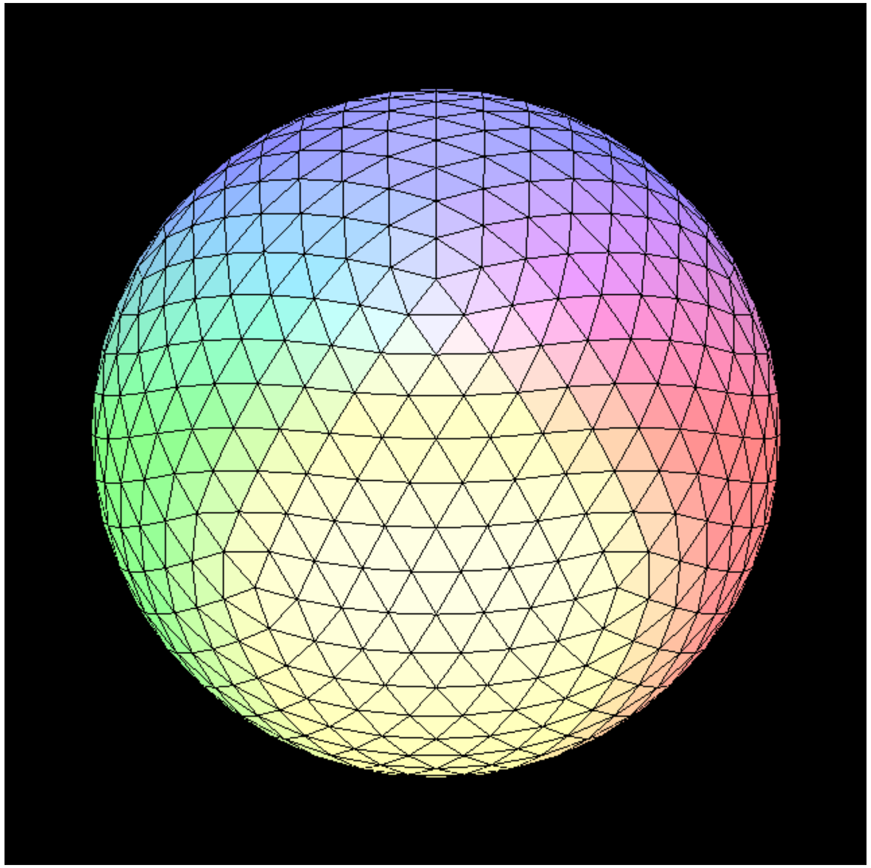
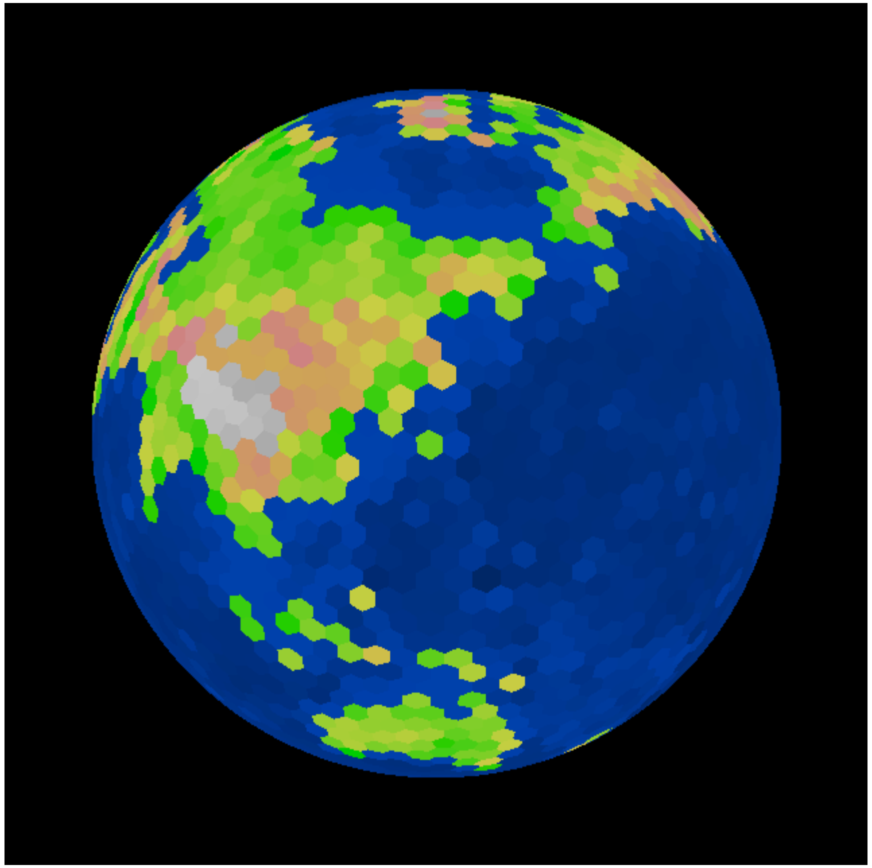
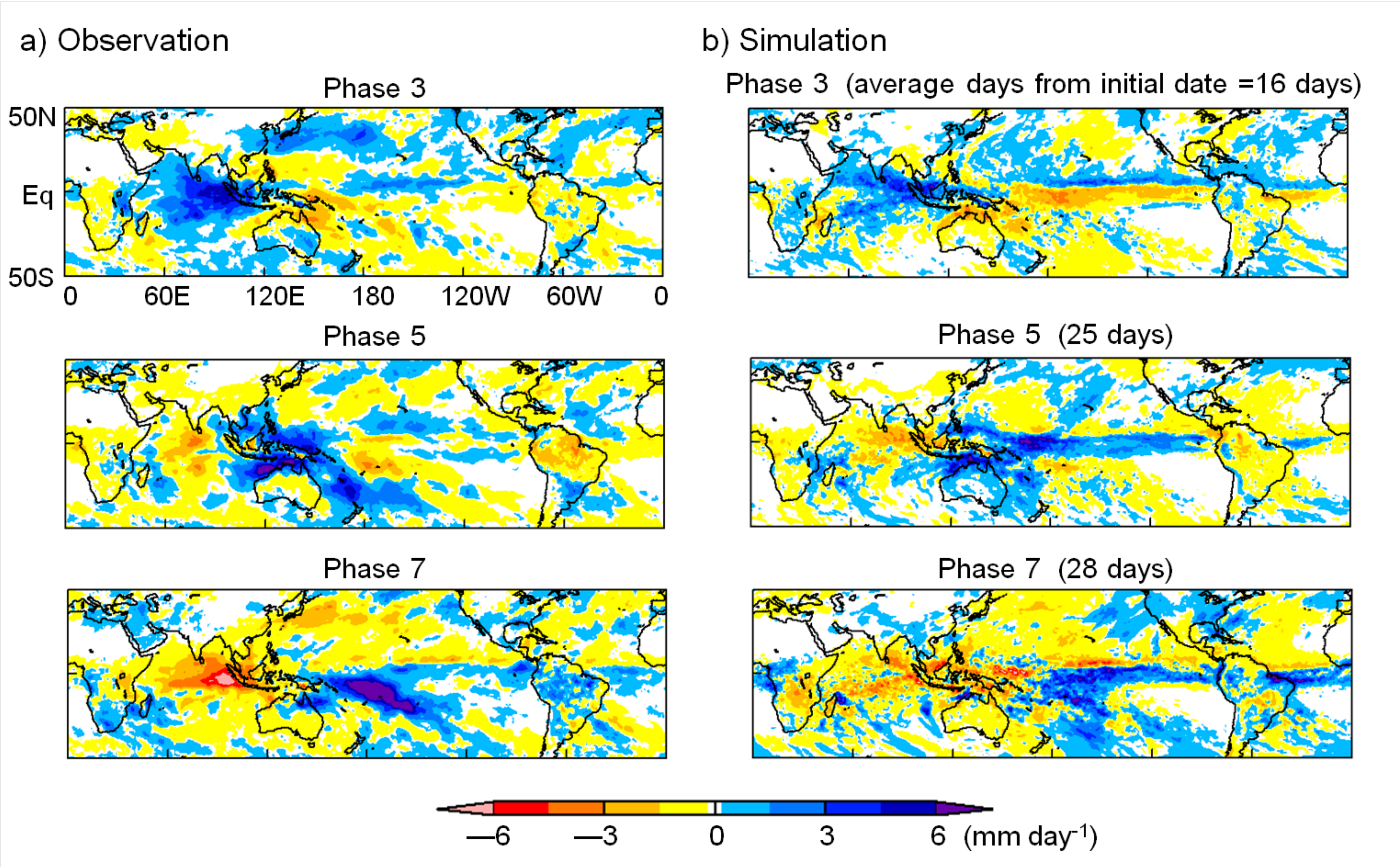
Recent progress in the icosahedral dynamical core development in Japan

Hiroaki Miura (University of Tokyo)

