The ITCZ and Hadley Circulation: A Mesoscale Powerplant with a Planetary Scale Supporting Structure

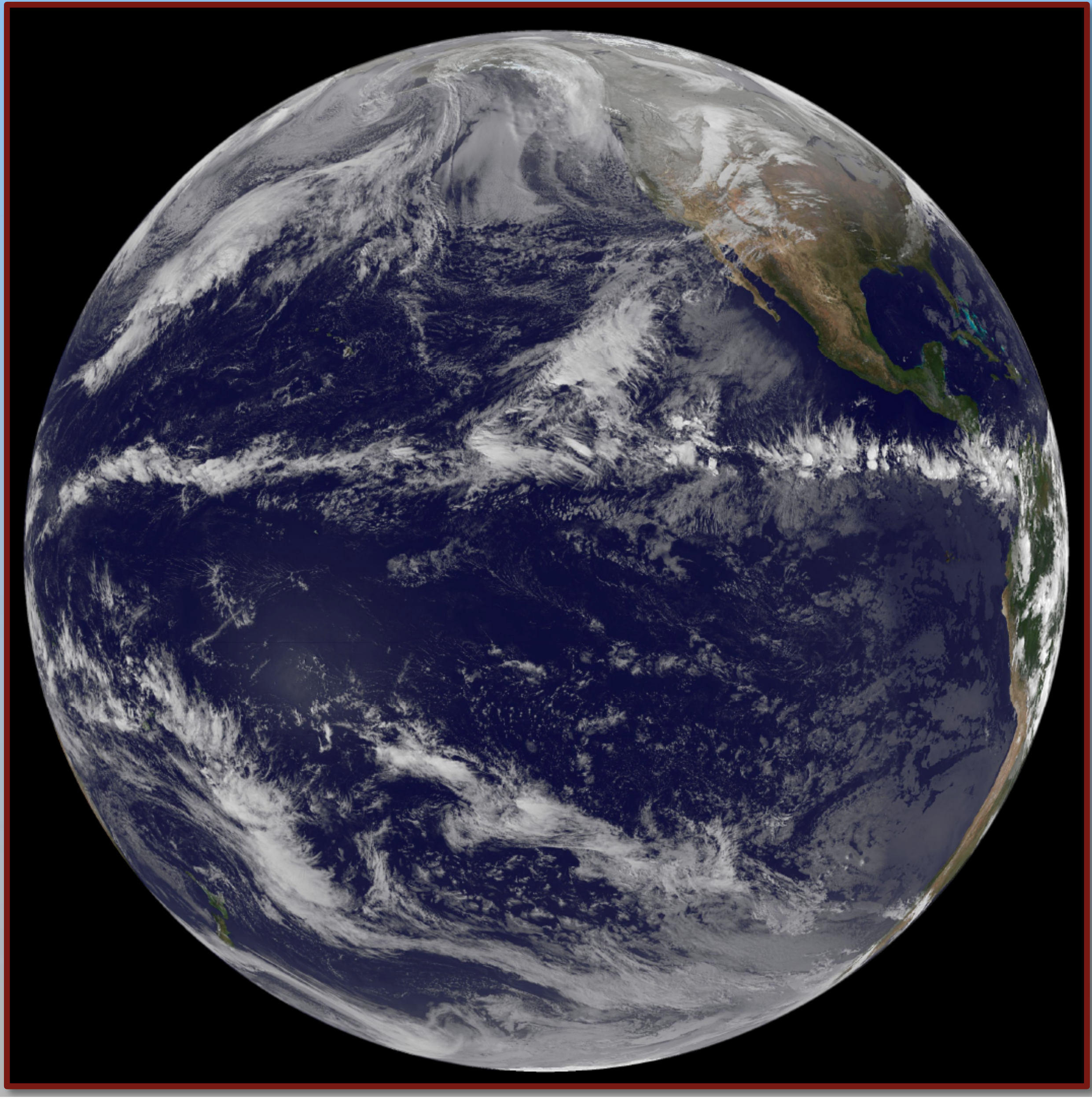
Wayne Schubert, Alex Gonzalez,
Gabriela Mora Rojas, and Richard Taft

Dept. of Atmospheric Science
Colorado State University

CMMAP
January 7, 2014

Motivation: Visible Satellite Imagery

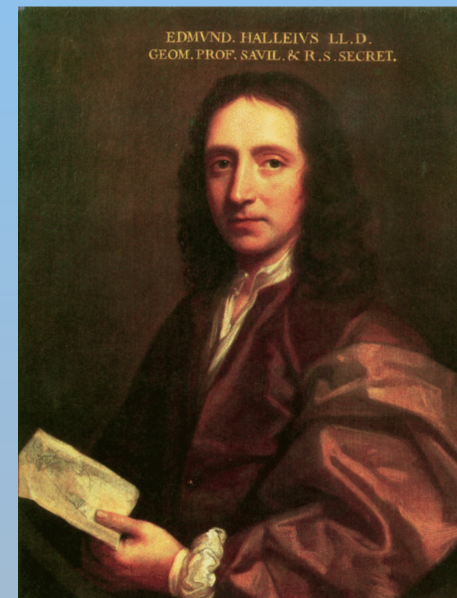
Nov. 24, 2010
00:00 UTC



From NASA GSFC
GOES Project
website

Historical Perspective: Edmond Halley

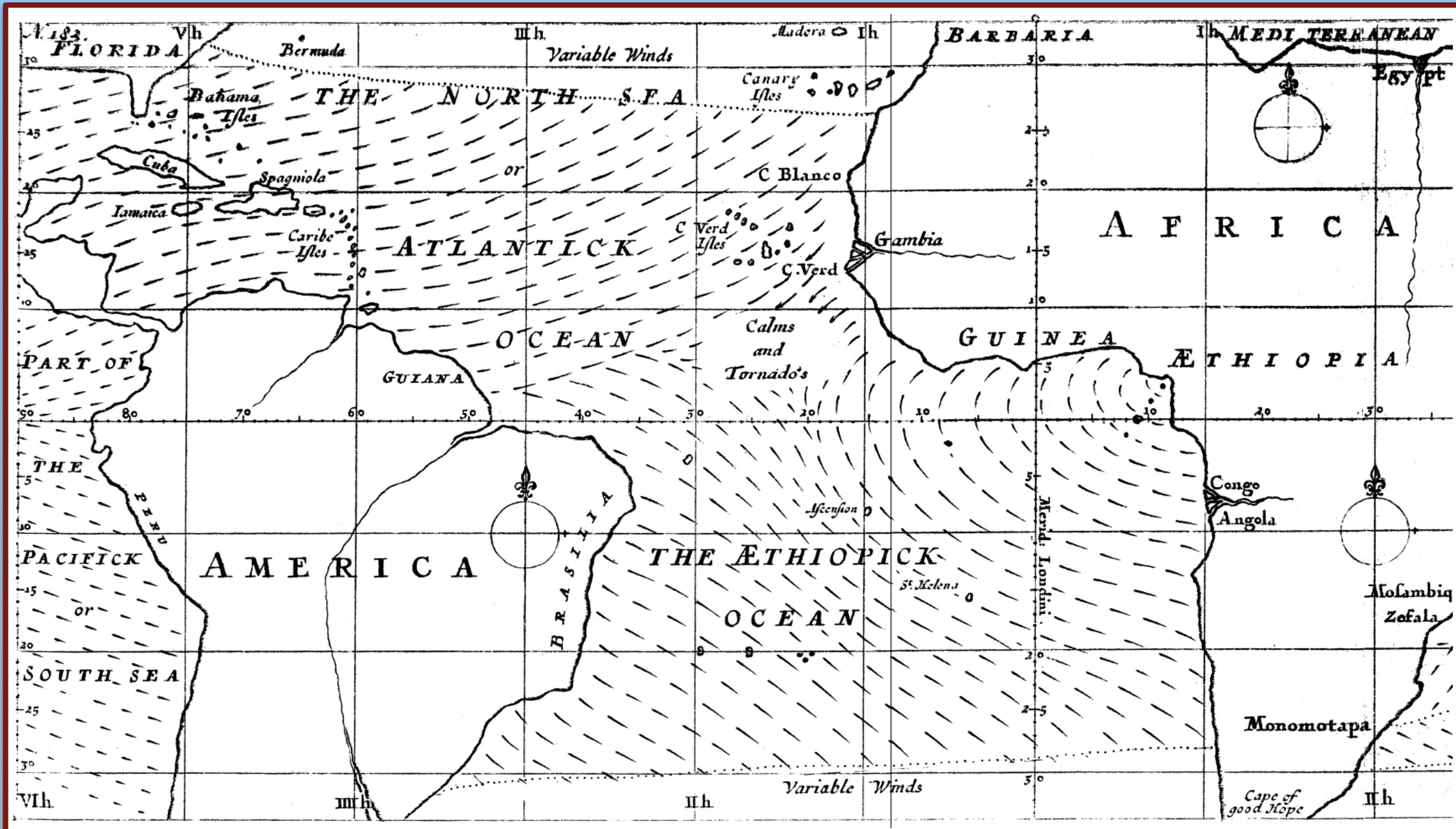
- English astronomer, geophysicist, mathematician, meteorologist, and physicist
- Best known for computing the orbit of Halley's comet
- Encouraged Newton to publish and funded the first publishing of *Philosophiæ Naturalis Principia Mathematica* in 1687
- Studied and sought to explain the Trade Winds



1656 - 1742

An Hiſtorical Account of the Trade Winds, *and* Monſoons, *obſervable in the Seas between and near the* Tropicks, *with an attempt to aſſign the Phiſical cauſe of the ſaid Winds, by* E. Halley.

(published in *Phil. Tran.* in 1686)

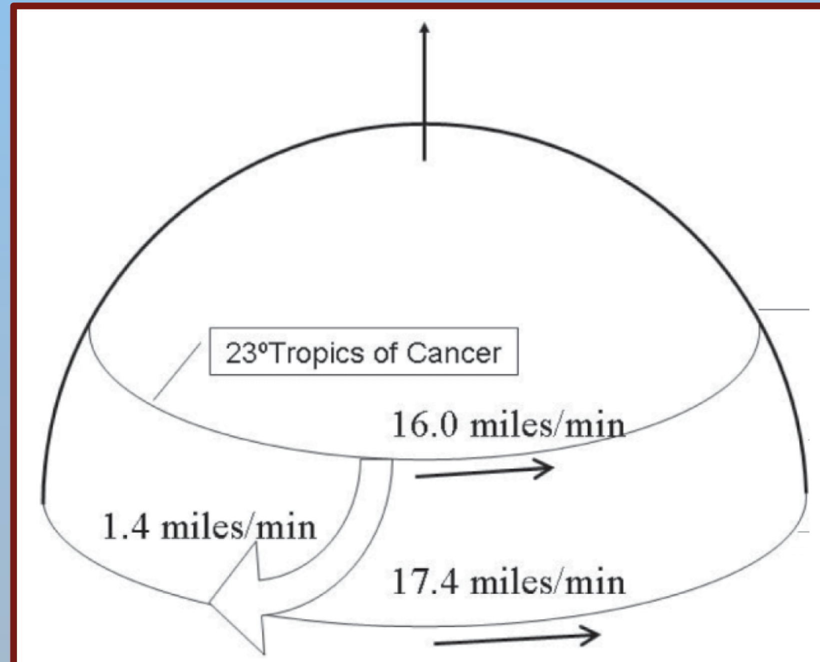


Edmond Halley (1686)

Theſe are particulars that merit to be conſidered more at Large, and furniſh a ſufficient Subject for a juſt Volume; which will be a very commendable Task for ſuch, who being uſed to Philoſophick Contemplation, ſhall have leaſure to apply their ſerious thoughts about it.

Historical Perspective: George Hadley

- English lawyer and amateur meteorologist (1685 – 1768)
- More interested in mechanical and physical studies than in legal work
- Proposed an atmospheric mechanism to explain the Trade Winds



from Perrson (2008) in *Weather*

VI. *Concerning* the Caufe *of the* General Trade-Winds: *By* Geo. Hadley, *Efq;* *F. R. S.*

(published in *Phil. Tran.* in 1735)

Halley (1680's)

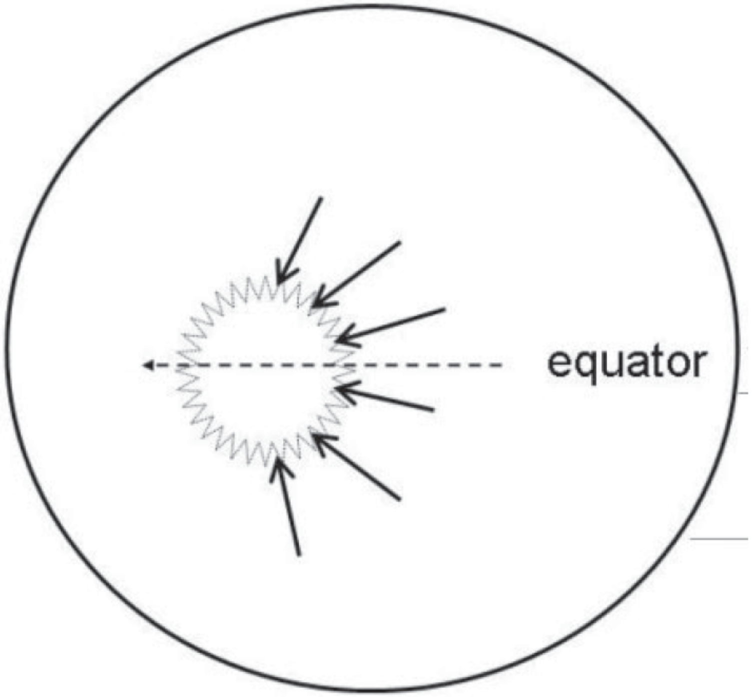


Figure 2. Halley's explanation of the easterly Trade Winds: as the maximum heating of the sun at the Earth's surface during the day moves westward, air will be sucked in from behind and replace the air that has been heated and risen.

Hadley (1730's)

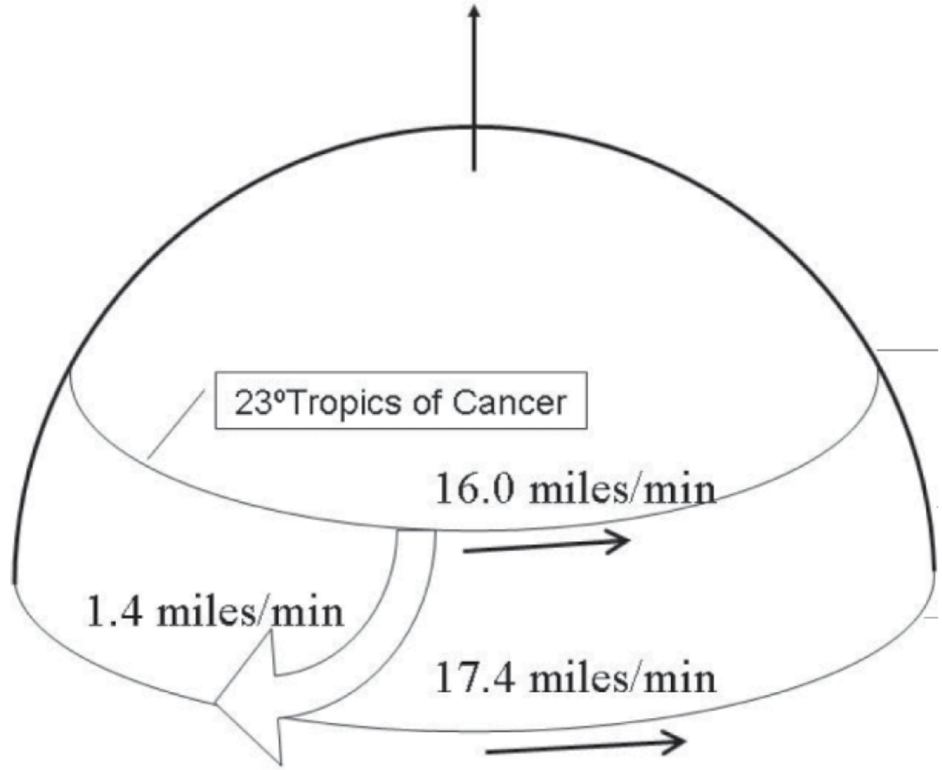


Figure 3. Hadley's explanation of the Trade Winds (with his own units): air moving from higher latitudes to lower to replace air that has been heated and risen will conserve its absolute velocity (16.0 miles/minute) and, when entering latitudes with higher velocities (17.4 miles/minute), appear to lag behind.

from Perrson (2008) in *Weather*

Hadley's Argument:

Conservation of Absolute Velocity (Linear Momentum):

$$u_f + \Omega a \cos \phi_f = u_i + \Omega a \cos \phi_i$$

Assume:

$$\phi_i = 23.5^\circ \quad u_i = 0 \quad \Rightarrow \quad \phi_f = 0^\circ$$

Then:

$$u_f = -\Omega a \left[1 - \cos\left(23.5^\circ\right)\right] = -38.5 \ \ \mathrm{m\ s^{-1}}$$

Correct Argument:

Conservation of Absolute Angular Momentum:

$$\left(u_f + \Omega a \cos \phi_f\right) a \cos \phi_f = \left(u_i + \Omega a \cos \phi_i\right) a \cos \phi_i$$

Assume:

$$\phi_i = 23.5^\circ \quad u_i = 0 \quad \Rightarrow \quad \phi_f = 0^\circ$$

Then:

$$u_f = -\Omega a \left[1 - \cos^2\left(23.5^\circ\right)\right] = -70.9 \ \ \mathrm{m\ s^{-1}}$$

Historical Perspective:

Yale Mintz & Akio Arakawa

Historical Perspective:

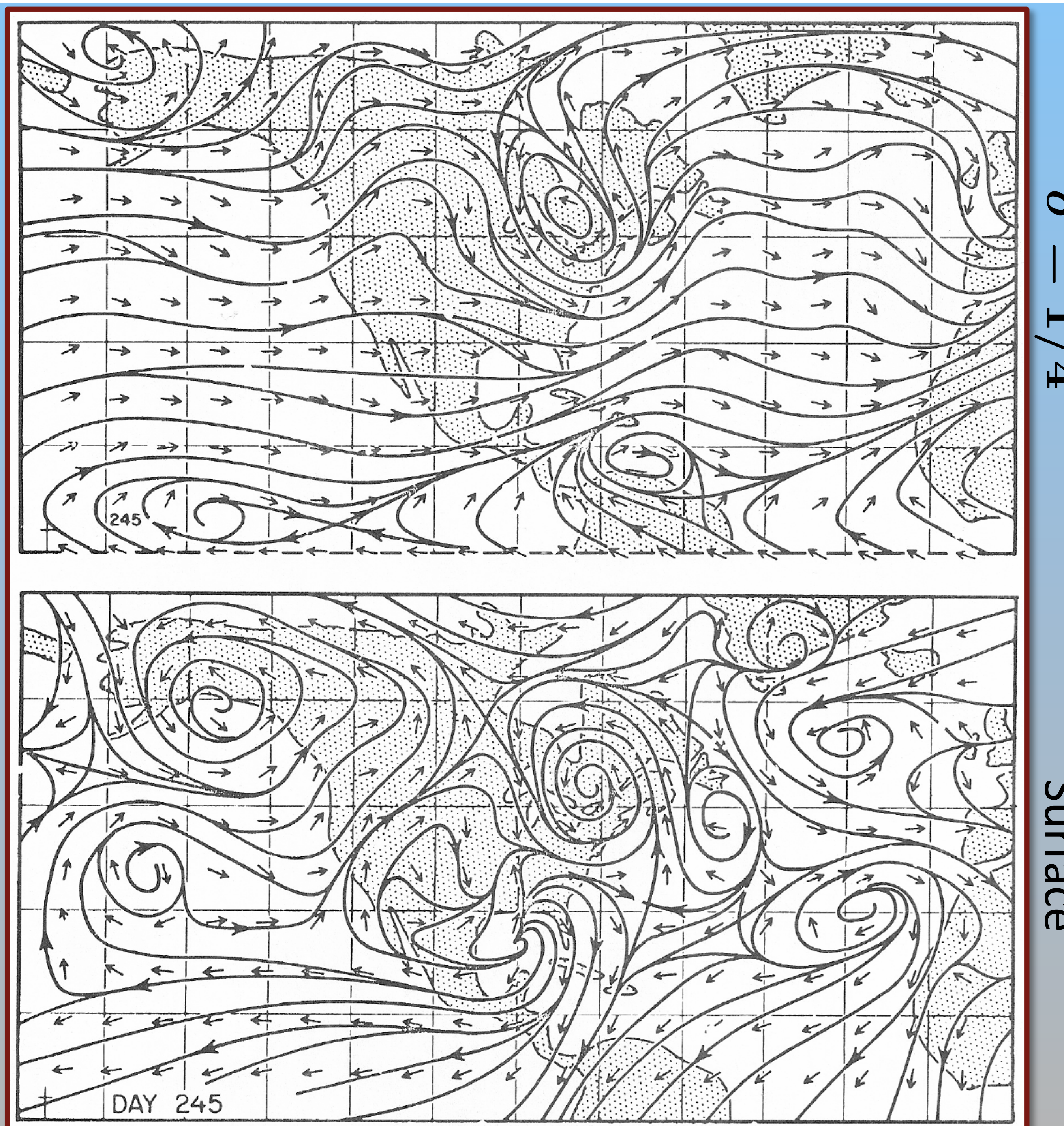
The UCLA 2-Level GCM
(1961 – 1963)

- Primitive Equation model
- 2 level σ-coordinate model: $\sigma = 1/4$ (nominally 400 mb)
  $\sigma = 3/4$ (nominally 800 mb)
- Spatial discretization designed by Arakawa
  - Allowed for "very long-term integrations" (~ 300 days)
  - Arakawa later published his classic "Jacobian" paper (1966)
- Horizontal resolution: $\Delta\lambda = 9^\circ$ and $\Delta\phi = 7^\circ$
  - Global grid was 40 x 25 = 1000 points
  - Only ~7 grid points/wavelength for baroclinic waves
- Experiments without and with mountains (perpetual NH winter)
- Model took 100 minutes (IBM 7094) to simulate 1 day and about 25 days to simulate a year

