A parameterization of turbulence enstrophy for VVM

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Introduction

Turbulence parameterization

- Traditional approach: solving turbulence kinetic equation (TKE) equation with difficulties involving parameterization of the pressure redistribution term
- New approach: solving the turbulence enstrophy equation (TE) equation

TE equation

$$\frac{\partial e}{\partial t} = -\frac{\partial\left(\overline{u}_i e\right)}{\partial x_j} - \frac{\partial\left(\overline{u_i' e'}\right)}{\partial x_j} + \overline{\xi_i'\xi_j'}\frac{\partial \overline{u}_i}{\partial x_j} - \overline{\xi_i' u_j'}\frac{\partial \overline{\xi}_i}{\partial x_j} - \overline{\xi_2'\frac{\partial B'}{\partial x_1}} + \overline{\xi_1'\frac{\partial B'}{\partial x_2}}$$

$$+\overline{\xi}_j\overline{\xi_i'\frac{\partial u_i'}{\partial x_j}} + \overline{\xi_j'\xi_i'\frac{\partial u_i'}{\partial x_j}} + \varepsilon_e.$$

The terms on the right-hand side are

Mean wind advection, perturbation advection, shear prodution, buoyancy procuction, twisting and deformation, and dissipation, respectively

Turbulence advection

$$\overline{u_i'e'} = -k_m \frac{\partial e}{\partial x_i},$$

$k_m = c_1 l^2 e^{1/2}$: the subgrid-scale eddy coefficient

Shear production

$$\overline{\xi_i'\xi_j'}\frac{\partial \overline{u_i}}{\partial x_j} = -\left[k_\xi \left(\frac{\partial \overline{\xi_i}}{\partial x_j} + \frac{\partial \overline{\xi_j}}{\partial x_i} \right) - \frac{2}{3}\delta_{ij} e \right] \frac{\partial \overline{u_i}}{\partial x_j},$$

and $$-\overline{\xi_i' u_j'}\frac{\partial \overline{\xi_i}}{\partial x_j} = -\left(k_\xi \frac{\partial \overline{u_j}}{\partial x_i} + k_m \frac{\partial \overline{\xi_i}}{\partial x_j} - \frac{1}{3}\delta_{ij}\overline{u_l}\,\overline{\xi_l} \right) \frac{\partial \overline{\xi_i}}{\partial x_j}$$

$k_\xi = c_2 l e^{1/2}$: subgrid-scale eddy coefficient for vorticity.

Buoyancy production

$$-\overline{\xi_2' \frac{\partial B'}{\partial x_1}} + \overline{\xi_1' \frac{\partial B'}{\partial x_2}} = k_{h\xi} \frac{\partial^2 \overline{B}}{\partial x_1 \partial x_3} - k_{h\xi} \frac{\partial^2 \overline{B}}{\partial x_2 \partial x_3}$$

$k_{h\xi} = (1 + 2l / \Delta s) \cdot k_\xi$:

eddy coefficient for scalar quantities.

Δs: grid-spacing

Twisting and Deformation

$$\overline{\xi_j}\,\overline{\xi_i'\frac{\partial u_i'}{\partial x_j}} = -k_\xi \overline{\xi_j}\frac{\partial \overline{u}_i}{\partial x_j}$$

$\overline{\xi_j'\xi_i'\frac{\partial u_i'}{\partial x_j}}$ is one-order of magnitude smaller,

and is neglected.

Dissipation

$$\varepsilon_{\mathrm{e}} = c_3\, e/\tau$$

$c_3 = 0.19 + 0.51 l / \Delta s$: a coefficient

$\tau = \dfrac{1}{e^{1/2}}$: a dissipation time-scale.

Turbulence fluxes and circulation

$$\overline{w'T'} \leftrightarrow \overline{\xi_1'T'} = \overline{\left(\frac{\partial w'}{\partial y} - \frac{\partial v'}{\partial z}\right)T'}$$

$$\overline{w'T'} = -k_h \frac{\partial \overline{T}}{\partial z} \quad \text{and} \quad \overline{\xi_1'T'} = -k_{h\xi} \frac{\partial \overline{T}}{\partial z}$$

$$\overline{w'T'} = l\overline{\xi_1'T'}$$

Nonlocal circulation

$$\overline{\xi_1' T'} = -k_{h\xi} \left(\frac{\partial \overline{T}}{\partial z} - \gamma \right)$$

nonlocal term $\gamma = c\,\overline{w' T_s'} / \left(z_i w_s \right)$

Conclusions and discussion

- Derived and parameterized TE equation and various terms

1. Avoid pressure redistribution term
2. Obtain dissipation time-scale directly

- Discussed turbulence and nonlocal circulation