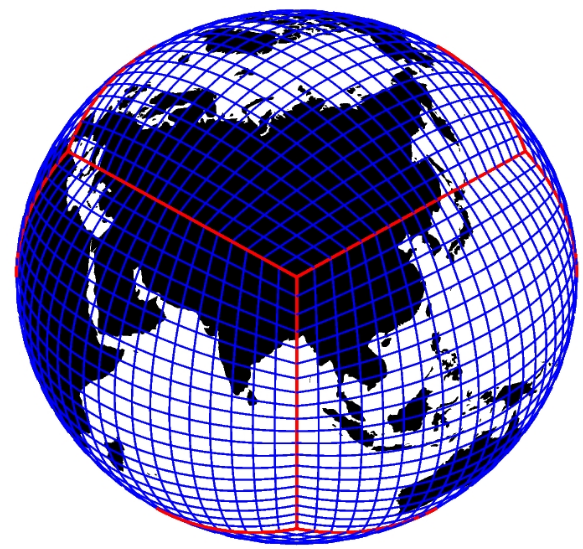
Separating physics, dynamics and tracer grids in CAM-SE

(Community Atmosphere model - Spectral Elements)

Peter Hjort Lauritzen
National Center for Atmospheric Research
Boulder, Colorado, USA



Collaborators:
M.A. Taylor, P.A. Ullrich,
S. Goldhaber, J. Bacmeister

CMMAP (Center for Multiscale Modeling of Atmospheric Processes) meeting
January 7, 2016
Boulder, Colorado, USA

Getting away from the lat-lon grid …

CAM=NCAR's Community Atmosphere Model

- Scalability
- Static mesh-refinement capability
- …

CAM-FV (finite volume)

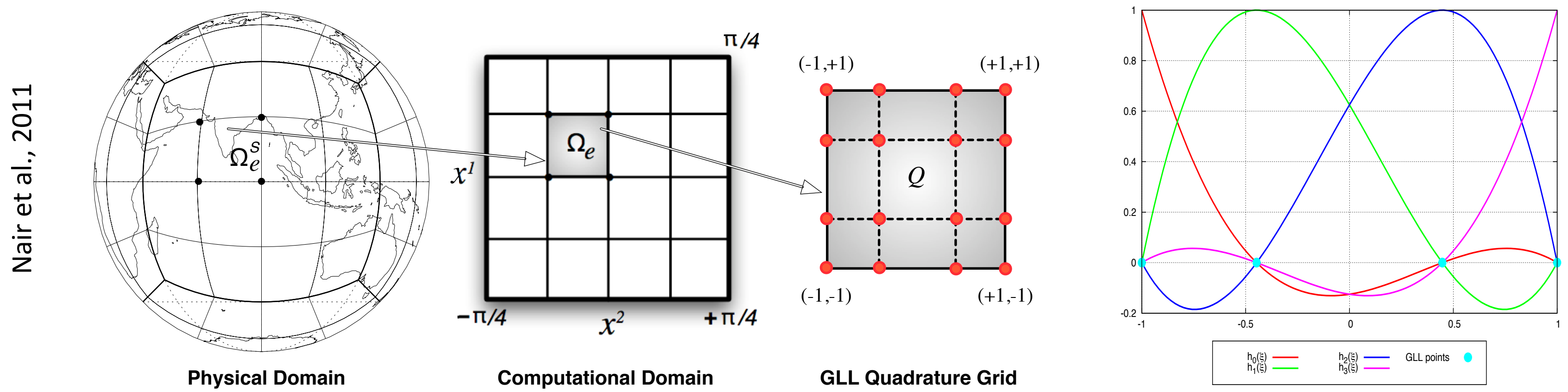
Lin (2004)

CAM-SE (spectral elements)

Taylor et al., (1997)
Dennis et al., (2012)

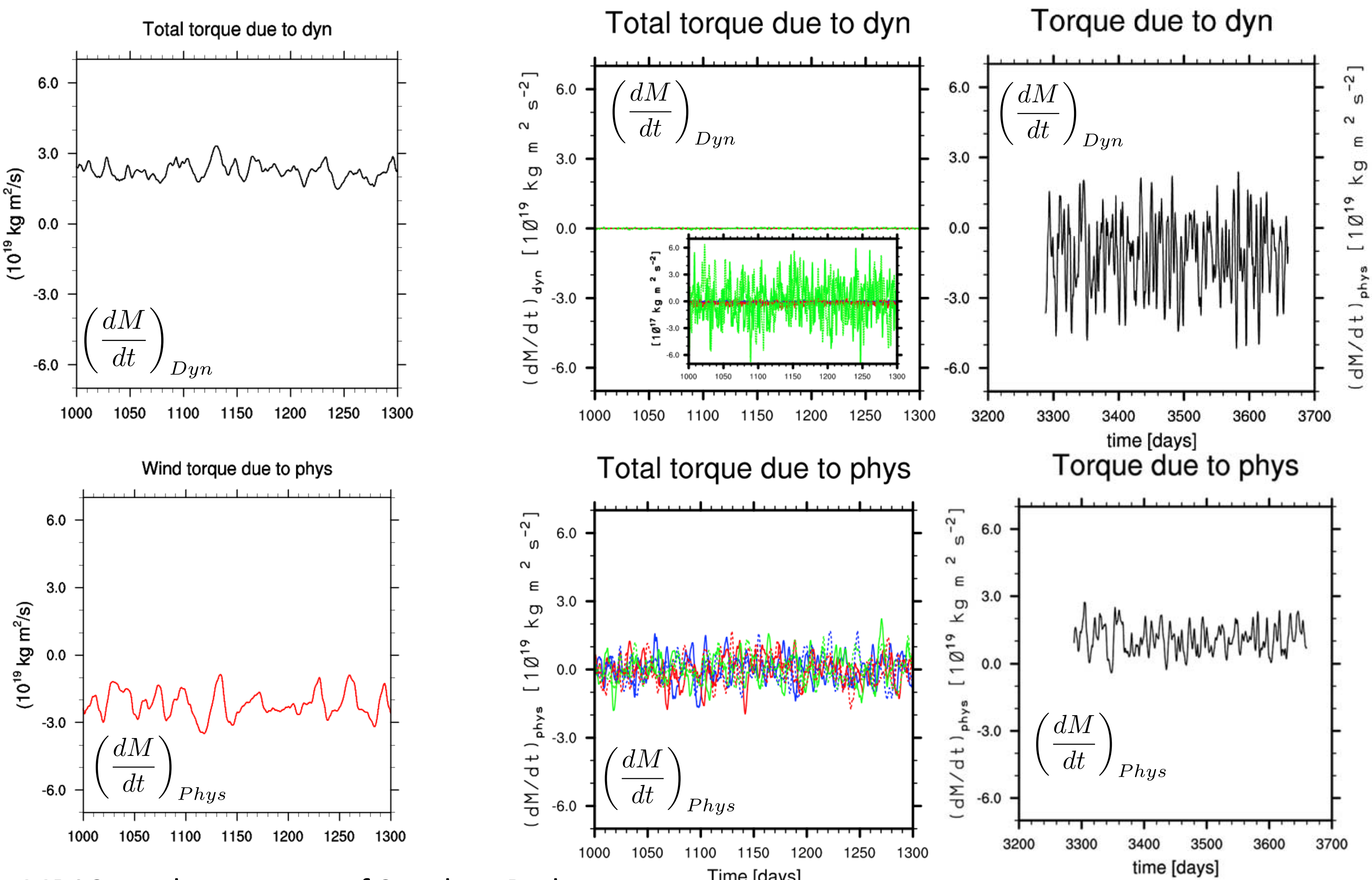
CAM-SE: NCAR Community Atmosphere Model with Spectral Elements dynamical core

Continuous Galerkin finite-element method (Taylor et al., 1997) on a cubed-sphere:



- Discretization is mimetic => mass-conservation & total energy conservation
- Conserves axial angular momentum very well (Lauritzen et al., 2014)
- Support static mesh-refinement and retains formal order of accuracy!
- Highly scalable to at least O(100K) processors (Dennis et al., 2012)
- Competitive “AMIP-climate” (Evans et al., 2012)
- **Lower computational throughput for many-tracer applications**
- **Tracer transport accuracy?**

MPAS CAM-SE CAM-FV



MPAS results courtesy of Sanghun Park

Lauritzen et al., 2014

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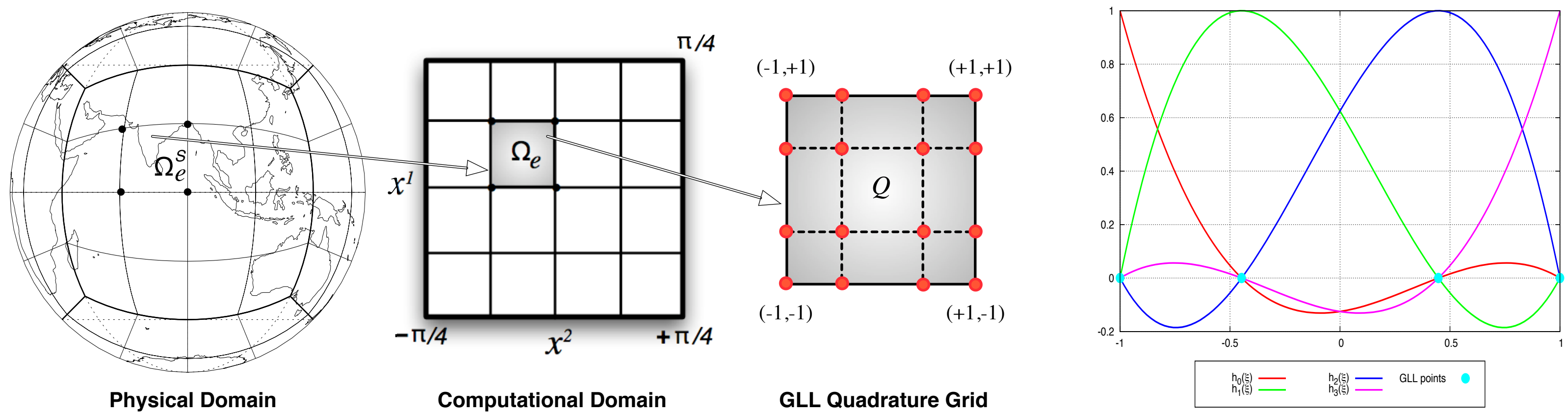

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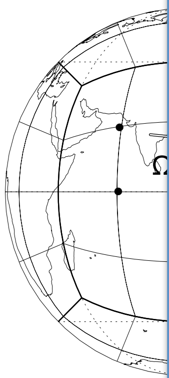
CAM-SE: NCAR Community Atmosphere Mode ore

Continuous



Nair et al., 2011

- Discretiz
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- Support
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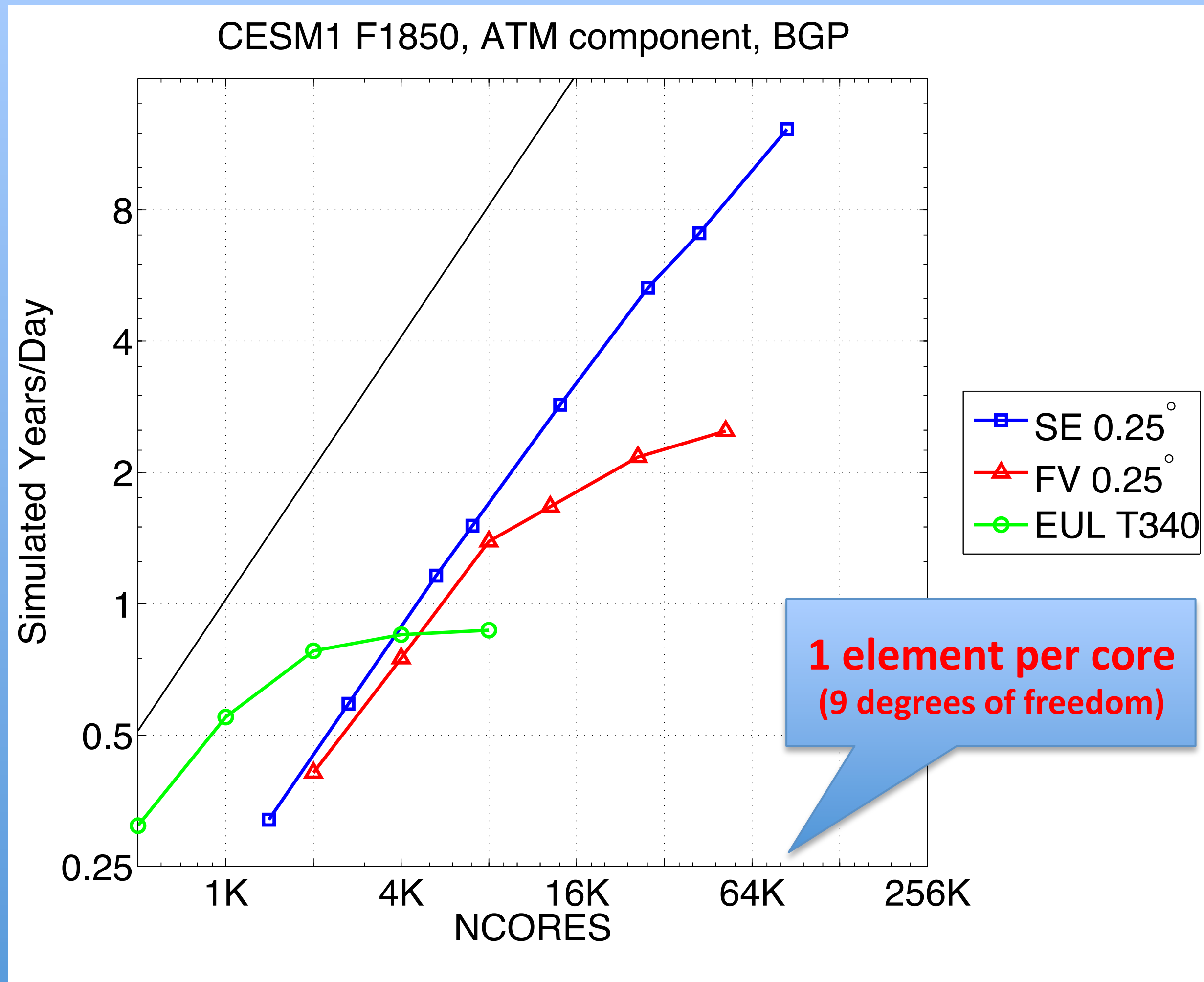

Figure from Dennis et al. (2012)

NCAR

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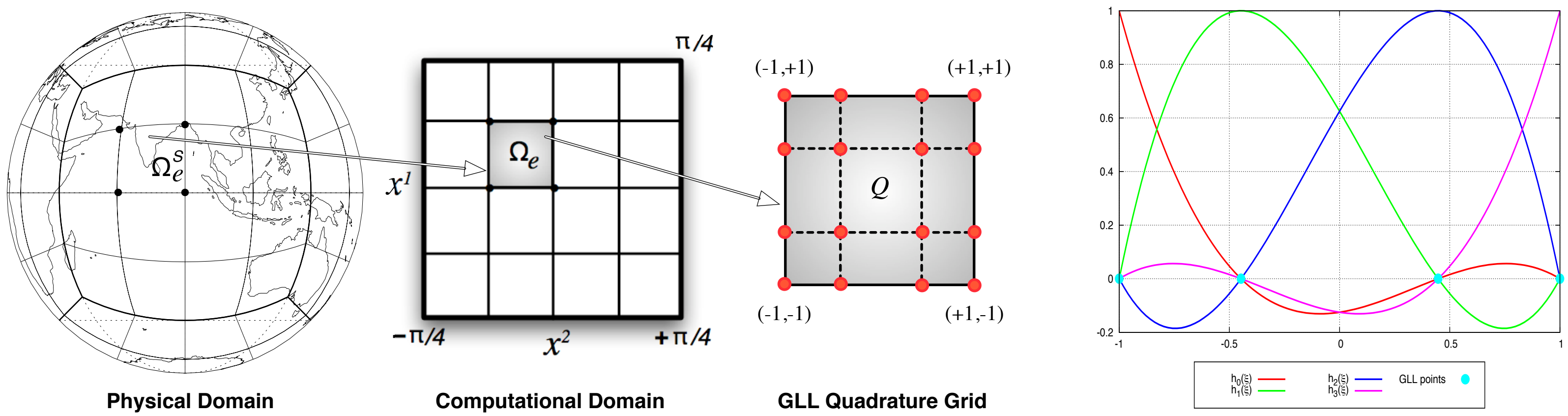

