Progress in the development of a zonal channel version of the vector vorticity model

Hiroaki Miura (CSU)

Thanks to David Randall, Akio Arakawa, Celal Konor, Joon-Hee Jung, and Ross Heikes

- Motivation
- A parallel Poisson solver
- Current configuration of the model
- Test results
  - Cold bubble experiment
  - Held-Suarez-like test
- Summary

Motivation

regional ↓ global

Jung and Arakawa (2008)

- A new CRM using the vorticity equation (VVM)
- Cyclic conditions in X and Y
- Not parallelized

Celal and MingXuan's model

- Upgrading the original model
- Cyclic conditions in X and Y
- Parallelized (FFT)

My work

- Zonal channel (Cyclic in X, walls in Y)
- Parallelized (Multigrid)

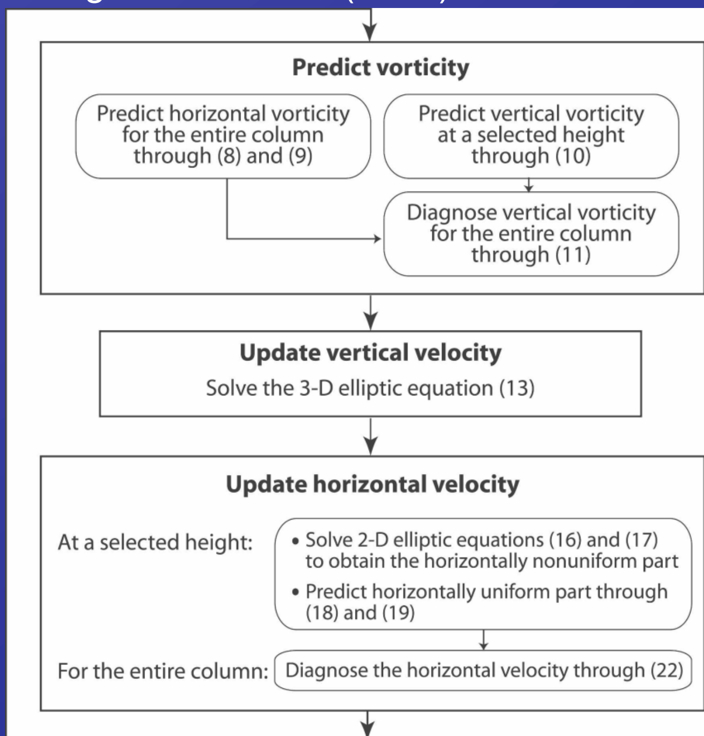
VVM on the spherical geodesic grid (future)

- Celal is working on a hexagonal VVM
- Ross is working on the Multigrid method

global

Flow for updating dynamical variables

Jung and Arakawa (2008)



Predict vorticity and scalars

$$\frac{\partial \xi}{\partial t} = -\left[\frac{\partial}{\partial x}(u\xi)+\frac{\partial}{\partial y}(v\xi)+\frac{\partial}{\partial z}(w\xi)\right] + \xi\frac{\partial u}{\partial x} + \eta\frac{\partial u}{\partial y} + \zeta\frac{\partial u}{\partial z} + f\frac{\partial u}{\partial z} + \frac{\partial B}{\partial y} + \frac{\partial F_w}{\partial y} - \frac{\partial F_v}{\partial z}$$

$$\frac{\partial \eta}{\partial t} = -\left[\frac{\partial}{\partial x}(u\eta)+\frac{\partial}{\partial y}(v\eta)+\frac{\partial}{\partial z}(w\eta)\right] + \xi\frac{\partial v}{\partial x} + \eta\frac{\partial v}{\partial y} + \zeta\frac{\partial v}{\partial z} + f\frac{\partial v}{\partial z} - \frac{\partial B}{\partial x} + \frac{\partial F_u}{\partial z} - \frac{\partial F_w}{\partial x}$$

$$\frac{\partial \zeta}{\partial t} = -\left[\frac{\partial}{\partial x}(u\zeta)+\frac{\partial}{\partial y}(v\zeta)+\frac{\partial}{\partial z}(w\zeta)\right] + \xi\frac{\partial w}{\partial x} + \eta\frac{\partial w}{\partial y} + \zeta\frac{\partial w}{\partial z} - f\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + \frac{\partial F_v}{\partial x} - \frac{\partial F_u}{\partial y}$$

$$\zeta_k = -\int_{z_T}^{z_k}\left[\frac{\partial \xi}{\partial x} + \frac{\partial \eta}{\partial y}\right]dz + \zeta_T$$

$$\frac{\partial \theta}{\partial t} = -\frac{1}{\rho_0}\left[\frac{\partial}{\partial x}(\rho_0 u\theta)+\frac{\partial}{\partial y}(\rho_0 v\theta)+\frac{\partial}{\partial z}(\rho_0 w\theta)\right] + \left(\frac{\partial \theta}{\partial t}\right)_{phys}$$

$$\frac{\partial q_x}{\partial t} = -\frac{1}{\rho_0}\left[\frac{\partial}{\partial x}(\rho_0 u q_x)+\frac{\partial}{\partial y}(\rho_0 v q_x)+\frac{\partial}{\partial z}(\rho_0 w q_x)\right] + \left(\frac{\partial q_x}{\partial t}\right)_{phys}$$

Diagnose velocity

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)w + \frac{\partial}{\partial z}\left[\frac{1}{\rho_0}\frac{\partial}{\partial z}(\rho_0 w)\right] = -\frac{\partial \eta}{\partial x} + \frac{\partial \xi}{\partial y}$$

$$u = u_\psi + u_\chi, \quad v = v_\psi + v_\chi$$

$$u_\psi \equiv -\frac{\partial \psi}{\partial y}, \quad u_\chi \equiv \frac{\partial \chi}{\partial x}, \quad v_\psi \equiv \frac{\partial \psi}{\partial x}, \quad v_\chi \equiv \frac{\partial \chi}{\partial y}$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\psi = \zeta, \quad \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\chi = \delta$$

$$u_k = \int_{z_T}^{z_k}\left(\frac{\partial w}{\partial x} + \eta\right)dz + u_T$$

$$v_k = \int_{z_T}^{z_k}\left(\frac{\partial w}{\partial y} - \xi\right)dz + v_T$$

One 3D and two 2D Poisson equations need to be solved.

