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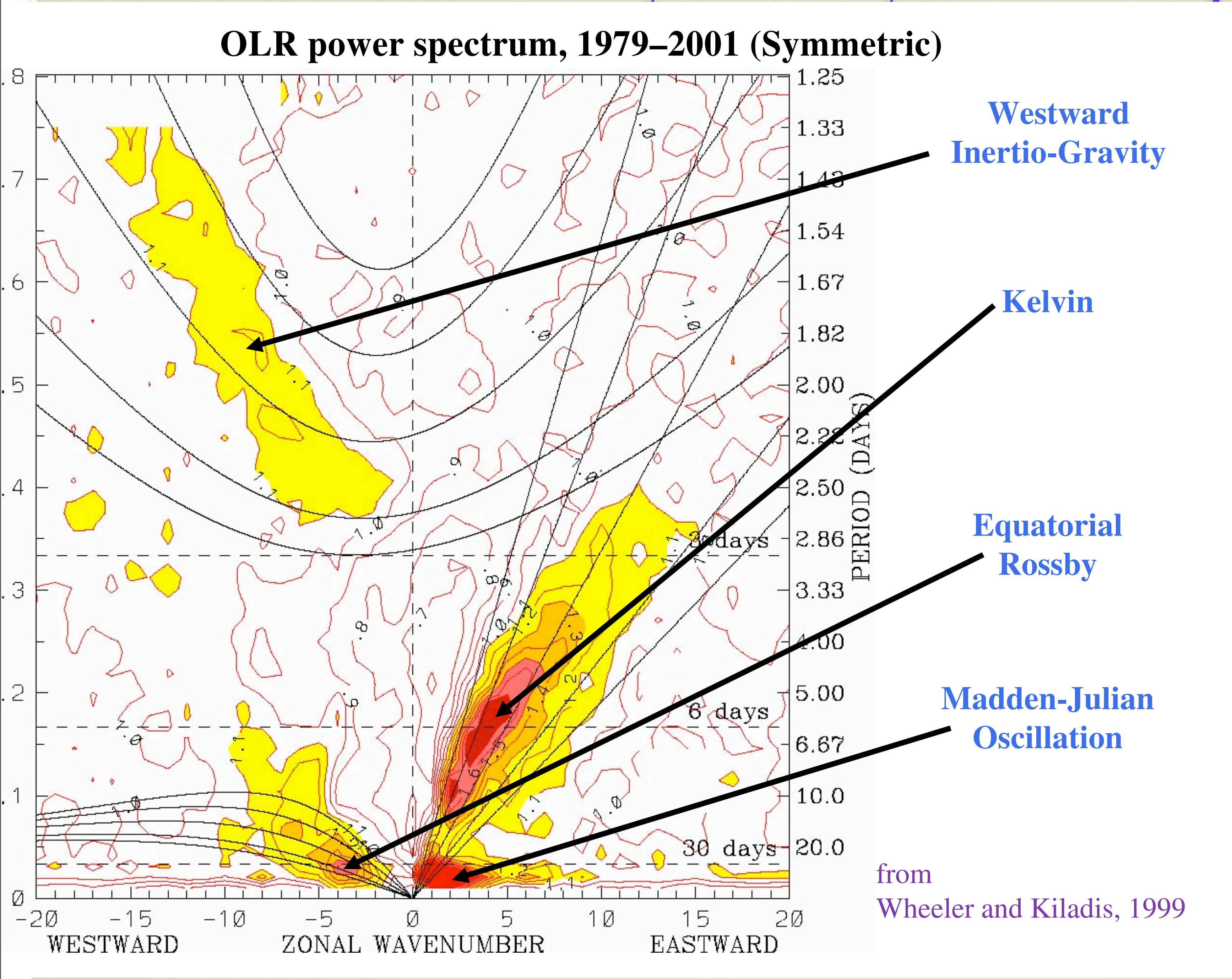
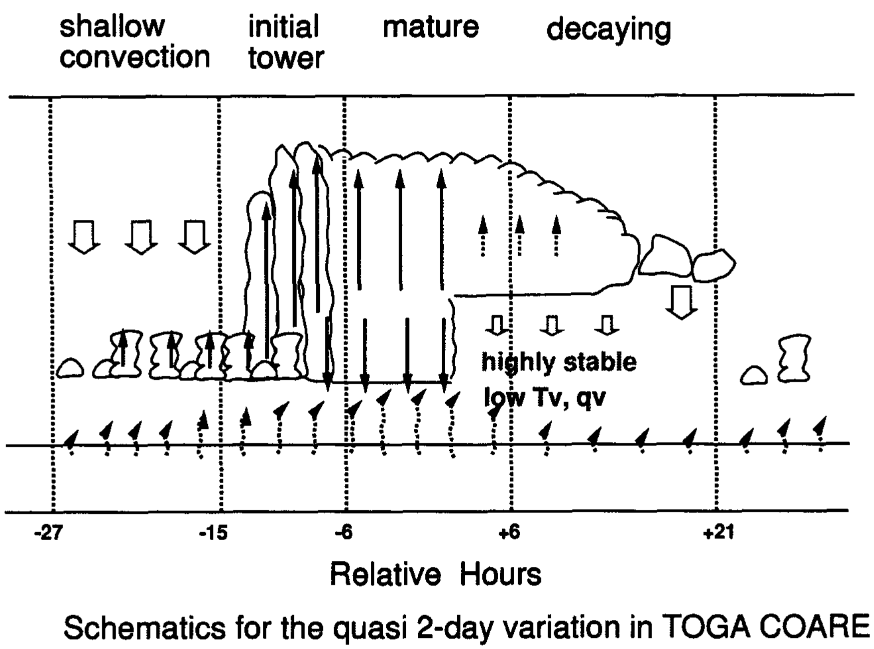
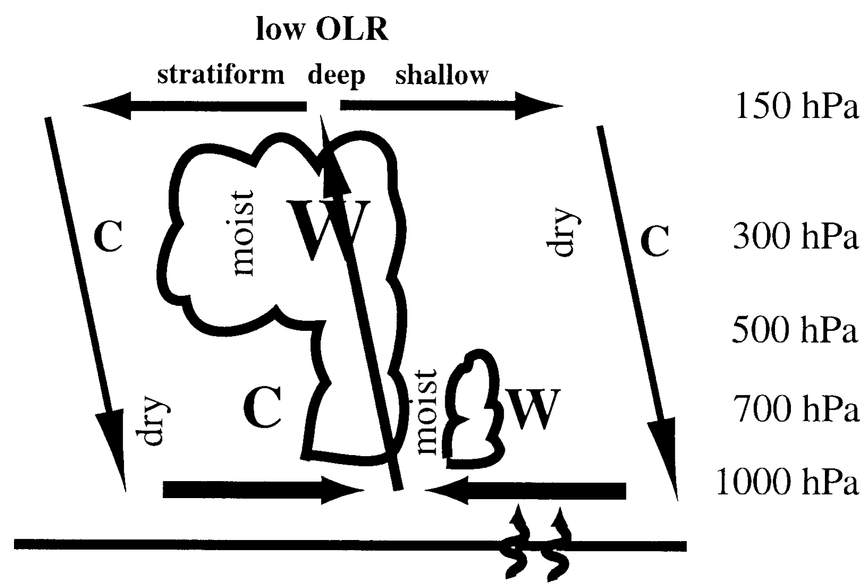
Linear Stability Analysis Compared to Obs.



Observations of Convectively Coupled Waves (CCWs)



Takayabu et al. (1996)

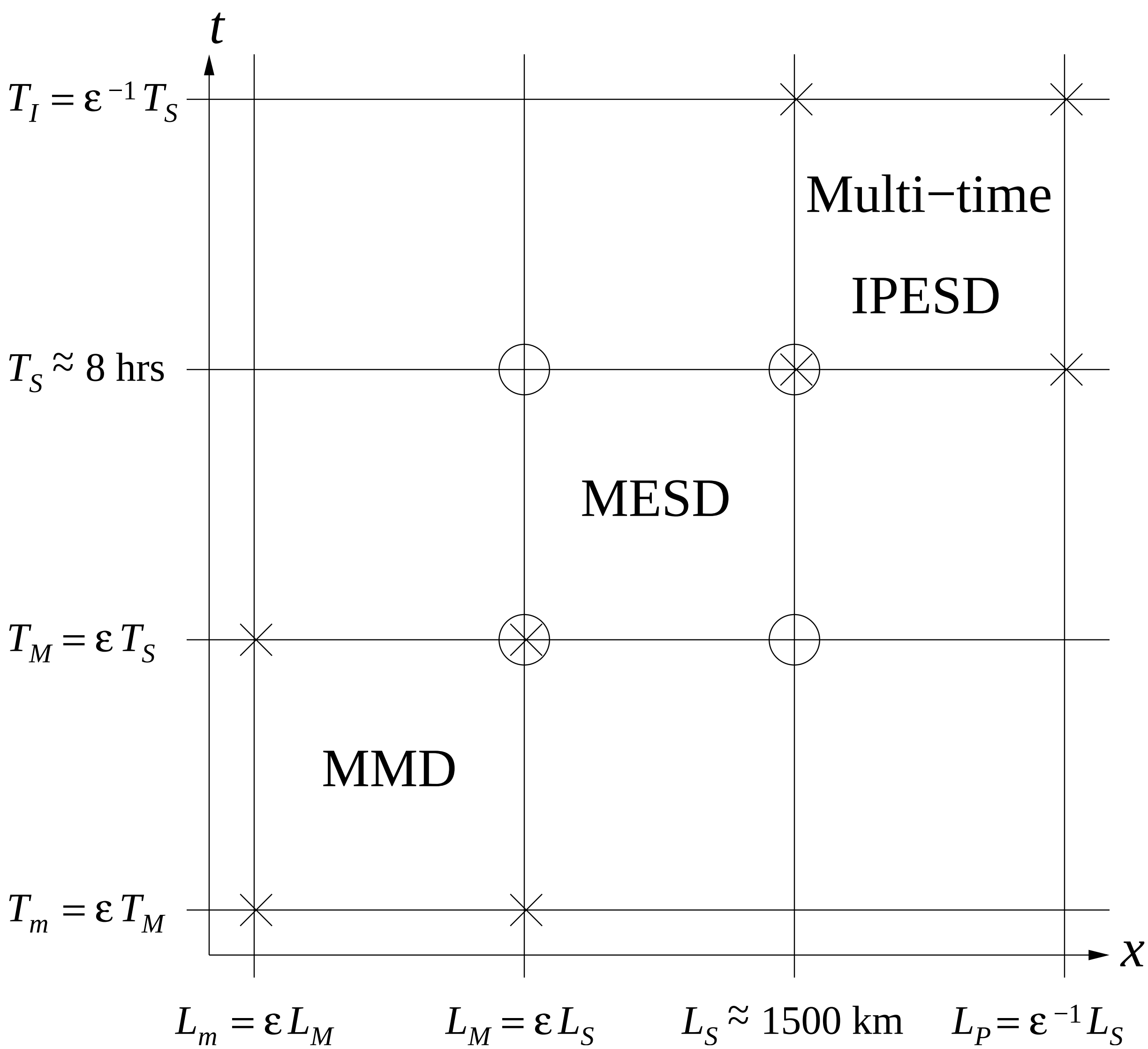


Straub and Kiladis (2003)

Progression from congestus to deep convective to stratiform clouds

Hierarchical Multi-Scale Models
for Theory, Observations, and Numerical Strategies

Majda & Klein (2003); Klein & Majda (2006); Majda (2006a,b)



IPESD – Intraseasonal Planetary Equatorial Synoptic Scale Dynamics

MESD – Mesoscale Equatorial Synoptic Dynamics

MMD – Microscale Mesoscale Dynamics

A Simple Dynamical Model with Features of Convective Momentum Transport

Andrew Majda (NYU) and Sam Stechmann (UCLA)

(paper in press, J. Atmos. Sci.)