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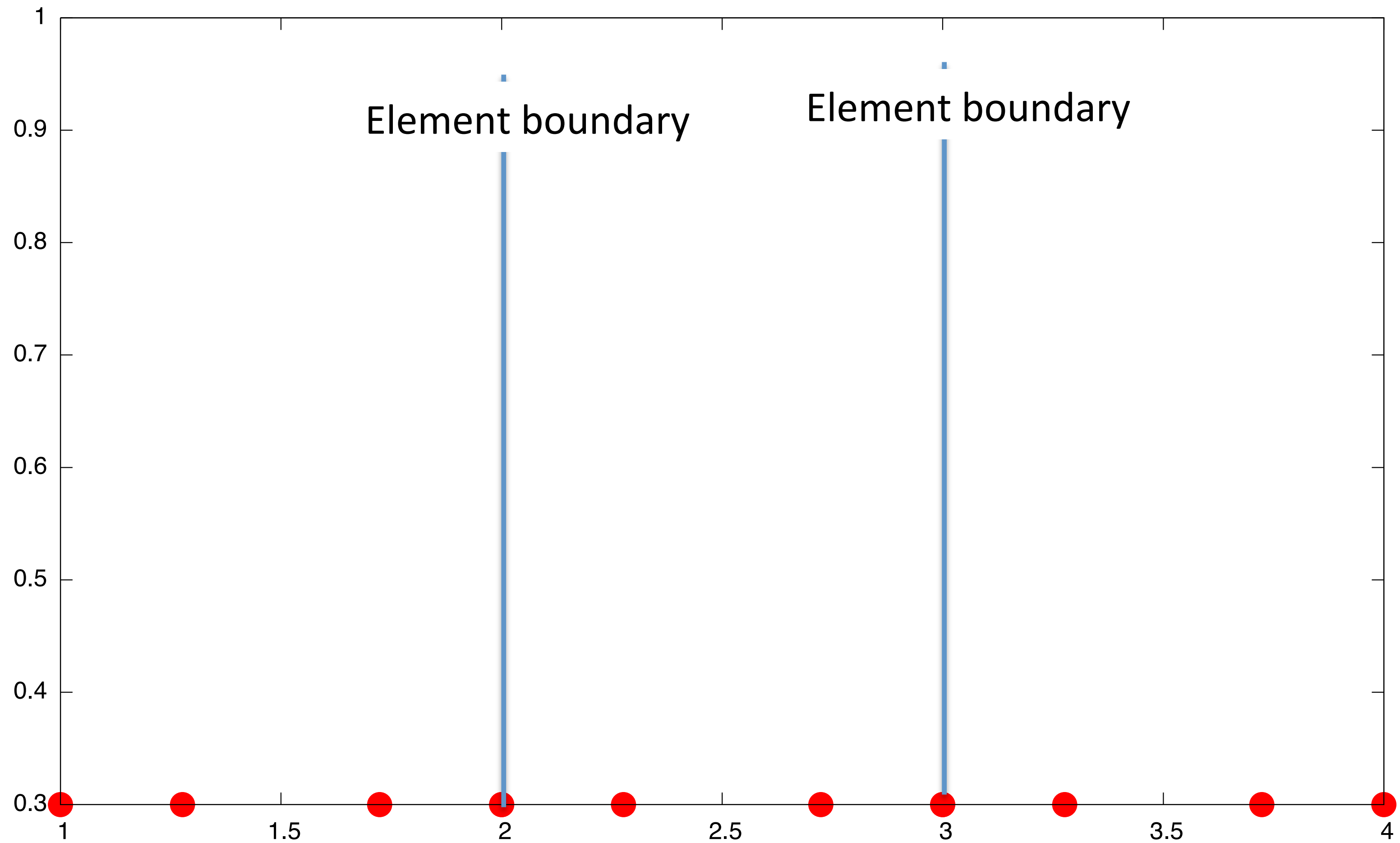
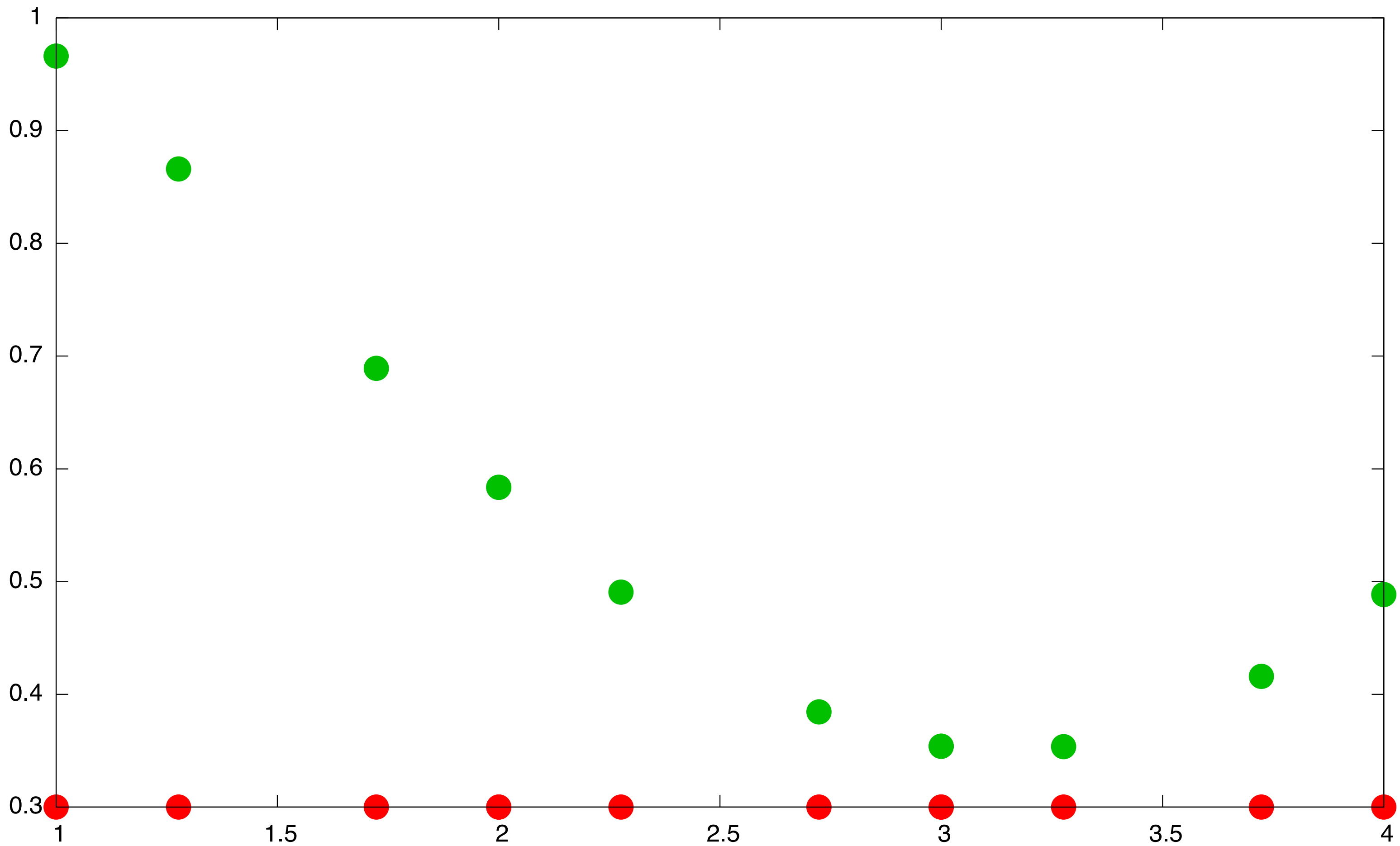
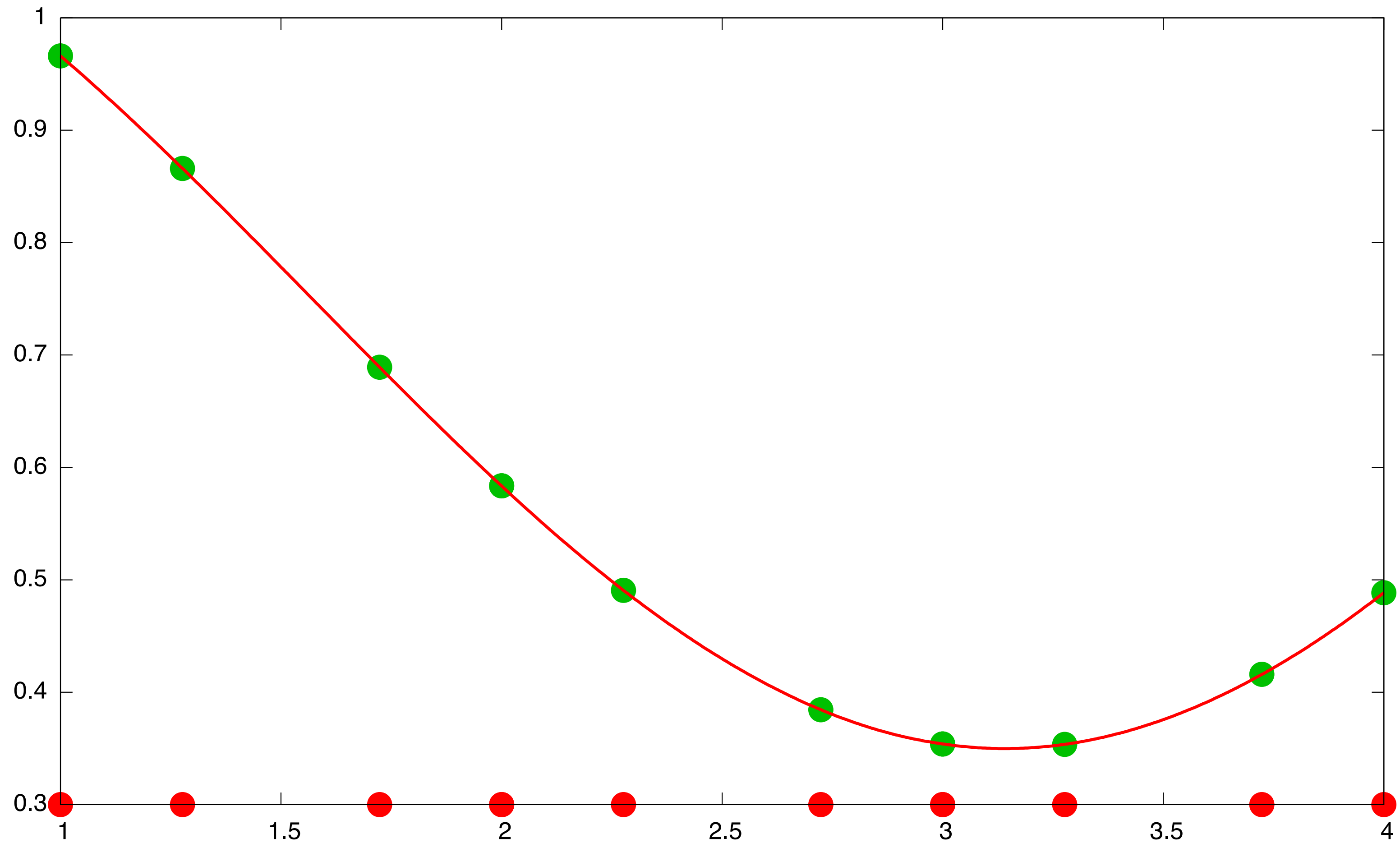
- Computational grid: 3 elements, 4 quadrature points in each element (np=4)
- This quadrature will integrate polynomials of degree 3 exactly
- Note: quadrature points are duplicated on element edges



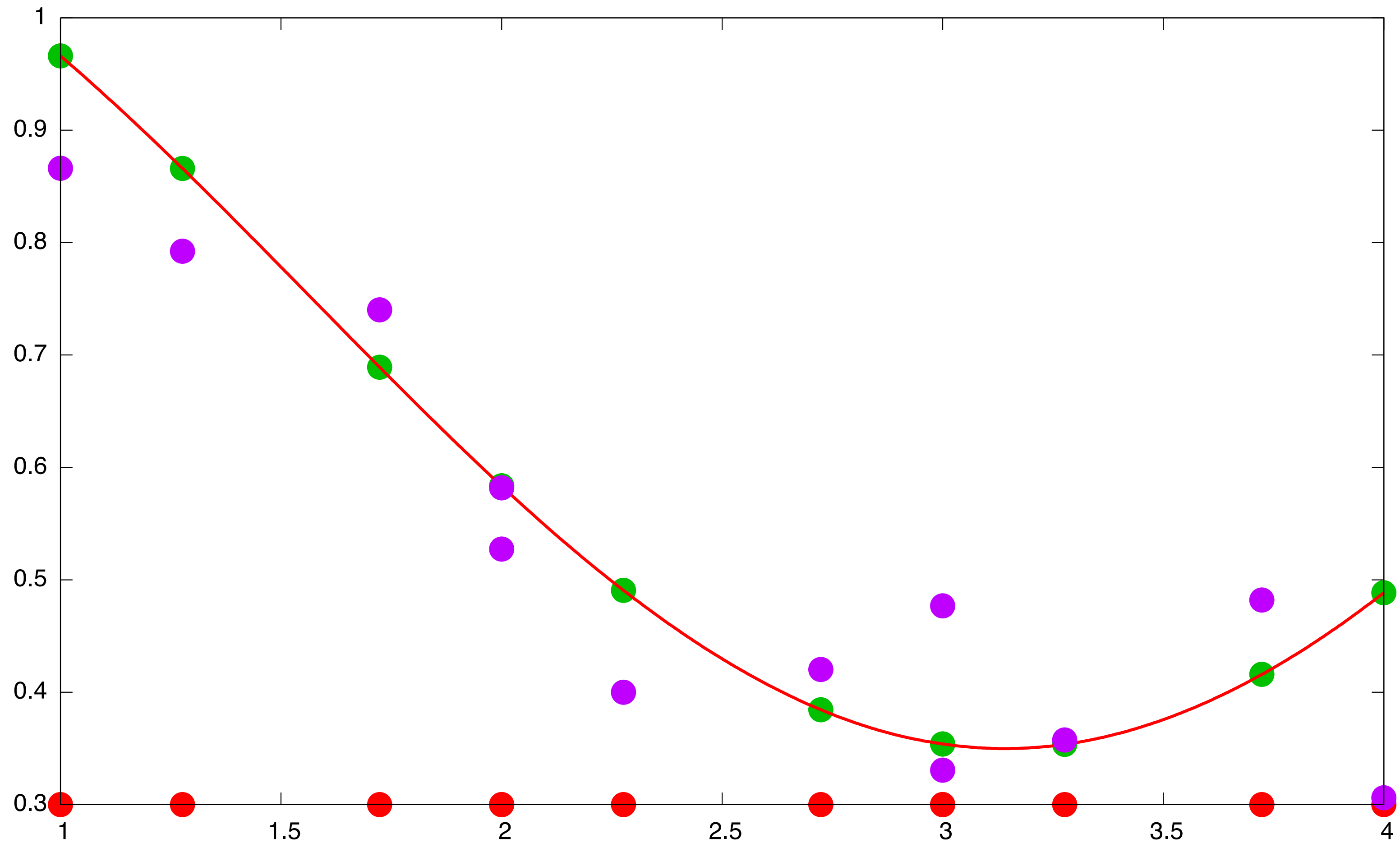
- Let the initial condition for GLL point values be a degree 3 polynomial



- Let the initial condition for GLL point values be a degree 3 polynomial
- The polynomial basis exactly represents initial condition



- Within each element the dynamical core advances one Runga-Kutta step
- Note each element advances the solution in time independently



