A Review of Dynamical Core Development Under CMMAP

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The 16th CMMAP Team Meeting, 7-9 January 2014, Newport Beach, CA

Professor Arakawa has made key contributions to all dynamical, physical and numerical aspects of atmosphere modeling.

In that sense, he is definitely a <u>one-man-band</u> in atmosphere modeling.

More than that, he is a <u>great unifier</u> of atmosphere modeling.

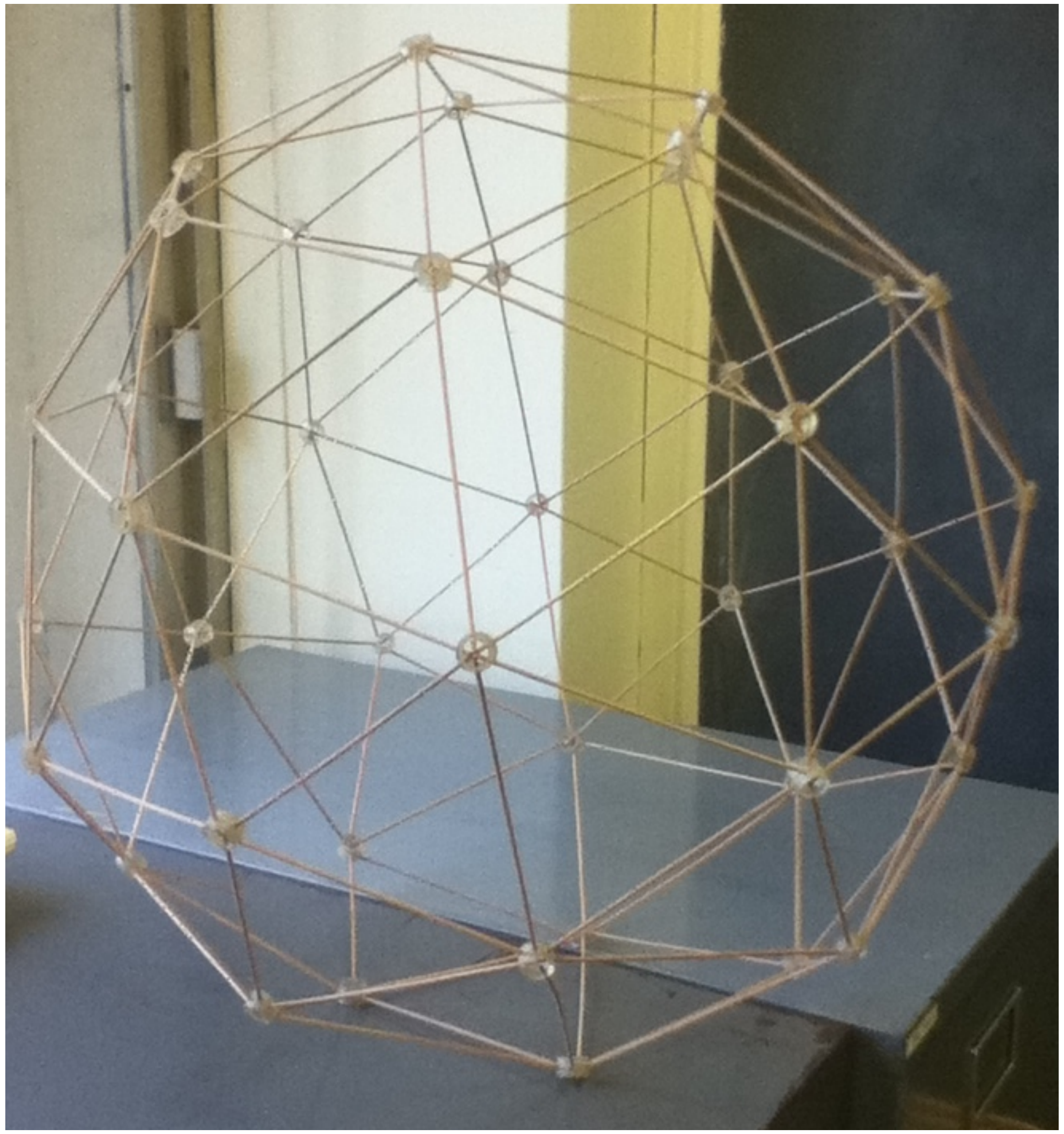
Parts of this talk summarize his recent contributions to the model dynamics.

A musician (one man band) performing in Santa Monica pier

Icosahedral dynamical core development

1-Grid optimization and shallow-water model

A 3D model of icosahedral grid used by Arakawa in his classes

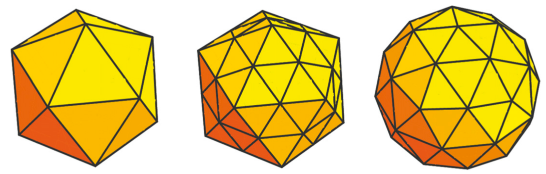
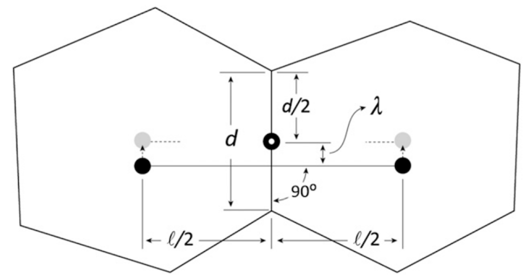
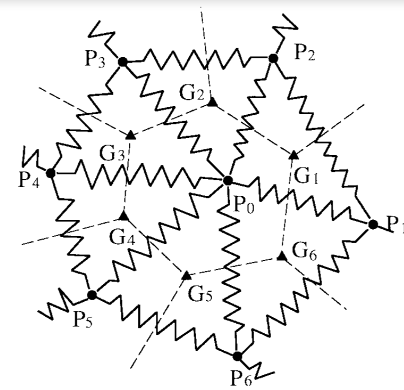
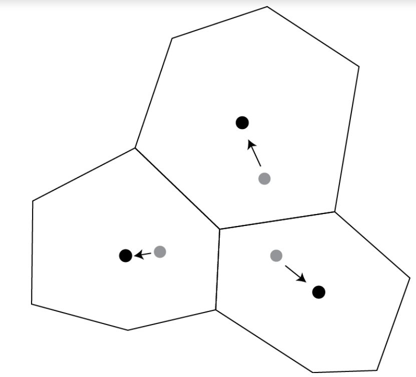
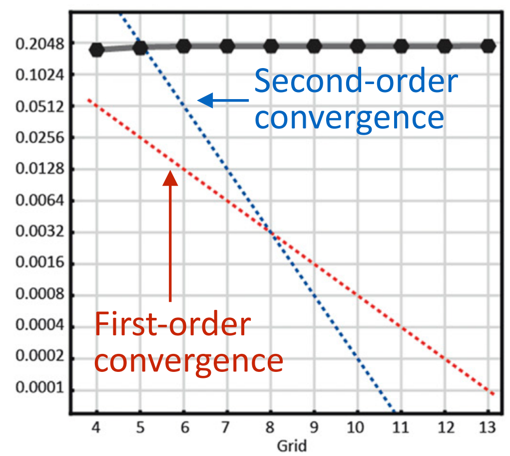
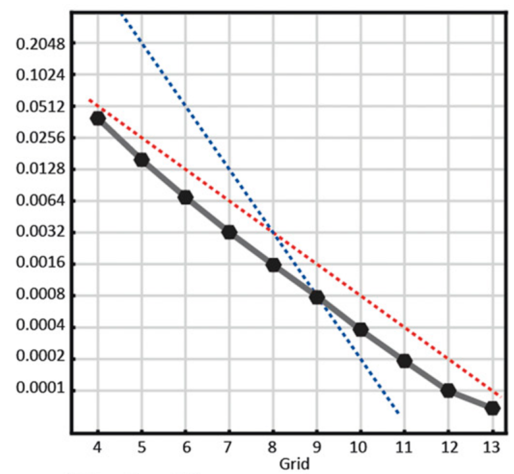
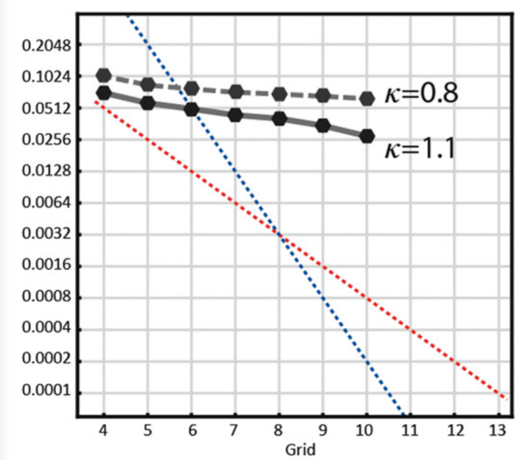
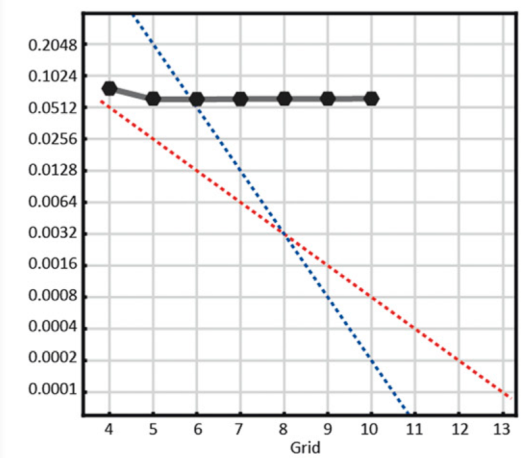


Professor Arakawa's office, Sep 23, 2011

(It could also be Sep 23, 1985)

Optimization of (Icosahedral) Voronoi grids

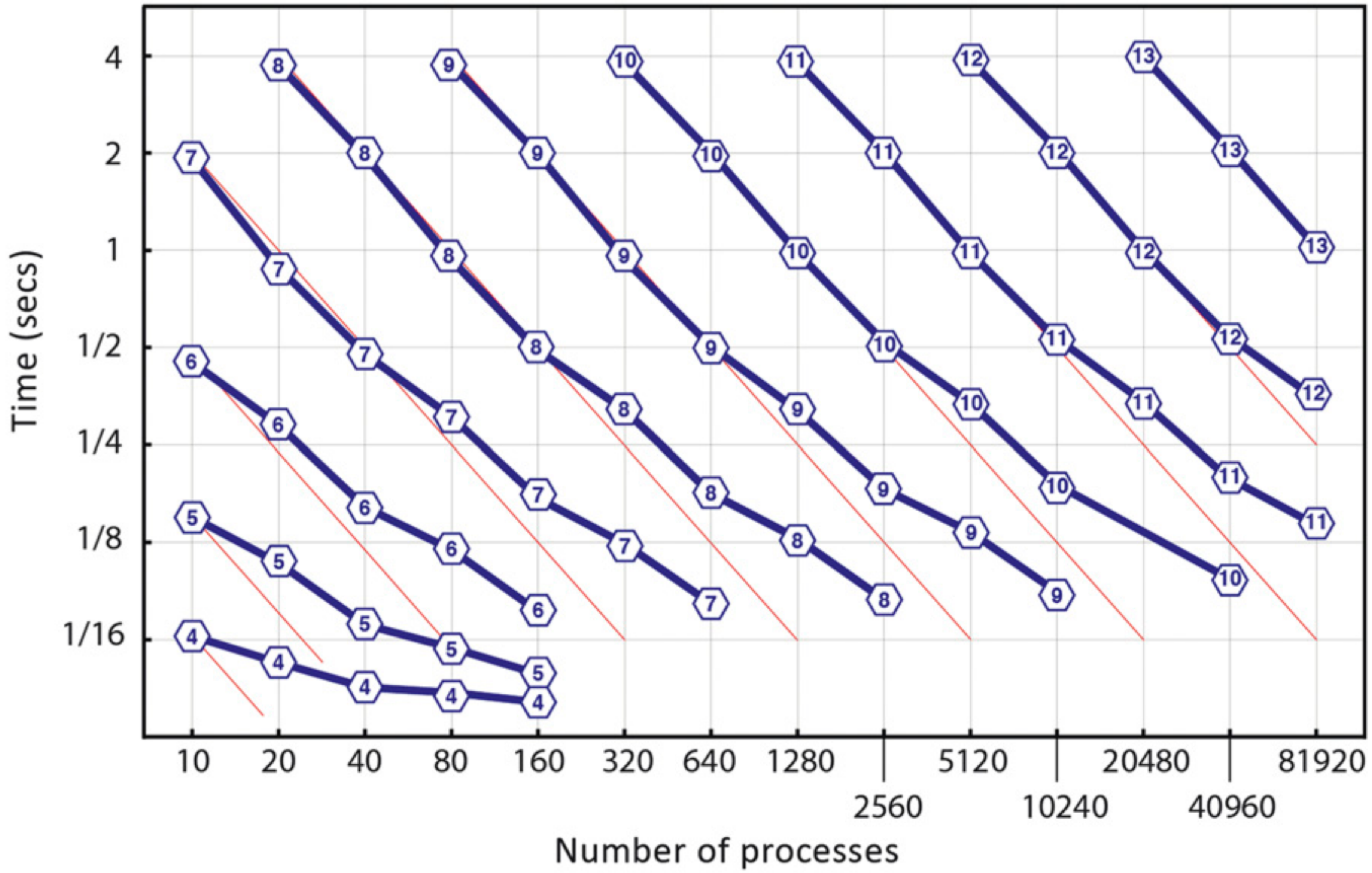
[Results from Heikes et al. (Dec 2013, *MWR*)]

Grid	Raw-grid	<u>Tweaked-grid</u>	Spring-grid	CVT-grid (Centroidal Voronoi Tessellation)
Optimization	(1) (2) (3)  Icosahedron			
L_∞-error (Laplacian)				
Remarks on cell sizes		Uniform distribution of cell areas Smallest and largest cells neighbors	Smooth transition from smallest to largest cells Big size difference between smallest and largest cells	Sharp transition from smallest to larger cells Big size difference between smallest and largest cells
References	Vestine et al. (1963) Sadourny et al. (1968) Williamson (1968)	UZIM Heikes et al. (*MWR*, 2013)	NICAM (Does not use Voronoi cells) Tomita et al. (2002)	MPAS (Does not start from an icosahedron) Du et al. (1999) Ju et al. (2011)

Horizontal discretization of dynamical cores

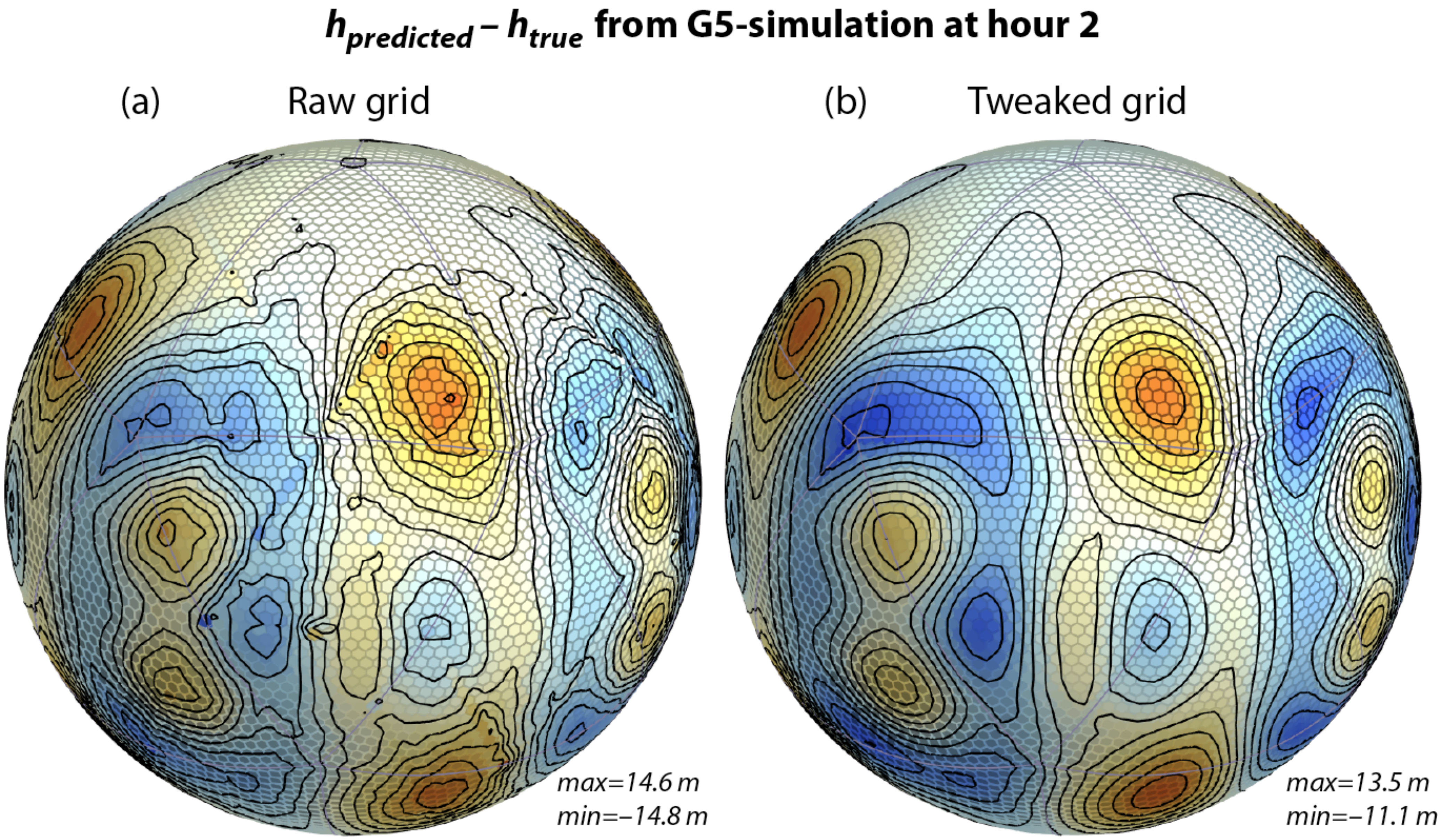
- Z-grid: Vorticity and divergence predicted for cell centers.
- Using 2D elliptic solvers, streamfunction and velocity potential are diagnosed from vorticity and divergence, respectively.