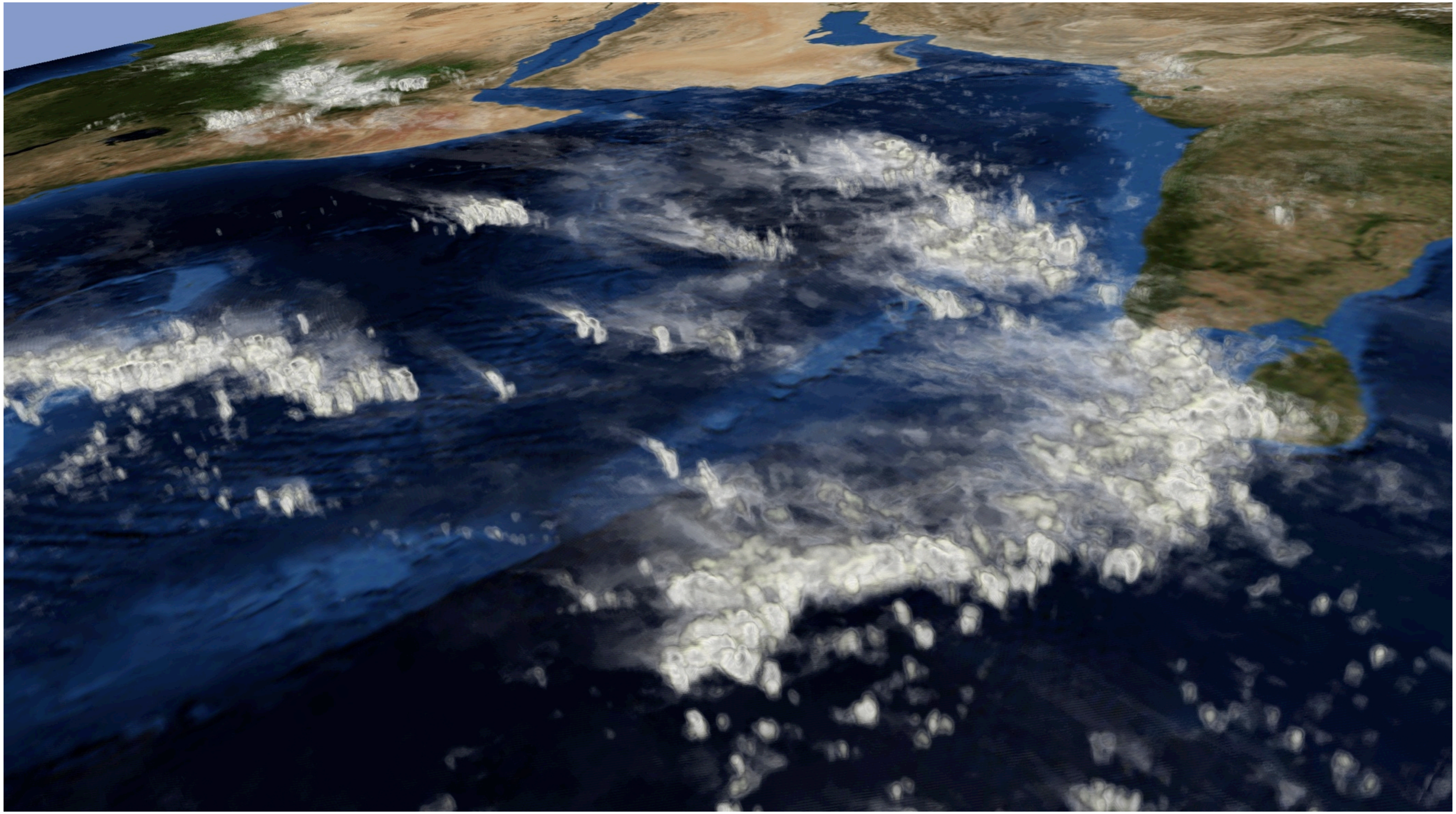



Nonhydrostatic Icosahedral Atmosphere Model (NICAM)



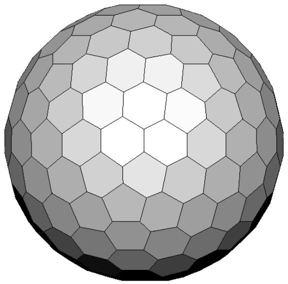
(Miyakawa et al. 2014, *Nature Comm.*)

Nonhydrostatic Icosahedral Atmosphere Model (NICAM)

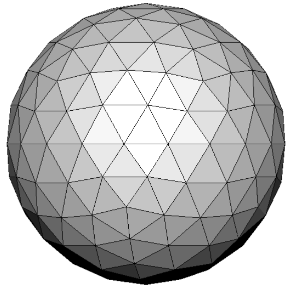


$\Delta x \sim 870$ m

(Miyamoto et al. 2013, *Geophys. Res. Lett.*)



No demand?



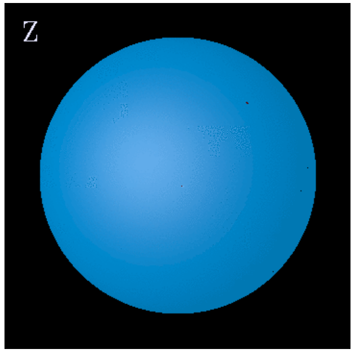
Is NICAM sufficiently good?

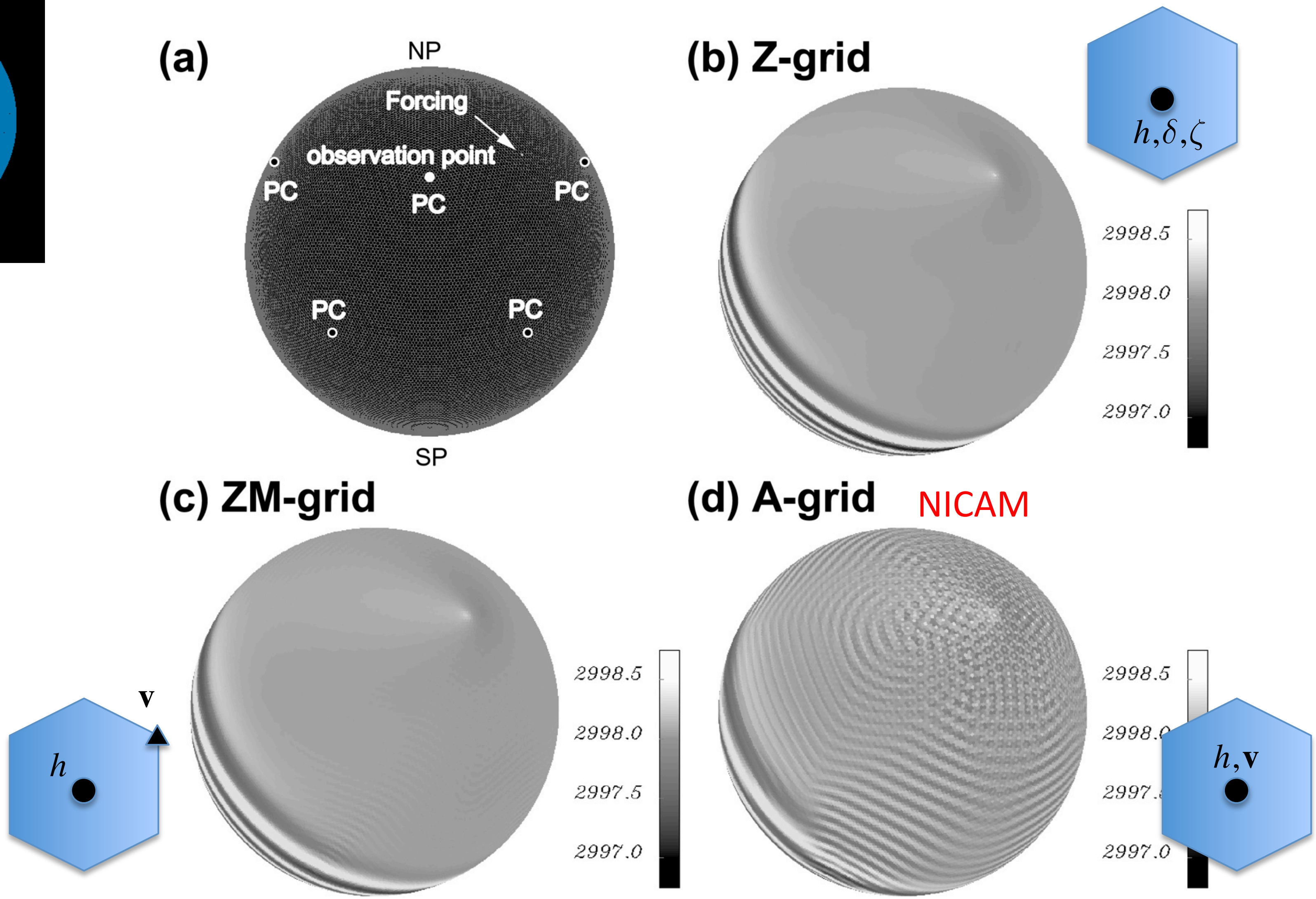
- I think so.
- Nothing is better than a working model!

But...

