Representation of topography by partial steps using immersed boundary method in VVM

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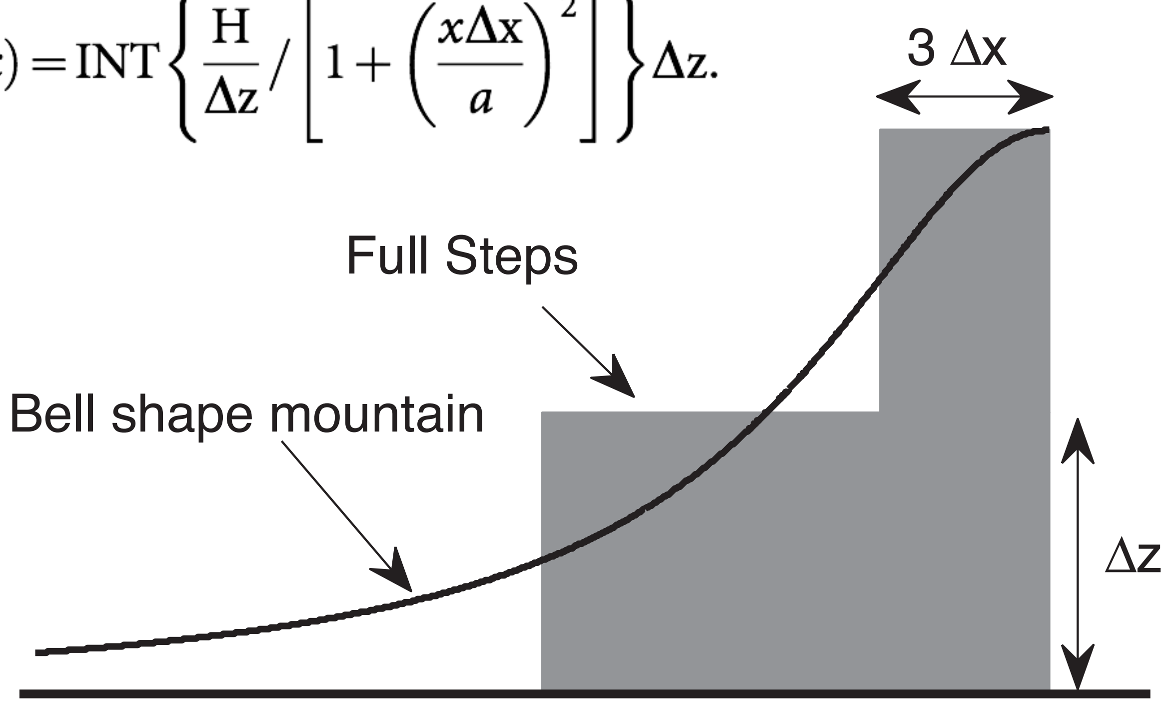
Topography in full step approach

- Representation of bell shape mountain in the full step approach is restricted to vertical grid size.

Bell shape mountain in full step approach

$$h(x) = \mathrm{INT}\left\{\frac{\mathrm{H}}{\Delta \mathrm{z}} \Big/ \left[1 + \left(\frac{x\Delta \mathrm{x}}{a}\right)^2\right]\right\}\Delta \mathrm{z}.$$