AGU Fall Meeting

San Francisco, California

December 19, 2008

2 important multi-scale effects

$$\frac{\partial u}{\partial t} + u\partial_x u + w\partial_z u = \cdots$$

$$u = \bar{u} + u'$$

1. Eddy momentum flux

   "Convective momentum transport" (CMT)

$$\frac{\partial \bar{u}}{\partial t} = -\partial_z \overline{w'u'} + \cdots$$

2. Background wind shear

$$\frac{\partial u'}{\partial t} + \bar{u}\partial_x u' + w'\partial_z \bar{u} = \cdots$$

Multi-scale organized convection

Key questions:

1. What are the physical mechanisms of the MJO?

   - MJO $\longrightarrow$ CCW?
   - MJO $\longleftarrow$ CCW?
   - MJO $\longleftrightarrow$ CCW?

2. What is the missing physics of the MJO in GCMs?

   Proper representation of

   - CCW?
   - *interactions* between CCW and the larger-scale environment?

Effect of CCW on MJO envelope:

Convective Momentum Transport (CMT)

$$\frac{\partial \bar{u}}{\partial t} = -\partial_z \overline{w'u'} + \cdots$$

Mesoscales and smaller:

CMT from squall lines and other mesoscale convection

- Moncrieff (1981), LeMone (1983), Moncrieff (1992), Wu and Yanai (1994), Tung and Yanai (2002), Moncrieff (2004)

Synoptic scales:

CMT from convectively coupled waves (CCW)

- Can change velocity on the planetary scales (and MJO)
- Majda and Biello (2004), Biello and Majda (2005)

Diagnostic multi-scale model

including CMT due to CCW

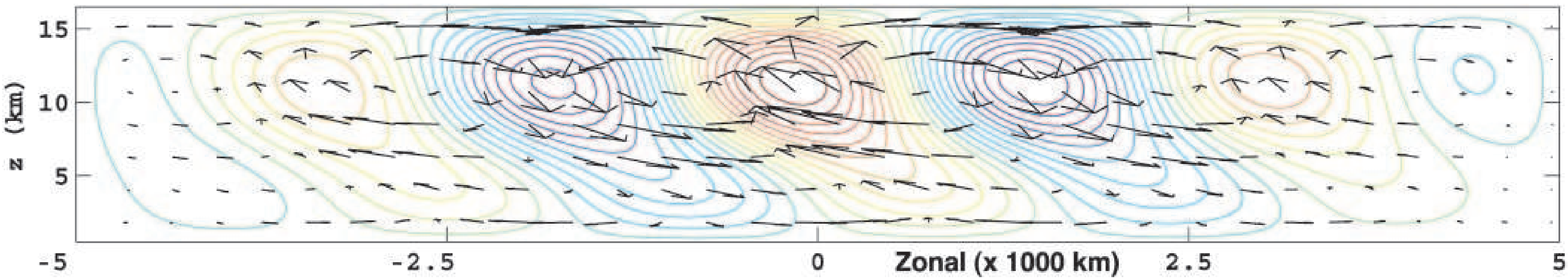
Majda and Biello (2004), Biello and Majda (2005)

- Diagnostic model for CCW:

$$w' = S'_\theta(X, x, z, t), \text{ etc.}$$

- CMT from CCW drives mean flow:

$$\partial_t \bar{U} = -\partial_z(\overline{w'u'}) + \cdots, \text{ etc.}$$

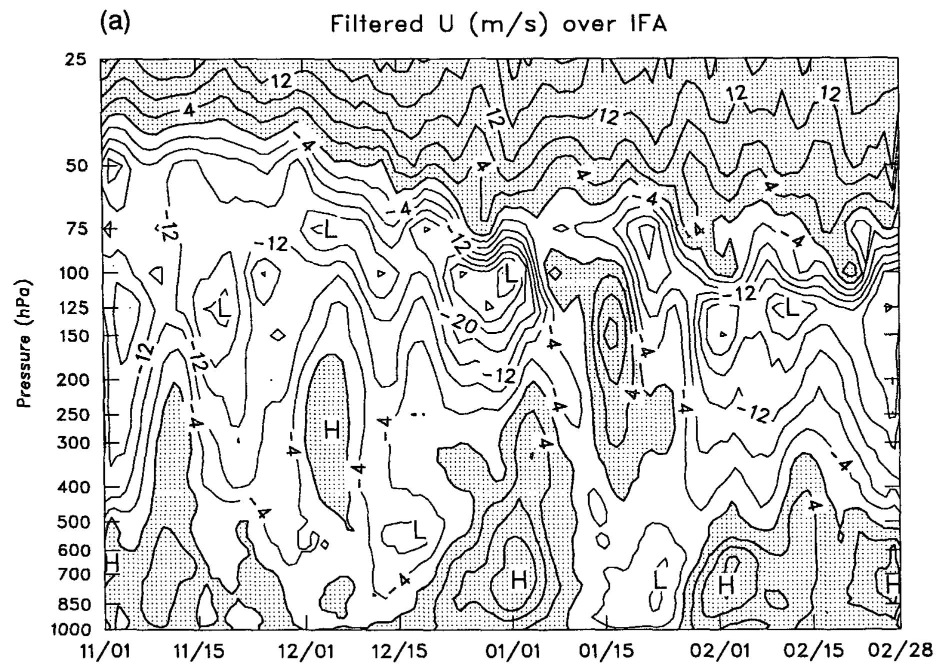


Diagnostic multi-scale model

including CMT due to CCW

Majda and Biello (2004),
Biello and Majda (2005):

CMT from CCW drives
the westerly wind burst aloft



Lin and Johnson (1996)

Majda and Stechmann (2009):

also include effect of mean flow $\bar{U}$ on CCW to give *dynamic* multi-scale model with two-way interactions between CCW and mean flow

Dynamic model for convectively coupled wave–mean flow interaction

$$\frac{\partial \bar{U}}{\partial T} + \frac{\partial}{\partial z}\langle\overline{w'u'}\rangle = 0$$

$$\frac{\partial u'}{\partial t} + \bar{U}\frac{\partial u'}{\partial x} + w'\frac{\partial \bar{U}}{\partial z} + \frac{\partial p'}{\partial x} = S'_{u,1}$$

(with similar equations for other variables)

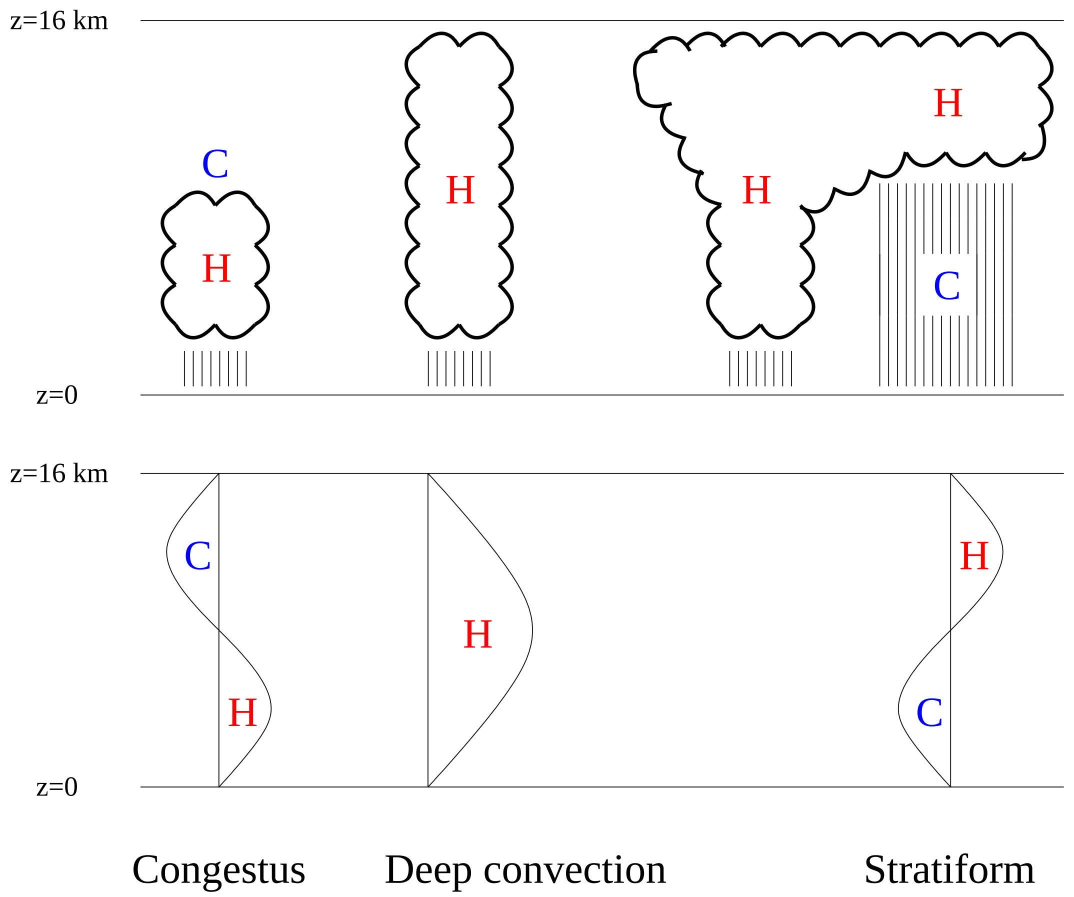
Key features of the model:

- Eddy flux convergence of wave momentum, $\partial_z\langle\overline{w'u'}\rangle$, feeds the mean flow $\bar{U}$
- Advection of the waves u' by the mean flow $\bar{U}$
- Mean flow time scale $T = \epsilon^2 t$ is longer than that for the waves

Multiscale asymptotic derivation of model

Need convectively coupled waves with *tilts* to have nonzero $\partial_z\langle\overline{w'u'}\rangle$

