

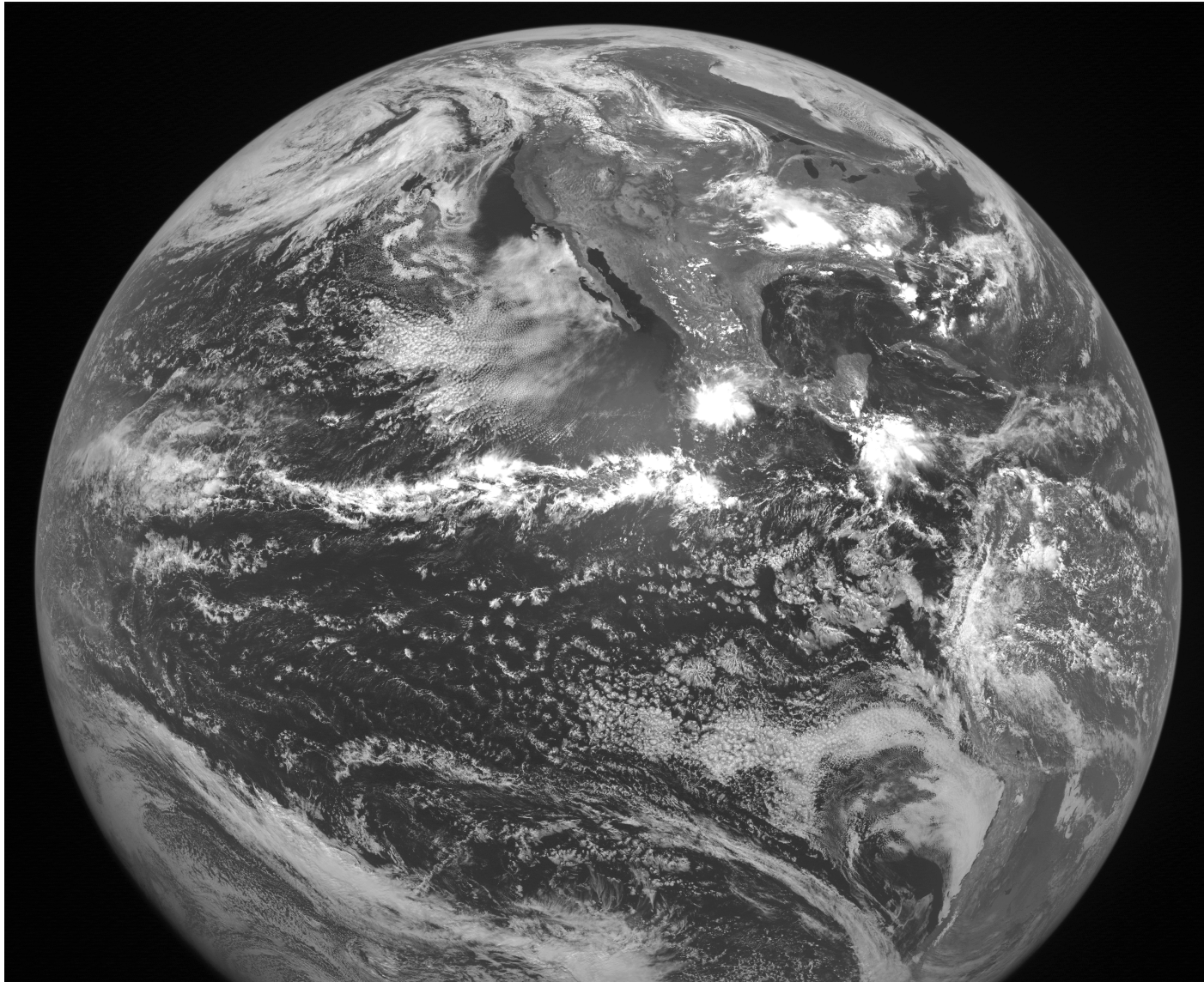


Slow manifolds and multiple equilibria in  
stratocumulus-capped boundary layers

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*JAMES*, 2, Art. #14, 2010

What do cloud-topped boundary layers remember?



# Approach

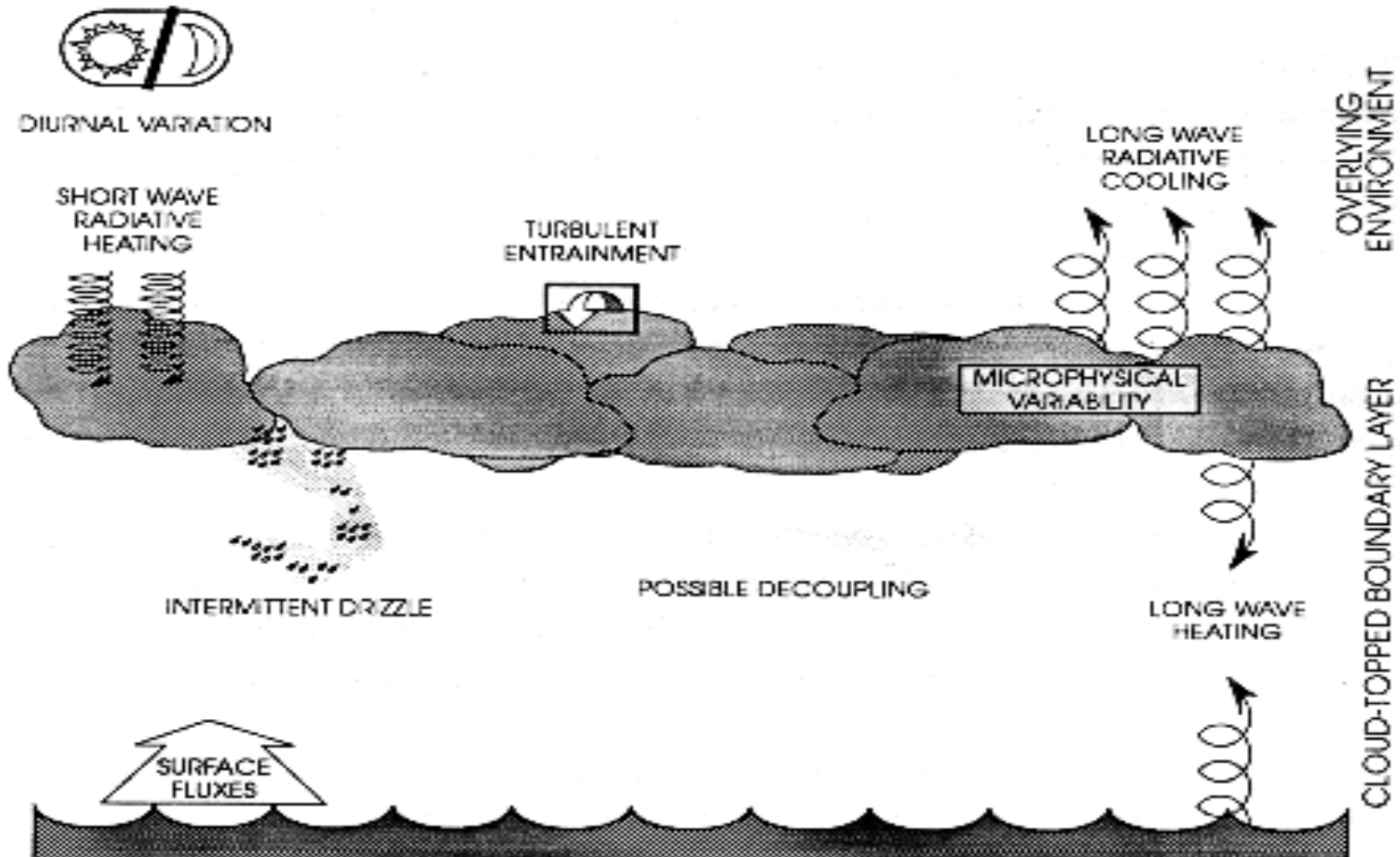
Goal: Identify fast and slow timescales of boundary-layer adjustment to changing 'forcings' (SST, surface wind, free-tropospheric temperature and moisture, vertical motion) and use these to our advantage.

Adjusts in  $< 1$  day  $\leftrightarrow$  Stays adjusted to typical changes

## Method

- A simple mixed layer model (MLM) of stratocumulus gives insight into Sc timescales and inspire 'slow-manifold' thinking, including the possibility of regime transitions and multiple equilibria.
- Transfer approach to large-eddy simulation (LES)...with wrinkles
- Implications for observations of Sc.