- A-grid
- Second-order centered transport
- Terrain-following coordinate

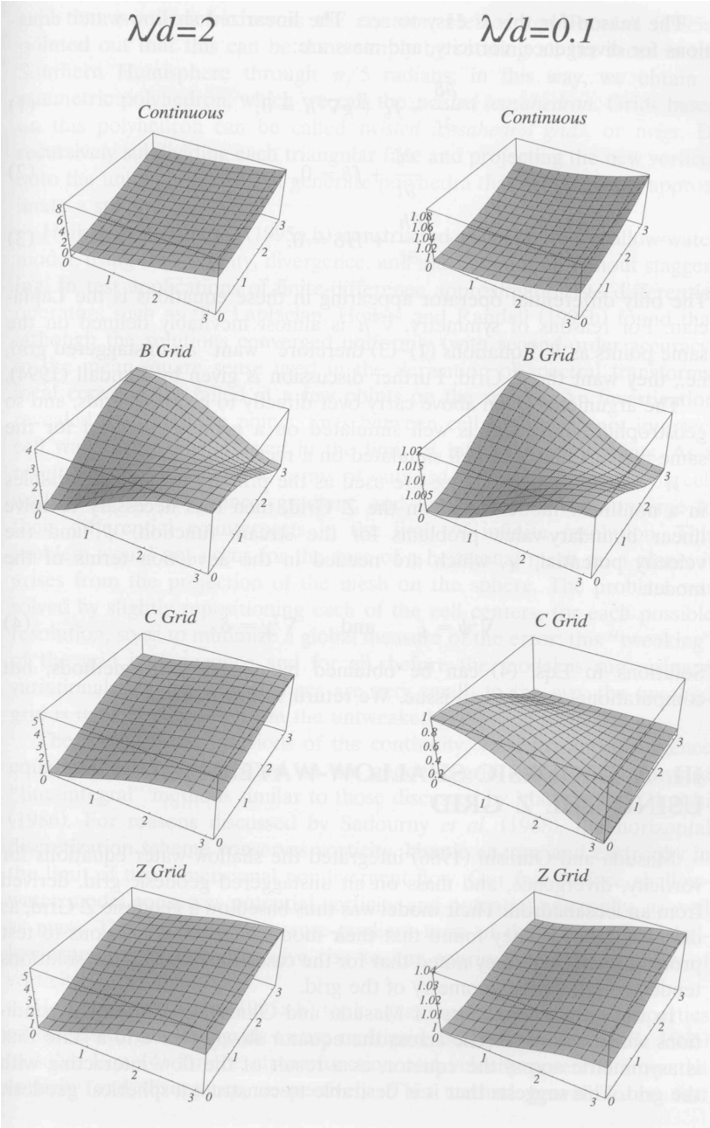
Arrangement of variables

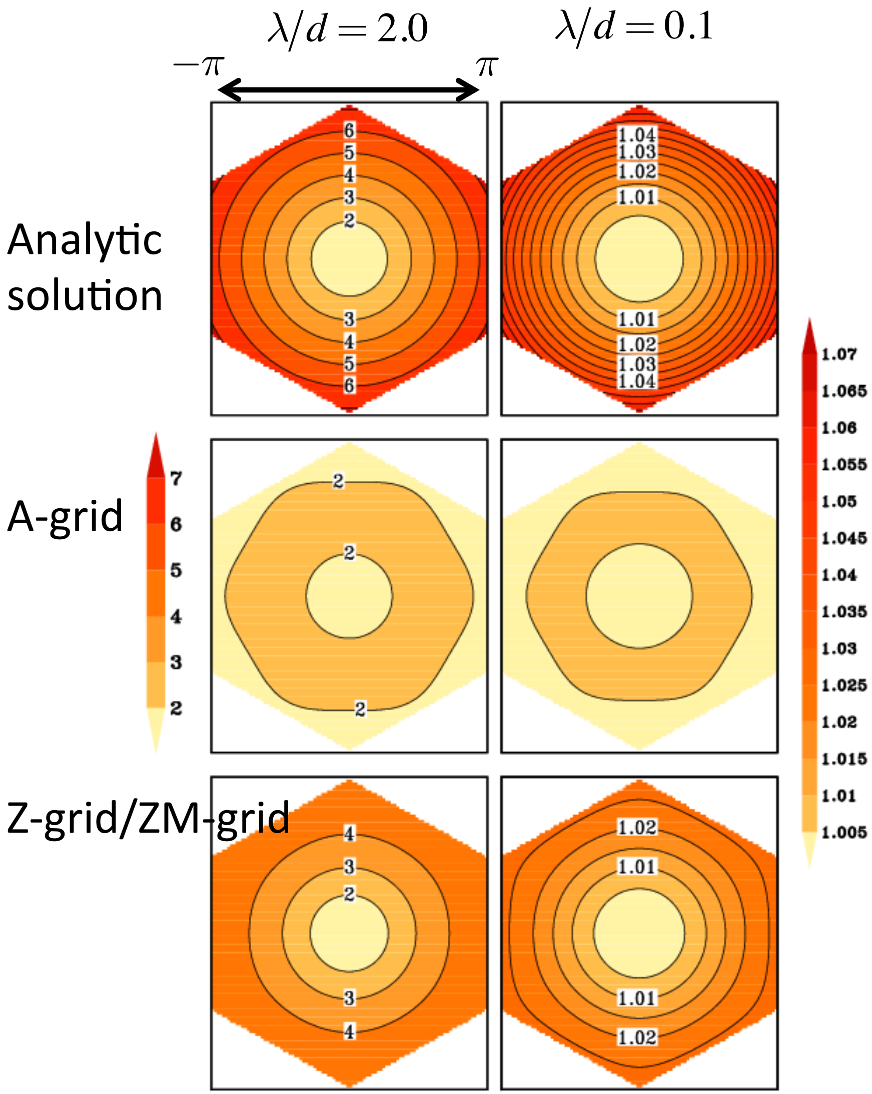




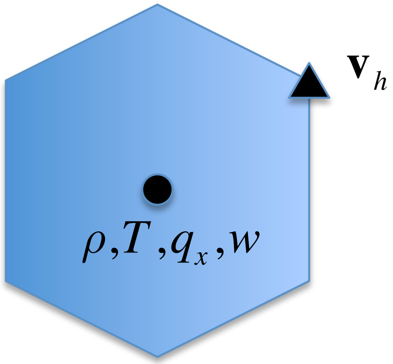
Dispersion relations of inertial-gravity waves

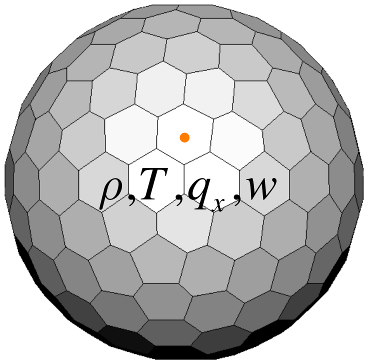
Randall (1994) : squared grid

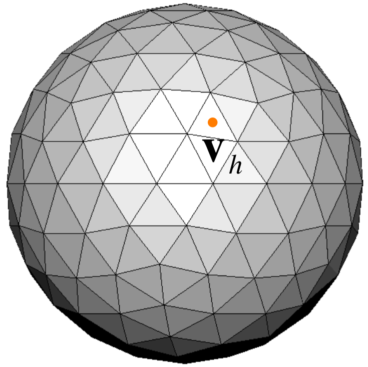




B(or ZM)-grid







Ringler and Randall (2002a,2002b)

- Good dispersion relation of gravity waves
- N_{tri} is almost twice as many as N_{hex}.