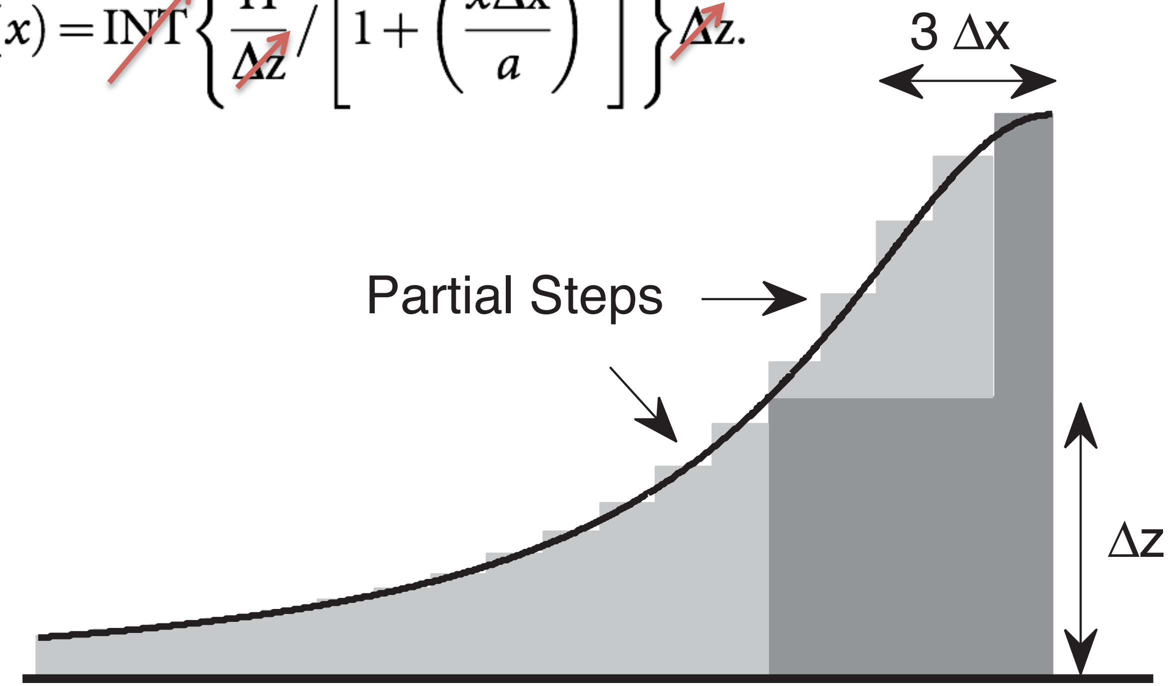
Wu and Arakawa 2011

Topography in partial step approach

- Partial steps mountain better capture topography effects with gentle slopes and micro mountains.

Bell shape mountain in ***partial*** step approach

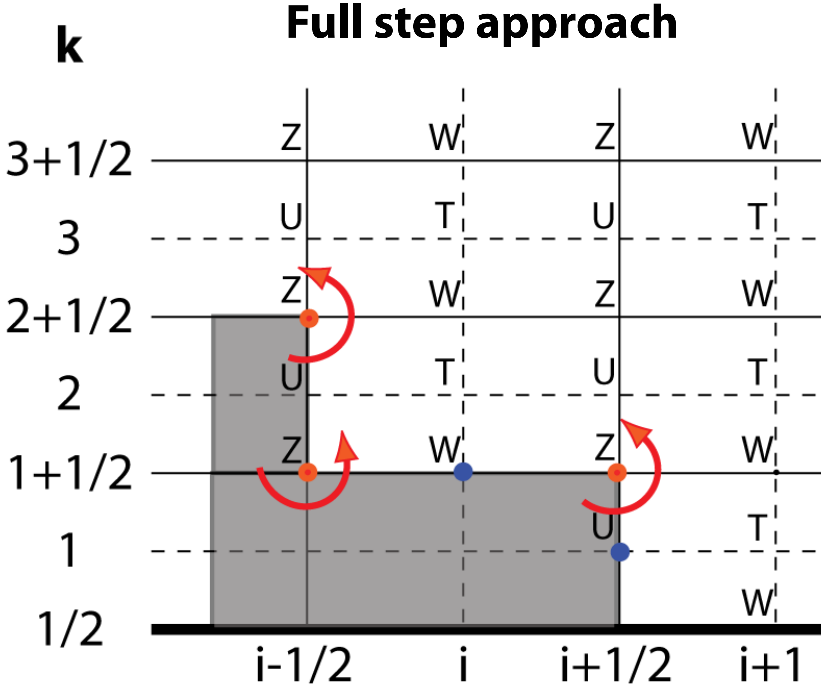
$$h(x) = \mathrm{INT}\left\{ \frac{\mathrm{H}}{\Delta z} / \left[1 + \left(\frac{x\Delta \mathrm{x}}{a} \right)^2 \right] \right\} \Delta z.$$



Inclusion of topography in VVM

•Topography effects are included with the point vorticities at the boundary (Wu and Arakawa 2011).