## Physical processes in subtropical Sc layers



- Surface heating and long-wave radiative cooling cause convective mixing
- Clouds are a strong feedback on turbulence.
- Strong capping inversion slows entrainment

# Profiles in a stratocumulus-capped mixed layer

‘Well-mixed’: Moist-conserved variables

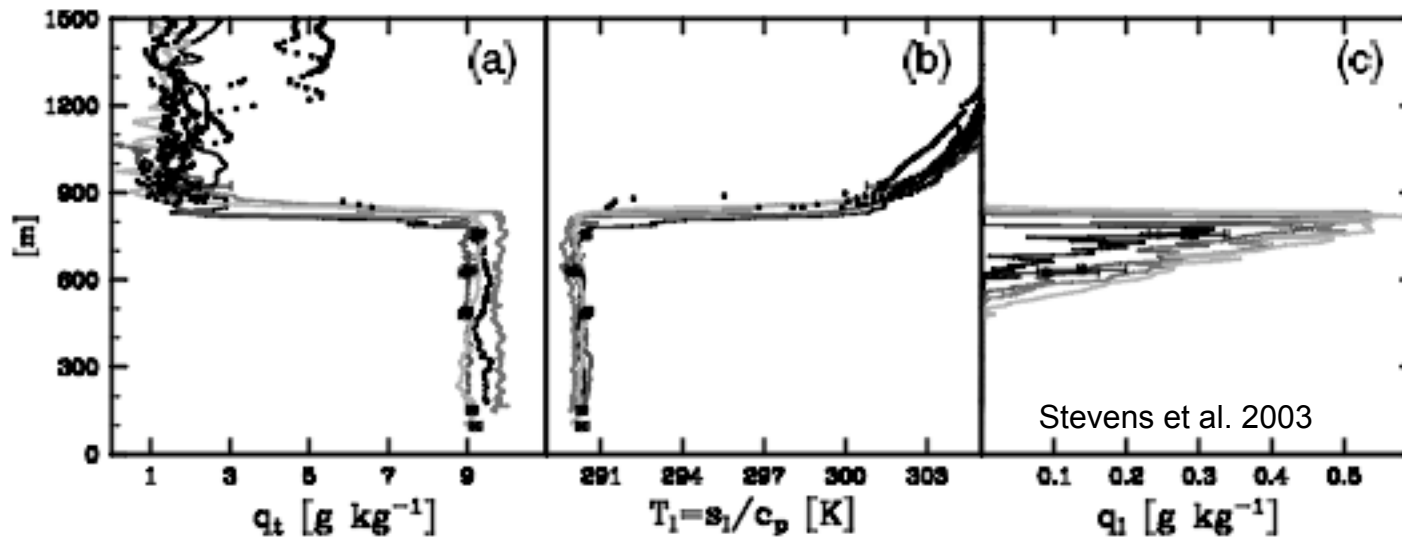
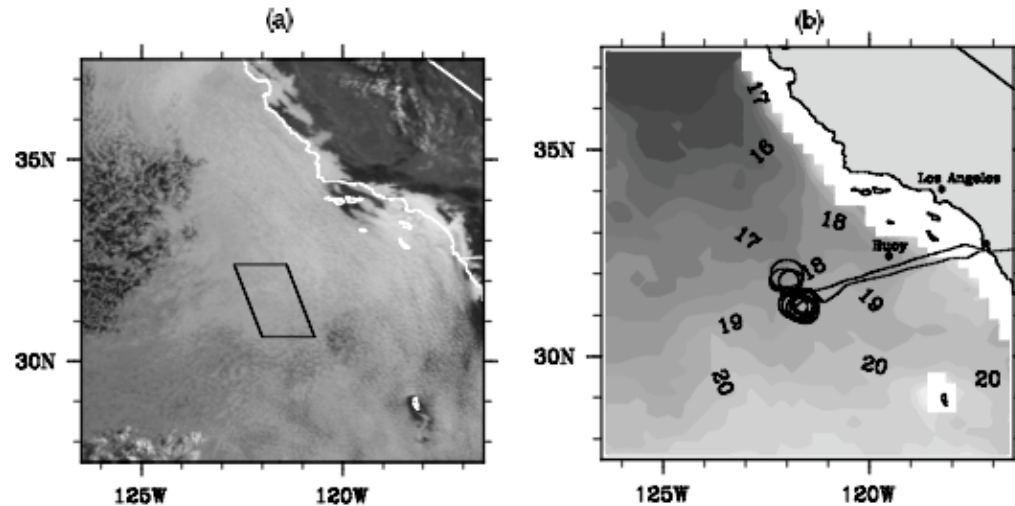
$$s_l = c_p T + gz - Lq_l,$$

$$q_t = q_v + q_l$$

$$h = c_p T + gz + Lq_v$$

$$= s_l + Lq_t$$

are nearly uniform with height within the MBL.



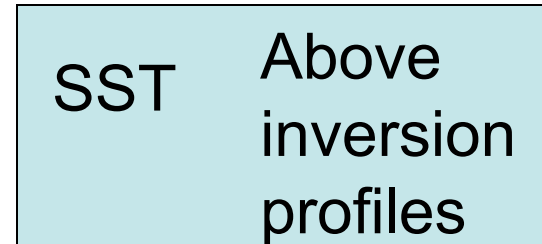
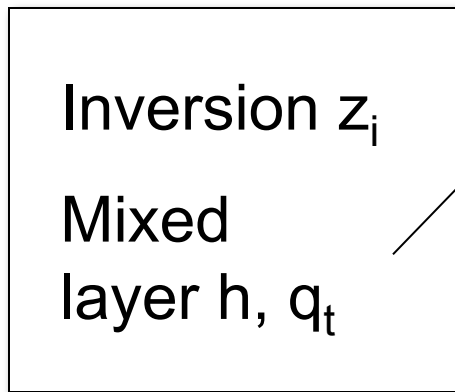
$q_l$  increases linearly with  $z$  above cloud base

Figure 1. Cloud-layer state as observed during RF01: (a) total-water specific humidity  $q_t$ , (b) liquid-water static energy temperature  $s_l/c_p$ , and (c) liquid-water specific humidity  $q_l$ . Lines are from soundings, darker indicating earlier, filled circles and bars denote level-leg means and standard deviations, and dots denote dropsonde data from the above-cloud portion of the descent.

# MLM

Bretherton and Wyant (1997)  
Caldwell et al. (2009)

3 prognostic variables



Cloud base

Drizzle & radiation flux

Sedimentation

Surface fluxes

Buoyancy flux

$$\frac{dz_i}{dt} = w_e - D z_i$$

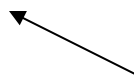
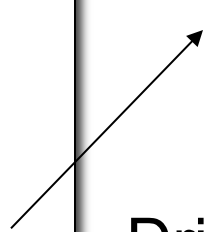
$$\frac{dq_t}{dt} = -\frac{\partial}{\partial z}(\overline{w'q'_t}(z) + F_p(z))$$

$$\frac{dh}{dt} = -\frac{\partial}{\partial z}(\overline{w'h'}(z) + F_R(z))$$

Entrainment rate  $w_e = A w_*^3 / \Delta b_i z_i$

$$A = a_1 \{ 1 + a_2 \chi_* (1 - \Delta b_2 / \Delta b_i) \exp(-a_{sed} w_{sed} / w_*) \}$$

$$w_* = \left( 2.5 \int_0^{z_i} \overline{w'b'} dz \right)^{1/3}$$

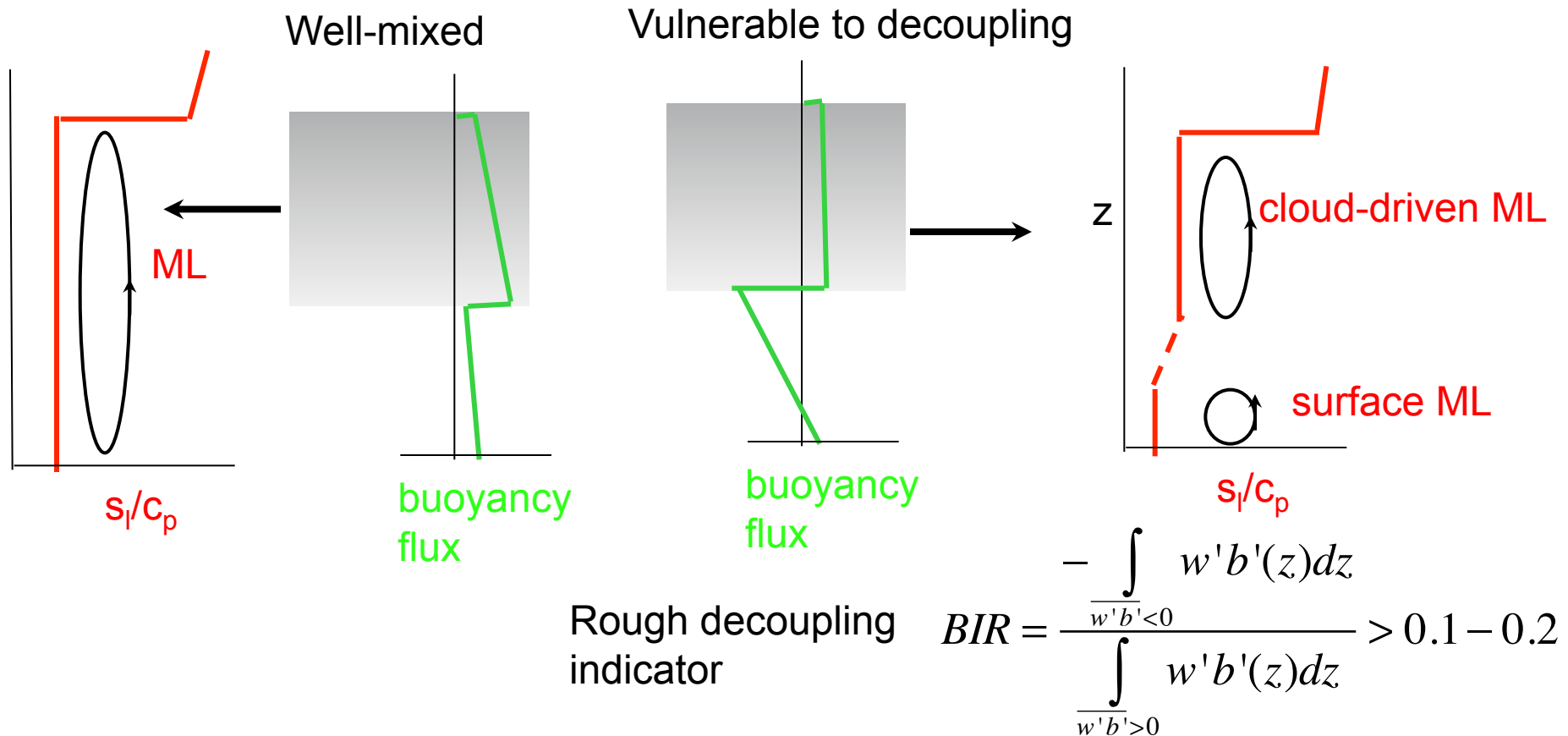


# Decoupling

Sc-capped boundary layers are driven by cloud top radiative/evaporative cooling and by surface heating. Two separate maxima of buoyancy flux (turbulence generation), separated by minimum at cloud base.

