Topographically Bound Balanced Motions

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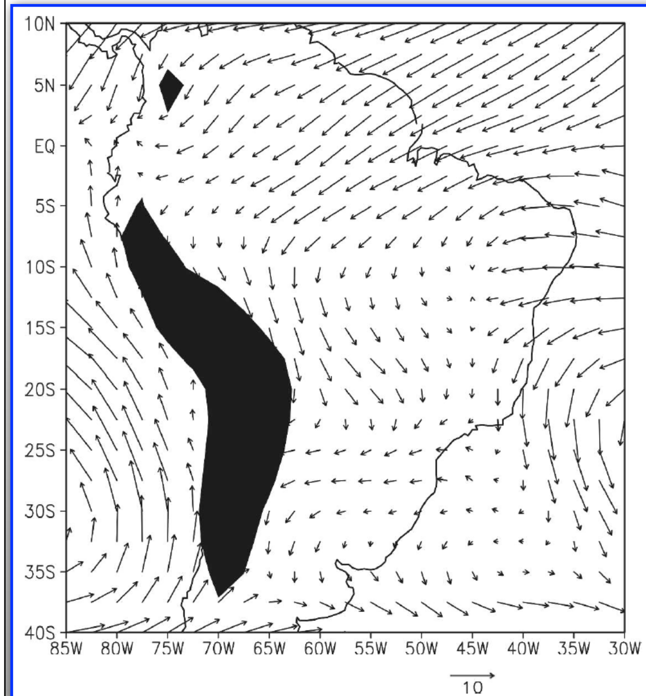
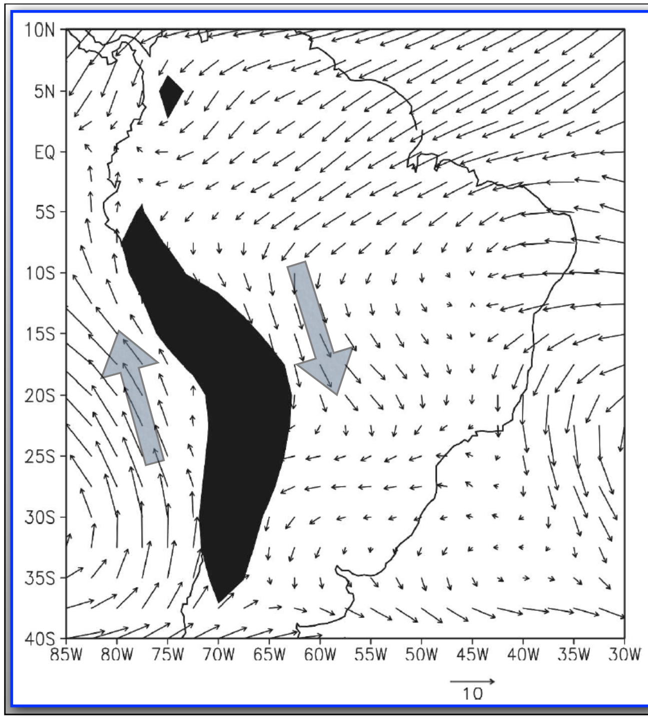


Jan 2003 Mean 925 hPa Wind Over South America

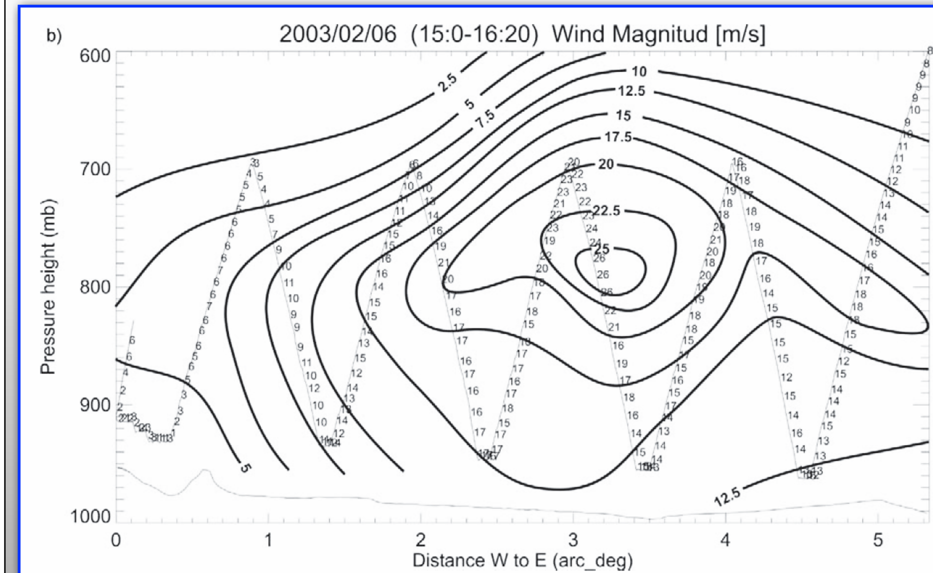
- NCEP-DOE Reanalysis II Data
- T62/L28 Resolution
- Shaded region indicates where the Andes lie above 925 hPa
- From Tarasova et al. (2006)