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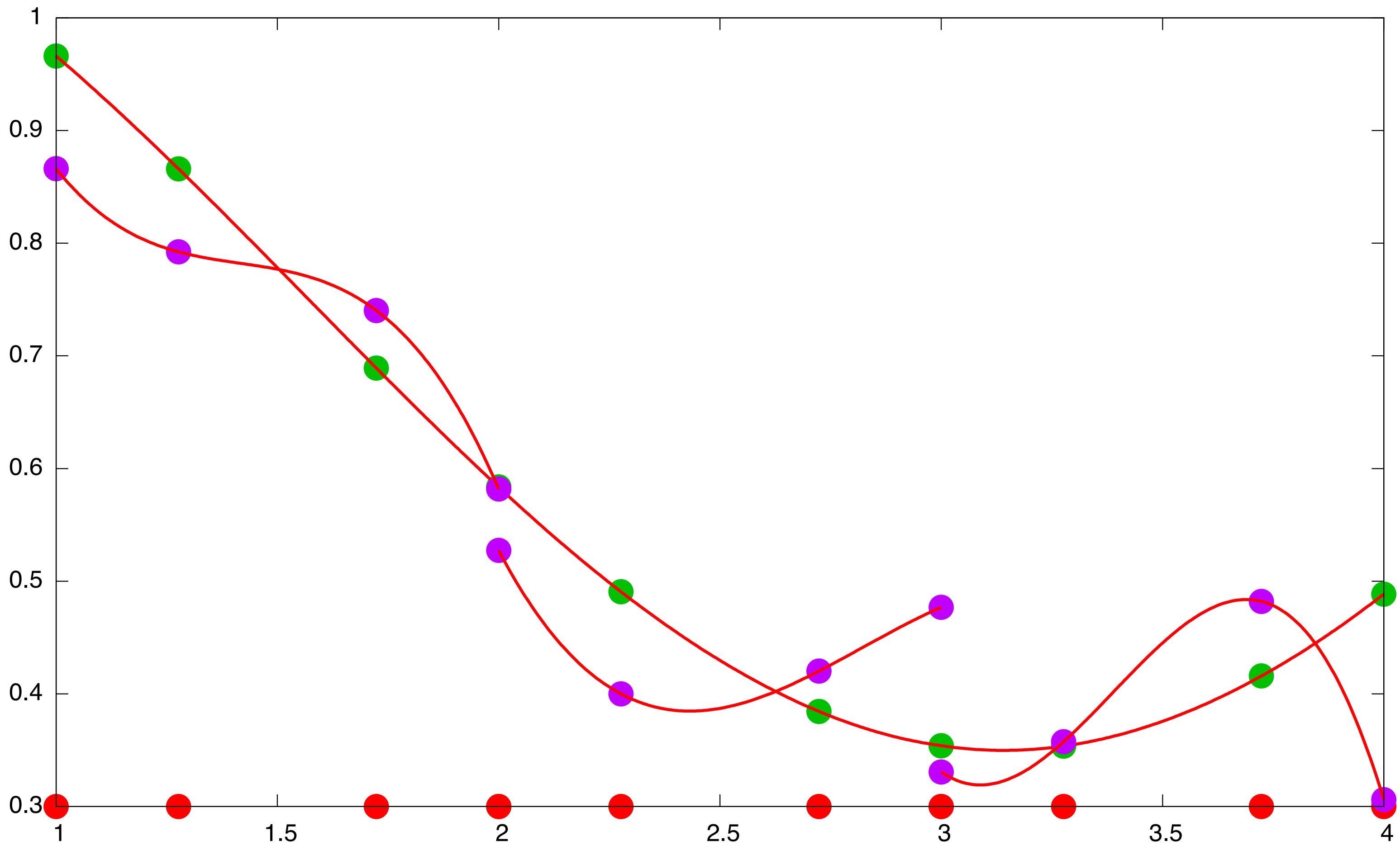
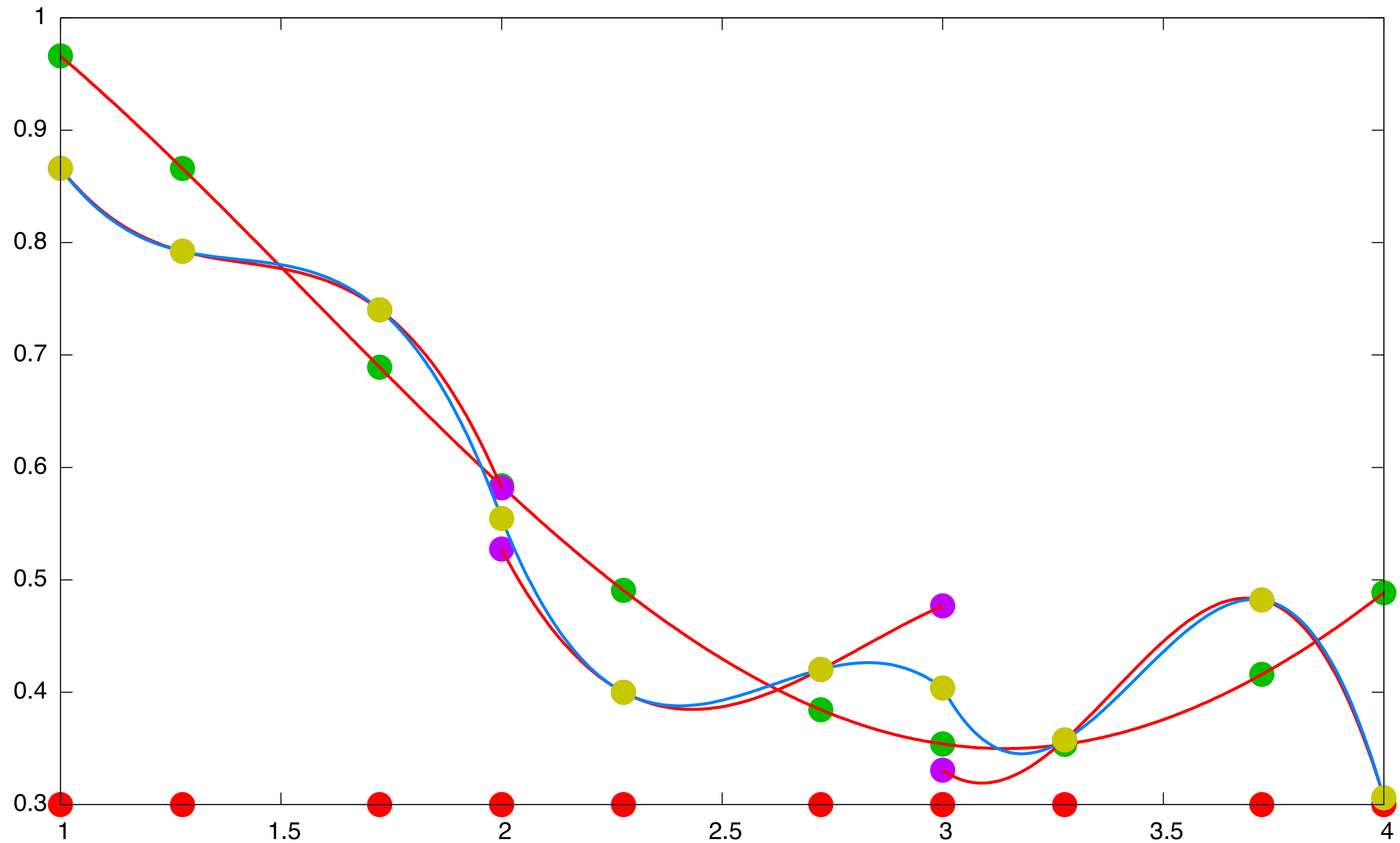
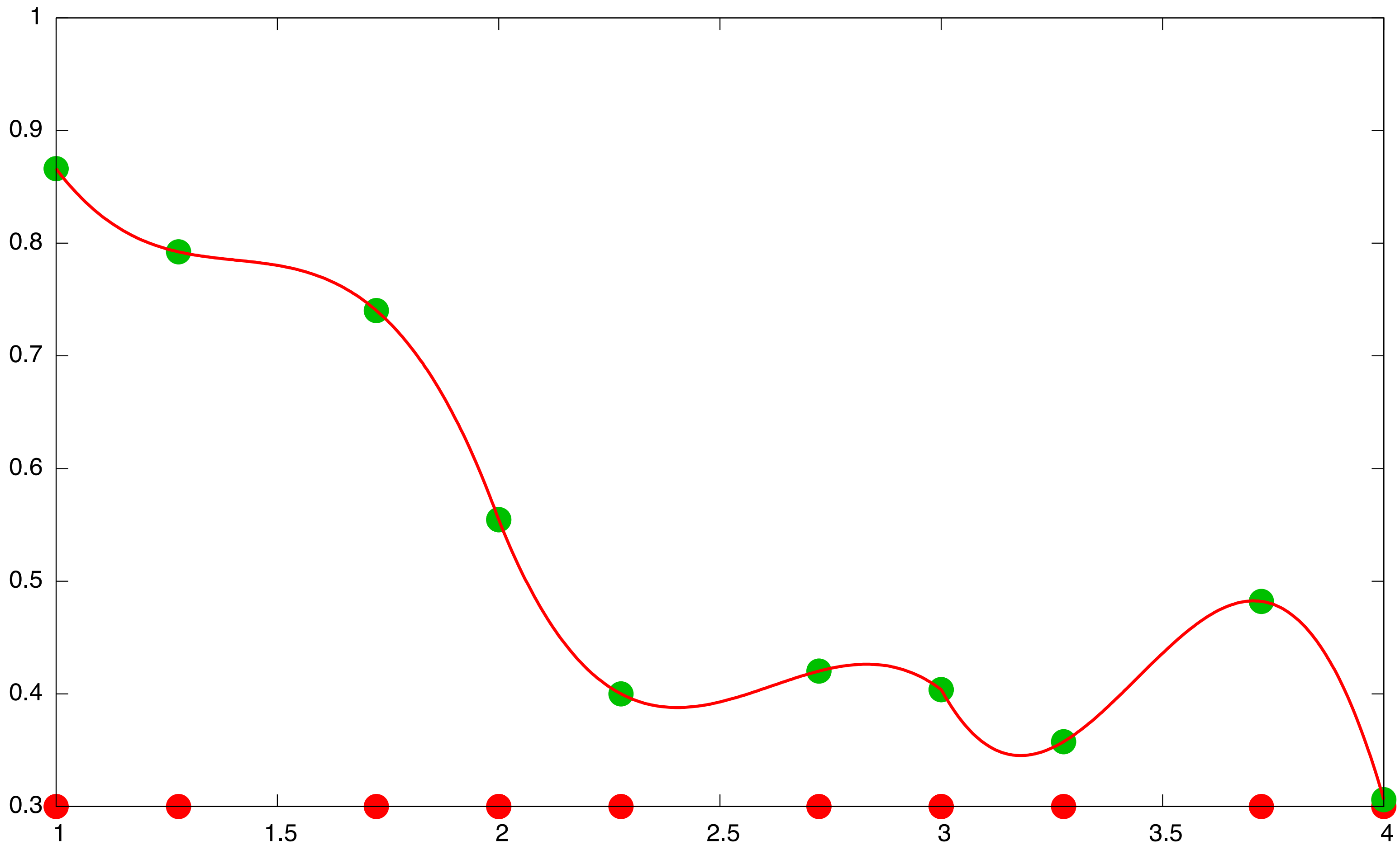


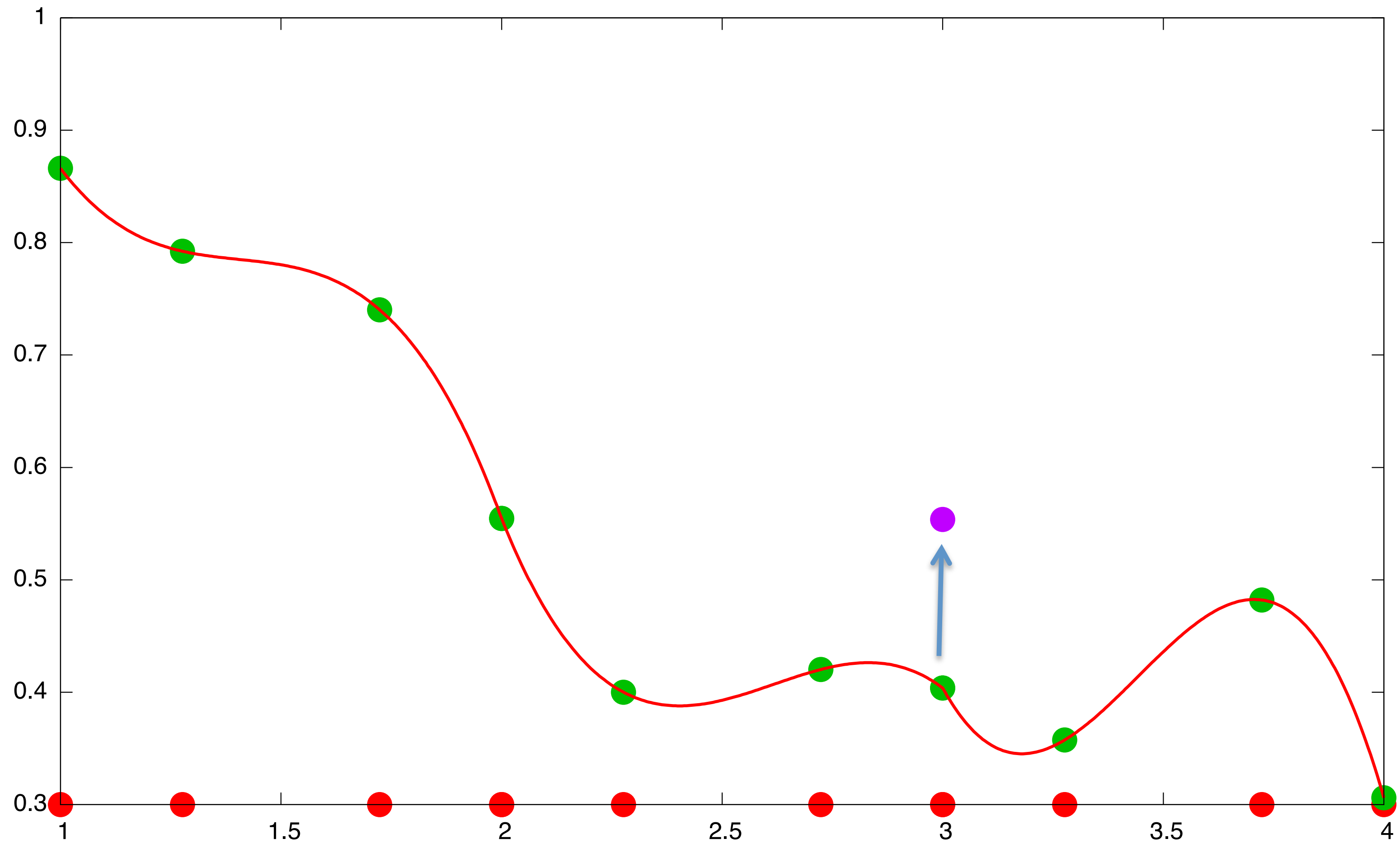
- Within each element the dynamical core advances one Runga-Kutta step
- Note each element advances the solution in time independently
- Discontinuities may develop at element edges **– averaging at element edges**



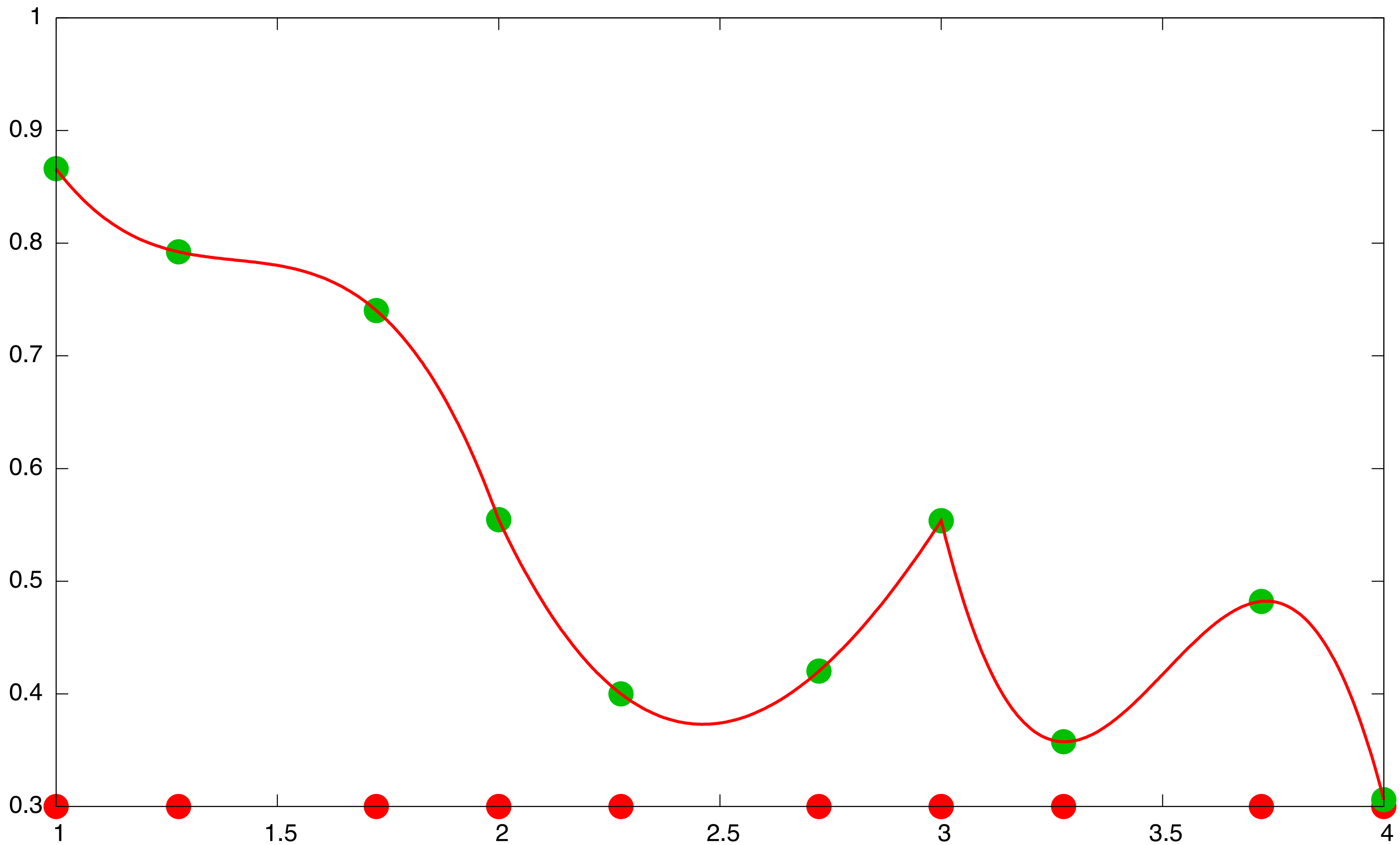
- This process is repeated for every Runga-Kutta stage (currently 5 times per dynamics time-step)
- **Physics is "run on GLL grid"**