Processes that stratify the boundary layer favor decoupling.

- More entrainment of dry/warm air or less cloud top cooling.
- Precipitation (thick cloud layer).



# LES...more realistic (mostly), also much more complex

6 prognostic variables  
per gridpoint

**Velocity**  $u, v, w$

**Liquid static energy**

$$s_{li} = c_p T + gz - Lq_l$$

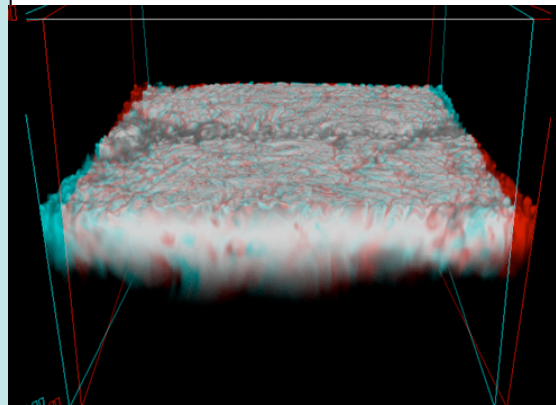
**Cloud microphysics**

$$q_t = q_v + q_l$$

( $q_l$ : saturation adjustment)

**Rain water**  $q_r$

(autoconv., collection, evap.)



← 6 km →

Parameterized  
processes

**Microphysics**

$q_l$ : saturation  
adjustment, droplet  
sedimentation

$q_r$ : fallspeed, autoconv.,  
collection, evap.

**Radiation**

**Subgrid turb. mixing**

**Large-scale advection**

**Surface fluxes**



# Key Insight from MLM

Response of an idealized MLM to a sudden  
2K SST increase:

Fast adjustment of thermodynamic structure:

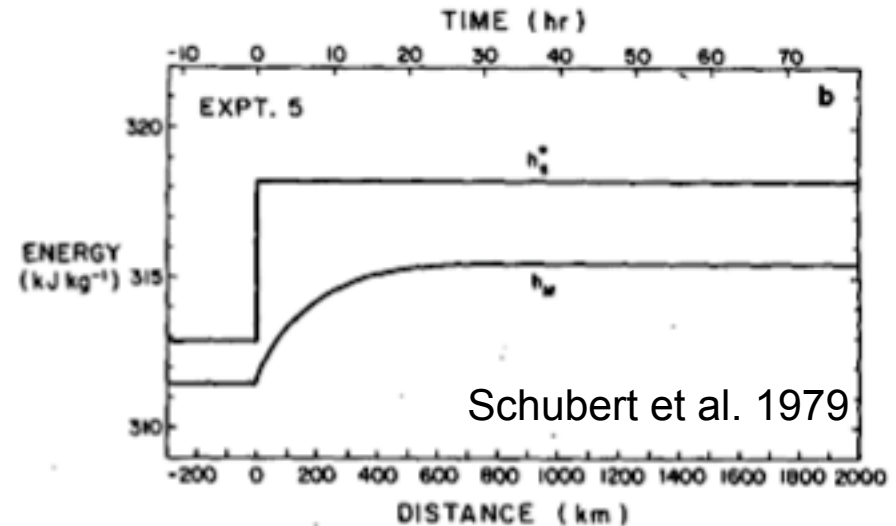
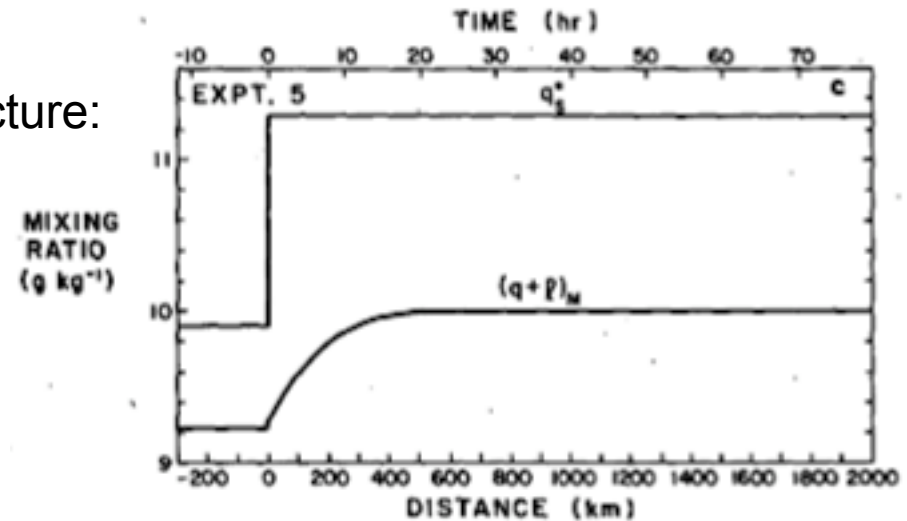
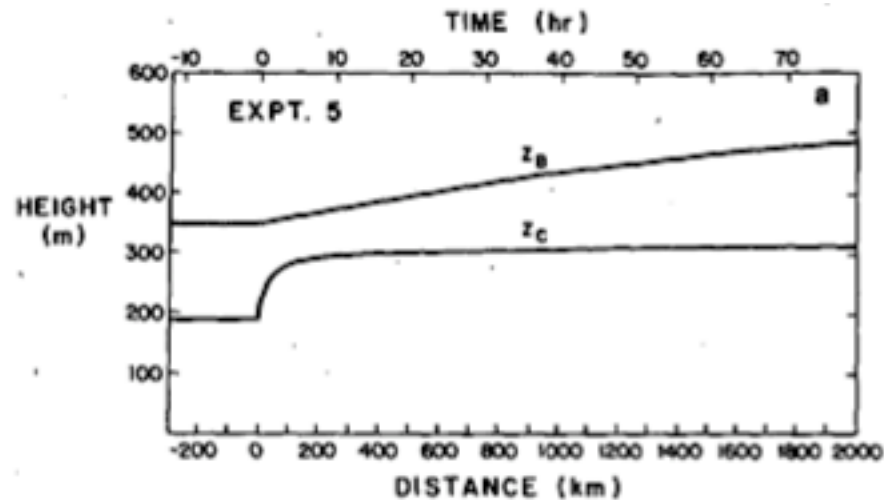
$$Dq_{tm}/Dt = w_s(q_s^* - q_{tm}) + w_e(q_t^+ - q_{tm})$$