Parallel Poisson solvers

From a lecture of Dr. H. S. Simon
http://www.cs.berkeley.edu/~demmel/cs267_Spr05/Lectures/Lecture25/Lecture_25_UnstructuredMultigrid_jd2005_v3.ppt

Algorithms for 2D Poisson Equation (N vars)

Algorithm	Serial	PRAM	Memory	#Procs
• Dense LU	N^3	N	N^2	N^2
• Band LU	N^2	N	$N^{3/2}$	N
• Jacobi	N^2	N	N	N
• Explicit Inv.	N	log N	N	N
• Conj.Grad.	$N^{3/2}$	$N^{1/2}$ *log N	N	N
• RB SOR	$N^{3/2}$	$N^{1/2}$	N	N
• Sparse LU	$N^{3/2}$	$N^{1/2}$	N*log N	N
• FFT	N*log N	log N	N	N
• Multigrid	N	$\log^2 N$	N	N
• Lower bound	N	log N	N	

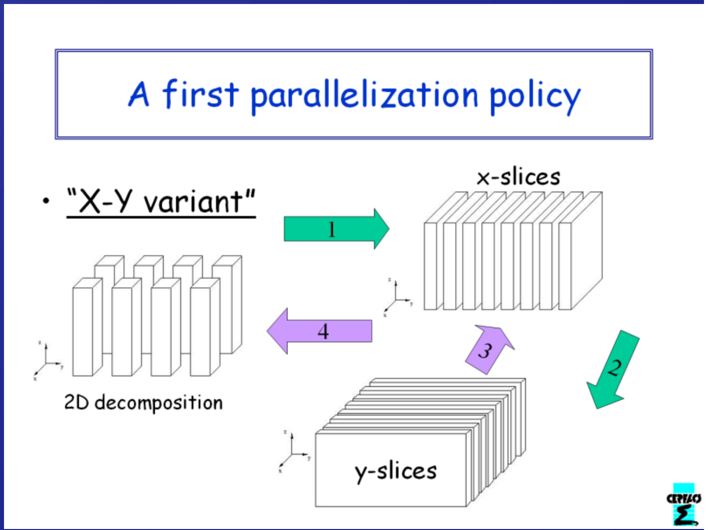
PRAM is an idealized parallel model with zero cost communication

Reference: James Demmel, Applied Numerical Linear Algebra, SIAM, 1997.

Why multigrid?

FFT can be faster even on parallel computers.

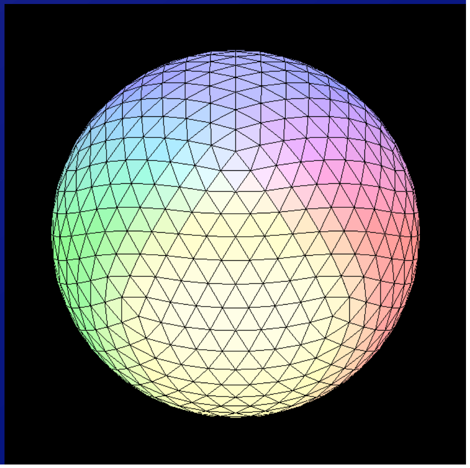
- Celal and MingXuan's model is testing a FFT solver.
- Other examples using FFT: SAM, meso-NH



From a presentation of Dr. L. Giraud
http://www.cerfacs.fr/~giraud/Talks/parCfd.pps

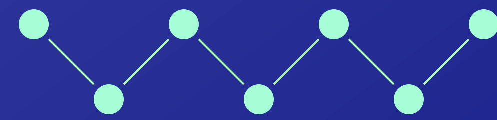
Merits of the multigrid method

- It is easier to code.
- Its computation is local.
    - We can code it using MPI_(I)SEND and MPI_(I)RECV only.
    - This may be desirable for large number of processors.
- We can use the same method on the spherical geodesic grid.
    - Heikes and Randall (1995)



A Poisson solver: Jacobi method

1D Poisson equation $$\frac{\partial^2 w}{\partial x^2} = F$$



Jacobi method:

$$\frac{w_{i-1}^{\tau} - 2w_i^{\tau+1} + w_{i+1}^{\tau}}{\Delta x^2} = F_i$$

$$w_i^{\tau+1} = \frac{1}{2}\left(w_{i-1}^{\tau} + w_{i+1}^{\tau} - F_i \Delta x^2\right)$$

$$\frac{w_{i-1}^{\tau} - 2w_i^{\tau+1} + w_{i+1}^{\tau}}{\Delta x^2} = 0$$

$$w_i^{\tau+1} = \frac{1}{2}\left(w_{i-1}^{\tau} + w_{i+1}^{\tau}\right)$$

	w(i-1)	w(i)	w(i+1)
t	-1	1	-1
t+1	1	-1	1

ω-Jacobi method:

$$\frac{w_{i-1}^{\tau} - \left[\alpha w_i^{\tau+1} + (2-\alpha) w_i^{\tau}\right] + w_{i+1}^{\tau}}{\Delta x^2} = F_i$$

$$w_i^{\tau+1} = w_i^{\tau} + \frac{2}{\alpha}\left[\frac{1}{2}\left(w_{i-1}^{\tau} + w_{i+1}^{\tau} - F_i \Delta x^2\right) - w_i^{\tau}\right]$$

$$\tilde{w}_i = \frac{1}{2}\left(w_{i-1}^{\tau} + w_{i+1}^{\tau} - F_i \Delta x^2\right)$$

$$\omega = 2/\alpha$$

$$w_i^{\tau+1} = w_i^{\tau} + \omega\left(\tilde{w}_i - w_i^{\tau}\right)$$

$$\alpha = 4.0$$

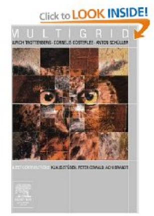
$$\frac{w_{i-1}^{\tau} + 2w_i^{\tau} + w_{i+1}^{\tau} - 4w_i^{\tau+1}}{\Delta x^2} = 0$$

$$w_i^{\tau+1} = 0$$

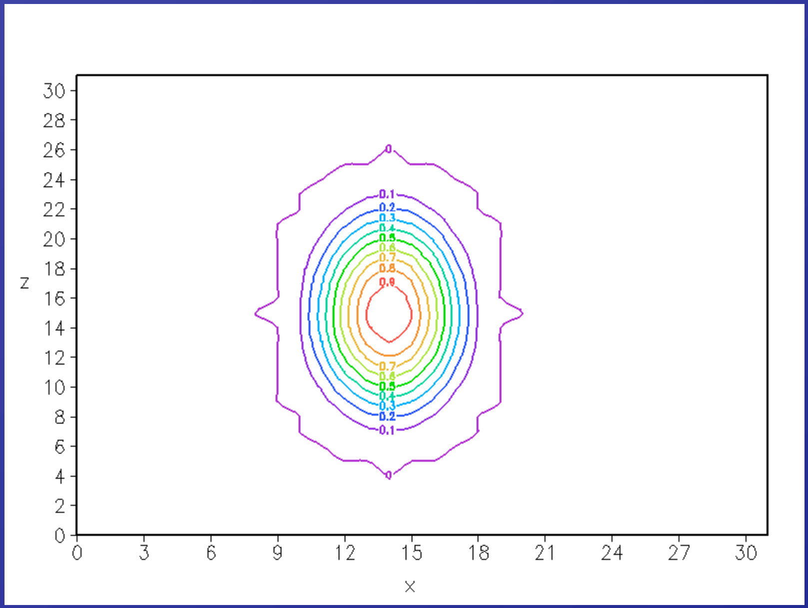
	w(i-1)	w(i)	w(i+1)
t	-1	1	-1
t+1	0	0	0

An optimum parameter:

$$\omega = 4/5 \;\rightarrow\; \alpha = 2.5$$



Test of 3D Poisson solvers



nx=ny=nz=32

dx=dy=dz=1.0

$$x_c = y_c = z_c = 16.0$$

$$r = \left[(x - x_c)/5.0\right]^2 + \left[(y - y_c)/5.0\right]^2 + \left[(z - z_c)/10\right]^2$$

$$w = \begin{cases} \cos^2\left(\pi/2 * \sqrt{r}\right) & \text{for } r \le 1.0 \\ 0 & \text{for } r > 1.0 \end{cases}$$

Method:

1. With a given w, compute x- and y-components of vorticity.
2. Then, reconstruct w by solving the 3D Poisson equation.

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) w + \frac{\partial}{\partial z}\left[\frac{1}{\rho_0}\frac{\partial}{\partial z}(\rho_0 w)\right] = -\frac{\partial \eta}{\partial x} + \frac{\partial \xi}{\partial y}$$

$w = 0$ at the top and bottom boundaries

Horizontal: Jacobi, Vertical: Implicit

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)(\rho_0 w)+\rho_0\frac{\partial}{\partial z}\left[\frac{1}{\rho_0}\frac{\partial}{\partial z}(\rho_0 w)\right]=\rho_0 F$$

$$\frac{\rho_{k+1/2}w^{\tau}_{i-1,j,k+1/2}-2\rho_{k+1/2}w^{\tau+1}_{i,j,k+1/2}+\rho_{k+1/2}w^{\tau}_{i+1,j,k+1/2}}{\Delta x^2}+\frac{\rho_{k+1/2}w^{\tau}_{i,j-1,k+1/2}-2\rho_{k+1/2}w^{\tau+1}_{i,j,k+1/2}+\rho_{k+1/2}w^{\tau}_{i,j+1,k+1/2}}{\Delta y^2}$$

$$+\rho_{k+1/2}\frac{1}{\Delta z_{k+1/2}}\left[\frac{\rho_{k+3/2}w^{\tau+1}_{i,j,k+3/2}-\rho_{k+1/2}w^{\tau+1}_{i,j,k+1/2}}{\rho_{k+1}\Delta z_{k+1}}-\frac{\rho_{k+1/2}w^{\tau+1}_{i,j,k+1/2}-\rho_{k-1/2}w^{\tau+1}_{i,j,k-1/2}}{\rho_k\Delta z_k}\right]$$

$$=\rho_{k+1/2}F_{i,j,k+1/2}$$

$$A_{k+1/2}\rho_{k-1/2}w^{\tau+1}_{i,j,k-1/2}+B_{k+1/2}\rho_{k+1/2}w^{\tau+1}_{i,j,k+1/2}+C_{k+1/2}\rho_{k+3/2}w^{\tau+1}_{i,j,k+3/2}=D^{\tau}_{i,j,k+1/2}$$

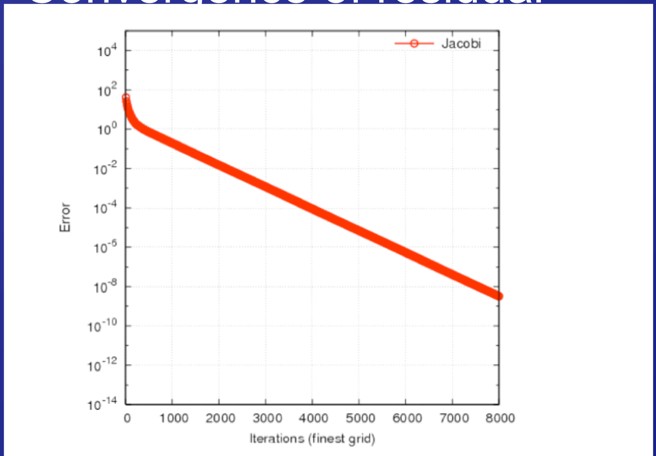
$$A_{k+1/2}=\frac{1}{\rho_k\Delta z_k}\frac{\rho_{k+1/2}}{\Delta z_{k+1/2}}$$

$$B_{k+1/2}=-2\left(\frac{1}{\Delta x^2}+\frac{1}{\Delta y^2}\right)-\left(A_{k+1/2}+C_{k+1/2}\right)$$

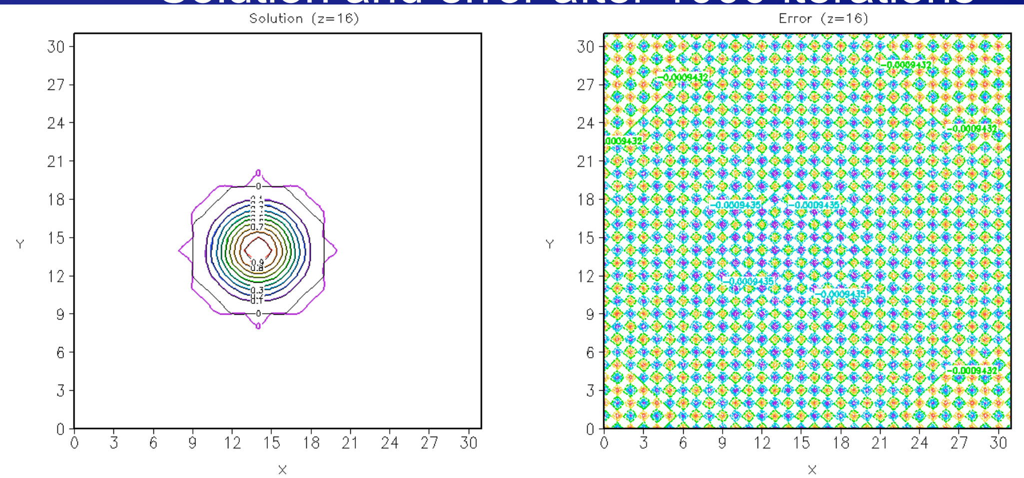
$$C_{k+1/2}=\frac{1}{\rho_{k+1}\Delta z_{k+1}}\frac{\rho_{k+1/2}}{\Delta z_{k+1/2}}$$

$$D^{\tau}_{i,j,k+1/2}=\rho_{k+1/2}F_{i,j,k+1/2}-\left(\frac{\rho_{k+1/2}w^{\tau}_{i-1,j,k+1/2}+\rho_{k+1/2}w^{\tau}_{i+1,j,k+1/2}}{\Delta x^2}+\frac{\rho_{k+1/2}w^{\tau}_{i,j-1,k+1/2}+\rho_{k+1/2}w^{\tau}_{i,j+1,k+1/2}}{\Delta y^2}\right)$$

Convergence of residual



Solution and error after 1000 iterations



Horizontal: ω-Jacobi, Vertical: Implicit

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)(\rho_0 w)+\rho_0\frac{\partial}{\partial z}\left[\frac{1}{\rho_0}\frac{\partial}{\partial z}(\rho_0 w)\right]=\rho_0 F$$

$$\frac{\rho_{k+1/2}w^{\tau}_{i-1,j,k+1/2}-\alpha\rho_{k+1/2}w^{\tau+1}_{i,j,k+1/2}-(2-\alpha)\rho_{k+1/2}w^{\tau}_{i,j,k+1/2}+\rho_{k+1/2}w^{\tau}_{i+1,j,k+1/2}}{\Delta x^2}$$
$$+\frac{\rho_{k+1/2}w^{\tau}_{i,j-1,k+1/2}-\alpha\rho_{k+1/2}w^{\tau+1}_{i,j,k+1/2}-(2-\alpha)\rho_{k+1/2}w^{\tau}_{i,j,k+1/2}+\rho_{k+1/2}w^{\tau}_{i,j+1,k+1/2}}{\Delta y^2}$$
$$+\rho_{k+1/2}\frac{1}{\Delta z_{k+1/2}}\left[\frac{\rho_{k+3/2}w^{\tau+1}_{i,j,k+3/2}-\rho_{k+1/2}w^{\tau+1}_{i,j,k+1/2}}{\rho_{k+1}\Delta z_{k+1}}-\frac{\rho_{k+1/2}w^{\tau+1}_{i,j,k+1/2}-\rho_{k-1/2}w^{\tau+1}_{i,j,k-1/2}}{\rho_k\Delta z_k}\right]=\rho_{k+1/2}F_{i,j,k+1/2}$$

$$A_{k+1/2}\rho_{k-1/2}w^{\tau+1}_{i,j,k-1/2}+B_{k+1/2}\rho_{k+1/2}w^{\tau+1}_{i,j,k+1/2}+C_{k+1/2}\rho_{k+3/2}w^{\tau+1}_{i,j,k+3/2}=D^{\tau}_{i,j,k+1/2}$$