Full step approach

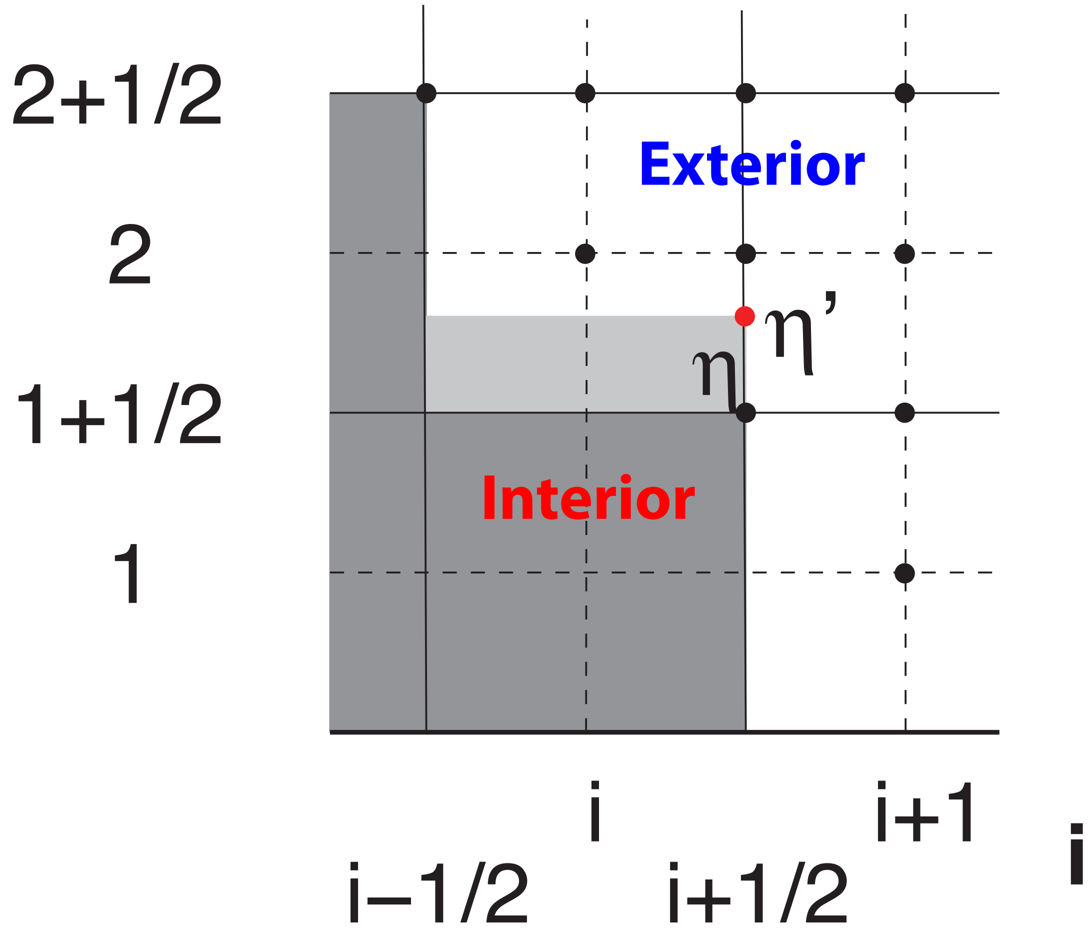


•The strength of the point vorticites is determined using its kinematic boundary condition.

$$\eta' = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \quad u' = w' = 0 \text{ at the boundary}$$

Topography in partial step approach

- Partial steps using immersed boundary method is done through interpolating the desired boundary conditions to the interior points.



