Curl to Curl-Curl

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Curl:

Dynamical core that predicts the three-dimensional vorticity
as in Jung and Arakawa (2008)

Curl-Curl:

Dynamical core that predicts the curl of the horizontal vorticity vector along with the vertical component of vorticity

Both have advantages in the implementation of filtered systems of equations (including the sound-proof, quasi-hydrostatic and unified systems) to dynamical cores

They are not directly applicable to the fully compressible system of equations

Curl	Curl-Curl
Cartesian coordinates	
$\omega_x \equiv -\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}$ $\omega_y \equiv \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}$ $\omega_z \equiv \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ $\frac{\partial \omega_z}{\partial z} = -\frac{\partial \omega_x}{\partial x} - \frac{\partial \omega_y}{\partial y}$ *is satisfied* $\nabla_H^2 w + \frac{\partial}{\partial z}\left[\frac{1}{\rho}\frac{\partial}{\partial z}(\rho w)\right] = -\left(\frac{\partial \omega_y}{\partial x} - \frac{\partial \omega_x}{\partial y}\right) + \mathcal{R}$ $\rho \equiv \rho_0(z)$ *and* $\mathcal{R} = 0$ *for anelastic equations* Jung and Arakawa (2008) $\rho \equiv \rho_{qs}(x,y,z,t)$ *for unified equations* $\mathcal{R}$ *is collection of terms for unified equations* Arakawa and Konor (2009)	$\Gamma \equiv \frac{\partial \omega_y}{\partial x} - \frac{\partial \omega_x}{\partial y}$ $\omega_z \equiv \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ $\frac{\partial (\omega_y)_{rot}}{\partial x} - \frac{\partial (\omega_x)_{rot}}{\partial y} = \Gamma$ $\frac{\partial (\omega_x)_{div}}{\partial x} + \frac{\partial (\omega_y)_{div}}{\partial y} = -\frac{\partial \omega_z}{\partial z}$ $\omega_x = (\omega_x)_{rot} + (\omega_x)_{div}$ $\omega_y = (\omega_y)_{rot} + (\omega_y)_{div}$ $\nabla_H^2 w + \frac{\partial}{\partial z}\left[\frac{1}{\rho}\frac{\partial}{\partial z}(\rho w)\right] = -\Gamma + \mathcal{R}$
Vector forms (Local coordinates)	
$\boldsymbol{\omega}_H \equiv \mathbf{k} \times \left(\frac{\partial \mathbf{v}}{\partial z} - \nabla_H w\right)$ $\omega_z \equiv \mathbf{k} \cdot \nabla_H \times \mathbf{v}$ $\frac{\partial \omega_z}{\partial z} = -\nabla_H \cdot \boldsymbol{\omega}_H$ $\nabla_H^2 w + \frac{\partial}{\partial z}\left[\frac{1}{\rho}\frac{\partial}{\partial z}(\rho w)\right] = -\mathbf{k} \cdot \nabla_H \times \boldsymbol{\omega}_H + \mathcal{R}$	$\Gamma \equiv \mathbf{k} \cdot \nabla_H \times \boldsymbol{\omega}_H$ $\omega_z \equiv \mathbf{k} \cdot \nabla_H \times \mathbf{v}$ $\mathbf{k} \cdot \nabla_H \times (\boldsymbol{\omega}_H)_{rot} = \Gamma$ $\nabla_H \cdot (\boldsymbol{\omega}_H)_{div} = -\frac{\partial \omega_z}{\partial z}$ $\omega_x = (\omega_x)_{rot} + (\omega_x)_{div}$ $\omega_y = (\omega_y)_{rot} + (\omega_y)_{div}$ $\nabla_H^2 w + \frac{\partial}{\partial z}\left[\frac{1}{\rho}\frac{\partial}{\partial z}(\rho w)\right] = -\Gamma + \mathcal{R}$

Curl	Curl-Curl

Traditional form (Equations based on Lipps-Hemler anelastic system)

Curl

$$\frac{\partial \omega_x}{\partial t} = -\frac{\partial}{\partial x}(\omega_x u) - \frac{\partial}{\partial y}(\omega_x v) - \frac{\partial}{\partial z}(\omega_x w) + \omega_x \frac{\partial u}{\partial x} + \frac{1}{2}(\omega_y R_z + \omega_z R_y) + f\frac{\partial u}{\partial z} + \frac{\partial B}{\partial y}$$

$$\frac{\partial \omega_y}{\partial t} = -\frac{\partial}{\partial x}(\omega_y u) - \frac{\partial}{\partial y}(\omega_y v) - \frac{\partial}{\partial z}(\omega_y w) + \omega_y \frac{\partial v}{\partial y} + \frac{1}{2}(\omega_x R_z + \omega_z R_x) + f\frac{\partial v}{\partial z} - \frac{\partial B}{\partial x}$$

$$\frac{\partial \omega_z}{\partial z} = -\frac{\partial \omega_x}{\partial x} - \frac{\partial \omega_y}{\partial y}$$

Jung and Arakawa (2008)

Curl-Curl

$$\frac{\partial \Gamma}{\partial t} = -\frac{\partial}{\partial x}(\Gamma u) - \frac{\partial}{\partial y}(\Gamma v) - \frac{\partial}{\partial z}(\Gamma w) + \frac{\partial}{\partial x}\left(\omega_x \frac{\partial u}{\partial y} - \omega_y \frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\omega_x \frac{\partial v}{\partial y} - \omega_y \frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial z}\left(\omega_x \frac{\partial w}{\partial y} - \omega_y \frac{\partial w}{\partial x}\right) + \frac{\partial}{\partial x}\left(\omega_y \frac{\partial v}{\partial y}\right) - \frac{\partial}{\partial y}\left(\omega_x \frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial x}\left[\frac{1}{2}(\omega_x R_z + \omega_z R_x)\right] - \frac{\partial}{\partial y}\left[\frac{1}{2}(\omega_y R_z + \omega_z R_y)\right] + \frac{\partial}{\partial x}\left(f\frac{\partial v}{\partial z}\right) - \frac{\partial}{\partial y}\left(f\frac{\partial u}{\partial z}\right) - \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)B$$

$$\frac{\partial \omega_z}{\partial t} = -\frac{\partial}{\partial x}\left[(\omega_z + f)u\right] - \frac{\partial}{\partial y}\left[(\omega_z + f)v\right] - \frac{\partial}{\partial z}\left[(\omega_z + f)w\right] + \frac{1}{2}(\omega_x R_y + \omega_y R_x) + (\omega_z + f)\frac{\partial w}{\partial z}$$

Compact form (Equations based on Lipps-Hemler anelastic system)

Curl

$$\frac{\partial \omega_x}{\partial t} = -\frac{\partial}{\partial z}\left[-(\omega_z + f)u + \omega_x w\right] + \frac{\partial}{\partial y}(-\omega_x v + \omega_y u) + \frac{\partial B}{\partial y}$$

$$\frac{\partial \omega_y}{\partial t} = \frac{\partial}{\partial z}\left[(\omega_z + f)v - \omega_y w\right] - \frac{\partial}{\partial x}(-\omega_x v + \omega_y u) - \frac{\partial B}{\partial x}$$

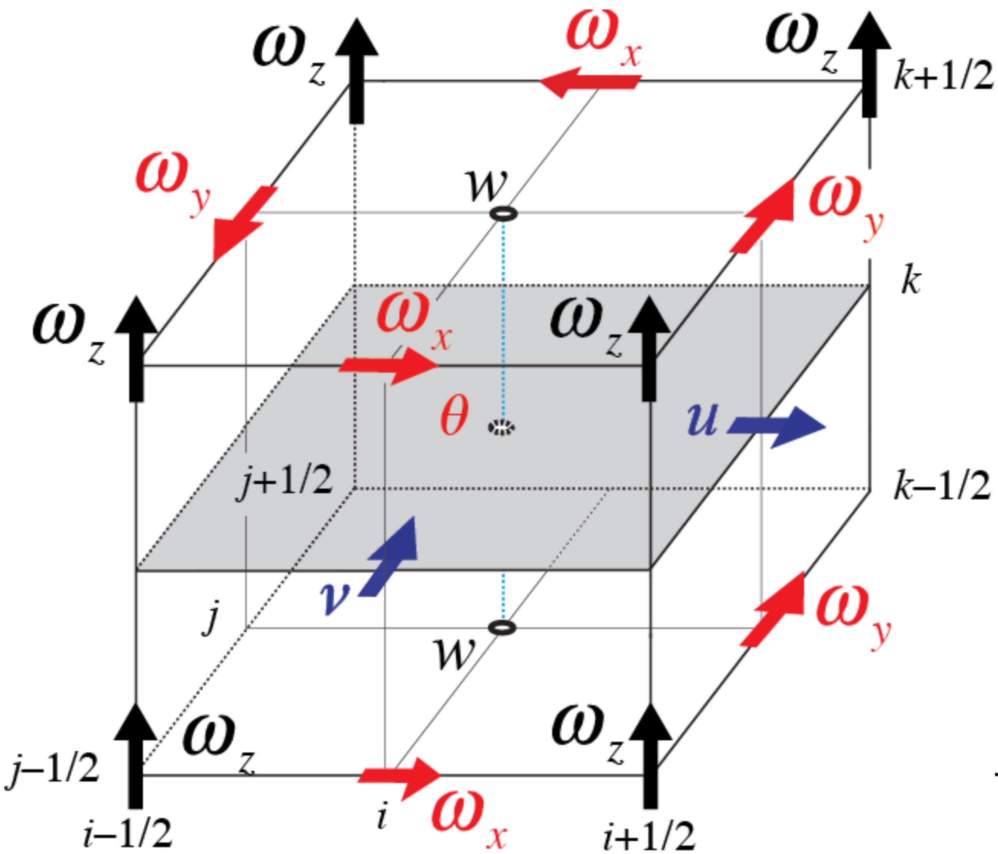
$$\frac{\partial \omega_z}{\partial z} = -\frac{\partial \omega_x}{\partial x} - \frac{\partial \omega_y}{\partial y}$$