UCLA 2-Level GCM Results:

Streamlines of the computed wind

Day 245



From Mintz (1965)

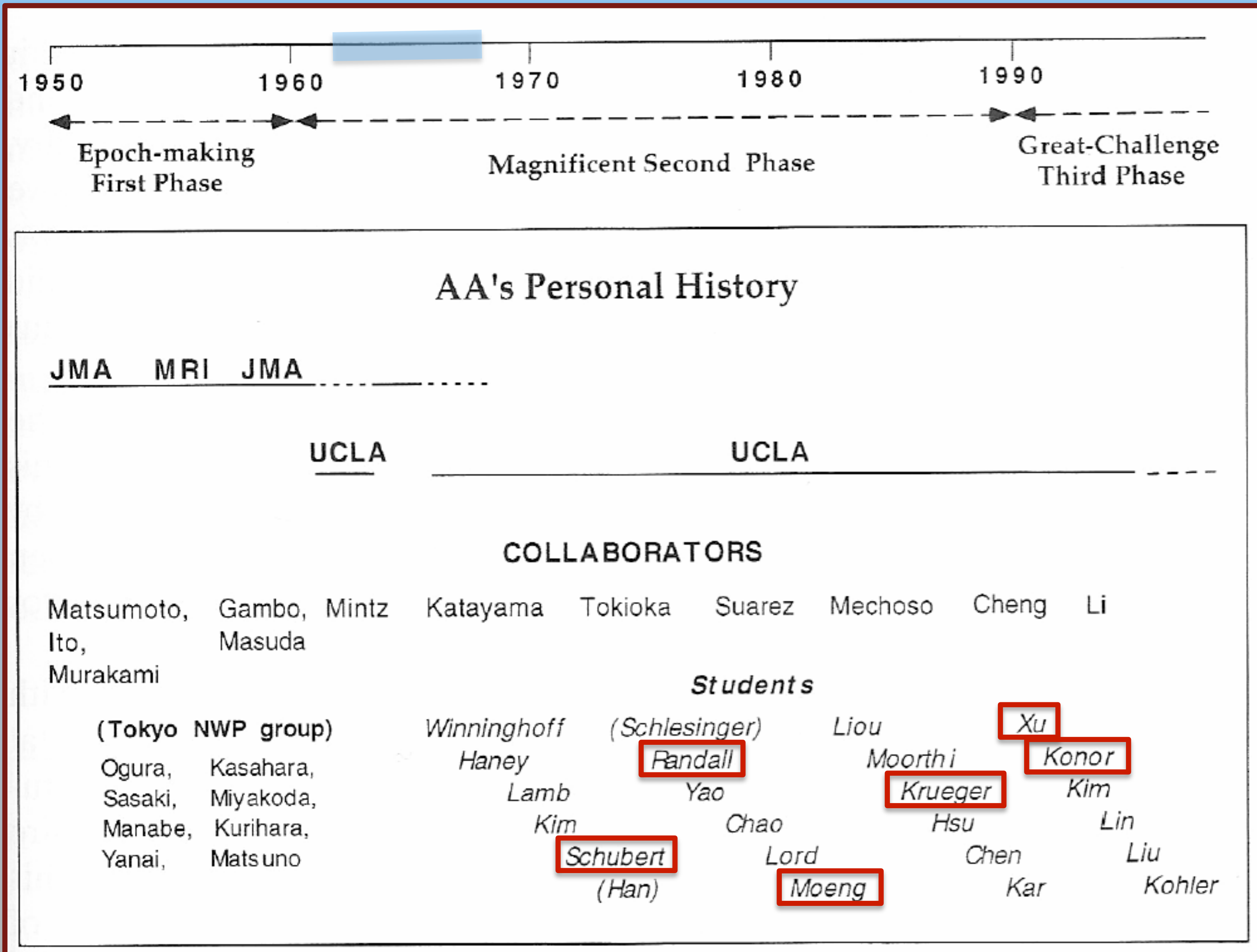
Good Understanding of Needed Improvements:

- Vertical resolution of 50 to 100 mb (finer in boundary layer)
- Horizontal resolution of 0.5 to 1 degree (for frontal zones)
- Carry water vapor as a dependent variable (improve ITCZ and Hadley cells)
- Improve treatment of radiation
- Include parameterized moist convection
- Raise upper boundary to include statosphere
- Include atmosphere/ocean coupling

Comment (Warning) about Terminology:

- WMO Technical Note by Yale Mintz (1965):
  - Very long-term global integration of the primitive equations of atmospheric motion.
  - ~300 day simulations using the UCLA 2-level GCM
- Follow-up work by W. E. Langlois (1971):
  - Digital simulation of the general atmospheric circulation using a very dense grid.
  - Increased horizontal resolution to $\Delta\lambda = 2.5^\circ$ and $\Delta\phi = 2^\circ$ (12816 grid points)

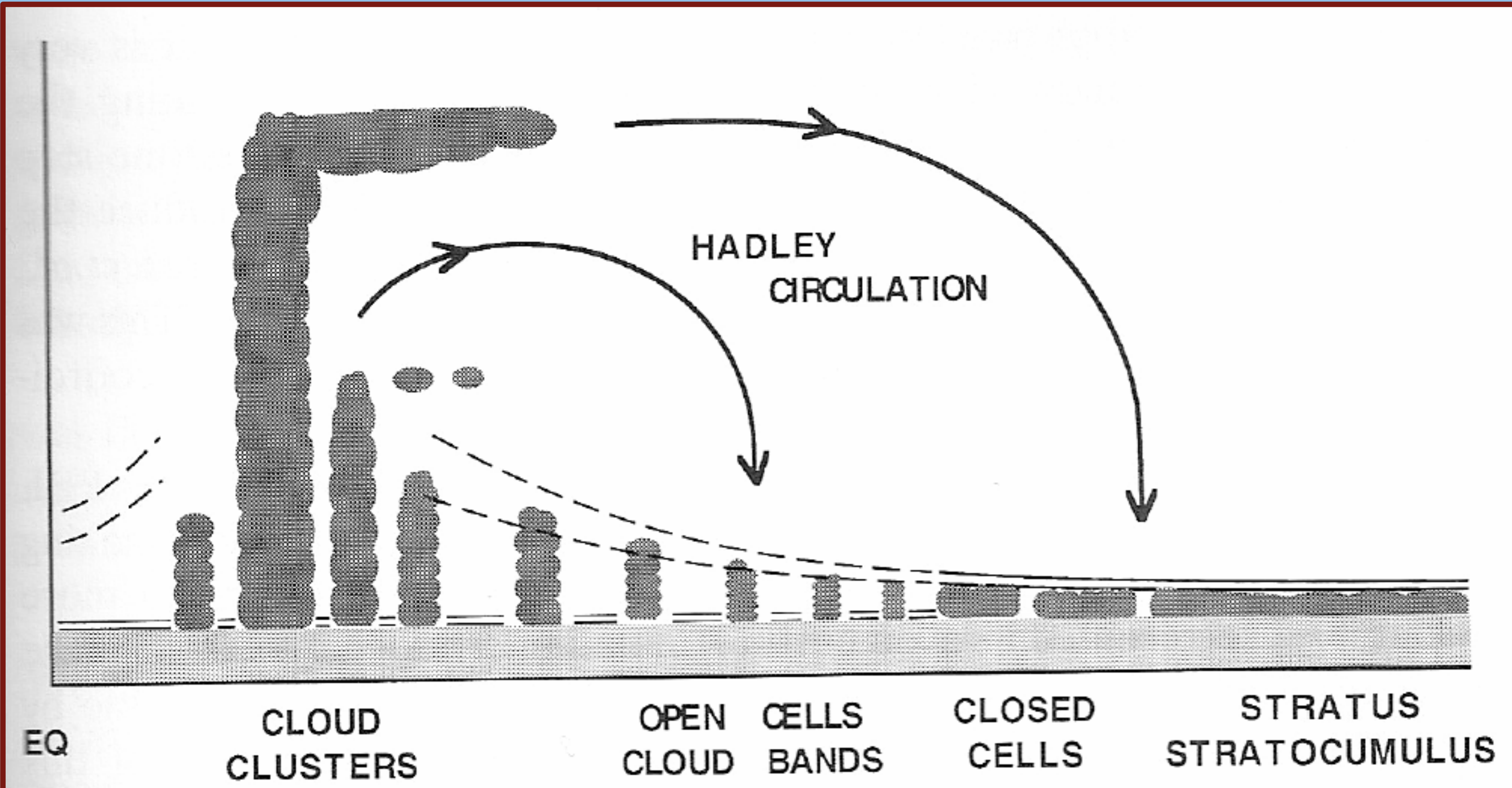
Historical Perspective: Akio Arakawa



From *General Circulation Model Development* edited by D. Randall (2000)

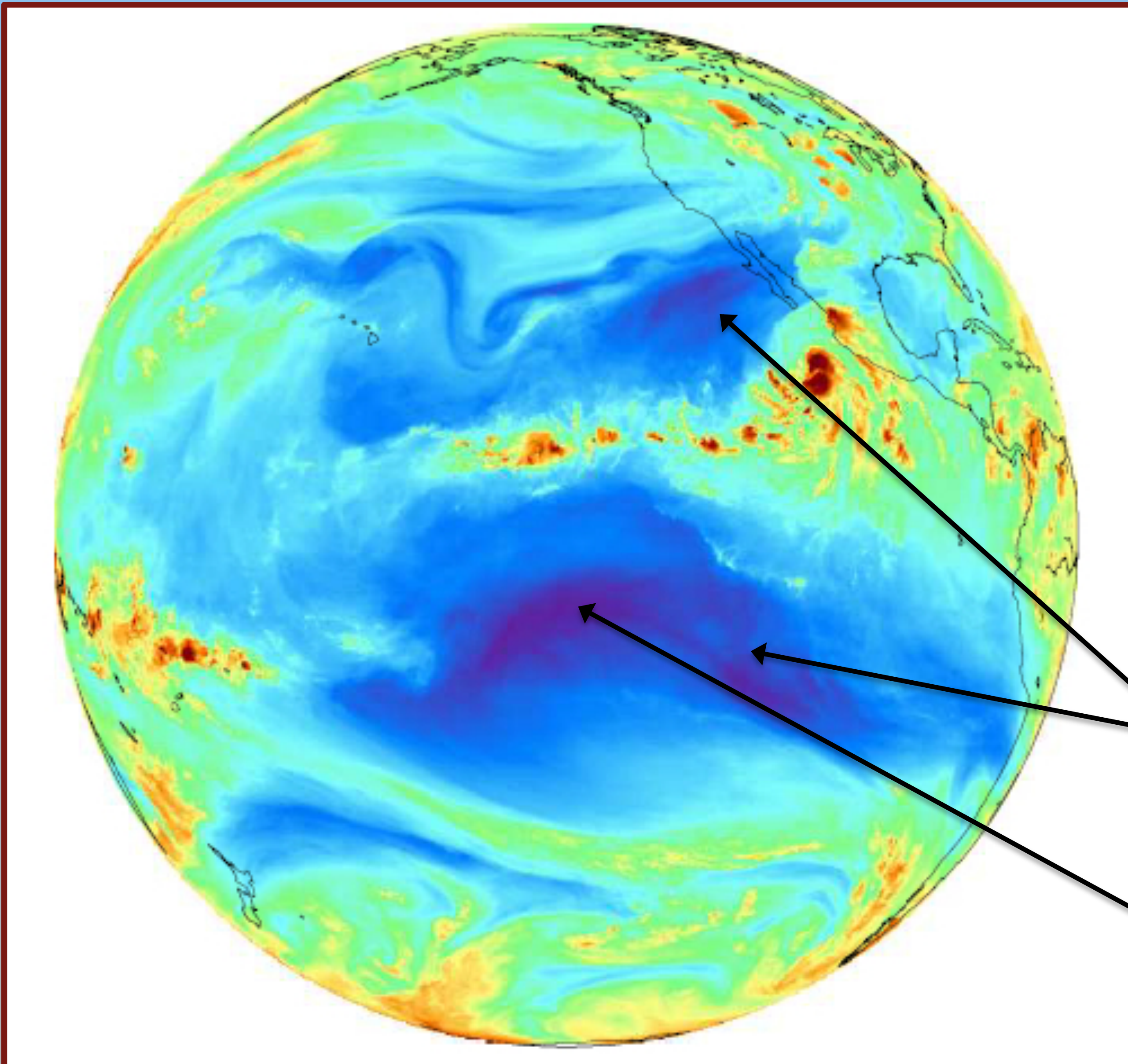
Historical Perspective:

Arakawa's Schematic of The Hadley Circulation



Adapted from Arakawa (1975)

Motivation: Water Vapor Imagery



Jun. 25, 2013
06:00 UTC

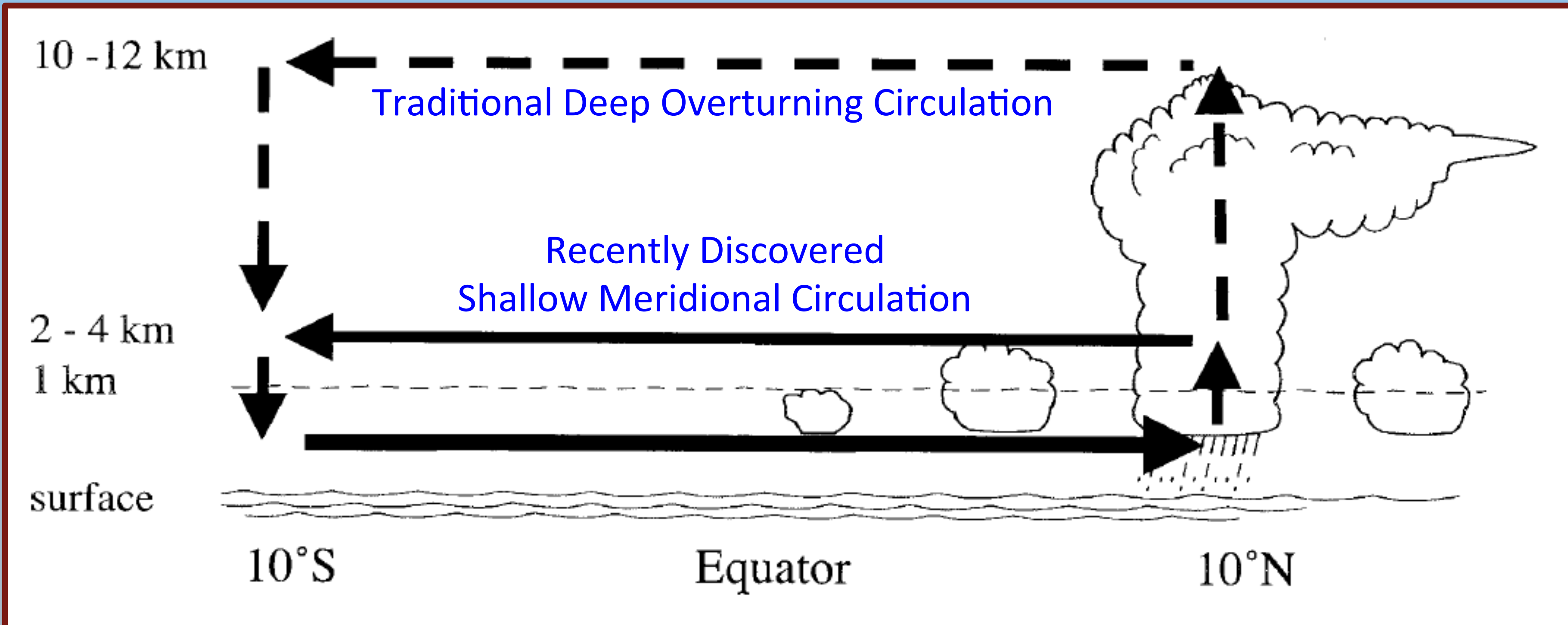
GOES West
Water Vapor
(6.7 μm)

Subsidence

Very Dry

Current Perspective:

The Hadley Circulation



From Zhang et al. (2004)

Analytical Modeling of the ITCZ and the Hadley Circulation

- Utilize the Zonally Symmetric Balanced Model and the Eliassen Meridional Circulation Equation
  - On the sphere and on the β-plane
- Two variable coefficients:
  - Inertial stability
  - Static stability
- Two forcings:
  - Diabatic heating (in the interior)
  - Boundary layer frictional pumping (lower BC)

Governing Equations for Zonally Symmetric Flow

- Assume weak: zonal & meridional flow and baroclinity
- For the inviscid interior: $0 \le z = H \log(p_0/p) \le z_T$

$$\frac{\partial u}{\partial t} - (2\Omega \sin\varphi)v = 0$$

Zonal momentum eq.

$$\frac{\partial v}{\partial t} + (2\Omega \sin\varphi)u + \frac{\partial \Phi}{a\partial\varphi} = 0$$

Meridional momentum eq.

$$\frac{\partial \Phi}{\partial z} = \frac{g}{T_0} T$$

Hydrostatic equation

$$\frac{\partial (v\cos\varphi)}{a\cos\varphi\,\partial\varphi} + \frac{\partial w}{\partial z} - \frac{w}{H} = 0$$

Continuity equation

$$\frac{\partial T}{\partial t} + \frac{T_0}{g} N^2 w = \frac{Q}{c_p}$$

Thermodynamic equation

Manipulations

- Eliminate u between zonal and meridional momentum equations:

$$\boxed{\frac{\partial}{a\partial\varphi}\left(\frac{\partial\Phi}{\partial t}\right)+\left(\frac{\partial^2}{\partial t^2}+4\Omega^2\sin^2\varphi\right)v=0}$$

- Eliminate T between hydrostatic and thermodynamic equations:

$$\boxed{\frac{\partial}{\partial z}\left(\frac{\partial\Phi}{\partial t}\right)+N^2 w=\frac{g}{c_pT_0}Q}$$

- Eliminate $\partial\Phi/\partial t$:

$$\boxed{N^2\frac{\partial w}{a\partial\varphi}-\left(\frac{\partial^2}{\partial t^2}+4\Omega^2\sin^2\varphi\right)\frac{\partial v}{\partial z}=\frac{g}{c_pT_0}\frac{\partial Q}{a\partial\varphi}}$$

- Express v and w in terms of a streamfunction ψ :

$$\boxed{e^{-z/H}v=-\frac{\partial\psi}{\partial z}\quad\text{and}\quad e^{-z/H}w=\frac{\partial(\psi\cos\varphi)}{a\cos\varphi\,\partial\varphi}}$$

Meridional Circulation Problem

- Single equation for the streamfunction:

$$\left(\frac{\partial^2}{\partial t^2} + 4\Omega^2 \sin^2\varphi\right)\frac{\partial}{\partial z}\left(e^{z/H}\frac{\partial \psi}{\partial z}\right) + N^2 e^{z/H}\frac{\partial}{a\partial\varphi}\left(\frac{\partial(\psi\cos\varphi)}{a\cos\varphi\,\partial\varphi}\right) = \frac{g}{c_p T_0}\boxed{\frac{\partial Q}{a\partial\varphi}}$$

Diabatic Forcing

- Boundary conditions:

$$\psi = 0 \ \text{ at } \ \varphi = \pm\pi/2 \qquad \text{and} \qquad \psi = 0 \ \text{ at } \ z = z_T$$

$$\left(\frac{\partial^2}{\partial t^2} + 4\Omega^2 \sin^2\varphi\right)\frac{\partial \psi}{\partial z} + g\frac{\partial}{a\partial\varphi}\left(\frac{\partial(\psi\cos\varphi)}{a\cos\varphi\,\partial\varphi}\right) = g\boxed{\frac{\partial \mathcal{W}}{a\partial\varphi}} \quad \text{at } \ z = 0$$

Boundary Layer Pumping

- Initial conditions:

$$\psi = 0 \ \text{ and } \ \frac{\partial \psi}{\partial t} = 0 \ \text{ at } \ t = 0$$

Additional Simplifications

- Meridional Circulation Equation:

$$\boxed{\begin{aligned}\left(\frac{\partial^2}{\partial t^2}+4\Omega^2\sin^2\varphi\right)\frac{\partial}{\partial z}\left(e^{z/H}\frac{\partial\psi}{\partial z}\right)&\\ +N^2e^{z/H}\frac{\partial}{a\partial\varphi}\left(\frac{\partial(\psi\cos\varphi)}{a\cos\varphi\,\partial\varphi}\right)&=\frac{g}{c_pT_0}\frac{\partial Q}{a\partial\varphi}\end{aligned}}$$

- Change meridional coordinate: $\varphi \quad\rightarrow\quad \mu=\sin\varphi$

- Define:

$$\boxed{\begin{aligned}\hat{\psi}(\mu,z,t)&=\psi(\mu,z,t)e^{z/2H}\\ \hat{Q}(\mu,z,t)&=Q(\mu,z,t)e^{-z/2H}\end{aligned}}$$

Simplified Meridional Circulation Problem

- Single equation for the streamfunction:

$$\boxed{\left(\frac{\partial^2}{\partial t^2}+4\Omega^2\mu^2\right)\left(\frac{\partial^2\hat{\psi}}{\partial z^2}-\frac{\hat{\psi}}{4H^2}\right)+\frac{N^2}{a^2}\left\{\frac{\partial}{\partial\mu}\left((1-\mu^2)\frac{\partial\hat{\psi}}{\partial\mu}\right)-\frac{\hat{\psi}}{1-\mu^2}\right\} = \frac{g}{c_pT_0}(1-\mu^2)^{1/2}\frac{\partial\hat{Q}}{a\partial\mu}}$$

- Boundary conditions:

$$\hat{\psi}=0 \ \text{ at } \ \mu=\pm1 \quad \text{and} \quad \hat{\psi}=0 \ \text{ at } \ z=z_T$$

$$\left(\frac{\partial^2}{\partial t^2}+4\Omega^2\mu^2\right)\left(\frac{\partial\hat{\psi}}{\partial z}-\frac{\hat{\psi}}{2H}\right)+\frac{g}{a^2}\left\{\frac{\partial}{\partial\mu}\left((1-\mu^2)\frac{\partial\hat{\psi}}{\partial\mu}\right)-\frac{\hat{\psi}}{1-\mu^2}\right\} = g(1-\mu^2)^{1/2}\frac{\partial\mathcal{W}}{a\partial\mu} \qquad \text{at } \ z=0$$