$$\eta = D\ (\delta z, \eta')$$

η' :point vorticity

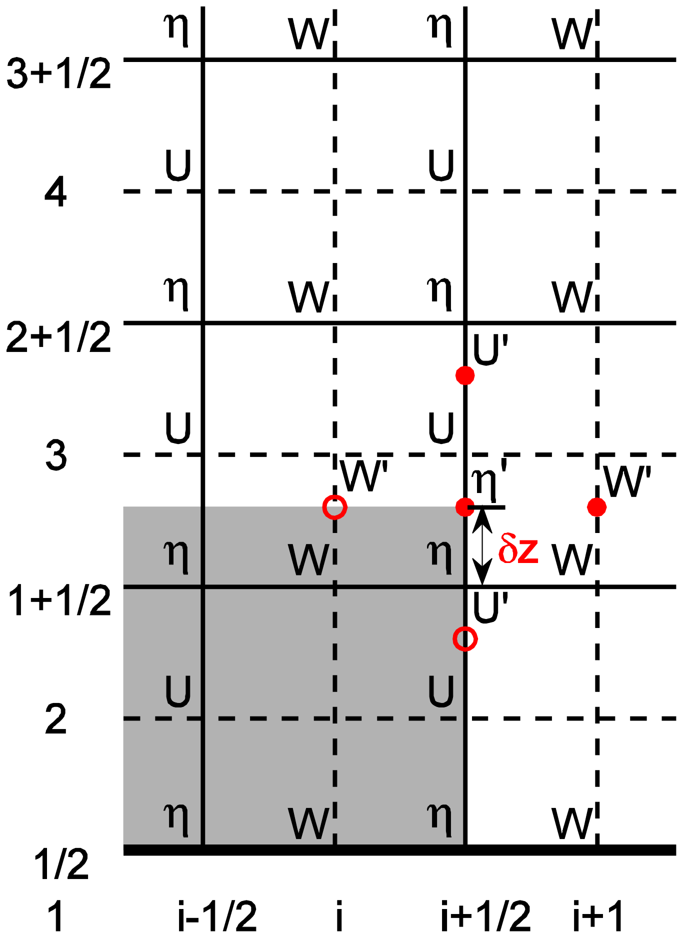
D :interpolation method

η :topography forcing

Procedures in partial step approach

- Vorticities at the boundary of topography is now determined through velocities related associated with the partial steps

Partial step approach



- The meridional vorticity

$$\eta'\Big|_{z+\delta z} = \frac{\partial W'}{\partial x} - \frac{\partial U'}{\partial z}$$

With δz above origin η point.

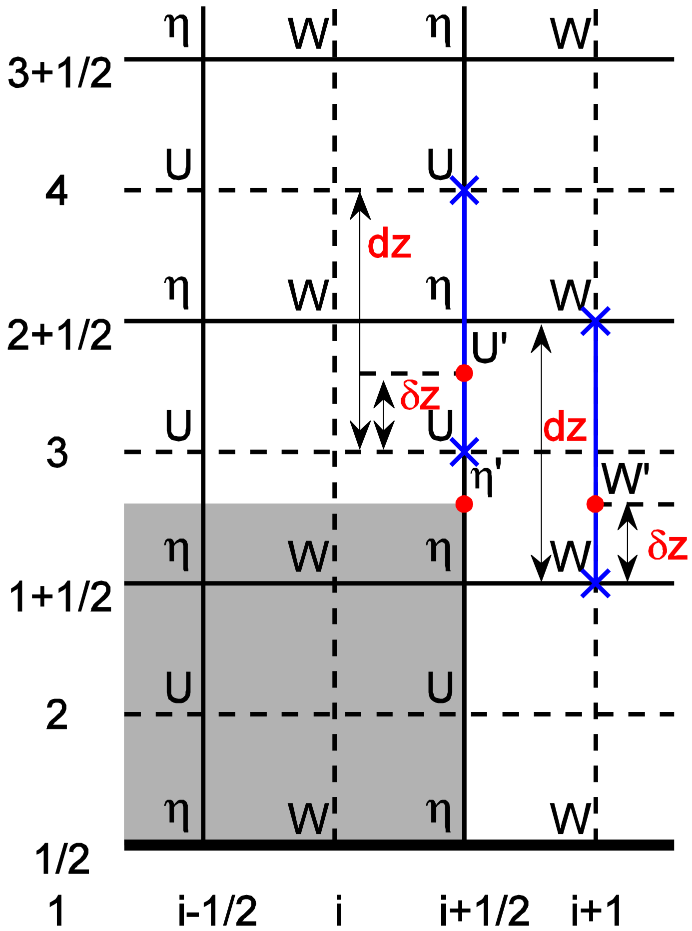
Where $W' = U' = 0$ at the physical boundary.

The U′ and W′ in the computational region apply linear interpolation.

Procedures in partial step approach

- The associated velocities is determined through linear interpolation of adjacent points in z direction.