Jan 2003 Mean 925 hPa Wind Over South America

- Note the strong cyclonic flow centered near the Andes
- Involves 2 low level jets:
  - South American Low-Level Jet (SALLJ)
  - Coastal LLJ

Cross Section Through the SALLJ on 6 Feb 2003



Data from SALLJEX

From Vera et al. (2006)

Deriving a PV Invertibility Principle

- Modeling assumptions:
  - compressible, stratified fluid on an f-plane
  - inviscid, quasi-hydrostatic, y-independent motions
- Potential vorticity (PV):

$$\frac{DP}{Dt} = 0 \quad \text{where} \quad P = \left(f + \frac{\partial v}{\partial x}\right)\left(-\frac{1}{g}\frac{\partial p}{\partial \theta}\right)^{-1}$$

- Far-field flow vanishes:

$$p(x,\theta) \longrightarrow \tilde{p}(\theta) \quad \text{and} \quad P(x,\theta) \longrightarrow \tilde{P}(\theta)$$

- Far-field PV:

$$\tilde{P} = f\left(-\frac{1}{g}\frac{\partial \tilde{p}}{\partial \theta}\right)^{-1}$$

Deriving a PV Invertibility Principle

- Ratio of PV's:

$$
\begin{aligned}
f\frac{P}{\tilde{P}} &= \left(f + \frac{\partial v}{\partial x}\right)\left(\frac{\partial \tilde{p}/\partial \theta}{\partial p/\partial \theta}\right) \\
&= \left(f + \frac{\partial v}{\partial x}\right)\left(\frac{\tilde{\rho}(d\tilde{\Pi}/d\tilde{p})(\partial \tilde{p}/\partial \theta)}{\rho(d\Pi/dp)(\partial p/\partial \theta)}\right) \\
&= \left(f + \frac{\partial v}{\partial x}\right)\left(\frac{\partial \tilde{\Pi}/\partial \theta}{\partial \Pi/\partial \theta}\right)
\end{aligned}
$$

where the Exner function is

$$\Pi = c_p(p/p_0)^{R/c_p} \quad \text{and} \quad \tilde{\Pi} = c_p(\tilde{p}/p_0)^{R/c_p}$$

Deriving a PV Invertibility Principle

- Define the buoyancy frequency:

$$N^2(\theta) = \frac{g^2}{\theta^2}\left(-\frac{d\tilde{\Pi}}{d\theta}\right)^{-1}$$

- The PV ratio then leads to:

$$\frac{\partial v}{\partial x} + \left(\frac{f\theta^2 N^2 P}{g^2 \tilde{P}}\right)\frac{\partial \Pi'}{\partial \theta} = f\left(\frac{P}{\tilde{P}} - 1\right)$$

where the Exner function anomaly is

$$\Pi'(x,\theta) = \Pi(x,\theta) - \tilde{\Pi}(\theta)$$

Deriving a PV Invertibility Principle

- Including topography:
  - geopotential along topography: $\phi_S(x)$
  - potential temperature along topography: $\theta_S(x)$
  - boundary of massless layer on a theta surface: $x_S(\theta)$
- Massless layer: $\theta_B < \theta < \theta_S(x)$
- Additional assumptions:
  - symmetry of $\phi_S(x)$ and $\theta_S(x)$ about $x = 0$
  - geostrophic and hydrostatic balance
  - PV is uniform on each isentropic surface above the massless layer