- Initial conditions: $\boxed{\hat{\psi}=0 \ \text{ and } \ \frac{\partial\hat{\psi}}{\partial t}=0 \ \text{ at } \ t=0}$

Similarly: Simplified Meridional Circulation Problem on the β-Plane

- Single equation for the streamfunction:

$$\left(\frac{\partial^2}{\partial t^2}+\beta^2 y^2\right)\left(\frac{\partial^2 \hat{\psi}}{\partial z^2}-\frac{\hat{\psi}}{4H^2}\right)+N^2\frac{\partial^2 \hat{\psi}}{\partial y^2}=\frac{g}{c_p T_0}\frac{\partial \hat{Q}}{\partial y}$$

- Boundary conditions:

$$\hat{\psi}\to 0 \ \text{ as } \ y\to\pm\infty$$

$$\hat{\psi}=0 \ \text{ at } \ z=z_T$$

$$\left(\frac{\partial^2}{\partial t^2}+\beta^2 y^2\right)\left(\frac{\partial \hat{\psi}}{\partial z}-\frac{\hat{\psi}}{2H}\right)+g\frac{\partial^2 \hat{\psi}}{\partial y^2}=g\frac{\partial \mathcal{W}}{\partial y} \ \text{ at } \ z=0$$

- Initial conditions: $\hat{\psi}=0 \ \text{ and } \ \dfrac{\partial \hat{\psi}}{\partial t}=0 \ \text{ at } \ t=0$

Analytical Solution Method #1

- Assume that the diabatic forcing $Q(y, z, t)$ and boundary layer forcing $\mathcal{W}(y, t)$ vary slowly in time
- Then can neglect the $\partial^2/\partial t^2$ terms
- The problem becomes an elliptic diagnostic problem that does not include gravity waves
- The solution $\psi(y, z, t)$ has no memory of past forcing and is diagnostically determined by the current forcing only
- Solution steps:
  1. Perform vertical transform of the diagnostic problem
  2. Use Green's functions to solve the resulting horizontal structure equations

Method #1: Elliptic Diagnostic Problem

- Elliptic equation for the streamfunction:

$$N^2 \frac{\partial^2 \hat{\psi}}{\partial y^2} + \beta^2 y^2 \left(\frac{\partial^2 \hat{\psi}}{\partial z^2} - \frac{\hat{\psi}}{4H^2} \right) = \frac{g}{c_p T_0} \frac{\partial \hat{Q}}{\partial y}$$

- Boundary conditions:

$$\hat{\psi} \to 0 \ \text{ as } \ y \to \pm\infty$$

$$\hat{\psi} = 0 \ \text{ at } \ z = z_T$$

$$g \frac{\partial^2 \hat{\psi}}{\partial y^2} + \beta^2 y^2 \left(\frac{\partial \hat{\psi}}{\partial z} - \frac{\hat{\psi}}{2H} \right) = g \frac{\partial \mathcal{W}}{\partial y} \qquad \text{at } \ z = 0$$

Method #1: Vertical Transform

- Seek solutions via the vertical transform pair:

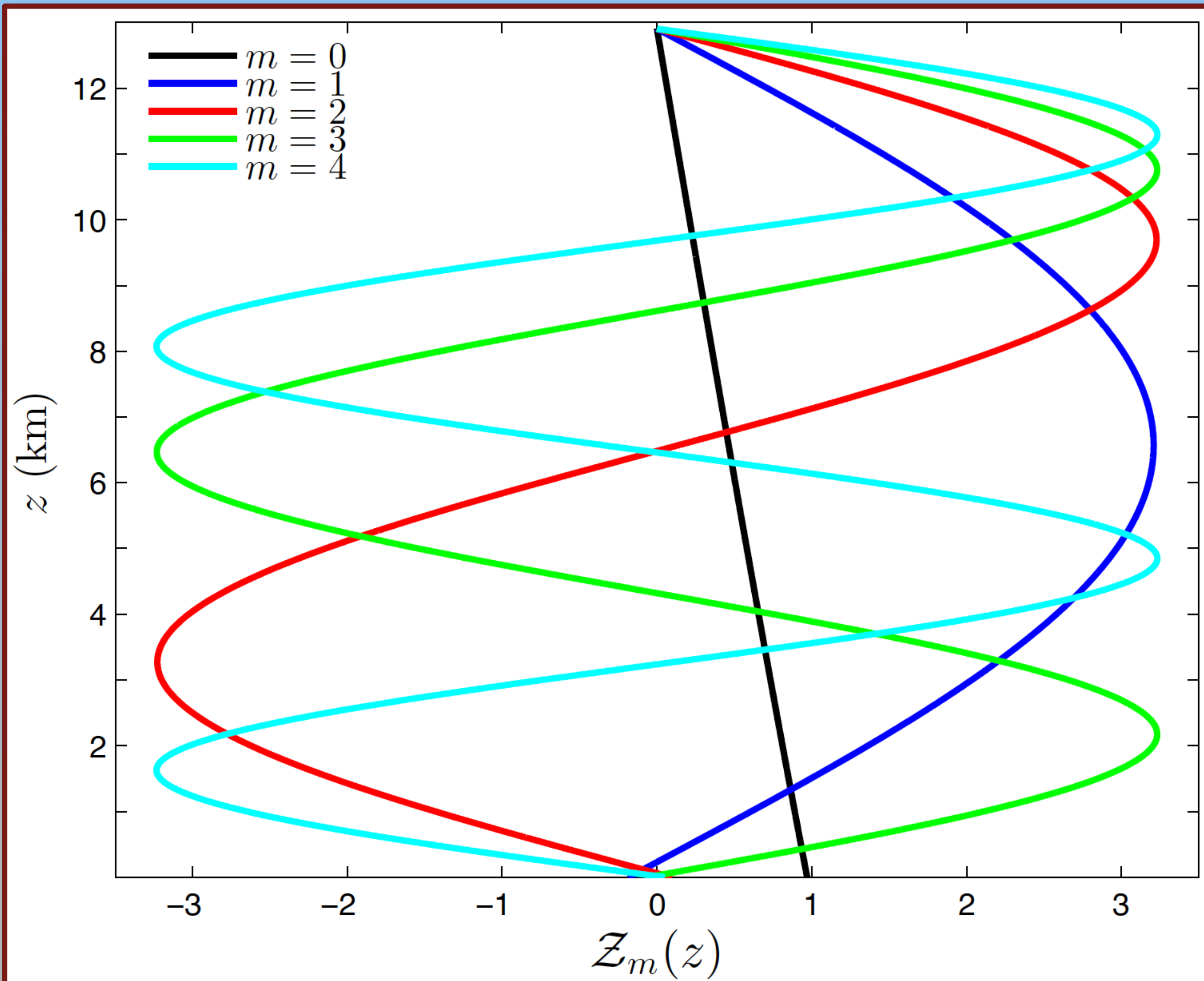
$$\boxed{\begin{aligned}\hat{\psi}(y,z,t) &= \sum_{m=0}^{\infty} \hat{\psi}_m(y,t)\,\mathcal{Z}_m(z) \\ \hat{\psi}_m(y,t) &= \frac{1}{g}\int_0^{z_T} \hat{\psi}(y,z,t)\mathcal{Z}_m(z)N^2(z)dz + \hat{\psi}(y,0,t)\mathcal{Z}_m(0)\end{aligned}}$$

- The vertical structure functions $\mathcal{Z}_m$ are solutions of the Sturm-Liouville eigenvalue problem:

$$\boxed{\begin{aligned}&\frac{d^2\mathcal{Z}_m}{dz^2} - \frac{\mathcal{Z}_m}{4H^2} = -\frac{N^2\mathcal{Z}_m}{gh_m} \\ &\mathcal{Z}_m = 0 \ \text{ at } \ z = z_T \\ &\frac{d\mathcal{Z}_m}{dz} - \frac{\mathcal{Z}_m}{2H} = -\frac{\mathcal{Z}_m}{h_m} \ \text{ at } \ z = 0\end{aligned}}$$

with eigenvalues (or equivalent depths) denoted by h_m

Vertical Structure Functions



Vertical Structure Information

m	h_m (m)	$(gh_m)^{1/2}$ (m s^{-1})	b_m (km)
0	7099	263.8	2400
1	229.8	47.46	1018
2	61.42	24.53	732.0
3	27.66	16.46	599.7
4	15.63	12.38	519.9

External mode

Internal modes

Vertical Mode — Equivalent Depth — Gravity Wave Speed — Rossby Length

$p_0 = 900$ hPa $H = 8581$ m $N = 1.2 \times 10^{-2}$ s^{-1}

$T_0 = 293$ K $z_T = 13$ km

Method #1: Meridional Structure Equations

- Vertical transform of the elliptic diagnostic problem:

$$\boxed{\begin{gathered}\frac{\partial^2 \hat{\psi}_m(y,t)}{\partial y^2} - \frac{y^2}{4b_m^4}\hat{\psi}_m(y,t) = \frac{\partial F_m(y,t)}{\partial y} \\ \hat{\psi}_m(y,t) \to 0 \text{ as } y \to \pm\infty\end{gathered}}$$

for each vertical mode m

- The forcing term now combines the interior diabatic heating and the boundary layer pumping:

$$\boxed{F_m(y,t) = \int_0^{z_T} \frac{\hat{Q}(y,z,t)}{c_p T_0} \mathcal{Z}_m(z)\, dz + \mathcal{W}(y,t)\mathcal{Z}_m(0)}$$

Method #1: Solution via Green's Functions

- The meridional structure equations are solved using the Green's functions $G_m(y, y')$ which are solutions of:

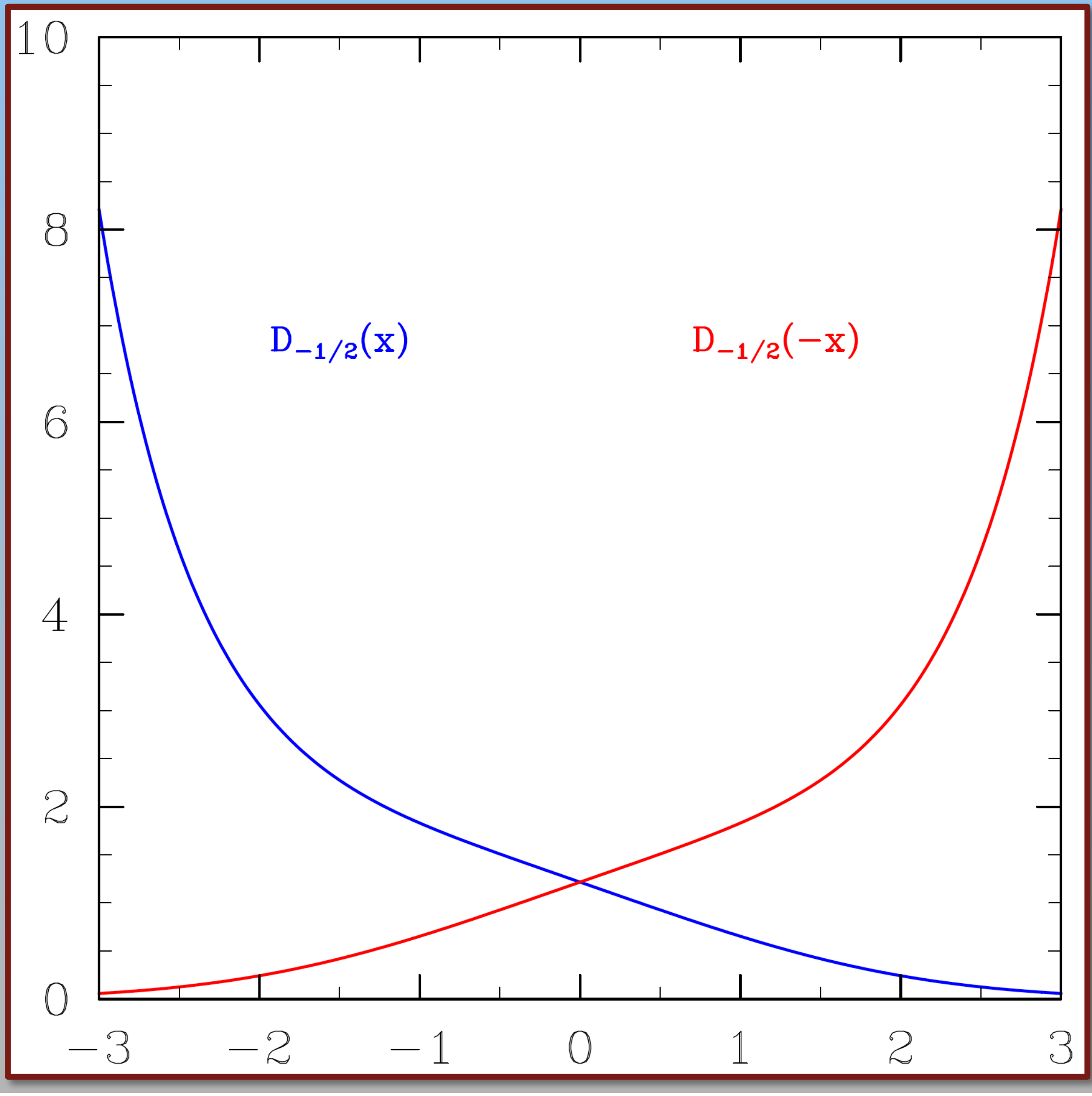
$$\boxed{\begin{gathered} \frac{d^2 G_m}{dy^2} - \frac{y^2}{4b_m^4} G_m = -\frac{1}{b_m^2}\delta\left(\frac{y-y'}{b_m}\right) \\[2ex] G_m(y, y') \to 0 \text{ as } y \to \pm\infty \end{gathered}}$$

- The Green's functions are constructed from parabolic cylinder functions $D_\nu(x)$ which satisfy:

$$\boxed{\frac{d^2 D_\nu}{dx^2} + \left(\nu + \frac{1}{2} - \frac{1}{4}x^2\right) D_\nu = 0}$$

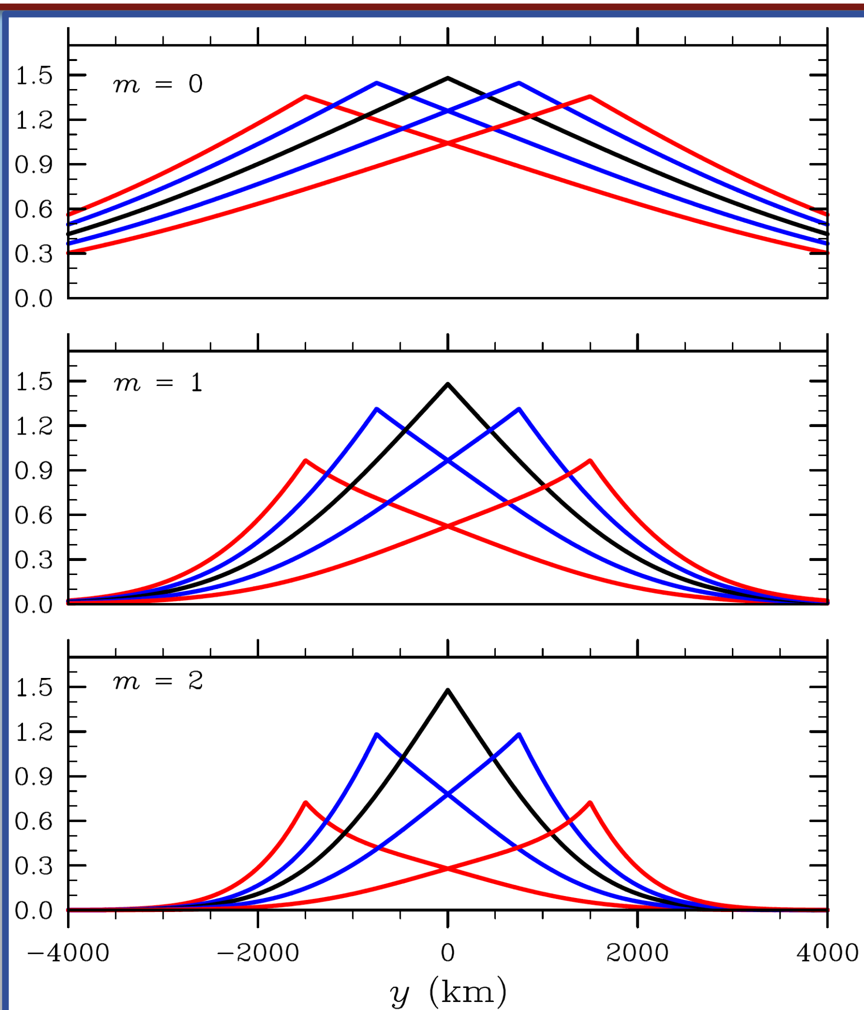
- In particular, $D_{-1/2}(y/b_m)$ and $D_{-1/2}(-y/b_m)$ are used to construct the desired Green's functions

Parabolic Cylinder Functions



Method #1: The Desired Green's Functions

$$G_m(y,y') = \frac{1}{\sqrt{2}} \begin{cases} D_{-1/2}(y'/b_m)D_{-1/2}(-y/b_m) & \text{if } -\infty < y \leq y' \\ D_{-1/2}(-y'/b_m)D_{-1/2}(y/b_m) & \text{if } y' \leq y < \infty \end{cases}$$



Black curve:

$$y' = 0 \text{ km}$$

Blue curves:

$$y' = \pm 750 \text{ km}$$

Red curves:

$$y' = \pm 1500 \text{ km}$$

Method #1: Summary of Solution Method

- The solution of the meridional circulation problem is a superposition of vertical modes:

$$\boxed{\psi(y,z,t) = e^{-z/2H} \sum_{m=0}^{\infty} \hat{\psi}_m(y,t)\, \mathcal{Z}_m(z)}$$

- Each coefficient $\hat{\psi}_m(y,t)$ is a superposition of Green's functions weighted by the forcing:

$$\boxed{\hat{\psi}_m(y,t) = -b_m \int_{-\infty}^{\infty} \frac{\partial F_m(y',t)}{\partial y'} G_m(y,y')\, dy'}$$

- Two idealized ITCZ forcings that lead to simple analytical solutions:
  1. $m = 1$ and diabatic heating prescribed as a step function in y
  2. $m \leq m_{\max}$ and BL pumping prescribed as a step function in y

Method #1: (Case #1) Production of Deep Overturning Circulations Through Diabatic Forcing

- Consider the following ITCZ experiment:
  - The diabatic heating is restricted to the ITCZ region
  - The diabatic heating is independent of y and has a vertical dependence that only projects onto the first internal mode
- Then:

$$\hat{Q}(y,z,t) = \begin{cases} \tilde{Q}(t)\mathcal{Z}_1(z) & \text{if } y_1 < y < y_2 \\ 0 & \text{otherwise} \end{cases}$$

and

$$F_m(y,t) = \frac{g\tilde{Q}(t)}{c_p T_0 N^2} \begin{cases} 1 & \text{if } m = 1 \text{ and } y_1 < y < y_2 \\ 0 & \text{otherwise} \end{cases}$$

- The analytical solution becomes:

$$\psi(y,z,t) = \frac{g b_1 \tilde{Q}(t)}{c_p T_0 N^2} e^{-z/2H} \mathcal{Z}_1(z) \left[G_1(y,y_2) - G_1(y,y_1)\right]$$

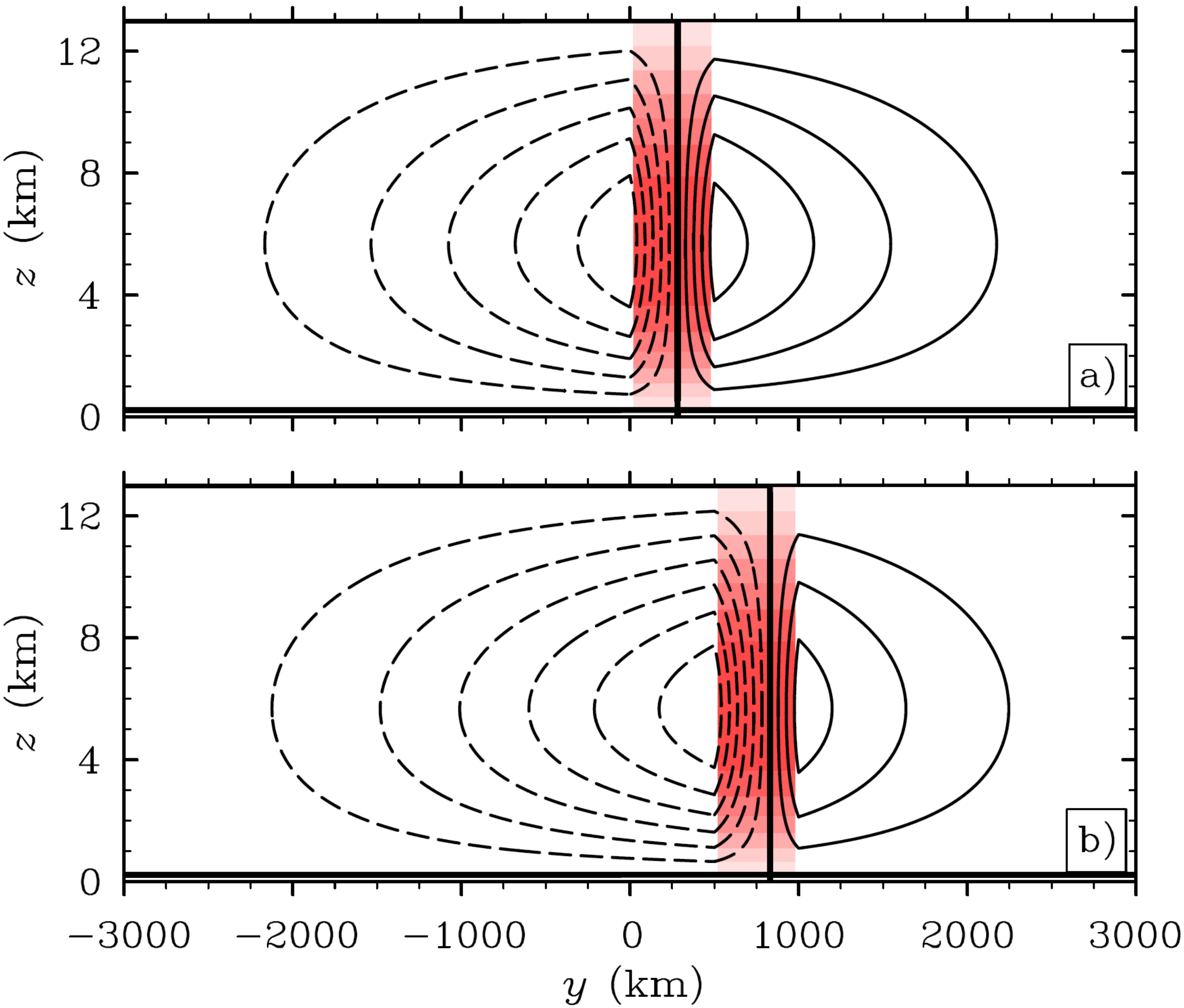
Method #1: Streamfunction and Diabatic Heating (Case #1)

