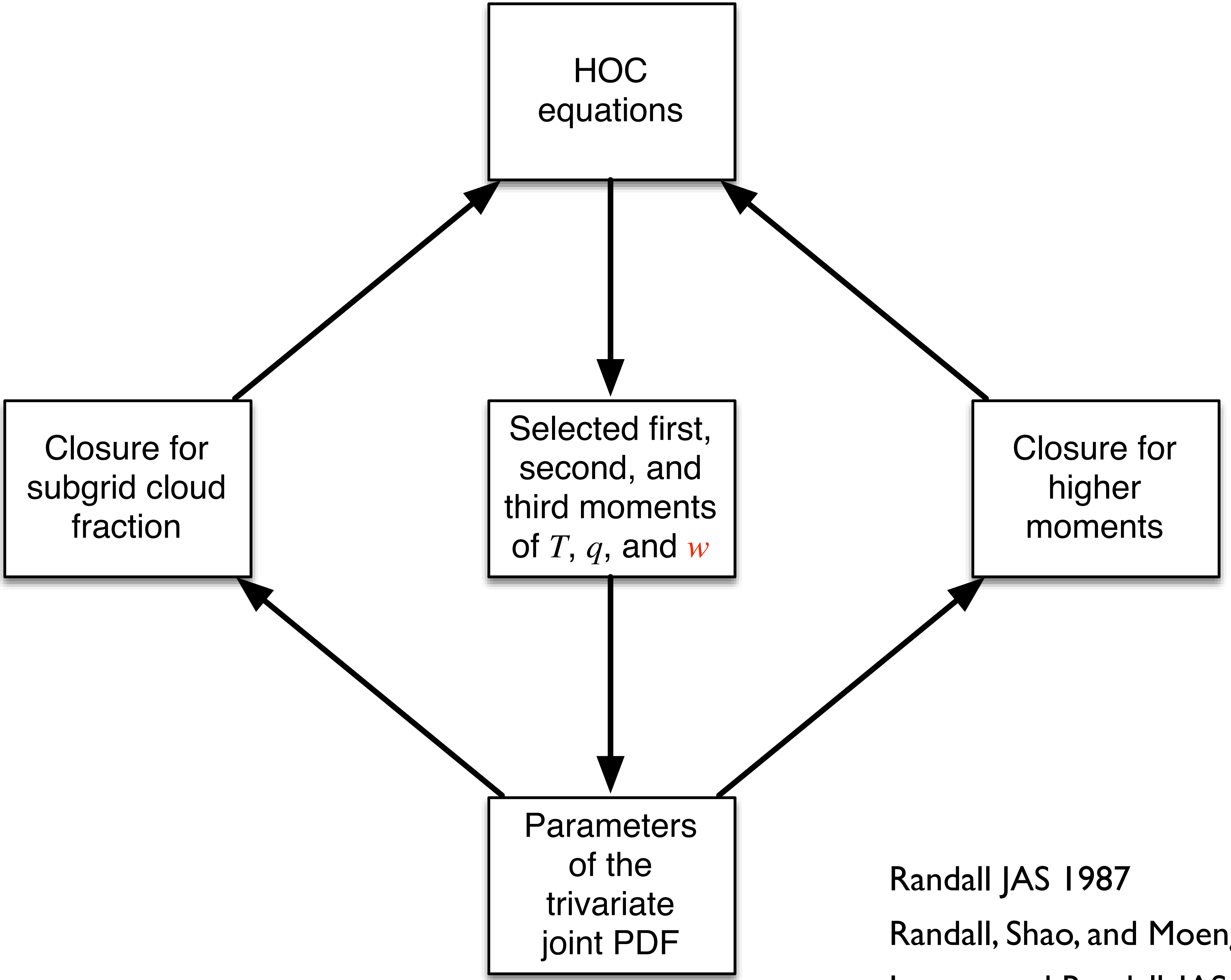
Double Gaussians without tears

Thanks to Grant Firl

Assumed distributions with HOC



Randall JAS 1987

Randall, Shao, and Moeng, JAS 1992

Lappen and Randall, JAS 2001

Closures Found

1) Closures for the effects of higher moments that are not predicted, e.g., as mentioned above, the fourth moments in a third-order closure model. ✔

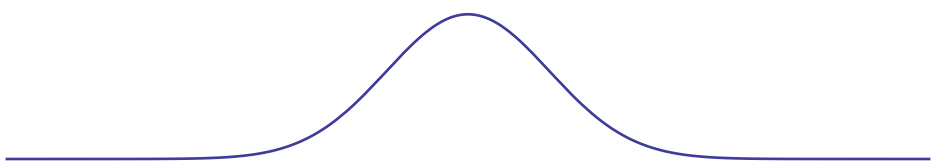
2) Closures for moments involving the pressure, which occur in the equations for moments that involve velocity components.