Curl-Curl

$$\frac{\partial \Gamma}{\partial t} = \frac{\partial}{\partial z}\left\{\frac{\partial}{\partial x}\left[(\omega_z + f)v - \omega_y w\right] + \frac{\partial}{\partial y}\left[-(\omega_z + f)u + \omega_x w\right]\right\} - \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)(-\omega_x v + \omega_y u) - \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)B$$

$$\frac{\partial \omega_z}{\partial t} = -\frac{\partial}{\partial x}\left[(\omega_z + f)u\right] - \frac{\partial}{\partial y}\left[(\omega_z + f)v\right] + \frac{\partial}{\partial y}(\omega_y w) + \frac{\partial}{\partial x}(\omega_x w)$$

Curl

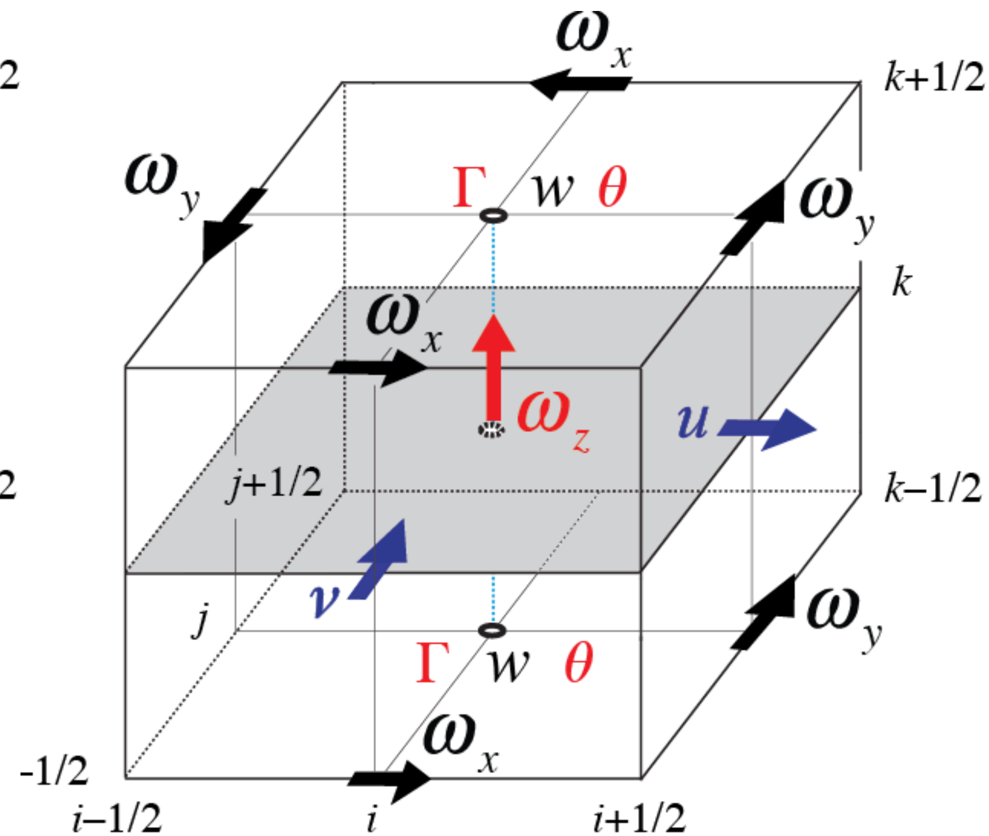


"C-grid" dynamics

Originally L-grid thermodynamics

Jung and Arakawa (2008)

Curl-Curl



"Z-grid" dynamics

Originally CP-grid thermodynamics

Red and black indicate prognostic and diagnostic variables, respectively

Blue indicates diagnostic horizontal velocity

Curl:

Successful simulations on the Cartesian grid (Jung and Arakawa, 2008)

Successful simulations over the steep surface topography

Questionable on the icosahedral grid due to the existence of a computational mode

Curl-Curl:

Better on the icosahedral grid because of its computational-mode-free “Z-grid” staggering

Computationally 10%-15% slower because two horizontally 2D elliptic solvers are needed to diagnose horizontal vorticity

Sigma coordinate version is needed to handle surface topography

Curl-Curl in Limited Area Modeling

Horizontal mean of horizontal vorticity needs to be predicted

$$\frac{\partial \bar{\omega}_x}{\partial t} = -\overline{\cancel{\frac{\partial}{\partial x}\left(\omega_x u\right)}} - \underbrace{\overline{\frac{\partial}{\partial y}\left(\omega_x \upsilon\right)}}_{\substack{=0 \\ \textit{if } \upsilon=0 \textit{ at boundaries}}} - \overline{\frac{\partial}{\partial z}\left(\omega_x w\right)}$$

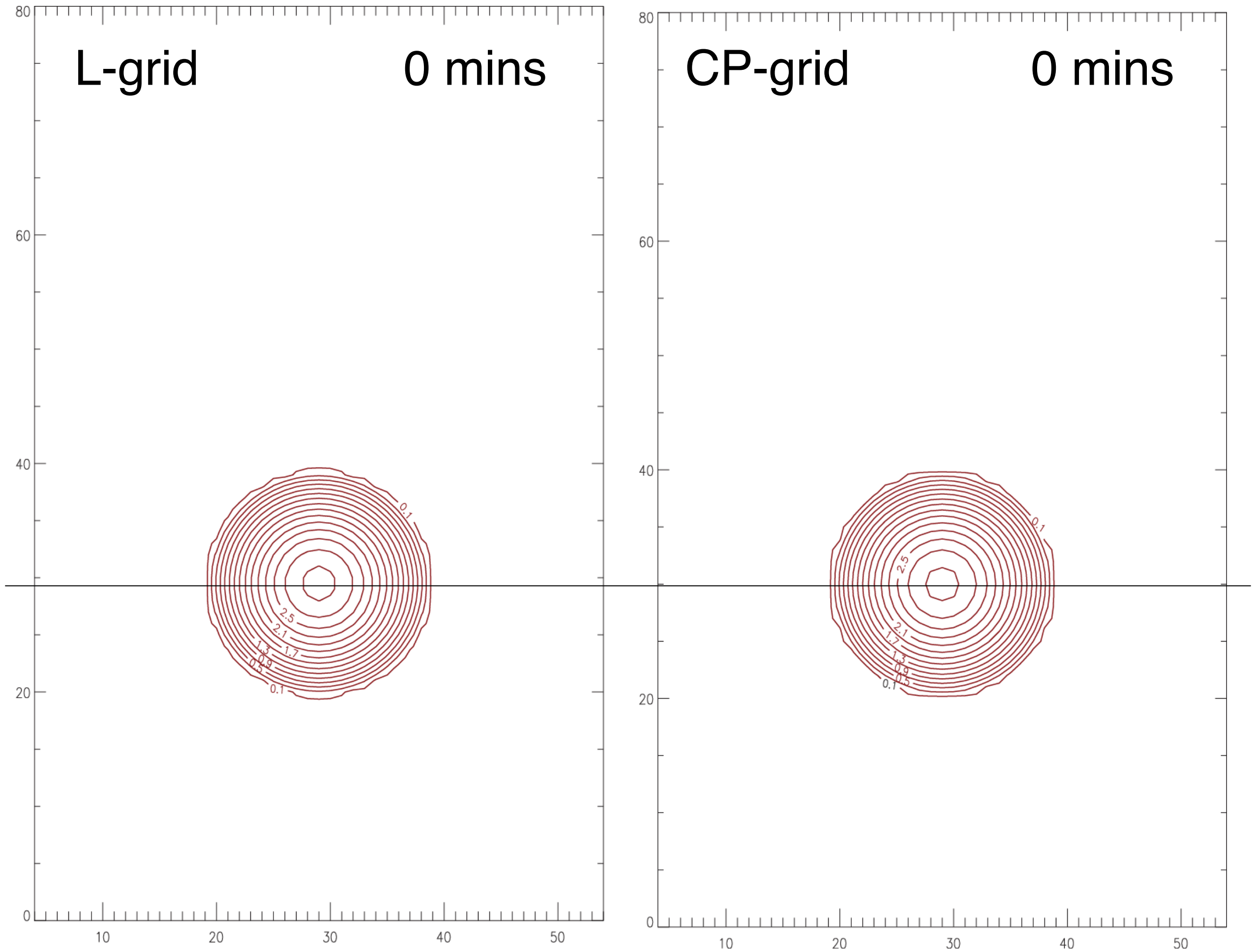
$$+\overline{\frac{1}{2}\left(\omega_y R_z + \omega_z R_y\right)} + \overline{\omega_x \frac{\partial u}{\partial x}} + \overline{f \frac{\partial u}{\partial z}} + \underbrace{\overline{\frac{\partial B}{\partial y}}}_{\substack{=0 \\ \textit{for doubly priodic domain}}}$$