$$t_{fast} = z_i / (w_s + w_e) \sim 0.5 \text{ d}$$

Slow adjustment of inversion structure

$$Dz_i/Dt = w_e - Dz_i$$

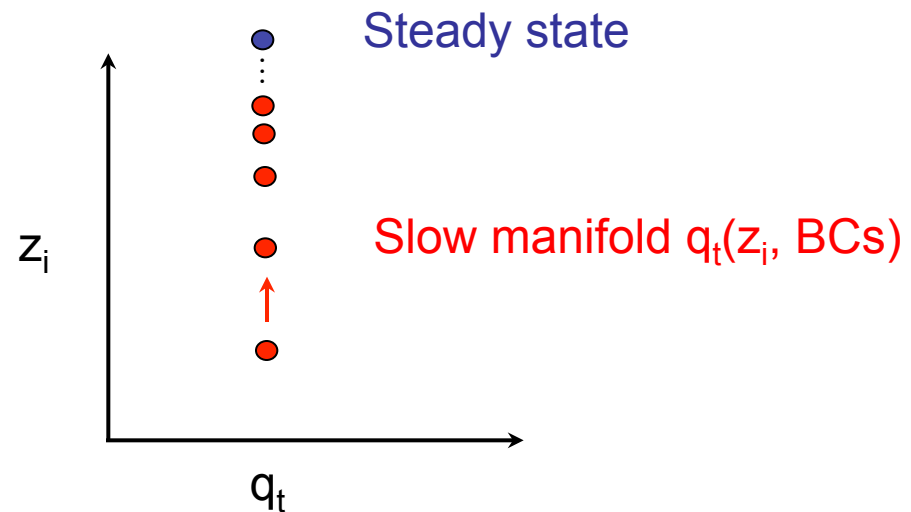
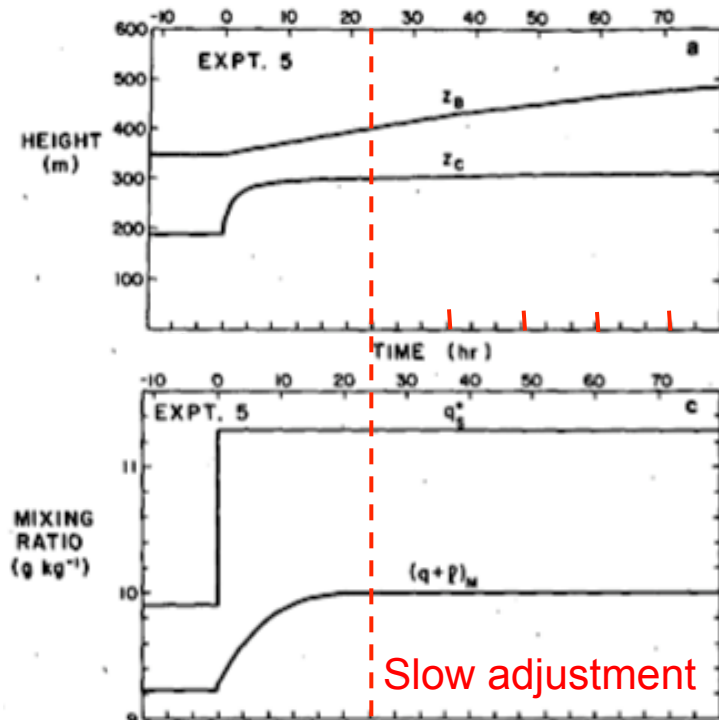
$$t_{inv} = D^{-1} \sim 3 \text{ d}$$



Schubert et al. 1979

# Implication: A 'slow-stable' manifold (after Leith 1980)

- If boundary conditions (BCs) are fixed, after a day or so, the solution locks onto a 1D 'slow-stable' manifold in which all variables ( $h_m$ ,  $q_{tm}$ ,  $z_i$ ) covary while slowly approaching a steady state.



- In the real world, the BCs vary over timescales of days due to column advection and weather. Hence the boundary layer stays thermodynamically adjusted (i. e. on slow manifold) but  $z_i$  will not get near equilibrium.

# Slow manifold behavior in MLM and LES

GEWEX Cloud System Study DYCOMS-II RF01 case (Stevens et al. 2005)

- Nonprecipitating well-mixed nocturnal Sc
- Entrainment rate, liquid water path, turbulence statistics well-observed.
- Initialized with cloud-topped mixed layer, depth  $z_i = 840$  m,  $N = 150 \text{ cm}^{-3}$ .
- Linear temperature, moisture profiles above cloud layer, initial  $\Delta\theta = 9$  K.
- $D = 3.75 \times 10^{-6} \text{ s}^{-1}$ , SST = 292.5 K.
- Simplified dependence of radiative cooling on cloud structure

Identical initialization and forcing of LES and MLM.

Original goal: Test whether LES and MLM have the same response to changing cloud droplet concentration  $N$  (a measure of aerosol pollution)

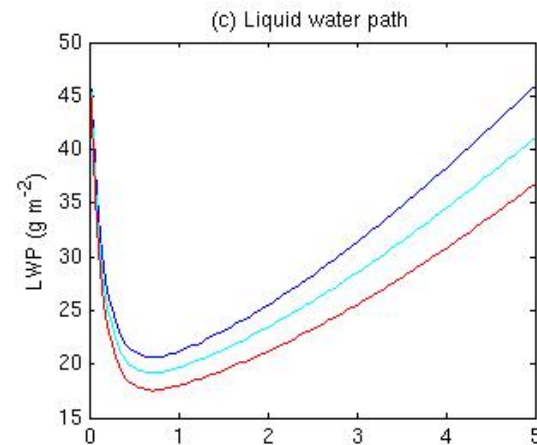
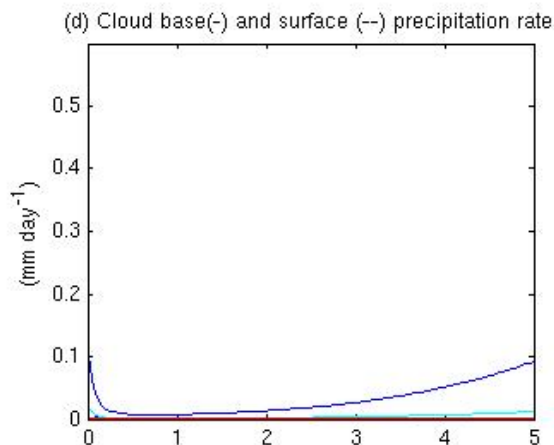
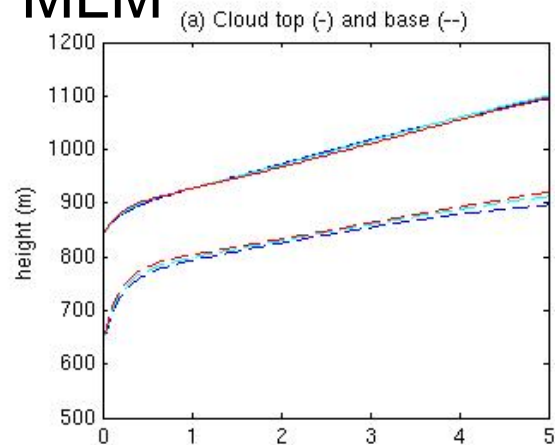
Here: Analyze approach of the two models to steady-state, esp. long times.

SAM6.7 LES:  $\Delta x = \Delta y = 25$  m,  $\Delta z = 5$  m up to 1500 m,  $L_x = L_y = 2.4$  km, periodic BCs (More details: Uchida et al. 2010, ACP).

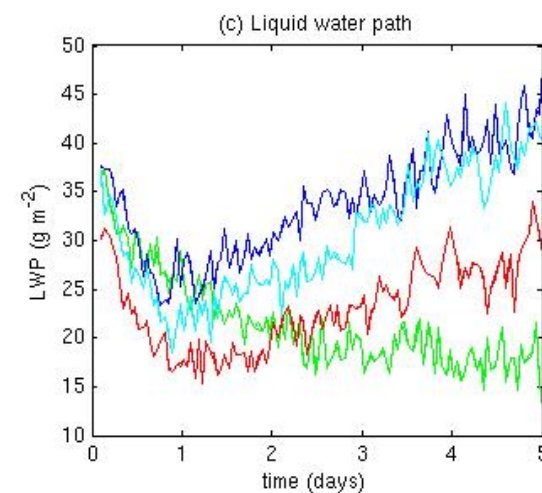
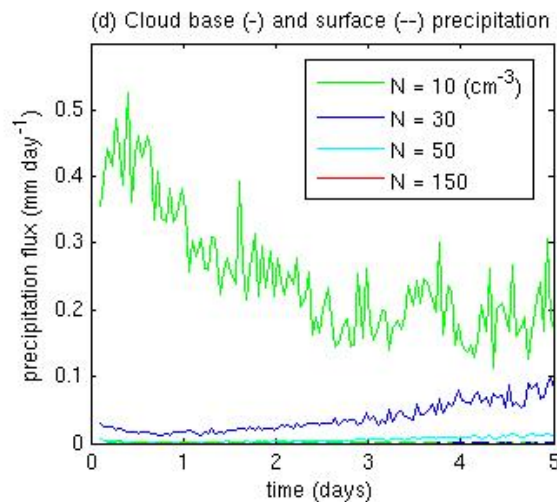
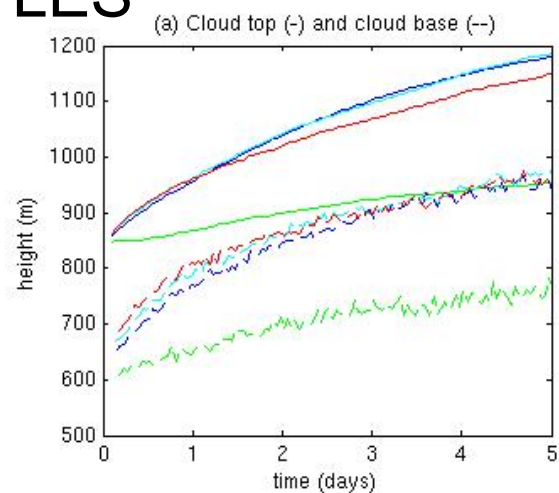
MLM of Caldwell et al. (2009) with LES-tuned entrainment and drizzle