Streamfunction:

- $\psi(y, z)$
- Black contours
- Interval is $400\ \mathrm{m^2\ s^{-1}}$

Diabatic heating:

- $\dfrac{Q(y,z)e^{-z/H}}{c_p}$
- Red shading
- Interval is $0.5\ \mathrm{K\ day^{-1}}$
- Max. value is $3.496\ \mathrm{K\ day^{-1}}$



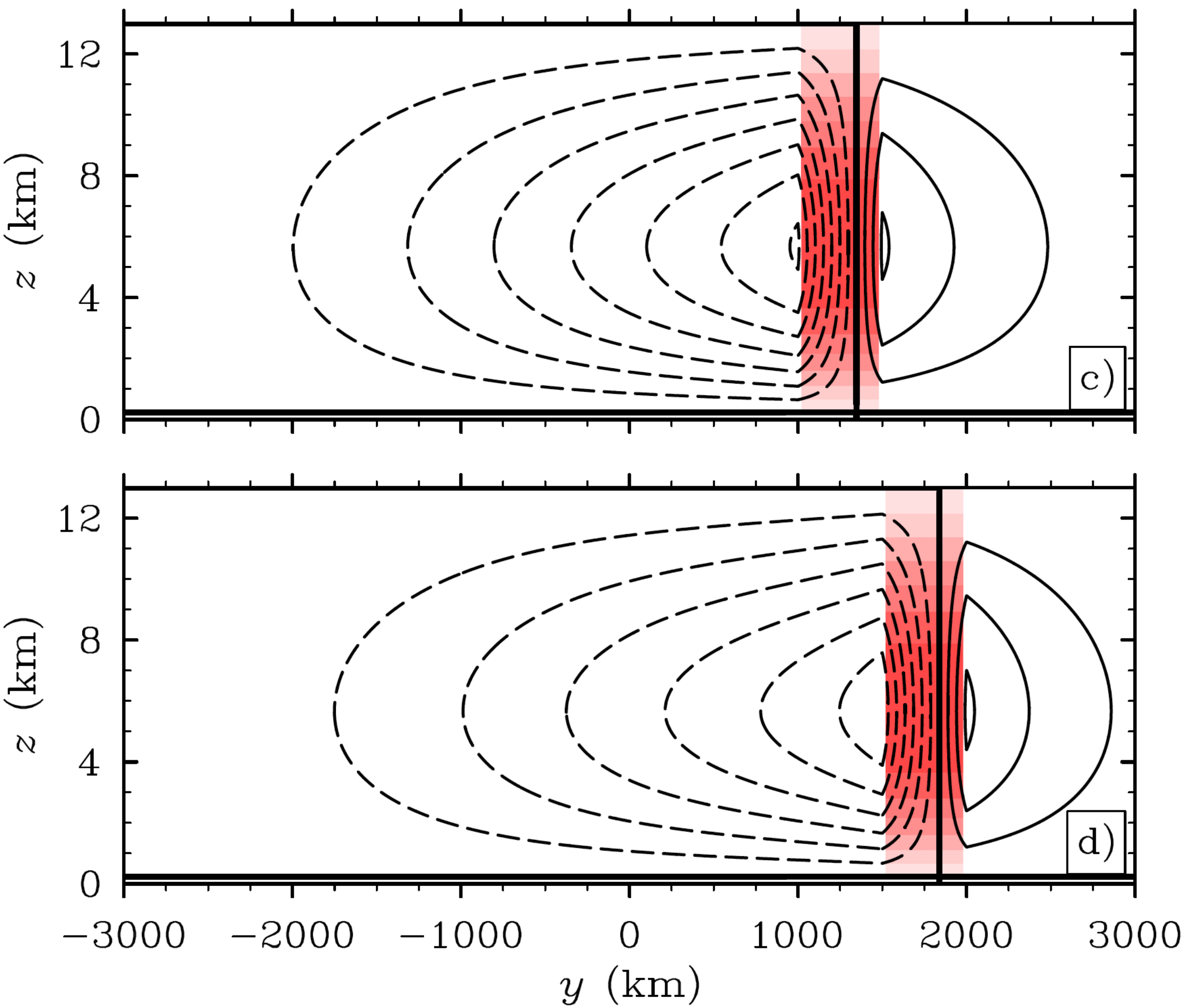
Method #1: Streamfunction and Diabatic Heating
(Case #1)

Streamfunction:

- $\psi(y,z)$
- Black contours
- Interval is $400\ \mathrm{m^2\ s^{-1}}$

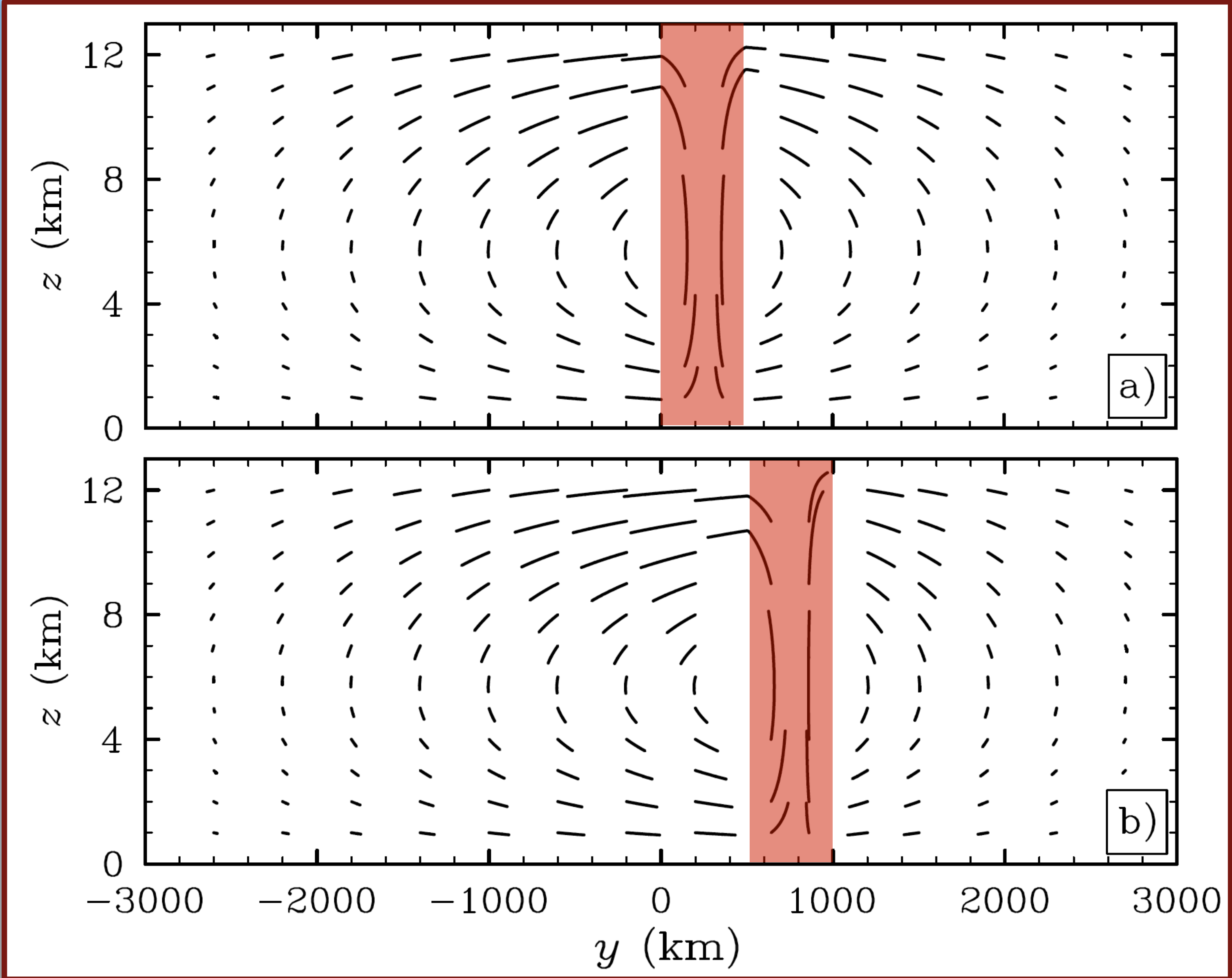
Diabatic heating:

- $\dfrac{Q(y,z)e^{-z/H}}{c_p}$
- Red shading
- Interval is $0.5\ \mathrm{K\ day^{-1}}$
- Max. value is $3.496\ \mathrm{K\ day^{-1}}$



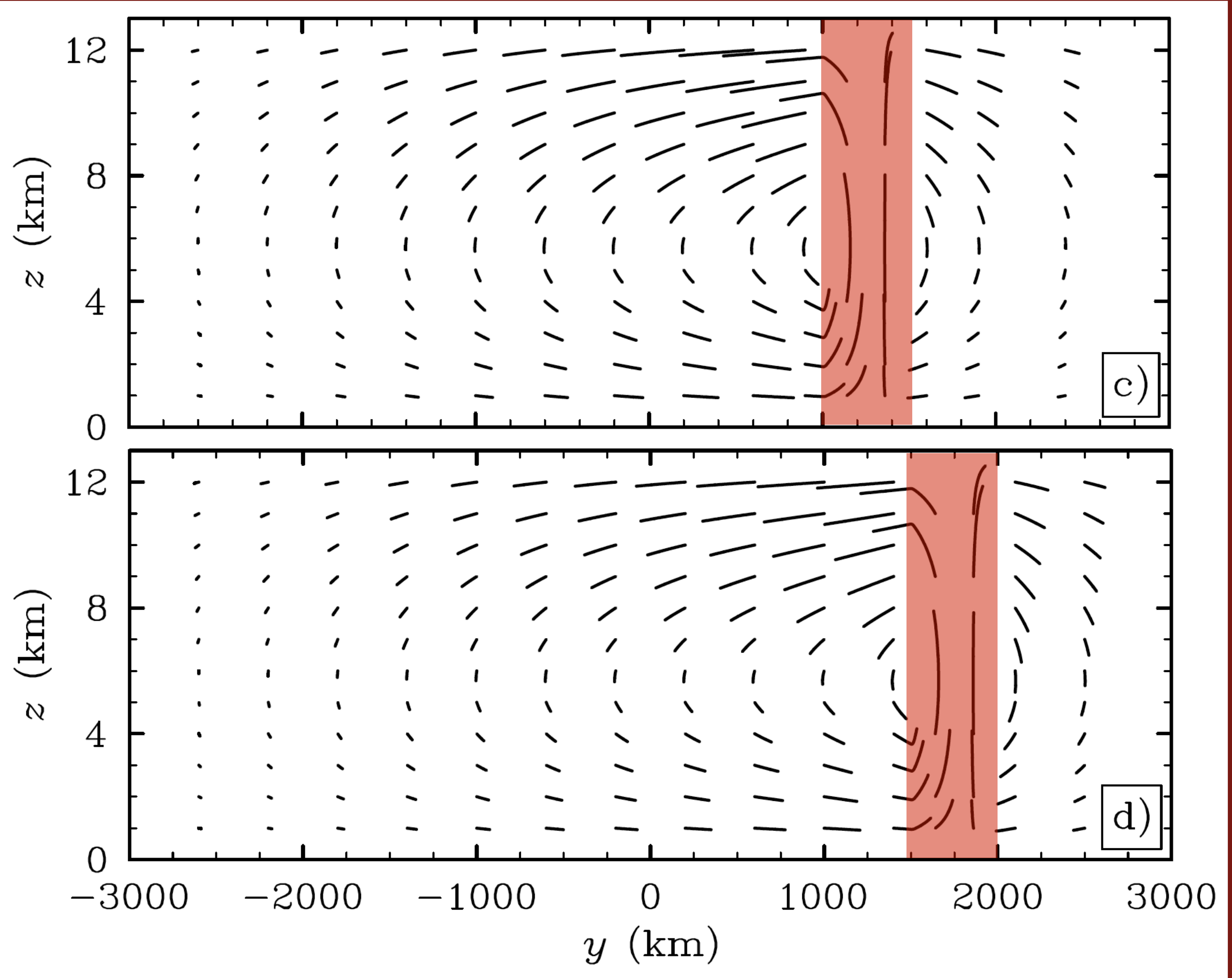
Method #1:
(Case #1)

3-Day Parcel Trajectories



Method #1: (Case #1)

3-Day Parcel Trajectories



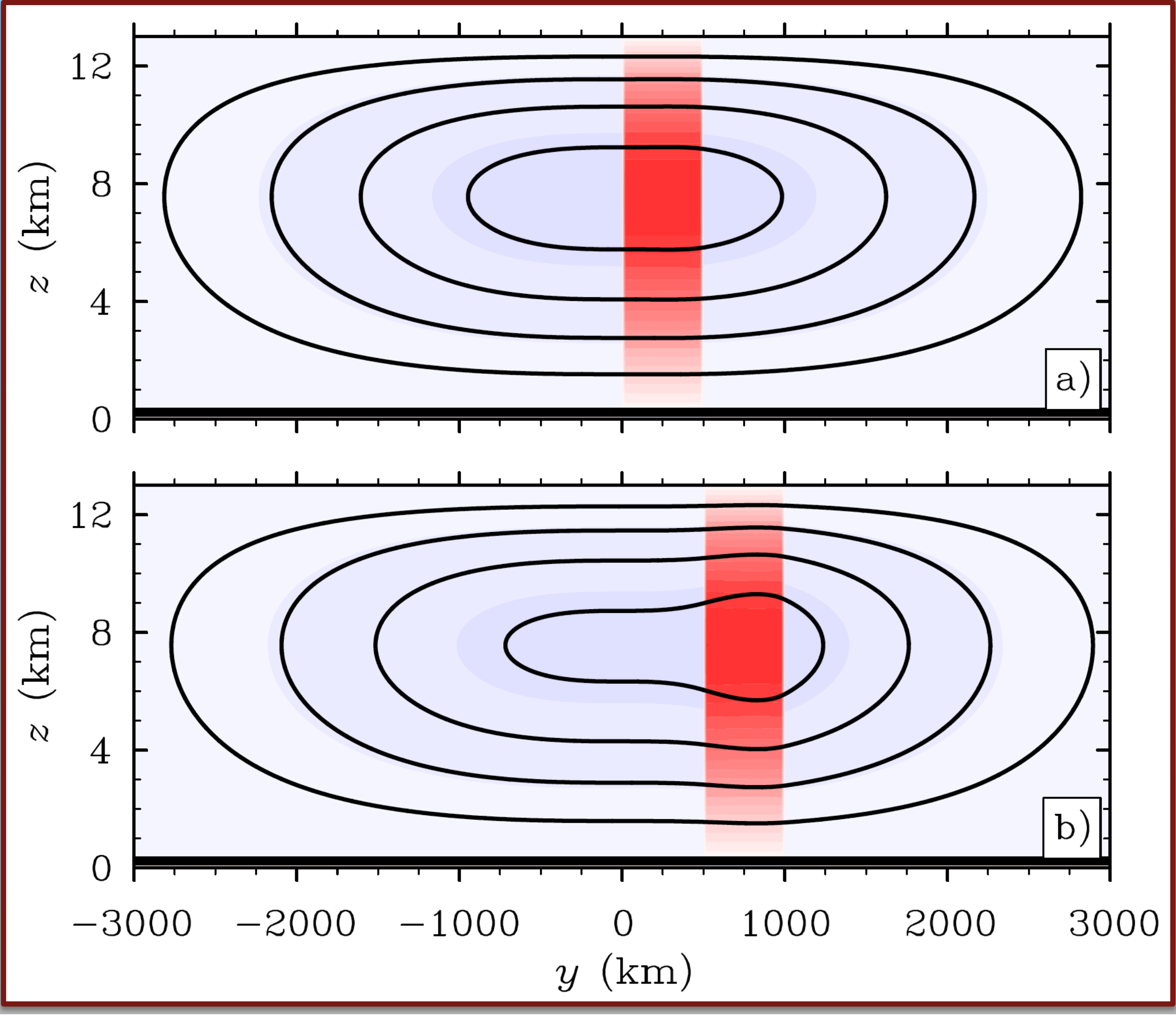
Method #1: Temperature Tendency and Vertical Velocity (Case #1)

Temp. Tendency:

- Black contours
- Interval is $0.2\ \mathrm{K\ day}^{-1}$

Vertical Velocity:

- Shading:
  $w > 0$
  $w < 0$
- Interval is $1\ \mathrm{mm\ s}^{-1}$
- Max. value is $18.01\ \mathrm{mm\ s}^{-1}$



Method #1: Temperature Tendency and Vertical Velocity
(Case #1)

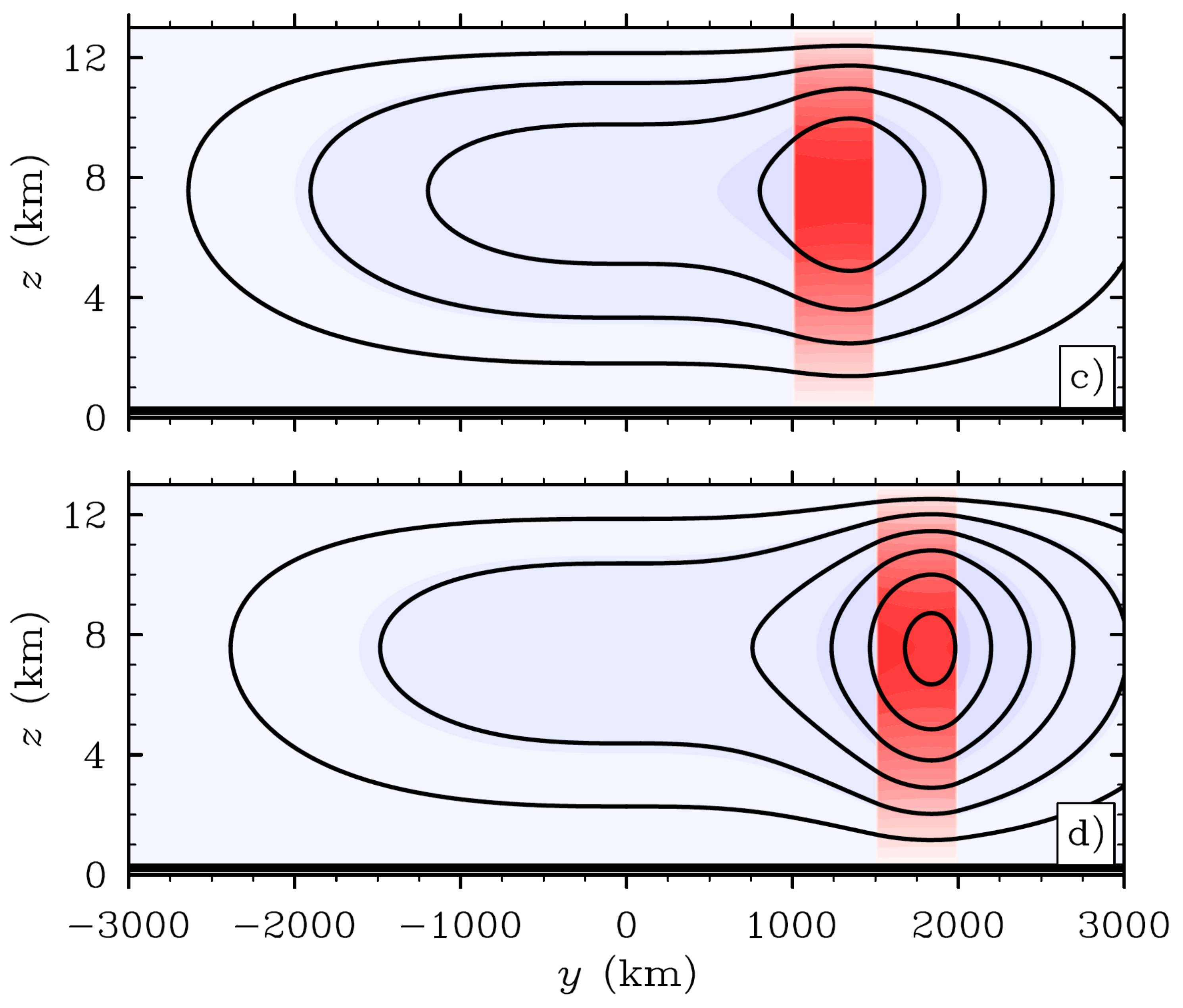
Temp. Tendency:

- Black contours
- Interval is $0.2\ \mathrm{K\ day^{-1}}$

Vertical Velocity:

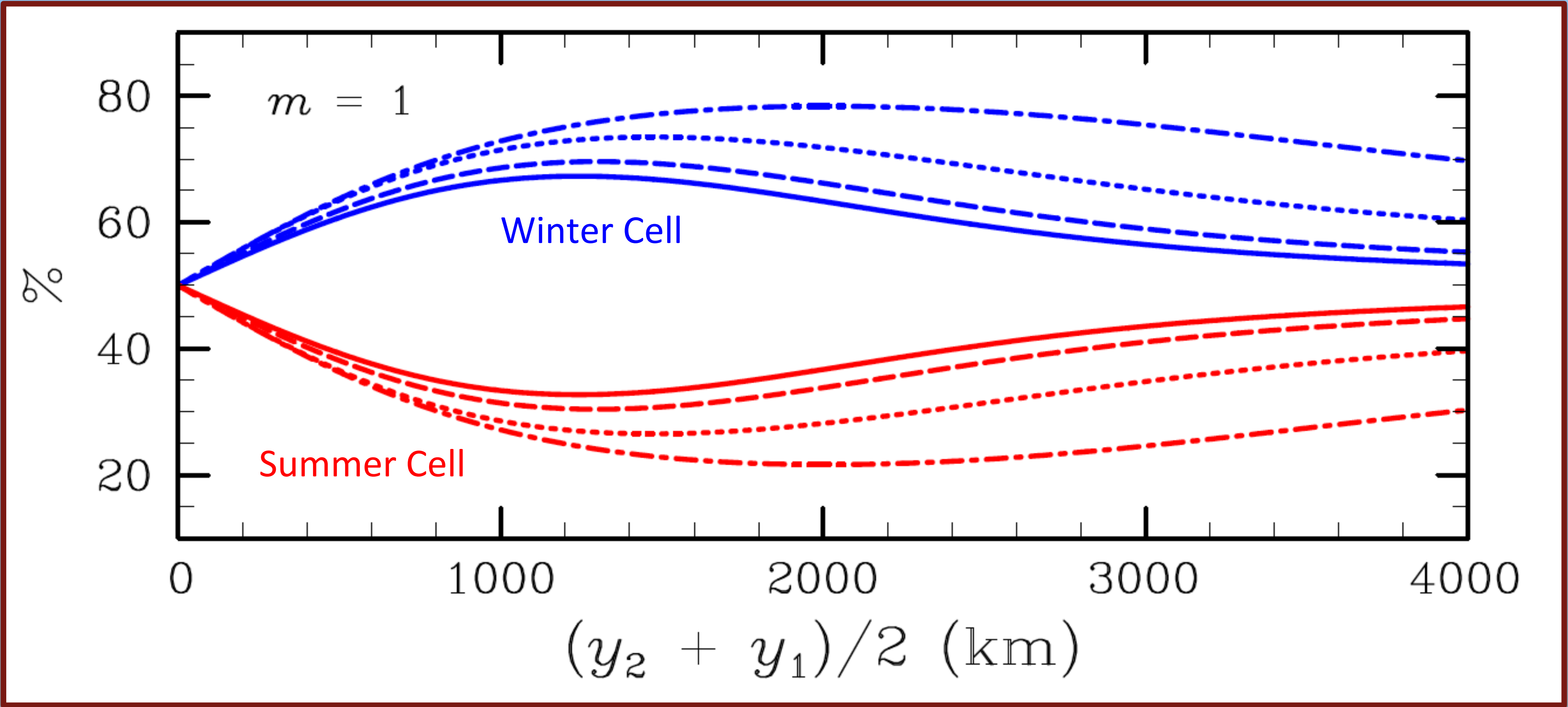
- Shading:
  - $w > 0$
  - $w < 0$
- Interval is $1\ \mathrm{mm\ s^{-1}}$
- Max. value is $18.01\ \mathrm{mm\ s^{-1}}$

Heating becomes more efficient and more localized as the heating source moves poleward

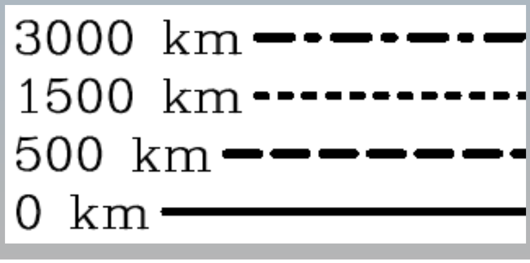


Method #1: (Case #1)

Partition of Total Mass Flux Between the Summer and Winter Hadley Cells



4 ITCZ Thicknesses:



Asymmetry between the summer and winter hemisphere Hadley cells depends on:

- Location of the ITCZ
- Thickness of the ITCZ

Method #1: (Case #2) Production of Shallow Overturning Circulations Through Ekman Pumping

- Consider the following ITCZ experiment:
  - No diabatic heating: $\hat{Q}(y,z,t) = 0$
  - Ekman pumping is restricted to the ITCZ region and is independent of y there
- Then:

$$\mathcal{W}(y,t) = \begin{cases} \mathcal{W}_{\text{ave}} & \text{if } \; y_1 < y < y_2 \\ 0 & \text{otherwise} \end{cases}$$

and

$$F_m(y,t) = \begin{cases} \mathcal{W}_{\text{ave}} \mathcal{Z}_m(0) & \text{if } \; y_1 < y < y_2 \\ 0 & \text{otherwise} \end{cases}$$

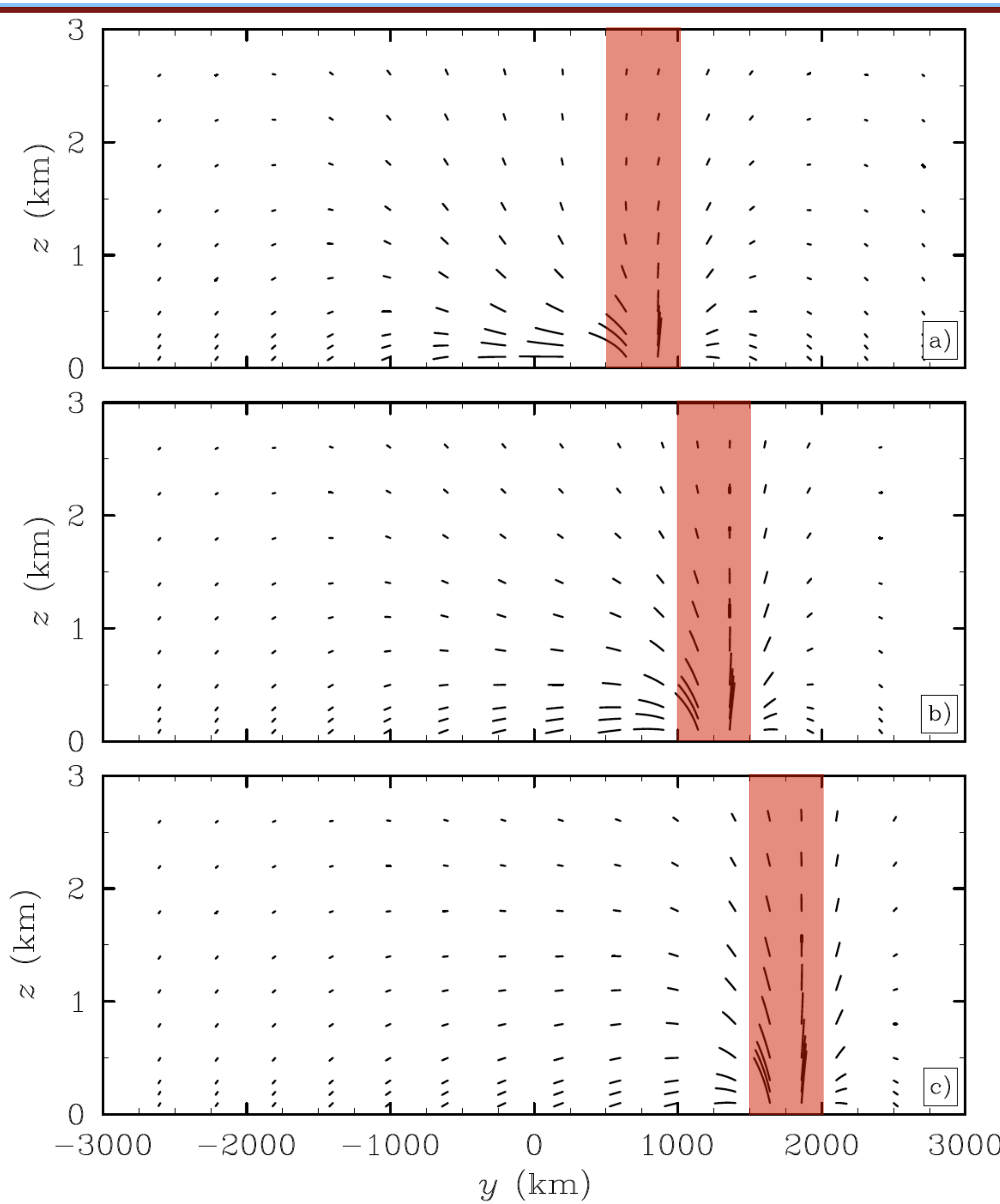
- The analytical solution becomes:

$$\psi(y,z,t) = \mathcal{W}_{\text{ave}} e^{-z/2H} \sum_{m=0}^{m_{\max}} b_m \mathcal{Z}_m(0) \mathcal{Z}_m(z) \left[G_m(y,y_2) - G_m(y,y_1) \right]$$

Method #1: (Case #2)

3-Day Parcel Trajectories

$$\mathcal{W}_{\mathbf{ave}} = \mathbf{4\ mm\ s^{-1}}$$



Note smaller vertical scale

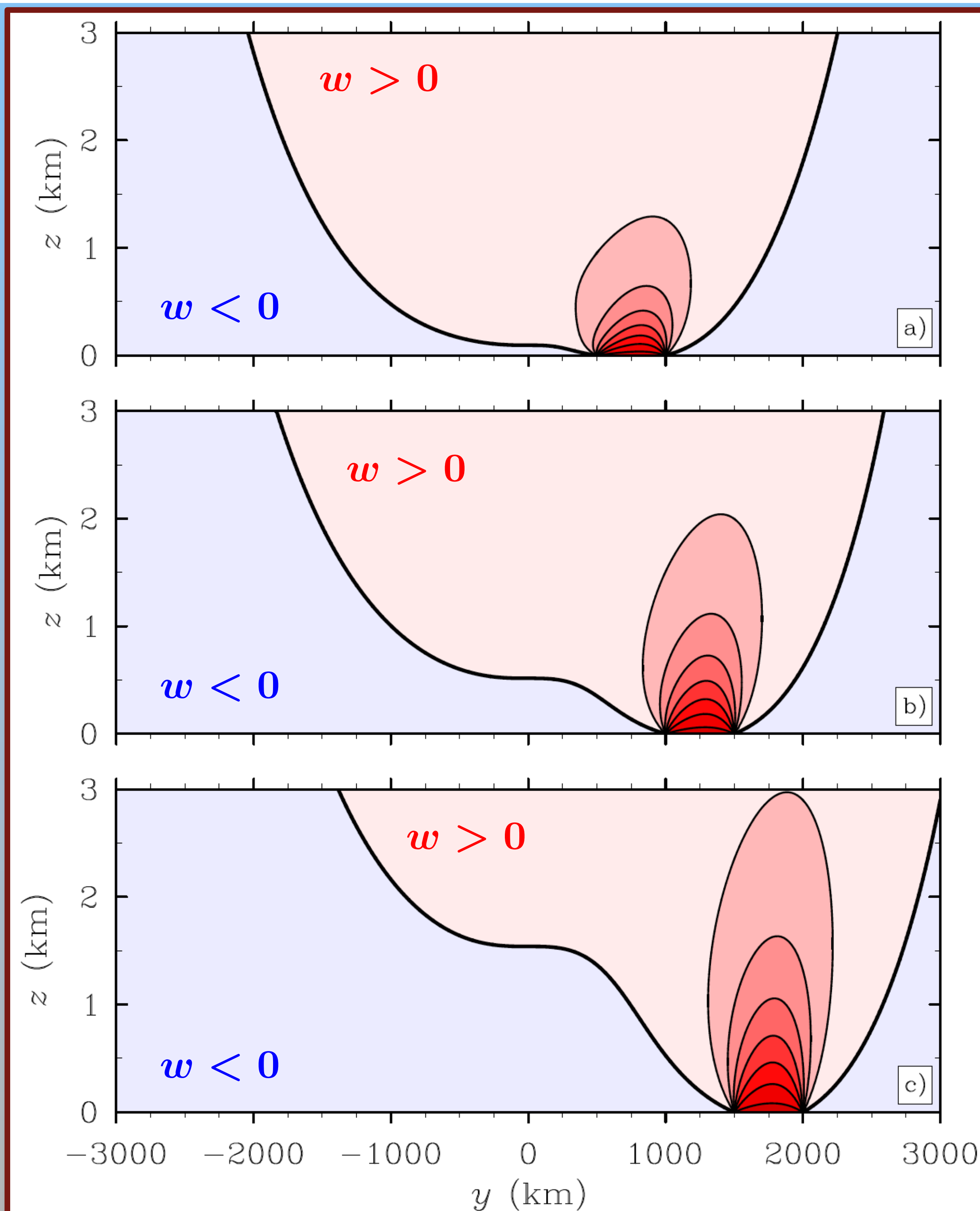
Method #1: (Case #2)

Vertical Velocity

Contour interval:
$\mathbf{0.5\ mm\ s^{-1}}$

Max. magnitude:
$\mathbf{3.774\ mm\ s^{-1}}$

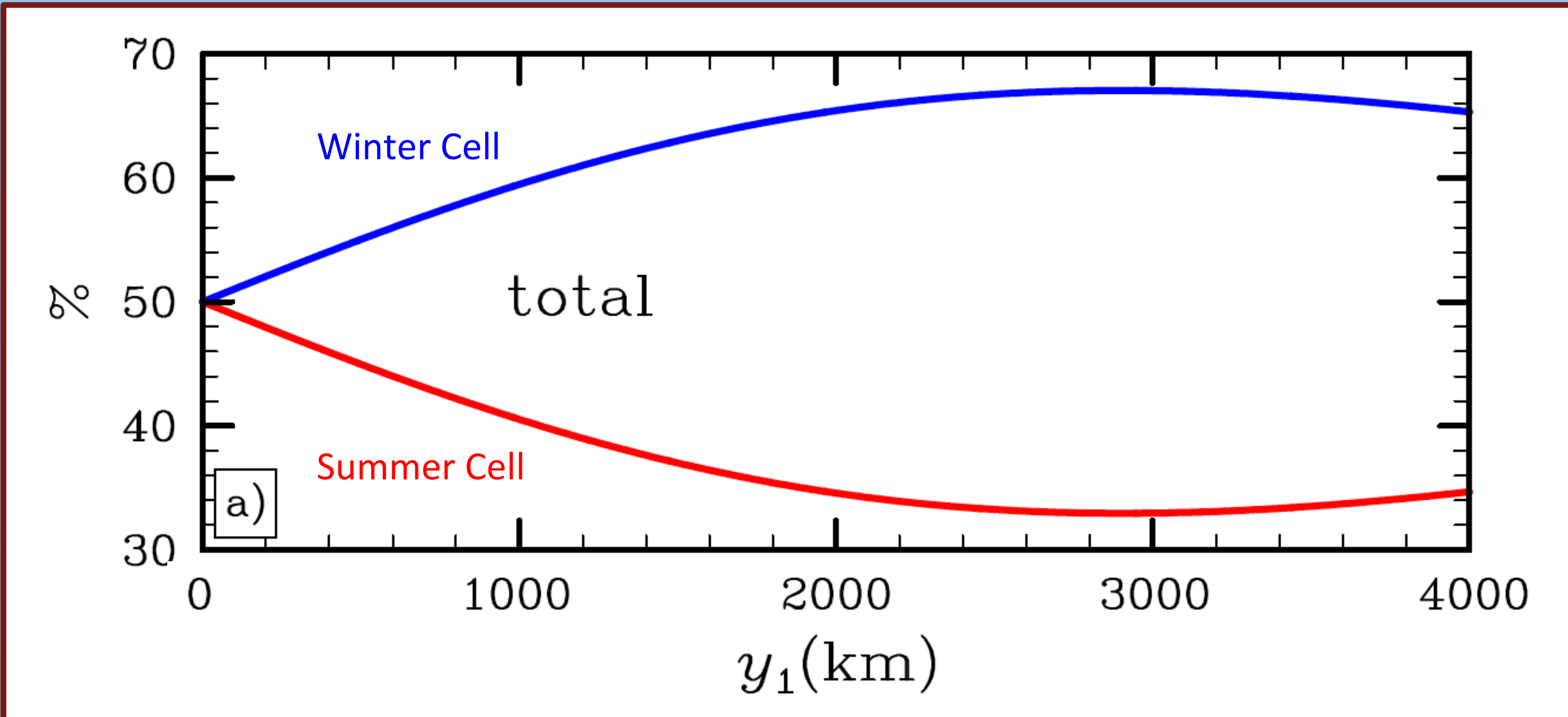
Ekman pumping penetrates deeper away from the equator, so shallow circulations are more efficient near the equator



Note smaller vertical scale

Method #1: (Case #2)

Partition of Total Mass Flux Between the Summer and Winter Hadley Cells



Infinitesimally thin ITCZ

Contributions from all vertical modes (but dominated by $m = 0$)

Recall: **Simplified Meridional Circulation Problem on the β-Plane**

- Single equation for the streamfunction:

$$\boxed{\left(\frac{\partial^2}{\partial t^2}+\beta^2y^2\right)\left(\frac{\partial^2\hat{\psi}}{\partial z^2}-\frac{\hat{\psi}}{4H^2}\right)+N^2\frac{\partial^2\hat{\psi}}{\partial y^2}=\frac{g}{c_pT_0}\frac{\partial\hat{Q}}{\partial y}}$$

- Boundary conditions:

$$\hat{\psi}\to 0 \ \text{ as } \ y\to\pm\infty$$

$$\hat{\psi}=0 \ \text{ at } \ z=z_T$$

$$\left(\frac{\partial^2}{\partial t^2}+\beta^2y^2\right)\left(\frac{\partial\hat{\psi}}{\partial z}-\frac{\hat{\psi}}{2H}\right)+g\frac{\partial^2\hat{\psi}}{\partial y^2}=g\frac{\partial\mathcal{W}}{\partial y} \ \text{ at } \ z=0$$

- Initial conditions: $\boxed{\hat{\psi}=0 \ \text{ and } \ \frac{\partial\hat{\psi}}{\partial t}=0 \ \text{ at } \ t=0}$

Analytical Solution Method #2

- Retain the $\partial^2/\partial t^2$ terms
- No need to assume that forcing varies slowly in time
- Solution now includes gravity waves
- Meridional circulation problem remains a prognostic problem, but no integration in time is required to get solutions
- Solution steps:
  1. Perform vertical transform of the meridional circulation problem (same vertical transform as in Method #1)
  2. Perform meridional Hermite tranform of the resulting horizontal structure equations to obtain a set of second order ODE's in time

Method #2: Vertical Transform (Same as Method #1)

- Seek solutions via the vertical transform pair:

$$\boxed{\begin{aligned}\hat{\psi}(y,z,t) &= \sum_{m=0}^{\infty} \hat{\psi}_m(y,t)\,\mathcal{Z}_m(z) \\ \hat{\psi}_m(y,t) &= \frac{1}{g}\int_0^{z_T} \hat{\psi}(y,z,t)\mathcal{Z}_m(z)N^2(z)dz + \hat{\psi}(y,0,t)\mathcal{Z}_m(0)\end{aligned}}$$

- The vertical structure functions $\mathcal{Z}_m$ are solutions of the Sturm-Liouville eigenvalue problem:

$$\boxed{\begin{gathered}\frac{d^2\mathcal{Z}_m}{dz^2} - \frac{\mathcal{Z}_m}{4H^2} = -\frac{N^2\mathcal{Z}_m}{gh_m} \\ \mathcal{Z}_m = 0 \ \text{ at } \ z = z_T \\ \frac{d\mathcal{Z}_m}{dz} - \frac{\mathcal{Z}_m}{2H} = -\frac{\mathcal{Z}_m}{h_m} \ \text{ at } \ z = 0\end{gathered}}$$

with eigenvalues (or equivalent depths) denoted by h_m

Method #2: Meridional Structure Equations

- Vertical transform of the meridional circulation problem:

$$\boxed{\begin{gathered} \frac{\partial^2 \hat{\psi}_m}{\partial t^2} - g h_m \left(\frac{\partial^2}{\partial y^2} - \frac{y^2}{\bar{b}_m^4} \right) \hat{\psi}_m = -g h_m \frac{\partial F_m}{\partial y} \\ \hat{\psi}_m \to 0 \text{ as } y \to \pm\infty \\ \hat{\psi}_m = 0 \quad \text{and} \quad \frac{\partial \hat{\psi}_m}{\partial t} = 0 \ \text{ at } \ t = 0 \end{gathered}}$$

for each vertical mode m

- The interior diabatic heating and the boundary layer pumping again merge into a single forcing term:

$$\boxed{F_m(y,t) = \int_0^{z_T} \frac{\hat{Q}(y,z,t)}{c_p T_0} \mathcal{Z}_m(z)\, dz + \mathcal{W}(y,t) \mathcal{Z}_m(0)}$$

Method #2: Meridional Hermite Transform

- Seek solutions via the following Hermite transform pair:

$$\hat{\psi}_m(y,t) = \sum_{n=0}^{\infty} \hat{\psi}_{mn}(t)\, \mathcal{H}_n^m(y)$$

$$\hat{\psi}_{mn}(t) = \frac{1}{\bar{b}_m} \int_{-\infty}^{\infty} \hat{\psi}_m(y,t)\, \mathcal{H}_n^m(y)\, dy$$

- The meridional structure functions are related to Hermite polynomials by:

$$\mathcal{H}_n^m(y) = \left(\pi^{\frac{1}{2}} 2^n n!\right)^{-\frac{1}{2}} H_n(y/\bar{b}_m)\, e^{-\frac{1}{2}(y/\bar{b}_m)^2}$$

- The meridional structure functions satisfy the following eigenproblem:

$$\left(\frac{d^2}{dy^2} - \frac{y^2}{\bar{b}_m^4}\right) \mathcal{H}_n^m(y) = -\left(\frac{2n+1}{\bar{b}_m^2}\right) \mathcal{H}_n^m(y)$$

Meridional Structure Functions

Eigenfunctions:

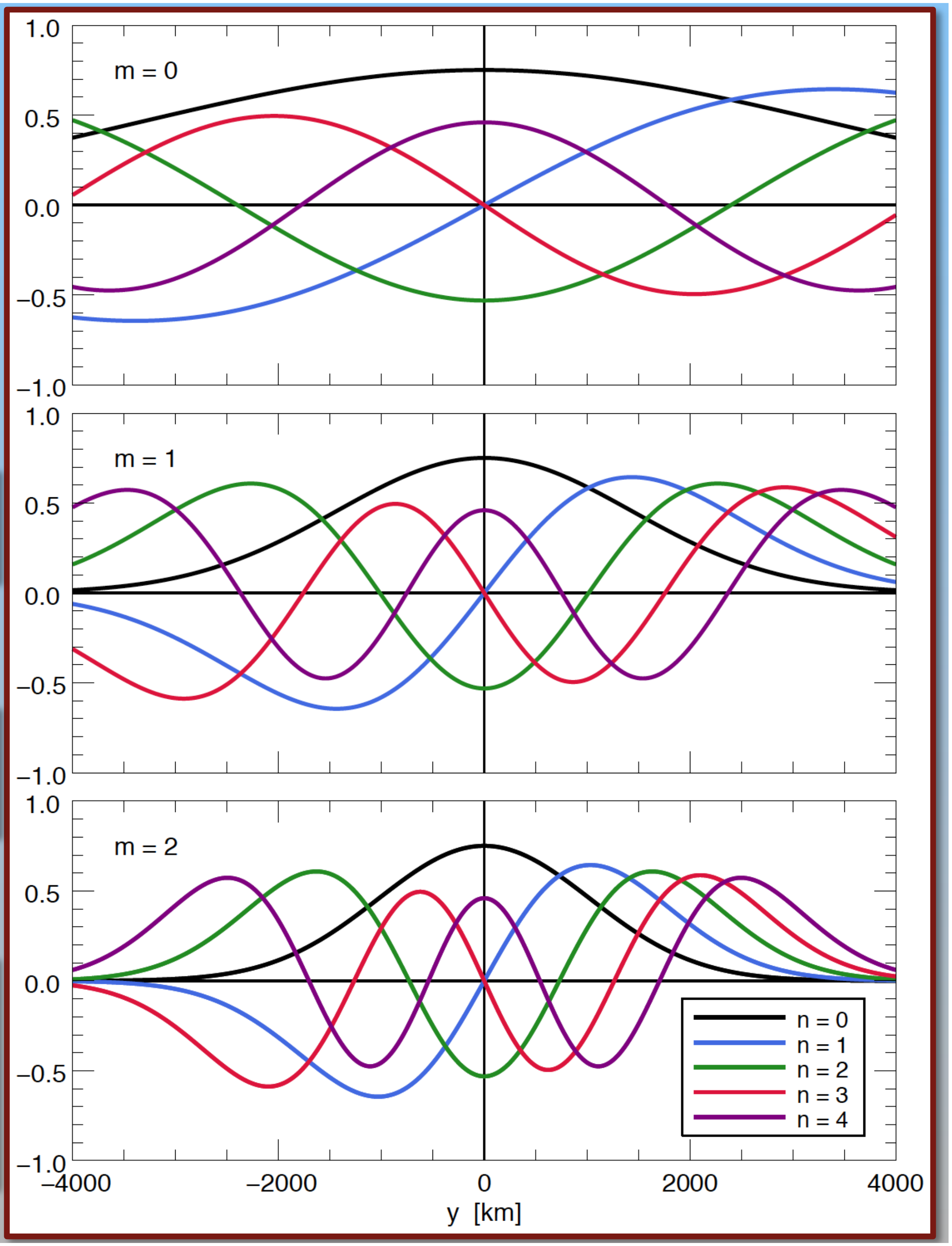
$$\left(\frac{d^2}{dy^2} - \frac{y^2}{\bar{b}_m^4}\right)\mathcal{H}_n^m(y) = -\left(\frac{2n+1}{\bar{b}_m^2}\right)\mathcal{H}_n^m(y)$$

Orthogonal:

$$\int_{-\infty}^{\infty} \mathcal{H}_n^m(y)\,\mathcal{H}_{n'}^m(y)\,dy = \begin{cases} \bar{b}_m & n' = n \\ 0 & n' \neq n \end{cases}$$

Recurrence Relation:

$$\mathcal{H}_{n+1}^m(y) = \left(\frac{2}{n+1}\right)^{1/2}\left(\frac{y}{\bar{b}_m}\right)\mathcal{H}_n^m(y) - \left(\frac{n}{n+1}\right)^{1/2}\mathcal{H}_{n-1}^m(y)$$



Method #2: Solution via Hermite Transform

- Meridional Hermite transform of the meridional structure equations give a set of 2nd order ODE's in time:

$$\boxed{\begin{gathered}\frac{d^2\hat{\psi}_{mn}}{dt^2} + \nu_{mn}^2\hat{\psi}_{mn} = -gh_m F'_{mn} \\[2em] \hat{\psi}_{mn} = 0 \;\text{ and }\; \frac{d\hat{\psi}_{mn}}{dt} = 0 \;\text{ at }\; t = 0\end{gathered}}$$

For each vertical mode m and each meridional mode n

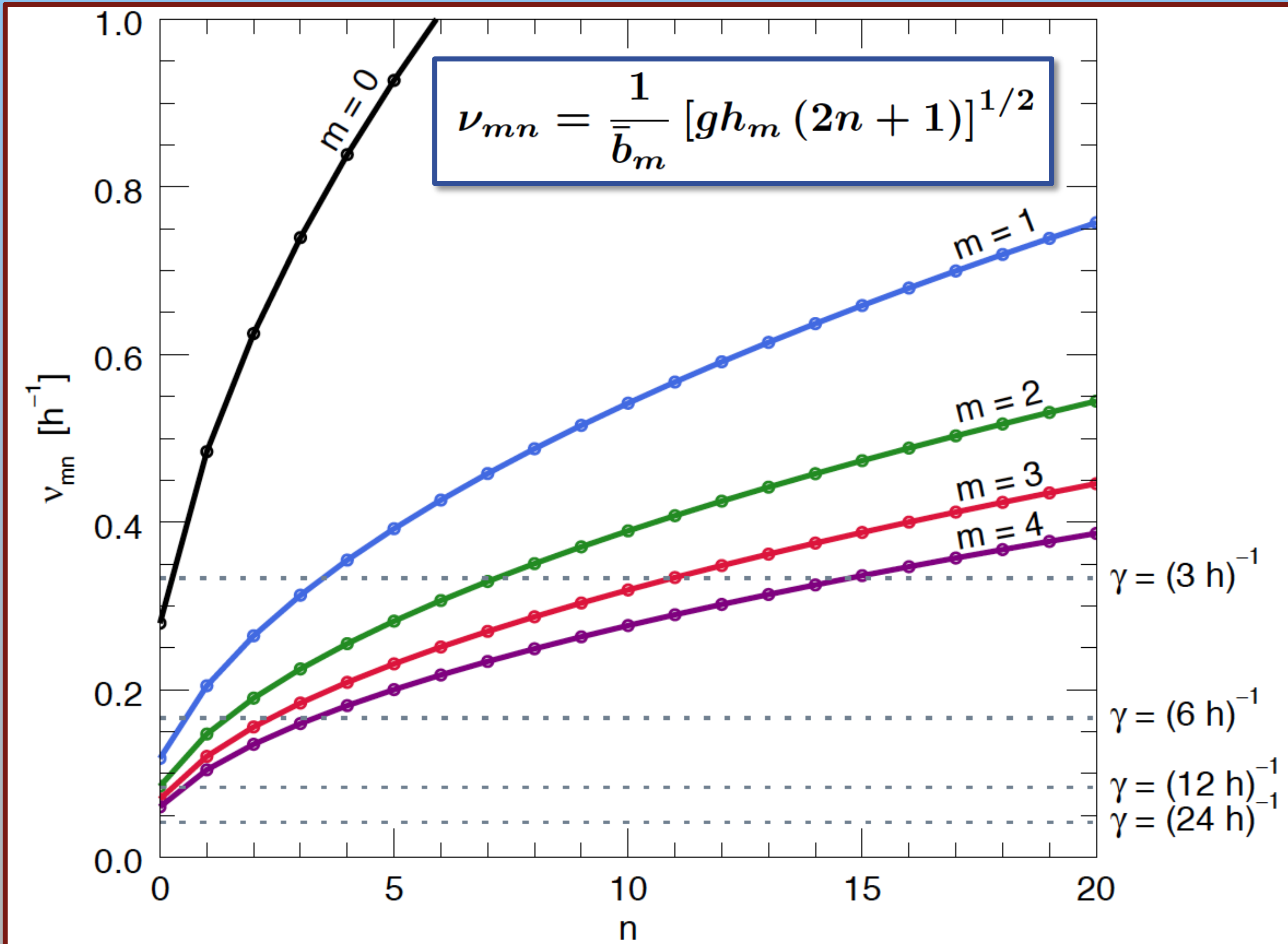
- The inertia-gravity wave frequencies are given by:

$$\boxed{\nu_{mn} = \frac{1}{\bar{b}_m}\left[gh_m\left(2n+1\right)\right]^{1/2}}$$

- The forcing terms are given by:

$$\boxed{F'_{mn}(t) = \frac{1}{\bar{b}_m}\int_{-\infty}^{\infty}\frac{\partial F_m(y,t)}{\partial y}\mathcal{H}_n^m(y)\,dy}$$

Method #2: Inertia-Gravity Wave Frequencies



Method #2: (Special Case) — Transient Hadley Circulations Forced by a Switch-On of ITCZ Convection

- Consider diabatic heating of the following form:
  - Just within the ITCZ and independent of y within it
  - Smoothly switched on to a steady state value

$$\hat{Q}(y,z,t) = \mathcal{T}(t)\begin{cases} \sum_{m=0}^{\infty} \tilde{Q}_m \mathcal{Z}_m(z) & \text{if } y_1 < y < y_2 \\ 0 & \text{otherwise} \end{cases}$$

where $\mathcal{T}(t) = 1 - (1+\gamma t)e^{-\gamma t}$

- The forcing terms then become:

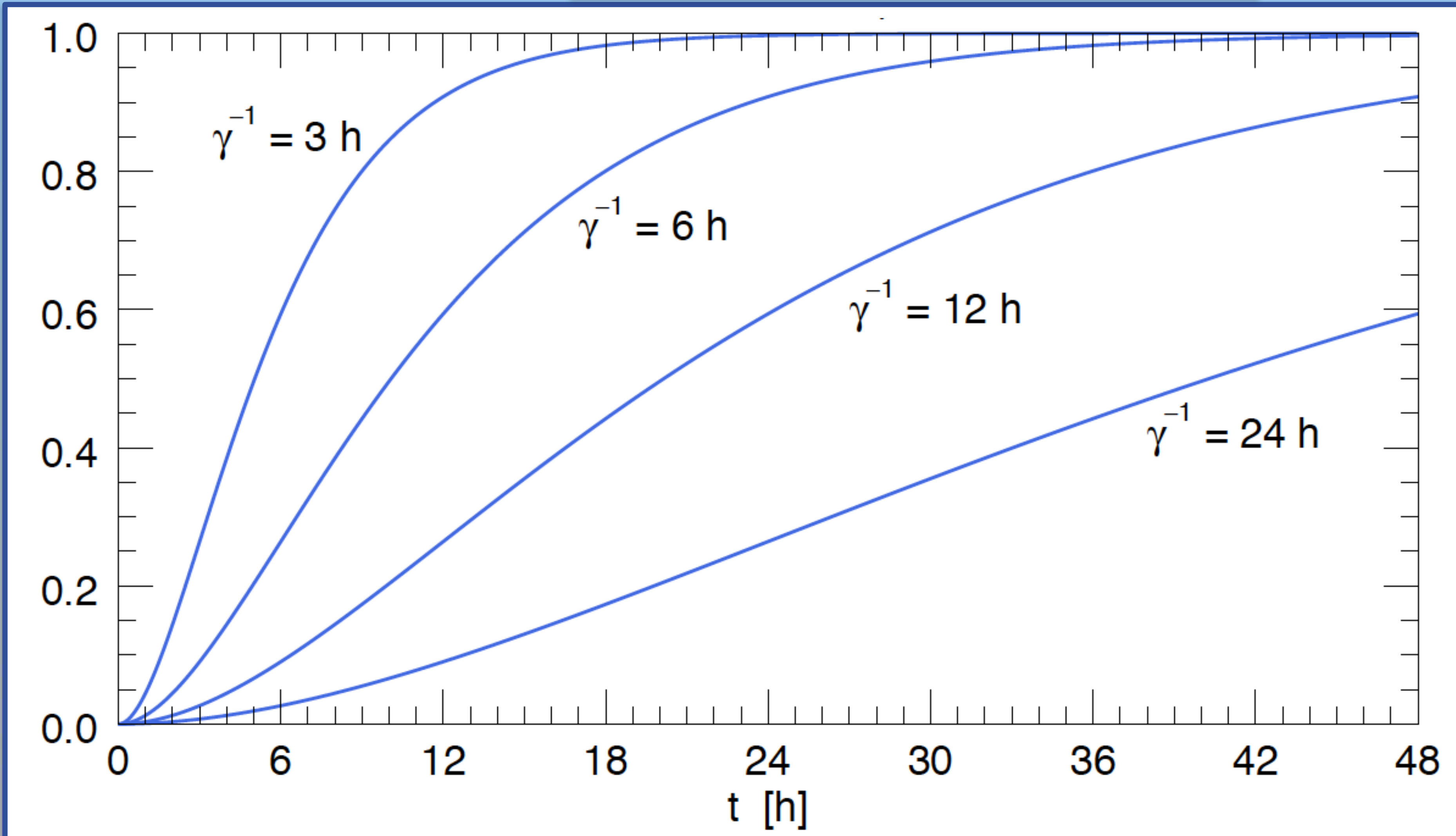
$$F'_{mn}(t) = \mathcal{T}(t)\mathcal{F}_{mn}$$

where $\mathcal{F}_{mn} = \frac{g\tilde{Q}_m}{c_p T_0 N^2 \bar{b}_m}\left[\mathcal{H}_n^m(y_1) - \mathcal{H}_n^m(y_2)\right]$

Method #2: (Special Case)

Switch-On Function

$$\mathcal{T}(t) = 1 - (1 + \gamma t)e^{-\gamma t}$$



Method #2: (Special Case)

Transient Hadley Circulations Forced by a Switch-On of ITCZ Convection

- Solution of the transformed problem:

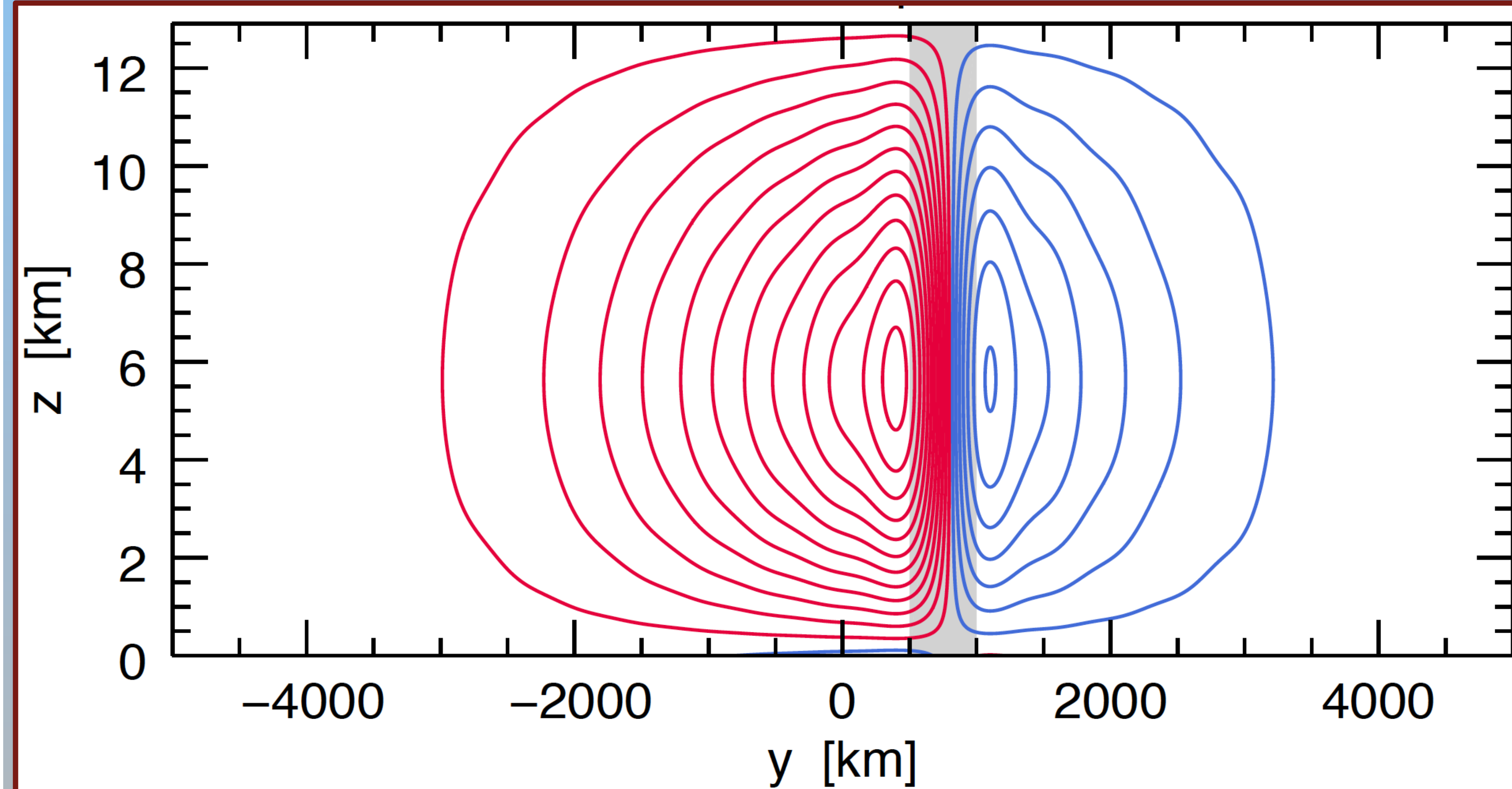
$$\hat{\psi}_{mn}(t) = -\frac{g h_m \mathcal{F}_{mn}}{\nu_{mn}^2} \left\{ \left(\frac{(\nu_{mn}^2 - \gamma^2)\gamma^2}{(\nu_{mn}^2 + \gamma^2)^2} \right) \cos(\nu_{mn} t) - \left(\frac{2\gamma^3 \nu_{mn}}{(\nu_{mn}^2 + \gamma^2)^2} \right) \sin(\nu_{mn} t) + 1 - \left(\frac{\nu_{mn}^2 + 3\gamma^2}{\nu_{mn}^2 + \gamma^2} + \gamma t \right) \left(\frac{\nu_{mn}^2 e^{-\gamma t}}{\nu_{mn}^2 + \gamma^2} \right) \right\}$$

- Solution of the meridional circulation problem:

$$\psi(y,z,t) = e^{-z/2H} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \hat{\psi}_{mn}(t)\, \mathcal{H}_n^m(y)\, \mathcal{Z}_m(z)$$

Method #2:
(Special Case)

Streamfunction at t = 120 h



ITCZ: $500\ \text{km} \leq y \leq 1000\ \text{km}$

Switch On: $\gamma = (24\ \text{h})^{-1}$ (slow)

Modes: $m = 1$

$n = 0, 1, \ldots, 200$

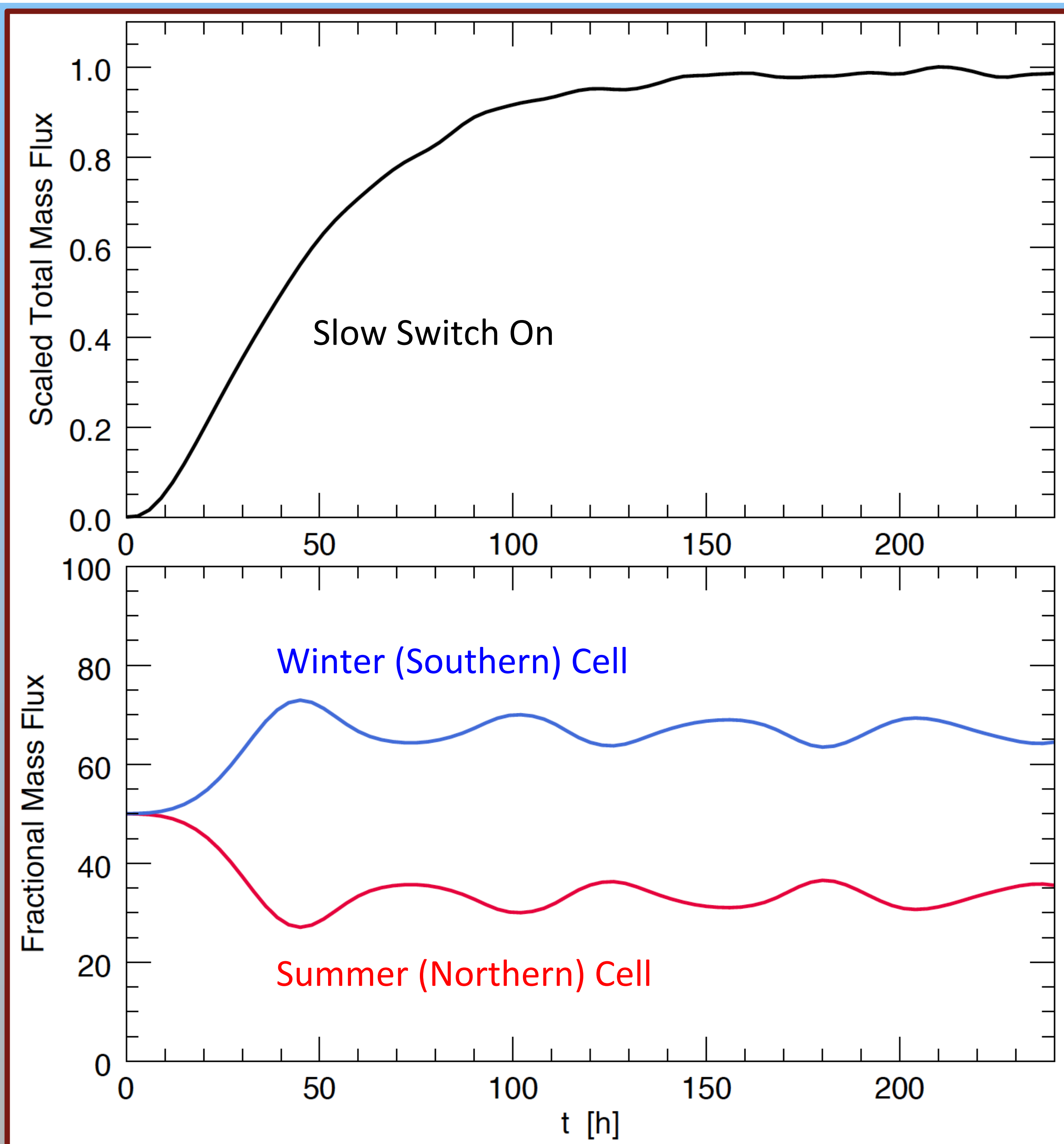
Method #2: (Special Case)