$$\frac{\partial \bar{\omega}_y}{\partial t} = -\overline{\cancel{\frac{\partial}{\partial x}\left(\omega_y u\right)}} - \underbrace{\overline{\frac{\partial}{\partial y}\left(\omega_y \upsilon\right)}}_{\substack{=0 \\ \textit{if } \upsilon=0 \textit{ at boundaries}}} - \overline{\frac{\partial}{\partial z}\left(\omega_y w\right)}$$

$$+\overline{\frac{1}{2}\left(\omega_x R_z + \omega_z R_x\right)} + \overline{\omega_y \frac{\partial \upsilon}{\partial y}} + \overline{f \frac{\partial \upsilon}{\partial z}} - \cancel{\overline{\frac{\partial B}{\partial x}}}$$

Curl　　　　Curl-Curl

Potential temp. perturbation (K)



dx=dy=dz=100 m

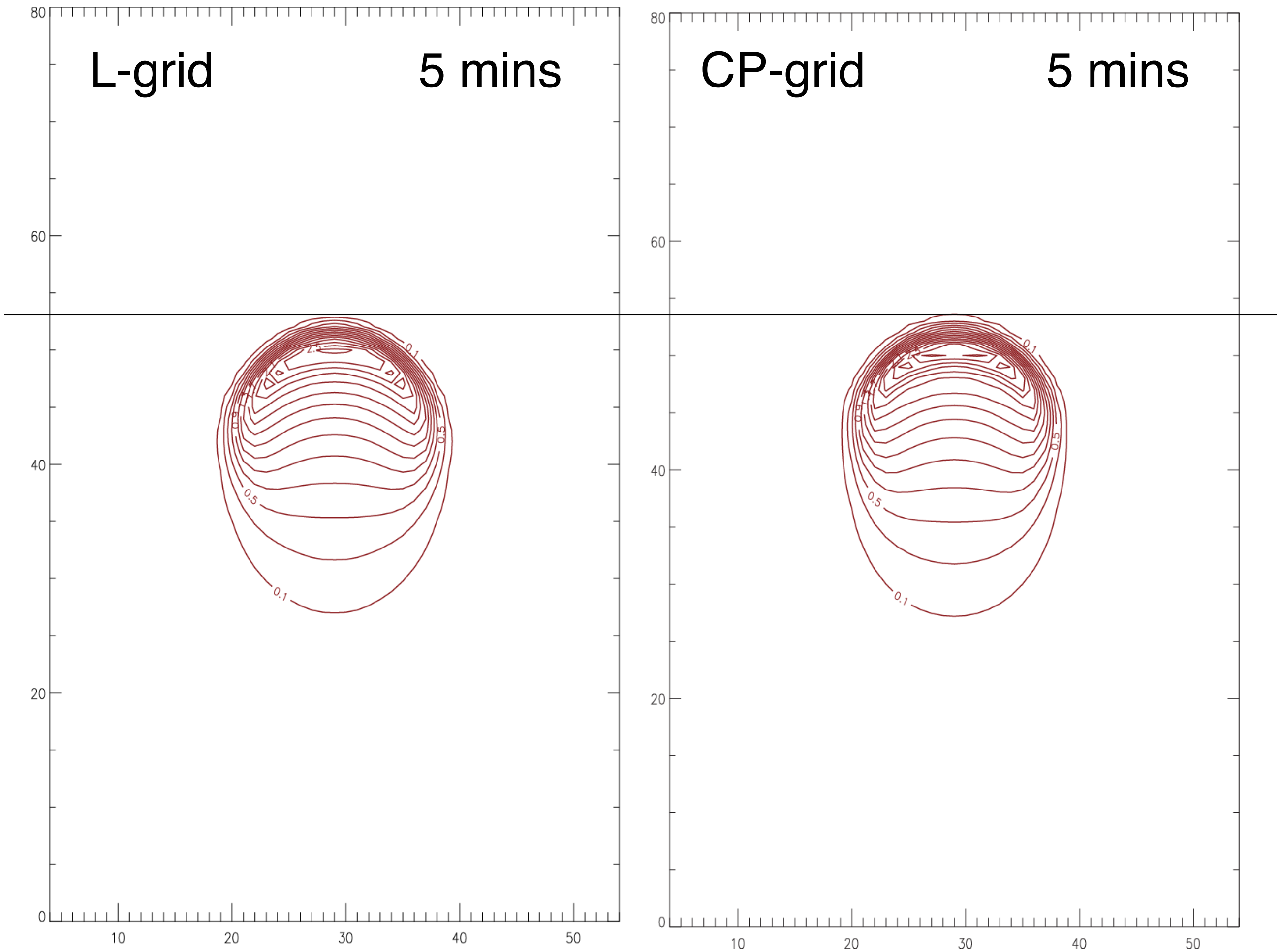
Domain height=8000 m

Domain length and width=5000 m

3K potential temperature perturbation max

Curl   Curl-Curl

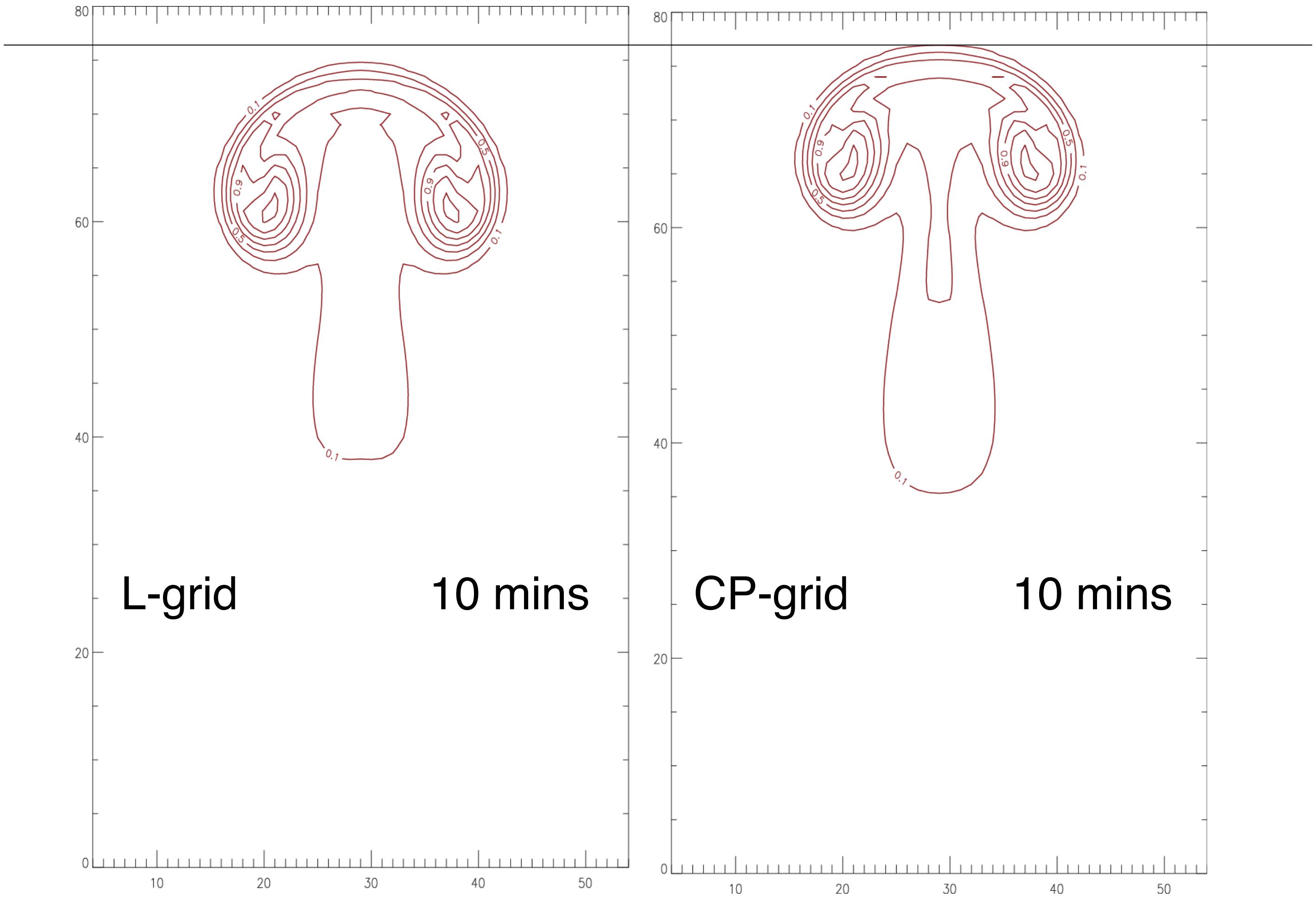
Potential temp. perturbation (K)