Partial step approach



- Let $\alpha = \delta z / dz$

$$U' = interp(U_3, U_4) = (1-\alpha)U_3 + \alpha U_4$$

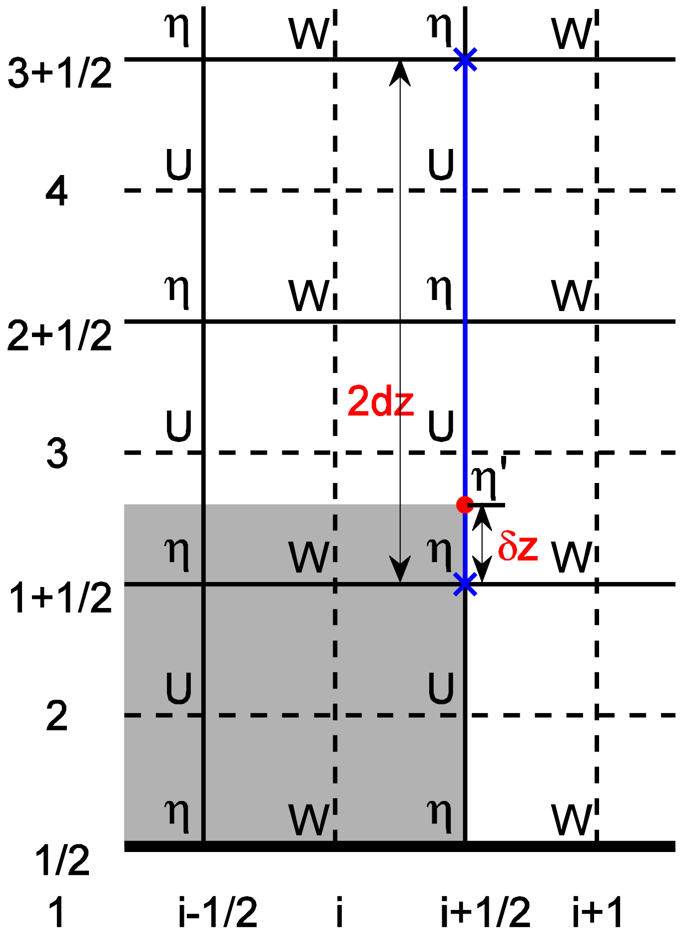
$$W' = interp\left(W_{1+1/2}, W_{2+1/2}\right) = (1-\alpha)W_{1+1/2} + \alpha W_{2+1/2}$$

$$\eta' = \frac{W'}{dx} - \frac{U'}{dz}$$

Procedures in partial step approach

- Vorticities used for the topography forcing is determined through extrapolation of two points above.

Partial step approach



- Apply $\eta_{3+1/2}$ and η' to extrapolate the $\eta_{1+1/2}$

Let $\beta = \delta z / 2dz$

$$\eta' = (1-\beta)\eta_{1+1/2} + \beta\eta_{3+1/2}$$

$$\eta_{1+1/2} = \eta' - \beta\eta_{3+1/2}/(1-\beta)$$

Procedures in partial step approach

- Velocity fields are obtained by solving the w-equation with added topography forcing at the boundary

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)w+\frac{\partial}{\partial z}\left[\frac{1}{\rho_0}\left(\frac{\partial}{\partial z}\rho_0 w\right)\right]=-\underline{\frac{\partial\eta}{\partial x}+\frac{\partial\xi}{\partial y}}$$

- The w-equation is then solved through Portable Extensible Toolkit for Scientific Computing (PETSc) for better efficiency in parallel codes.