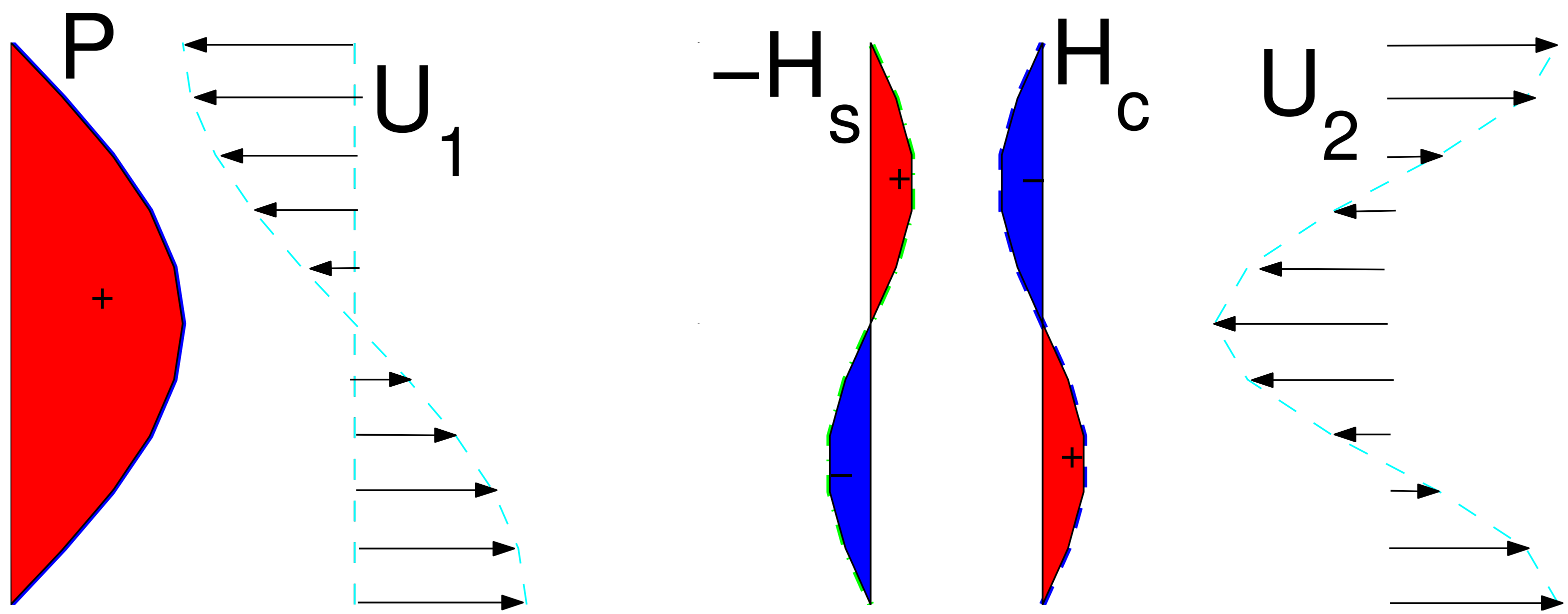
The Multicloud Model (Khouider and Majda 2006)
(a model for CCW)



Two vertical baroclinic modes ⇒ waves with vertical tilts

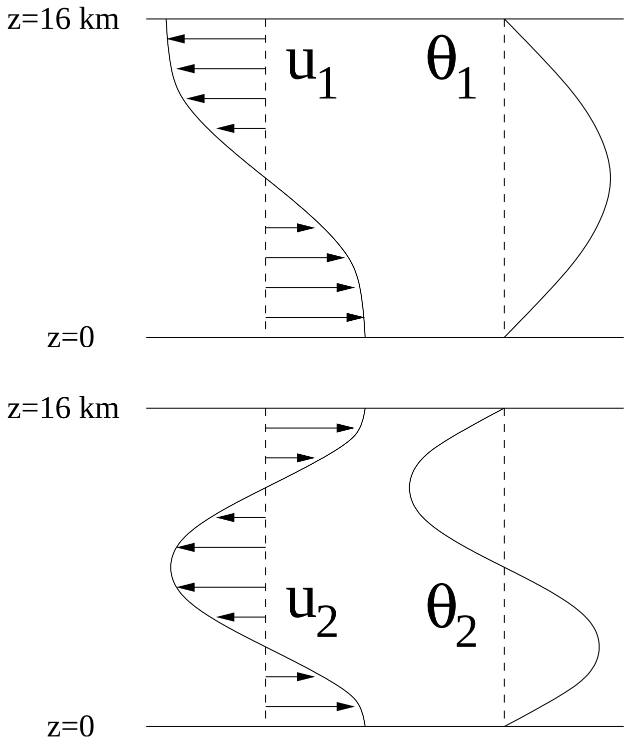
Multi-scale effects: add nonlinear advection and a 3rd baroclinic mode

Three Cloud Type Models



Equations of the multicloud model

Two linear shallow water systems, coupled through nonlinear source terms:



$$\begin{cases} \dfrac{\partial u_1}{\partial t} - \dfrac{\partial \theta_1}{\partial x} = -\dfrac{1}{\tau_u} u_1 \\ \dfrac{\partial \theta_1}{\partial t} - \dfrac{\partial u_1}{\partial x} = H_d - R_1 \end{cases}$$

$$\begin{cases} \dfrac{\partial u_2}{\partial t} - \dfrac{\partial \theta_2}{\partial x} = -\dfrac{1}{\tau_u} u_2 \\ \dfrac{\partial \theta_2}{\partial t} - \dfrac{1}{4}\dfrac{\partial u_2}{\partial x} = H_c - H_s - R_2 \end{cases}$$

$H_d =$ Deep convective heating

$H_c =$ Congestus heating

$R =$ Radiative cooling

$H_s =$ Stratiform heating

+ 4 more prognostic equations for θ_{eb}, q, H_s, H_c

+ diagnostic equations for some source terms

Dynamic model for convectively coupled wave–mean flow interaction

$$\frac{\partial \bar{U}}{\partial T} + \frac{\partial}{\partial z}\langle\overline{w'u'}\rangle = 0$$

$$\frac{\partial u'}{\partial t} + \bar{U}\frac{\partial u'}{\partial x} + w'\frac{\partial \bar{U}}{\partial z} + \frac{\partial p'}{\partial x} = S'_{u,1}$$

(with similar equations for other variables)

Key features of the model:

- Eddy flux convergence of wave momentum, $\partial_z\langle\overline{w'u'}\rangle$, feeds the mean flow $\bar{U}$
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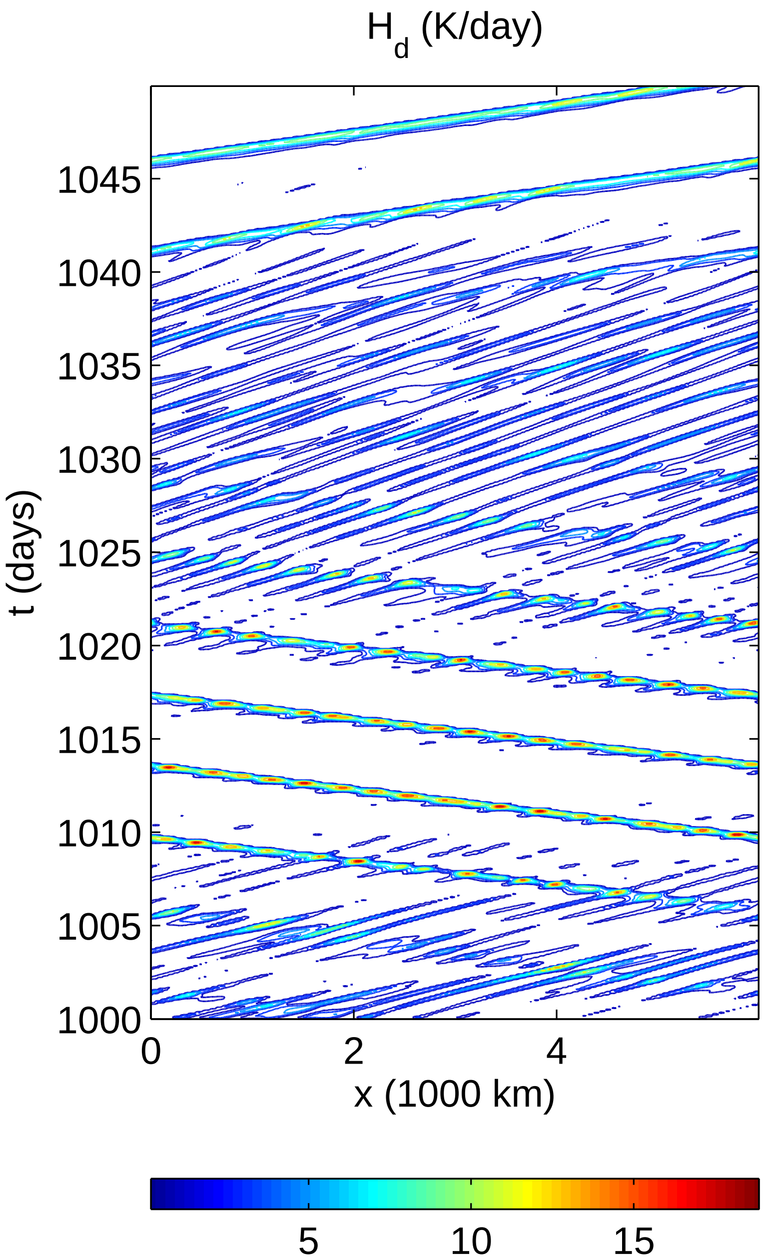
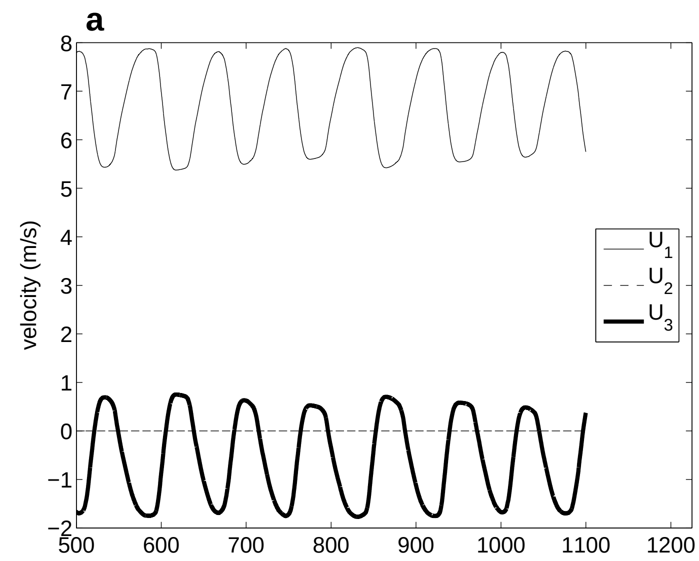
Multiscale asymptotic derivation of model

Need convectively coupled waves with *tilts* to have nonzero $\partial_z\langle\overline{w'u'}\rangle$