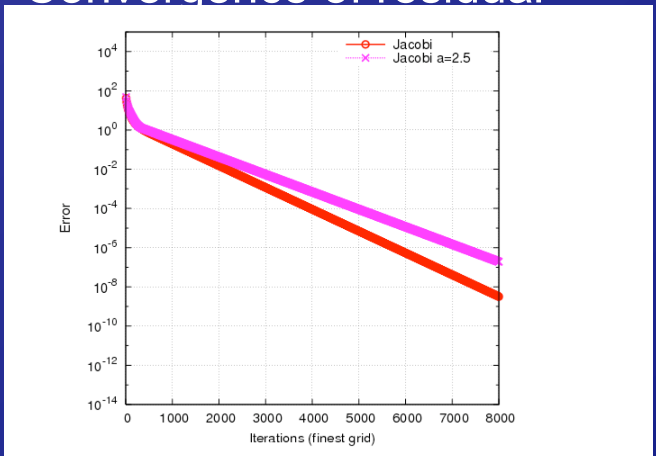
$$A_{k+1/2}=\frac{1}{\rho_k\Delta z_k}\frac{\rho_{k+1/2}}{\Delta z_{k+1/2}}$$

$$B_{k+1/2}=-\alpha\left(\frac{1}{\Delta x^2}+\frac{1}{\Delta y^2}\right)-\left(A_{k+1/2}+C_{k+1/2}\right)$$

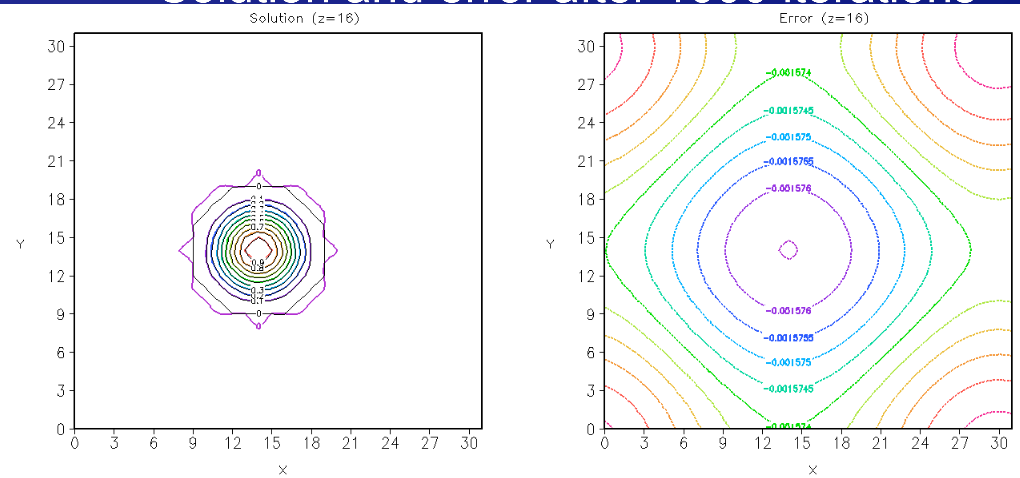
$$C_{k+1/2}=\frac{1}{\rho_{k+1}\Delta z_{k+1}}\frac{\rho_{k+1/2}}{\Delta z_{k+1/2}}$$

$$D^{\tau}_{i,j,k+1/2}=\rho_{k+1/2}F_{i,j,k+1/2}-\left[\frac{\rho_{k+1/2}w^{\tau}_{i-1,j,k+1/2}+(2-\alpha)\rho_{k+1/2}w^{\tau}_{i,j,k+1/2}+\rho_{k+1/2}w^{\tau}_{i+1,j,k+1/2}}{\Delta x^2}+\frac{\rho_{k+1/2}w^{\tau}_{i,j-1,k+1/2}+(2-\alpha)\rho_{k+1/2}w^{\tau}_{i,j,k+1/2}+\rho_{k+1/2}w^{\tau}_{i,j+1,k+1/2}}{\Delta y^2}\right]$$

Convergence of residual



Solution and error after 1000 iterations



Multigrid method

$\nabla_0^2 w_0 = F_0$ ← Poisson equation to be solved

$e_0 = w_0 - \tilde{w}_0^{\tau}$ ← correction

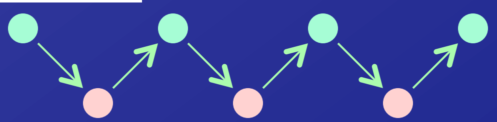
$r_0 = \nabla_0^2 \tilde{w}_0^{\tau} - F_0$ ← residual

If "correction" is estimated from residual, we can update an approximation of w.

$$\begin{aligned}\nabla_0^2 e_0 &= \nabla_0^2 w_0 - \nabla_0^2 \tilde{w}_0^{\tau} \\ &= F_0 - \nabla_0^2 \tilde{w}_0^{\tau} \\ &= -r_0\end{aligned} \longrightarrow \tilde{w}_0^{\tau+1} = \tilde{w}_0^{\tau} + e_0$$

In the multigrid method, "correction" is estimated by solving a Poisson equation on a coarser grid to adjust larger-scale efficiently.

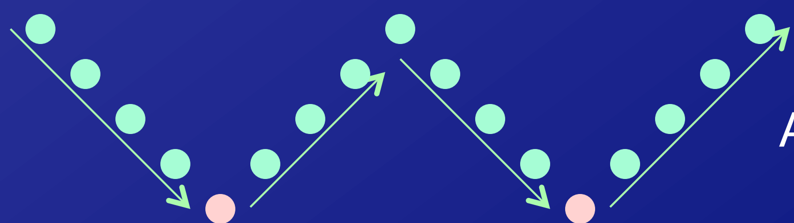
$\nabla_0^2 w_0 = F_0$ A Poisson solver is a "smoother."



$\nabla_1^2 w_1 = F_1$ A Poisson solver is a solver.

↘ Restriction (fine to coarse): $F_1 = \mathrm{R}(-r_0)$

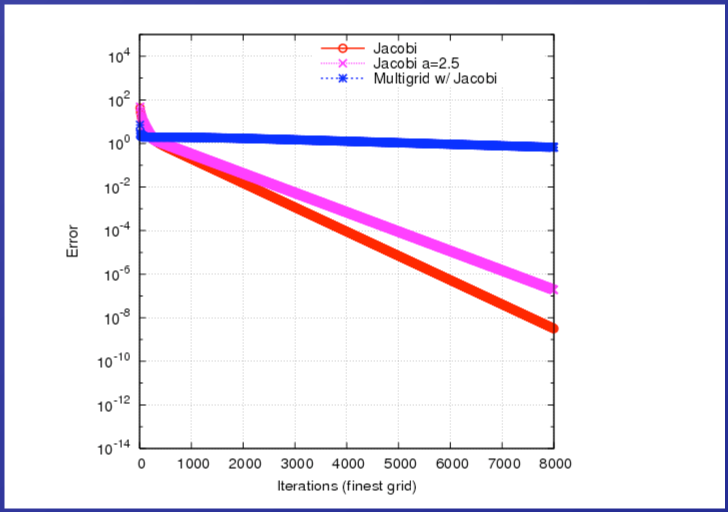
↗ Prolongation (coarse to fine): $e_0 = \mathrm{P}(\tilde{w}_1)$



A solution is obtained by repeating "V-cycle."