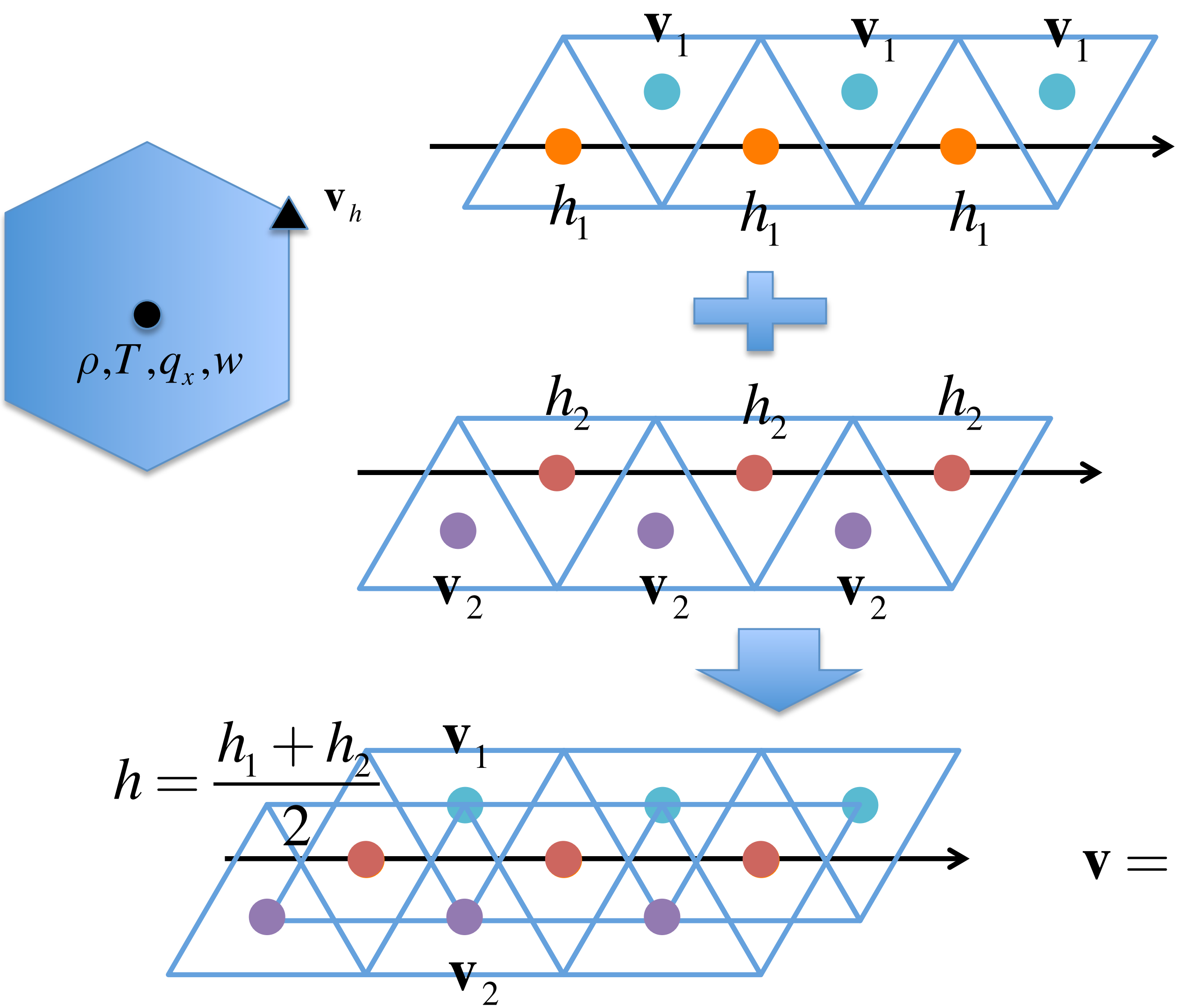
→ Computational mode in horizontal velocity

No working model on the sphere.

I developed a model, but there were several serious problems.

Flux-divergence and Gradient operators

B(or ZM)-grid: computational mode



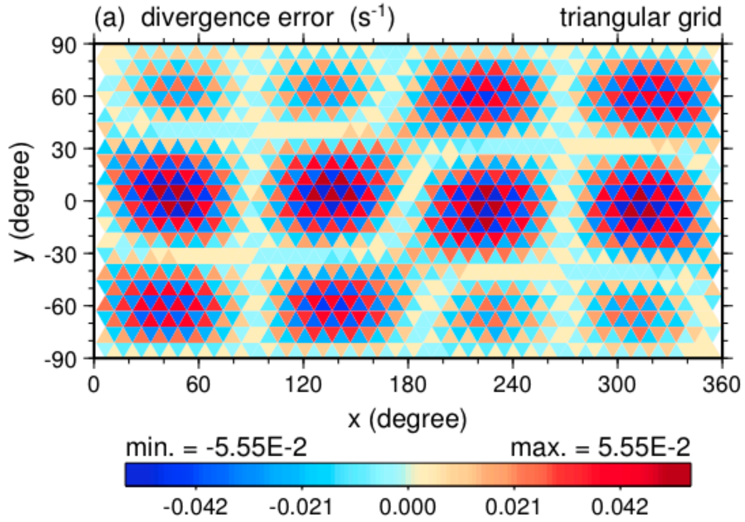
$$\mathbf{v} = \frac{(\mathbf{v} + \delta\mathbf{v}) + (\mathbf{v} - \delta\mathbf{v})}{2}$$

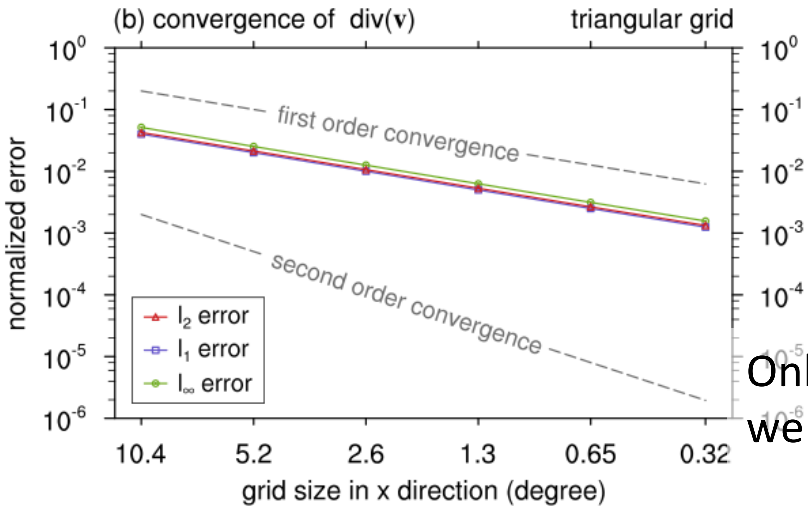
- C-grid like for divergent motions
- Degeneracy of mass points

  （Blue and purple modes are independent.）

Wan et al. (2013): ICON-1.2 (MPI)

Error of divergence





Only first-order schemes were available.

Walko and Avissar (2008)

A transport test

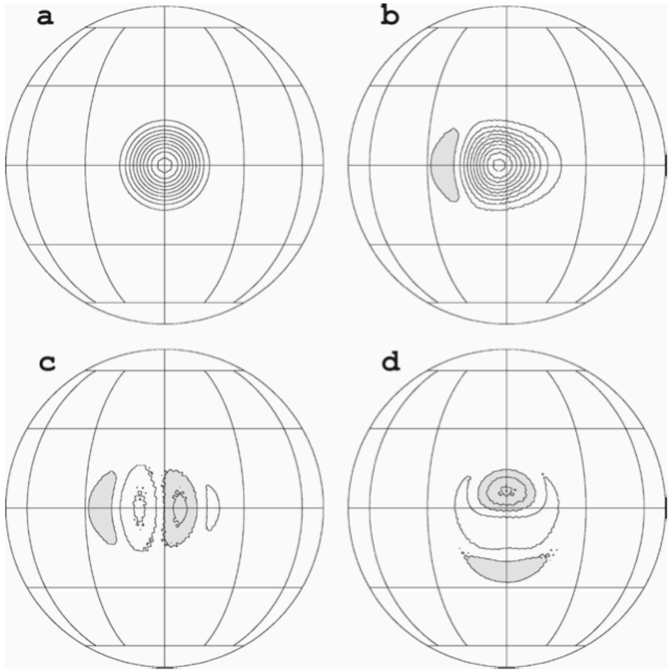
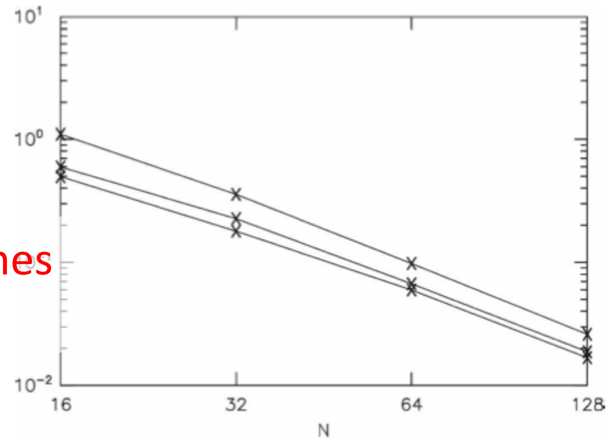
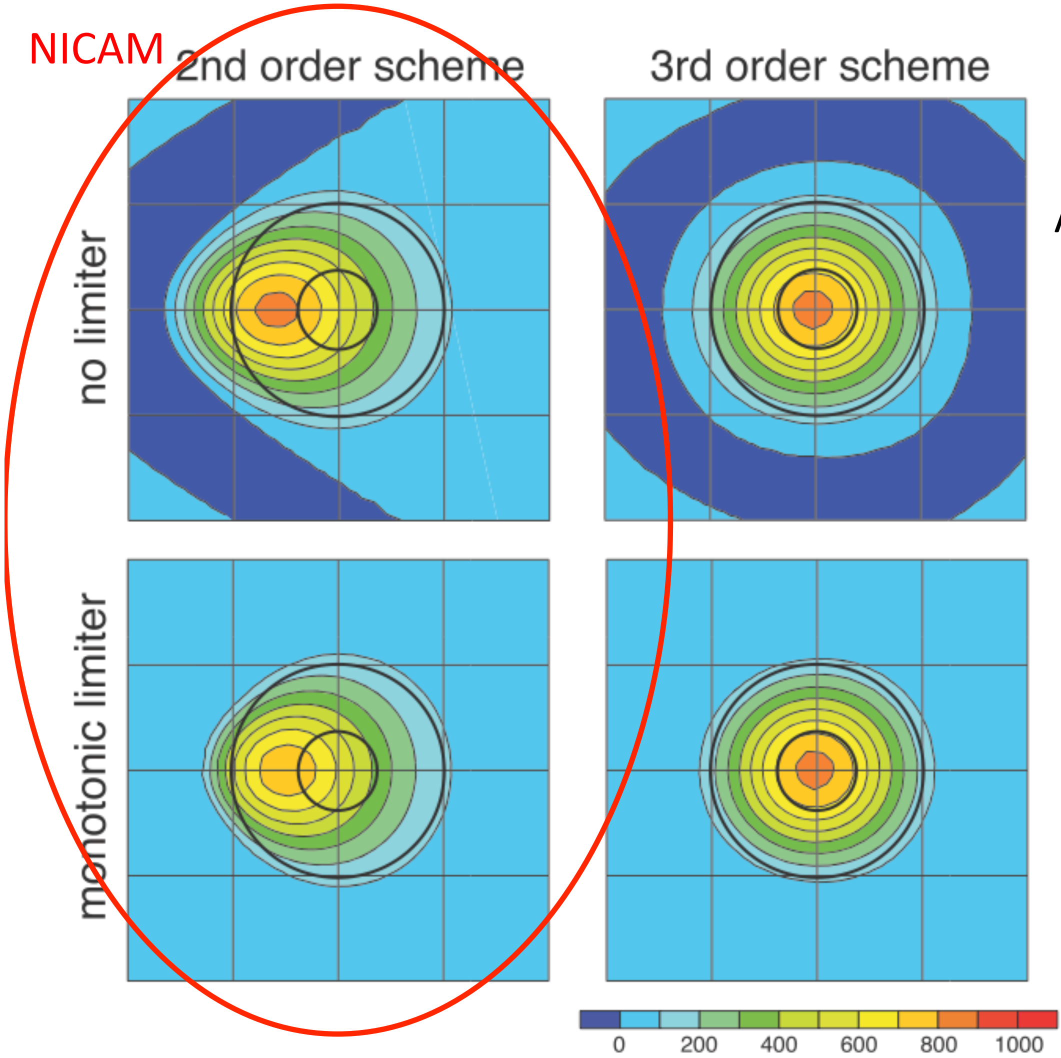