- Physics update: say it perturbs one point value



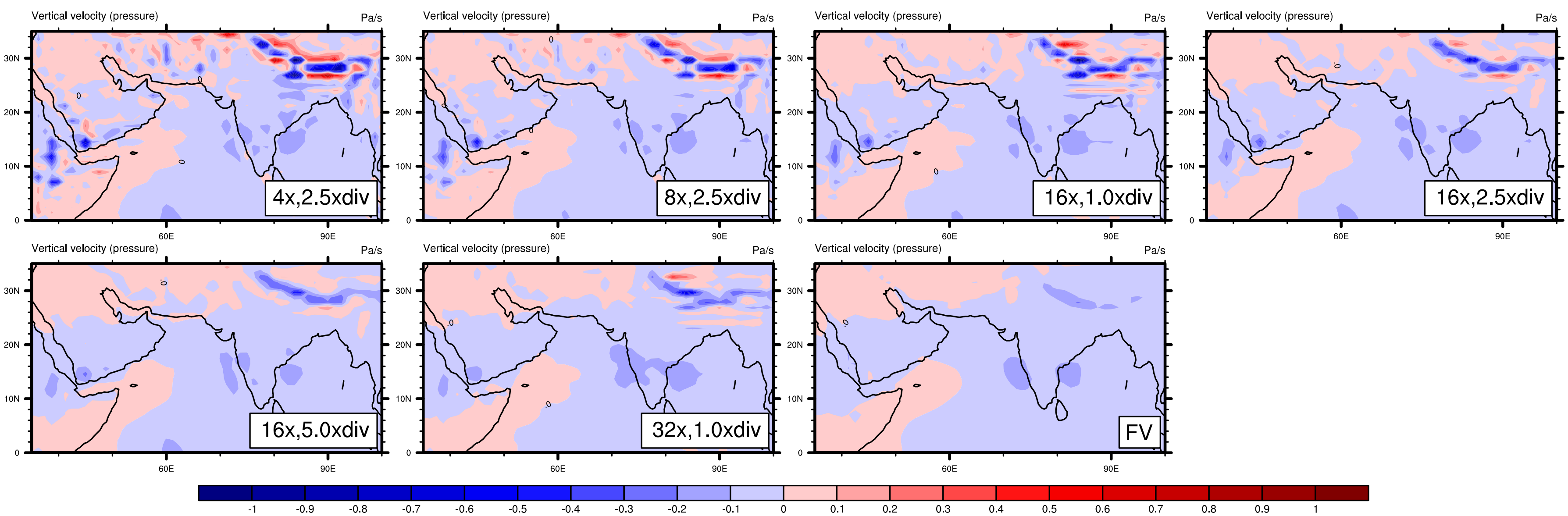
- Physics update: say it perturbs one point value
- **Polynomial basis changed in element 2**
- **Basis functions only C^0 at element edges**



Topography smoothing in CAM

30 year AMIP simulations

OMEGA, JJA, model level 16 (approximately 323 hPa)



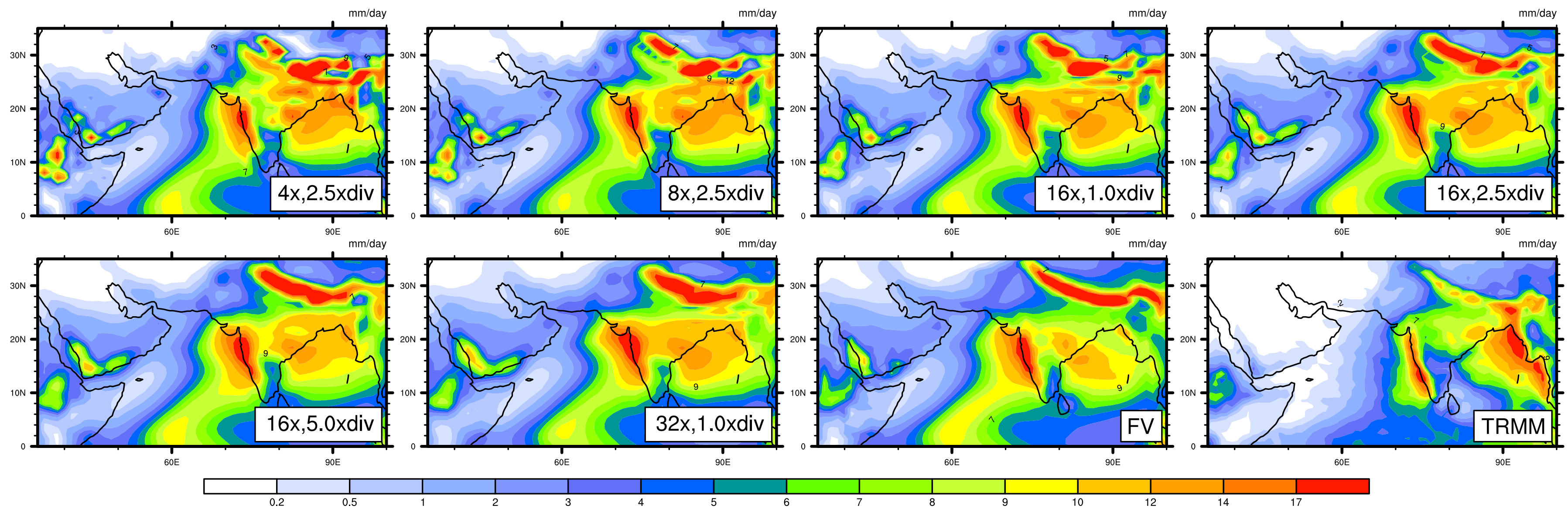
Notation: 2.5xdiv = 2.5^2 times more divergence damping than vorticity damping
4x, 8x, …, 32x = smoothing of surface geopotential height

Lauritzen et al., (2015): *NCAR Global Model Topography Generation Software for Unstructured Grids*

Topography smoothing in CAM

30 year AMIP simulations

Total precipitation rate



Mean sea level pressure differences, DJF, diff

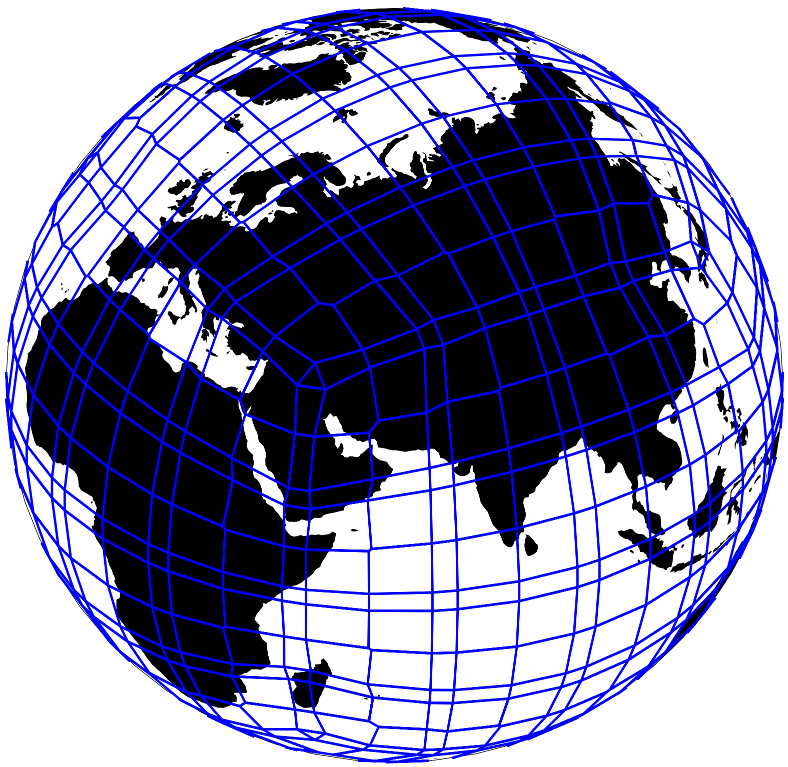
Lauritzen et al., (2015): *NCAR Global Model Topography Generation Software for Unstructured Grids*

How do we (/should we?) couple the dynamical core with sub-grid scale parameterizations (physics)?

Traditionally physics and dynamics grids are collocated

- smoothly varying grid in terms of grid size
- Much higher resolution near poles, however, dynamical core usually has filter in the polar regions to filter out small scales
- Aside: Lat-lon grid is “optimal” for minimizing zonal flow errors! … when grid is no longer zonally aligned errors get rather large ….

Traditionally physics and dynamics grids are collocated

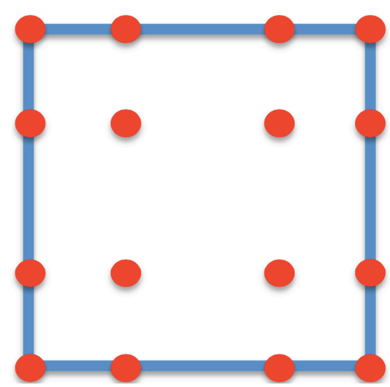


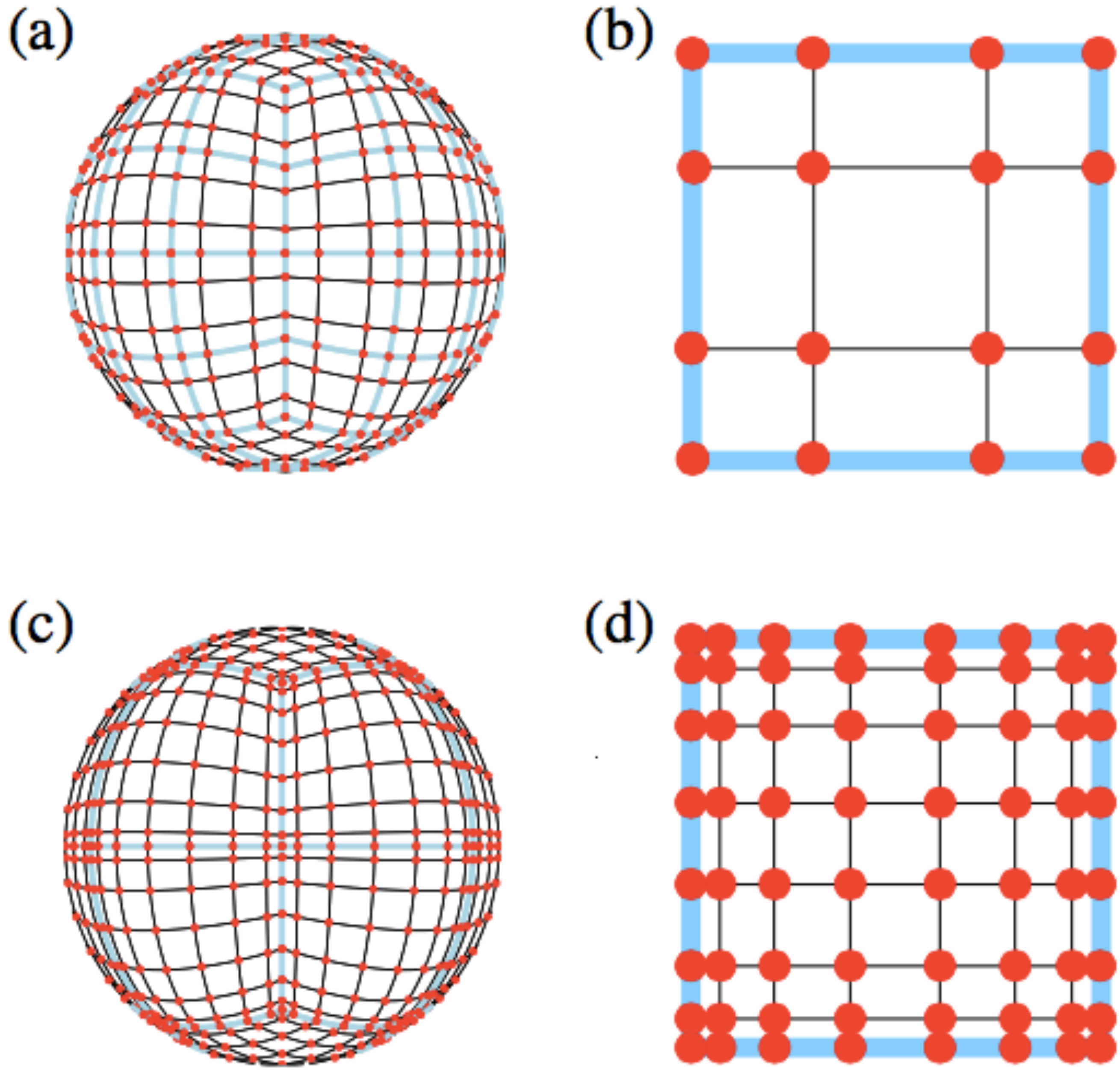
Np=4

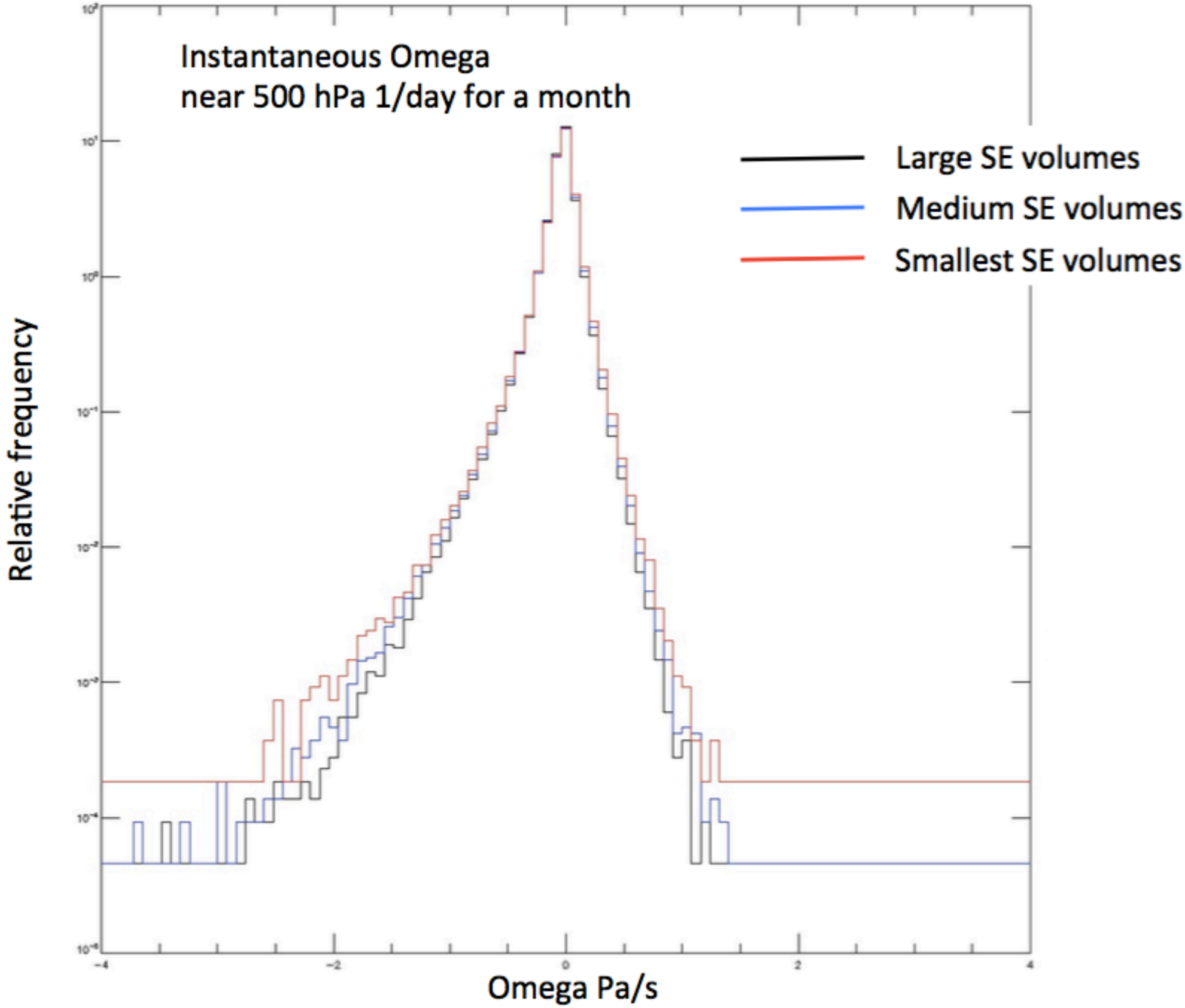
If you construct control volumes around the quadrature points so that the area of the control volumes equals the Gaussian quadrature weight (times metric term) then a very anisotropic grid results

Gets “worse” with:

- mesh-refined grids
- increasing polynomial order



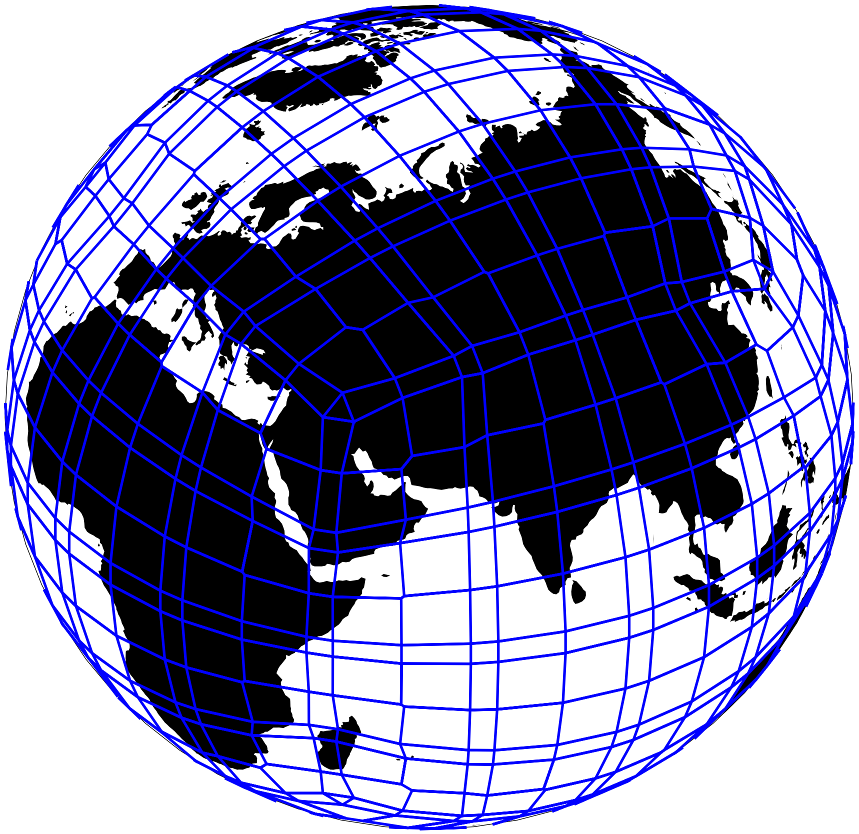

(a)
(b)
(c)
(d)



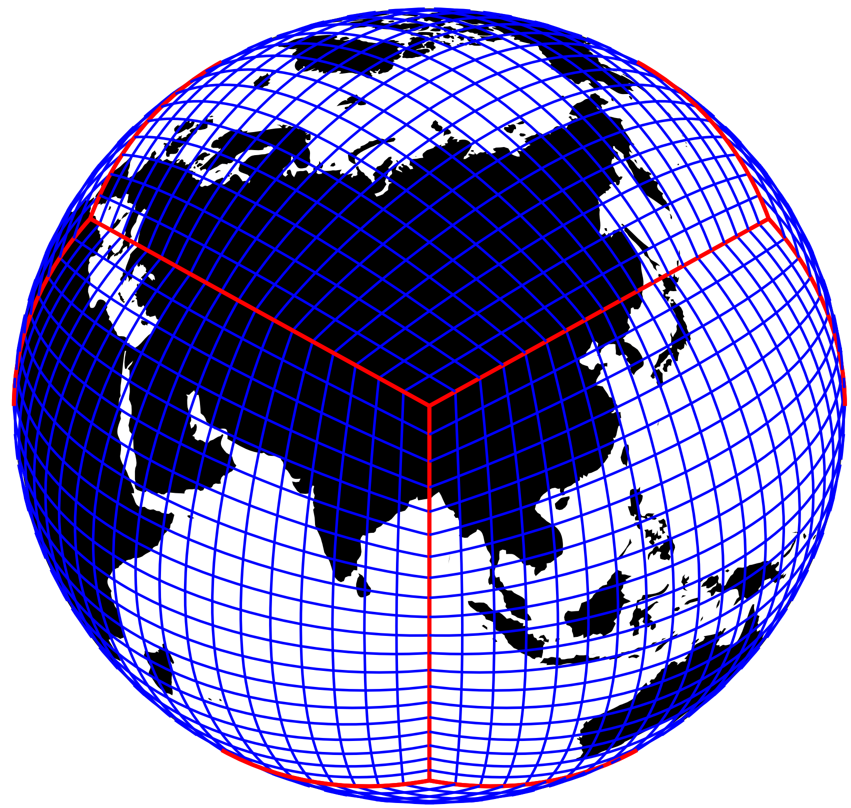
Instantaneous Omega
near 500 hPa 1/day for a month
Large SE volumes
Medium SE volumes
Smallest SE volumes
Relative frequency
Omega Pa/s

Separate physics-dynamics grids?

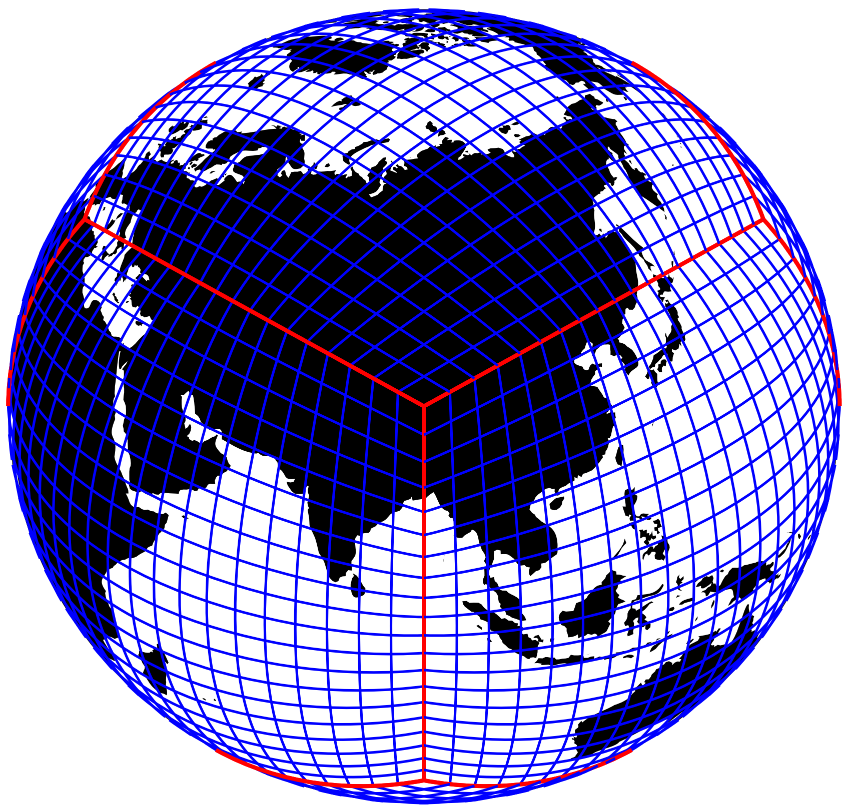
Current physics/“coupler” grid

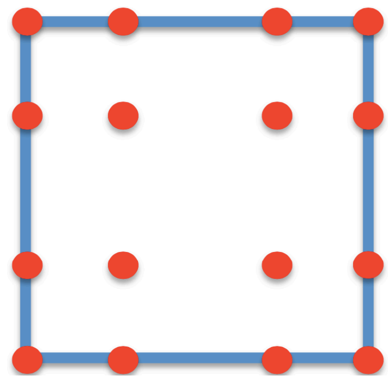


Finite-volume equi-angular gnomonic grid



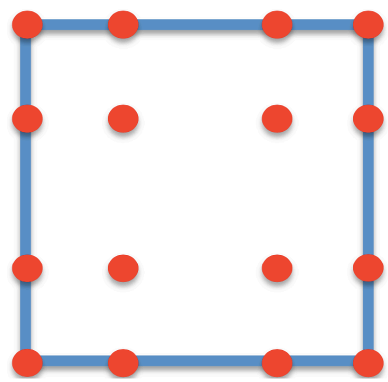
CAM-SE default configuration



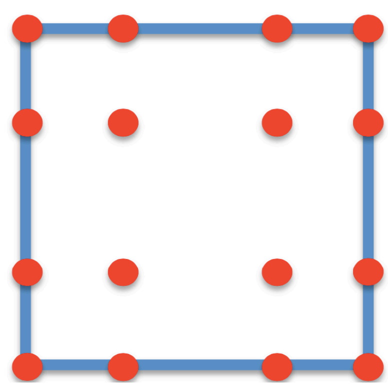


Dynamics: Spectral-element dynamics on Gauss-Lobatto-Legendre (GLL) nodal values

(4x4 GLL point in each element; degree 3 Lagrange polynomials)

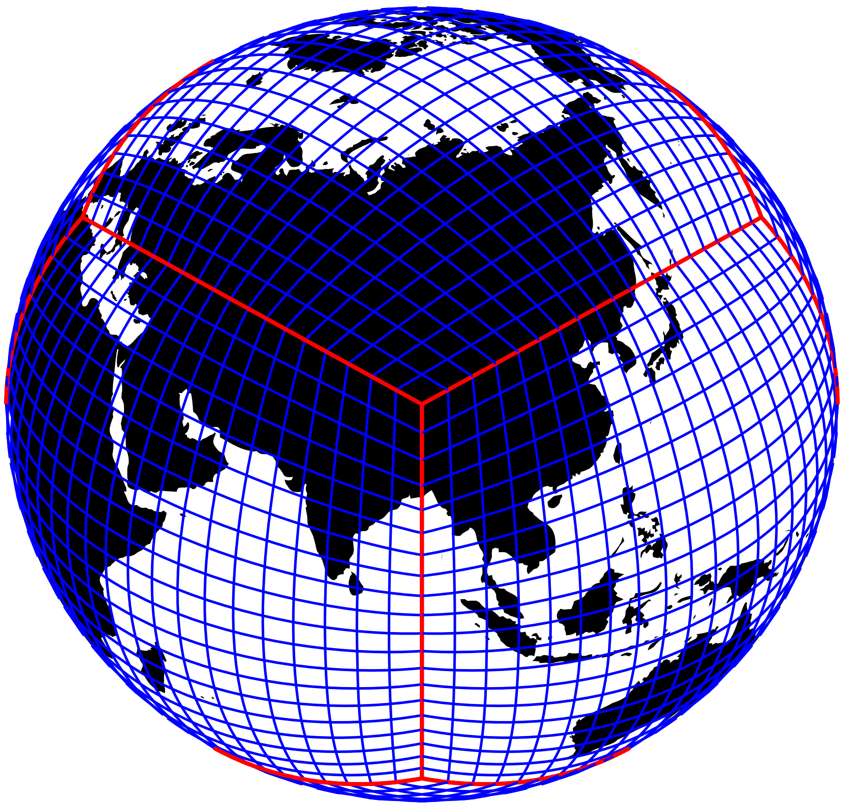


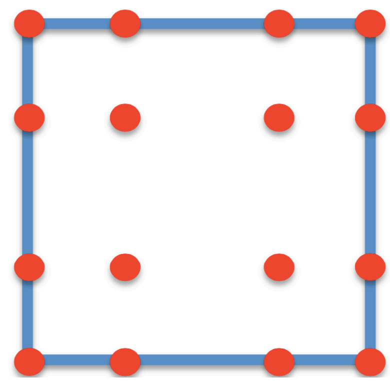
Tracer advection: Spectral-element method that is element-wise conservative and shape-preserving at the node level



Physics: Physics columns computed at GLL nodal values

CAM-SE-CSLAM configuration





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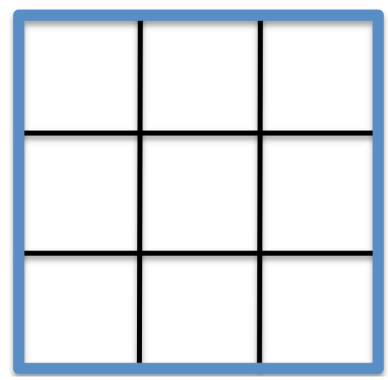
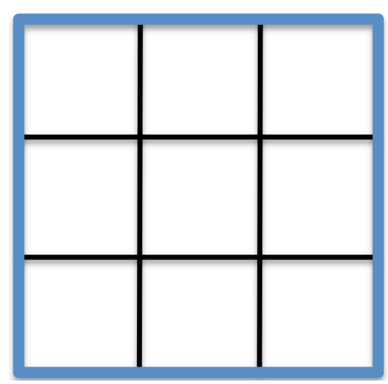
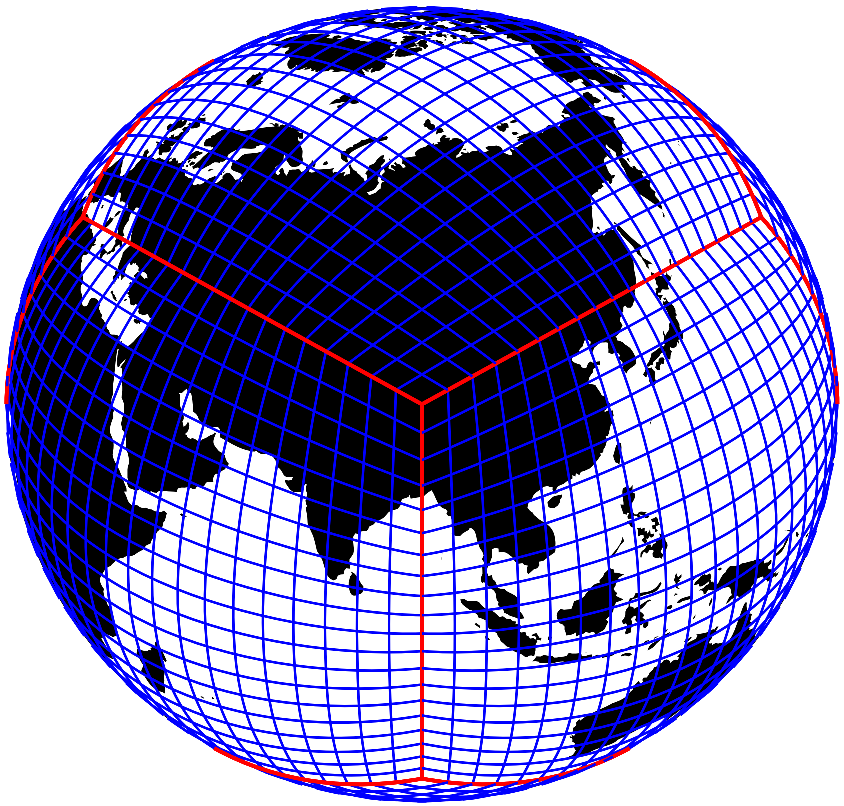


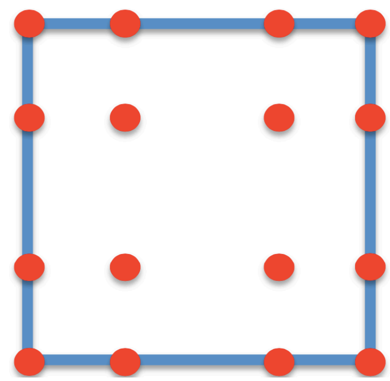
Tracer advection: Conservative Semi-Lagrangian Multi-tracer transport scheme (CSLAM)



Physics: Physics columns using cell-averaged state of atmosphere

CAM-SE-physgrid configuration





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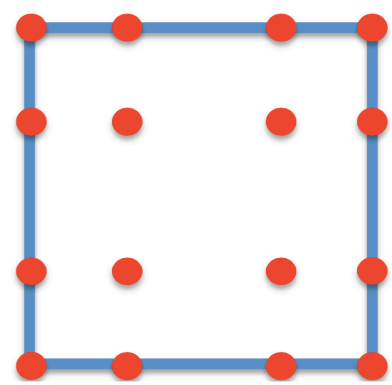

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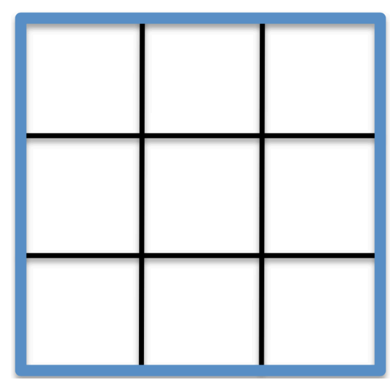