Irregular intraseasonal oscillation with multiscale waves



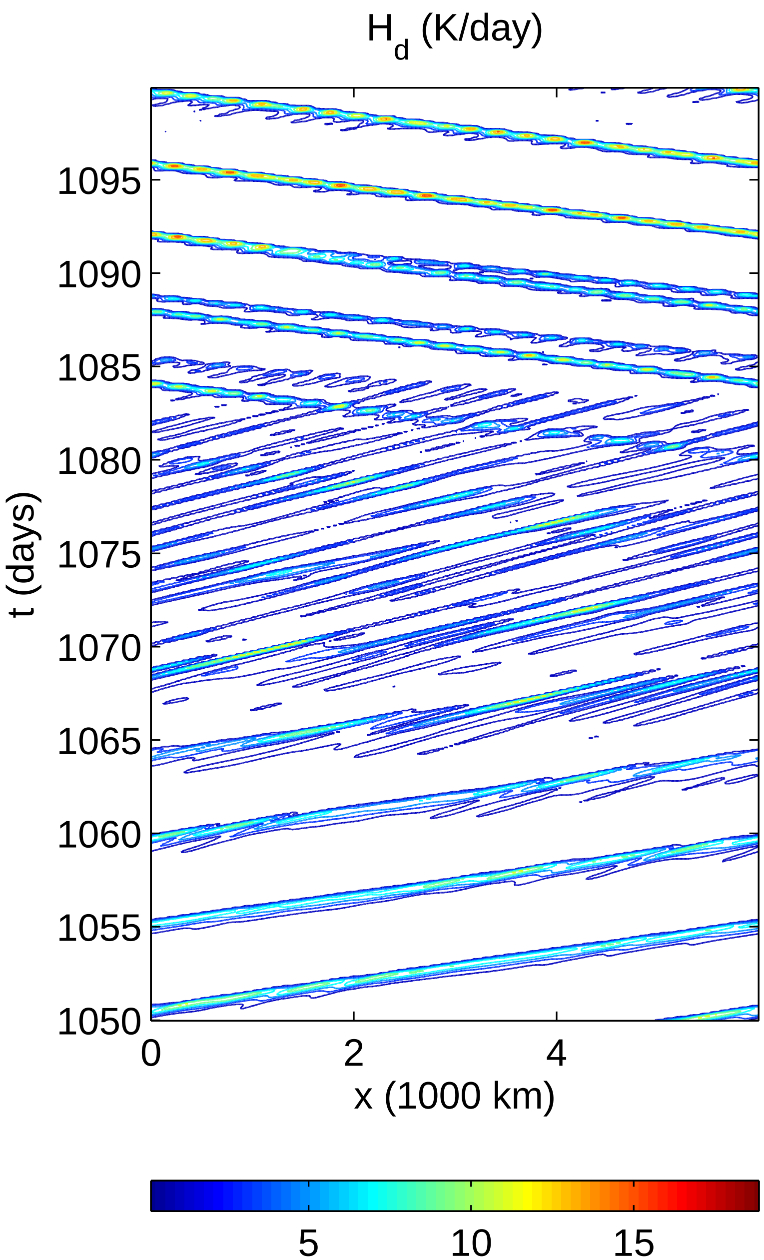
Mean wind $\bar{U}_j(t)$:



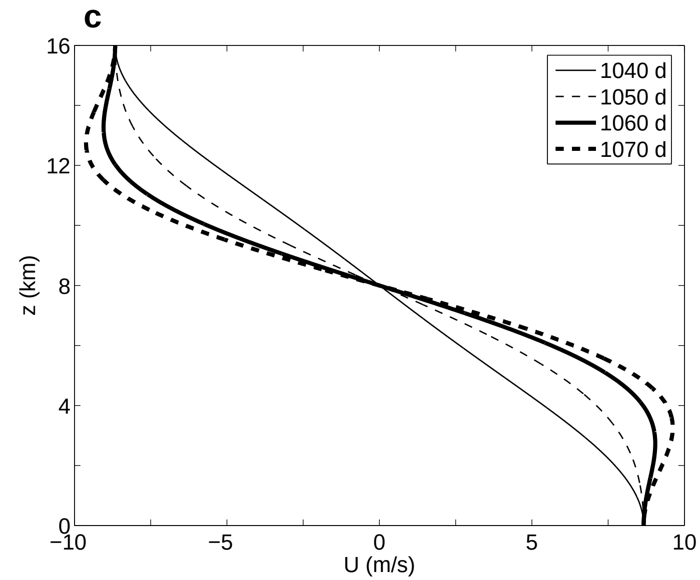
time (days)

- Climate base state with low-level westerlies
- Either coherent or multiscale waves depending on mean wind

Westerly Wind Burst Intensification



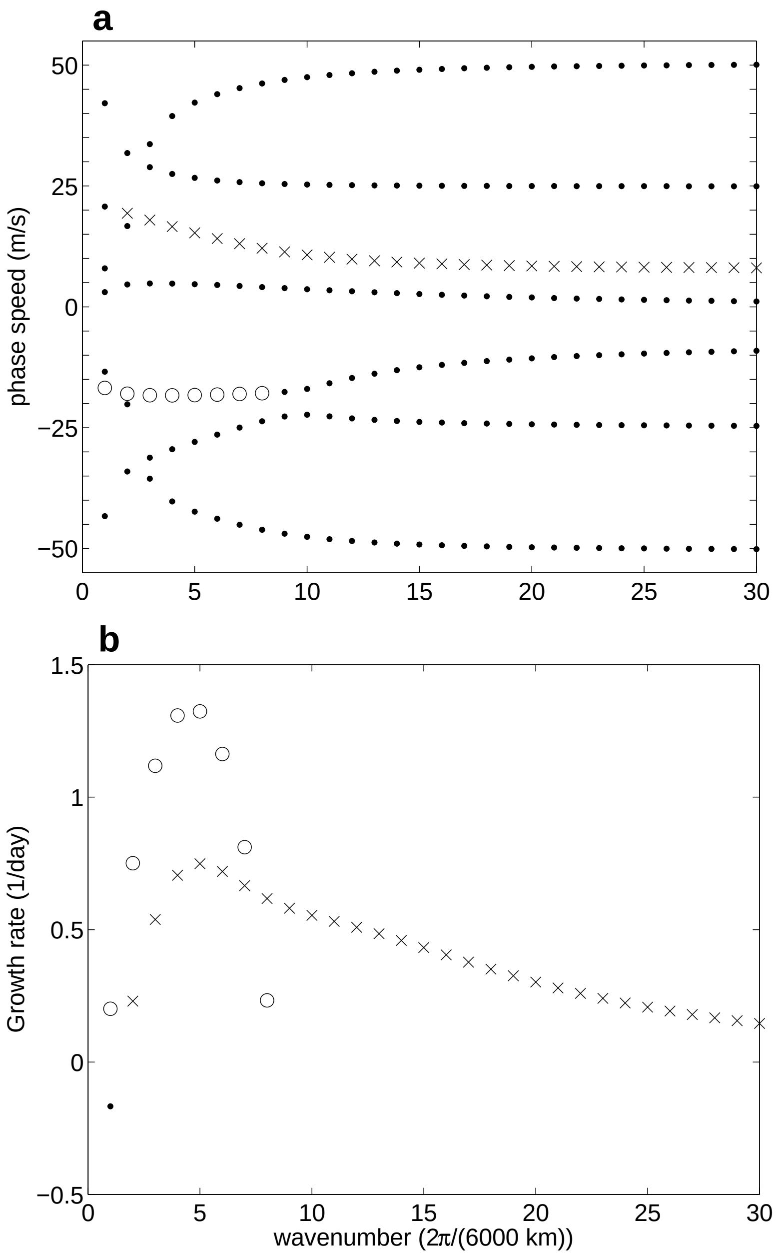
Mean wind $\bar{U}(z)$:

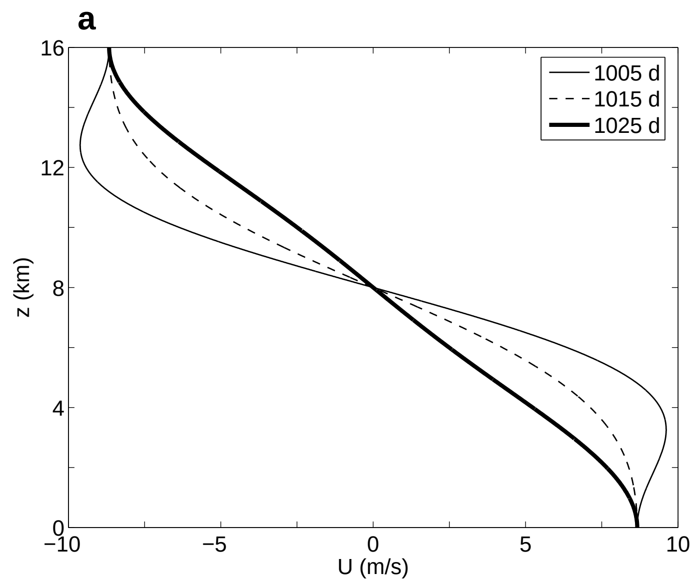


- Climate base state with low-level westerlies
- Upscale CMT from eastward-moving CCW accelerates WWB aloft

Linear Stability Theory

$t = 1005$ days





- Westward-propagating CCW favored at larger scales
- Eastward-propagating CCW favored at smaller scales

Summary

- Dynamic multi-scale model for CCW–mean flow interactions
  - Two-way interactions
  - Asymptotic derivation and intraseasonal time scale

- Intraseasonal oscillations of CCW and mean flow
  - westerly wind burst intensification as in MJO
  - either coherent or multiscale waves depending on mean wind
  - linear theory

Majda and Stechmann (2009) J. Atmos. Sci., in press

Cloud-Resolving Model (CRM) simulations of CCW:

What is the role of CMT from mesoscale convection?