# MLM and LES evolution, sensitivity to N are comparable

## MLM



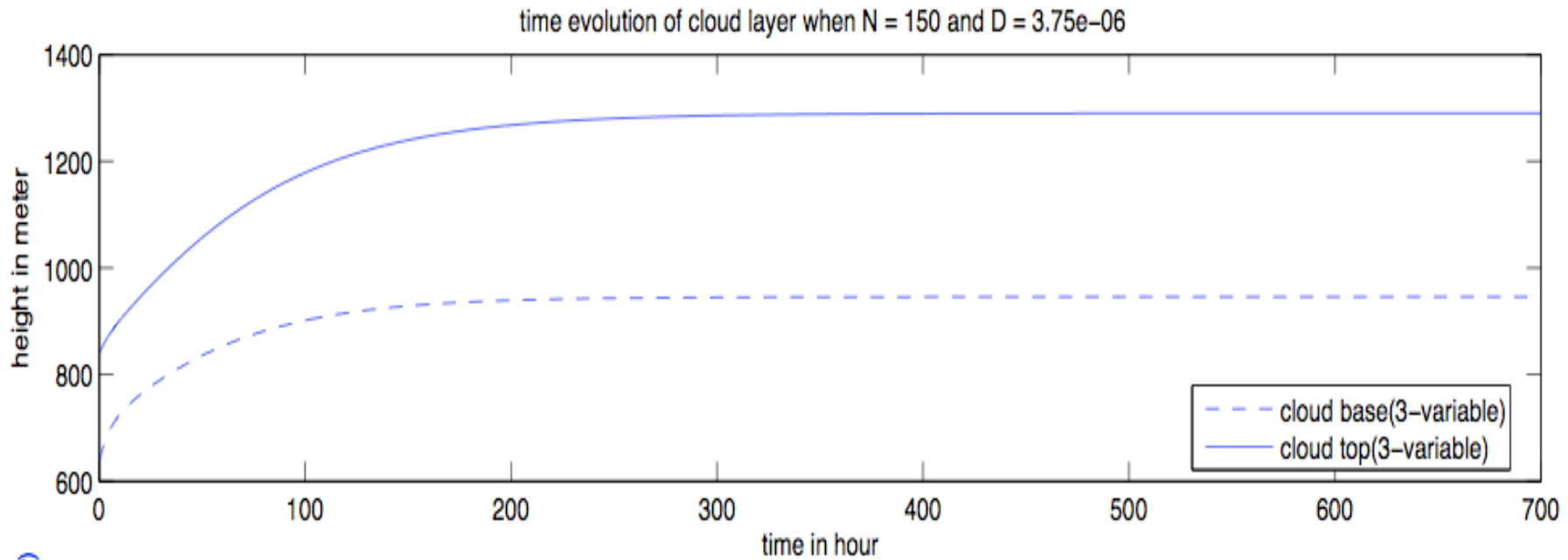
## LES



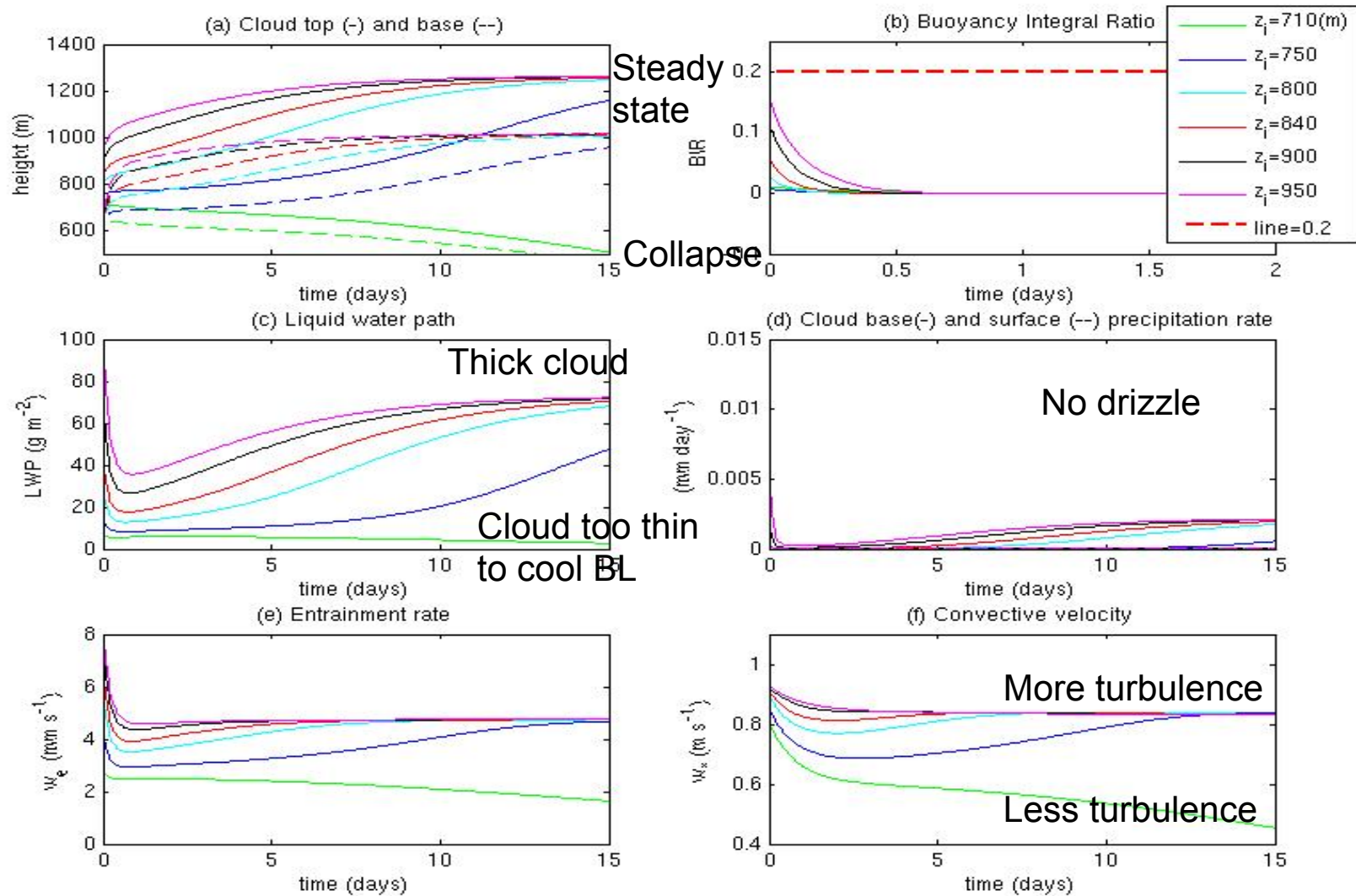
Uchida et al. 2010 ACP

# MLM simulation

Deepens in ~10 days to a steady state from DYCOMS ICs  
Fast adjustment not clearly distinguishable



# MLM equilibrium depends on initial boundary layer depth



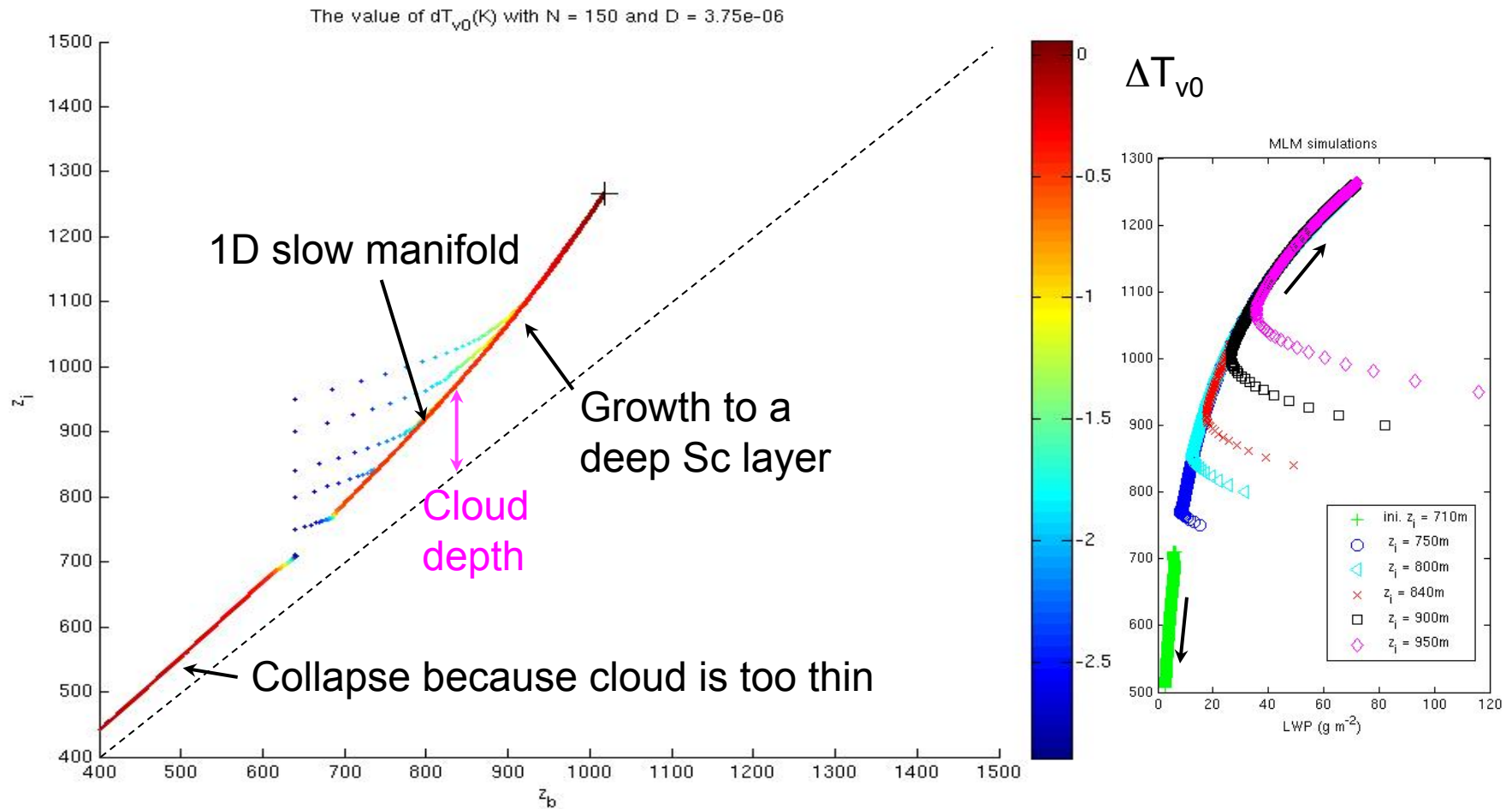
...a bit like Randall and Suarez (1984)