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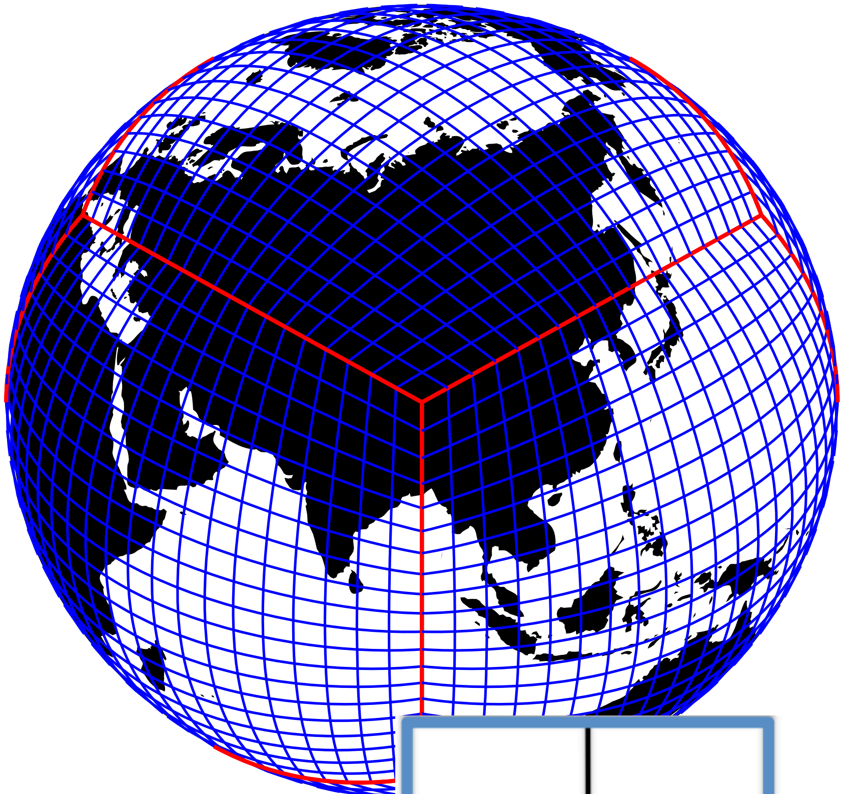

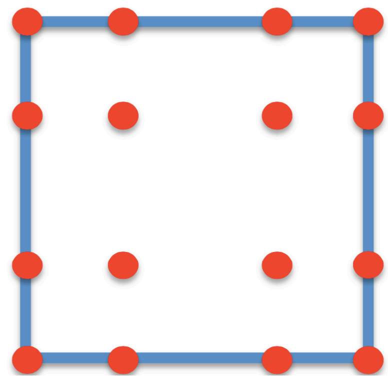

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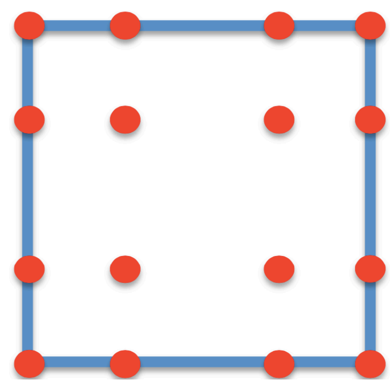
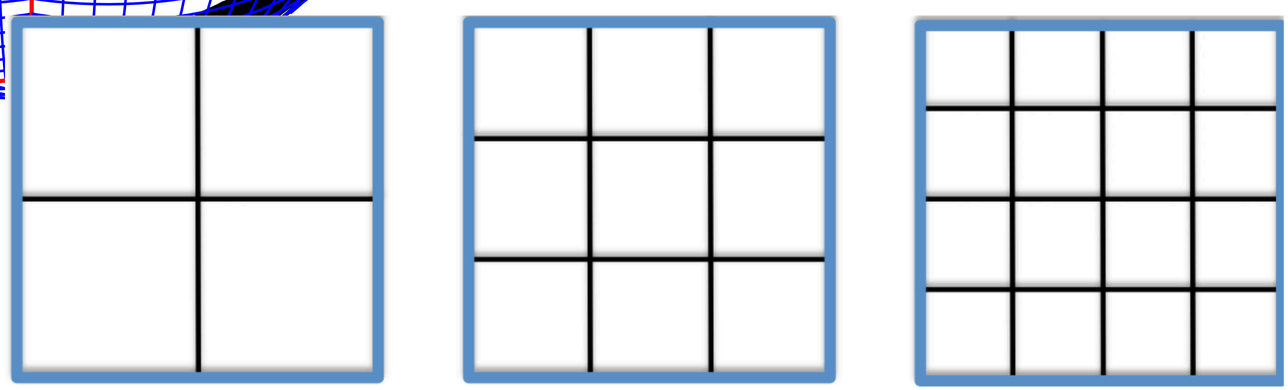
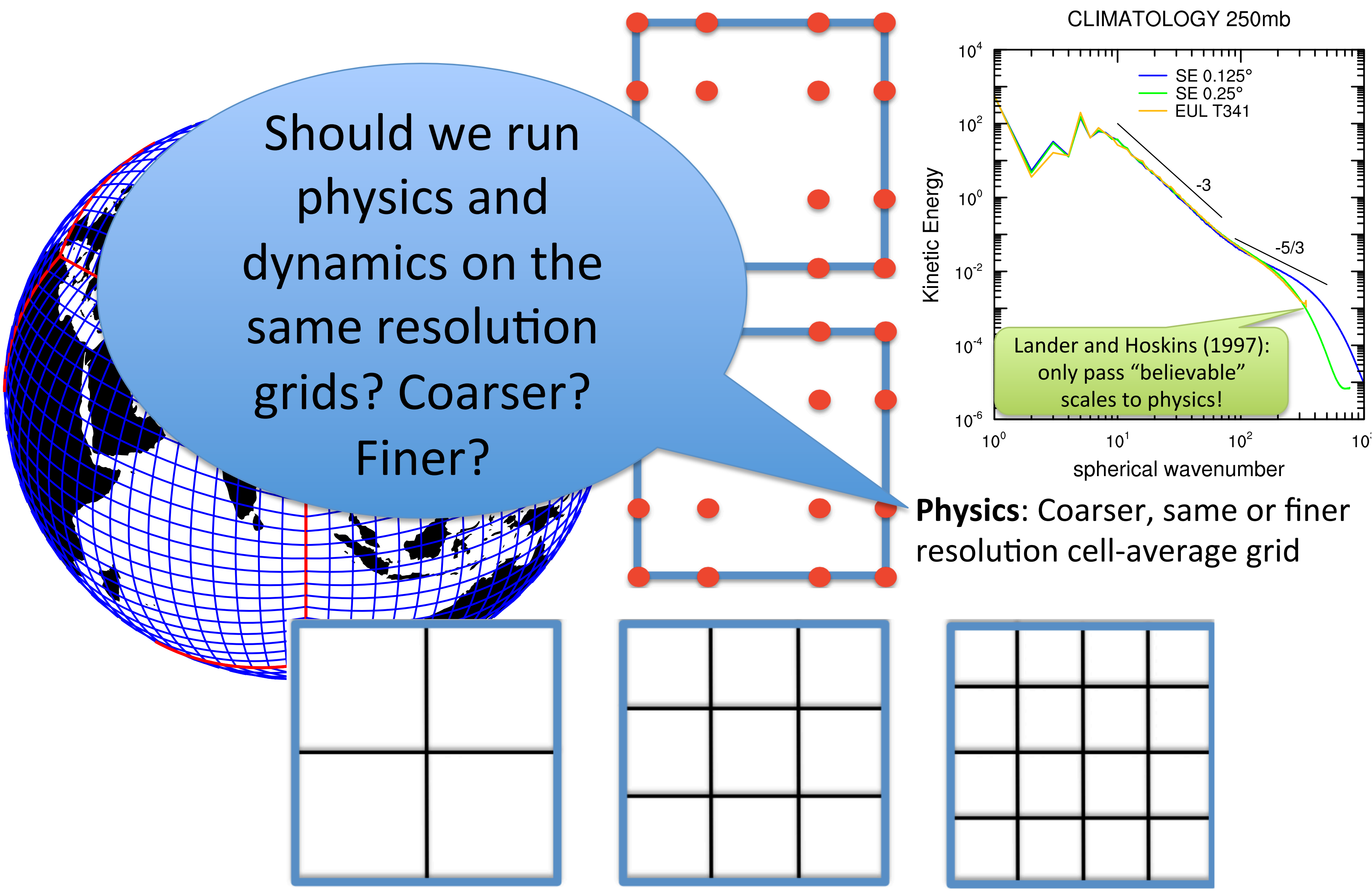


Tracer advection: Spectral-element method

Physics: Coarser, same or finer resolution cell-average grid



CAM-SE-physgrid configuration



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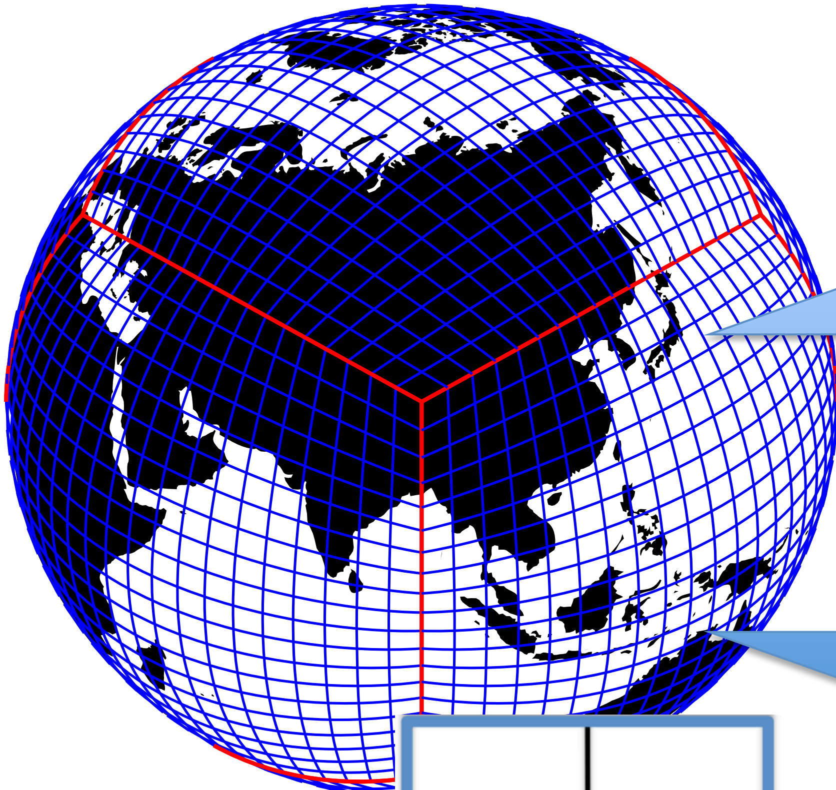

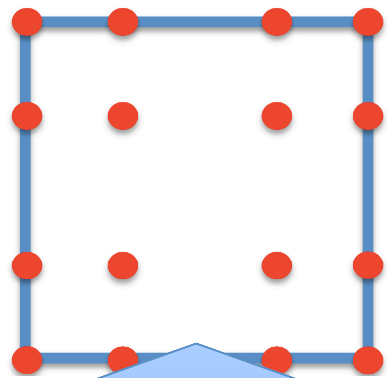

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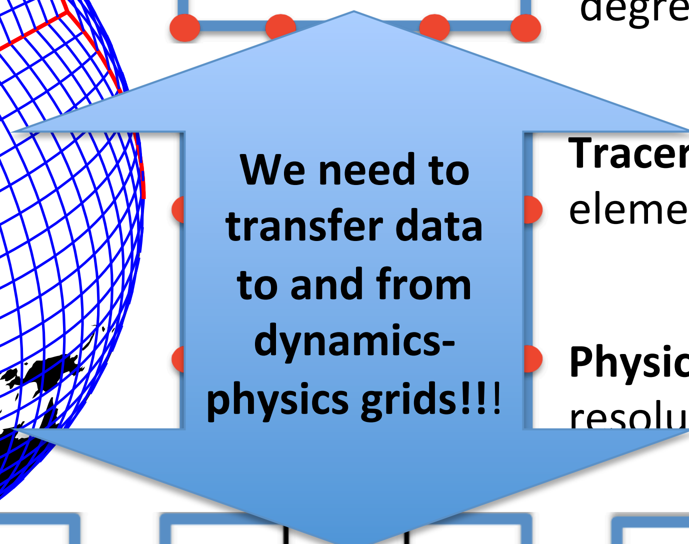

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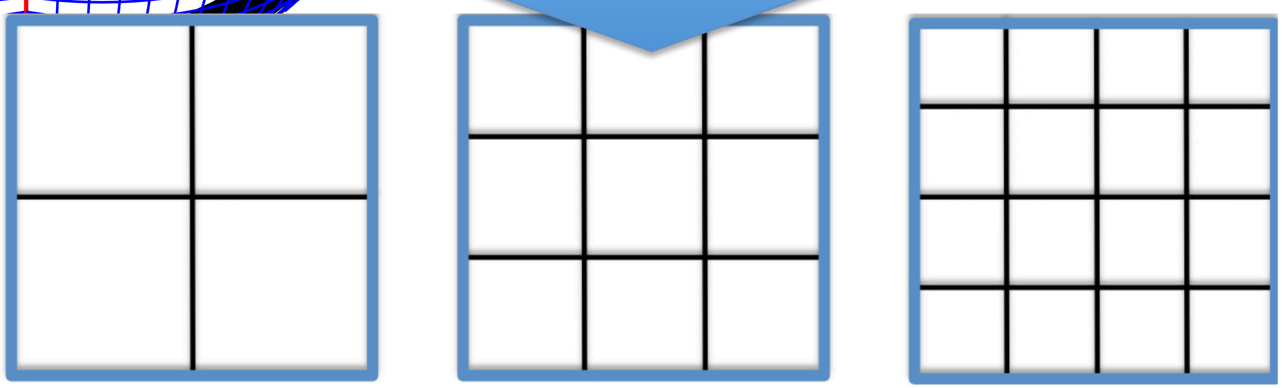

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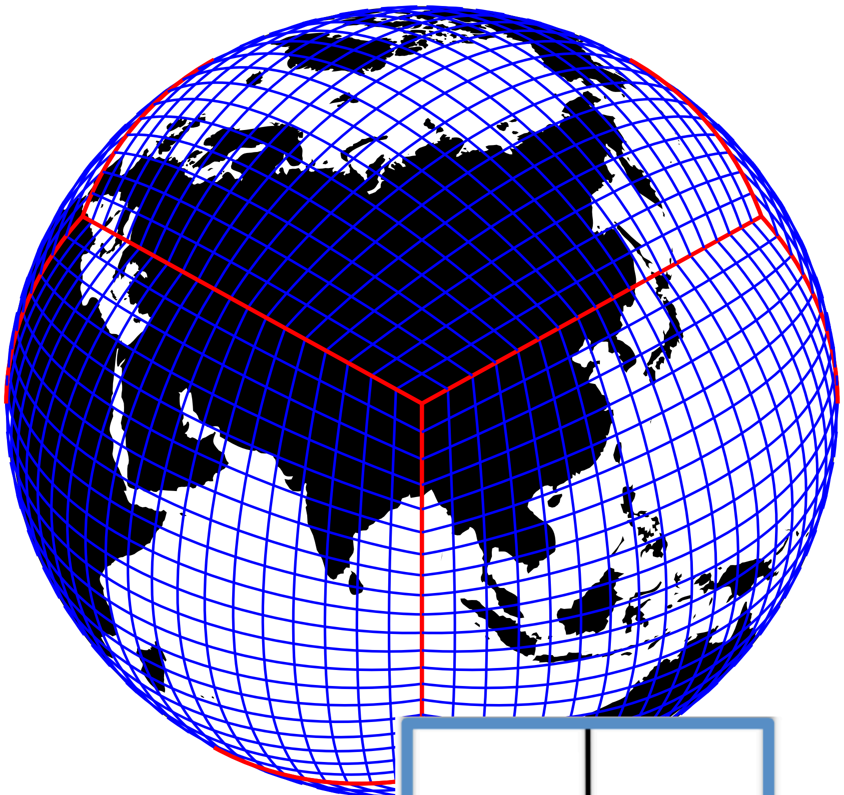
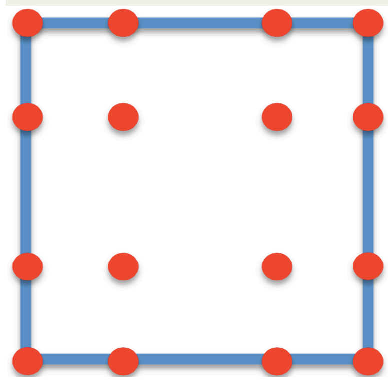


Notation

- **NE*NE elements on each cubed-sphere panel**
- **NP*NP quadrature points in each element (note quadrature points are duplicated on the element boundary)**
- **NC*NC physics grid columns in each element**