Results vary depending on strength of momentum damping:

$$\frac{\partial u}{\partial t} = -\frac{1}{\tau} u + \cdots$$

- Held et al. (1993): No momentum damping: Long-time oscillation develops
  - Is this due to CMT interactions or stratospheric interactions?
- Grabowski & Moncrieff (2001): Weak momentum damping: CCW develop with significant CMT
- Tulich et al. (2007): Stronger momentum damping: CCW develop with little or no CMT
- Held et al. (1993): Intense momentum damping: Convection shut down except at a few grid points

An MJO-like Wave with the Multicloud Model

Boualem Khouider (UVic) Andy Majda (NYU) Sam Stechmann (NYU)

Exploit self-similarity idea in multicloud model:

Deep convection	$\tau_{conv} = 12$ hours
Stratiform	$\tau_s = 7$ days
Congestus	$\tau_c = 7$ days

Unstable wave at 5 m/s

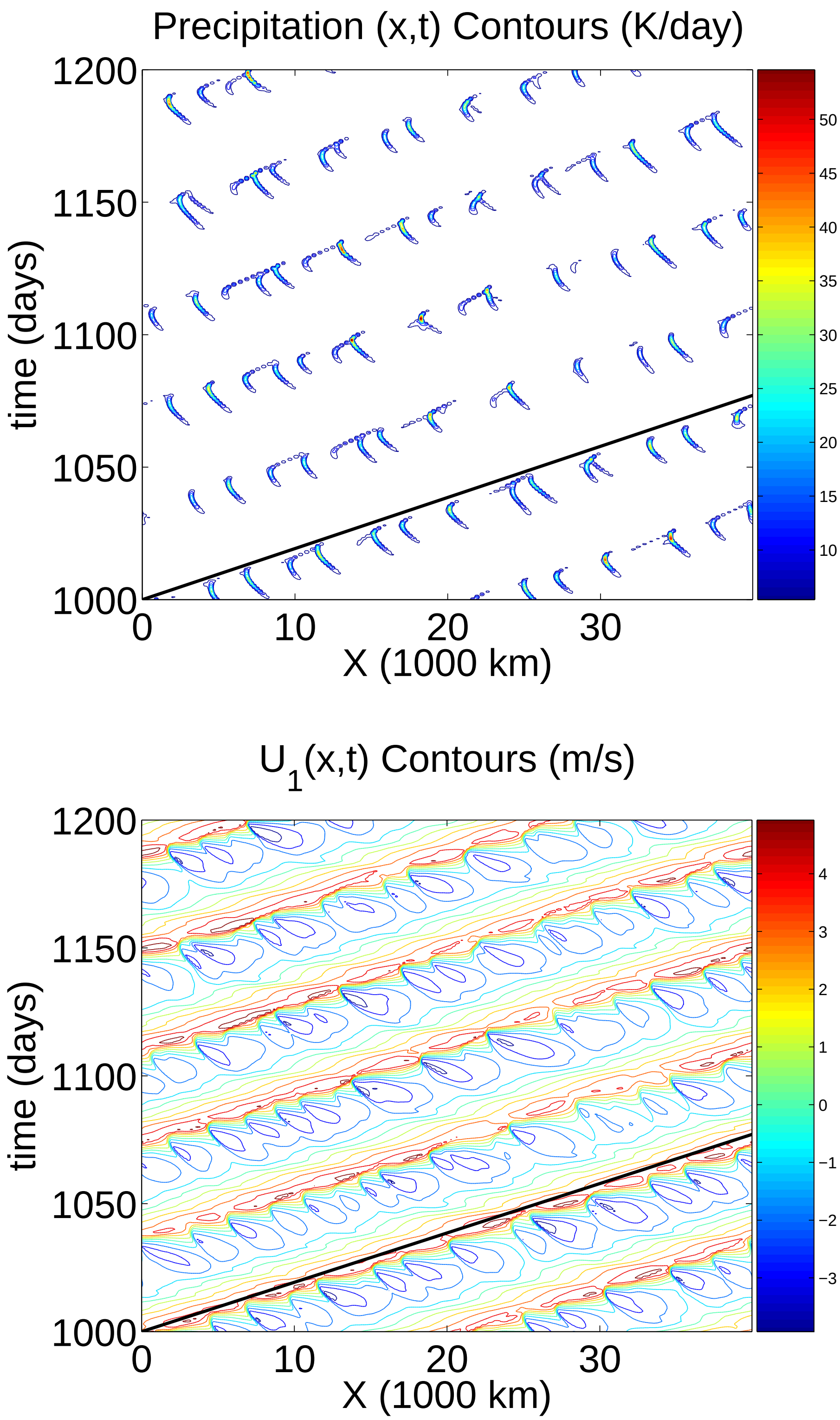
Preliminary results, no rotation

What features should a realistic MJO analog have?

1. An actual propagation speed of 4–7 m s^{-1} predicted by theory,
2. A wavenumber-2 or -3 structure for the low-frequency planetary scale envelope with distinct active and inactive phases of deep convection,
3. An intermittent turbulent chaotic multi-scale structure within the wave envelope involving embedded westward- and eastward-propagating deep convection events,
4. Qualitative features of the low-frequency averaged planetary-scale envelope in agreement with the observational record in terms of vertical structure of heating, westerly wind burst, etc.

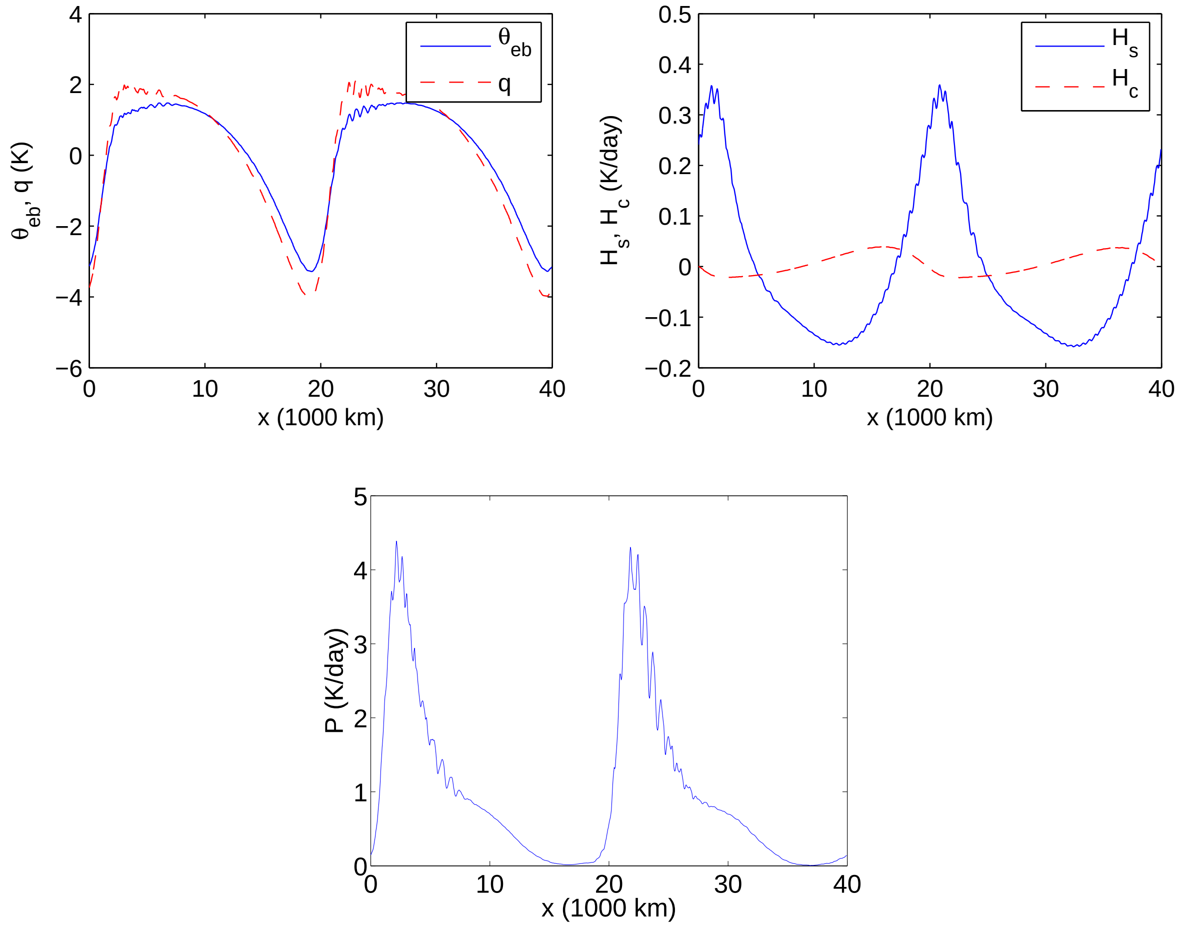
Which physical mechanisms are absent from the multi-cloud model?

1. WISHE (wind-induced surface heat exchange)
2. Wave-CISK
3. Boundary layer frictional convergence
4. Active radiation
5. Eddy flux divergences of momentum
6. Active atmosphere–ocean coupling
7. Rotation (future work)



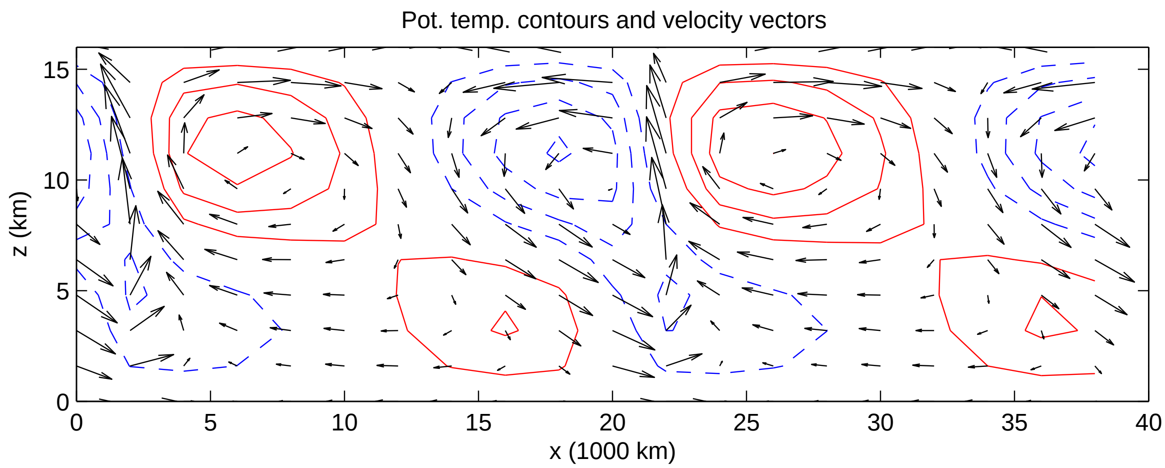
Solid black line: 6 m/s

Moving Averages

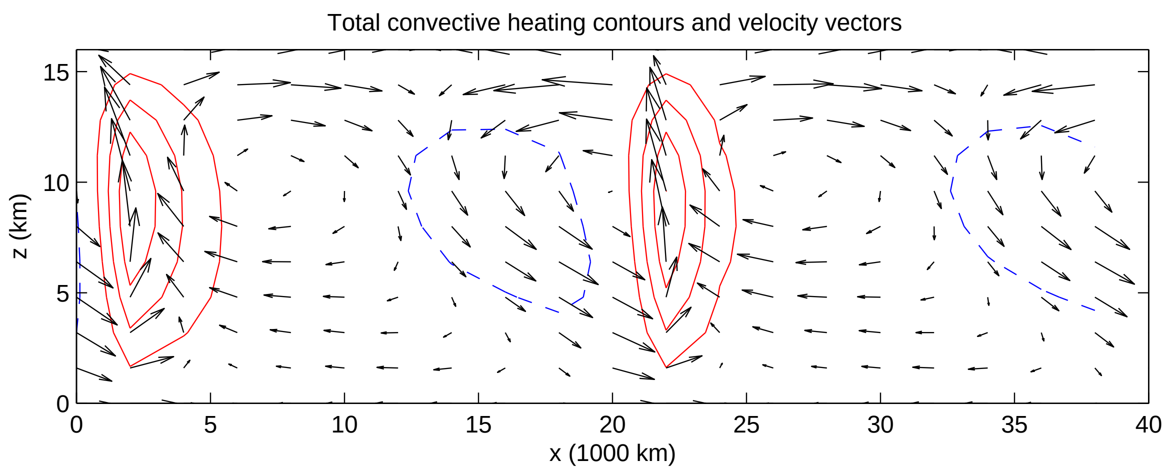


Average in frame moving at 6 m/s from $t = 1000$ to 1500 days

Congestus leading, with
Moisture preconditioning,
Deep convection,
Stratiform trailing.