Strong parallel scalability of the V-cycled multigrid



Heikes et al. (2013 *MWR*)

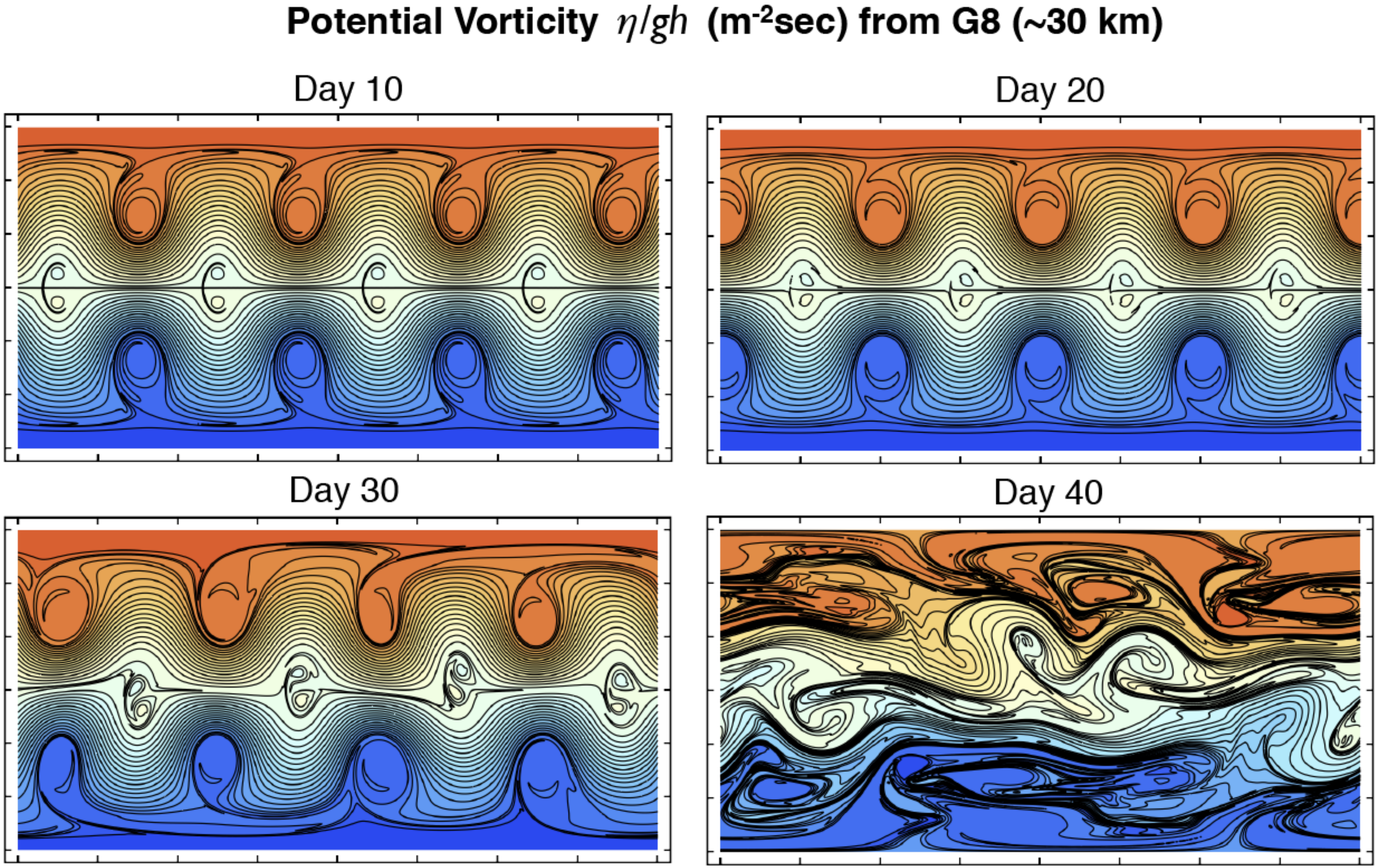
Wavenumber-4 Rossby-Haurwitz wave simulations (shallow water)



The "true-solution" is constructed by rotating the initial height field approximately 1 deg per 2 hours eastward, which is estimated from the propagation speed of the wavenumber-4 RH wave.

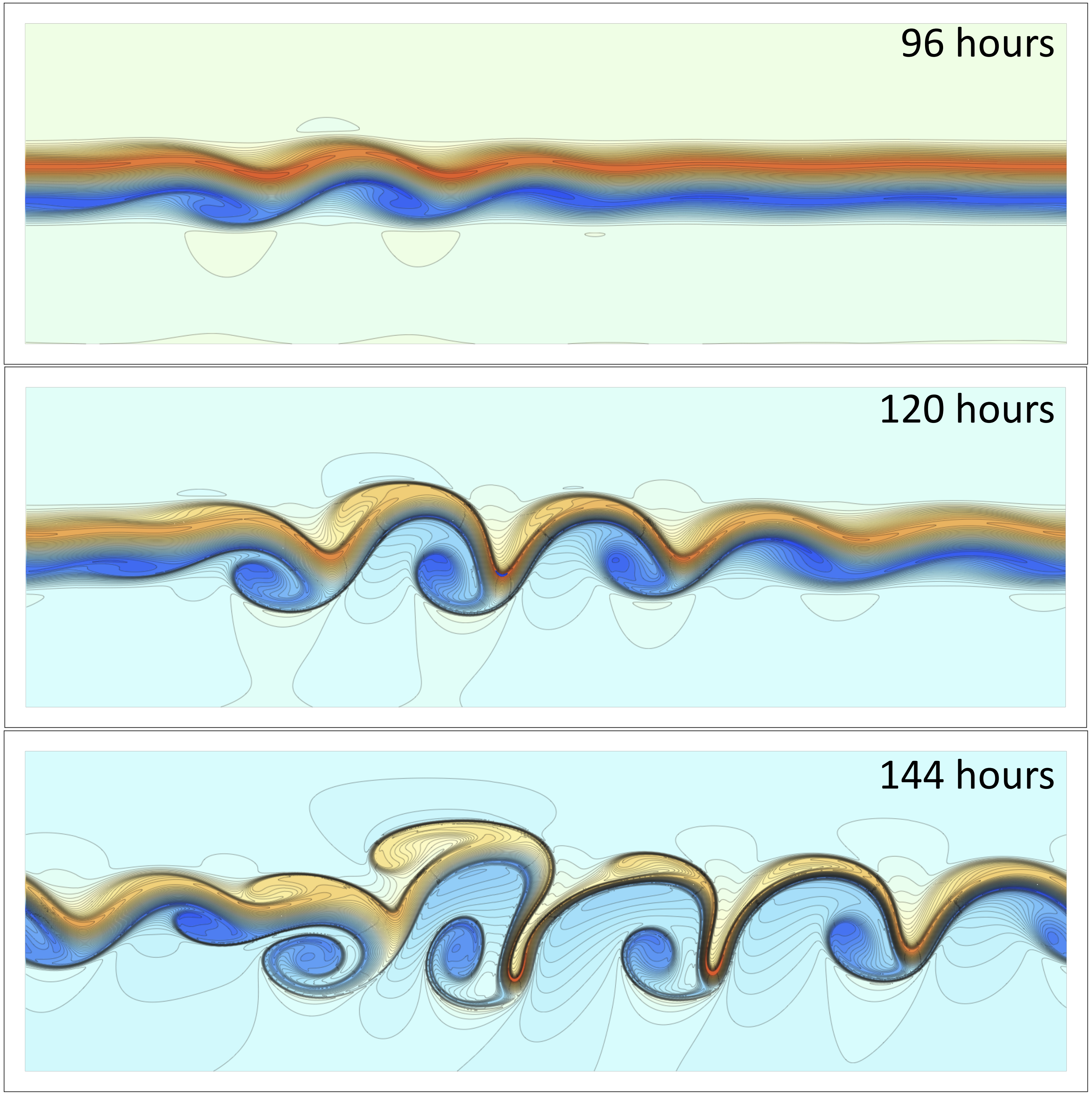
Resolution: G5

Wavenumber-4 Rossby-Haurwitz wave simulations (shallow water)



Resolution: G8

Shallow-water simulations (setup from Galewsky et al., 2004 *Tellus*)



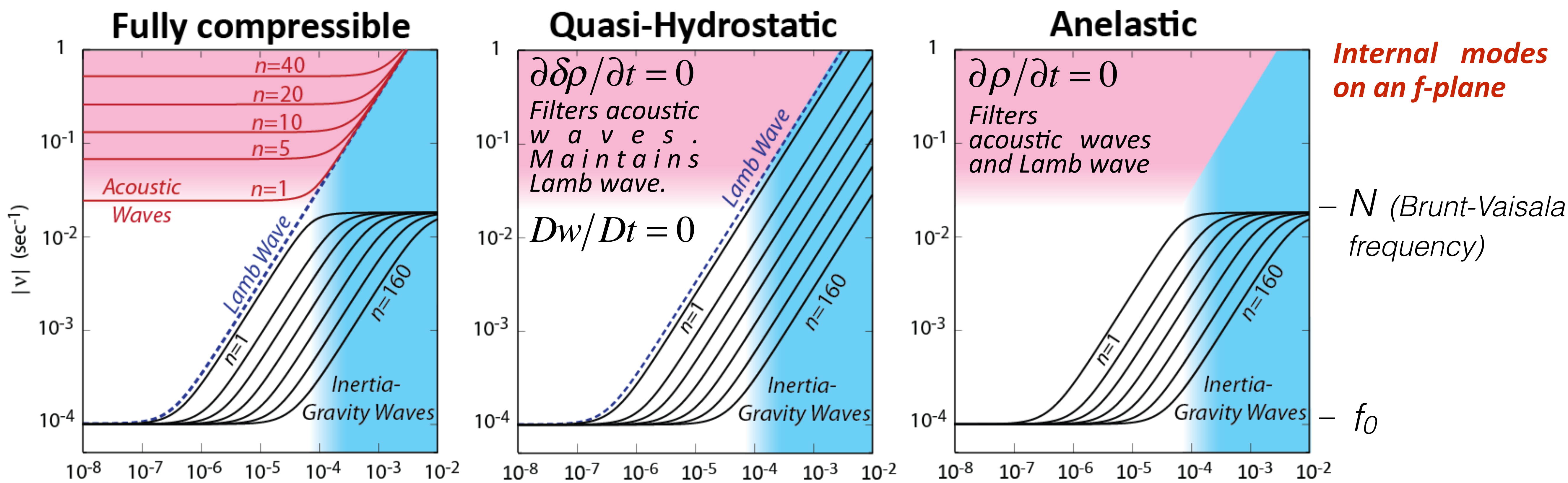
Resolution: G9 (~15 km)

2-Unified System of Equations

- It has been designed for use in global cloud resolving models.
- It filters vertically propagating acoustic waves.
- It does not require a basic state. It conserves energy.
- It can be added on any quasi-hydrostatic model.
- For description, see Arakawa and Konor (2009 *MWR*) and Konor (2014 *MWR*).
- Various aspects of the unified system resemble Durran (1989 *JAS*, 2008 *JFM*); Laprise (1992 *MWR*); and Lipps and Hemler (1982 *JAS*).
- Linear aspects of the system are tested by Dukowicz (2013 *MWR*).
- Filtered systems, not including unified, are tested by Davies et al. (2003 *QJRMS*).

Rationale Behind the Unified System

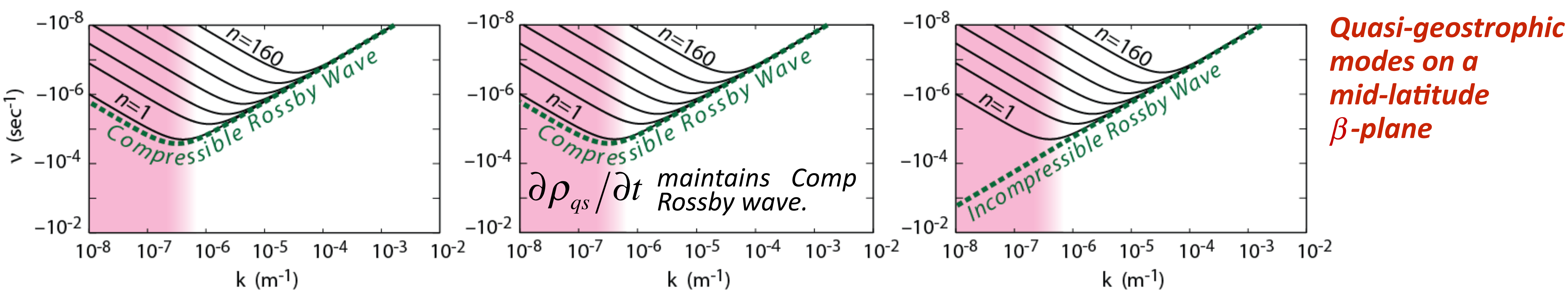
Frequency of normal modes as a function of horizontal wavenumber



$\delta\rho$: *Nonhydrostatic part of density*

ρ_{qs} : *Quasi-hydrostatic part of density*

n is vertical wavenumber



Dw/Dt *is needed for nonhydrostatic effects*

$\partial \rho_{qs} / \partial t$ *is needed for large-scale elasticity*

$\partial \delta\rho / \partial t = 0$ *is enough to filter vertically propagating acoustic waves*

Blue shading: Dw/Dt *is not negligible*

Pink shading: $\partial \rho_{qs} / \partial t$ *is not negligible*

Fully compressible equations

θ and z are not split

$$p \equiv p_{qs} + \delta p \qquad \rho \equiv \rho_{qs} + \delta\rho$$
$$\pi \equiv \pi_{qs} + \delta\pi \qquad T \equiv T_{qs} + \delta T$$

State equation:

$$p = \rho R \pi \theta \qquad \theta \equiv T/\pi \qquad \pi \equiv \left(p/p_{00}\right)^{\kappa}$$

Horizontal momentum equation:

$$\frac{D\mathbf{v}}{Dt} + f\mathbf{k}\times\mathbf{v} = -c_p\theta\nabla_H\left(\pi_{qs} + \delta\pi\right)$$

Vertical momentum equation:

$$\frac{Dw}{Dt} = -c_p\theta\frac{\partial\left(\pi_{qs} + \delta\pi\right)}{\partial z} - g \quad\Rightarrow\quad \frac{Dw}{Dt} = -c_p\theta\frac{\partial\delta\pi}{\partial z}$$

$\Downarrow$

"Hydrostatic" equation:

$$0 = -c_p\theta\frac{\partial\pi_{qs}}{\partial z} - g$$

Thermodynamic equation:

$$\frac{D\theta}{Dt} = 0$$

Continuity equation:

$$\frac{\partial\left(\rho_{qs} + \delta\rho\right)}{\partial t} = -\nabla_H\cdot\left[\left(\rho_{qs} + \delta\rho\right)\mathbf{v}\right] - \frac{\partial\left[\left(\rho_{qs} + \delta\rho\right)w\right]}{\partial z}$$

Unified equations

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$$p_{qs} = \rho_{qs} R \pi_{qs} \theta \qquad T_{qs} = \pi_{qs}\theta \qquad \pi_{qs} \equiv \left(p_{qs}/p_{00}\right)^{\kappa}$$

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$\Downarrow$

"Hydrostatic" equations:

$$0 = -c_p\theta\frac{\partial\pi_{qs}}{\partial z} - g \qquad 0 = -\frac{1}{\rho_{qs}}\frac{\partial p_{qs}}{\partial z} - g$$

Thermodynamic equation:

$$\frac{D\theta}{Dt} = 0$$

Continuity equation:

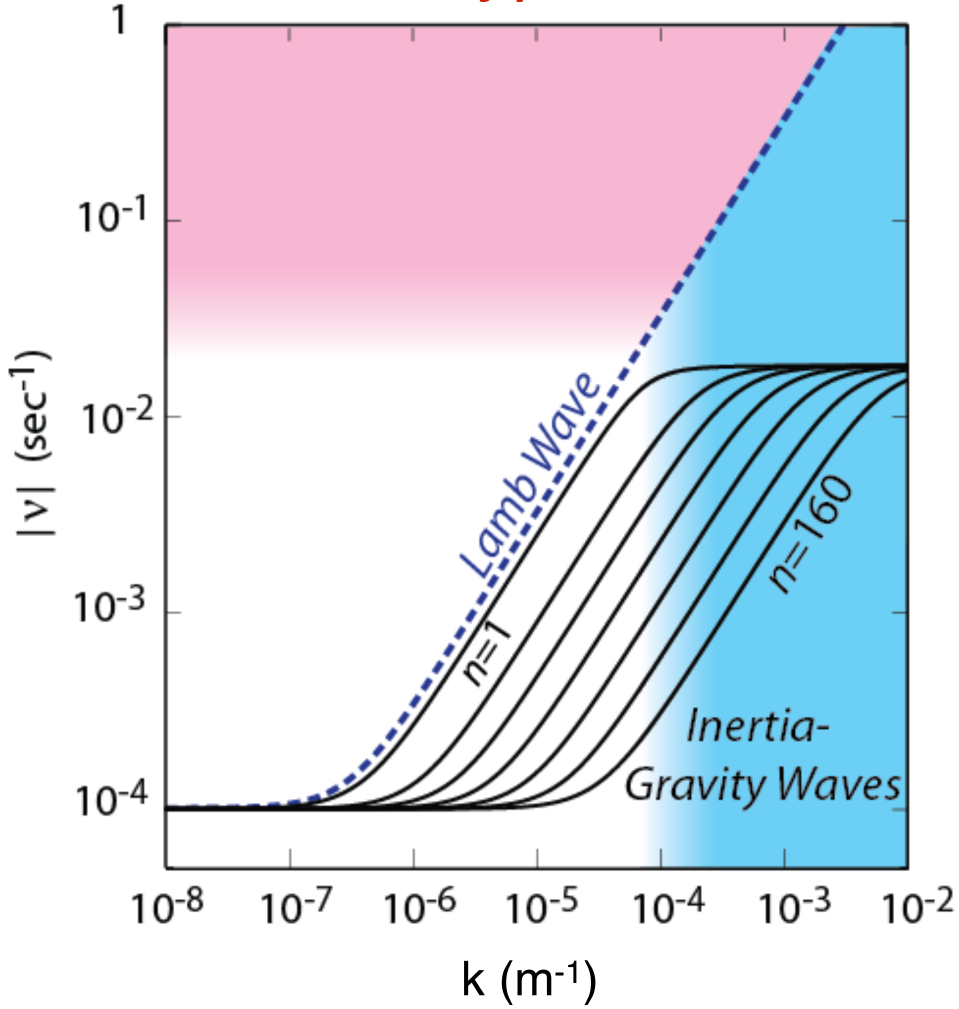
$$\frac{\partial\rho_{qs}}{\partial t} = -\nabla_H\cdot\left(\rho_{qs}\mathbf{v}\right) - \frac{\partial\left(\rho_{qs}w\right)}{\partial z}$$

$\rho_{qs} \gg \delta\rho$ is assumed

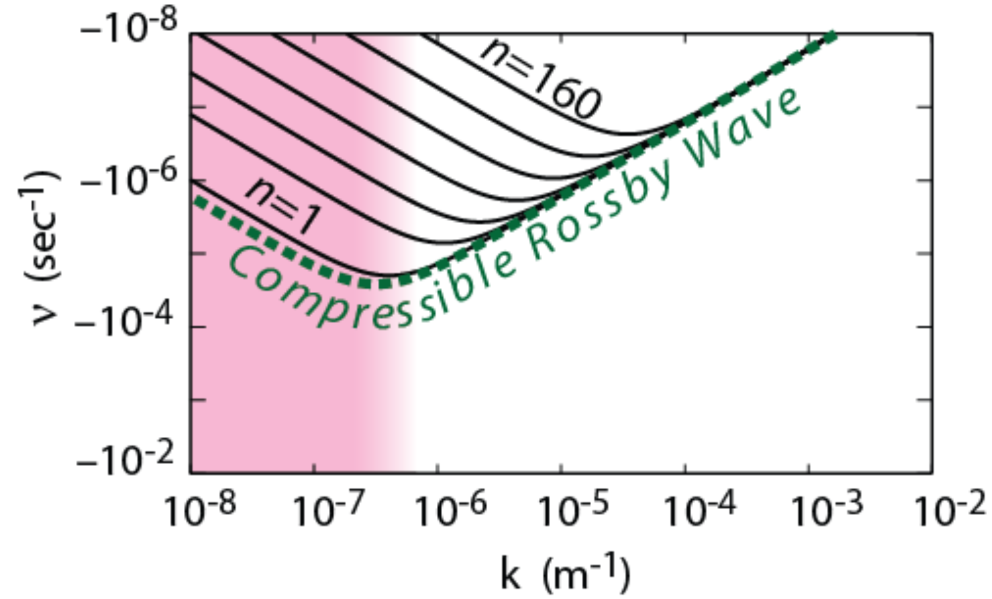
Unified System

Frequency of normal modes as a function of horizontal wavenumber

Internal modes on an f-plane



Quasi-geostrophic modes on a mid-latitude β-plane



n is vertical wavenumber

Dw/Dt *and* $\partial \rho_{qs}/\partial t$ *are maintained*

$\partial \delta\rho/\partial t = 0$ *is used to filter vertically propagating acoustic waves*

… "The atmospheric case is much more sensitive to approximation. The recent "unified" approximation of Arakawa and Konor is one of the most accurate"…

Dukowicz (Dec 2013, *MWR*)

Solution algorithm for the unified equations (z vertical coordinate)

θ *and* z *are not split* $\qquad \pi \equiv \pi_{qs} + \delta\pi$

Prediction of potential temperature:

$$\frac{D\theta}{Dt} = 0$$

Diagnosis and prediction of "quasi-hydrostatic" variables:

$$\Downarrow$$

$$0 = -c_p \theta \frac{\partial \pi_{qs}}{\partial z} - g$$

$$\frac{\partial (\pi_{qs})_S}{\partial t} - \frac{\partial (\pi_{qs})_T}{\partial t} = \frac{g}{c_p} \int_{z_T}^{z_S} \theta^{-2} \frac{\partial \theta}{\partial t} dz$$

$$\Leftarrow$$

$$0 = -\frac{1}{\rho_{qs}} \frac{\partial p_{qs}}{\partial z} - g \qquad \frac{\partial \rho_{qs}}{\partial t} = -\nabla_H \cdot (\rho_{qs} \mathbf{v}) - \frac{\partial (\rho_{qs} w)}{\partial z}$$

$$\frac{\partial (p_{qs})_S}{\partial t} - \frac{\partial (p_{qs})_T}{\partial t} = g \int_{z_T}^{z_S} \nabla_H \cdot (\rho_{qs} \mathbf{v}) dz$$

$$\Downarrow$$

$$\frac{\partial \rho_{qs}}{\partial t} = \rho_{qs} \left(\frac{1-\kappa}{\kappa} \frac{1}{\pi_{qs}} \frac{\partial \pi_{qs}}{\partial t} - \frac{1}{\theta} \frac{\partial \theta}{\partial t} \right) \qquad \frac{\partial^2 \rho_{qs}}{\partial t^2} = \frac{\partial}{\partial t} \left[\rho_{qs} \left(\frac{1-\kappa}{\kappa} \frac{1}{\pi_{qs}} \frac{\partial \pi_{qs}}{\partial t} - \frac{1}{\theta} \frac{\partial \theta}{\partial t} \right) \right]$$

Diagnosis of nonhydrostatic Exner pressure: $\Downarrow$

$$\nabla_H \cdot (\rho_{qs} c_p \theta \nabla_H \delta\pi) + \frac{\partial}{\partial z} \left(\rho_{qs} c_p \theta \frac{\partial \delta\pi}{\partial z} \right) =$$

$$- \nabla_H \cdot (\rho_{qs} c_p \theta \nabla_H \pi_{qs}) - \nabla_H \cdot (\rho_{qs} \mathbf{G}_H) - \frac{\partial}{\partial z} (\rho_{qs} G_z) + \frac{\partial^2 \rho_{qs}}{\partial t^2}$$

Prediction of horizontal momentum:

$$\Downarrow$$

$$\frac{D\mathbf{v}}{Dt} + f\mathbf{k} \times \mathbf{v} = -c_p \theta \nabla_H (\pi_{qs} + \delta\pi)$$

Diagnosis of ver momentum through continuity equation:

$$\Downarrow$$

$$\frac{\partial (\rho_{qs} w)}{\partial z} = -\nabla_H \cdot (\rho_{qs} \mathbf{v}) - \frac{\partial \rho_{qs}}{\partial t}$$

Energetics of the unified system

Energy equation and conservation of energy:

$$\frac{\partial}{\partial t}\left[\rho_{qs}\left(E_{qs}+c_p\delta T\right)\right]+\nabla_H\cdot\left[\left(\rho_{qs}E_{qs}+p\right)\mathbf{v}\right]+\frac{\partial}{\partial z}\left[\left(\rho_{qs}E_{qs}+p\right)w\right]=\frac{1}{\kappa}\frac{p_{qs}}{\pi_{qs}}\frac{\partial}{\partial t}\delta\pi$$

where $E_{qs}\equiv\frac{1}{2}\left(\mathbf{v}^2+w^2\right)+gz+c_v T_{qs}$ *is the quasi-hydrostatic energy*

Conservation of energy requires $\overline{\overline{\frac{p_{qs}}{\pi_{qs}}\frac{\partial}{\partial t}\delta\pi}}=0$ *($\overline{\overline{}}$ denotes 3D domain mean)*

The use of domain mean nonhydrostatic Exner pressure to conserve energy :

$(\delta\pi)^{(0)}$ *is the nh Exner pressure obtained by solving the 3D elliptic equation*

$(\delta\pi)^{(*)}=(\delta\pi)^{(0)}-\overline{\overline{(\delta\pi)^{(0)}}}$ *is the nh Exner pressure with no domain mean*

$\delta\pi=(\delta\pi)^{(*)}+\overline{\overline{\delta\pi}}$ *is the nh Exner pressure which conserves energy*

Then $\frac{\partial}{\partial t}\overline{\overline{\delta\pi}}=-\overline{\overline{\frac{p_{qs}}{\pi_{qs}}\frac{\partial}{\partial t}(\delta\pi)^{(*)}}}\Big/\overline{\overline{\frac{p_{qs}}{\pi_{qs}}}}$ *conserves energy*

Implication of energy conservation:

$$T\equiv T_{qs}+\delta T=T_{qs}\left\{1+\frac{1}{\pi_{qs}}\left[(\delta\pi)^{(*)}+\overline{\overline{\delta\pi}}\right]\right\}$$

Enforcing energy conservation through this approach impacts mean temperature, and therefore model physics

Unified equations

Continuity equation:

$$\nabla_H \cdot (\rho_{qs}\mathbf{v}) + \frac{\partial(\rho_{qs} w)}{\partial z} = -\frac{\partial \rho_{qs}}{\partial t}$$

(Generalized) pseudo-incompressible equations

Pseudo-incompressible continuity equation:

$$\nabla_H \cdot (\bar{\theta}\bar{\rho}\mathbf{v}) + \frac{\partial(\bar{\theta}\bar{\rho} w)}{\partial z} = \frac{\bar{\rho} Q}{c_p \bar{\pi}\bar{\theta}}$$

where $\bar{\theta} = \bar{\theta}(z)$ $\bar{\rho} = \bar{\rho}(z)$ Durran (1989)

Generalized pseudo-incompressible continuity equation:

3D time dependent basic state

$\bar{\theta} = \bar{\theta}(x,y,z,t)$ $\bar{\rho} = \bar{\rho}(x,y,z,t)$ Durran (2008)

Issues:

Durran (2008) does not describe an algorithm for the determination of $\bar{\theta} = \bar{\theta}(x,y,z,t)$ and $\bar{\rho} = \bar{\rho}(x,y,z,t)$.

It proposes to prescribe $\bar{\theta} = \bar{\theta}(x,y,z,t)$ and $\bar{\rho} = \bar{\rho}(x,y,z,t)$ from a low-resolution global model (GCM).

Two problems with this:

1- Psudo-incompressible model cannot influence the GCM.

2- Double counting of physics.

Fully compressible equations in a quasi-hydrostatic pressure coordinate

Laprise (1992 *MWR*) approach

Coordinate transformation $z \Rightarrow p_{qs}$ (*)

State equation:

$$p = \rho R \pi \theta \qquad \theta \equiv T/\pi \qquad \pi \equiv \left(p/p_{00}\right)^{\kappa}$$

Coordinate transformation:

$$\nabla_H \equiv \nabla_{p_{qs}} + g\rho\left(\nabla_{p_{qs}} z\right)\frac{\partial}{\partial p_{qs}}$$

$$\frac{\partial}{\partial z} \equiv -g\rho\frac{\partial}{\partial p_{qs}}$$

$$\frac{\partial p_{qs}}{\partial z} \equiv -g\rho$$

Thermodynamic equation:

$$\frac{D\theta}{Dt} = 0 \qquad \frac{D}{Dt} = \left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)_{p_{qs}} + \omega\frac{\partial}{\partial p_{qs}}$$

Unified equations in a quasi-hydrostatic pressure coordinate

Arakawa and Konor (2009 *MWR*)

θ and z are not split

$$p \equiv p_{qs} + \delta p \qquad \rho \equiv \rho_{qs} + \delta\rho$$

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$$\frac{\partial}{\partial z} \equiv -g\rho_{qs}\frac{\partial}{\partial p_{qs}}$$

$$\frac{\partial p_{qs}}{\partial z} = -g\rho_{qs}$$

Thermodynamic equation:

$$\frac{D\theta}{Dt} = 0 \qquad \frac{D}{Dt} = \left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)_{p_{qs}} + \omega\frac{\partial}{\partial p_{qs}}$$

difference! ($g\rho$ vs. $g\rho_{qs}$ in ∇_H)

difference! ($g\rho$ vs. $g\rho_{qs}$ in $\partial/\partial z$)

Source of the difference! ($-g\rho$ vs. $-g\rho_{qs}$ in $\partial p_{qs}/\partial z$)

(*) p_{qs} in Laprise (1992) is different than that in Arakawa and Konor (2009)

Fully compressible equations in a quasi-hydrostatic pressure coordinate

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Horizontal momentum equation:

$$\frac{D\mathbf{v}}{Dt} + f\mathbf{k}\times\mathbf{v} = -\frac{1}{\rho}\nabla_{p_{qs}} p - g\left(\nabla_{p_{qs}} z\right)\frac{\partial p}{\partial p_{qs}}$$

Equation that is used to diagnose *z*:

$$\frac{\partial z}{\partial p_{qs}} \equiv -\frac{1}{g\rho}$$

Consistent!

Continuity equation:

$$\frac{D\rho}{Dt} + \rho D_3 = 0 \quad \text{or} \quad \frac{D}{Dt}\left(\frac{1}{\rho}\right) - \frac{1}{\rho} D_3 = 0$$

where

$$D_3 \equiv \nabla_{p_{qs}} \cdot \mathbf{v} - g\left(\nabla_{p_{qs}} z\right)\cdot\frac{\partial \mathbf{v}}{\partial p_{qs}} - g\frac{\partial(\rho w)}{\partial p_{qs}}$$

Equation that is used to diagnose omega:

$$\nabla_{p_{qs}} \cdot \mathbf{v} + \frac{\partial \omega}{\partial p_{qs}} = 0$$

Obtained by requiring $\rho\frac{\partial z}{\partial p_{qs}} \equiv const.$

Vertical momentum equation:

$$\frac{Dw}{Dt} = g\frac{\partial p}{\partial p_{qs}} - g$$

Unified equations in a quasi-hydrostatic pressure coordinate

Arakawa and Konor (2009 *MWR*)

State equations:

$$p = \rho R \pi \theta \qquad \theta \equiv T/\pi \qquad \pi \equiv \left(p/p_{00}\right)^{\kappa}$$

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Horizontal momentum equation:

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Equation that is used to diagnose *z*:

$$\frac{\partial z}{\partial p_{qs}} \equiv -\frac{1}{g\rho_{qs}} = -\frac{R\pi_{qs}\theta}{gp_{qs}}$$

Continuity equation:

$$\nabla_{p_{qs}} \cdot \mathbf{v} + \frac{\partial \omega}{\partial p_{qs}} = 0$$

Vertical momentum equation:

$$\frac{Dw}{Dt} = g\frac{\partial \delta p}{\partial p_{qs}} - g\frac{\delta\rho}{\rho_{qs}}$$

Fully compressible equations in a quasi-hydrostatic Exner pressure coordinate

Coordinate transformation $z \Rightarrow \pi_{qs}$

State equation:

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Coordinate transformation:

$$\nabla_z \equiv \nabla_{\pi_{qs}} + \frac{g}{c_p \theta}\left(\nabla_{\pi_{qs}} z\right)\frac{\partial}{\partial \pi_{qs}}$$

$$\frac{\partial}{\partial z} = -\frac{g}{c_p \theta}\frac{\partial}{\partial \pi_{qs}}$$

$$\frac{\partial \pi_{qs}}{\partial z} \equiv -\frac{g}{c_p \theta}$$

Thermodynamic equation:

$$\frac{D\theta}{Dt} = 0 \qquad \frac{D}{Dt} \equiv \left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)_{\pi_{qs}} + \varpi \frac{\partial}{\partial p_{\pi_{qs}}} \qquad \varpi \equiv \frac{D\pi_{qs}}{Dt}$$

Unified equations in a quasi-hydrostatic Exner pressure coordinate

Coordinate transformation $z \Rightarrow \pi_{qs}$

θ *and* z *are not split*

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$$\frac{\partial}{\partial z} = -\frac{g}{c_p \theta}\frac{\partial}{\partial \pi_{qs}}$$

$$\frac{\partial \pi_{qs}}{\partial z} \equiv -\frac{g}{c_p \theta}$$

No difference!