Diffusion coefficient: 25 m^2/sec

Curl Curl-Curl

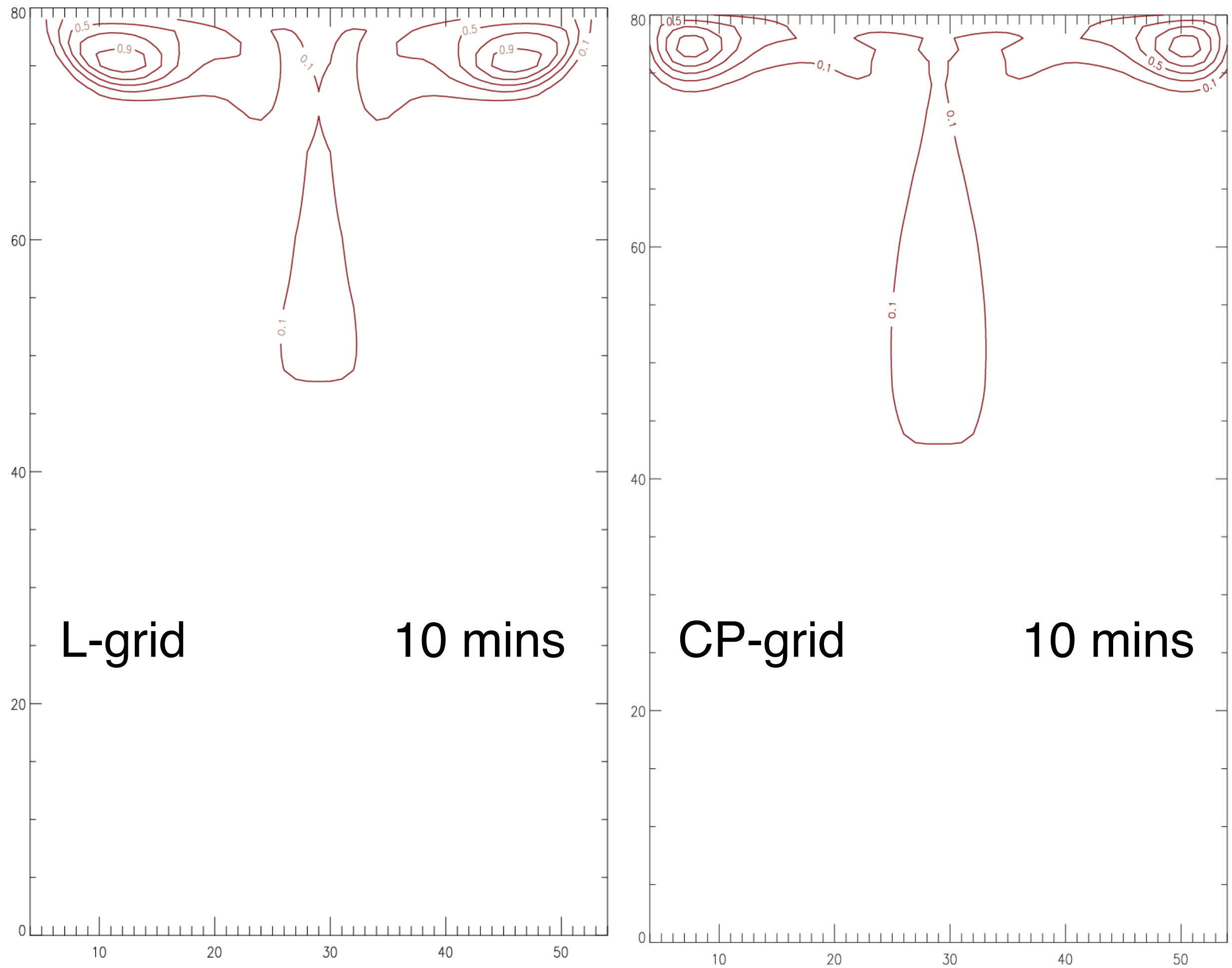
Potential temp. perturbation (K)



Diffusion coefficient: 25 m^2/sec

Curl   Curl-Curl

Potential temp. perturbation (K)



Diffusion coefficient: 25 m^2/sec

Curl (traditional form)