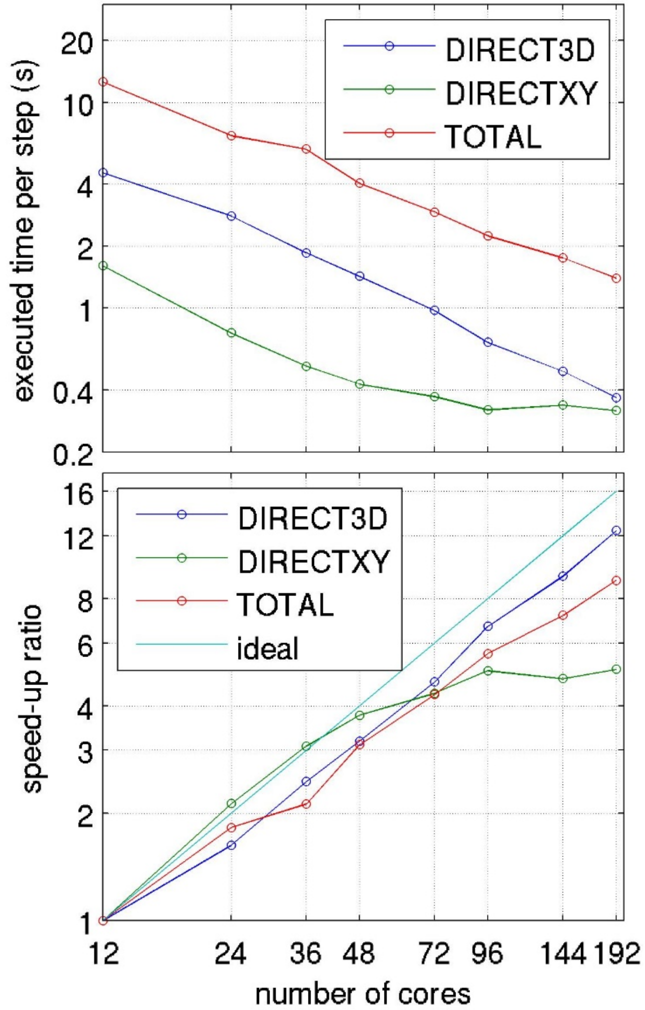
Portable Extensible Toolkit for Scientific Computing (PETSc)

http://www.mcs.anl.gov/petsc/index.html

A suite of routines for the parallel solution of partial differential equations

- System Size
  - Can afford 500B unknowns with Jaguar (225k cores)
  - 8M grid with 300+core for VVM
- Application in VVM
  - Conjugate gradient (CG)
  - Symmetric Successive Over Relaxation (SSOR)
  - Fitted for different shape of rectangle domain

Free to everyone (BSD-style license), open development



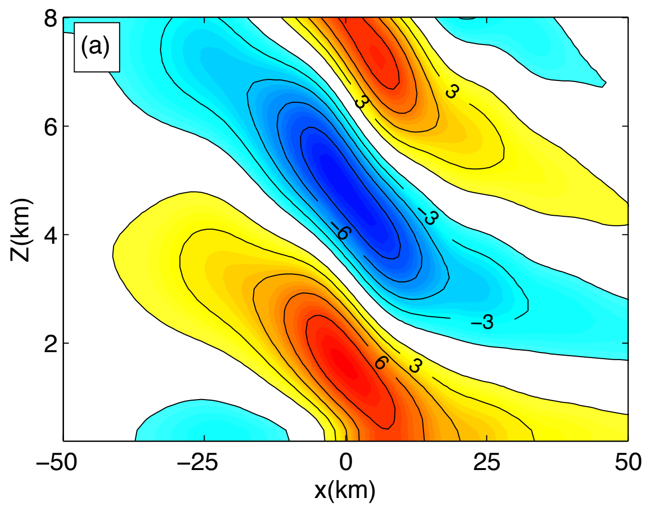
Speed-up test for VVM with 720x720x34 grid