# Phase plane/slow manifold perspective

MLM is a nonlinear autonomous system of 3 ODEs

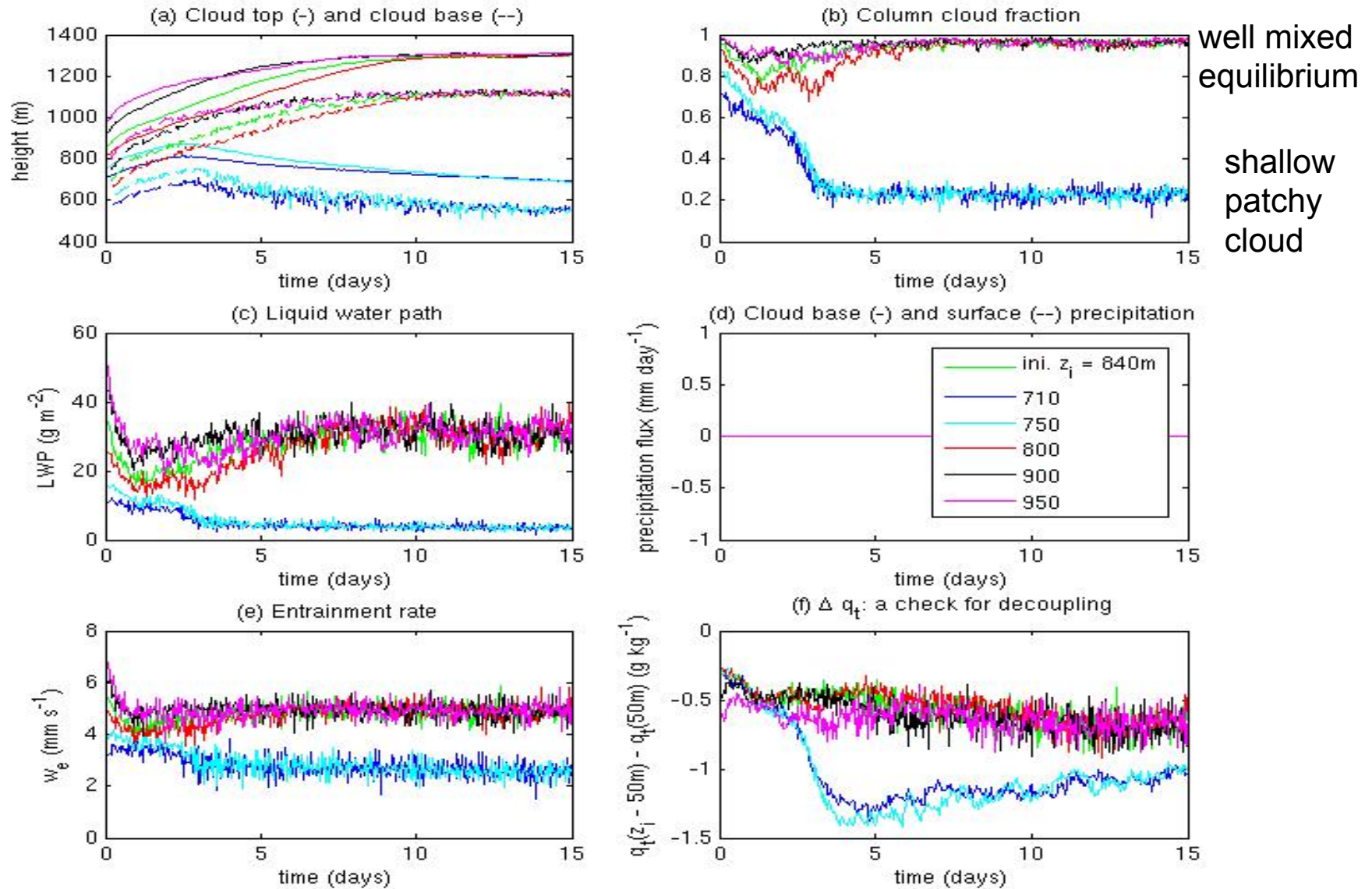
→ can study evolution using trajectories in 3-variable phase space

$\{\Delta T_{v0} = T_{vM}(0) - SST_v, z_b, z_i\}$  starting from the various  $z_i(0)$



Does LES show same slow-manifold behavior with multiple equilibria?

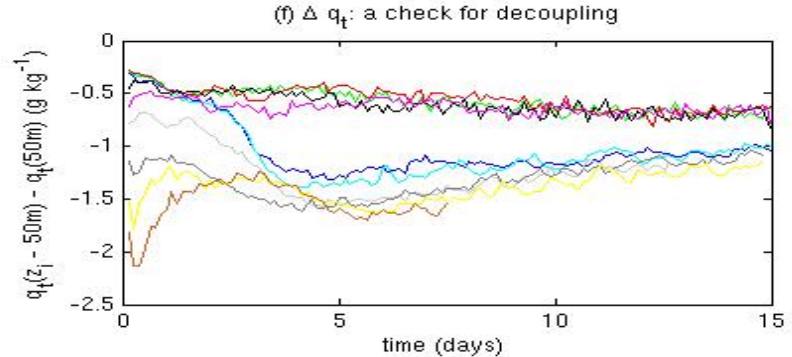
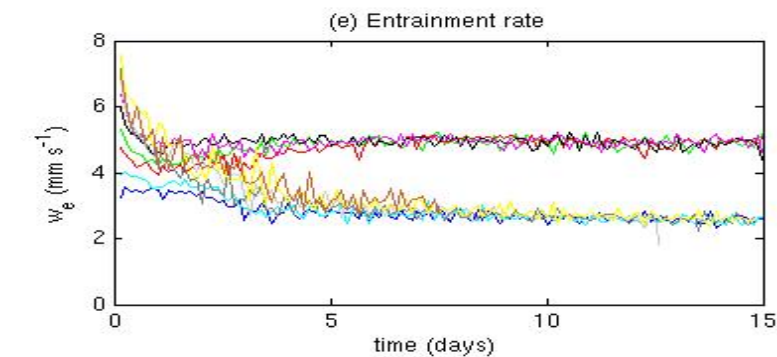
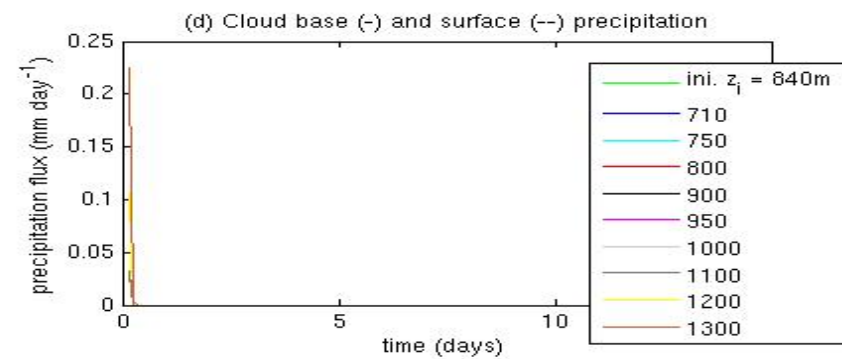
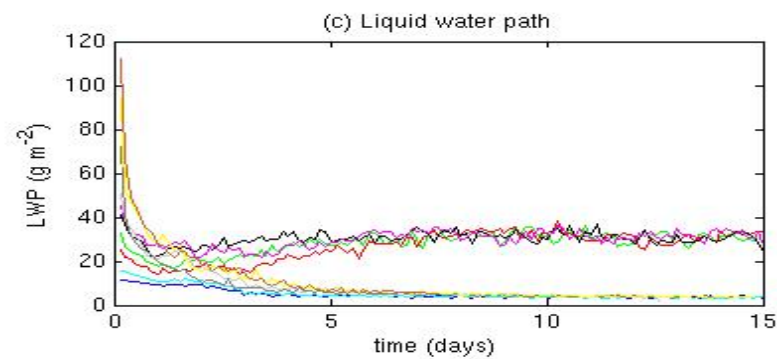
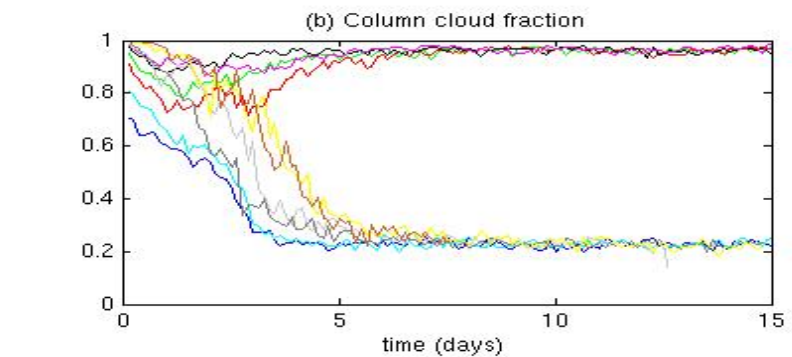
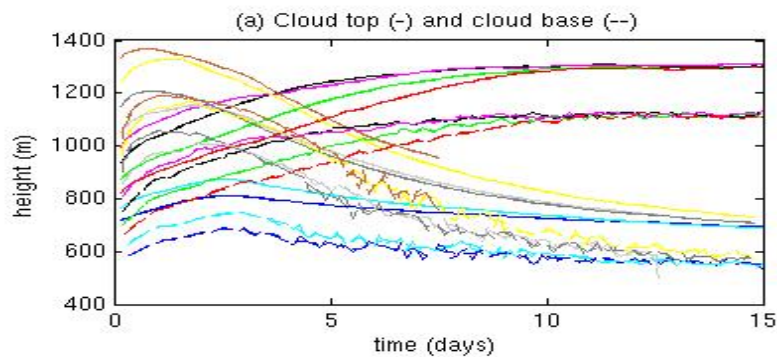
# LES results (time series)