PV Invertibility Principle

Elliptic problem:

$$\frac{\partial v}{\partial x} + \left(\frac{f\theta^2 N^2}{g^2}\right)\frac{\partial \Pi'}{\partial \theta} = 0 \quad \text{for} \quad x_S(\theta) < x < \infty,$$

$$\left(\frac{f\theta^2 N^2}{g^2}\right)\frac{\partial \Pi'}{\partial \theta} = f \quad \text{for} \quad 0 \le x < x_S(\theta),$$

$$f\frac{\partial v}{\partial \theta} - \frac{\partial \Pi'}{\partial x} = 0 \quad \text{for} \quad 0 \le x < \infty, \quad \theta_B \le \theta \le \theta_T$$

Horizontal BC's:

$$\left.\begin{array}{r} v \to 0 \\ \Pi' \to 0 \end{array}\right\} \quad \text{as} \quad x \to \infty$$

Vertical BC's:

$$\Pi' = 0 \qquad \text{at} \quad \theta = \theta_T,$$

$$f\left(v - \theta\frac{\partial v}{\partial \theta}\right) = \frac{d\phi_S(x)}{dx} \qquad \text{at} \quad \theta = \theta_B$$

Solving the PV Invertibility Principle

- Utilize Fourier integral transforms in x:

$$\hat{v}(k,\theta) = (2/\pi)^{1/2} \int_0^\infty v(x,\theta)\sin(kx)\,dx,$$

$$v(x,\theta) = (2/\pi)^{1/2} \int_0^\infty \hat{v}(k,\theta)\sin(kx)\,dk$$

and

$$\hat{\Pi}(k,\theta) = (2/\pi)^{1/2} \int_0^\infty \Pi'(x,\theta)\cos(kx)\,dx,$$

$$\Pi'(x,\theta) = (2/\pi)^{1/2} \int_0^\infty \hat{\Pi}(k,\theta)\cos(kx)\,dk$$

Transformed PV Invertibility Principle

- Fourier transform of the elliptic problem:

$$k\hat{v} + \left(\frac{f\theta^2 N^2}{g^2}\right)\frac{d\hat{\Pi}'}{d\theta} = F(k,\theta),$$

$$f\frac{d\hat{v}}{d\theta} + k\hat{\Pi}' = 0$$

with boundary conditions

$$\hat{\Pi}' = 0 \qquad \text{at} \quad \theta = \theta_T,$$

$$f\left(\hat{v} - \theta\frac{d\hat{v}}{d\theta}\right) = -k\hat{\phi}_S(k) \quad \text{at} \quad \theta = \theta_B$$

where

$$F(k,\theta) = (2/\pi)^{1/2}\int_0^{x_S(\theta)}\left(f + \frac{\partial v}{\partial x}\right)\cos(kx)\,dx$$

(measure of "absolute isentropic vorticity in the mtn.")

Special Cases

- Balanced wind and mass fields are forced in two ways:
  - by $F(k,\theta)$ and by $\hat{\phi}_S(k)$
- Trivial case:

$$\left.\begin{array}{r} F(k,\theta)=0 \\ \hat{\phi}_S(k)=0 \end{array}\right\} \Longrightarrow \left\{\begin{array}{r} \hat{v}(k,\theta)=0 \\ \hat{\Pi}'(k,\theta)=0 \end{array}\right.$$

Special Case	$F(k,\theta)$	$\hat{\phi}_S(k)$
• Flat topography • Variation of θ along bottom boundary	$\neq 0$	$= 0$
• Nonzero topography (Eliassen 1980) • No variation of θ along bottom boundary	$= 0$	$\neq 0$

Simple Analytical Solutions

- Topography:
  - mountain of height H and width a

$$\phi_S(x) = gHe^{-x^2/a^2} \quad \Longrightarrow \quad \hat{\phi}_S(k) = \frac{gHa}{\sqrt{2}} e^{-a^2k^2/4}$$