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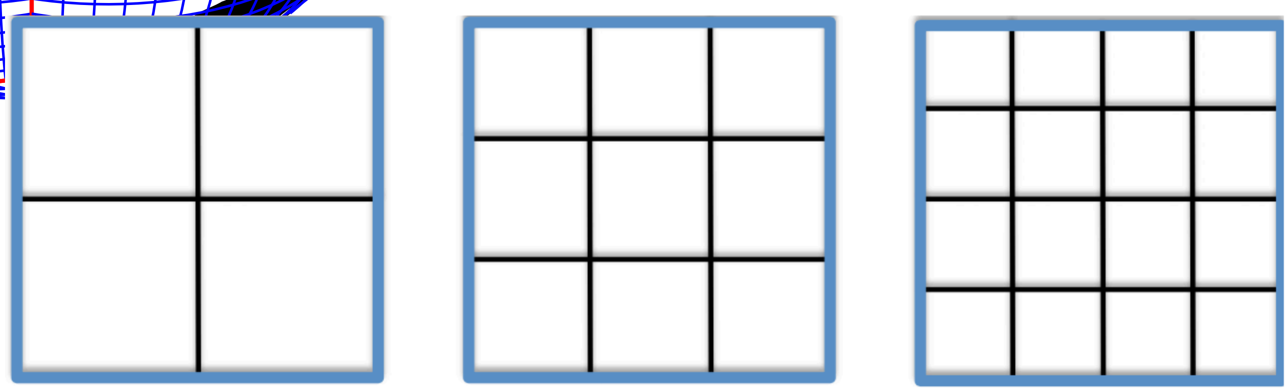
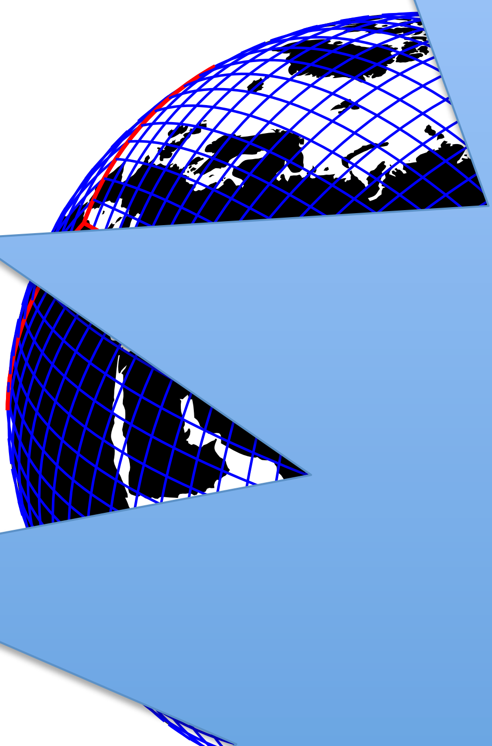


CAM-SE-physgrid configurati



Separating physics and dynamics grids was a major software engineering task in CAM – affected many parts of the code:

- **history (output)**
- **initialization/restart**
- **Some parameterizations assumed grids were collocated**

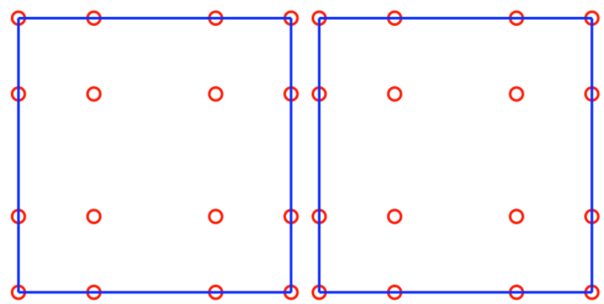
Interpolator properties: passing state to physics and returning tendencies to dynamics

- Conservation (coupled climate modeling)
- Shape-preservation (in particular, no negatives)
- Preserve tracer correlations (important for coupling with chemistry)
- Consistent (preserves a constant)
- Other? Total energy?

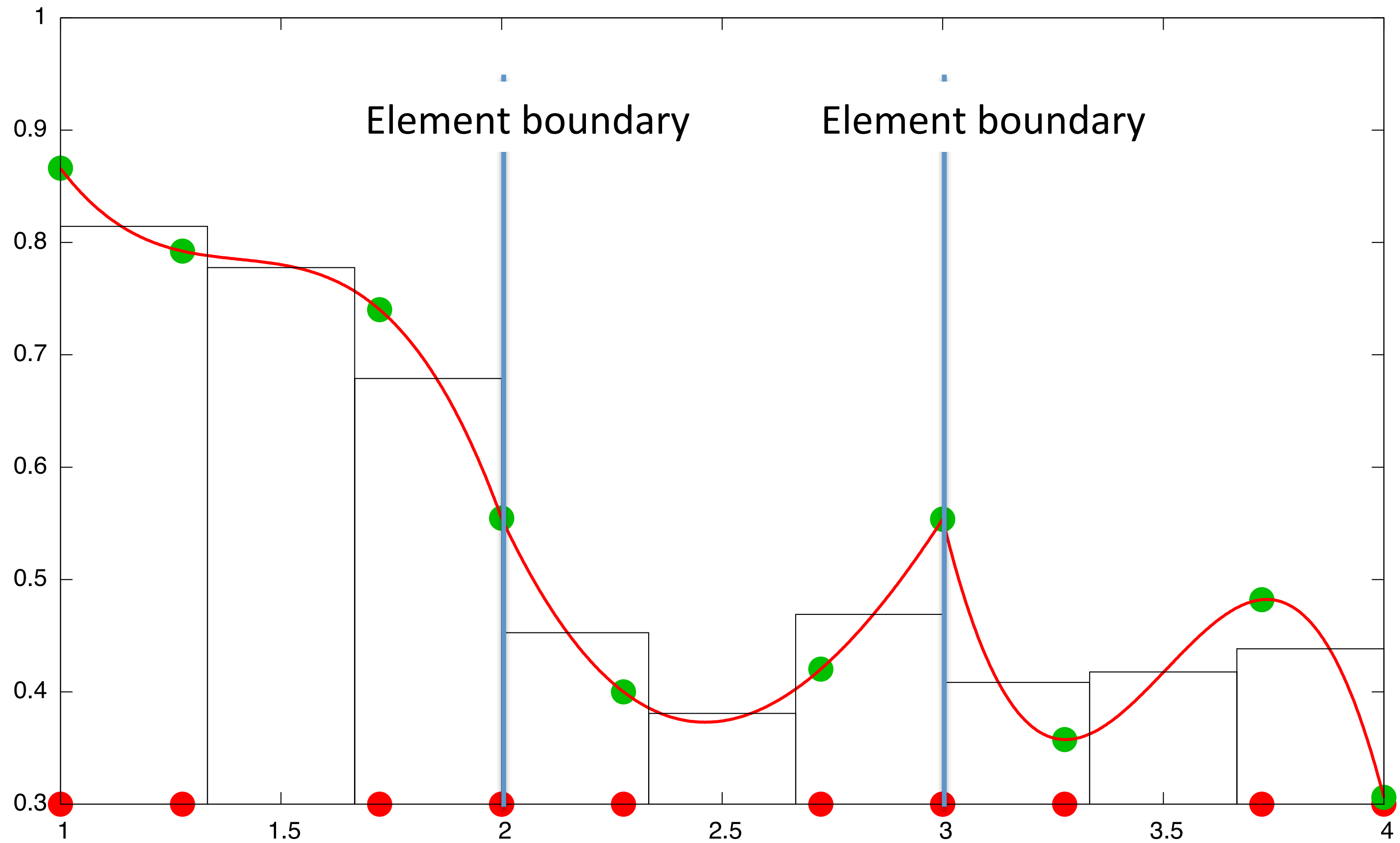
Implementation constraints/limitations (not "physical" limitations):

- Physics-grid must be a sub-grid of the element
  With some extra software engineering we can relax this constraint!
  (example application: mesh-refinement)

- To reduce MPI communication no halo exchange for physics-dynamics coupling except for boundary exchange at end of interpolation
  (could also be relaxed at the expense of computational cost)



Passing state (v,T,q,...) to physics: For conservation we interpolate dp*u, dp*T, dp*q



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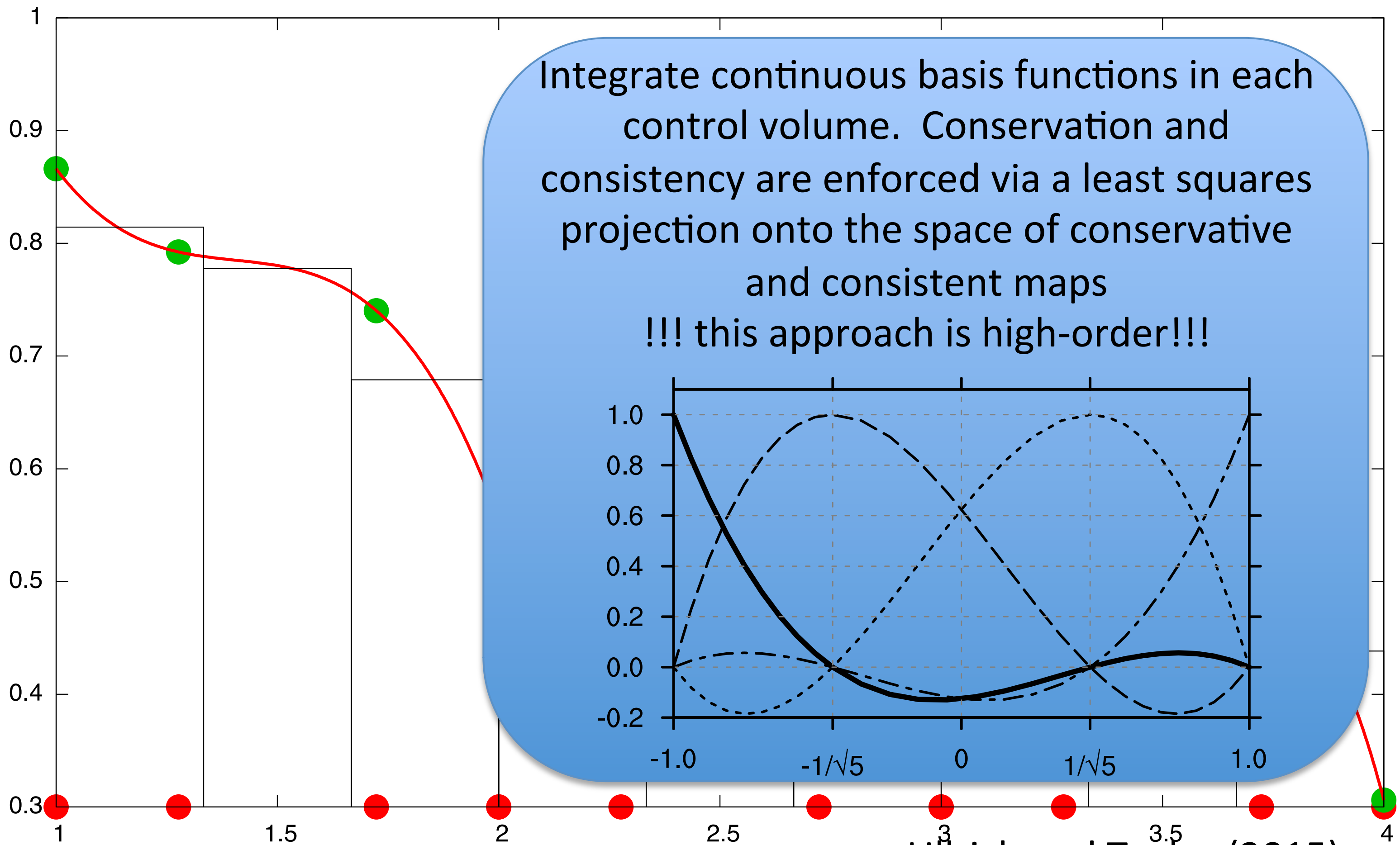

Ullrich and Taylor (2015)

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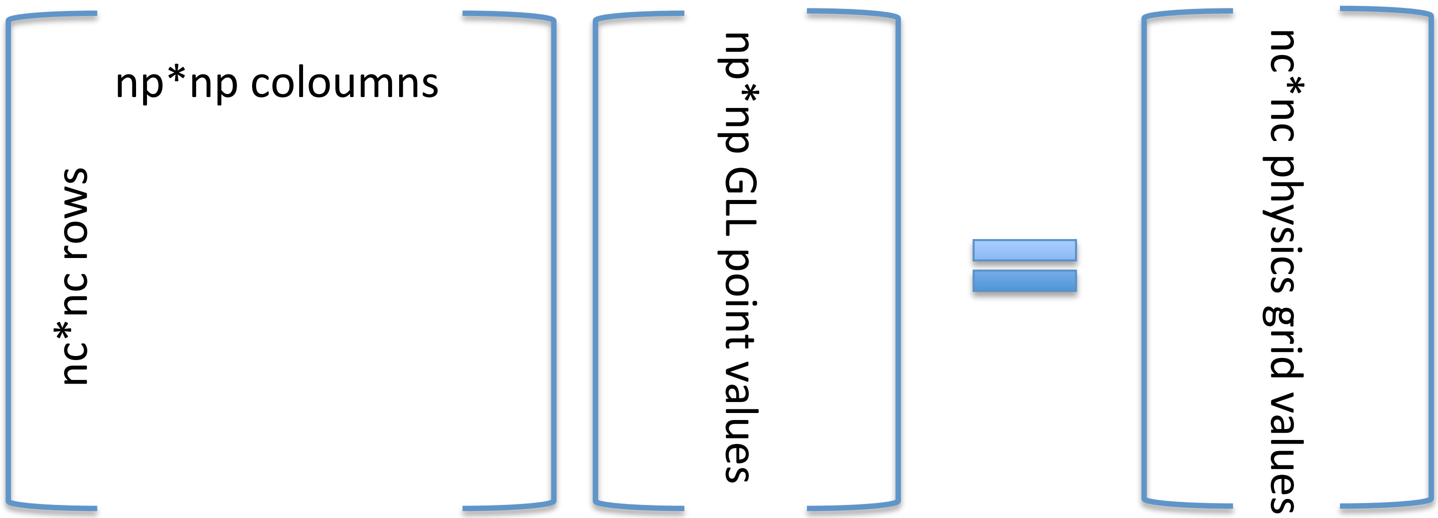
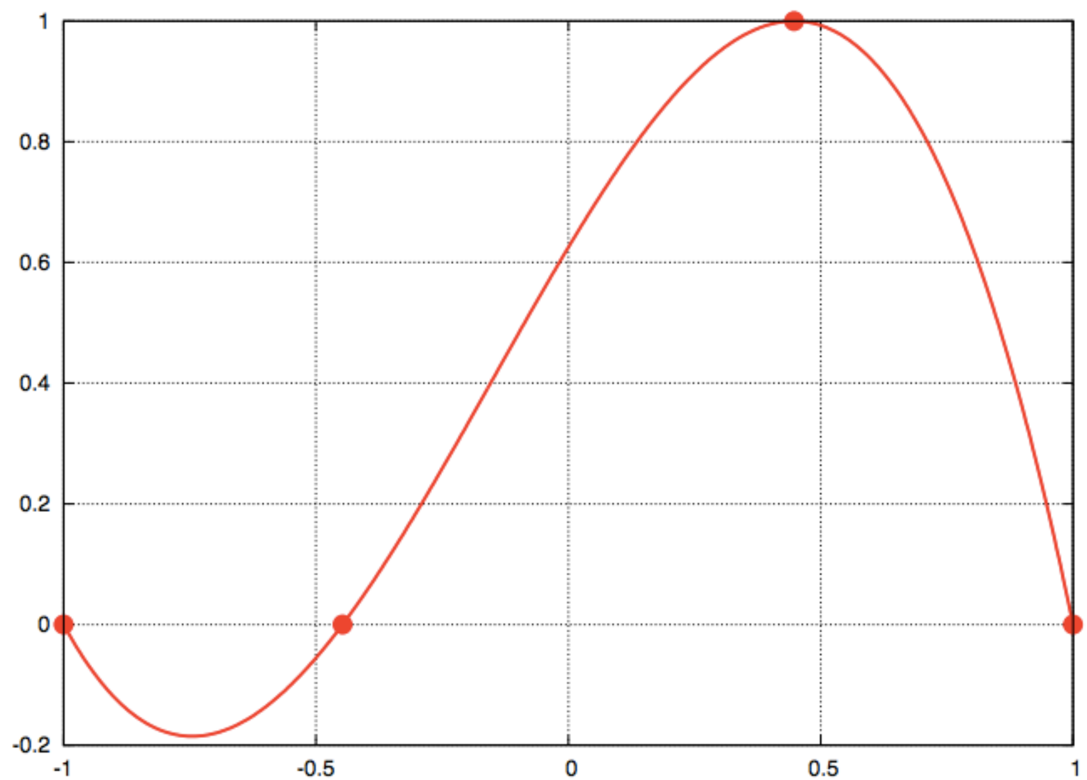


- Interpolation matrix can be pre-computed (it is a linear map)!!!
- After application of interpolation matrix there is a boundary exchange that averages point values on the element boundaries!