Diffusion of potential temperature and omega_x and omega_y

100 m^2/sec ⟶ 25 m^2/sec

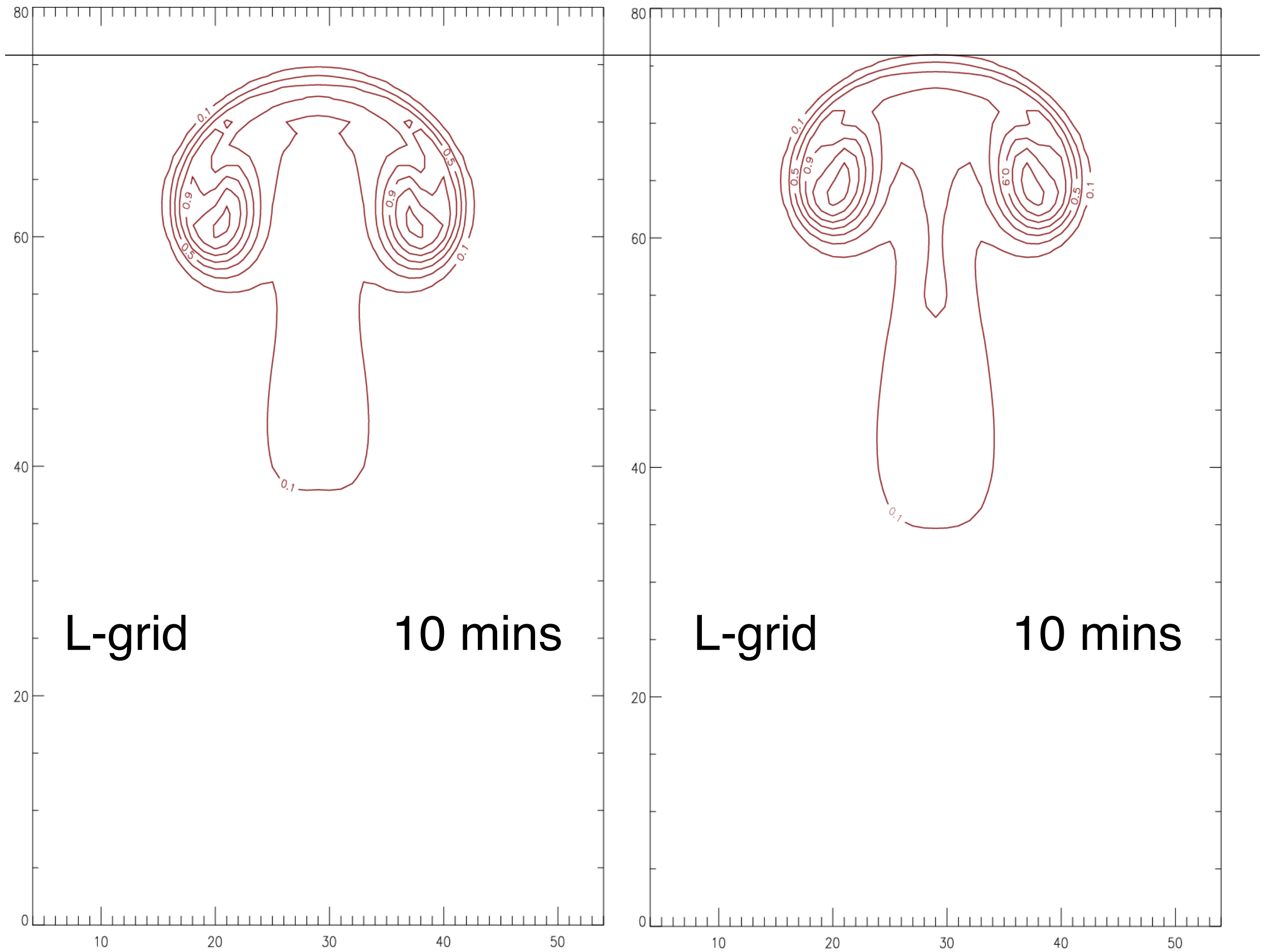
Curl-Curl

Diffusion of potential temperature and gamma

100 m^2/sec ⟶ 0 m^2/sec

Curl   Curl-Curl

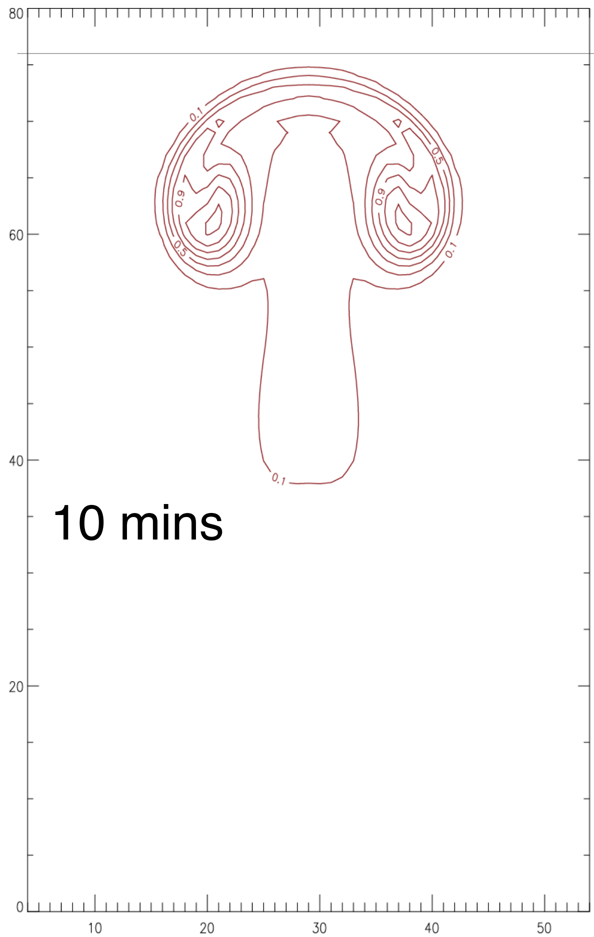
Potential temp. perturbation (K)



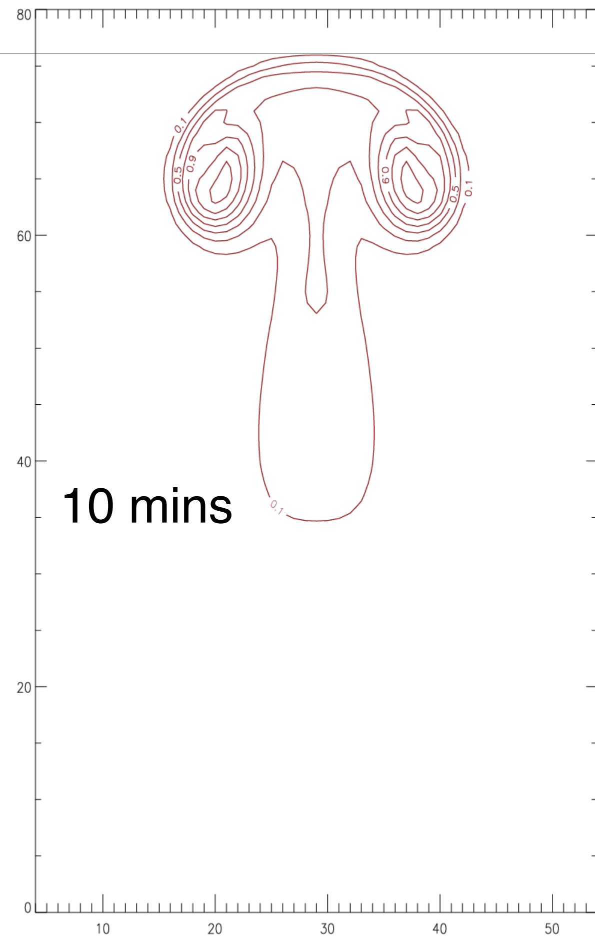
Curl **Curl-Curl** **Curl-Curl**

Potential temp. perturbation (K)