Pot. temp. contours and velocity vectors
z (km)
x (1000 km)
0
5
10
15
0
5
10
15
20
25
30
35
40



Total convective heating contours and velocity vectors
z (km)
x (1000 km)
0
5
10
15
0
5
10
15
20
25
30
35
40

New Efficient Sparse Space-Time Algorithms for Superparameterization on Mesoscales

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A.J. Majda and Y. Xing, New Multi-Scale models on mesoscales and squall lines, *Comm. Math. Sci.,* in press, 2008

Details of SP

Denote the large scale variables by:

$$Q = Q\ (X, Y, z, t)$$

the small scale variables by:

$$q = q\ (x, z, t)\ |_{(X,Y)}$$

The fundamental property of the two sets of variables

$$Q\ (X, Y, z, t) = <q\ (x, z, t)\ |_{(X,Y)}>$$

Horizontal averaging of a cloud scale dependent variable:

$$\langle \mathbf{q}(x,\ z,\ t)|_{(X,Y)} \rangle \equiv \frac{1}{L} \int_{-L/2}^{L/2} \mathbf{q}(\xi,\ z,\ t)|_{(X,Y)}\ d\xi,$$

Numerical algorithm

Large-scale model equations

$$\frac{\partial Q}{\partial t} = A_Q + S_Q + F_{CS}^Q$$

where $A_Q \equiv -\mathbf{U} \cdot \boldsymbol{\nabla} Q$ is the large-scale advection term

Small-scale model equations

$$\frac{\partial q}{\partial t} = a_q + s_q + f_{\mathrm{LS}}^q$$

where $a_q \equiv -\mathbf{u} \cdot \boldsymbol{\nabla} q$ ($\mathbf{u}$ is the small-scale flow)

An alternative formula of the algorithm

Switching the order to solve large and small scale models.

Original SP becomes:

1. The small scale model is solved from T to $T + N\Delta t = T + \Delta T$

$$\langle q|^{n+1}\rangle = \langle q|^{n}\rangle + \sum_{i=1}^{N} \Delta t (a_q + s_q)|_i^{i+1} + \sum_{i=1}^{N} \Delta t f^q_{LS,SO}|^n,$$

2. Define the small scale feedback as:

$$F^Q_{CS,SO}|^n = \frac{\langle q|^{n+1}\rangle - Q|^n}{\Delta T}.$$

and the large scale models are solved from T to T+ ΔT by

$$Q|^{n+1} = Q|^n + \Delta T (A_Q + S_Q)|_n^{n+1} + \Delta T F^Q_{CS,OS}|^n,$$

3. Define the large scale forcing to small scale as

$$f^q_{LS,OS}|^{n+1} = \frac{Q|^{n+1} - \langle q|^{n+1}\rangle}{\Delta T}$$

Comparable numerical results are obtained by the two forms of SP.

Reduced time and space strategy

Assume 1/p small scale time steps and spatial cells are both employed, the computational cost of small scale model is decreased by $1/p^2$.

We denote the new efficient algorithms by

Sparse Space-Time algorithms for SuperParameterization (SSTSP).

An alternative formula of the algorithm

Two main changes, compared with SP :

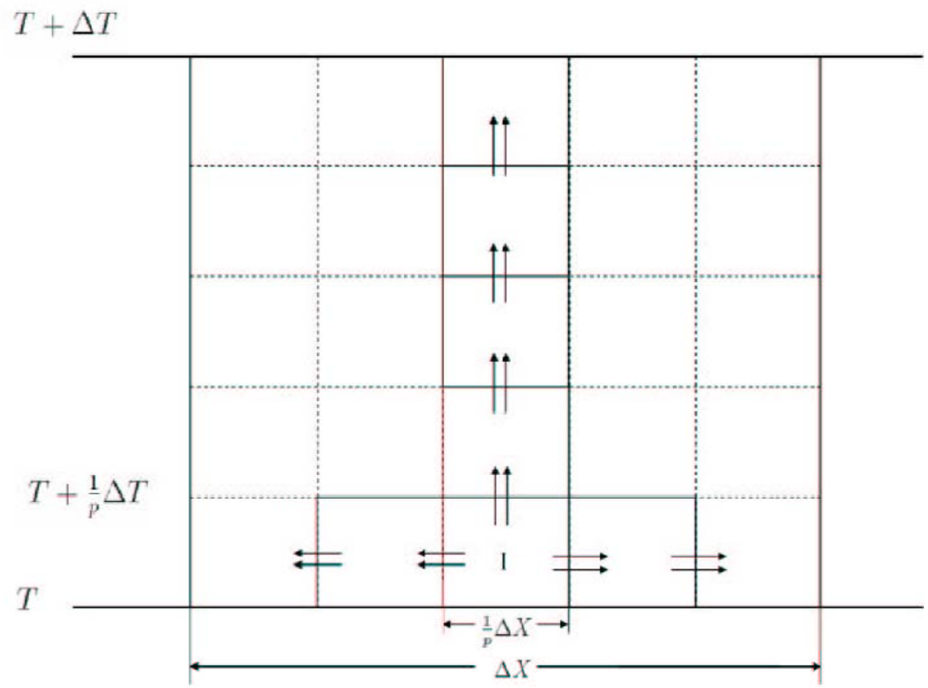
a) $q|^{n+1} = q|^{n+\frac{1}{p}}$

b) $F^{Q}_{CS,OS}|^{n} = \dfrac{\langle q|^{n+\frac{1}{p}}\rangle - Q|^{n}}{\Delta T/p} = p\dfrac{\langle q|^{n+1}\rangle - Q|^{n}}{\Delta T}.$

is p times bigger.

One explanation :

periodically extend the solution inside I to the big domain.



Squall line experiment setup

The 2D squall line experiment designed in [Grabowski 2006] is explored:

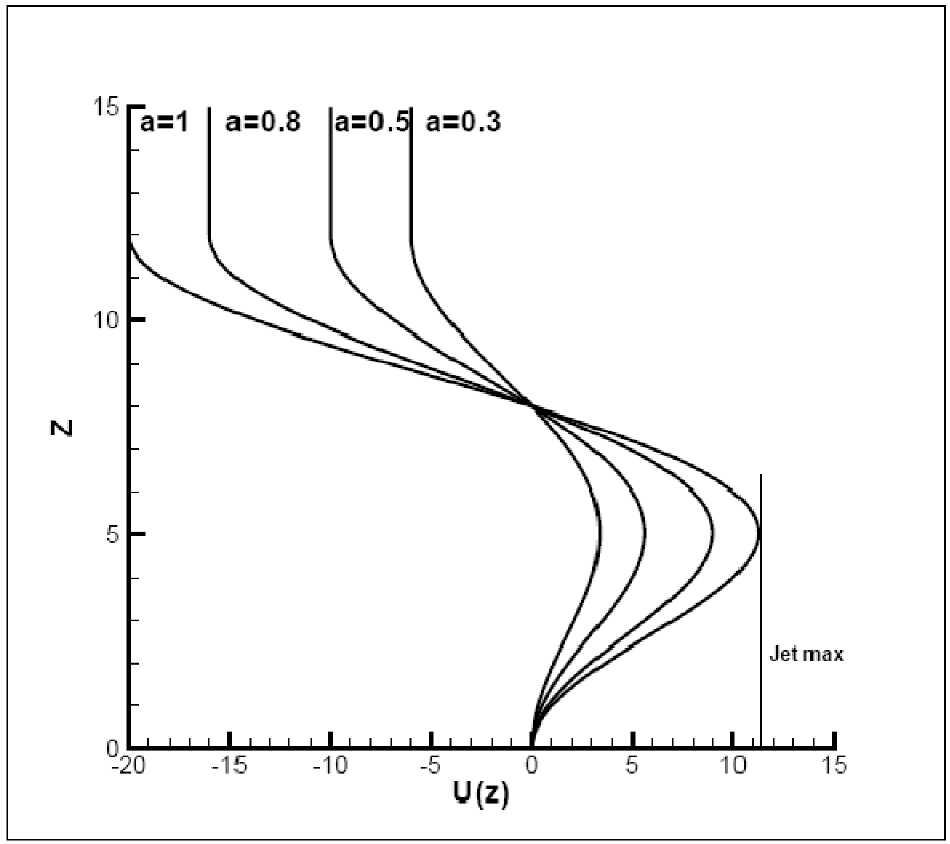
- 2D domain of 1024 km length and 25 km height.
- The initial temperature, humidity profiles and horizontal wind fields are based on the GARP GATE Phase-III mean sounding.
- A 4-km-deep, 512-km-long cold pool of $D\theta = -6.75$ K and $Dq_v = -3.5$ g kg^{-1} is placed in the domain on the initial data to initiate convection.
- Impose a large-scale forcing representing climatological background through the cooling and moistening rates for the first 6 hours, then remove it.
- A 10% amplitude random noise is added to the surface fluxes to provide small-scale excitation (important for the initial development of convection).

Test bed

The following initial large scale background shears are used:

$$\bar{U}(z) = \begin{cases} 10a(cos(\frac{\pi z}{12}) - cos(\frac{2\pi z}{12})) & \text{if } z < 12, \\ -20a & \text{otherwise.} \end{cases}$$

with a = 1, 0.8, 0.5, 0.3.



Test bed

a = 1, 0.8, 0.5: (strong or weak shear)

Propagating squall line emerges in these three cases.

a = 0.3: (weaker shear)

Dying scattered convection. No squall line is generated.

Application to squall lines

Five simulations:

- CRM: Cloud-Resolving Model
- SP: Original superparameterization simulation
- SSTSP2: Efficient algorithm with p=2
- SSTSP3: Efficient algorithm with p=3
- SSTSP6: Efficient algorithm with p=6

Application to squall lines

Setup for these simulations:

CRM: resolution ~ 1km, time step ~ 10 seconds.

SP, SSTSP2, SSTSP3, SSTSP6:
large scale horizontal domain size 1024 km with resolution 32 km
large time step ~ 60 seconds, small time step ~ 10 seconds.

SP: small scale horizontal domain size 32 km with resolution 1 km
small time models are solved 6 times in each large time step

SSTSP2: small scale horizontal domain size 16 km with resolution 1 km
small time models are solved 3 times in each large time step

SSTSP3: small scale horizontal domain size 10 km with resolution 1 km
small time models are solved 2 times in each large time step

SSTSP6: small scale horizontal domain size 6 km with resolution 1 km
small time models are solved 1 time in each large time step

Fig 1. The contours of the surface precipitation when a=1.
Top left: CRM; Top right: SP; Bottom left: SSTSP3; Bottom right: SSTSP6.