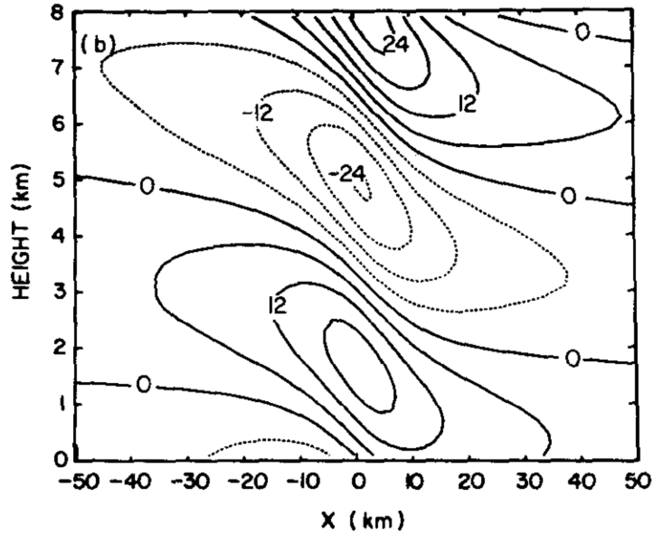
Results

Flow over 1 m bell-shaped mountain

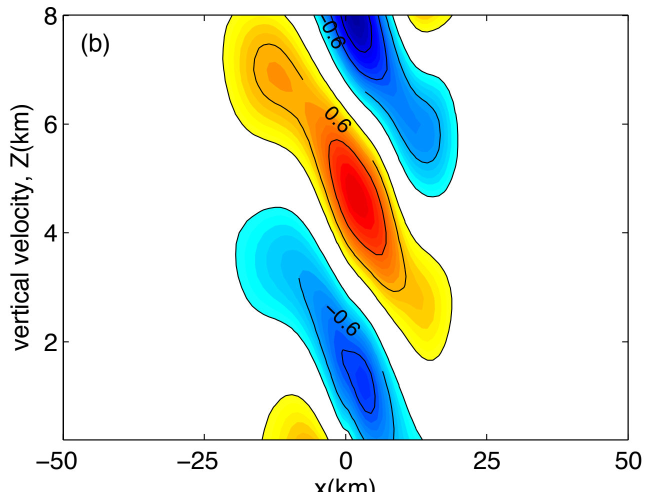
- The partial step approach produces reasonable shape with weaker amplitude compared with the analytical solution

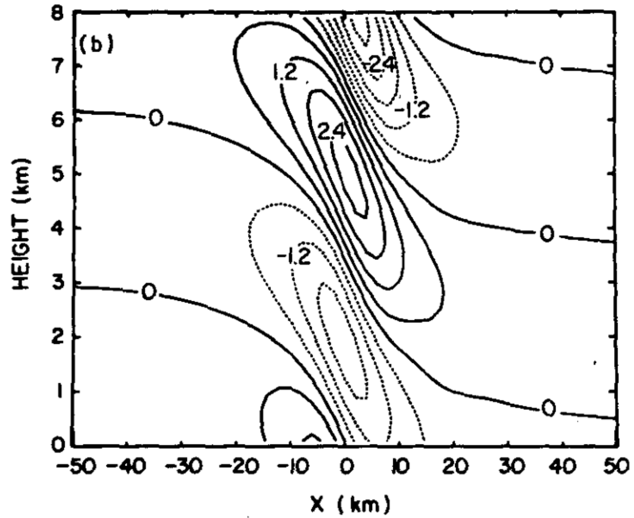
Horizontal velocity
[1e-3 m/s]





Vertical velocity
[1e-3 m/s]