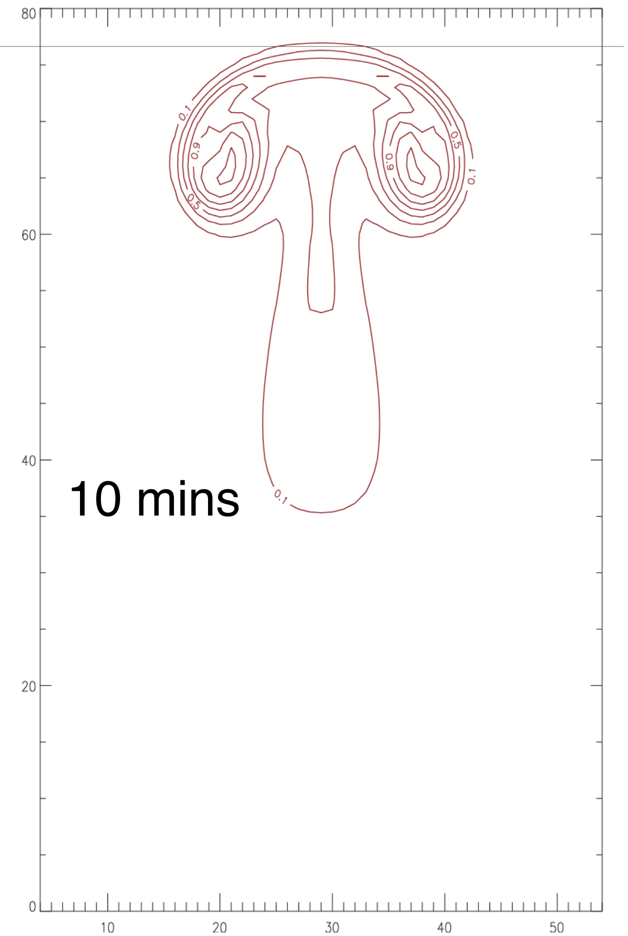
L-grid



L-grid

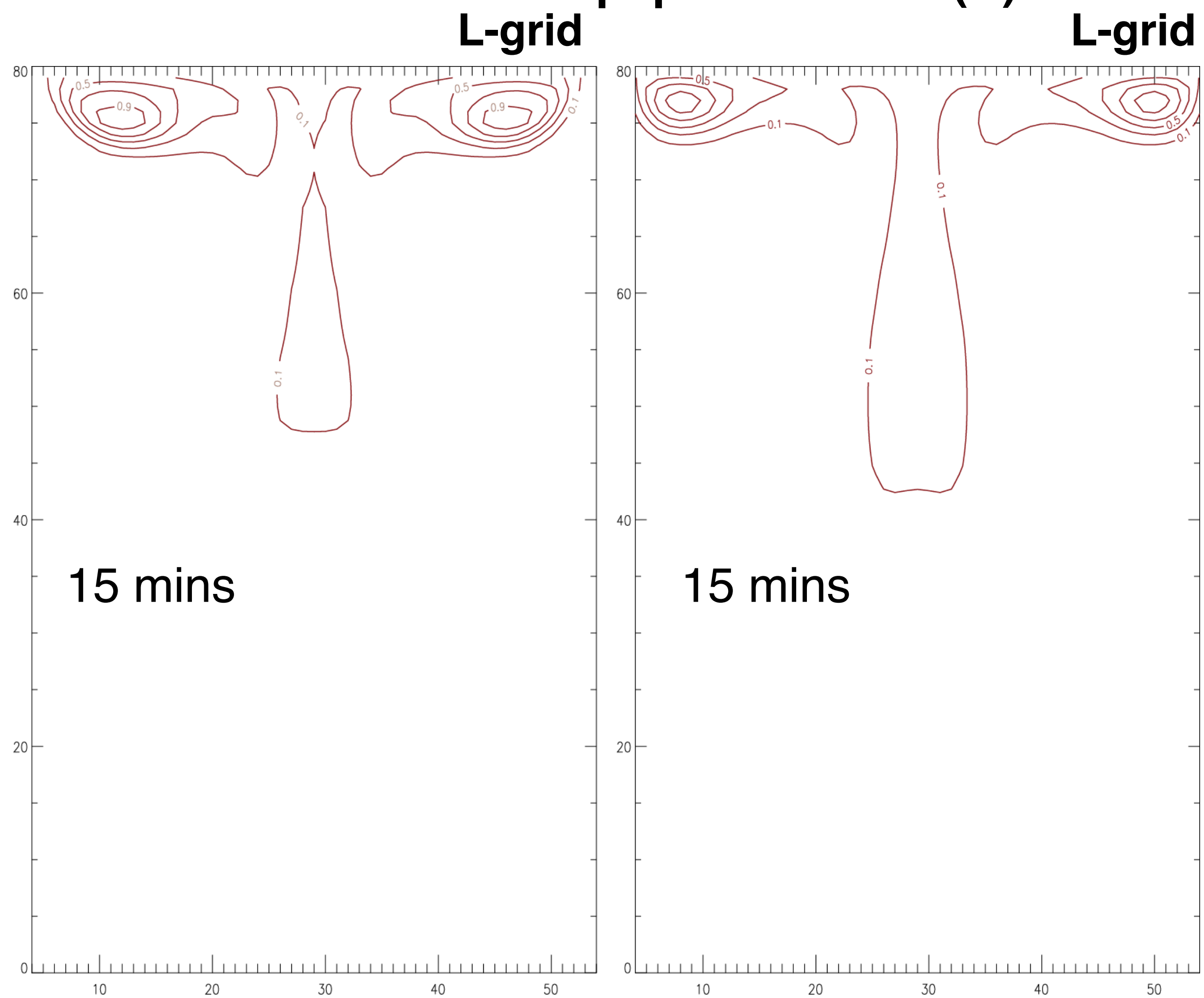


CP-grid



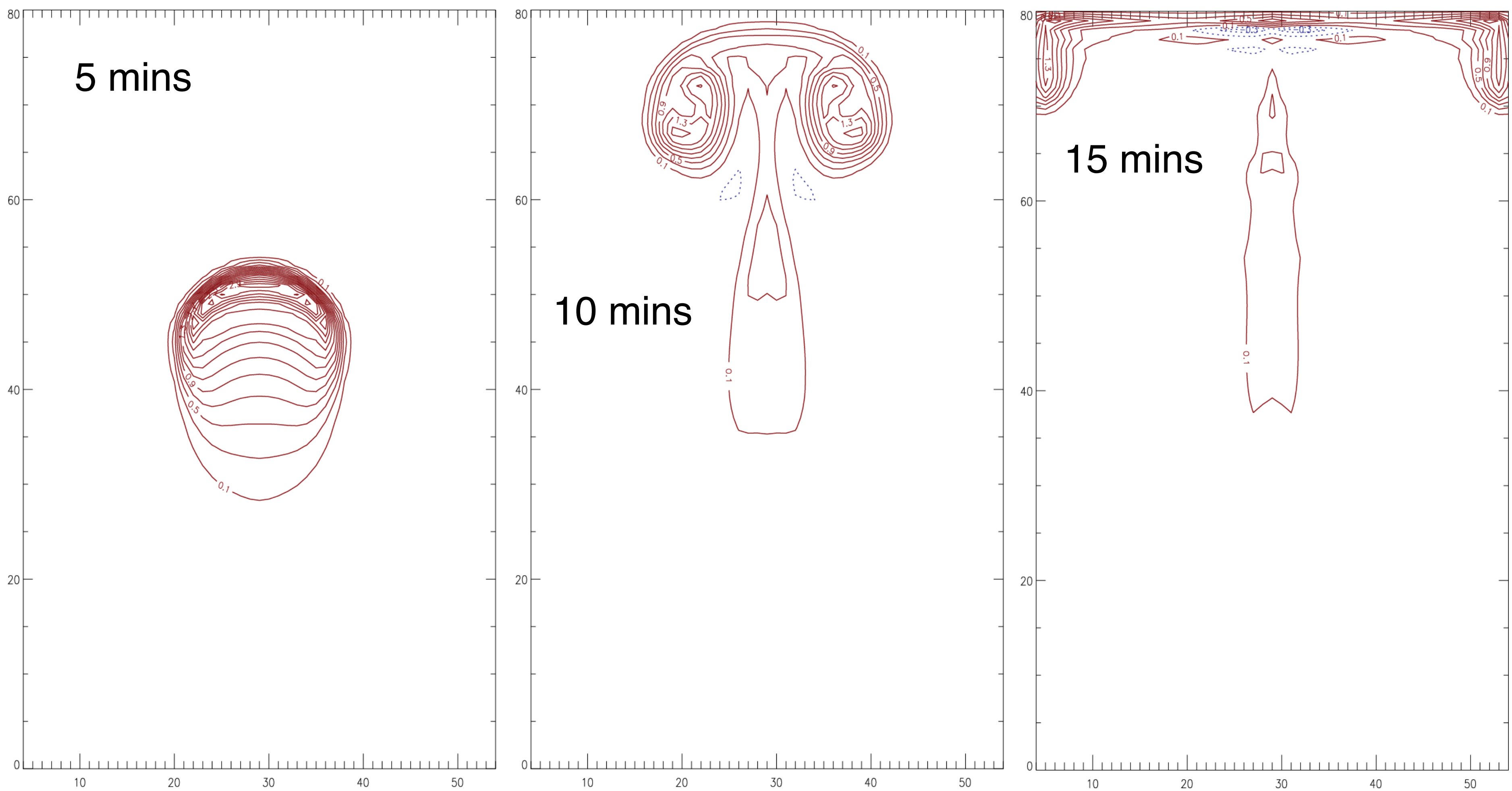
Curl   Curl-Curl

Potential temp. perturbation (K)



Curl-Curl

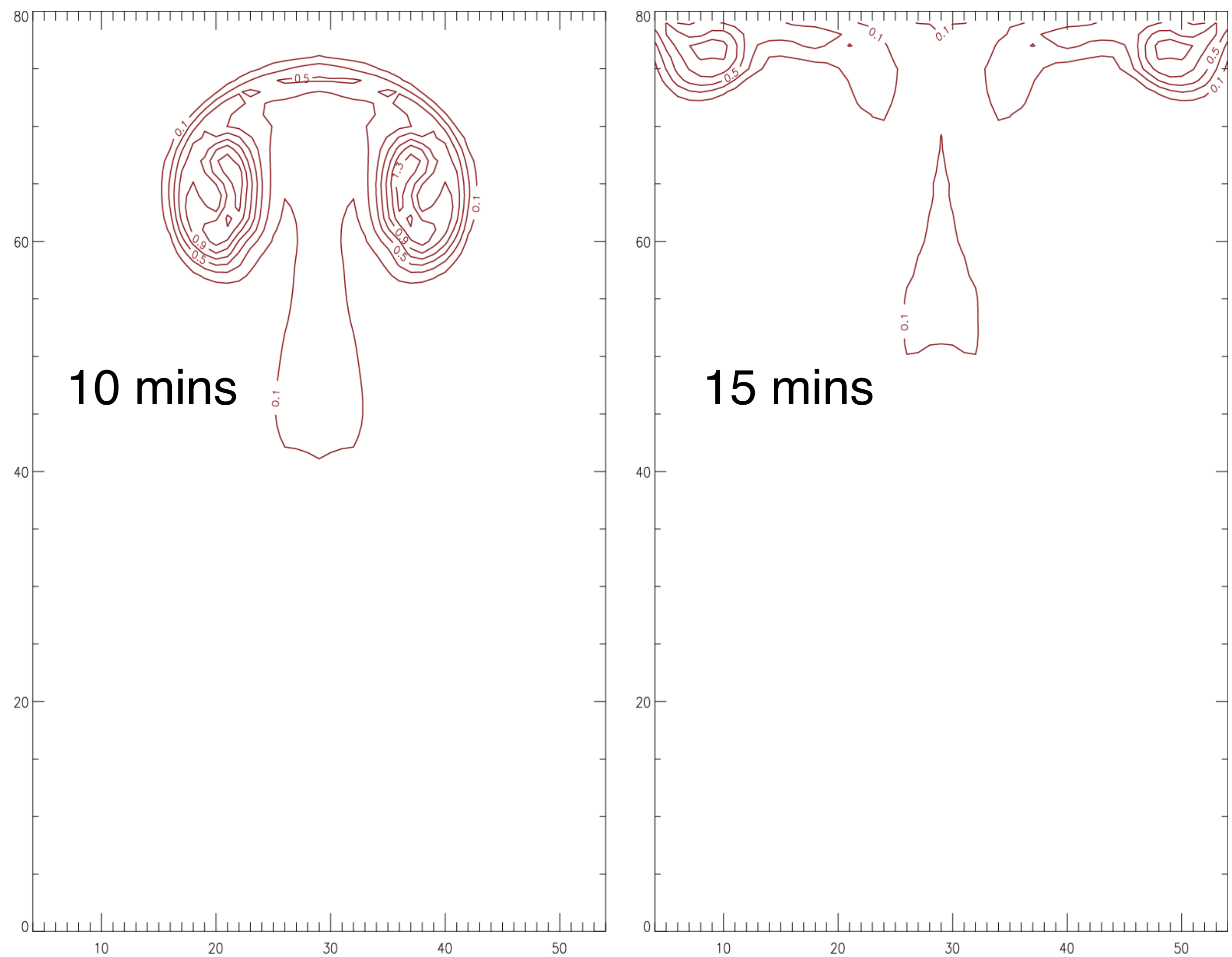
Potential temp. perturbation (K) **CP-grid**