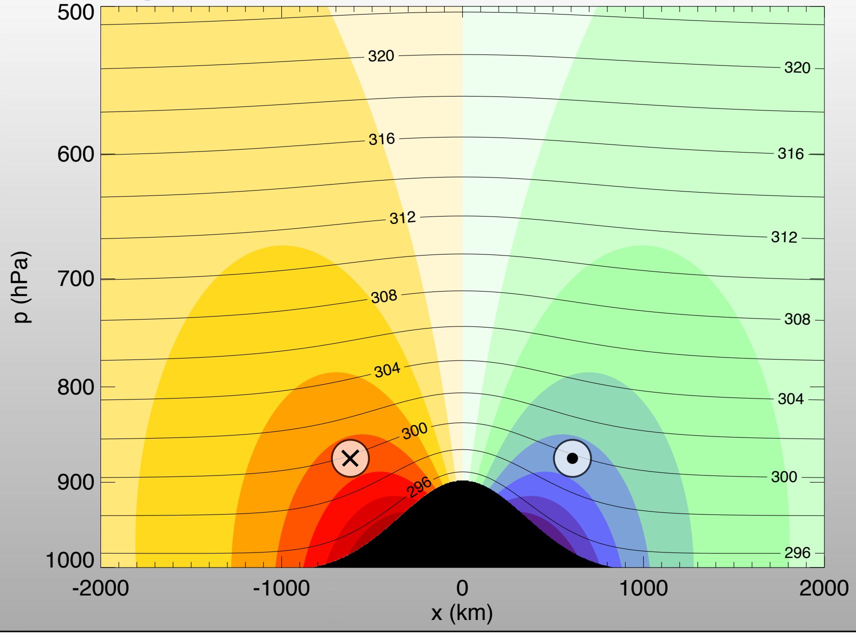
- Two simple reference state profiles:

(1) Buoyancy frequency is inversely proportional to θ

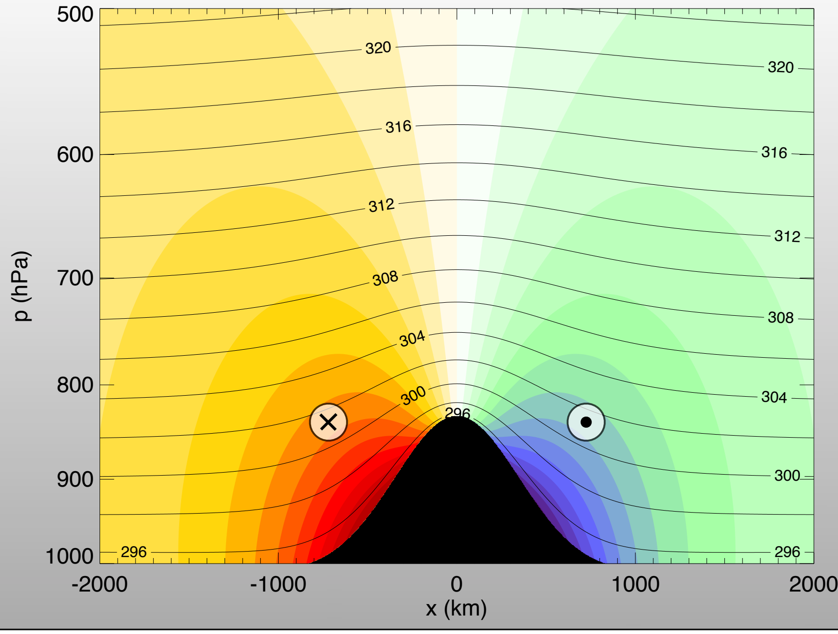
$$N(\theta) = N_1 \theta_B / \theta$$

(2) Buoyancy frequency is a constant $N(\theta) = N_2$

Isentropic Mountain with H = 1000 m



Isentropic Mountain with H = 1700 m



Summary & Conclusions

- The SALLJ and Coastal LLJ are not separate entities
- We anticipate that a balanced response to diabatically heated topography can explain such jets
- We have confirmed Eliassen's solutions for isentropic mountains
- Massless layers are a necessary ingredient in this approach
- Generalization of these results to the sphere and the use of more realistic topography will allow for more direct comparison with observations
- Concerning Knowledge Transfer, these analytical solutions can form the basis for the comparison of GCM solutions having a variety of horizontal and vertical discretizations