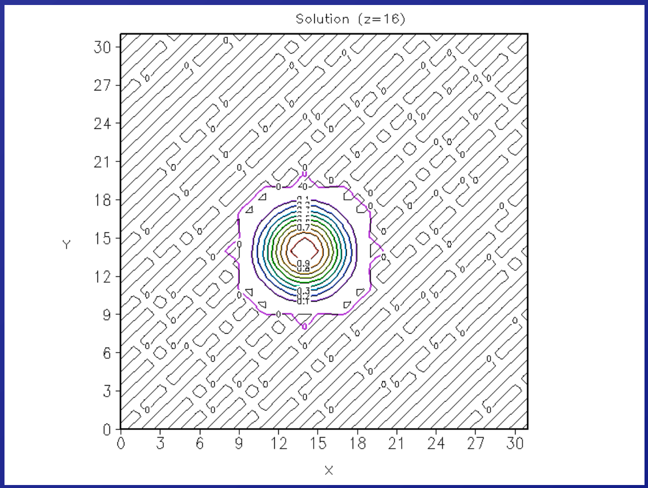
Horizontal: Multigrid + Jacobi, Vertical: Implicit

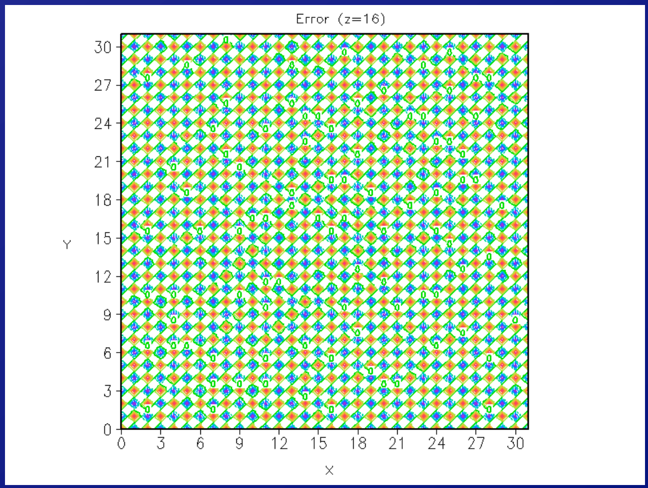
Convergence of residual



Convergence is even slower than the Jacobi method.

Solution and error after 250 V-cycles
(1004 iterations on the finest grid)

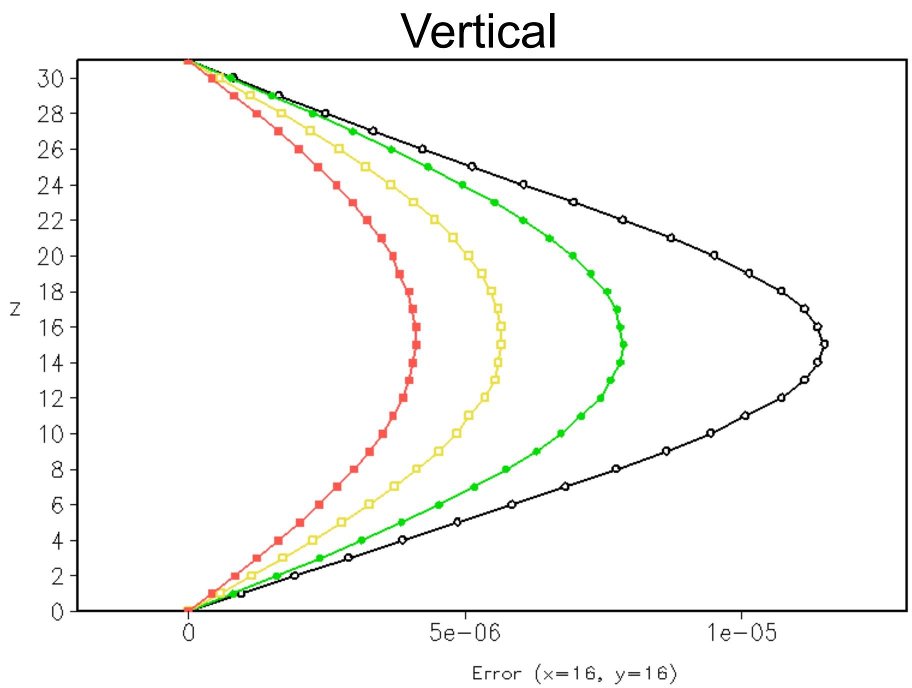


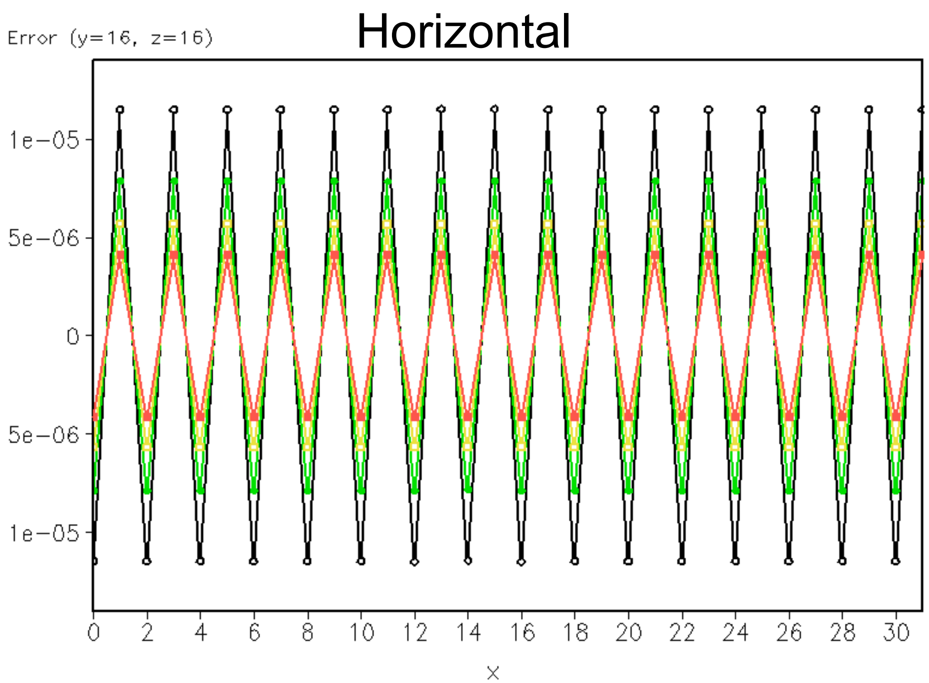


What happened?