3) Closures for dissipation rates, which are especially important in the equations governing variances.

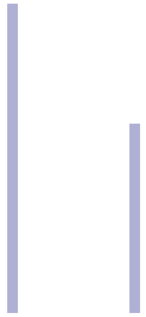
4) Closures to determine SGS phase changes (e.g., Sommeria and Deardorff, 1977; Mellor, 1977) and other microphysical processes (e.g., Larson et al., 2005), as well as radiative heating and cooling. ✔

Which assumed PDF?

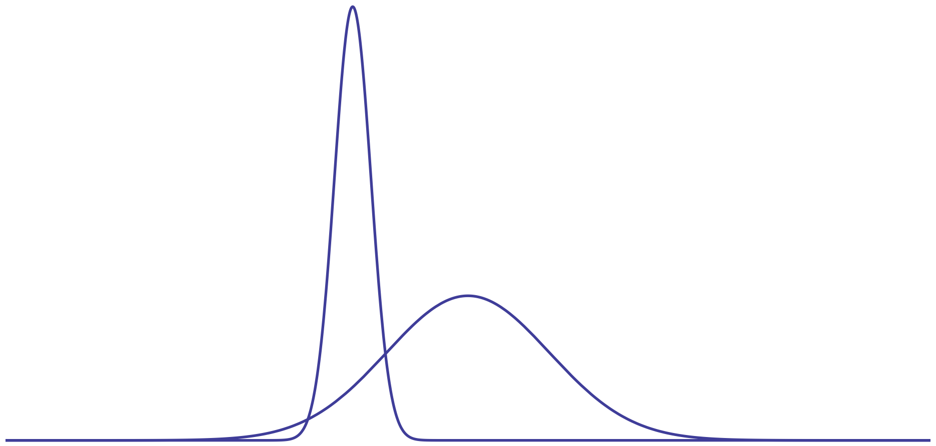
- Gaussian?
  No third moments.