Thermodynamic equation:

$$\frac{D\theta}{Dt} = 0 \qquad \frac{D}{Dt} \equiv \left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla\right)_{\pi_{qs}} + \varpi \frac{\partial}{\partial p_{\pi_{qs}}}$$

Fully compressible equations in quasi-hydrostatic Exner pressure coordinate

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Horizontal momentum equation:

$$\frac{D\mathbf{v}}{Dt} + f\mathbf{k} \times \mathbf{v} = -c_p \theta \nabla_{\pi_{qs}} p - g\left(\nabla_{\pi_{qs}} z \right) \frac{\partial p}{\partial \pi_{qs}}$$

Equation that is used to diagnose *z*:

$$\frac{\partial z}{\partial \pi_{qs}} \equiv -\frac{c_p \theta}{g}$$

Conflict!

Continuity equation:

TBD

Equation that is used to diagnose omega:

$$\nabla_{\pi_{qs}} \cdot \mathbf{v} + \frac{\partial \varpi}{\partial \pi_{qs}} = 0$$

Obtained by requiring $\rho \frac{\partial z}{\partial \pi_{qs}} \equiv const.$

Vertical momentum equation:

$$\frac{Dw}{Dt} = g \frac{\partial p}{\partial \pi_{qs}}$$

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Horizontal momentum equation:

$$\frac{D\mathbf{v}}{Dt} + f\mathbf{k} \times \mathbf{v} = -c_p \theta \nabla_{\pi_{qs}} p - g\left(\nabla_{\pi_{qs}} z \right) \frac{\partial p}{\partial \pi_{qs}}$$

Equation that is used to diagnose *z*:

$$\frac{\partial z}{\partial \pi_{qs}} \equiv -\frac{c_p \theta}{g}$$

Continuity equation:

TBD

Vertical momentum equation:

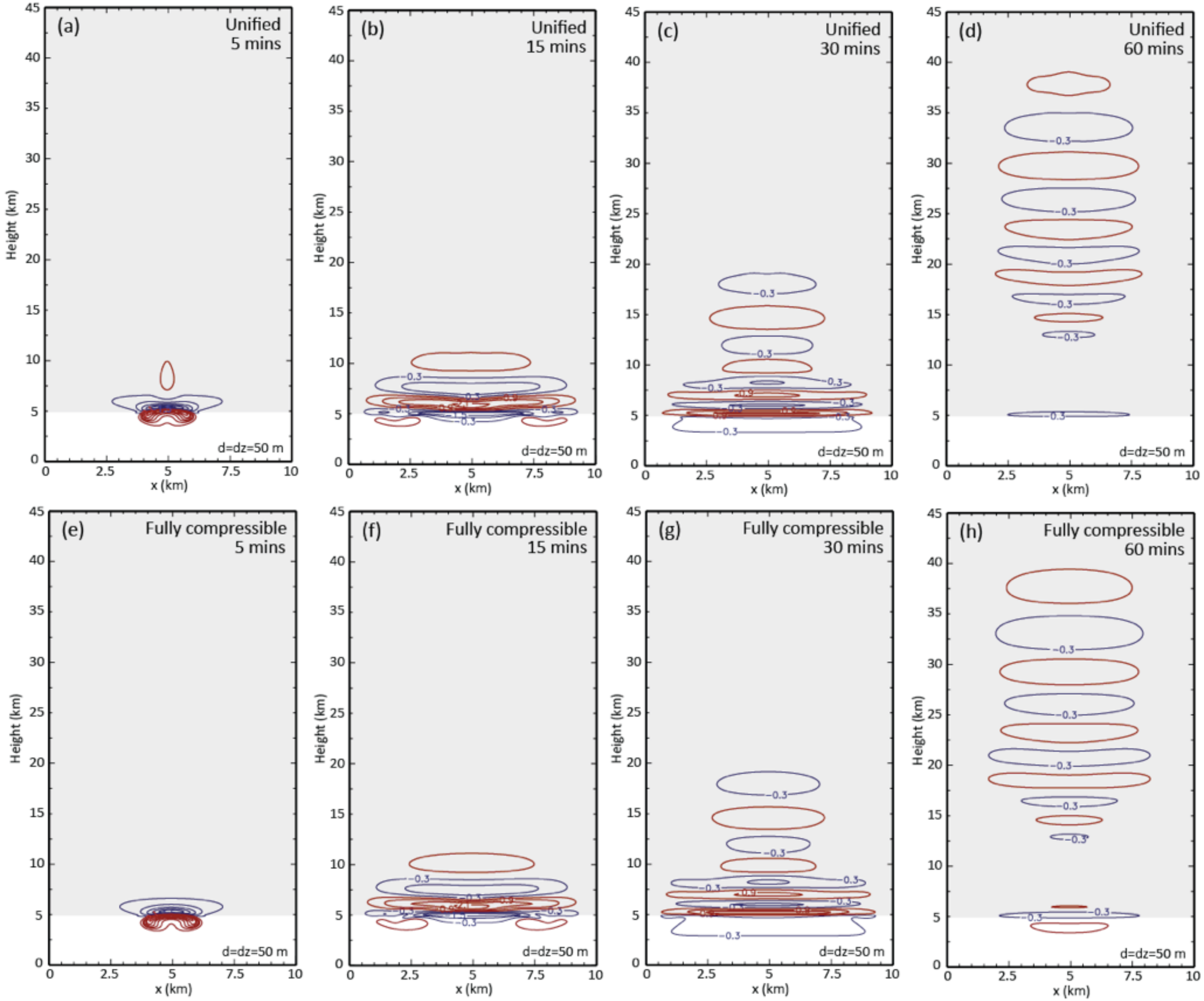
$$\frac{Dw}{Dt} = g \frac{\partial p}{\partial \pi_{qs}}$$

To be continued…

Cloud-scale simulations including a stratosphere with the unified, fully compressible, anelastic and pseudo-incompressible models (Warm bubble and stratosphere)

Unified

Fully compressible



Konor (2014, *MWR*)

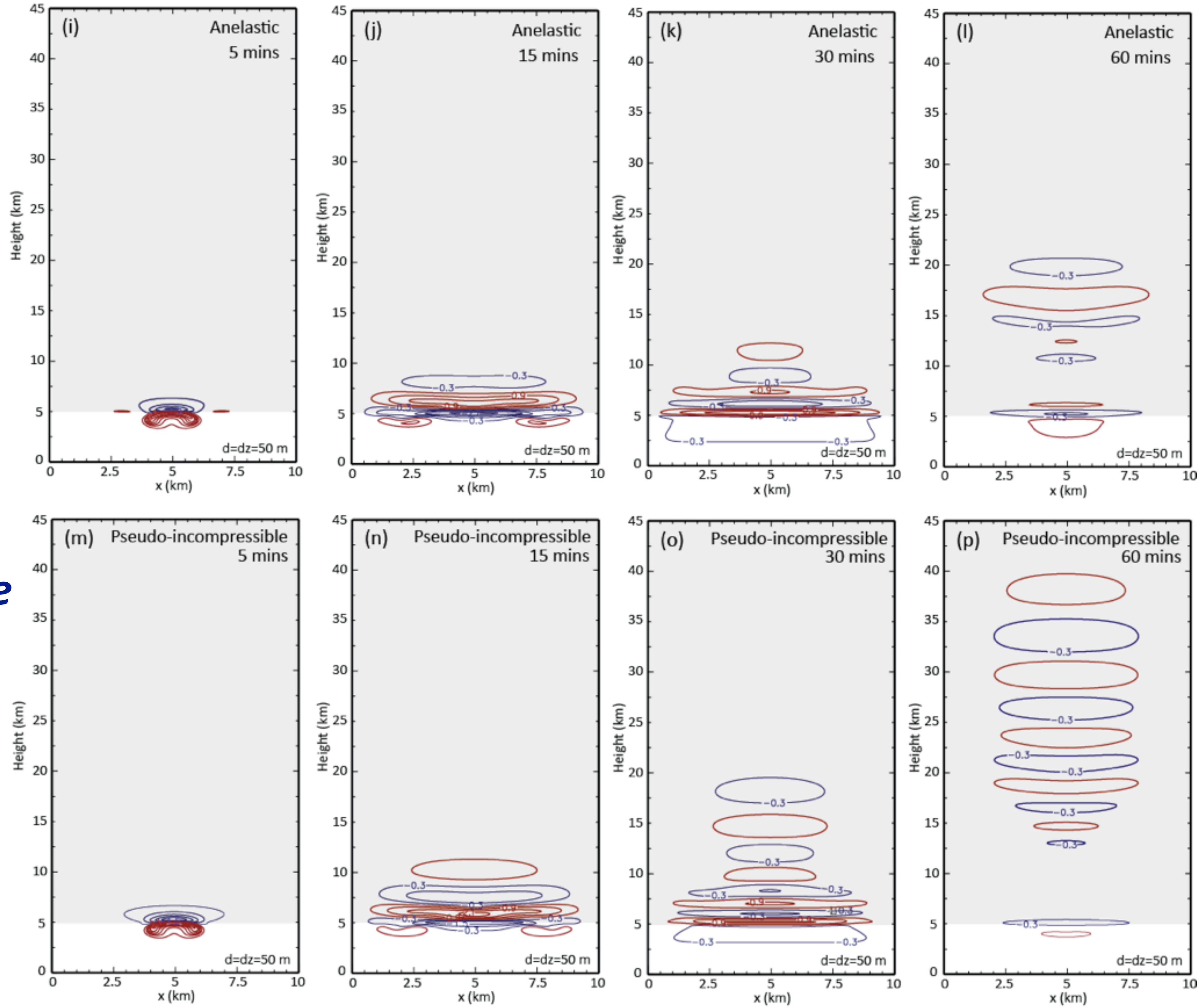
Cloud-scale simulations including a stratosphere with the unified, fully compressible, anelastic and pseudo-incompressible models (Warm bubble and stratosphere)

Anelastic (Lipps and Hemler)

Lipps and Hemler (1983)

Pseudo-incompressible

Durran (1989 and 2008)

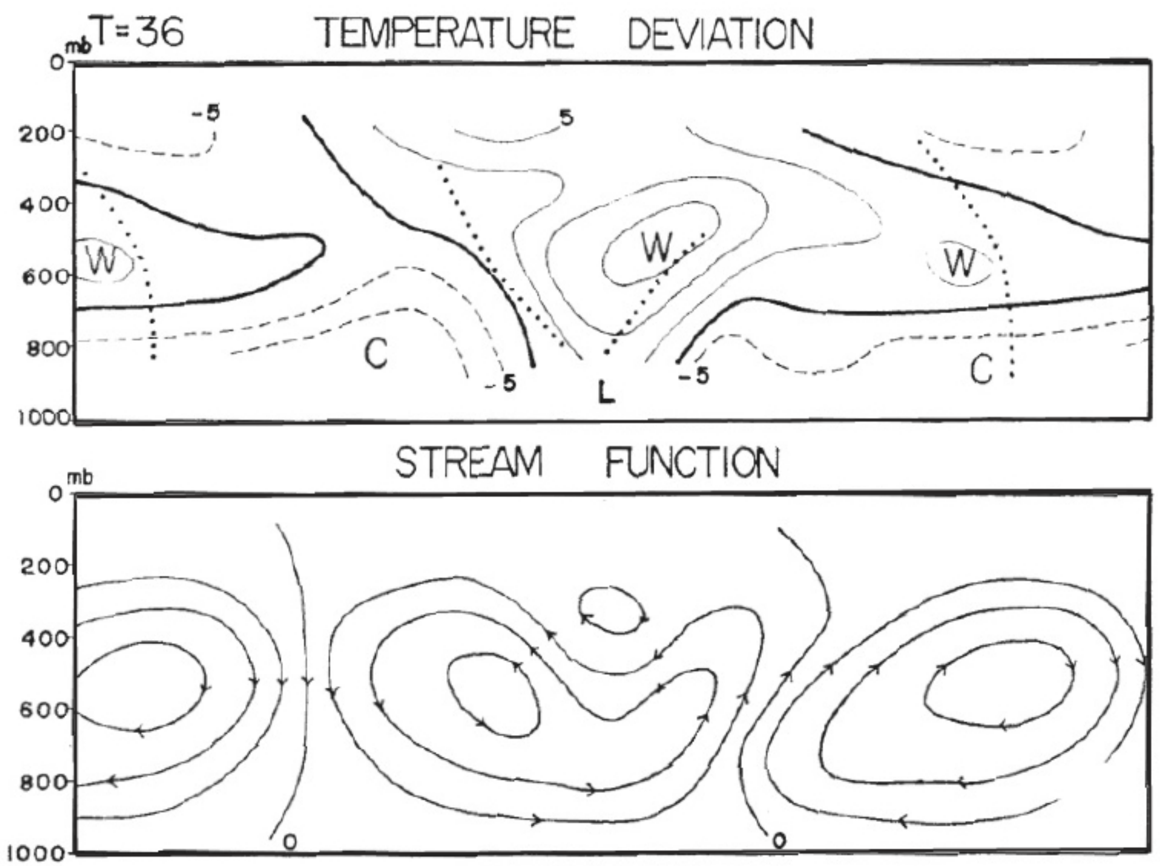


Konor (2014, *MWR*)

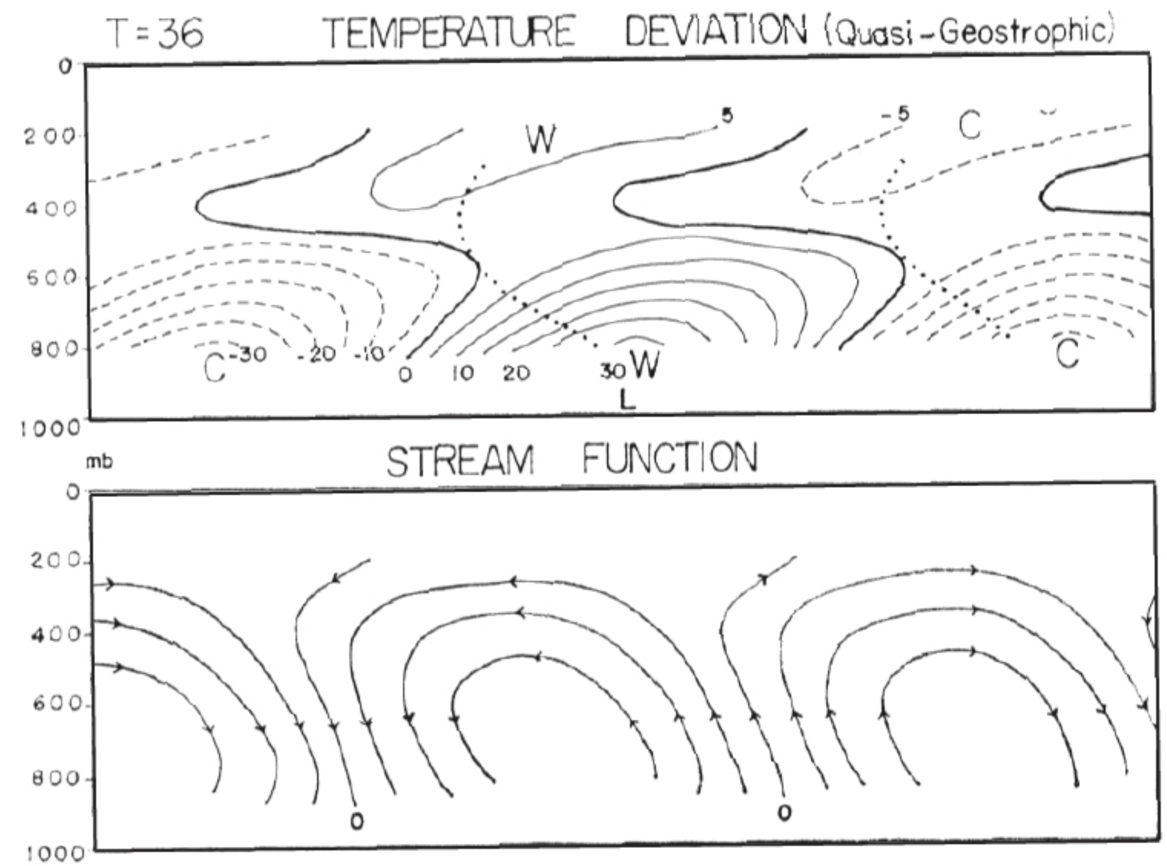
Non-geostrophic effects in the baroclinic prognostic equations