Change in error

V-cycle = 125, 250, 375, 500



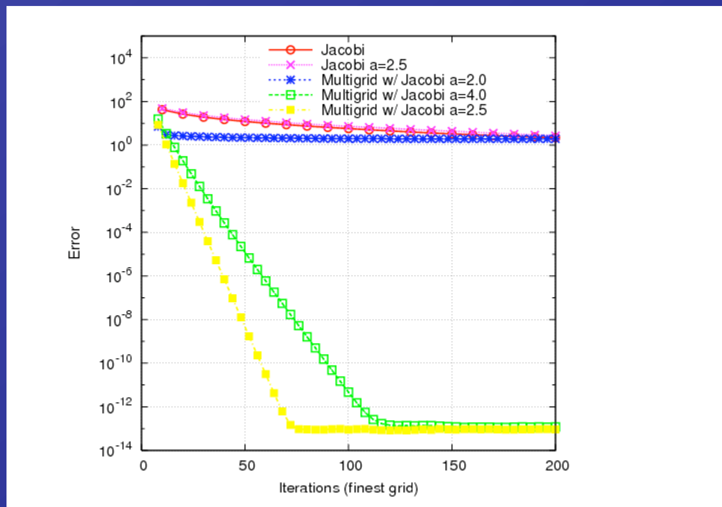


A vertical mode, which is friendly with horizontal 2-grid noise, is allowed in the system.

If the Jacobi method is applied vertically instead of the implicit form used, the vertical mode can be eliminated. But, vertical 2-grid noise appears in addition to horizontal one.

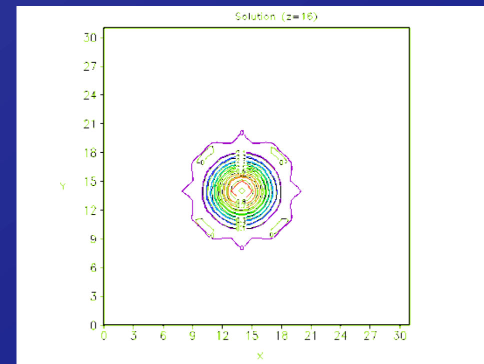
Horizontal: Multigrid + ω-Jacobi, Vertical: Implicit

Convergence of residual

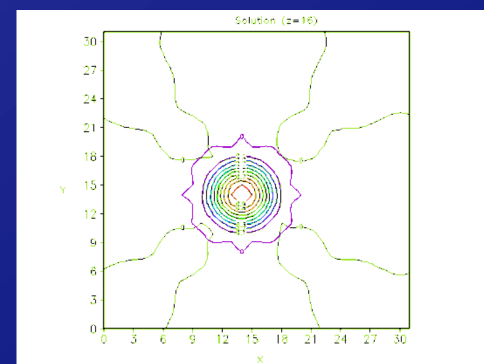


Solution

V-cycle=1



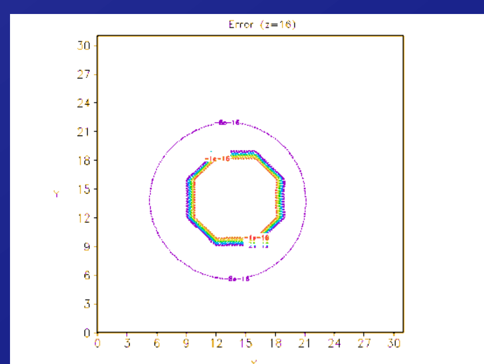
V-cycle=3



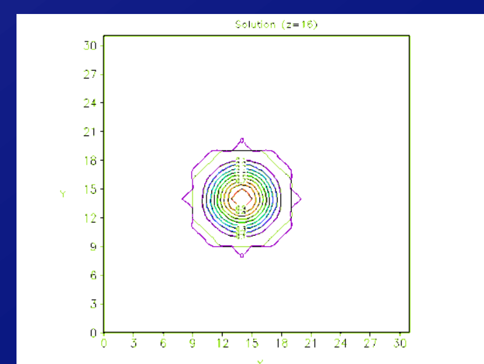
Convergence is much faster than the others.

10~20 V-cycles (40~80 iterations on the finest grid) are sufficient to achieve convergence.

Error



V-cycle=20

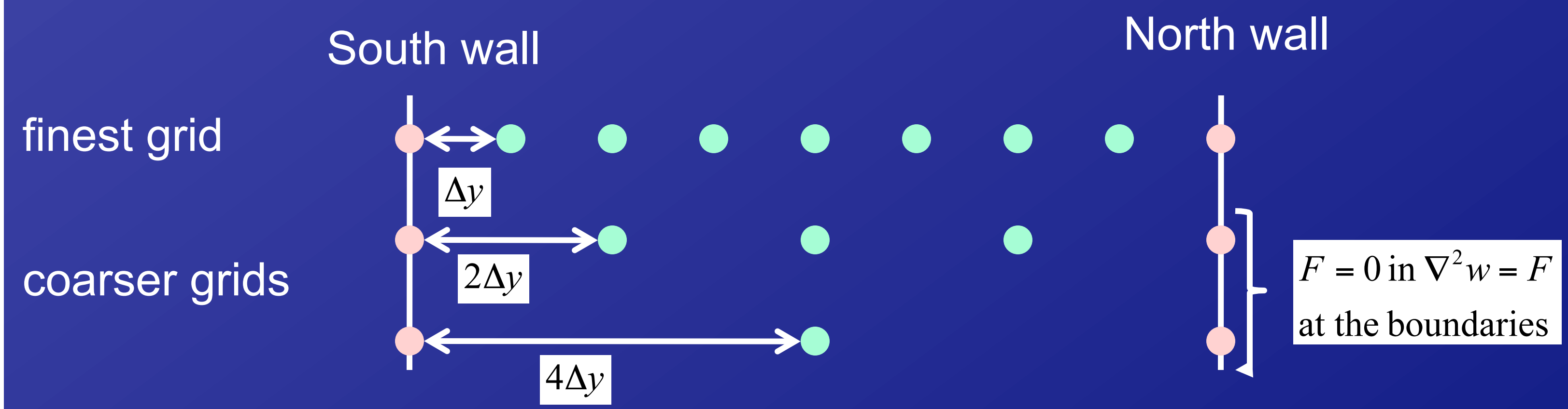


North and south boundaries

Different algorithms are used for Dirichlet and Neumann boundary conditions in restriction and prolongation .

Dirichlet condition

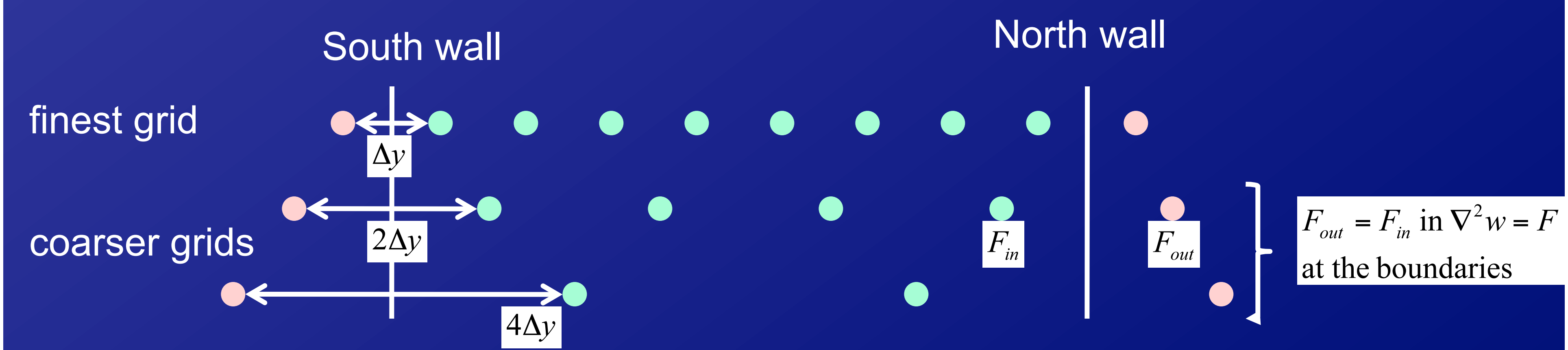
$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\psi = \zeta$$