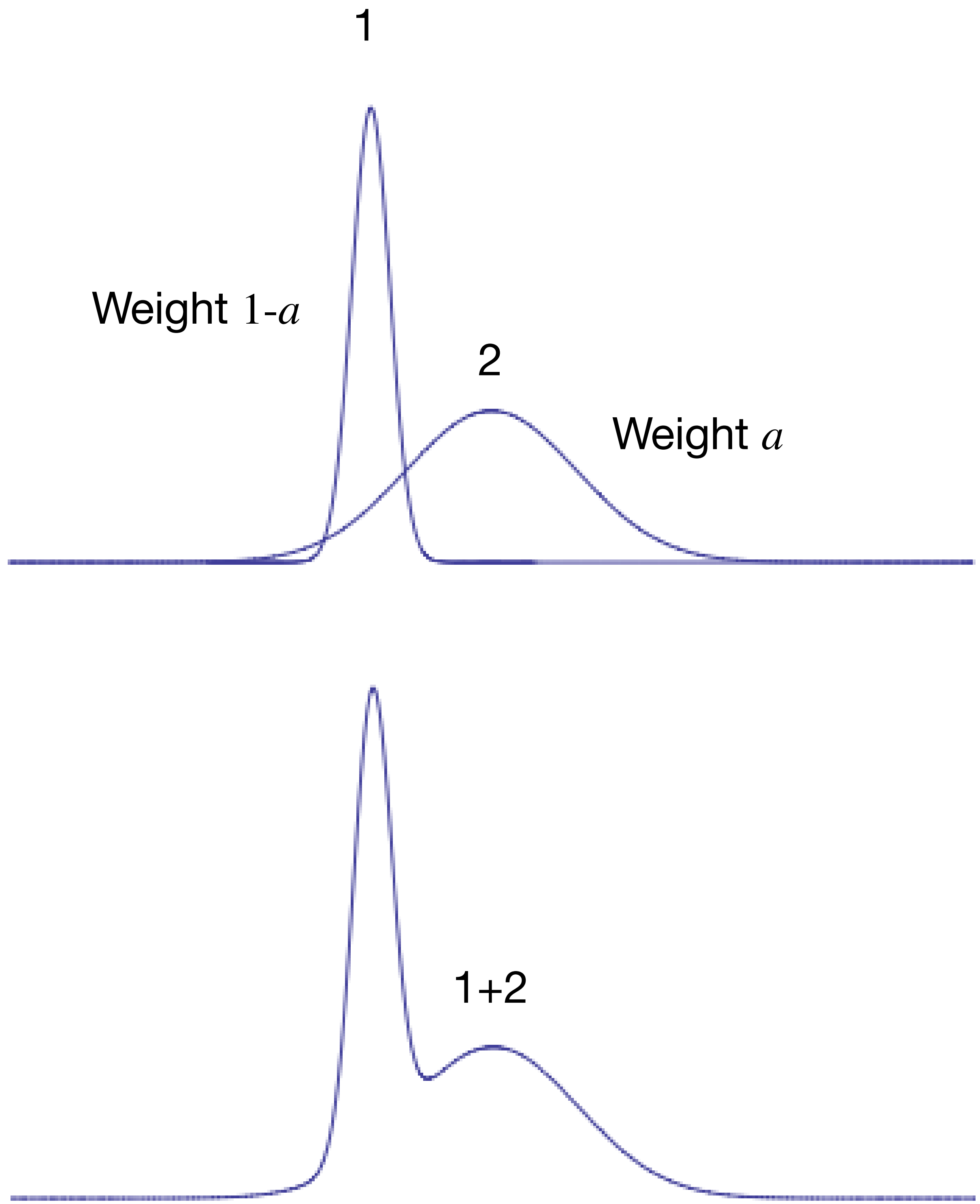


- Two delta functions?
  Not realistic enough.



- Two Gaussians?
  Good compromise,
  suggested by Lewellen and
  Yoh (1993) and implemented
  by Golaz et al., 2002.





We use trivariate double Gaussians, for w, T, and q.

Equations for the moments

$$\overline{x} = (1-a)\overline{x_1} + a\overline{x_2}$$

$$\overline{x'y'} = a(1-a)\delta\overline{x}\delta\overline{y} + (1-a)\left(\overline{x'y'}\right)_1 + a\left(\overline{x'y'}\right)_2$$

$$\begin{aligned}\overline{x'y'z'} &= a(1-a)(1-2a)\delta\overline{x}\delta\overline{y}\delta\overline{z} \\ &\quad + a(1-a)\left\{\delta\overline{x}\left[\left(\overline{y'z'}\right)_2 - \left(\overline{y'z'}\right)_1\right] + \delta\overline{y}\left[\left(\overline{x'z'}\right)_2 - \left(\overline{x'z'}\right)_1\right] + \delta\overline{z}\left[\left(\overline{x'y'}\right)_2 - \left(\overline{x'y'}\right)_1\right]\right\}\end{aligned}$$

$$\delta x \equiv \overline{x_2} - \overline{x_1}$$

Which moments are important?