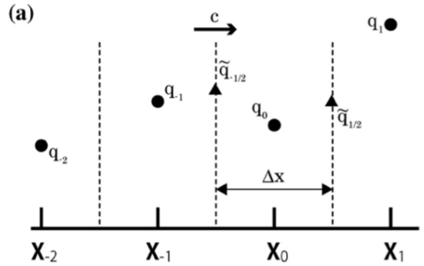
FIG. 4. Results for test case 1: (a) the initial scalar field, (b) the scalar field after 12 days for $\alpha = 0$, (c) the error field for $\alpha = 0$, and (d) the error field for $\alpha = \pi/2$. Contour levels for (a)–(d) are −0.05, 0.05, 0.15, etc., and values less than −0.05 are shaded.



Transport scheme (Hex)

Skamarock and Gassmann (2011)





Advective form (third-order)

$$-c\left.\frac{\partial q}{\partial x}\right|_{x_0} = -c\frac{2q_1+3q_0-6q_{-1}+q_{-2}}{6\Delta x}+O\left(\Delta x^3\right)$$

Flux form (third-order)

$$-c\left.\frac{\partial q}{\partial x}\right|_{x_0} = -\frac{c\tilde{q}_{1/2}-c\tilde{q}_{-1/2}}{\Delta x}+O\left(\Delta x^3\right)$$

$$\tilde{q}_{1/2} = \frac{2q_1+5q_0-q_{-1}}{6}$$

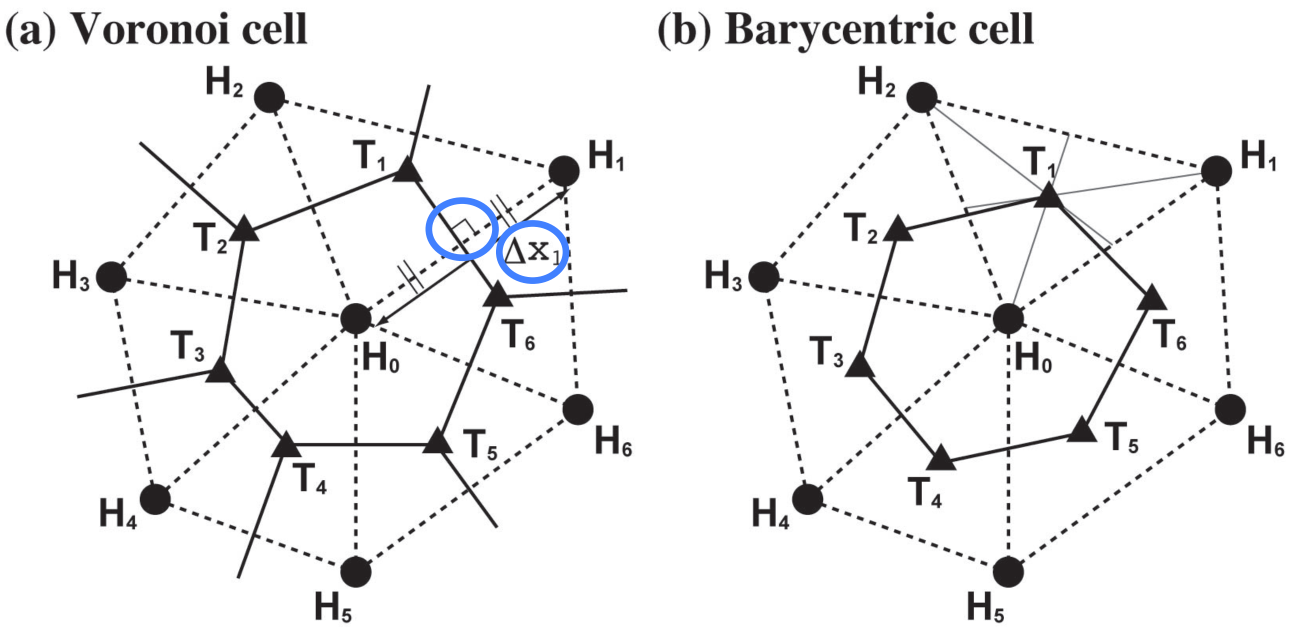
$$\tilde{q}_{1/2} = \frac{q_1+q_0}{2}-\frac{1}{12}(1+\beta)\Delta x^2\frac{q_1-2q_0+q_{-1}}{\Delta x^2}$$

$$\left.\frac{\partial^2 q}{\partial x^2}\right|_{x_0} = \frac{q_1-2q_0+q_{-1}}{\Delta x^2}+O\left(\Delta x^2\right)$$

A length scale

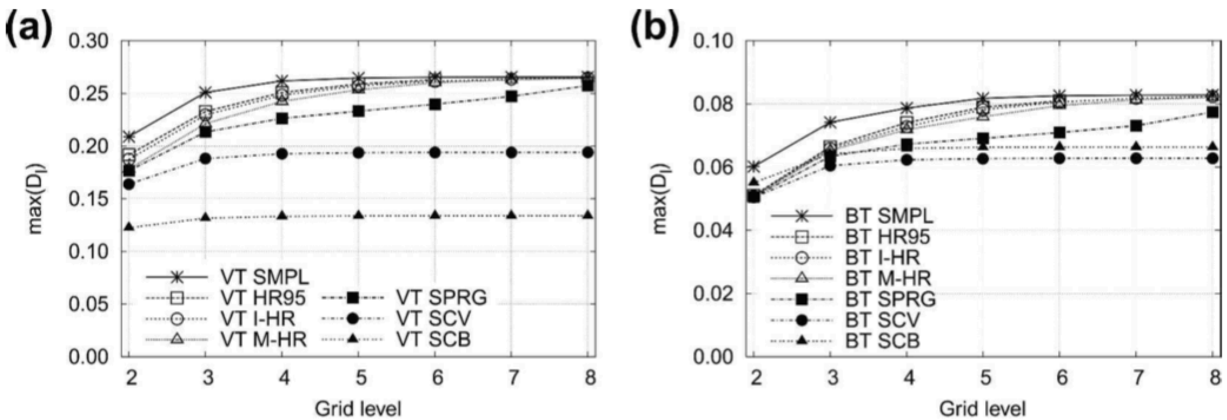
$$\tilde{q}_{1/2} = \frac{q_1+q_0}{2}-\frac{1}{12}(1+\beta)\Delta x^2\left.\frac{\partial^2 q}{\partial x^2}\right|_{x_0}$$

<u>Skamarock and Gassmann scheme is not usable in NICAM</u>



NCAR etc.

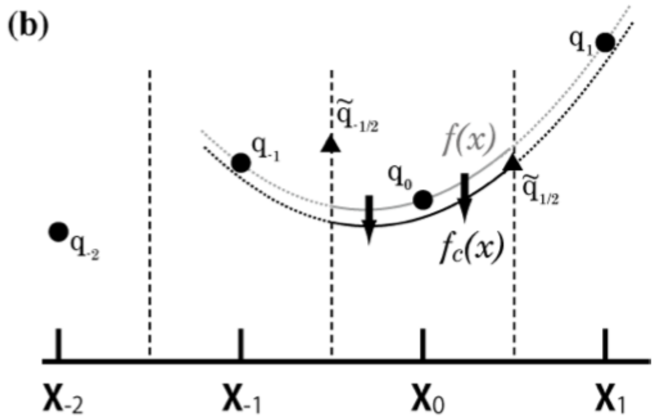
NICAM



Miura and Kimoto (2005)

- Less distorted
- No orthogonality → Δx ?