Neumann conditions

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\chi = \delta$$

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)w+\frac{\partial}{\partial z}\left[\frac{1}{\rho_0}\frac{\partial}{\partial z}(\rho_0 w)\right] = -\frac{\partial \eta}{\partial x}+\frac{\partial \xi}{\partial y}$$



Current configuration

- Parallelized using MPI (domain decomposition in X and Y)
- Boundaries
  - free-slip rigid walls in Z
  - Cyclic or free-slip rigid walls in X, Y
- Dynamics
  - Governing equations: an anelastic system (Jung and Arakawa, 2008)
  - Spatial discretization
    - Following an updated version of Jung and Arakawa (2005)
      - Arakawa C-grid
      - Lorenz grid in vertical
      - 2^{nd}-order centered schemes except for advection
      - 3^{rd}-order upwind biased advection
    - slope limiter for TVD (optional): min-mod limiter
    - flux limiter for monotonicity: N/A
    - 3D and 2D Poisson solvers
      - 2D Multigrid using ω-Jacobi or Red-Black solver
  - Temporal discretization
    - 3^{rd}-order or 2^{nd}-order Runge-Kutta scheme
- Physics
  - N/A

Cold bubble experiment

Settings following Straka et al. (1993)

No explicit diffusions here

dx(=dy)=dz=100 m
dt = 1 s
nx= 256 x 2 (processes)
nz = 60
np = 2

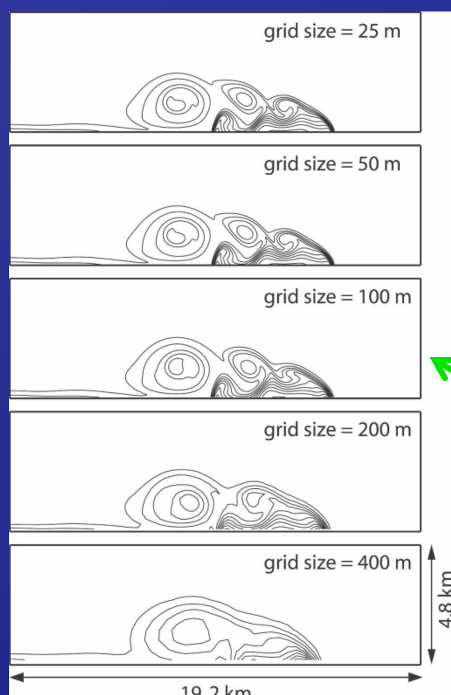
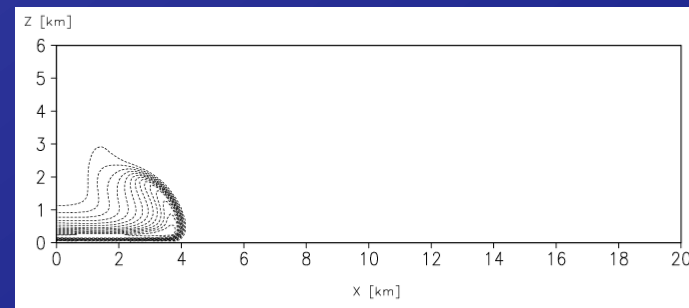
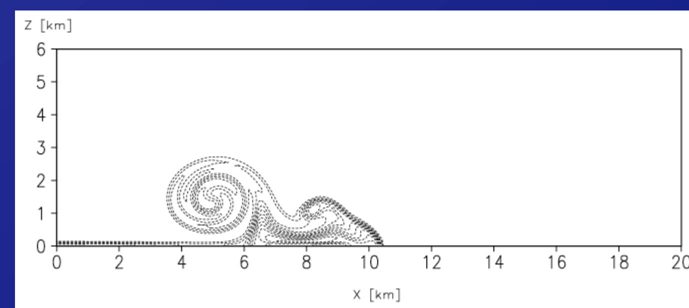


Fig. 3 of Jung and Arakawa (2008)

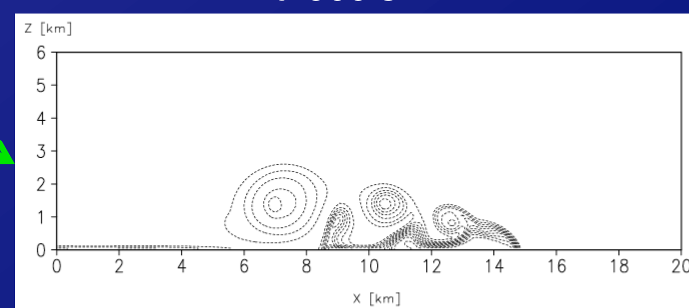
t=300 s



t=600 s



t=900 s



Difference may be attributed to the lack of the explicit diffusion.