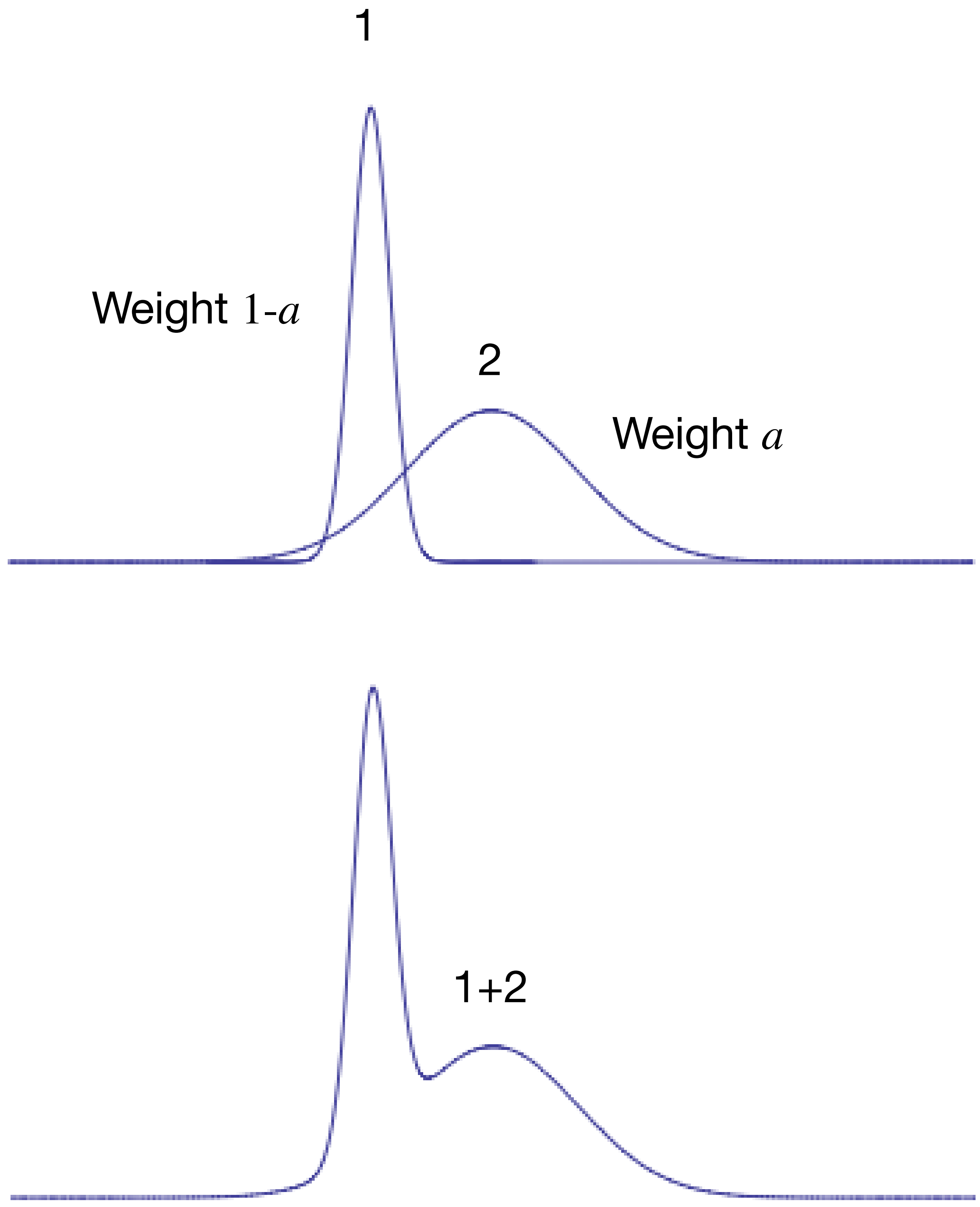
- Needed in HOC equations, directly, *or*
- Needed to determine the parameters of the PDF

Fluxes are needed in the first moment equations.

$$\overline{w'\theta'} \qquad \overline{w'q'}$$

The third moments measure the “distance” between the Gaussians.

$$\overline{w'w'w'} \qquad \overline{q'q'q'} \qquad \overline{\theta_l'\theta_l'\theta_l'}$$



The third moments measure the "distance" between the Gaussians.

Simpler case of two delta functions

$$\overline{x} = (1-a)\overline{x_1} + a\overline{x_2}$$

$$\overline{(x')^2} = a(1-a)\left(\delta\overline{x}\right)^2$$

$$\overline{(x')^3} = a(1-a)(1-2a)\left(\delta\overline{x}\right)^3$$

$$\delta x = \pm\sqrt{\frac{\overline{(x')^2}}{a(1-a)}}$$

$$\overline{(x')^3} = \pm a(1-a)(1-2a)\left[\frac{\overline{(x')^2}}{a(1-a)}\right]^{3/2}$$

$$\pm S_x = \frac{1-2a}{\sqrt{a(1-a)}}$$

$$S_x \equiv \frac{\overline{(x')^3}}{\left[\overline{(x')^2}\right]^{3/2}}$$

$$a = \frac{1}{2}\left(1 \pm \frac{S_x}{\sqrt{4+S_x^2}}\right)$$

$$\delta x = \mp\sqrt{\overline{(x')^2}\left(4+S_x^2\right)}$$

Parameters of the trivariate PDF

a , -1

$\overline{w_1}$, $\overline{w_2}$, $\overline{T_1}$, $\overline{T_2}$, $\overline{q_1}$, $\overline{q_2}$, -6