Diffusion coefficient: 0 m^2/sec

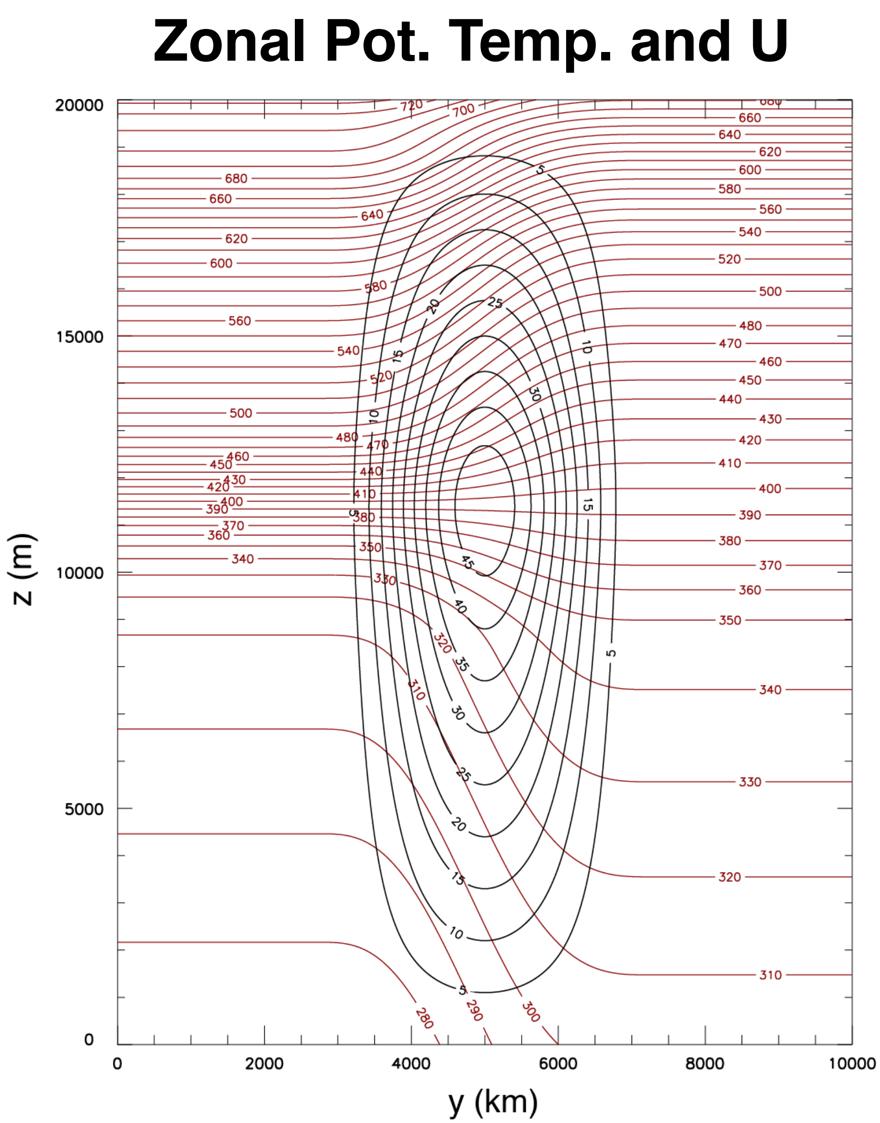
Curl (based on the compact form)

Potential temp. perturbation (K) **L-grid**



Diffusion coefficient: 5 m^2/sec

Extratropical cyclogenesis in a midlatitude channel on the β-plane



dx=dy=100 km   dz=400m

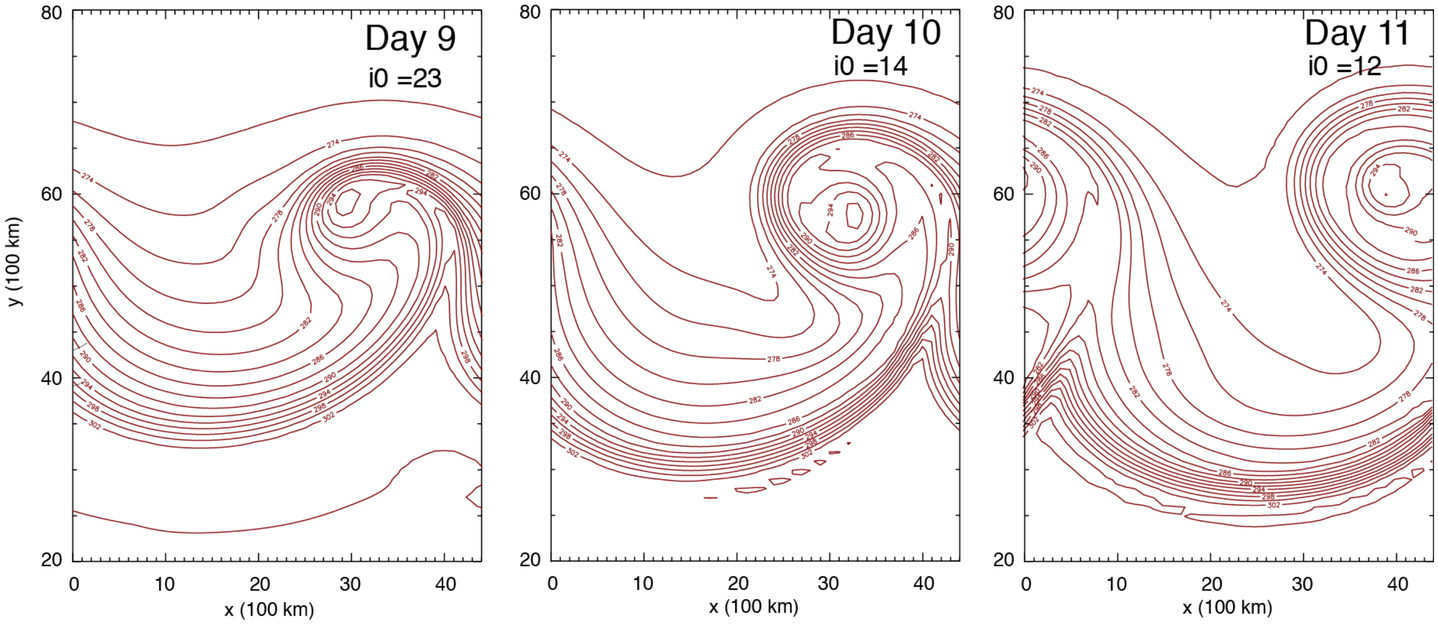
Domain height=20 km

Domain length=4500 km and width=10000 km

Wavenumber-1 perturbation (±3K) added on the surface potential temperature at the start

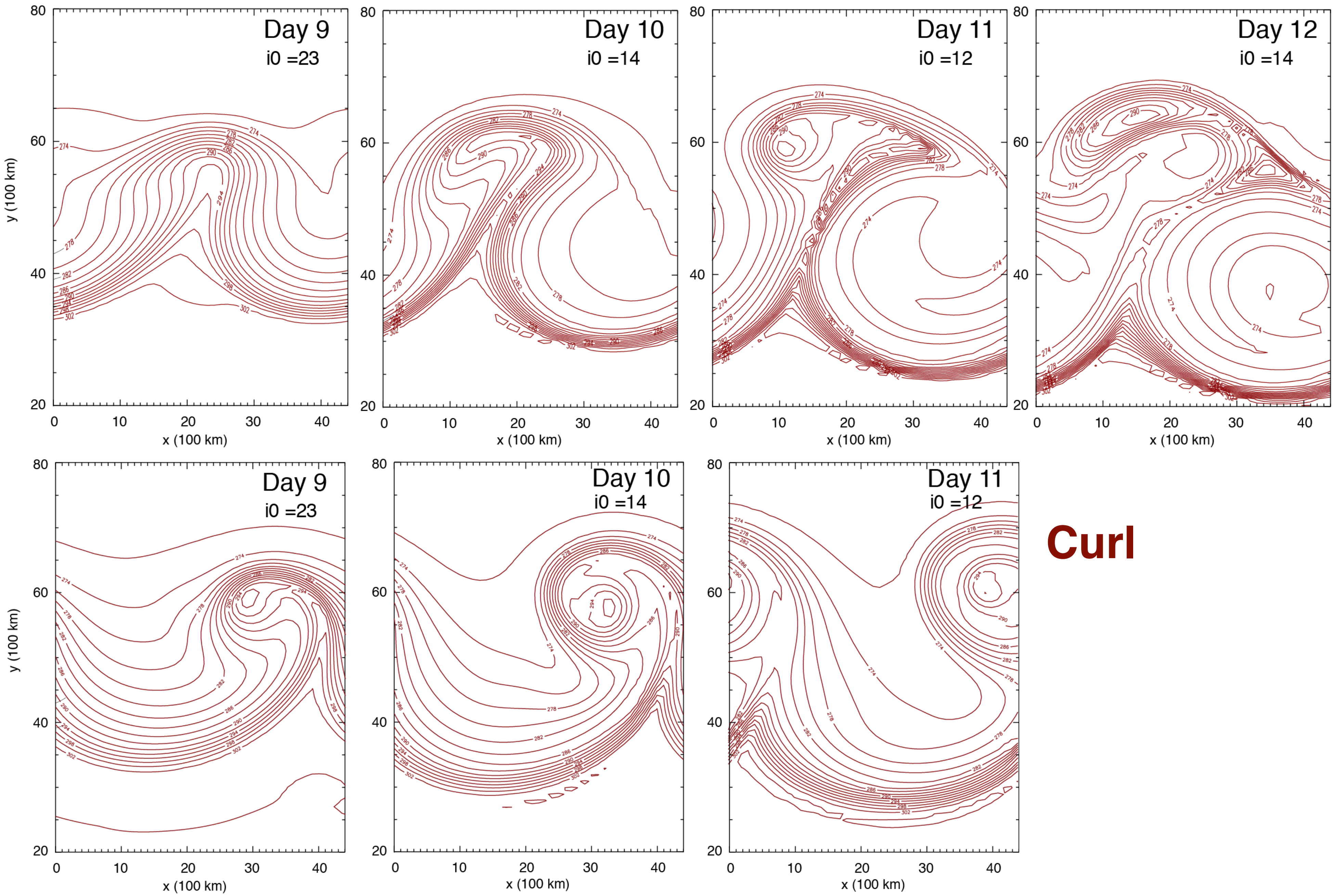
Curl (traditional form)