Dependence on V-cycle

dx(=dy)=dz=400 m
dt = 4 s
nx= 64 x 2 (processes)
nz = 15
np = 2

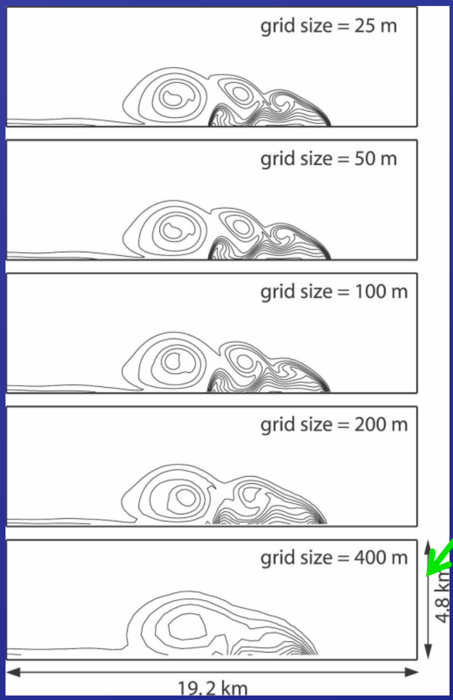
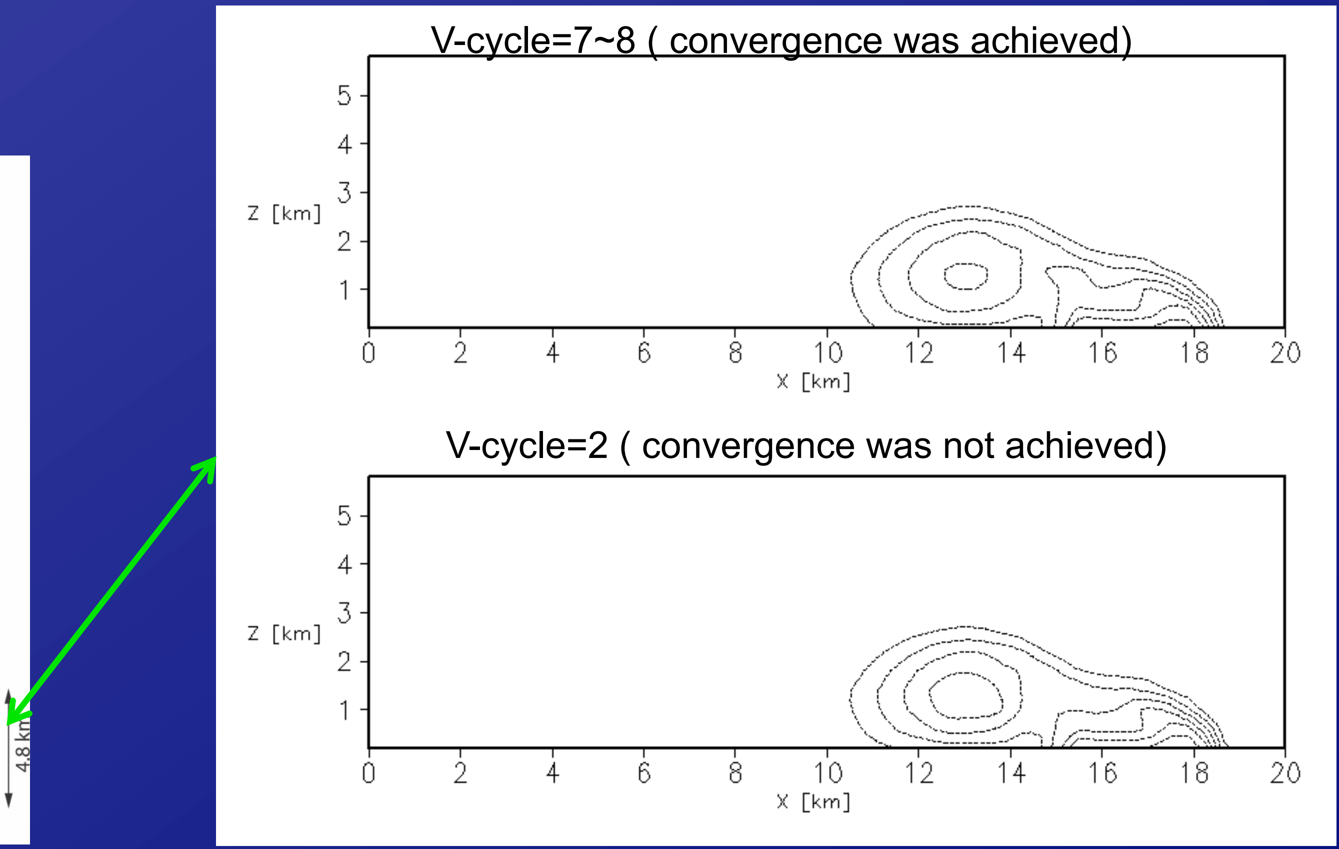


Fig. 3 of Jung and Arakawa (2008)



We can obtain a similar result even if convergence of the Poisson solver is insufficient.

Held-Suarez(-like) test

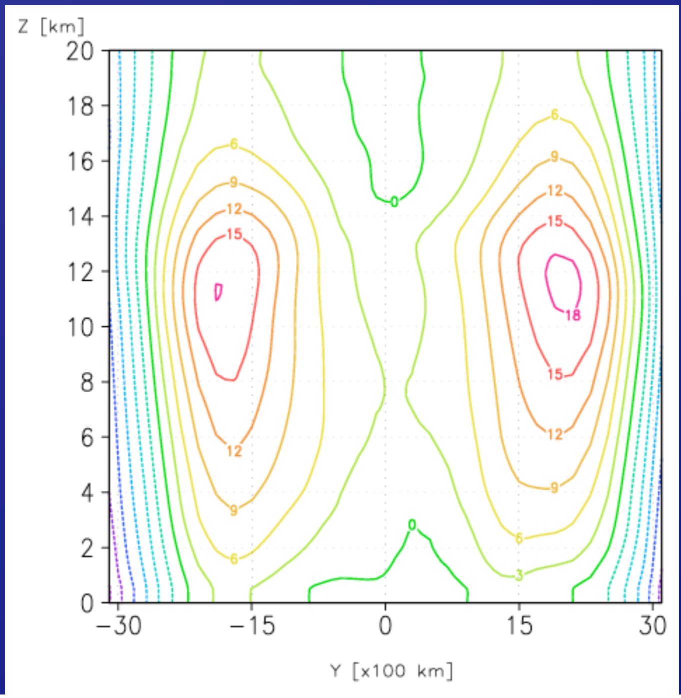
Following Held and Suarez (1994), but the forcing terms are modified to be a function of z because pressure is not diagnosed in my model currently.

Equatorial beta plane was assumed.

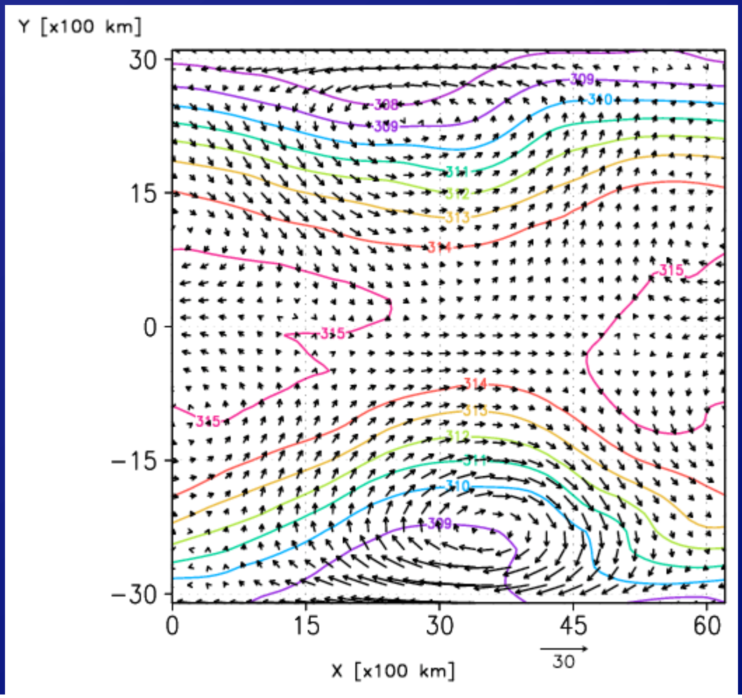
dx=dy= 200 km
dz= 500 m
dt = 600 s
nx= 16 x 2 (processes)
ny= 16 x 2 (processes)
nz = 60
np = 4

Day 30

Zonal mean zonal wind



Potential temperature and winds



Summary

- <u>A dry version of a zonal channel VVM is working.</u>
  - The model is parallelized using MPI.
  - A multigrid Poisson solver was developed.
  - Evolution of a cold bubble was simulated reasonably.
  - In a Held-Suarez-like test, at least for a 30 day integration, a jet was generated around 20S and 20N with maximum strength of about 20 m/s. But, easterly winds were unrealistically strong near the north and south boundaries.
- <u>Future issues</u>
  - "Opening" the south and north walls for a realistic flow
    - Following a document by Prof. Arakawa
  - Chaney-Phillips grid
  - Flux limiter
  - Efficiency of the multigrid solver
  - Physical parameterizations