Partitioning of Total Mass Flux

ITCZ:
$500 \ \text{km} \ \text{to} \ 1000 \ \text{km}$

Switch On:
$\gamma = (24 \ \text{h})^{-1}$
(slow)

Modes:
$m = 1$
$n = 0, 1, \ldots, 200$



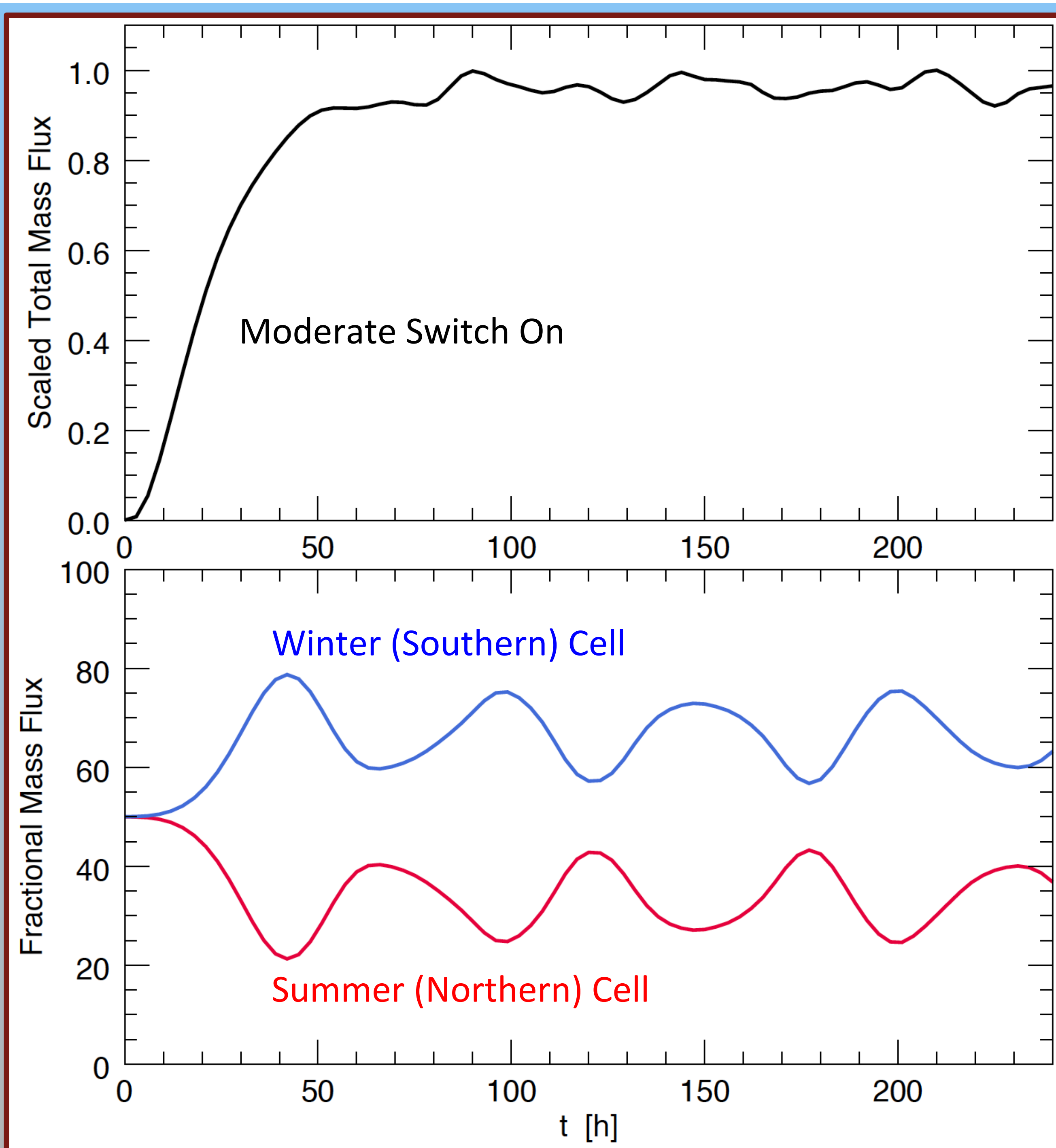
Method #2:
(Special Case)

Partitioning of Total Mass Flux

ITCZ:
$500 \;\; \text{km} \;\; \text{to}$
$1000 \;\; \text{km}$

Switch On:
$\gamma = (12\ \text{h})^{-1}$
(moderate)

Modes:
$m = 1$
$n = 0, 1, \ldots, 200$



Method #2: (Special Case)

Partitioning of Total Mass Flux

ITCZ:

$500\ \ \mathrm{km}\ \ \mathrm{to}$

$1000\ \ \mathrm{km}$

Switch On:

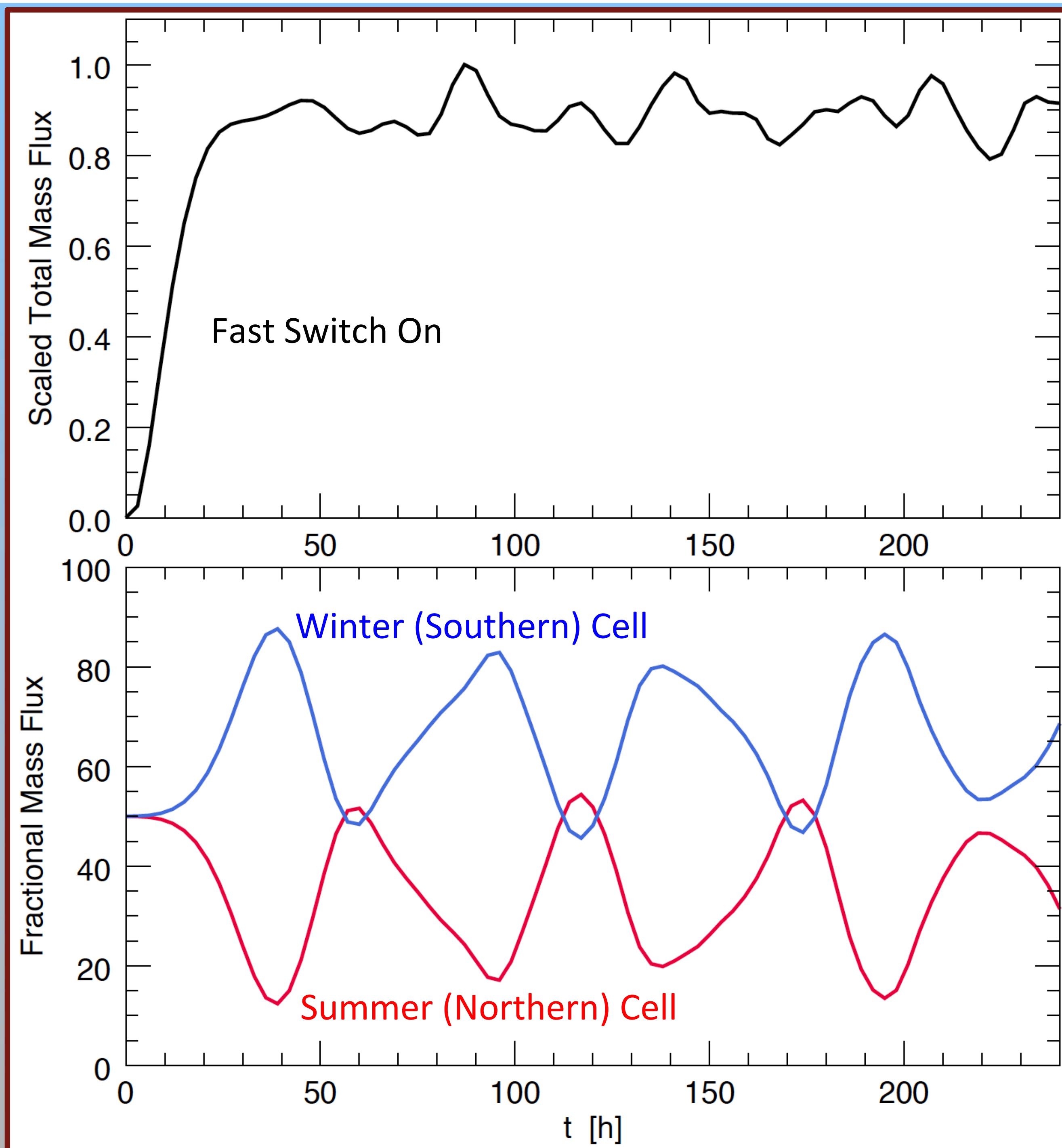
$\gamma = (6\ \mathrm{h})^{-1}$

(fast)

Modes:

$m = 1$

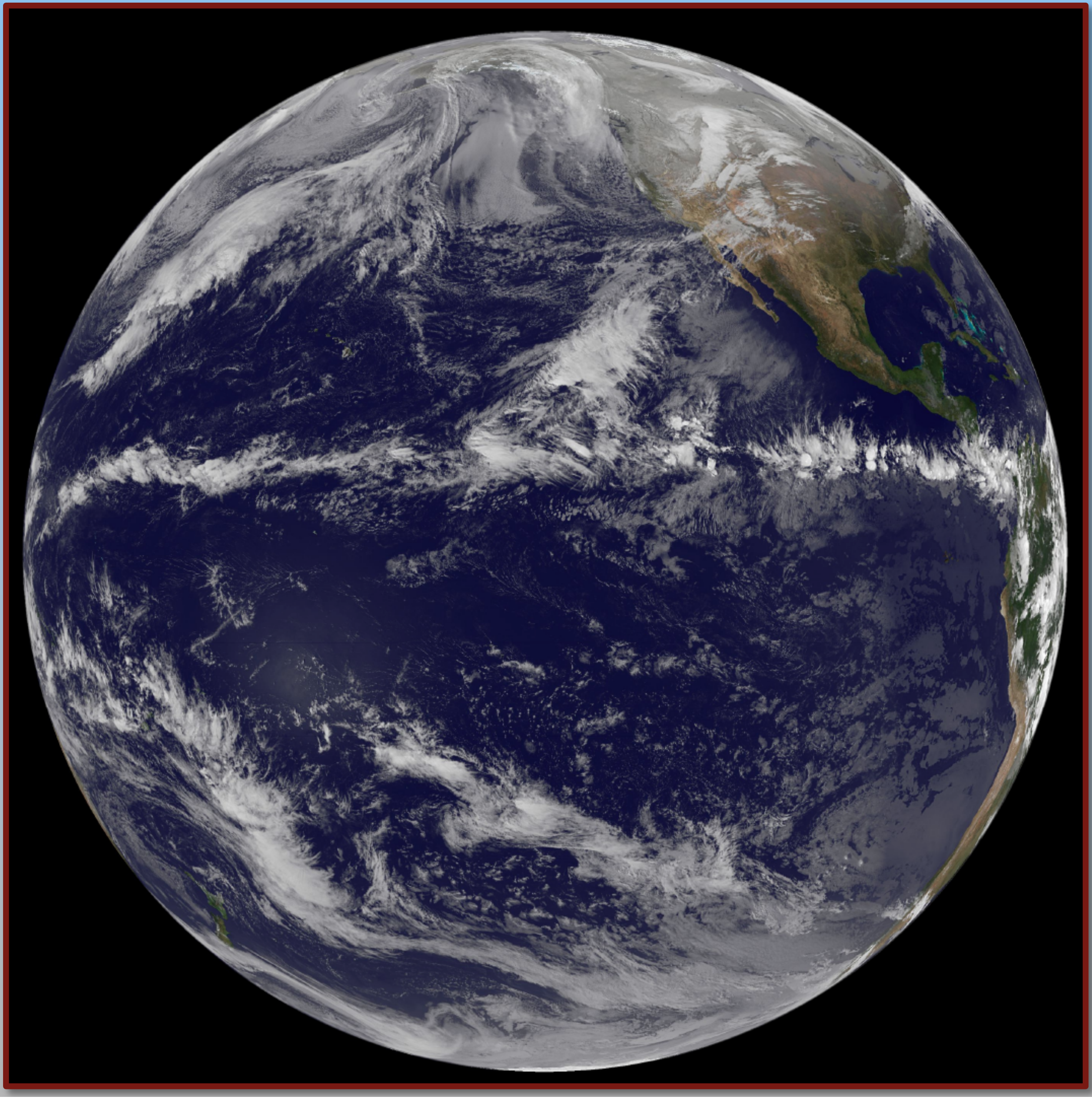
$n = 0, 1, \ldots, 200$



Additional Thoughts:

Why is the ITCZ so thin?

Nov. 24, 2010
00:00 UTC



From NASA GSFC GOES Project website

Summary & Conclusions

- Analytically solved the meridional circulation problem with two types of forcings:
  1. Meridional variation of the interior diabatic heating
  2. Ekman pumping at the top of the boundary layer

- A vertical transform of the meridional circulation equation converts the PDE in (y, z, t) into a system of PDE's in (y, t) for the meridional structures of each vertical mode

- The resulting meridional structure equations can then be solved by using a Green's function method (a diagnostic approach) or by using a Hermite transform method (a full prognostic approach)

Summary & Conclusions (Continued):

- Diabatic heating projected only onto the first internal mode (deep diabatic forcing) leads to deep overturning circulations in the absence of Ekman pumping
- Forcing by Ekman pumping in the absence of diabatic heating leads to shallow overturning circulations
- Both meridional circulations display an asymmetry between the winter and summer overturning cells due to the spatially varying inertial stability coefficient
- Shallow Hadley circulations tend to occur when Ekman pumping occurs close to the equator because the inertial stability is weak so there is little resistance to horizontal motion

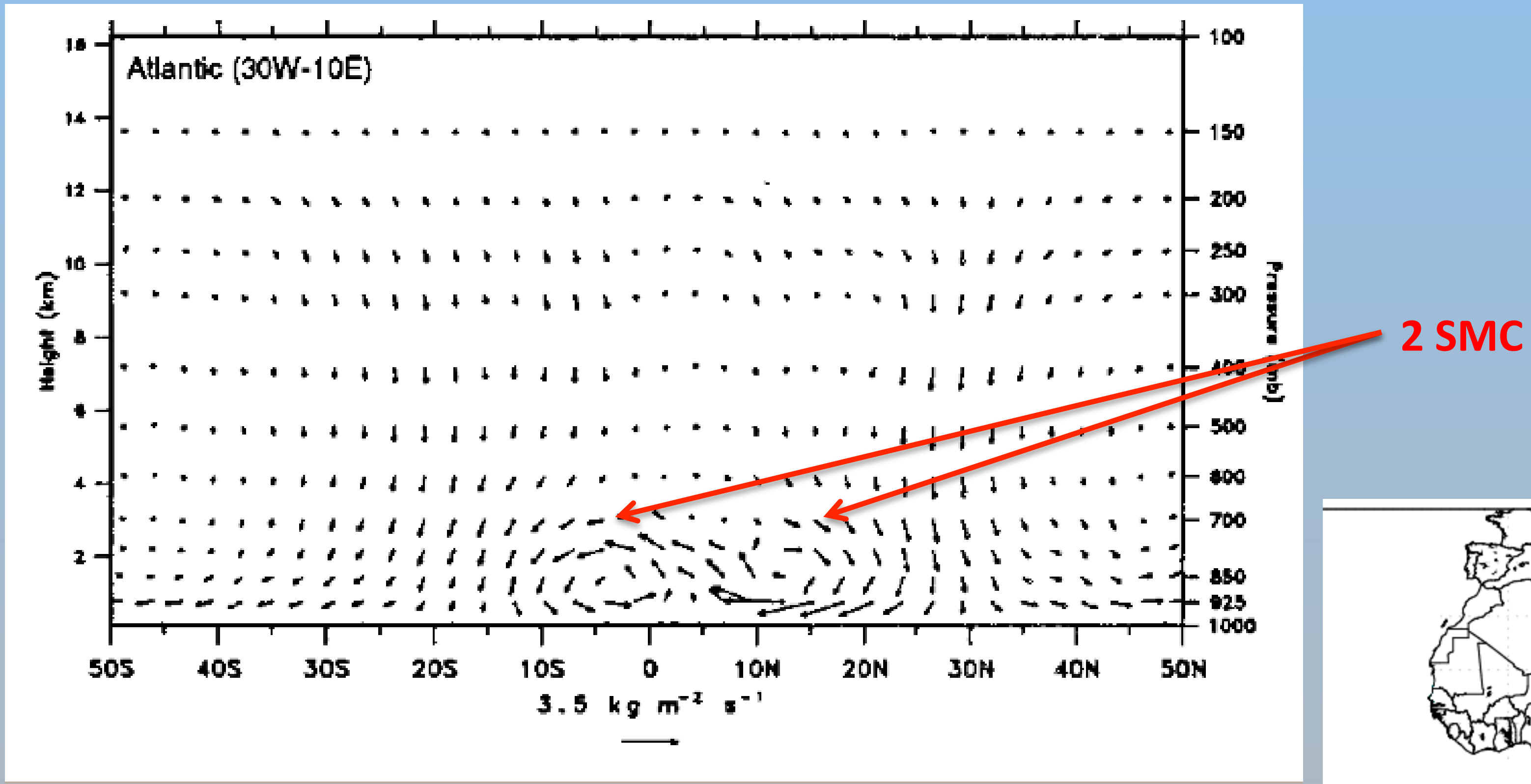
Summary & Conclusions (Continued):

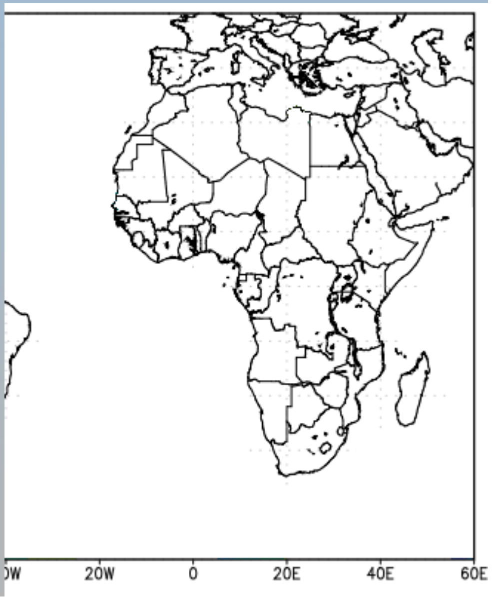
- This work has shown that Ekman pumping is a viable forcing mechanism for the Shallow Hadley Circulation
  - However, diabatic heating due to shallow precipitating convection and surface heating (in analogy with land/sea breezes, as discussed by Nolan et al. 2007, 2010) are also viable forcing mechanisms
  - Further research is needed to understand the relative importance of these three forcing mechanisms
- The narrowness of the ITCZ is due to the formation of shock-like structures in the boundary layer. How well do current GCM's simulate this?
- After fluctuations in the strength of the ITCZ, theory predicts that the tropical troposphere will ring with "trapped inertia-gravity wave activity." Do observations or GCM's show this effect?

Thank you for starting the "Magnificent Second Phase of Climate Modeling" in 1960!