Arakawa (1962)

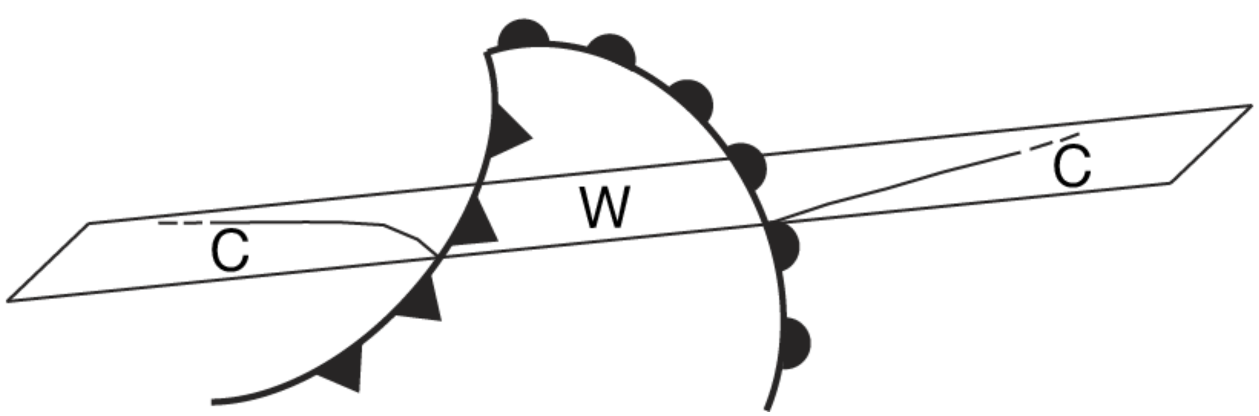
Model based on the balanced system of equations

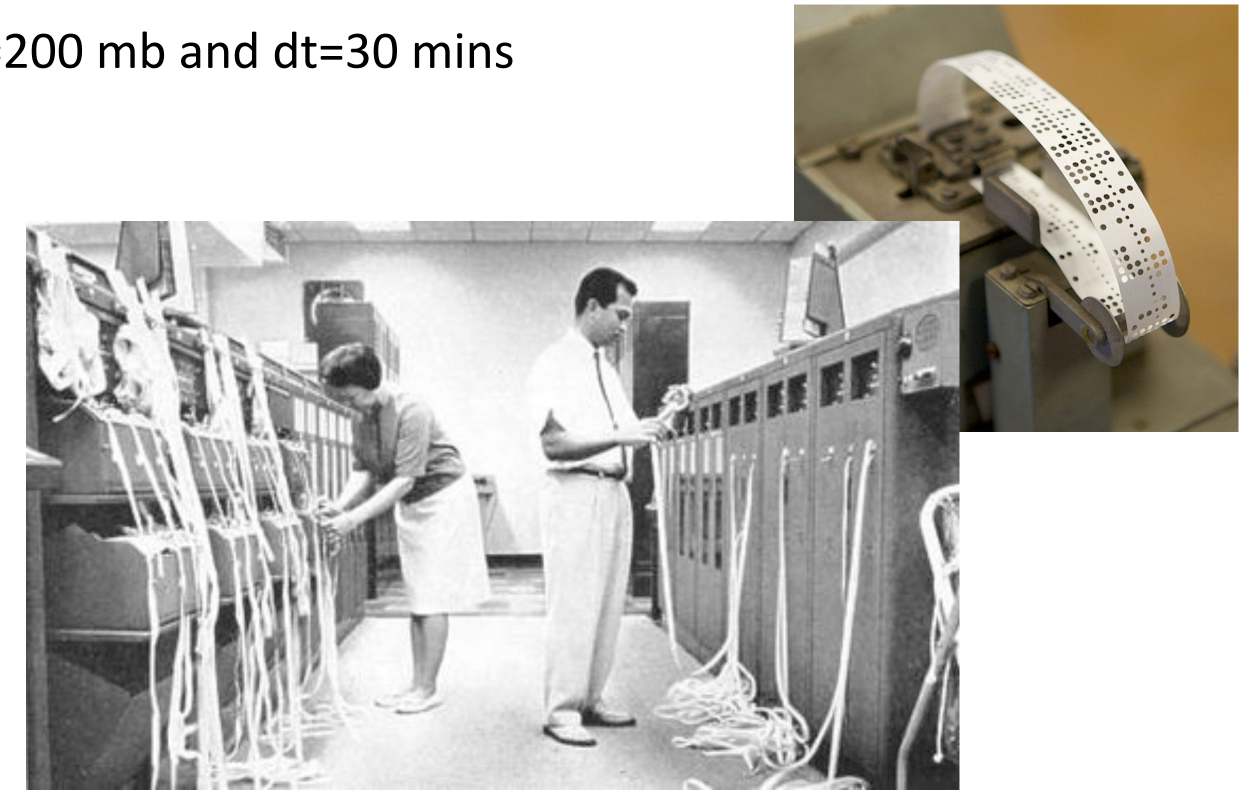


Quasi-geostrophic model

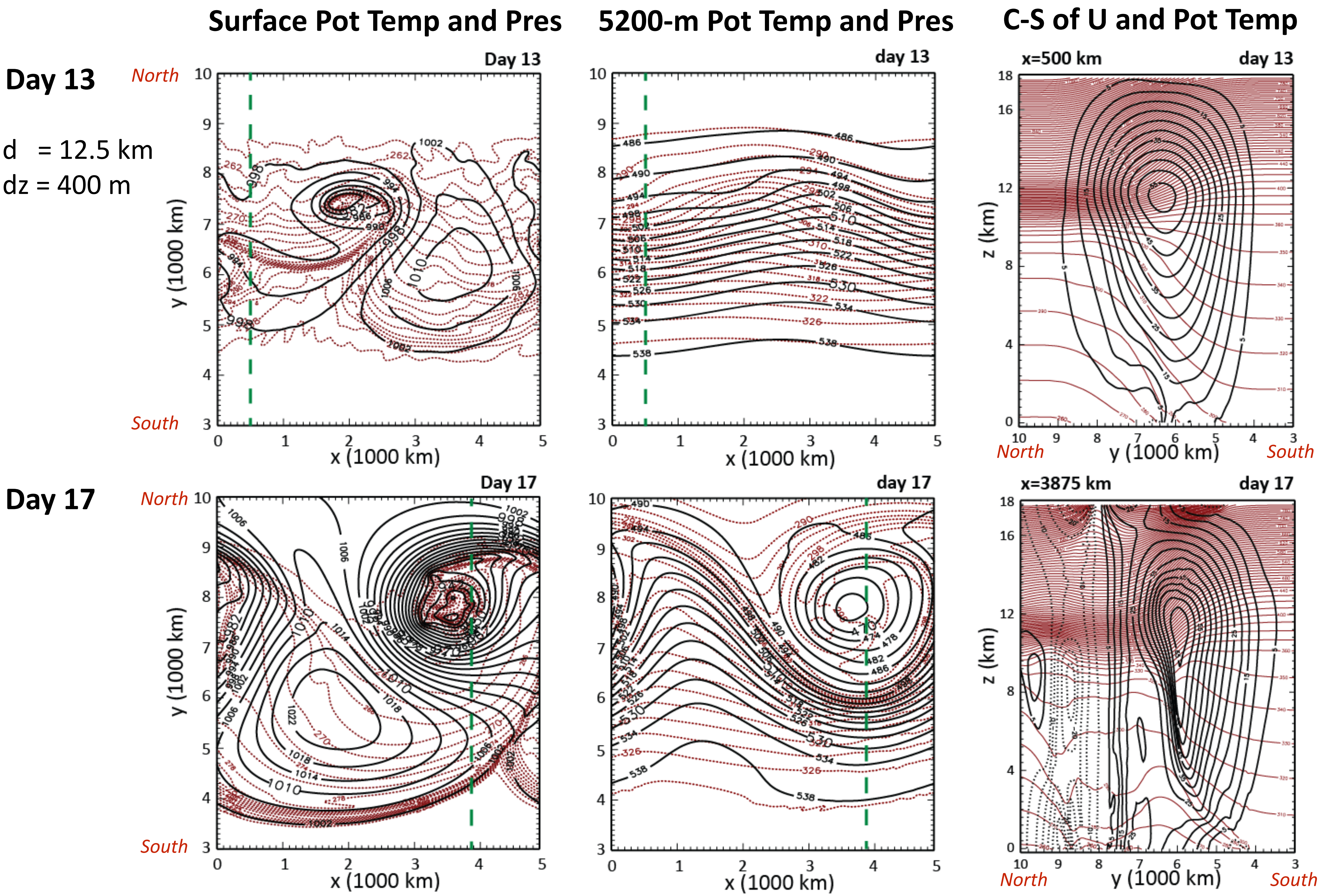


2D model (*x*-*p* plane) d=100 km, dp=200 mb and dt=30 mins





Idealized extratropical cyclogenesis simulated by a unified model



- From an idealized extratropical cyclogenesis simulation on a β-plane. Random initial perturbation on pot temp.
- The unified model used in the simulation has a height vertical coordinate. The C-grid and the L-grid are used in the horizontal and vertical discretizations, respectively.

Unified system in sigma (ς) vertical coordinate

Quasi-hydrostatic module

$$\varsigma \equiv \frac{(p_{qs})_S - p_{qs}}{m}$$ *(increases with height)*

$$m \equiv \frac{\partial p_{qs}}{\partial \varsigma} = (p_{qs})_S - (p_{qs})_T$$

$$\frac{D\theta}{Dt} = 0$$

$$\frac{\partial (p_{qs})_S}{\partial t} = -\int_{\varsigma_S}^{\varsigma_T} \nabla_\varsigma \cdot (m\mathbf{v})\, d\varsigma \quad \text{and} \quad \frac{\partial (p_{qs})_T}{\partial t} = 0$$

$$\frac{\partial z}{\partial \varsigma} = \frac{m}{g}\frac{\kappa \pi_{qs}}{p_{qs}} c_p \theta$$

$$\frac{\partial (m\dot{\varsigma})}{\partial \varsigma} = -\nabla_\varsigma \cdot (m\mathbf{v}) + \int_{\varsigma_S}^{\varsigma_T} \nabla_\varsigma \cdot (m\mathbf{v})\, d\varsigma$$

$$(m\dot{\varsigma})_S = (m\dot{\varsigma})_T = 0$$

$$\frac{D\mathbf{v}}{Dt} + f\mathbf{k} \times \mathbf{v} = - c_p \theta \nabla_\varsigma \pi_{qs} - g(\nabla_\varsigma z)$$

$$- c_p \theta \nabla_\varsigma \delta\pi + g(\nabla_\varsigma z)\frac{\rho_{qs}}{m} c_p \theta \frac{\partial}{\partial \varsigma}\delta\pi$$

Nonhydrostatic module

$$\frac{Dw}{Dt} = -\frac{g\rho_{qs}}{m} c_p \theta \frac{\partial \delta\pi}{\partial \varsigma}$$

(not a prognostic equation of the model, used only in the derivation of the elliptic equation)

$$w = w_c + \frac{1}{g\rho_{qs}} m\dot{\varsigma}$$

$$w_c \equiv \left(\frac{\partial}{\partial t}\right)_\varsigma z + \mathbf{v} \cdot \nabla_\varsigma z$$

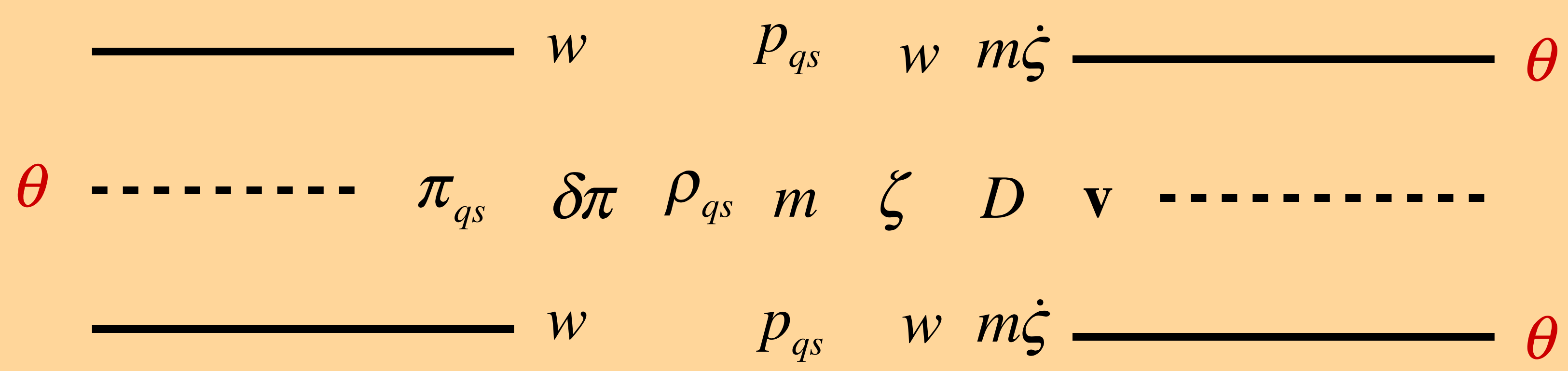
$$\left(\frac{\partial}{\partial t}\right)_\varsigma z = \int_{\varsigma_S}^{\varsigma} \left(\frac{\partial}{\partial t}\right)_\varsigma \left(\frac{mR\pi_{qs}\theta}{g p_{qs}}\right) d\varsigma$$

$$\nabla_\varsigma \cdot (m c_p \theta \nabla_\varsigma \delta\pi) - \nabla_\varsigma \cdot \left[g\rho_{qs} (\nabla_\varsigma z) c_p \theta \frac{\partial}{\partial \varsigma}\delta\pi \right] + \frac{\partial}{\partial \varsigma}\left(\frac{g^2 \rho_{qs}^2}{m} c_p \theta \frac{\partial \delta\pi}{\partial \varsigma} \right)$$
$$= -\nabla_\varsigma \cdot \left[m\left(c_p \theta \nabla_\varsigma \pi_{qs} + g\nabla_\varsigma z\right)\right]$$
$$- g\frac{\partial}{\partial \varsigma}\left[-w\frac{\partial \rho_{qs}}{\partial t} + \frac{\partial}{\partial t}(\rho_{qs} w_c) + \rho_{qs} G_z \right] - \nabla_\varsigma \cdot \left(m\mathbf{G}_H - \mathbf{v}\frac{\partial m}{\partial t}\right) + \frac{\partial^2 m}{\partial t^2}$$

3-Unified Z-grid Icosahedral Model (UZIM)

- Predicts vorticity and divergence
- UZIM-z is the height vertical coordinate version with the L-grid
- UZIM-sigma is the sigma vertical coordinate version with the CP-grid
- For hexagon/pentagon icosahedral grid optimization, see Heikes et al. (2013 *MWR*)

L-grid of UZIM-z **CP-grid of UZIM-*sigma***



CP-grid and L-grid difference

Arakawa and Moorthi (1988)

Arakawa and Konor (1996)

Characteristics of simulations shown here:

- 32 stretched layers in the vertical under 32 km

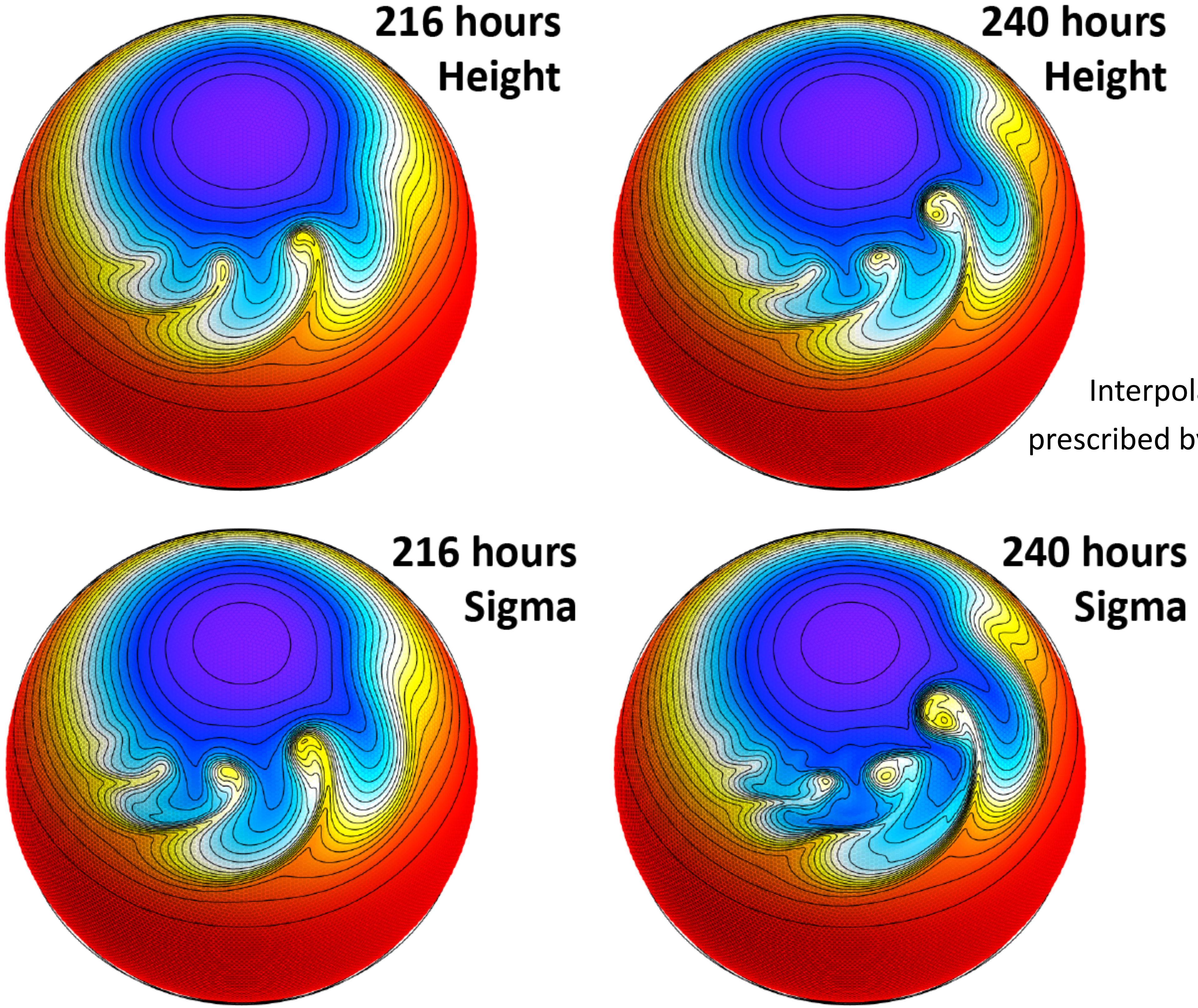
Grid	Number of cells	Horizontal grid distance
G6	40962	121.3 km
G7	163842	60.6 km

Idealized extratropical cyclogenesis (setup from Jablonowski and Williamson, 2006)

Surface Potential Temperature

UZIM-z

UZIM-sigma

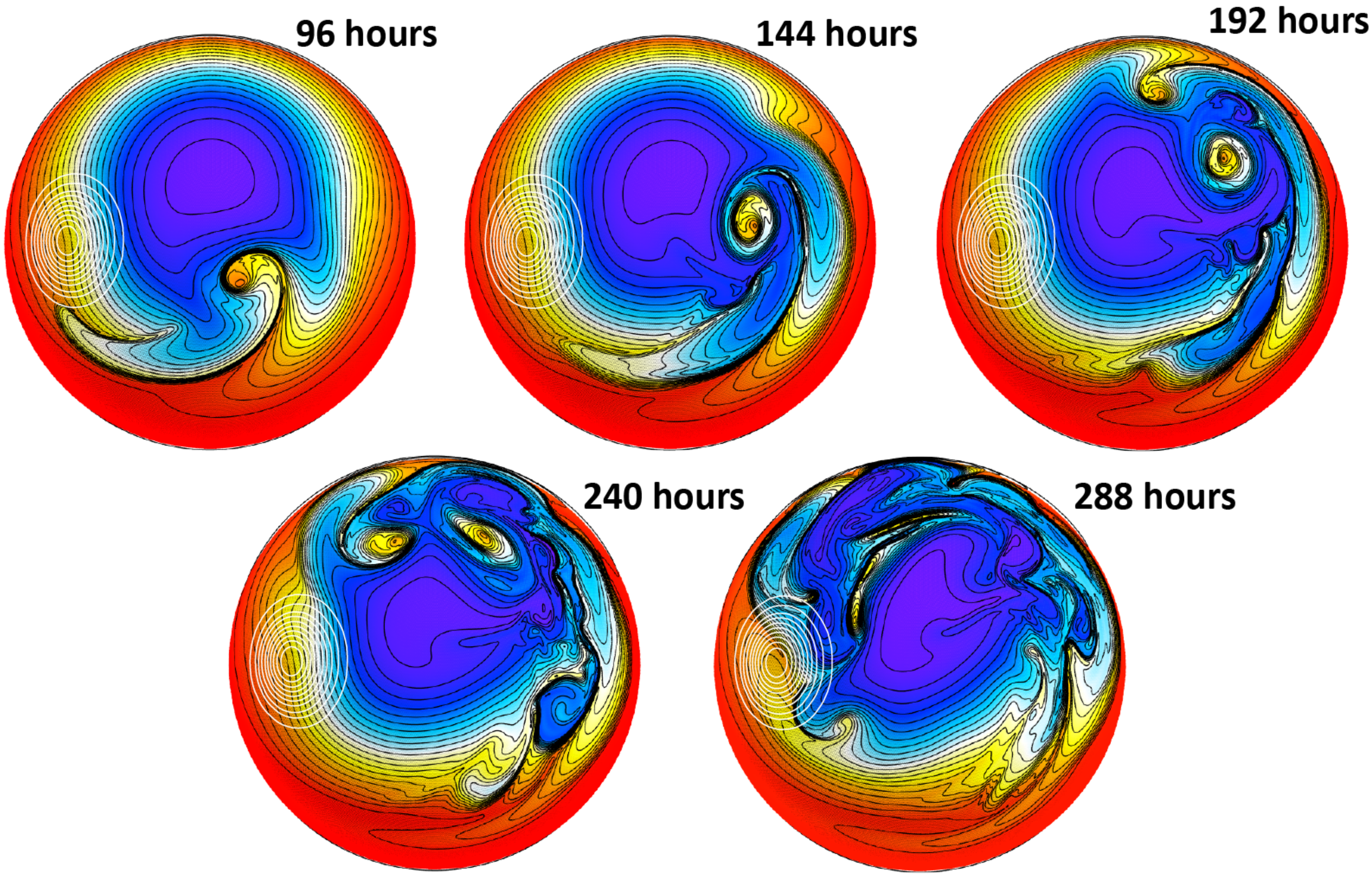


Interpolated to z_S
prescribed by JW2006

Resolution: G7-32L

Extratropical cyclogenesis modified by a mountain

Surface Potential Temperature

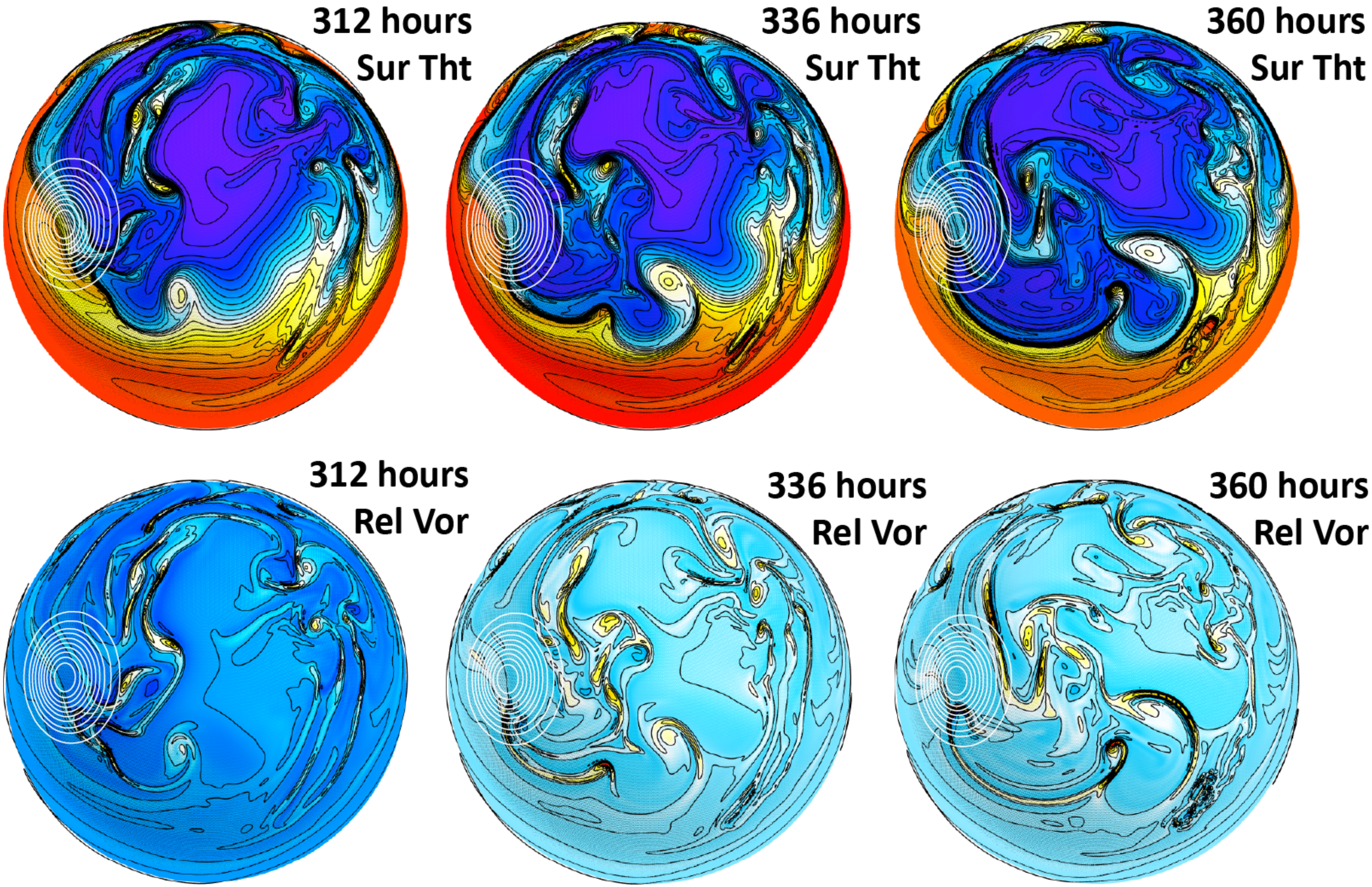


Mountain is 2000 m high
(No gravity wave drag)