Like MLM, LES has two possible evolutions depending on  $z_i(0)$   
 The first demonstration of multiple equilibria in LES of CTBL

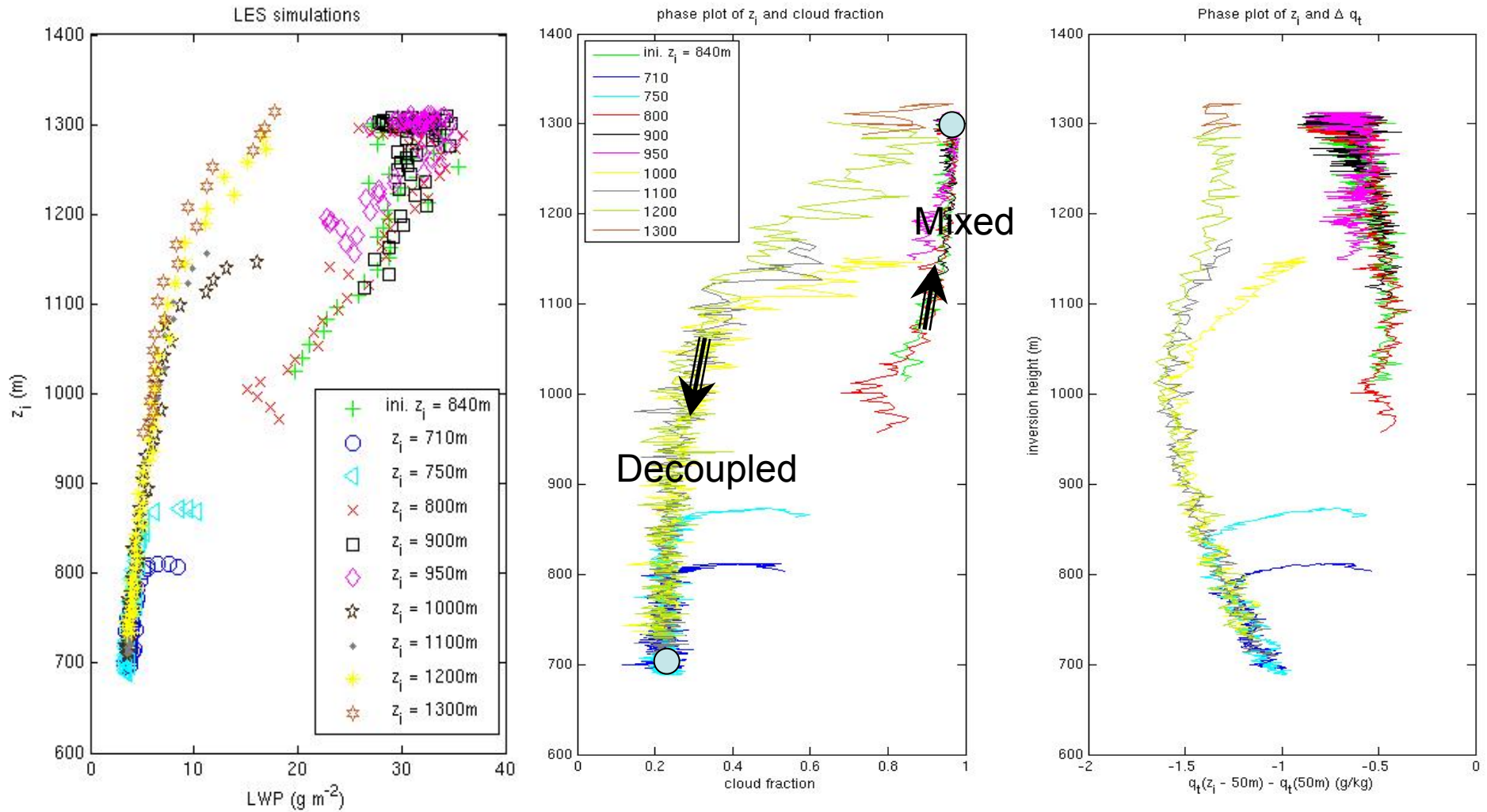


LES time series with deeper  $z_i(0)$ : The plot thickens  
 Deep  $z_i(0) = 1000\text{m}-1300\text{m}$  cases decouple, radiatively collapse  
 Medium depth cases deepen, shallow cases also collapse.

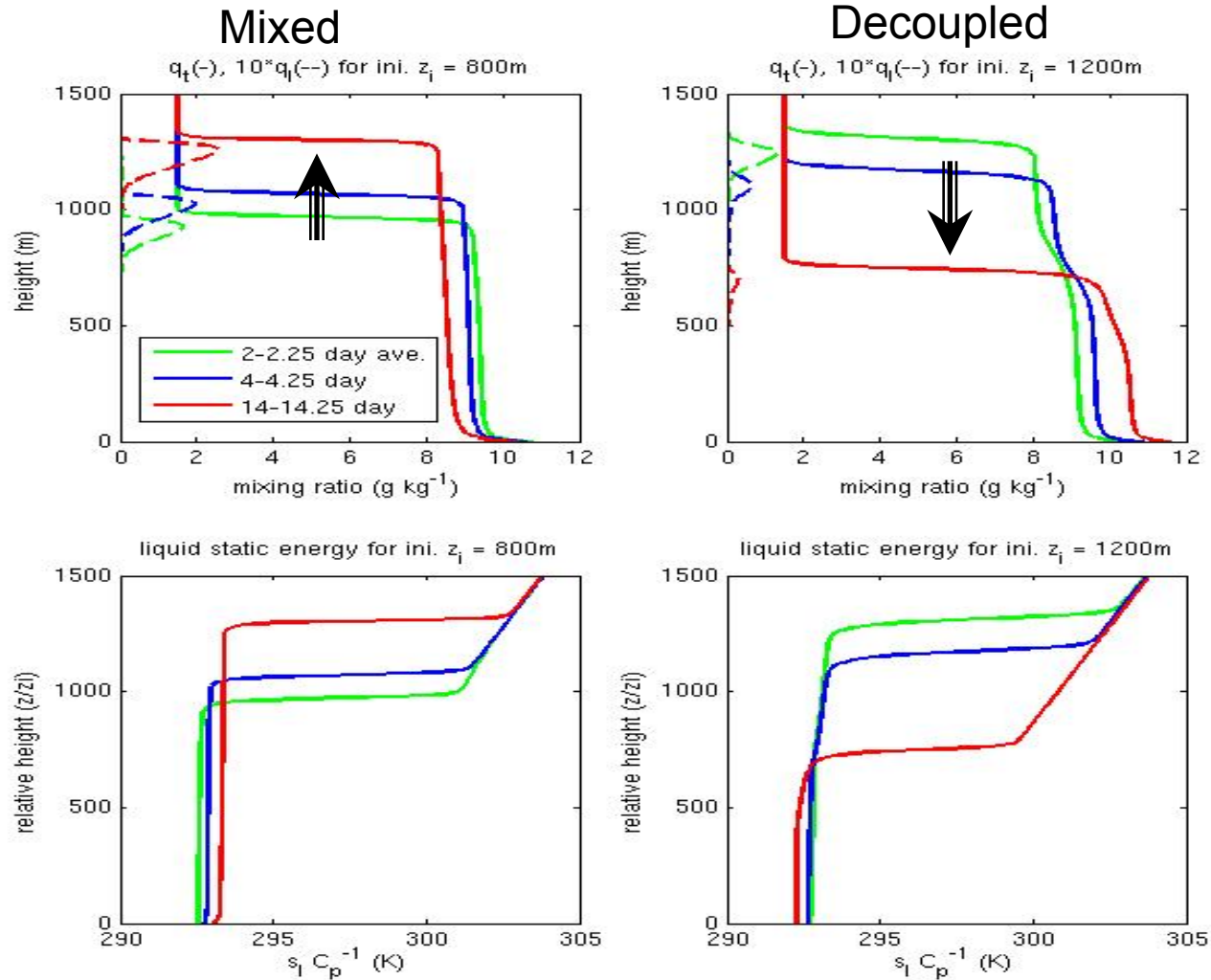


# Slow manifold analysis of LES (t > 2 days)

Similar to MLM, but two manifolds (‘decoupled’ and mixed)