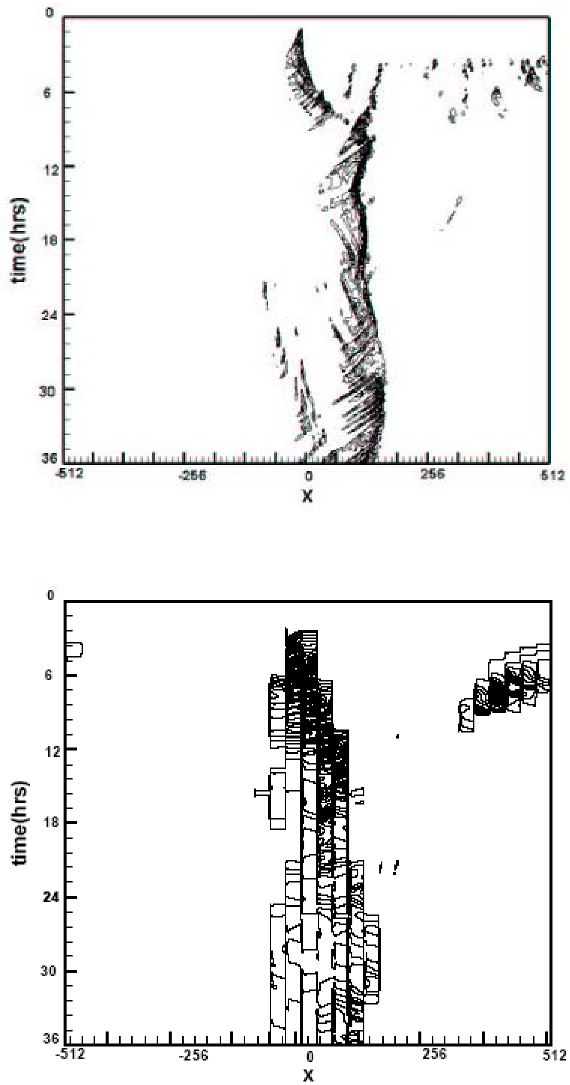


Fig 2. The contours of the large scale horizontal velocity when a=1.
Top left: CRM; Top right: SP; Bottom left: SSTSP3; Bottom right: SSTSP6.

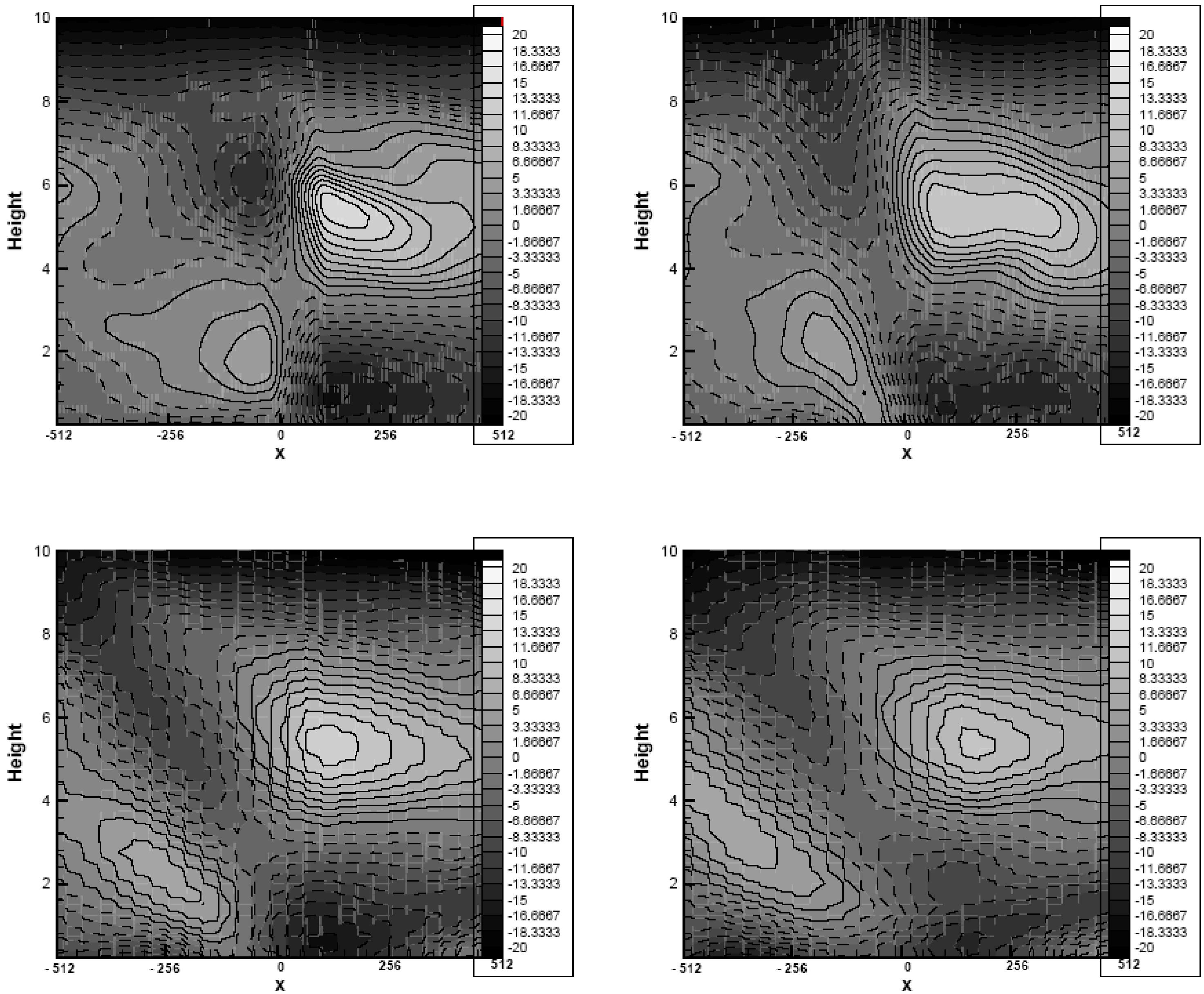
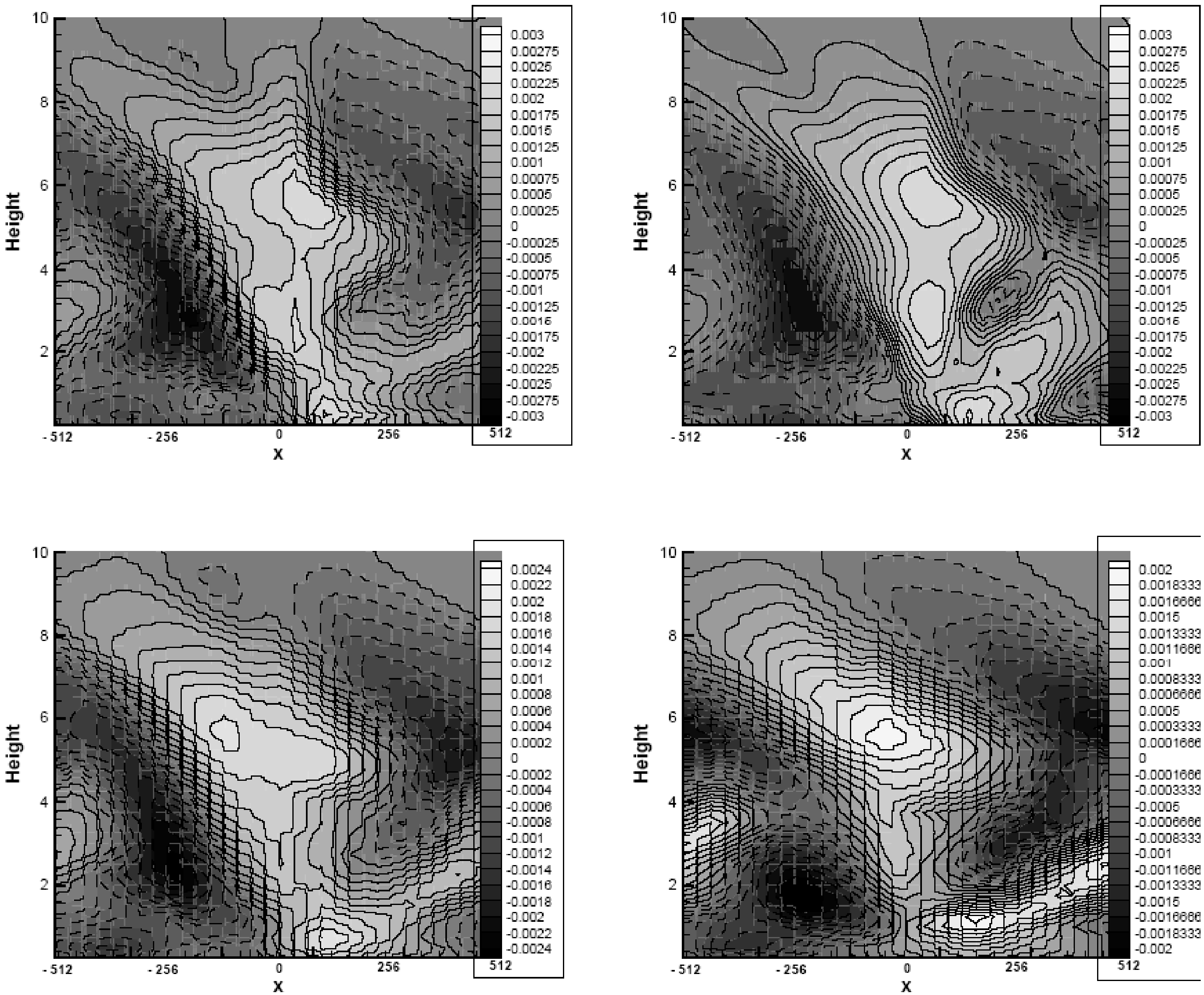


Fig 3. The contours of the large scale specific humidity when a=1.
Top left: CRM; Top right: SP; Bottom left: SSTSP3; Bottom right: SSTSP6.



Test case with a=1

We compute the correlation between these plots.

	Between SP and SSTSP2	Between SP and SSTSP3	Between SP and SSTSP6
$\langle \bar{q}_v \rangle$	0.9417	0.8738	0.7168
$\langle \bar{\theta} \rangle$	0.9104	0.7795	0.5878
$\langle \bar{u} \rangle$	0.9025	0.8621	0.5876
	Between CRM and SSTSP2	Between CRM and SSTSP3	Between CRM and SSTSP6
$\langle \bar{q}_v \rangle$	0.8521	0.7781	0.6571
$\langle \bar{\theta} \rangle$	0.8425	0.7325	0.5356
$\langle \bar{u} \rangle$	0.8735	0.8082	0.6215

These large scale variables are the most important thing to examine as the output in a squall line.