UZIM-sigma

Resolution: G7-32L

Extratropical disturbances induced by a mountain



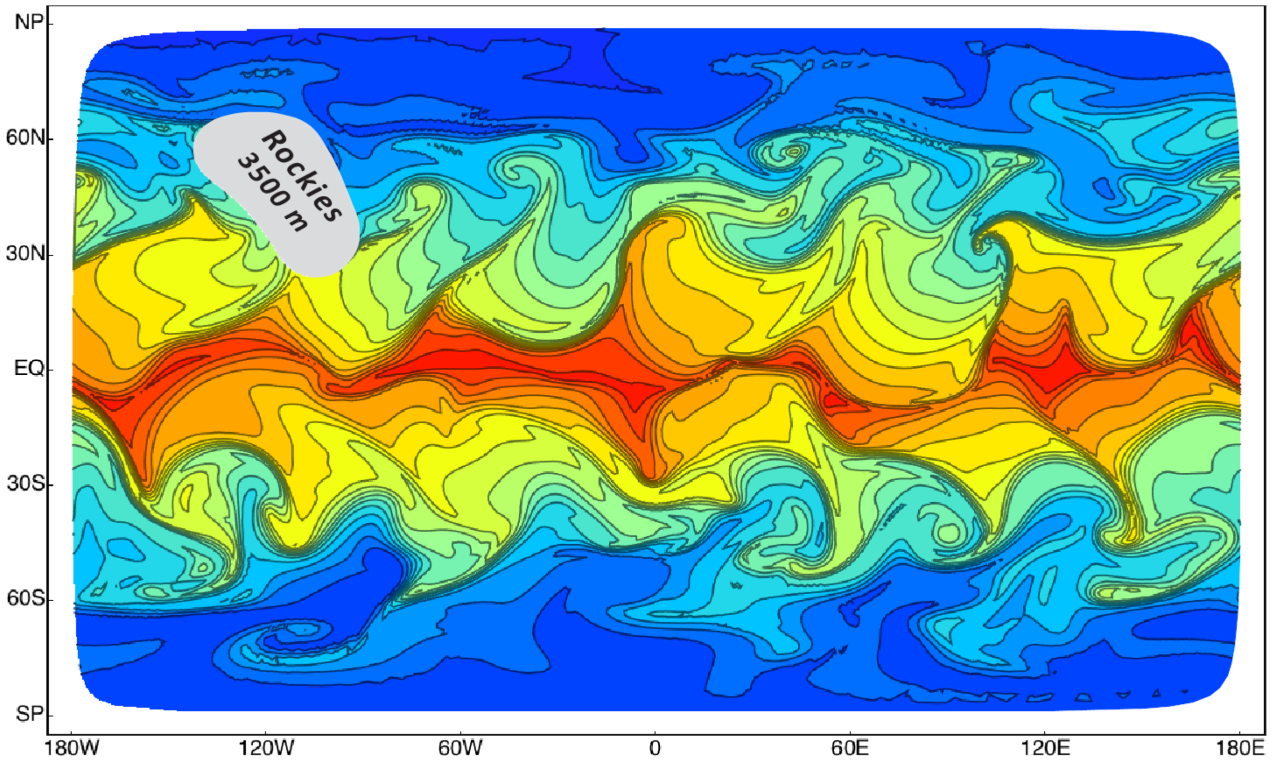
Mountain is 2000 m high

UZIM-sigma

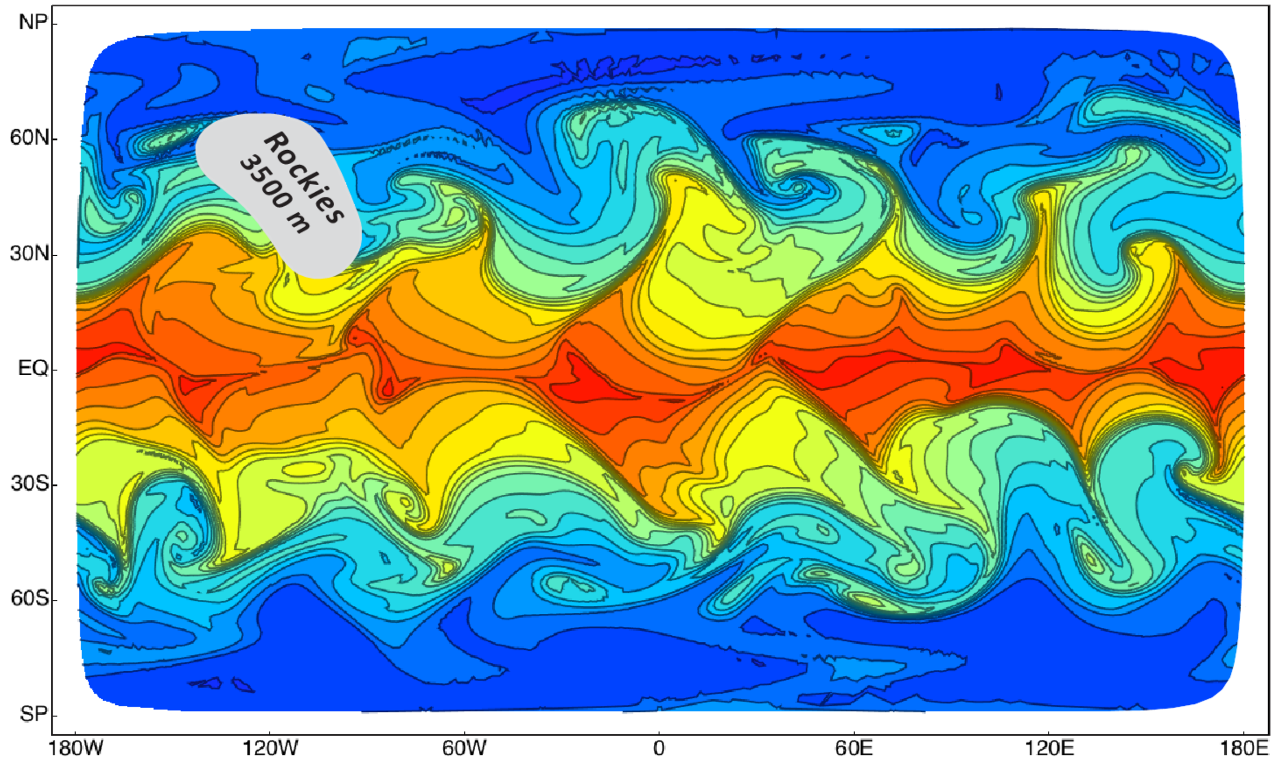
Resolution: G7-32L

Snapshots of surface potential temperature

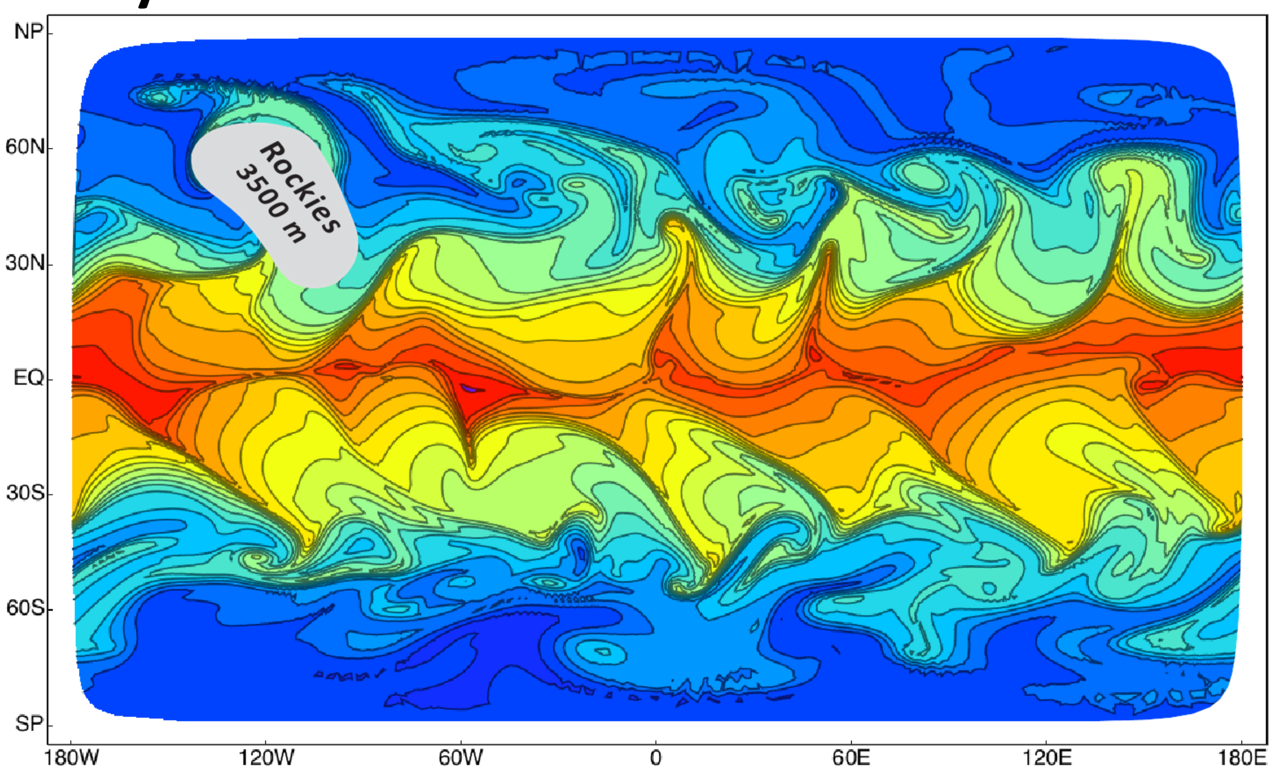
Day 1690



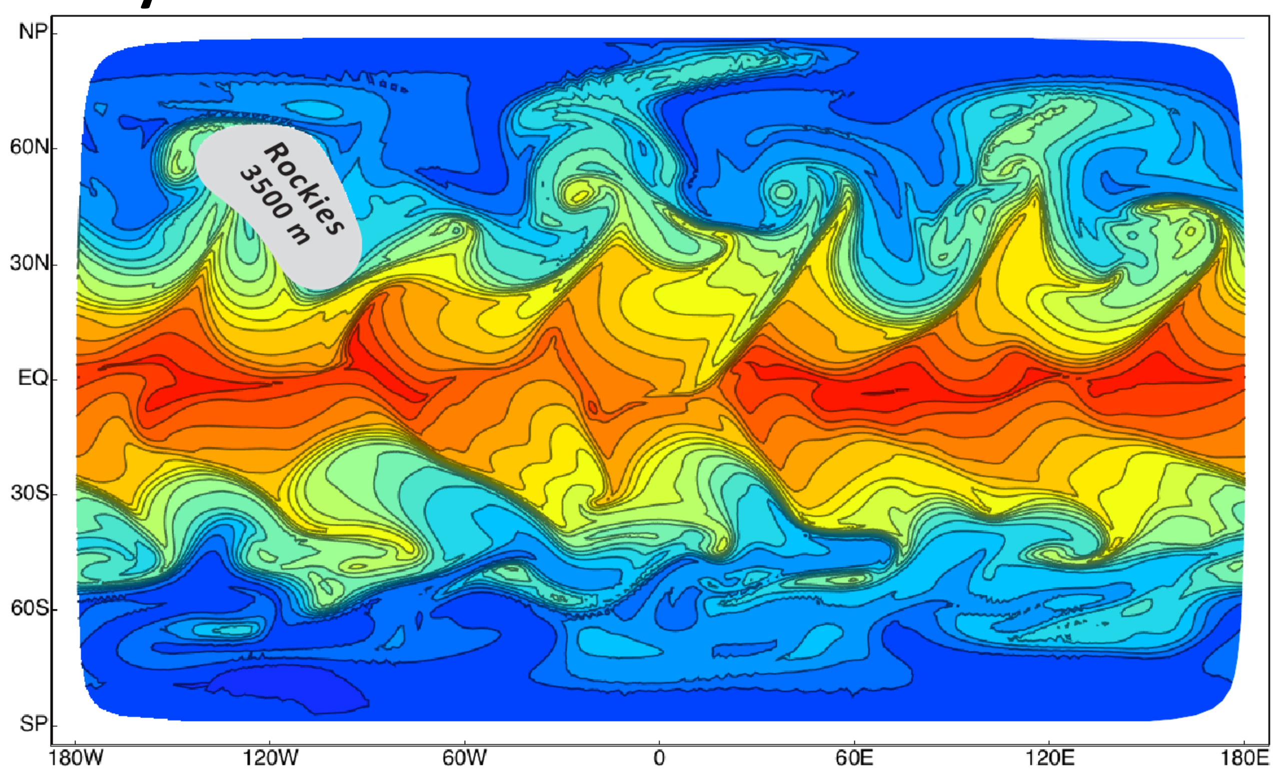
Day 1720



Day 1750



Day 1780

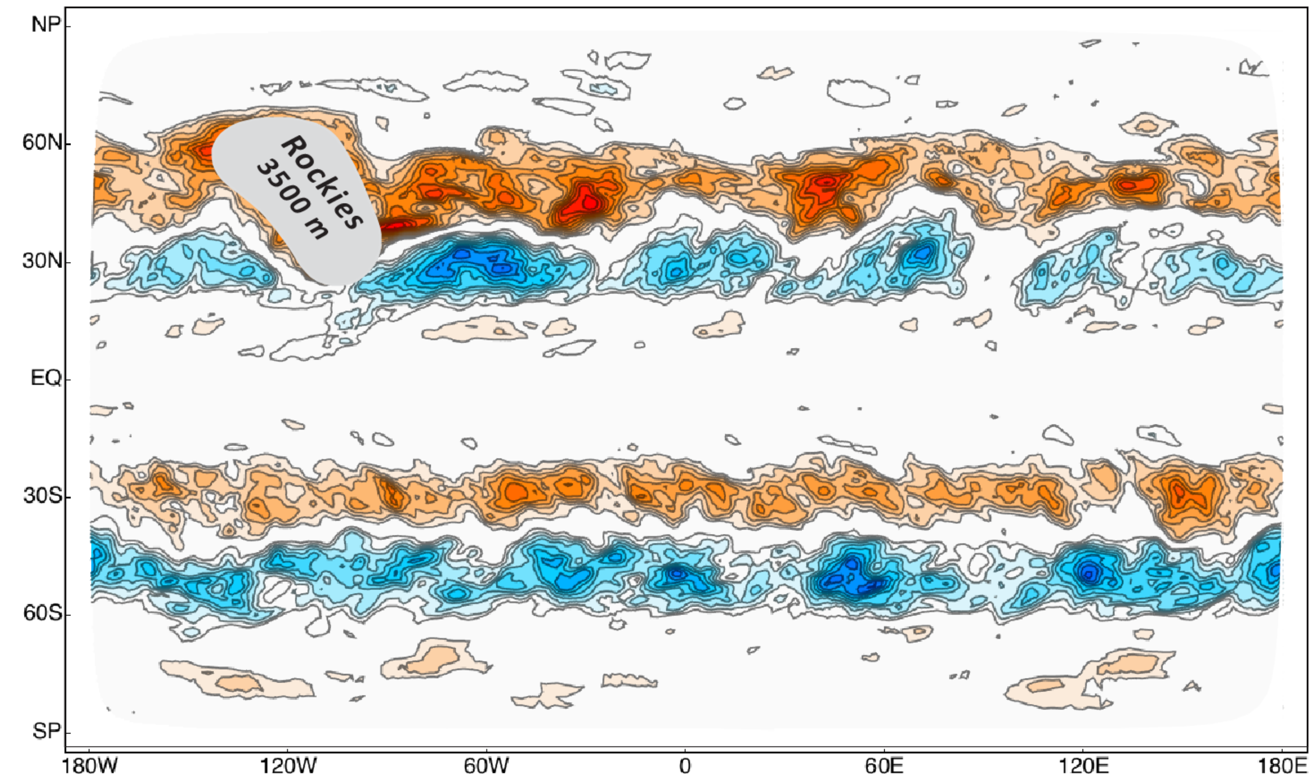


UZIM-sigma *(No gravity wave drag)*

Resolution: G6-32L

Idealized general circulation with Held-Suarez forcing and the "Rocky" mountains

Time Averaged Vorticity at 750 mb

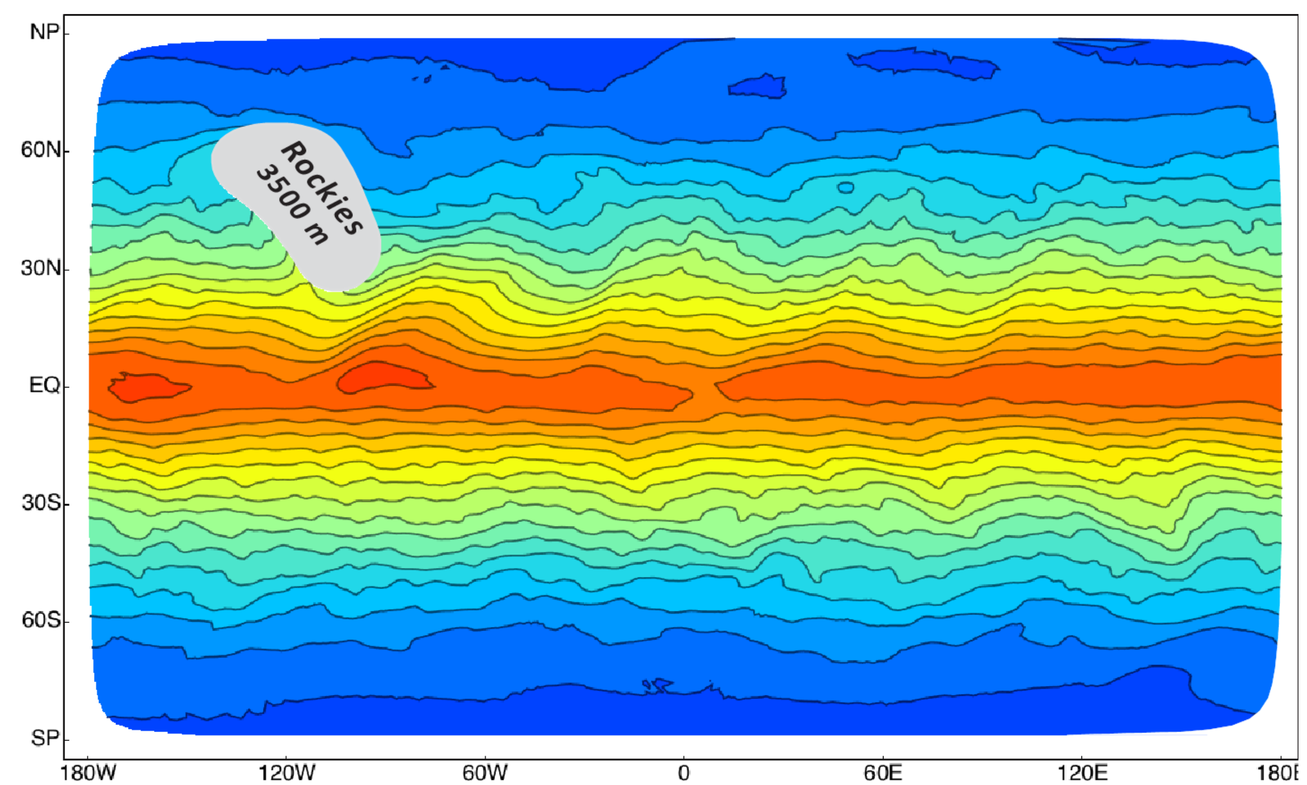


"Rockies" are 3500 m high

Averaging between days 1500 to 2000

No gravity wave drag

Time Averaged Surface Potential Temperature

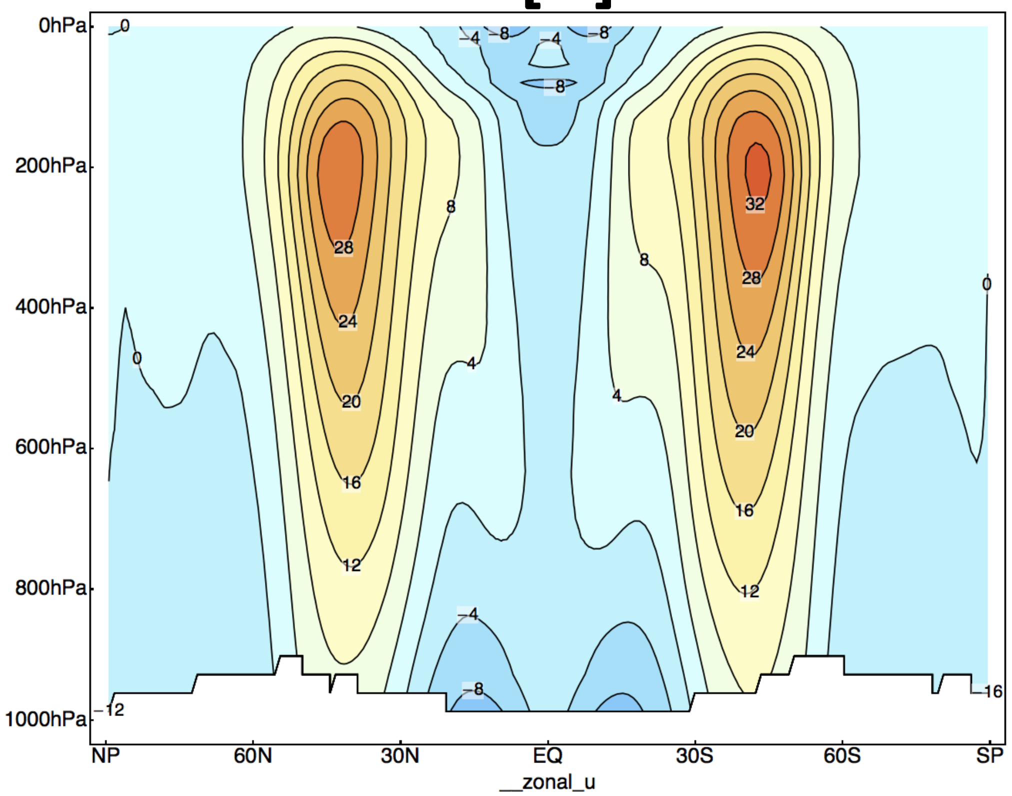


UZIM-sigma

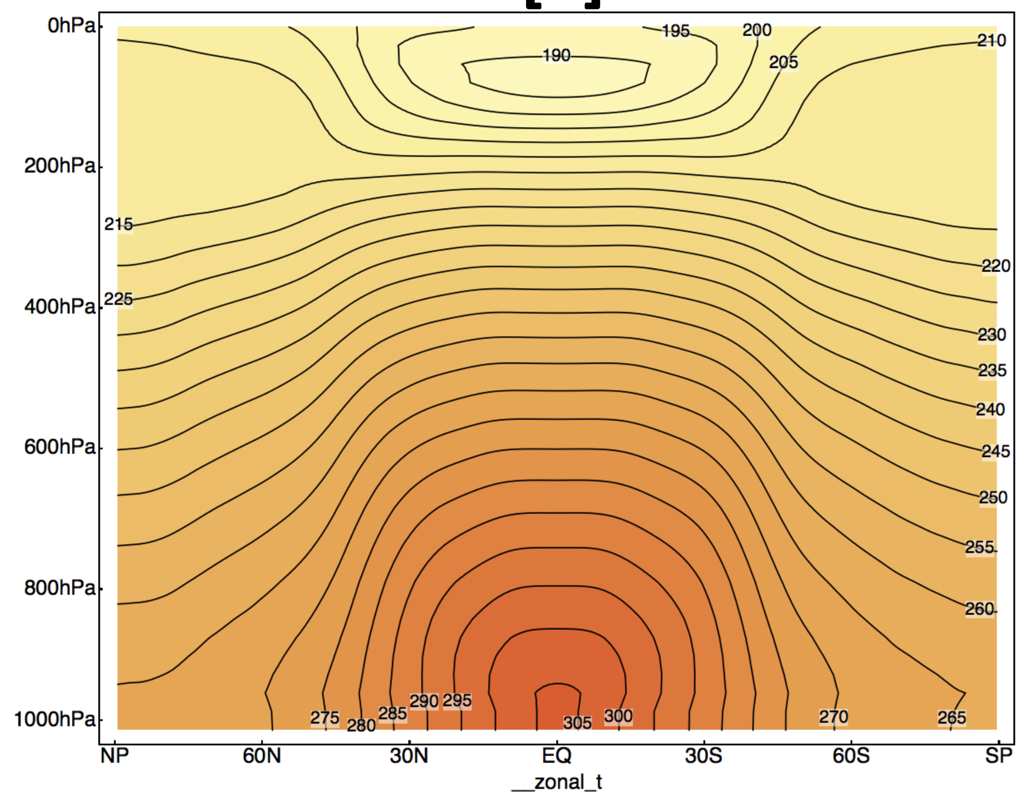
Resolution: G7-32L

Idealized general circulation with Held-Suarez forcing and the “Rocky” mountains

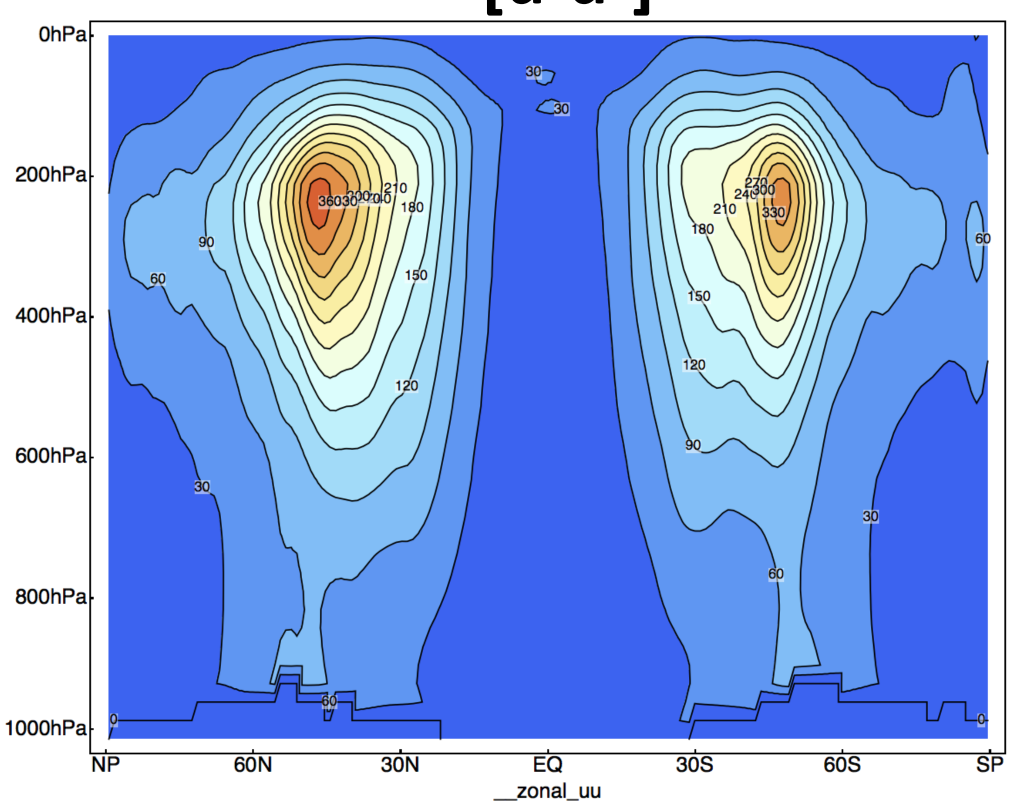
[U]



[T]



[u´u´]



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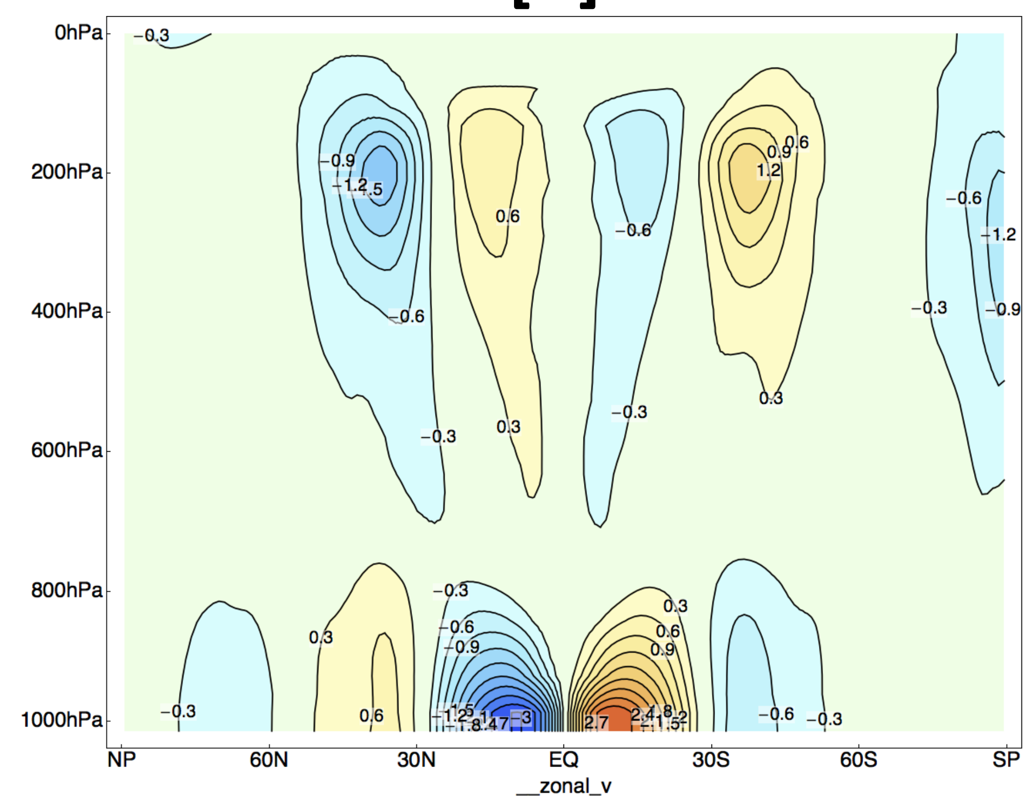
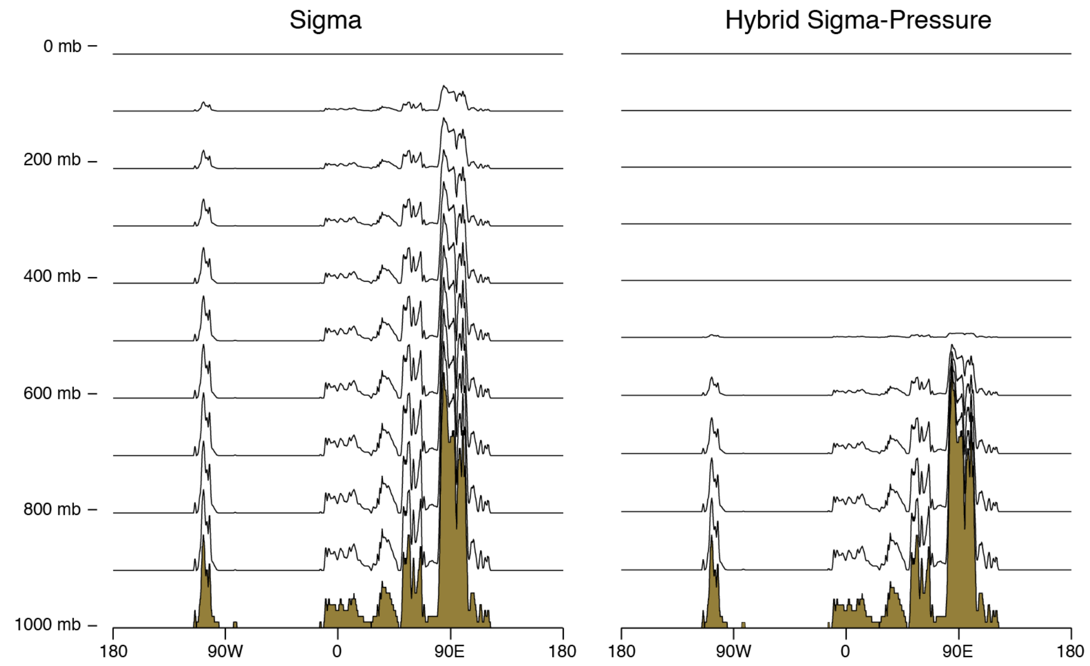