Surface Pot. Temp. (K)



Curl-Curl (traditional form)

Surface Pot. Temp. (K)



Curl

Midlatitude channel

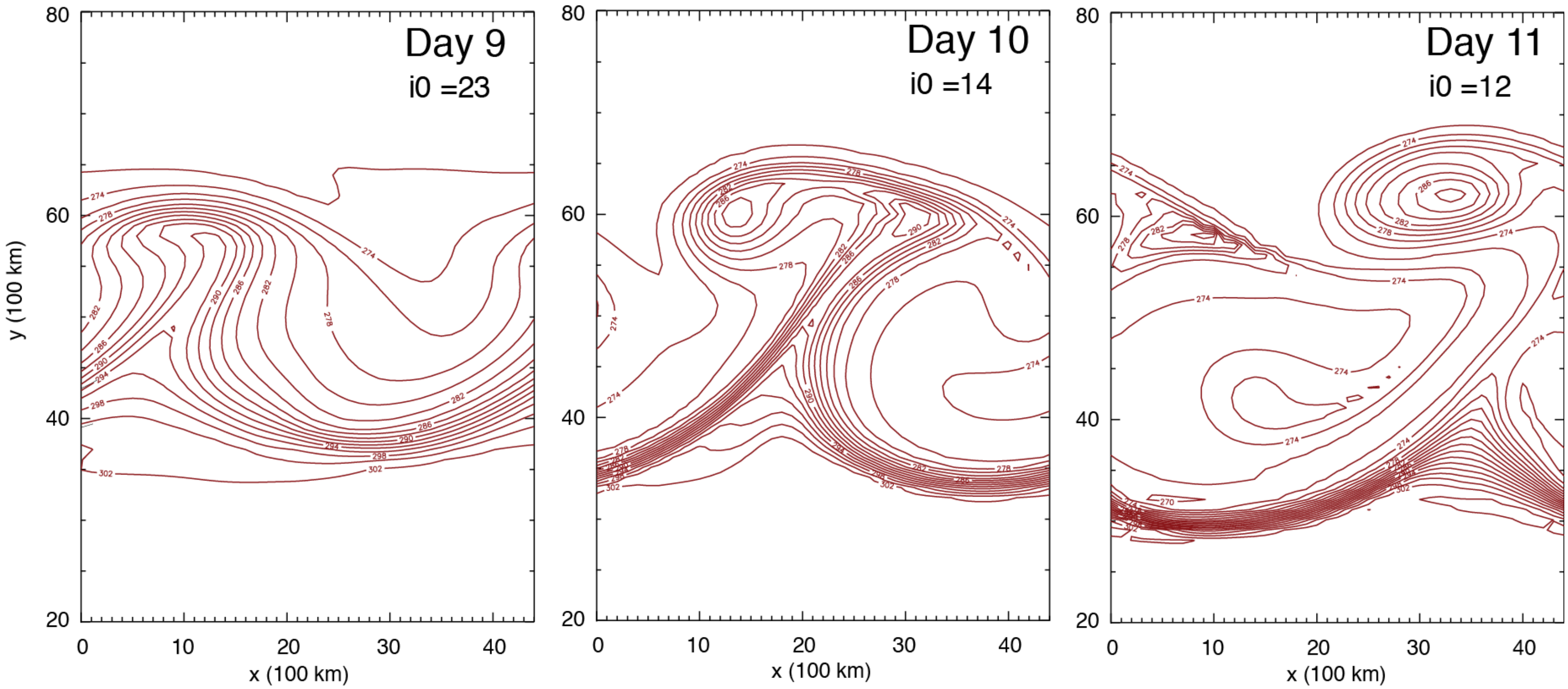
$$\frac{\partial \bar{\varpi}_x}{\partial t} = -\overline{\frac{\partial}{\partial z}(\omega_x w)} + \overline{\frac{1}{2}(\omega_y R_z + \omega_z R_y)} + \overline{\omega_x \frac{\partial u}{\partial x}} + \overline{f \frac{\partial u}{\partial z}} + \frac{B_{North} - B_{South}}{L_y}$$

which may yield $\bar{\varpi}_x \neq 0$

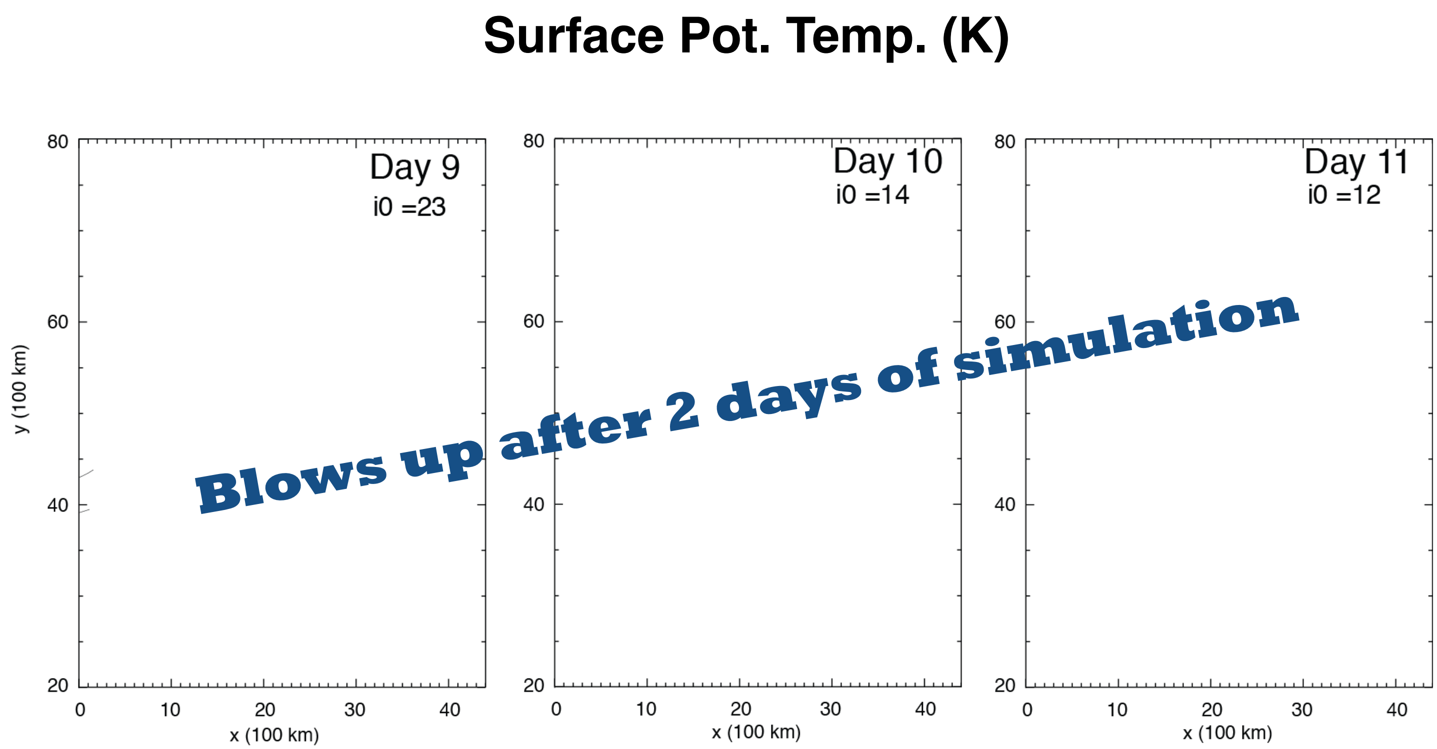
But Curl uses $(\bar{\varpi}_x)_{North} = (\bar{\varpi}_x)_{South} = 0$

Curl-Curl (traditional form)

Surface Pot. Temp. (K)



Curl (compact form)



Summary

Cartesian coordinates

Curl	Curl-Curl
Traditional form (Equations based on Lipps-Hemler anelastic system)	
Warm bubble test: OK (with diff) **Mid-lat channel run: OK**	**Warm bubble test: OK** **Mid-lat channel run: Suffers from failing boundary condition**
Compact form (Equations based on Lipps-Hemler anelastic system)	
Warm bubble test: OK (with low diff) **Mid-lat channel run: Blows up**	Warm bubble test: Remains to be made Mid-lat channel run: Remains to be made