Test case with a=1

The correlation shows nice structural agreement.

SSTSP3 captures the main large scale effect of the squall line experiment well in a statistically way, and save the computational cost by a factor of roughly 9, compared with the original SP.

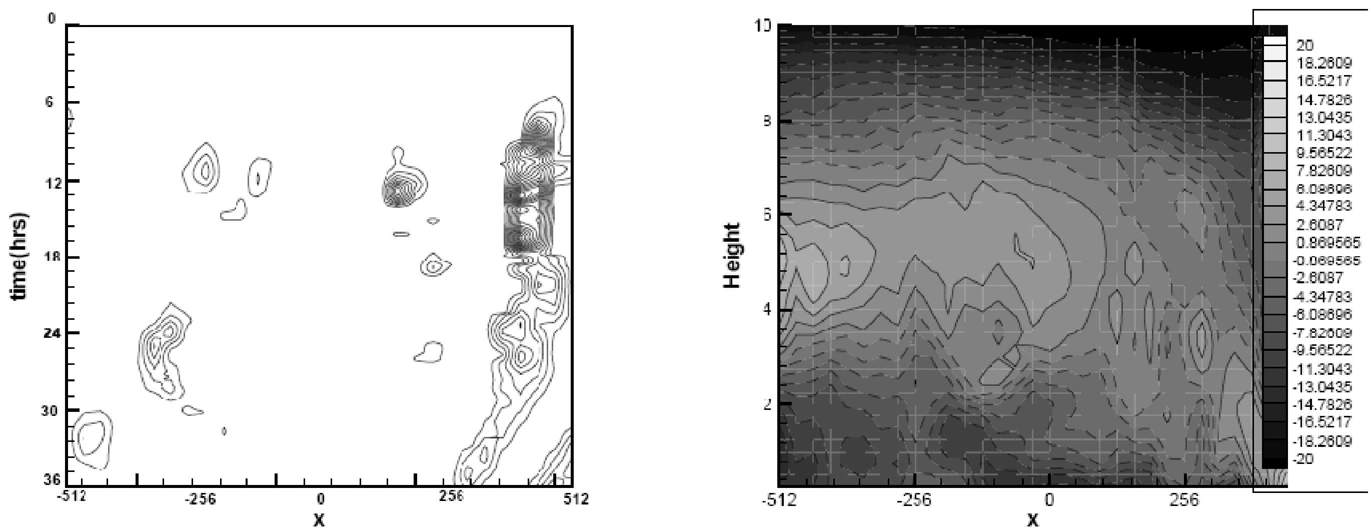
Even SSTSP6 has pattern correlation above 0.6 for velocity and humidity.

Test case with a=1

To emphasize that the small scale models play an important role in capturing the squall line:

Run the CRM code with very coarse 32 km resolution, the resolution for the large scale model of the SP test.

No squall line is developed on such coarse meshes. The resolution is too big to capture those cloud scale effects.



Left: Surface Precipitation; Right: Large scale horizontal velocity.

Test case with a=0.3

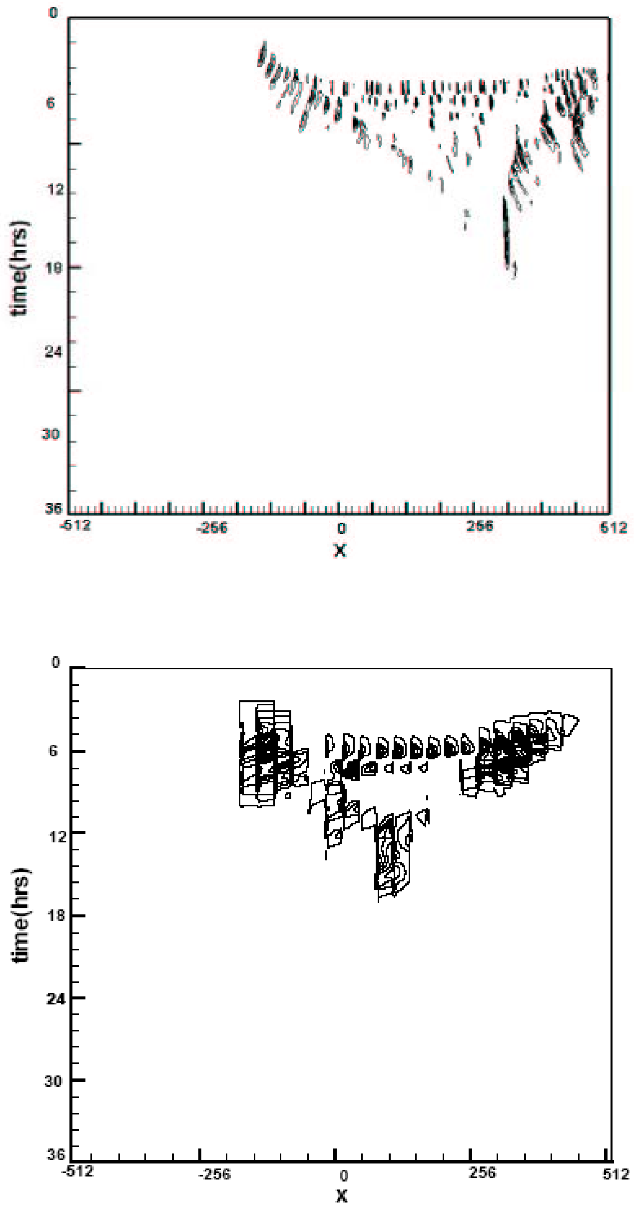
A weaker shear with a=0.3

CRM shows that the turbulent travelling wave will decay and not propagate in time.

The SSTSP3 and SSTSP6 both capture this fact.

SSTSP algorithms not only capture the large scale features when a squall line is developed, but also have significant skill in the situation when no quasi-steady squall line is formed.

Fig 5. The contours of the surface precipitation when a=0.3.
Top left: CRM; Top right: SP; Bottom left: SSTSP3; Bottom right: SSTSP6.



A propagating squall line

We modify this experiment to obtain a propagating squall line.

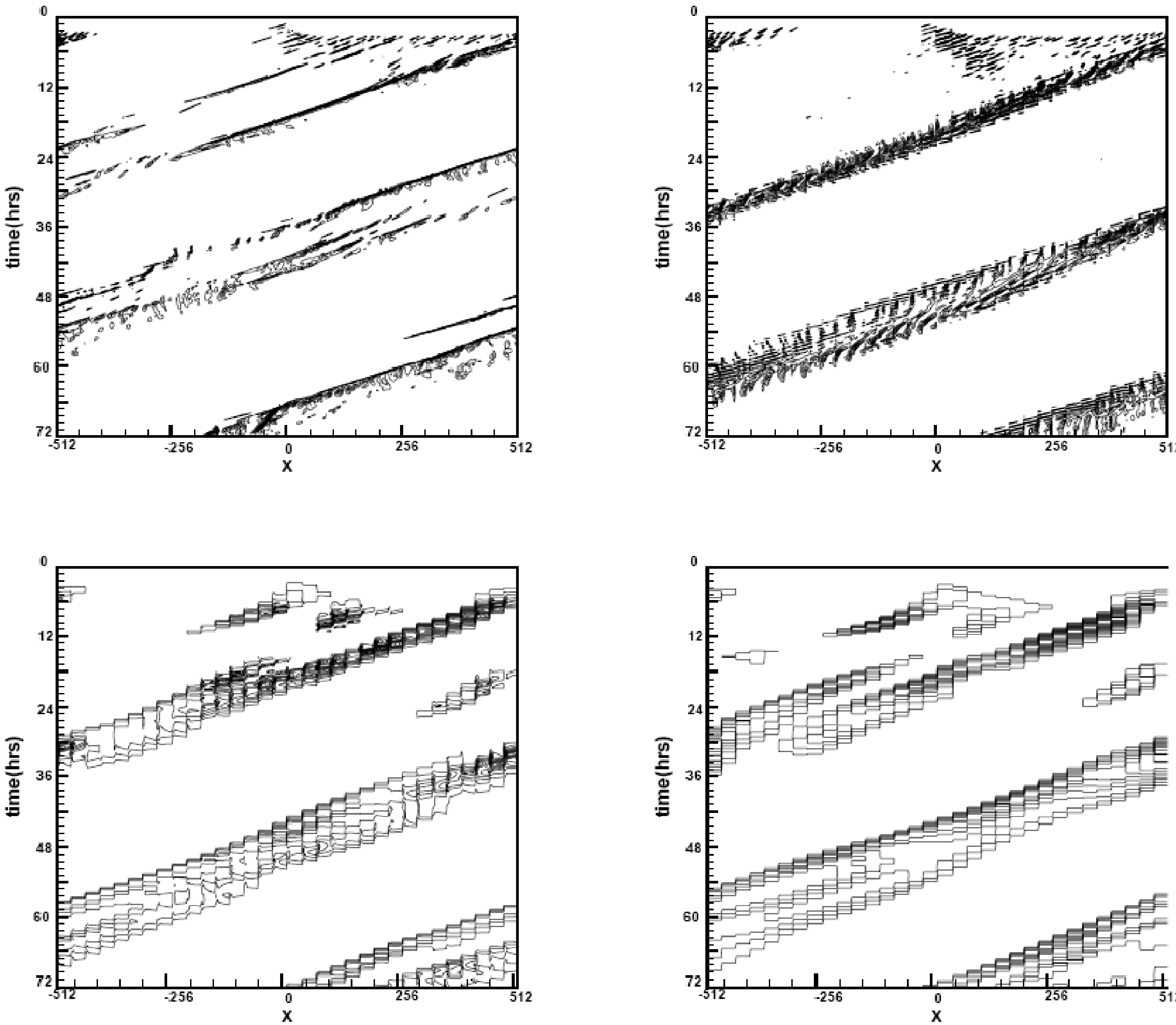
A different background shear U(z) (obtained from reality) is used.

We show the jet max and mean propagation speed of the squall line:

	CRM	SP	SSTSP2	SSTSP3	SSTSP6
Jet max(km)	14.1	14.1	14.1	14.1	14.1
Squall line speed (m/s)	11.5	10.5	10	10	10

Both the SSTSP3 and SSTSP6 cases reproduce the squall line speed in the CRM within 10%

Fig 10. The contours of the surface precipitation for a propagating squall line. Top left: CRM; Top right: SP; Bottom left: SSTSP3; Bottom right: SSTSP6.



Summary

We show:

- SSTSP3 results in a gain of roughly a factor of 10 in efficiency, and captures large scale variables, such as velocity and specific humidity, in a reasonably statistically accurate way (with correlation above 0.75).

- SSTSP6 algorithm, with 1/36 computational cost, has pattern correlation above 0.6 for large scale velocity and humidity.

- The structure of eddy momentum flux divergence, positive region below and negative region above, is captured qualitatively by SP and SSTSP3. SSTSP6 fails in capturing this fact.