New algorithm (1D example)



A quadratic profile: $f(x) = a_0 + a_1 x + a_2 x^2$

$$f(-\Delta x) = q_{-1},\ f(0) = q_0 \text{ and } f(\Delta x) = q_1$$

$$f(x) = q_0 + \frac{q_1 - q_{-1}}{2\Delta x} x + \frac{q_1 - 2q_0 + q_{-1}}{2\Delta x^2} x^2$$

<u>This does not recover the third-order scheme.</u>

$$\tilde{q}_{1/2} = f(\Delta x/2) = \frac{3q_1 + 6q_0 - q_{-1}}{8}$$

The quadratic profile is shifted: $f_c(x) = f(x) - a_0'$

The shift is determined to satisfy $\int_{-\Delta x/2}^{\Delta x/2} f_c(x) dx = q_0 \Delta x$. Then,

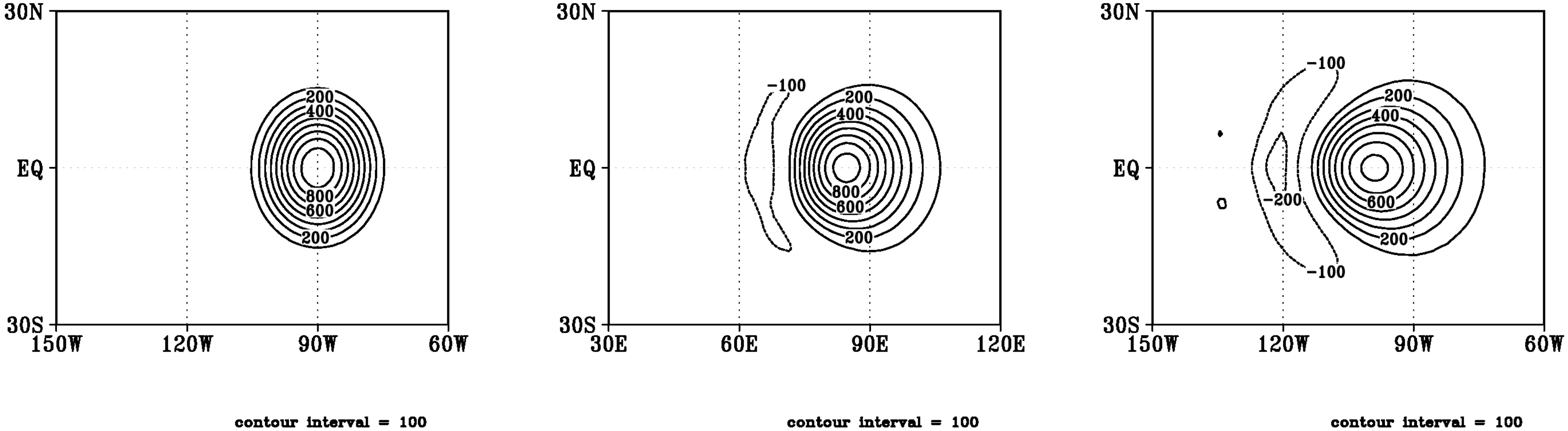
$$f_c(x) = q_0 - \frac{q_1 - 2q_0 + q_{-1}}{24} + \frac{q_1 - q_{-1}}{2\Delta x} x + \frac{q_1 - 2q_0 + q_{-1}}{2\Delta x^2} x^2$$

<u>This shifted profile recovers the third-order scheme.</u>

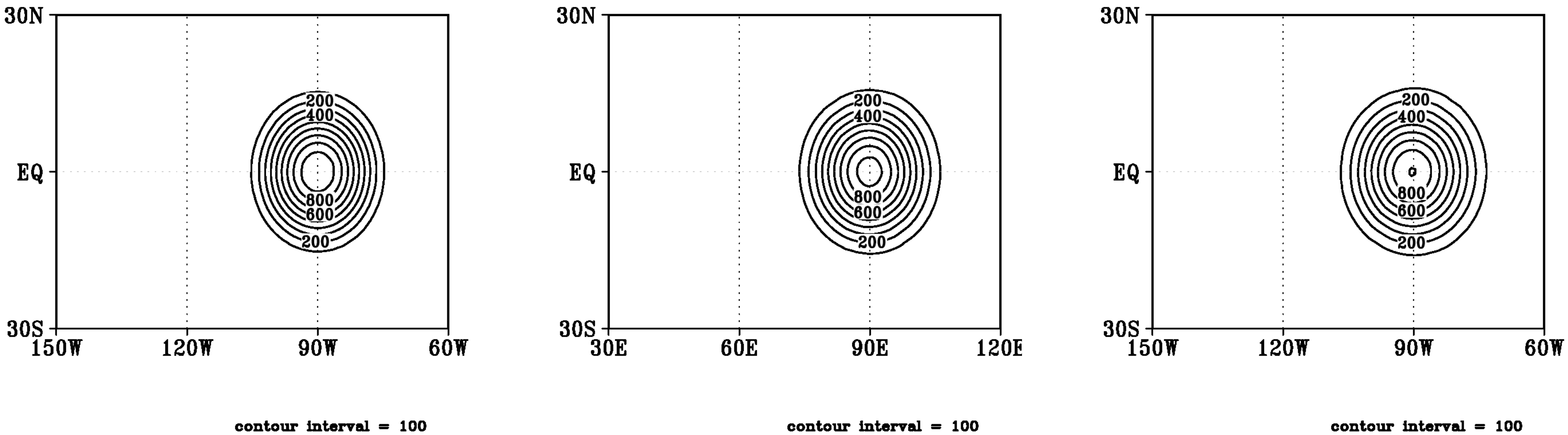
$$\tilde{q}_{1/2} = f_c(\Delta x/2) = \frac{2q_1 + 5q_0 - q_{-1}}{6}$$

Williamson et al. (1992), test case-1

2nd-order centered scheme



3rd-order upwind-biased scheme

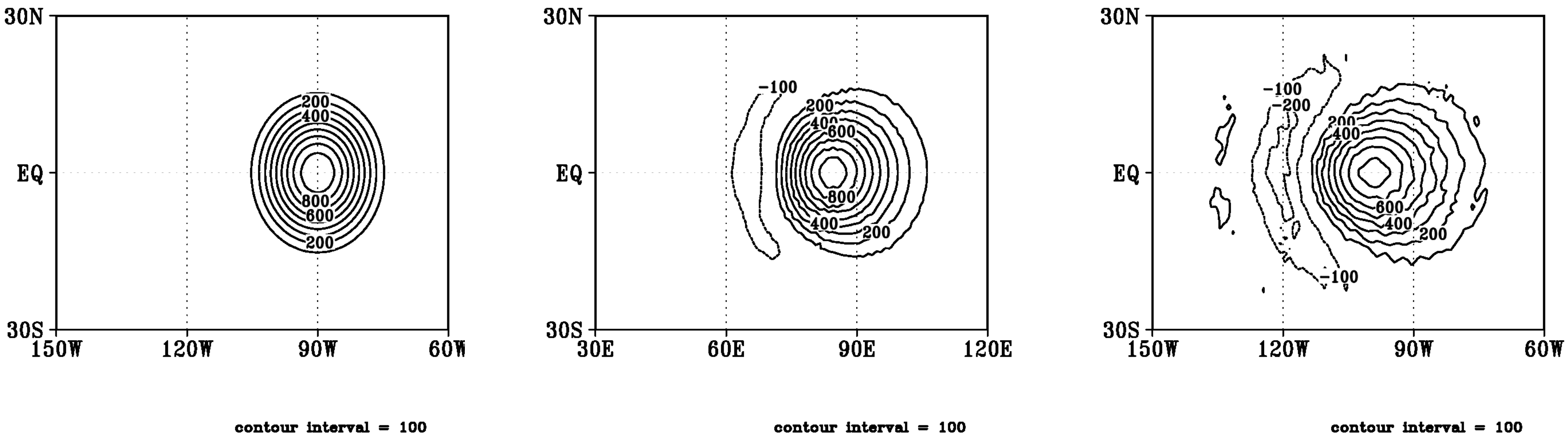


Miura (2013, Mon. Wea. Rev.)