Regional Meridional Cross Sections

January
(1979-1993)



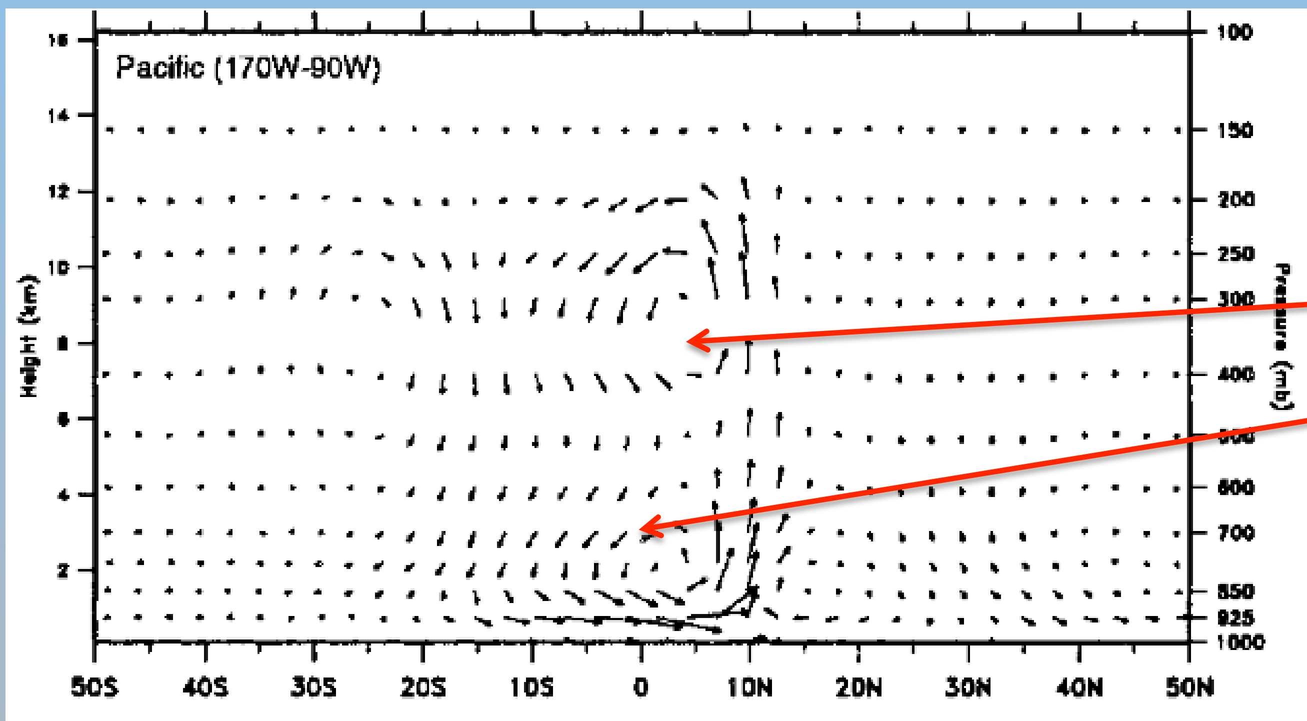


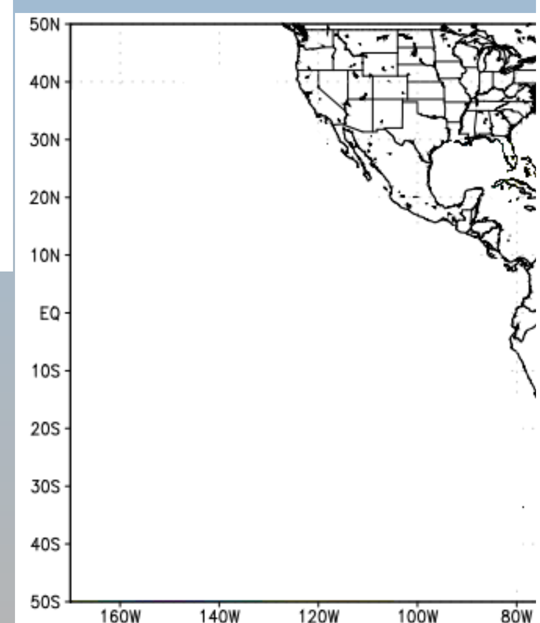
Divergent flow vectors (ECMWF Reanalysis)

From Trenberth et al., 2000

Regional Meridional Cross Sections

July
(1979-1993)





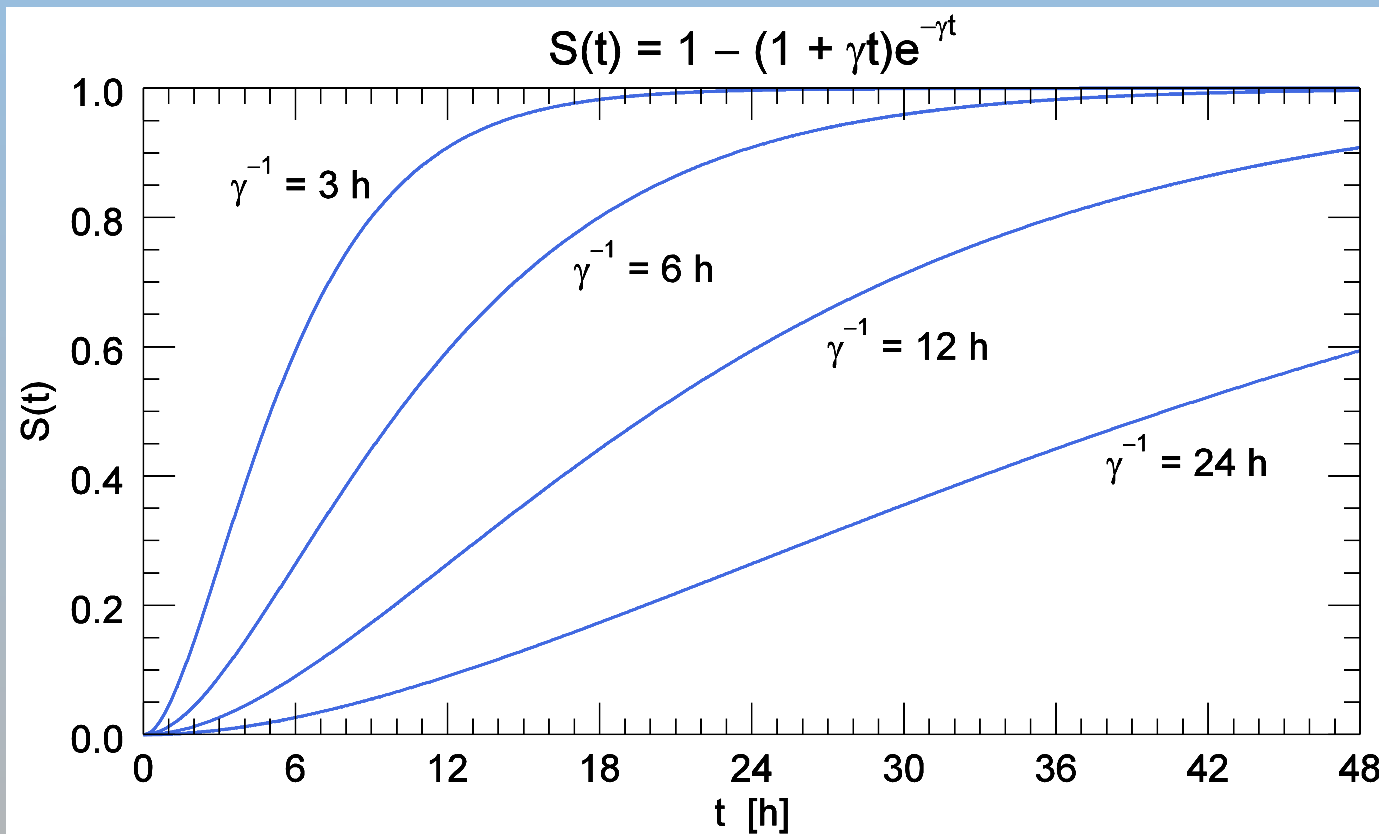
Divergent flow vectors (ECMWF Reanalysis)

From Trenberth et al., 2000

Time dependence

$$\boxed{S(t) = 1 - (1 + \gamma t)e^{-\gamma t}}$$

$\boxed{\gamma}$ sharpness of the switch-on function S(t)



Partition of Total Mass Flux Between the Summer and Winter Hemisphere Hadley Cells for the Deep Heating Case

- Fractional mass flux of summer hemisphere cell:

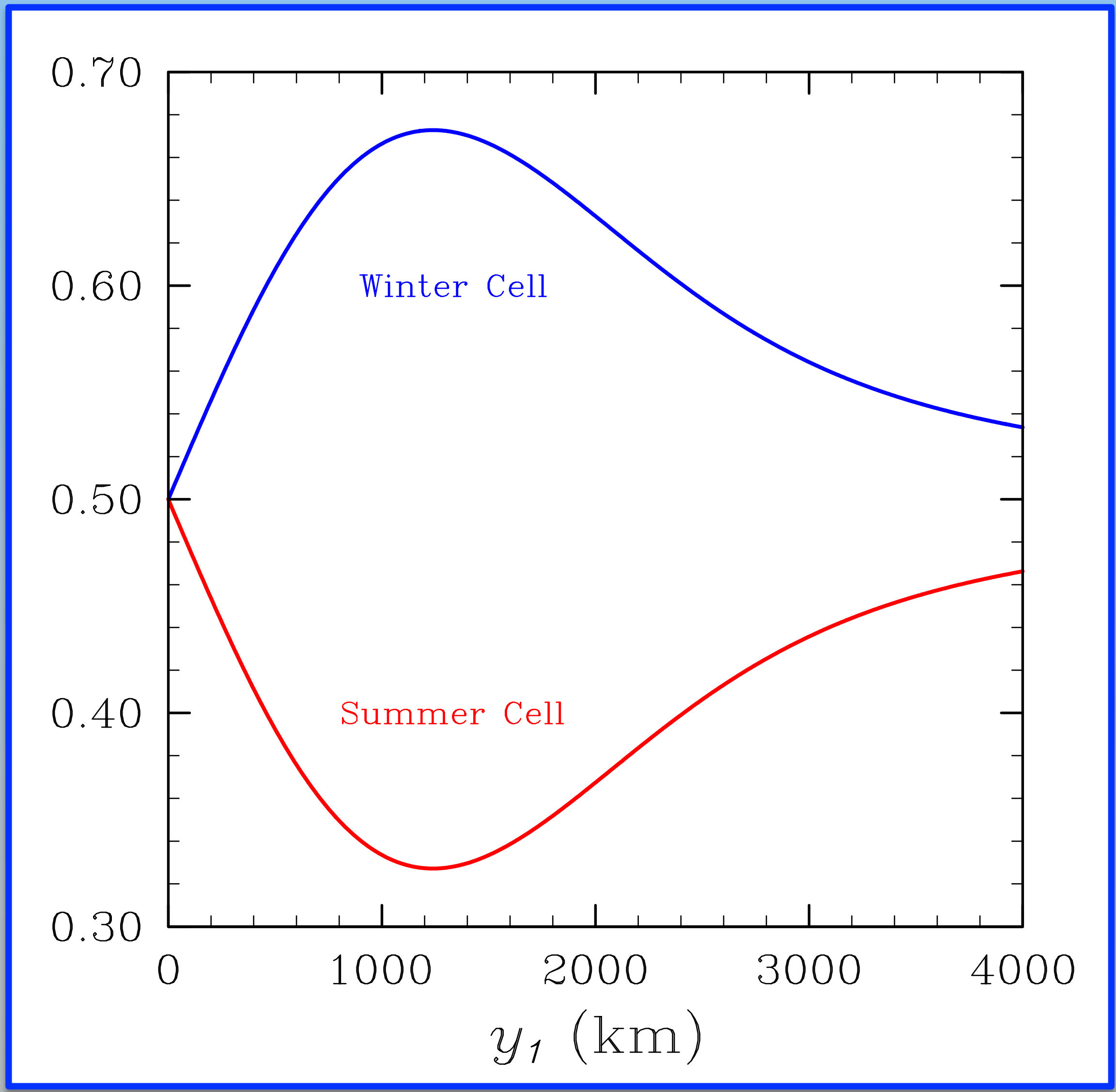
$$\boxed{\frac{\psi(y_1+,z,t)}{\psi(y_1+,z,t)-\psi(y_1-,z,t)}=\frac{1}{\sqrt{2}}D'_{-1/2}(-y_1/b_1)D_{-1/2}(y_1/b_1)}$$

- Fractional mass flux of winter hemisphere cell:

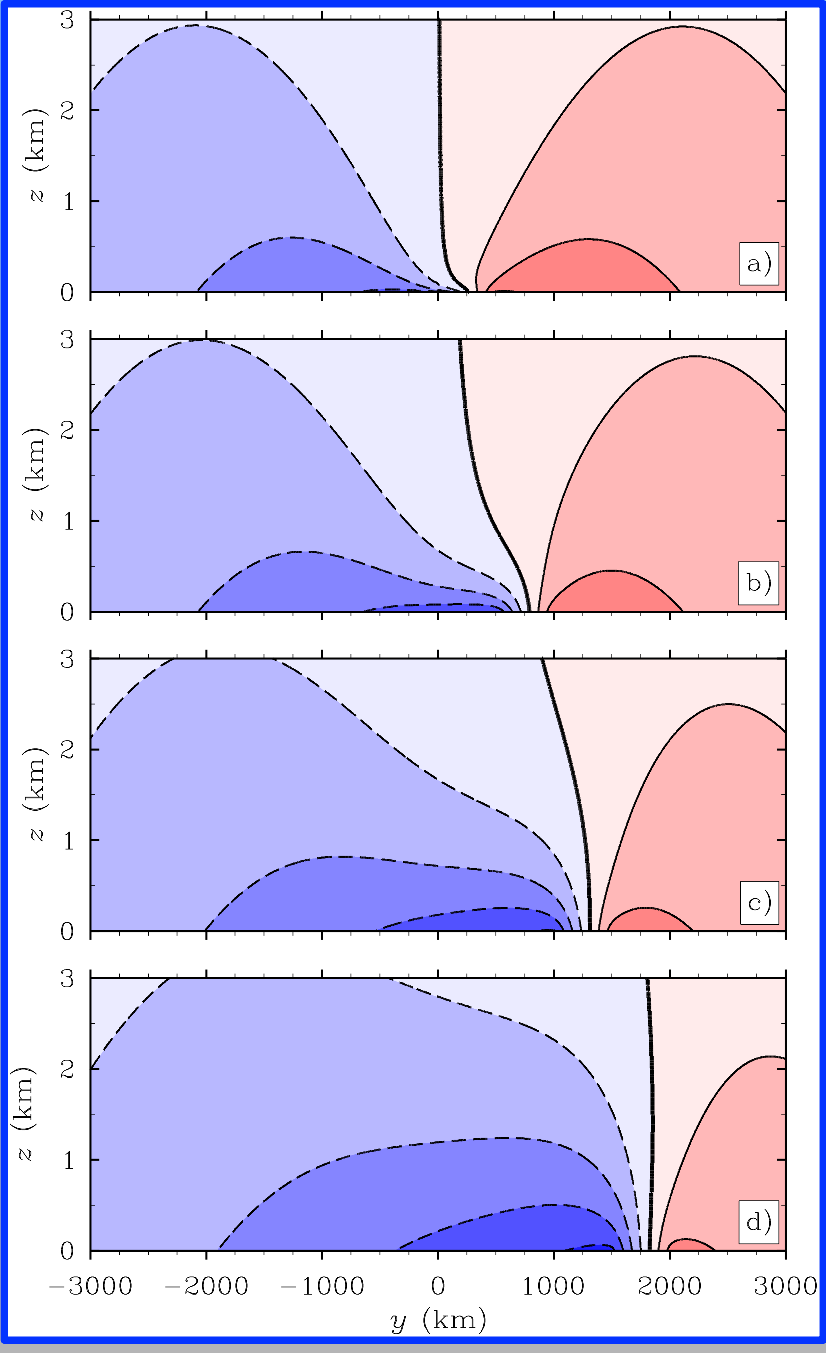
$$\boxed{\frac{-\psi(y_1-,z,t)}{\psi(y_1+,z,t)-\psi(y_1-,z,t)}=-\frac{1}{\sqrt{2}}D'_{-1/2}(y_1/b_1)D_{-1/2}(-y_1/b_1)}$$

- The maximum asymmetry between the two cells occurs when the ITCZ is located 1200-1300 km off the equator, in which case the winter cell carries approx. twice the mass flux of the summer cell

Partition of Total Mass Flux for the Deep Heating Case

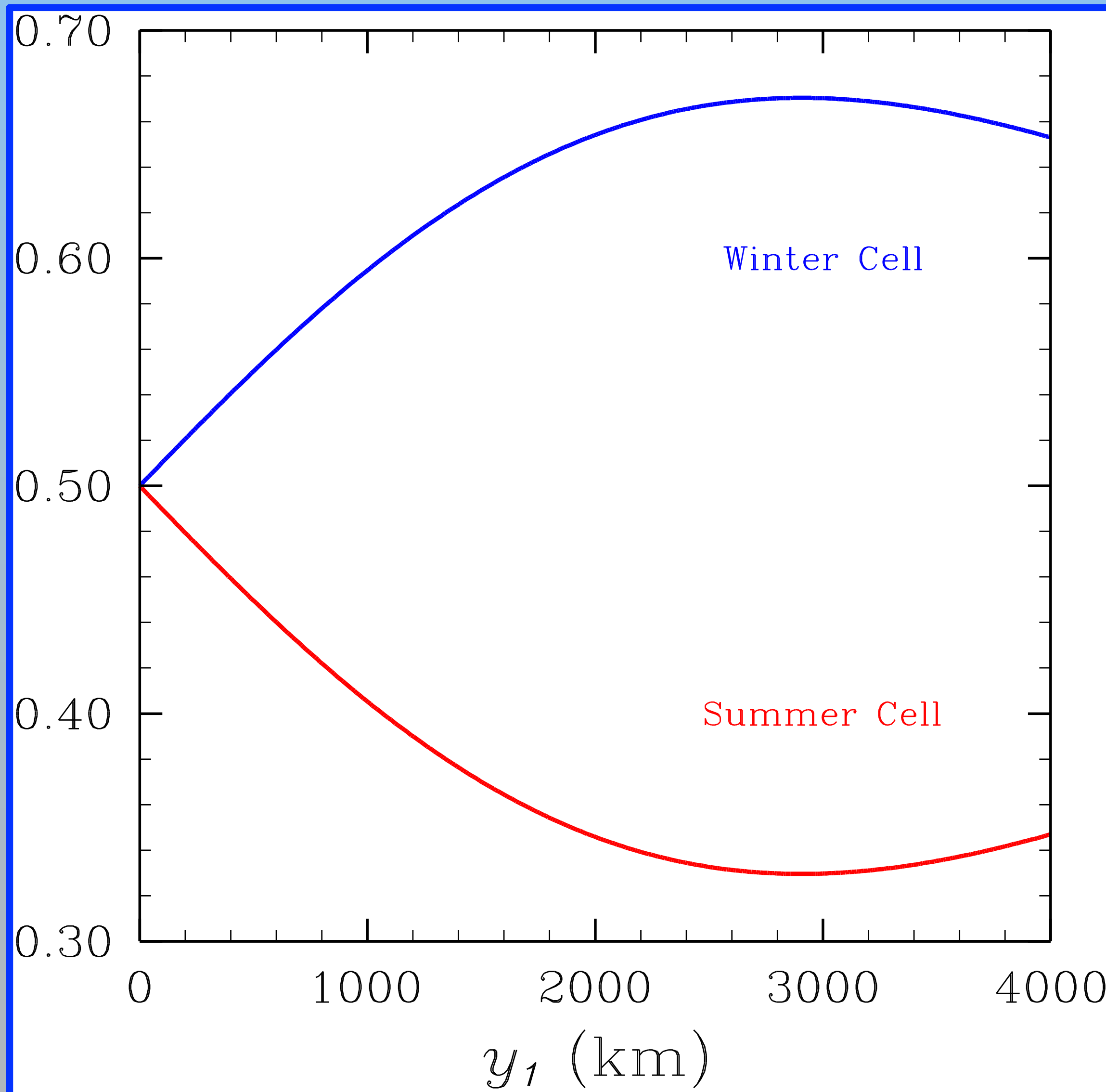


Streamfunction for the Shallow Circulation

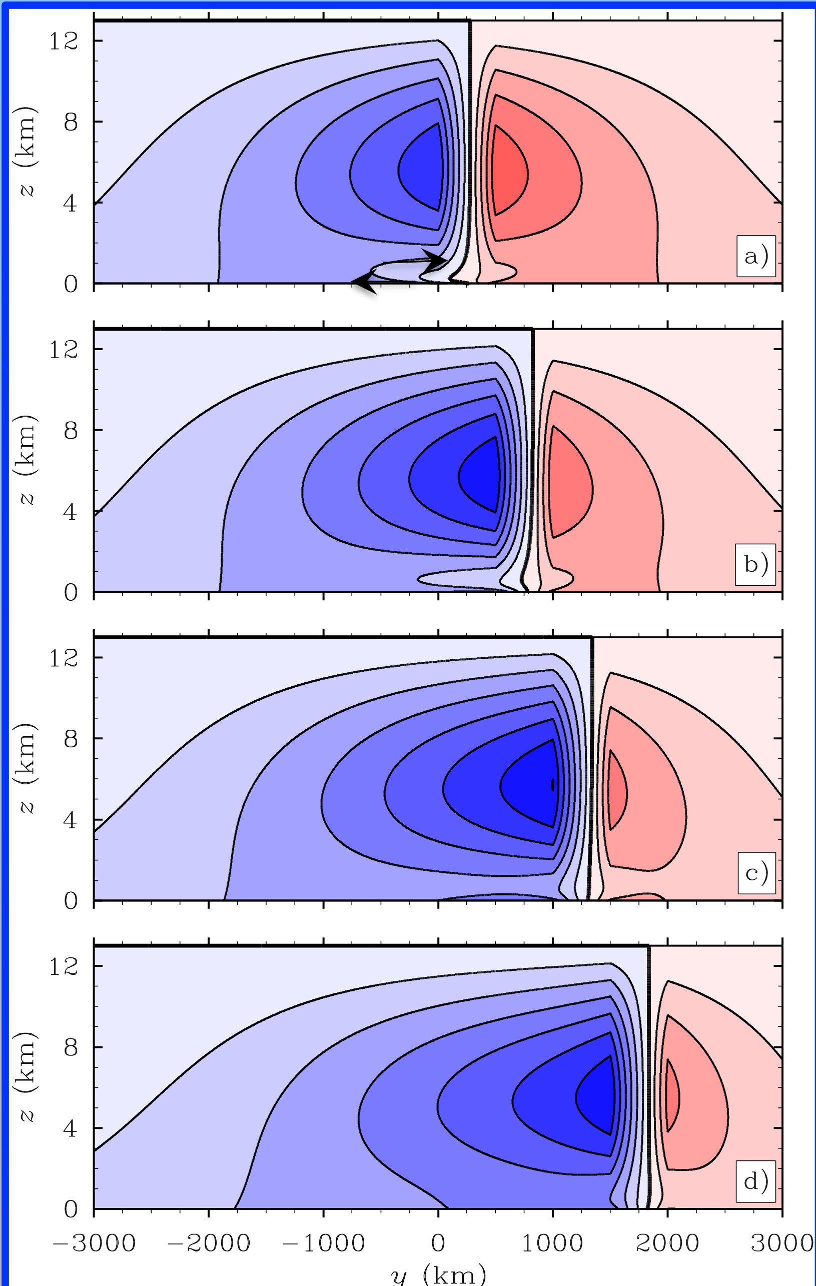


Asymmetric cells

Partition of Total ITCZ Mass Flux



Streamfunction for both cases



SMC patterns seem to be the most apparent when the ITCZ is close to the equator

Q_{max} = 5 K day^{-1}

$\mathcal{W}$ = 0.4 cms s^{-1}

One single deep cell

Meridional wind