$\overline{(w')_1^2}$, $\overline{(w')_2^2}$, $\overline{(T')_1^2}$, $\overline{(T')_2^2}$, $\overline{(q')_1^2}$, $\overline{(q')_2^2}$, -6

$\left(\overline{w'T'}\right)_1$, $\left(\overline{w'T'}\right)_2$, $\left(\overline{w'q'}\right)_1$, $\left(\overline{w'q'}\right)_2$, $\left(\overline{q'T'}\right)_1$, $\left(\overline{q'T'}\right)_2$,-6

Total: 19

Parameters of the trivariate PDF

Determine by predicting three first moments and four third moments

$$a\ ,\ \text{-1}$$

$$\overline{w}_1\ ,\ \overline{w}_2\ ,\ \overline{T}_1\ ,\ \overline{T}_2\ ,\ \overline{q}_1\ ,\ \overline{q}_2\ ,\ \text{-6}$$

$$\overline{(w')_1^2}\ ,\ \overline{(w')_2^2}\ ,\ \overline{(T')_1^2}\ ,\ \overline{(T')_2^2}\ ,\ \overline{(q')_1^2}\ ,\ \overline{(q')_2^2}\ ,\ \text{-6}$$

$$\left(\overline{w'T'}\right)_1,\ \left(\overline{w'T'}\right)_2,\ \left(\overline{w'q'}\right)_1,\ \left(\overline{w'q'}\right)_2,\ \left(\overline{q'T'}\right)_1,\ \left(\overline{q'T'}\right)_2\ ,\text{-6}$$

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Parameterize (phenomenology)

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Total: 19

Parameterize (phenomenology)

Coincidentally, a general trivariate joint PDF has exactly nineteen first, second, and third moments, namely

$$\overline{w}\,,\ \overline{T}\,,\ \overline{q}\ \text{-}3$$

$$\overline{w'w'}\,,\ \overline{T'T'}\,,\ \overline{q'q'}\ \text{-}3$$

$$\overline{w'T'}\,,\ \overline{w'q'}\,,\ \overline{T'q'}\ \text{-}3$$

$$\overline{w'w'w'}\,,\ \overline{T'T'T'}\,,\ \overline{q'q'q'}\ \text{-}3$$

$$\overline{w'T'q'}\ \text{-}1$$

$$\overline{w'w'T'}\,,\ \overline{w'w'q'}\,,\ \overline{T'T'w'}\,,\ \overline{T'T'q'}\,,\ \overline{q'q'w'}\,,\ \overline{q'q'T'}\ \text{-}6$$

Total: 19

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Let’s calculate the 19 moments from an LES, and solve for the 19 parameters of the double Gaussian.

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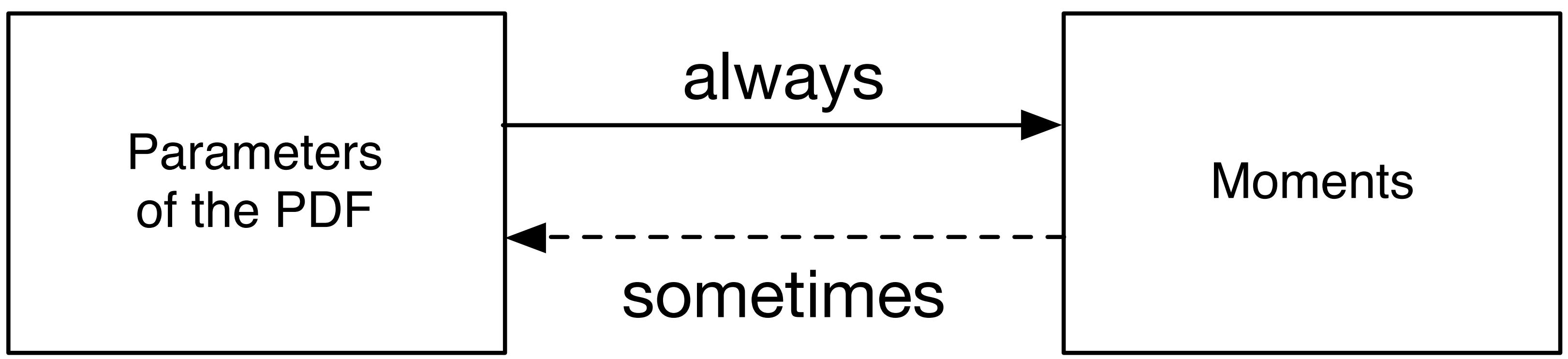

