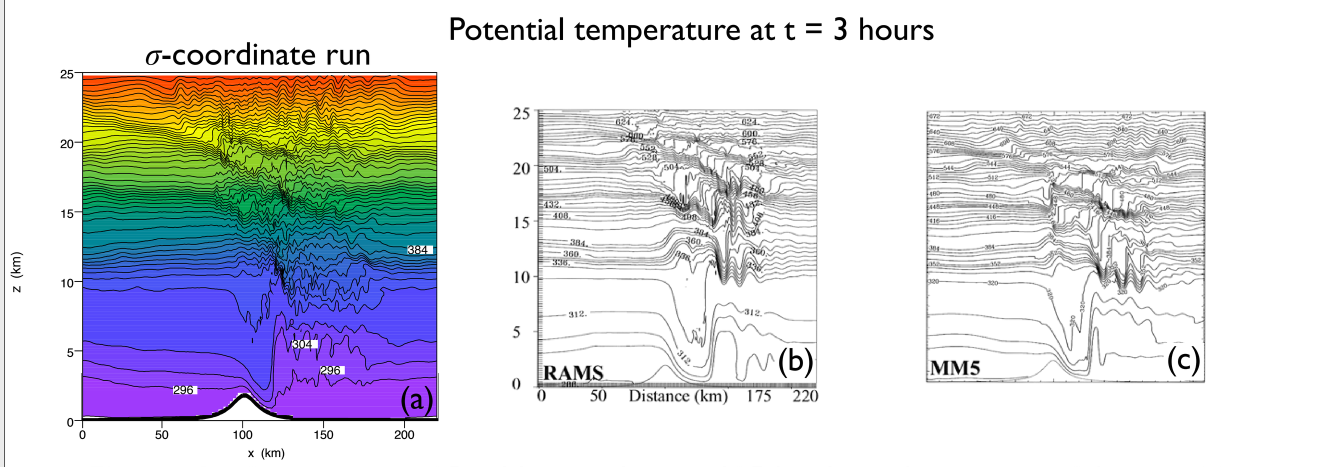

Figure 6: Comparison of Boulder windstorm 125 level simulation at t = 3 hours: (a) our model using pure σ-coordinates, (b) RAMS, and (c) MM5.
Note: (b) and (c) reproduced from Doyle et al. (2000).

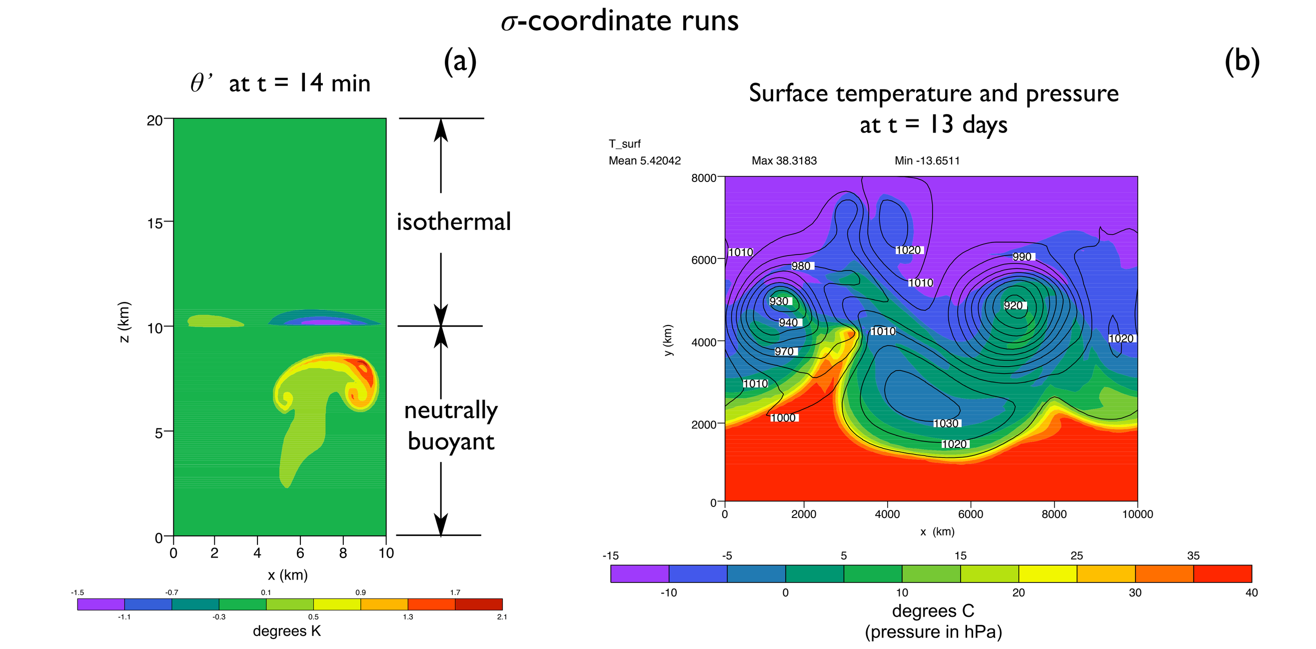

Figure 7: Snapshots of simulations with a previous σ-coordinate version of the model: a) a rising thermal in a sheared, neutrally buoyant environment capped by a stable layer, and b) an amplifying baroclinic disturbance on a β-plane.

When we successfully simulate these experiments using the hybrid-coordinate we will then introduce moist processes. It is intended to ultimately use the dynamical core in a future global cloud resolving climate model.

References

Doyle, J.D., D. R. Durran, C. Chen, B. A. Colle, M. Georgelin, V. Grubisic, W. R. Hsu, C. Y. Huang, D. Landau, Y. L. Lin, G. S. Poulos, W. Y. Sun, D. B. Weber, M. G. Wurtele, and M. Xue, 2000: An intercomparison of model-predicted wave breaking for the 11 January 1972 Boulder windstorm. *Mon. Wea. Rev.*, **128**, 901-914.

He, Z., 2002: A nonhydrostatic model with a generalized vertical coordinate. Ph.D. dissertation, University of Miami, 99 pp.

Konor, C.S., and A. Arakawa, 1997: Design of an atmospheric model based on a generalized vertical coordinate. *Mon. Wea. Rev.*, **125**, 1649-1673.

Acknowledgments

This research was supported by the Office of Science (BER), U. S. Department of Energy, Grant No. DE-FG02-02ER63370.

Colorado State University

Office of Science
U.S. DEPARTMENT OF ENERGY