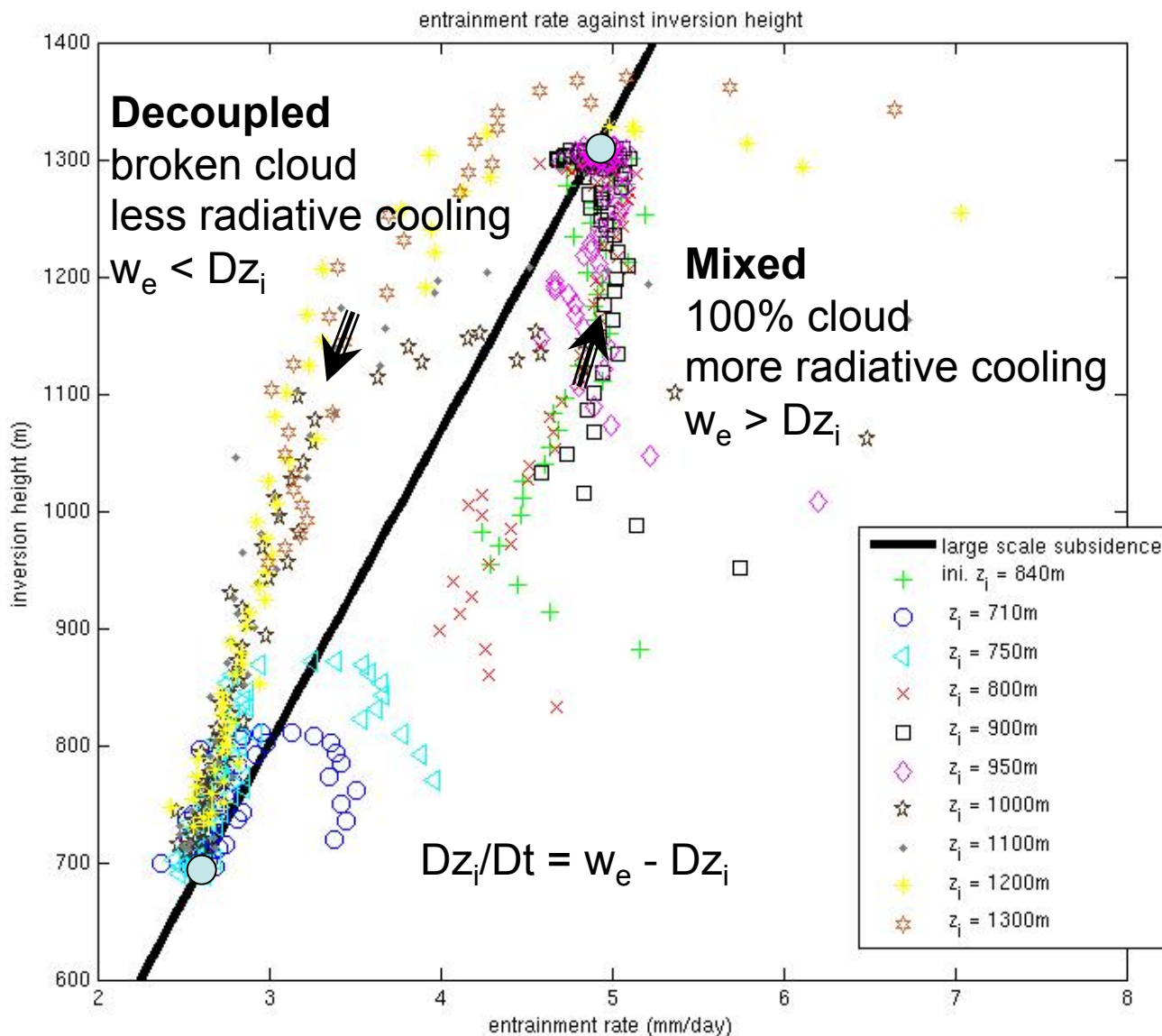


# Vertical structure along the two manifolds



- Modest T, humidity differences  $\rightarrow$  large cloud differences!

# Entrainment on the two manifolds

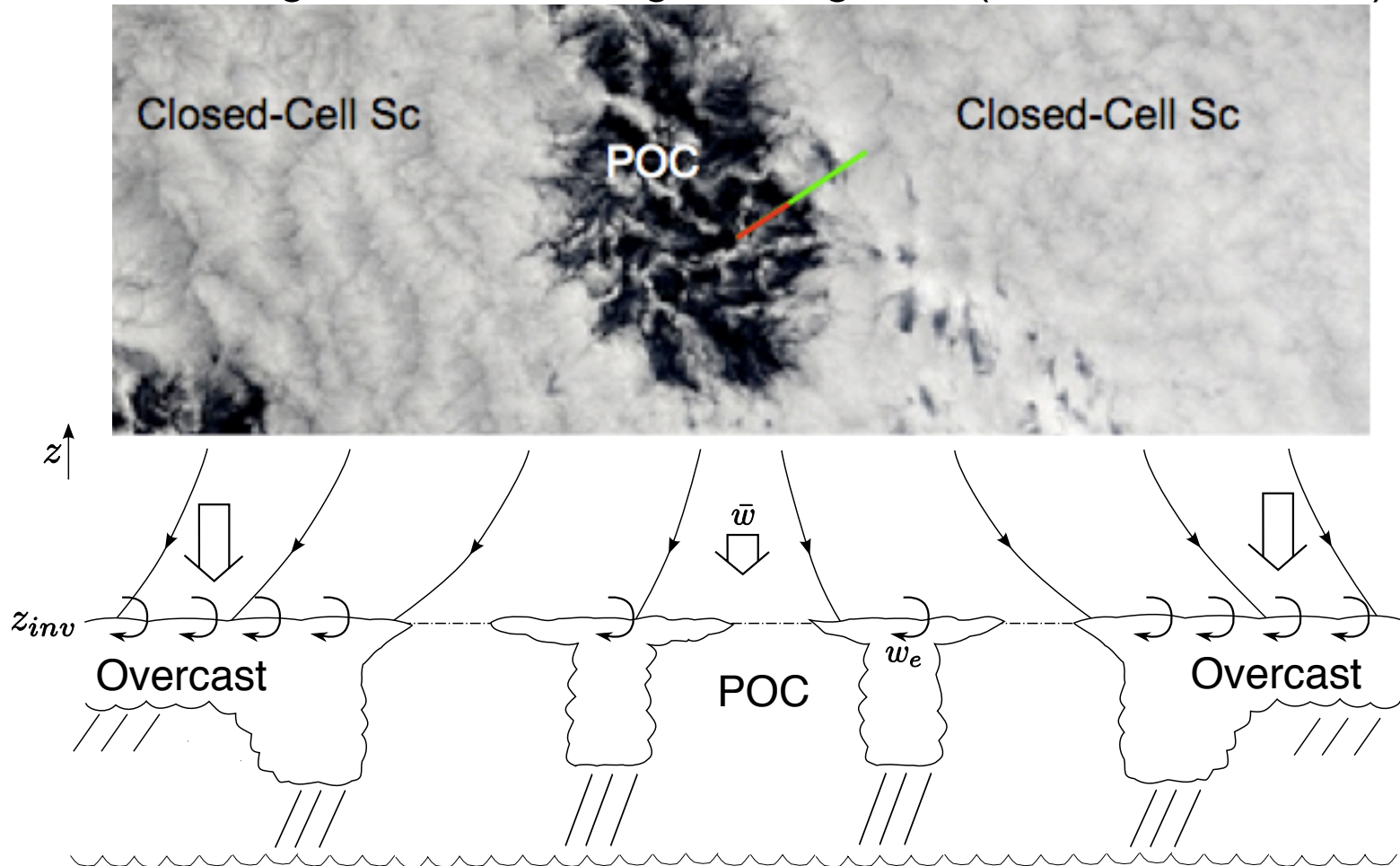


## Broader perspective

- In a GCSS-CFMIP column cloud feedbacks intercomparison case, we have also seen multiple equilibria for similar reason (positive cloud-radiation-entrainment feedback – see Anning’s and my Th talks)
- Can easily include diurnal cycle into the slow manifold
- If BL memory is mainly in inversion height:  
Daily observations of stratocumulus-capped boundary layers should collapse well and provide a tight model constraint if inversion height and perhaps aerosol characteristics are used as predictors in addition to key forcing parameters (EIS, cold advection, FT humidity)

# Interacting slow manifolds and POCs

Across a POC edge, two cloud regimes (slow manifolds?) interact. The broken cloud regime entrains less than the solid cloud regime, but compensating vertical motions keep the strong inversion flat and lock the inversion heights in the two regimes together (Berner et al. 2011).



# Conclusions

- Inspired by the fast and slow timescale adjustment of mixed layer models, we have introduced slow manifold thinking as a promising way to think about cloud-topped boundary layers. Unlike steady-state solutions, the slow manifold should be roughly realized in nature.
- On a slow manifold, the CTBL structure is slaved to the boundary conditions and the inversion height (which provides the ‘memory’).
- A single LES case can yield multiple equilibria (and even multiple slow manifolds) depending on initial conditions.
- This suggests there are situations in which the boundary layer structure is highly sensitive to initial or boundary conditions, including thin-cloud and thick-cloud cases.