U=20m/s, a=10km, h=1m,T=250k

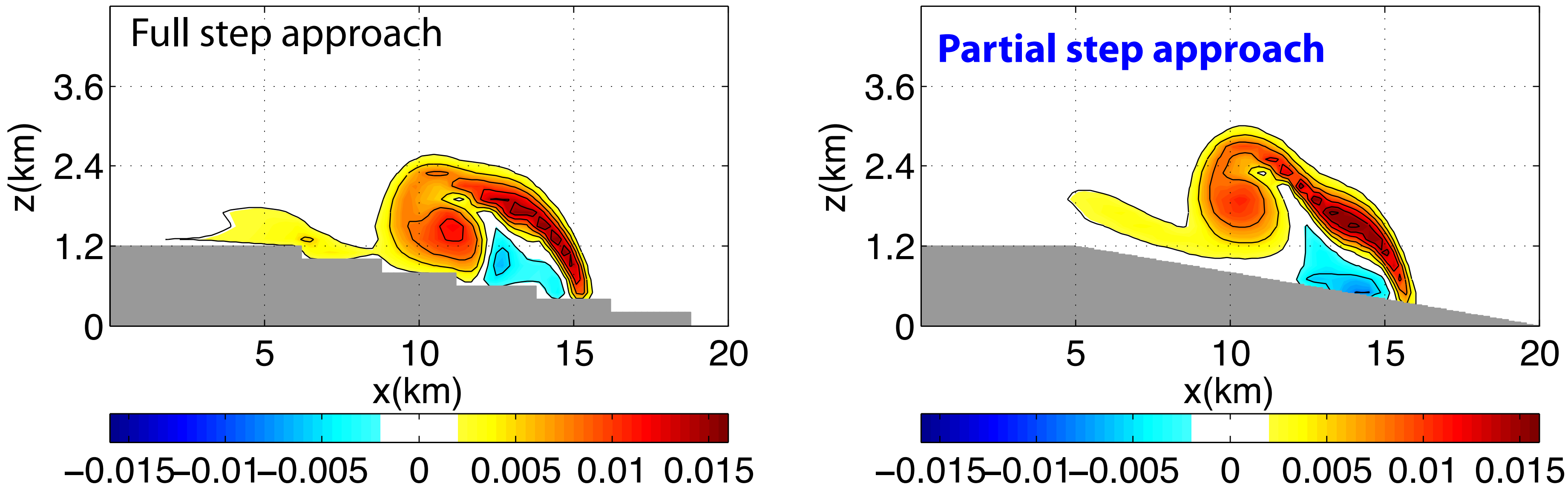
Durran and Klemp 1983

Results

Cold bubble over a gentle slope

- Vorticity in the partial step approach is smoother near surface due to continuous topography forcing along the slope compared to stair-like topography forcing in full step approach.

Horizontal vorticity (1/s)

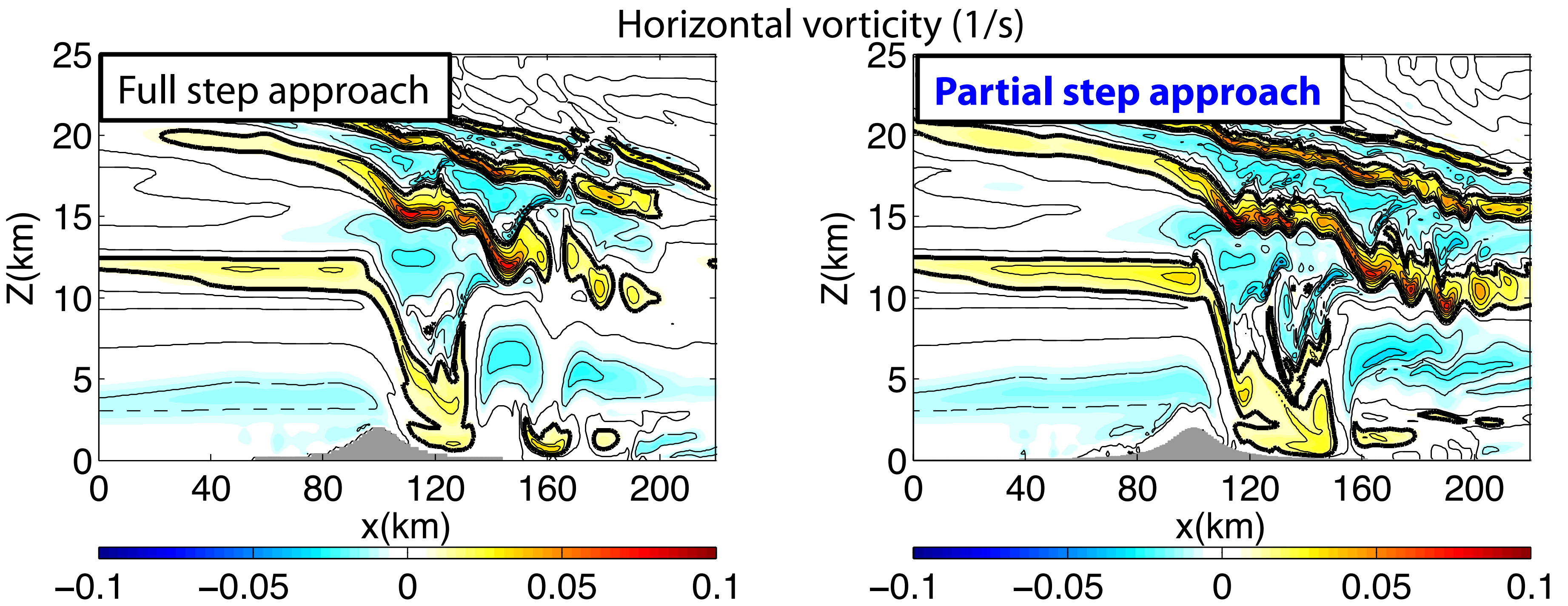


Cold bubble: -1K initially
Slope: 4.6 degree
Theta: 300K

Results

Boulder downslope windstorm

- The partial step approach captures the important features of the downslope wind storm.



Summary and future work

- The partial step approach in VVM better captures the topography effects in gentle slope and micro mountains due to additional consideration of topography.
- W-equation in VVM is now solved using PETSc with iterative methods (CG with SSOR preconditioning).
- A fully immersed boundary approach requires a better interpolation approach and more point vortices used.