UZIM-sigma

Zonal averaging excludes Rockies

No gravity wave drag

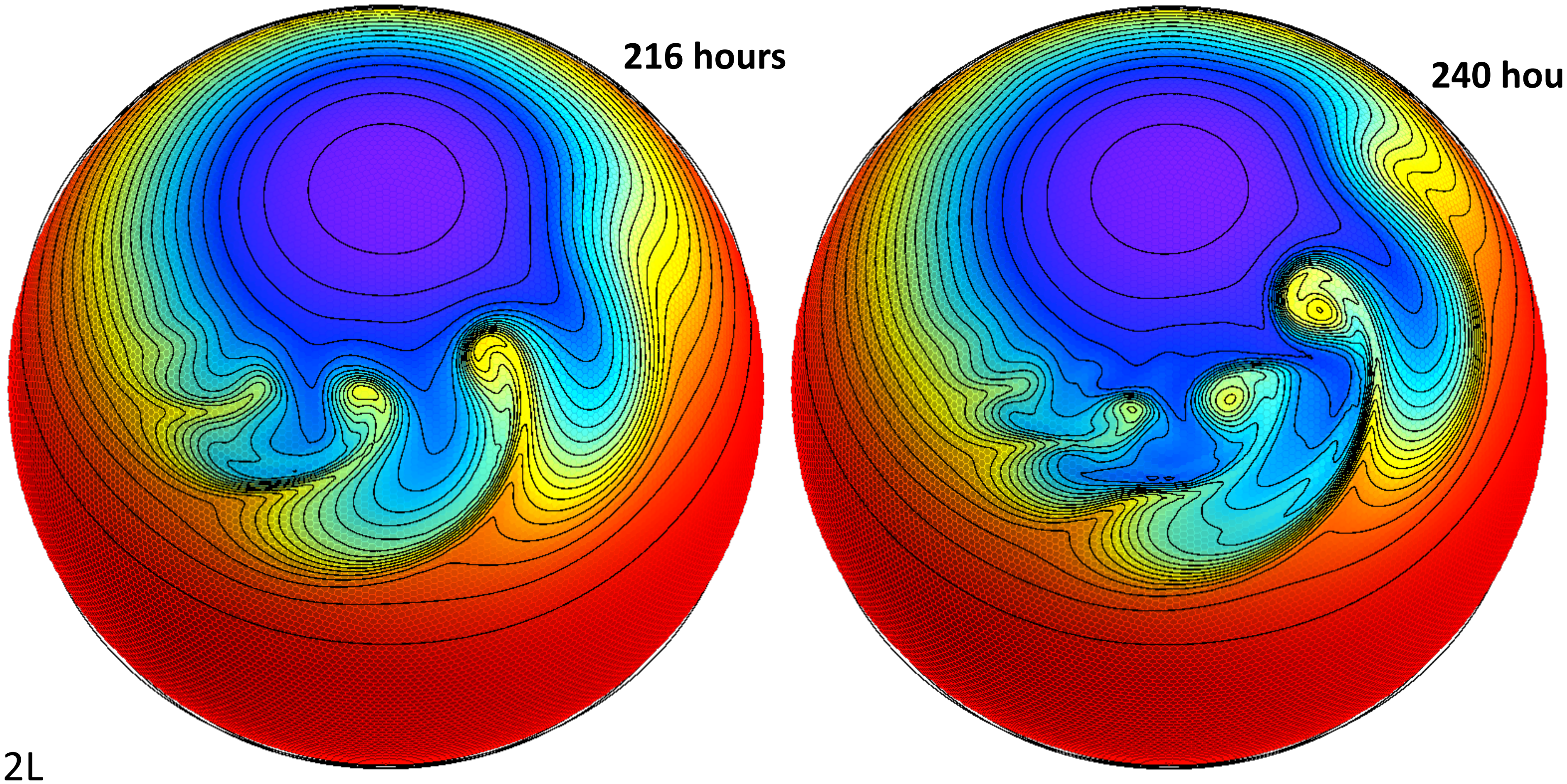
UZIM-sigma-p



Surface topography at 39^0 Latitude circle

Idealized extratropical cyclogenesis (Jablonowski and Williamson, 2006)

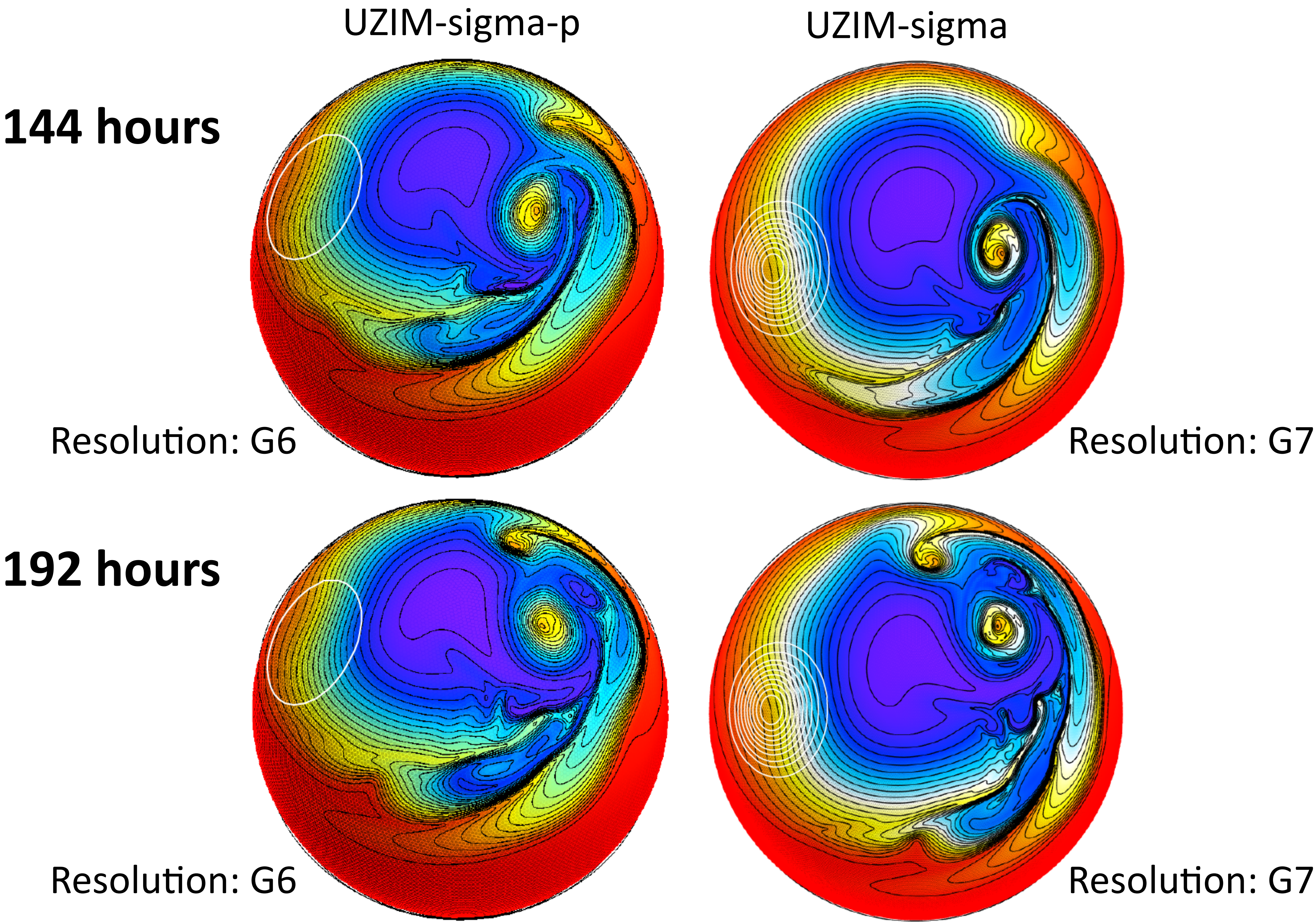
Surface Potential Temperature



Resolution: G6-32L

Extratropical cyclogenesis modified by a mountain

Surface Potential Temperature



4-Summary

- Icosahedral grid optimization, horizontal discretization and 2D multigrid solver, which are also used in the UZIM, have been updated under CMMAP. The updates include accuracy increases, computational performance improvements and extensions to higher resolutions (up to G13).

- The UZIM based on the height coordinate and the L-grid, and the UZIM based on the hybrid sigma-pressure coordinate and the CP-grid are ready for large-scale global applications.

- Our quest for a "unified" 3D elliptic solver is still continuing.

- The unified system (and UZIM) in the hybrid isentropic-sigma coordinate and the SP-UZIM (with surface topography and prototype MMF) are months away.

Thank you professor Arakawa for being an inspiration for us.