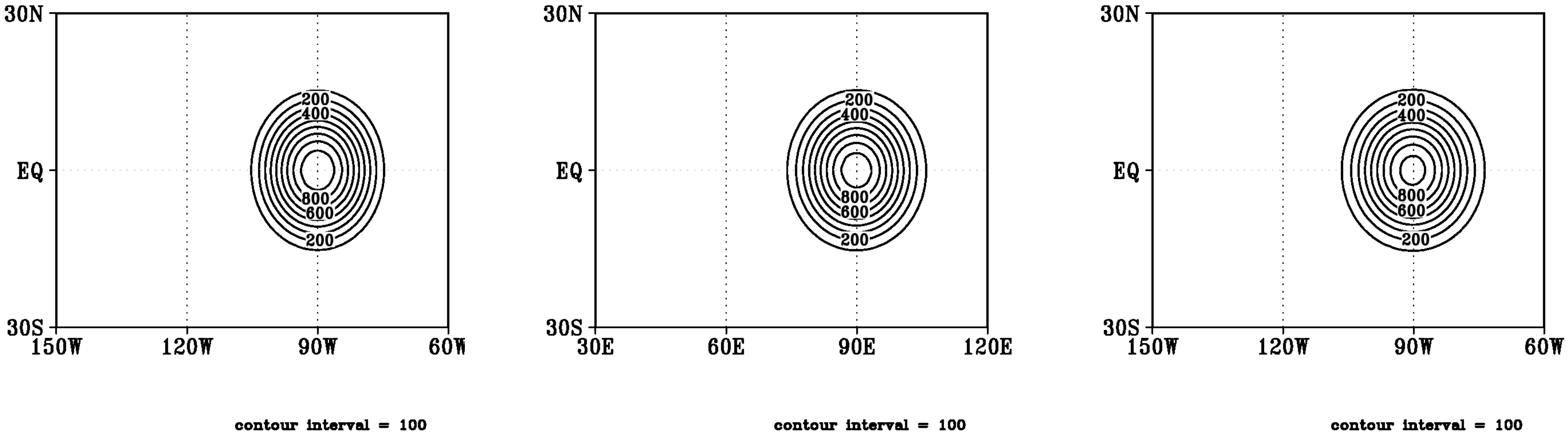
New algorithm is also applicable to the triangular mesh.

triangular mesh

2nd-order centered scheme

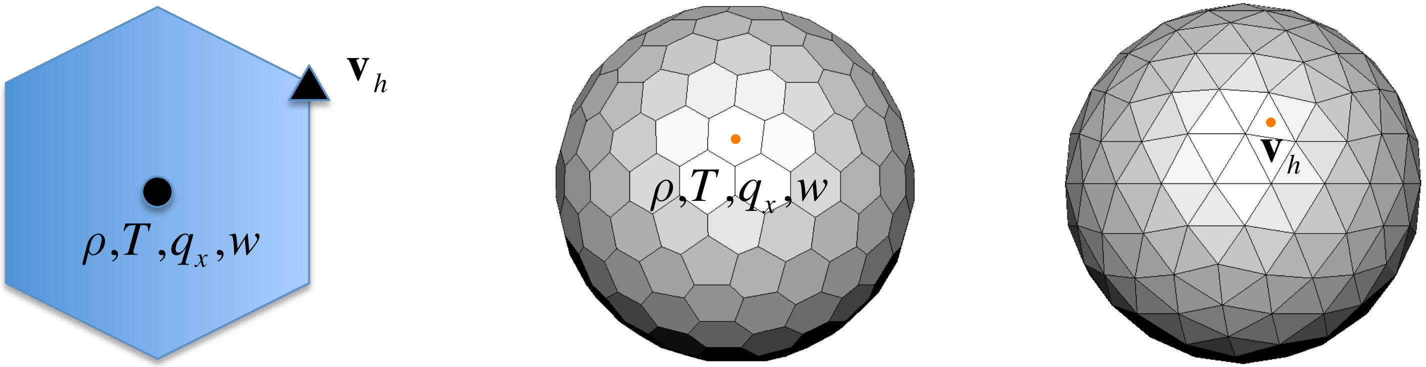


3rd-order upwind-biased scheme



Miura (2013, Mon. Wea. Rev.)

B(or ZM)-grid



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~~Flux-divergence~~ and Gradient operators

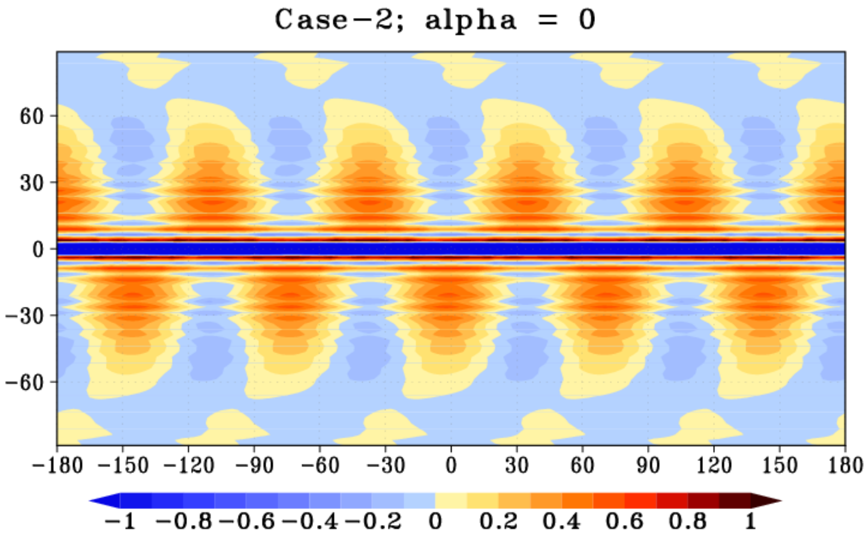
First-order accurate gradient

A result of the SWM model based on Ringler and Randall (2002).

$$\frac{\partial h}{\partial t} = -\nabla \bullet (h\mathbf{v})$$

$$\frac{\partial \mathbf{v}}{\partial t} = -(f+\zeta)\mathbf{k}\times\mathbf{v} - \nabla K - g\nabla(h+h_s)$$

Williamson et al. (1992): test case-2

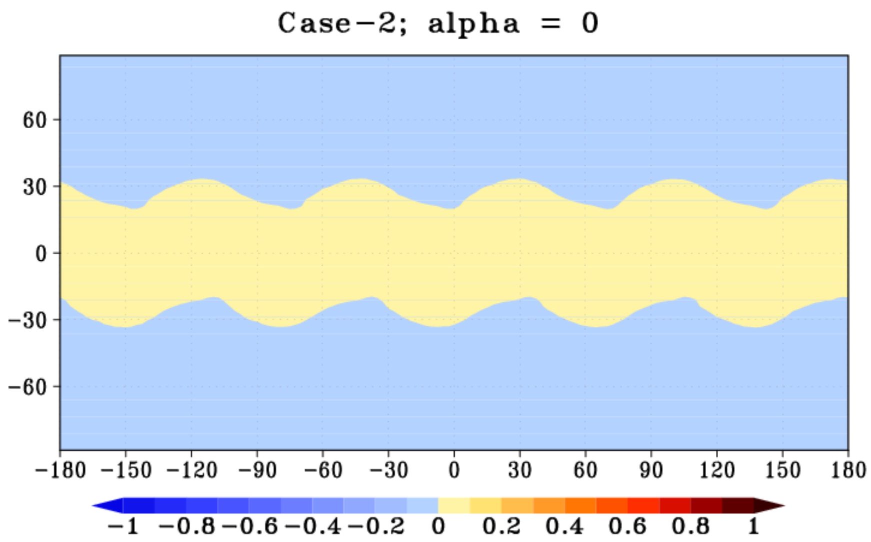


Spurious flow away from the equator.

Higher-order diffusion can damp the flow.

→ But, spurious loss/accumulation of mass at the poles remains.

Second-order accurate gradient

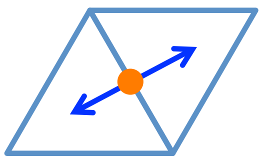


Second-order gradient can largely eliminate supurious flow.

But, algorithm based on the least-square fit is unstable for longer simulations.

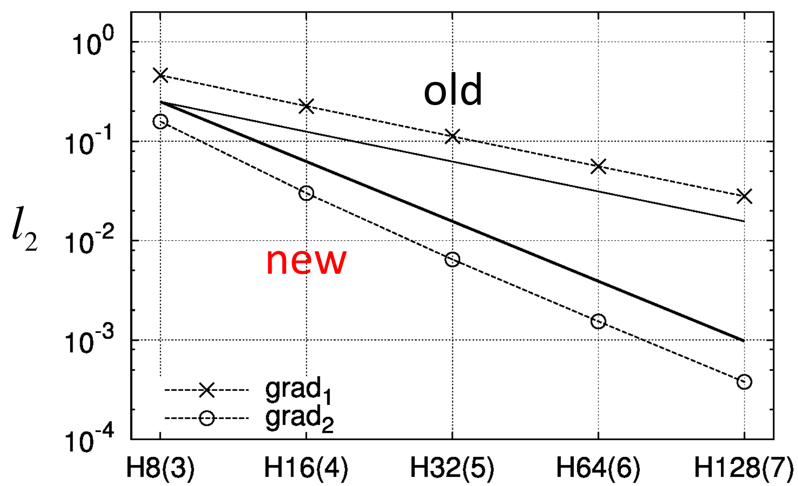
$$\oint \nabla p \, dA \neq 0$$

New algorithm based on the finite-volume method.