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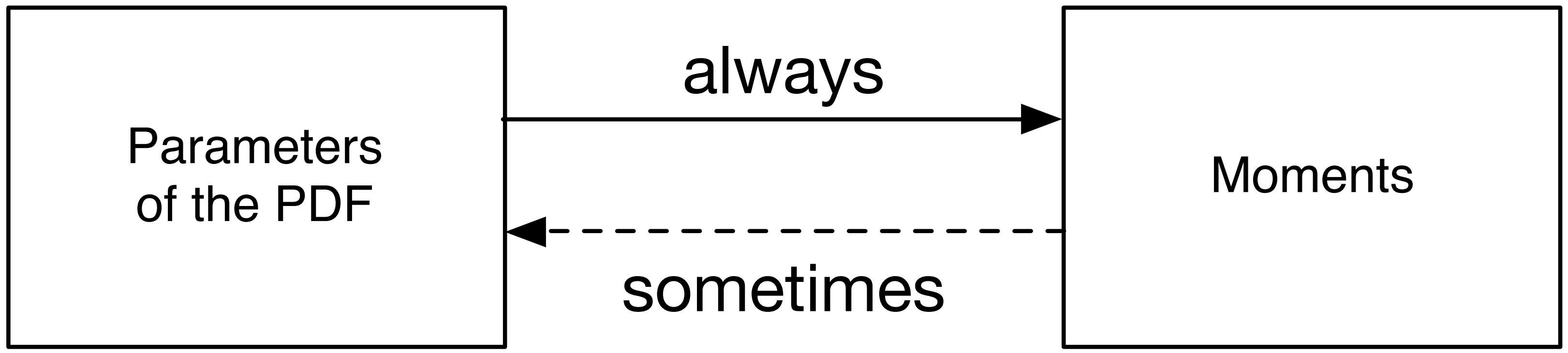

Can't do it. Along the way, there is a singular matrix.

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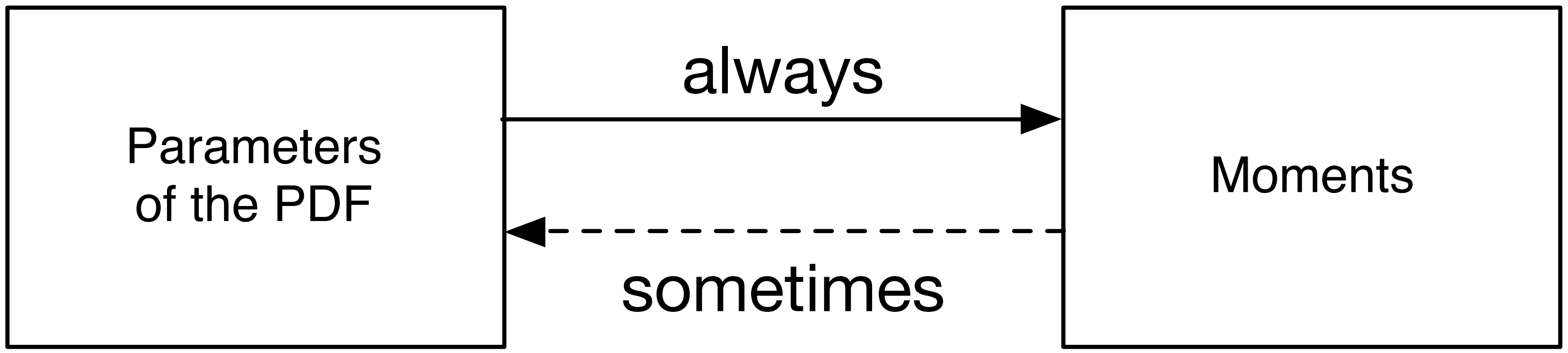

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Bummer.

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- It is useful to combine HOC with assumed distributions.
- A trivariate double Gaussian is a good assumed distribution.
- For determining the parameters of the distribution, the third moments are more important than the second moments.
- The second moments associated with the individual Gaussians should be (and must be) parameterized. This is where phenomenology can come in.