$$\oint \nabla p \, dA \sim 0$$

Heikes and Randall's test function

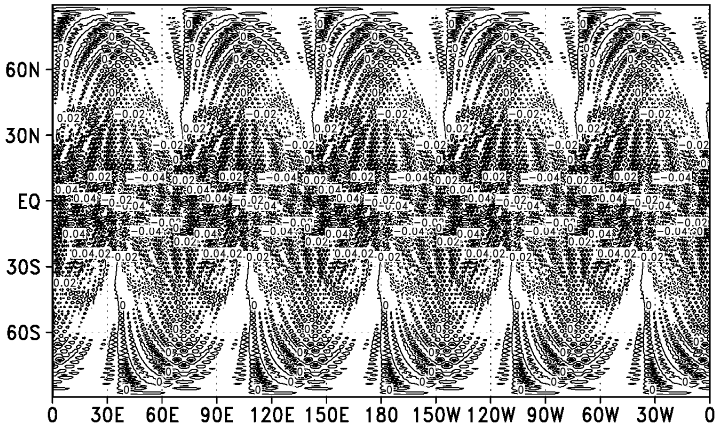


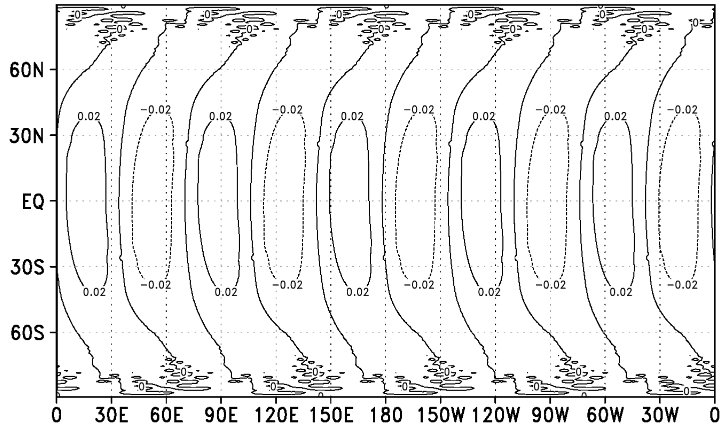
SWM results

Williamson et al. (1992): test case-2

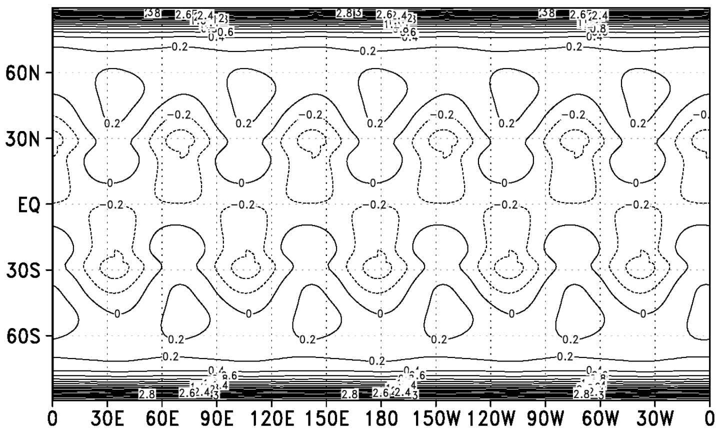
1st-order gradient

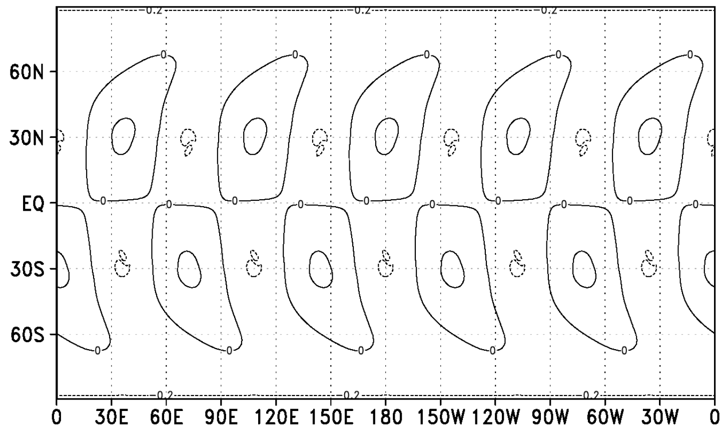
2nd-order gradient



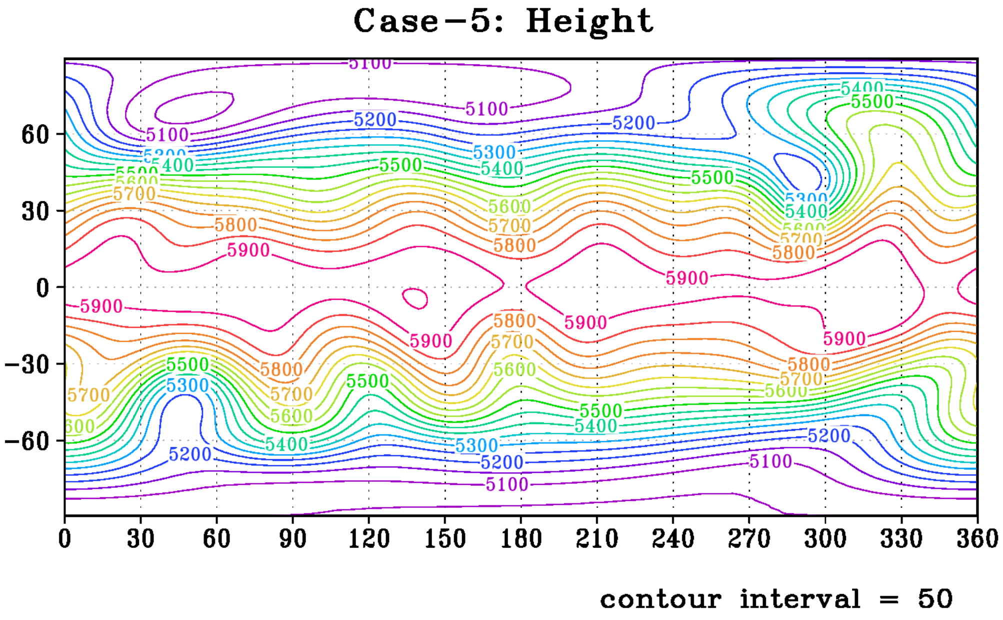


V error





H error

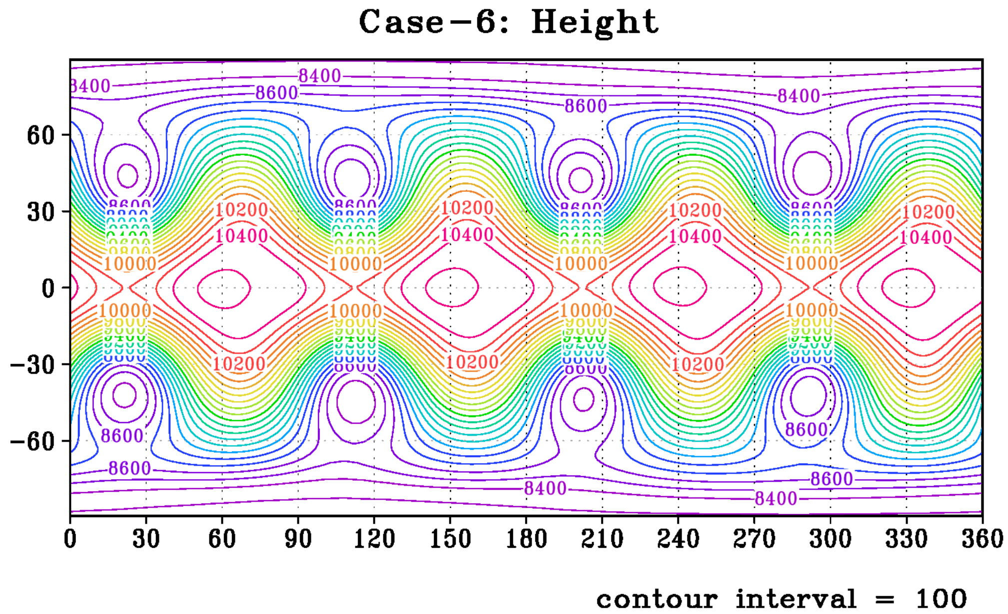


Williamson et al. (1992): test case-5

$\Delta x \sim 240$ km

Williamson et al. (1992): test case-6

$\Delta x \sim 240$ km



Summary

Stable long-term simulation using the same flux-form equations as NICAM

$$\frac{\partial h}{\partial t} = -\nabla \bullet (h\mathbf{v})$$

$$\frac{\partial h\mathbf{v}}{\partial t} = -\nabla \bullet (h\mathbf{v}\mathbf{v}) - f\mathbf{k} \times h\mathbf{v} - gh\nabla (h + h_s)$$

	NICAM	New
Arrangements	A-grid	B (or ZM)-grid
Transport	2^{nd}-order centered (Tomita et al. 2001)	2^{nd}-order centered/3^{rd}-order upwind-biased (Miura 2013)
Passive tracer transport	Piecewise linear-like (Miura 2007)	Piecewise parabolic-like (Miura and Skamarock 2013)
Gradient	2^{nd}-order (Tomita et al. 2001)	1^{st}-order (RR2002)→2^{nd}-order

Future plan

- 3D
    - The common dynamical core of NICAM and MIROC
    - The first (dry) version is working w/ topography.
- Selective damping of the computational mode