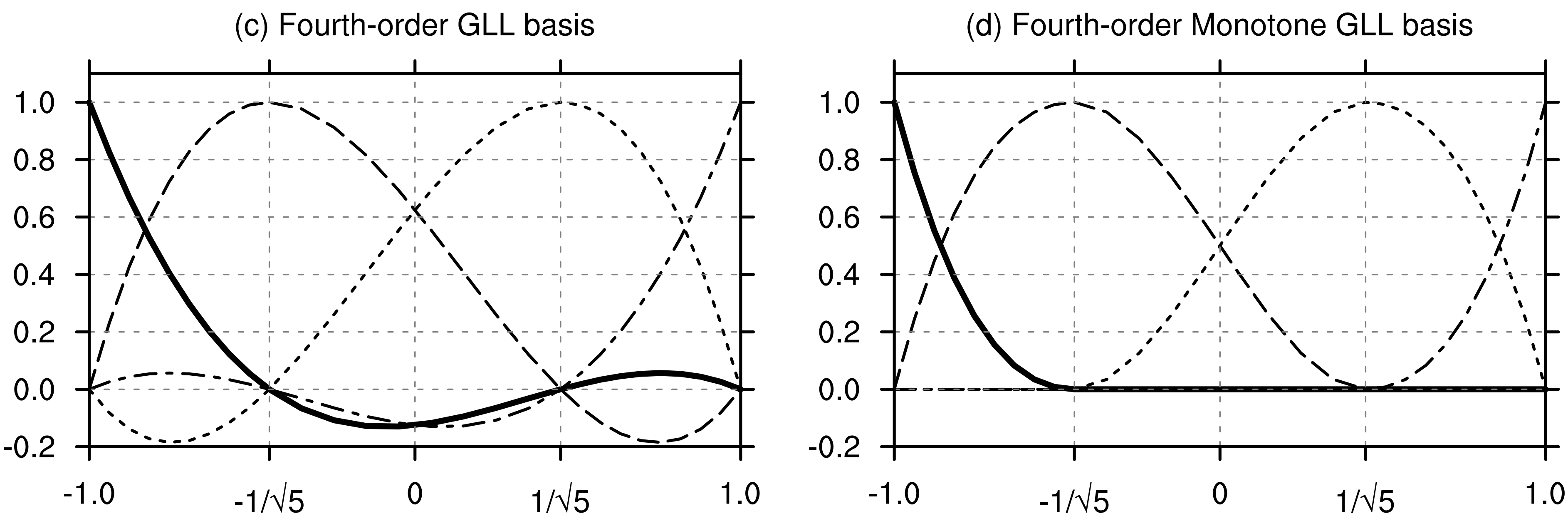
Ullrich and Taylor (2015)

Passing state (v,T,q,…) to physics: basis functions oscillatory!

Given GLL point values, $U_{j,k}(t)$ = {0,0,1,0} for k=0,..,3, the Lagrange "reconstruction" is shown on the Figure below:



Monotone linear map

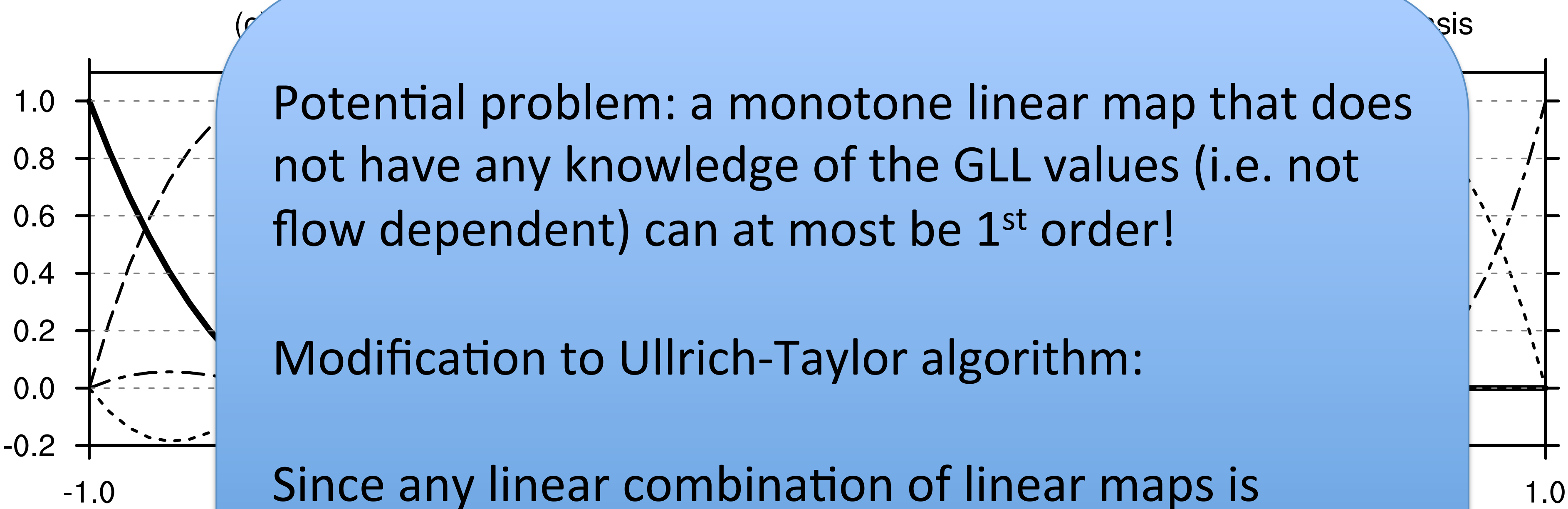


Monotonicity is enforced via a two-step procedure.

- instead of the regular FEM basis functions we use a set of monotone basis functions (ones whose range is [0,1]).
- This would be sufficient except for the fact that the least squares projection onto conservative/consistent maps could produce some (small) negative values in the mapping coefficients. To fix that problem we then "linearly interpolate" between the conservative/consistent map and the simplest first-order conservative/consistent/monotone map. This has roughly the effect of "borrowing mass" from other GLL nodes within the element.

Ullrich and Taylor (2015)

Monotone linear map



Potential problem: a monotone linear map that does not have any knowledge of the GLL values (i.e. not flow dependent) can at most be 1st order!

Modification to Ullrich-Taylor algorithm:

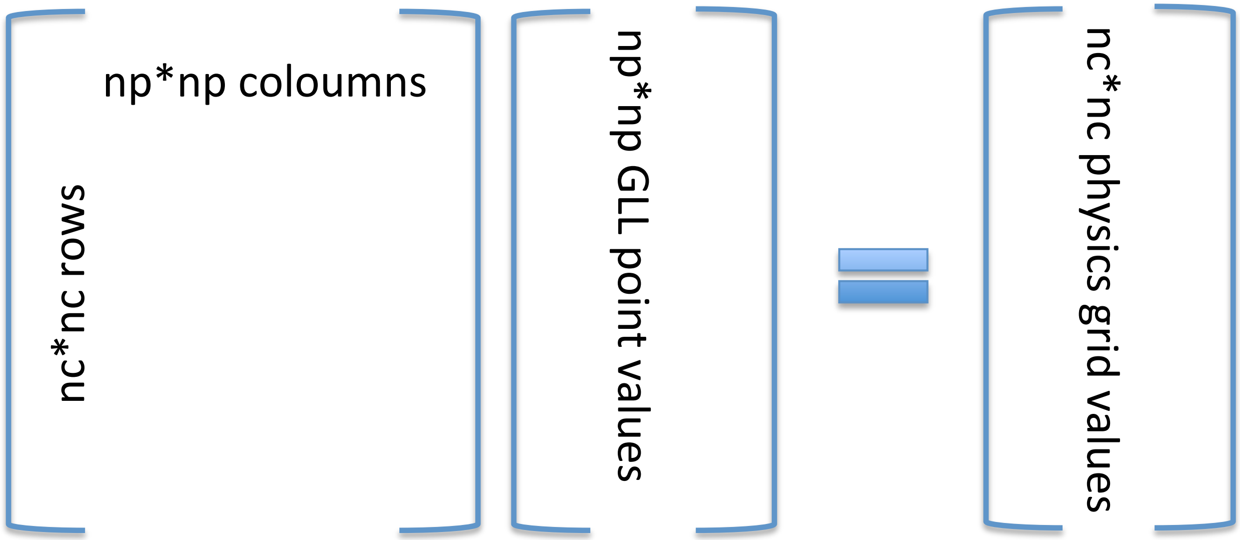
Since any linear combination of linear maps is conservative and consistent one may “optimally” blend the maps for shape-preservation (“FCT-like method”)

Monotonic

- instead functio
- This wo conserv mapping the conservative/consistent map and the simplest first-order conservative/ consistent/monotone map. This has roughly the effect of “borrowing mass” from other GLL nodes within the element.

Ullrich and Taylor (2015)

"FCT" version of Ullrich-Taylor algorithm



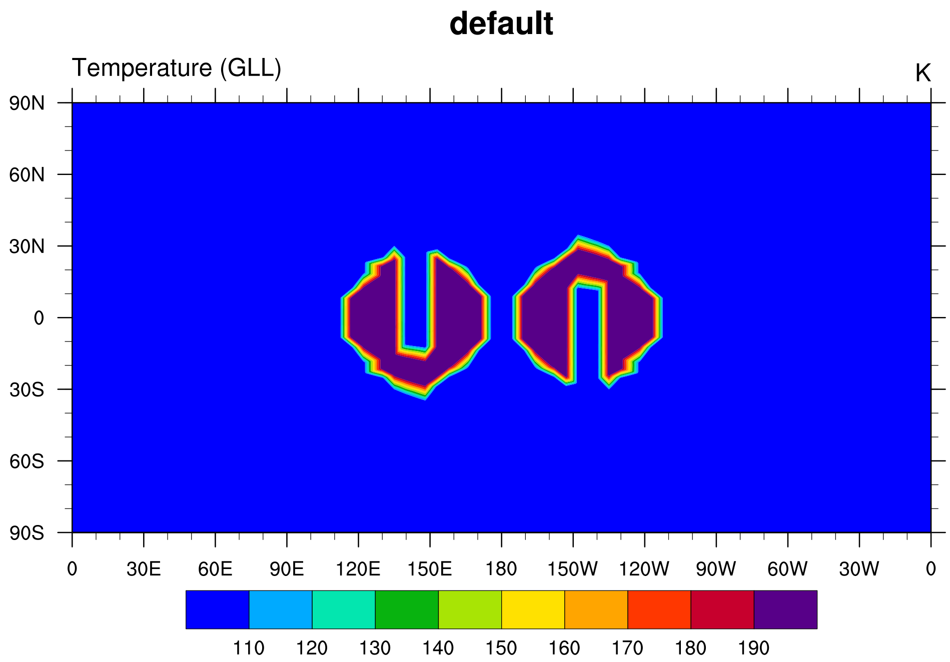
$A_{non\text{-}mono}$*GLL = $PHYS_{non\text{-}mono}$

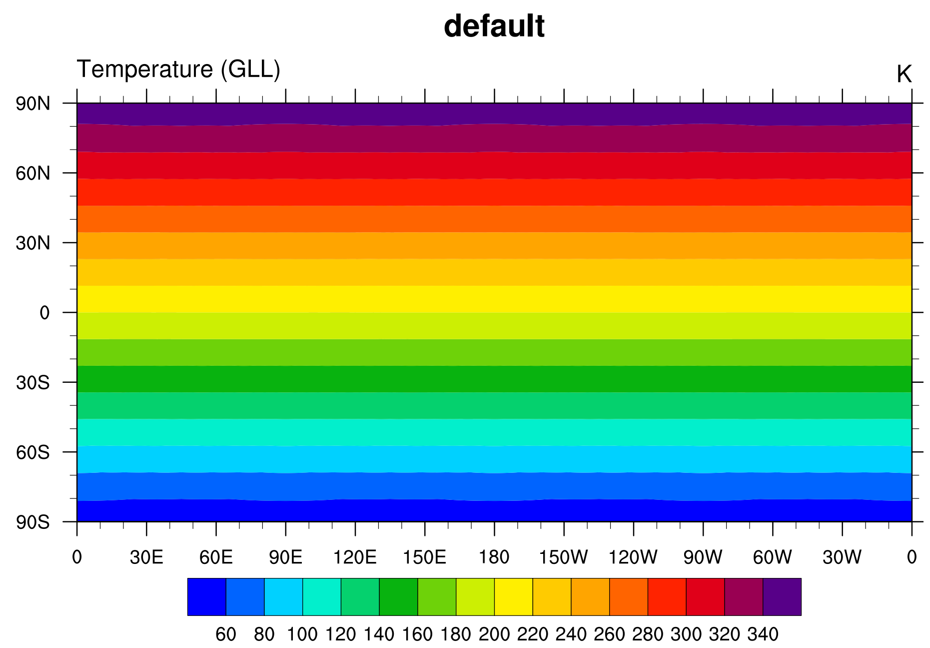
A_{mono}*GLL = $PHYS_{mono}$

$[\alpha A_{mono} + (1-\alpha) A_{non\text{-}mono} GLL] = PHYS_{mono}$

where $\alpha = (\max(GLL)-PHYS_{non\text{-}mono})/(PHYS_{mono} - PHYS_{non\text{-}mono})$ or
$\alpha = (\min(GLL)-PHYS_{non\text{-}mono})/(PHYS_{mono} - PHYS_{non\text{-}mono})$

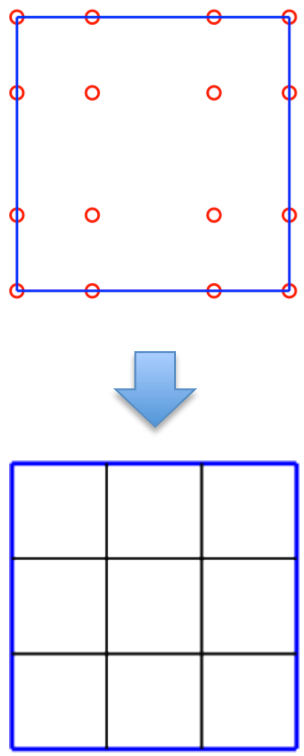
Dynamics to physics grid mapping

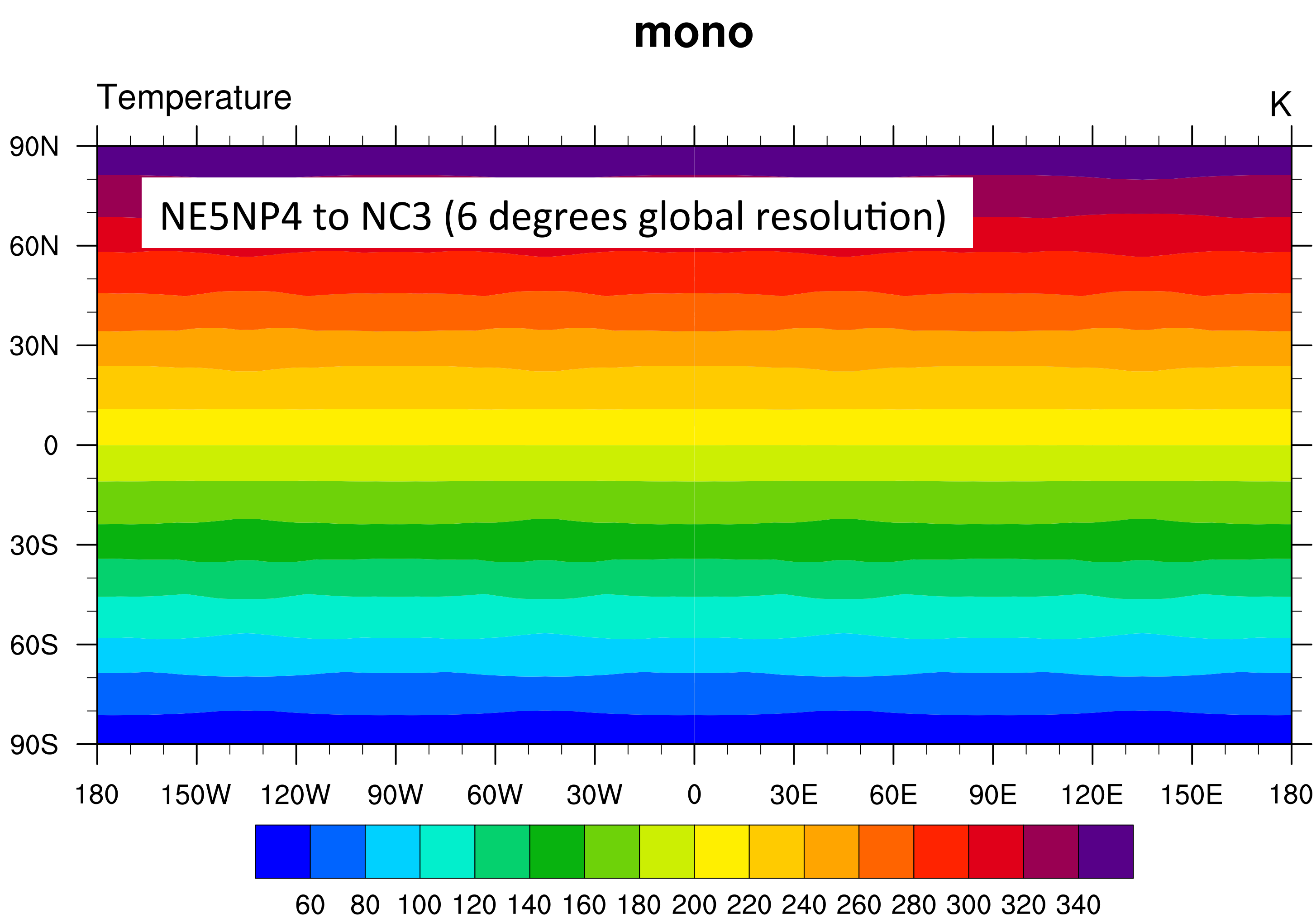




Properties we are looking for: Preserve smooth fields and at the same time not generate new extrema for rough distributions (and be mass-conservative and consistent)

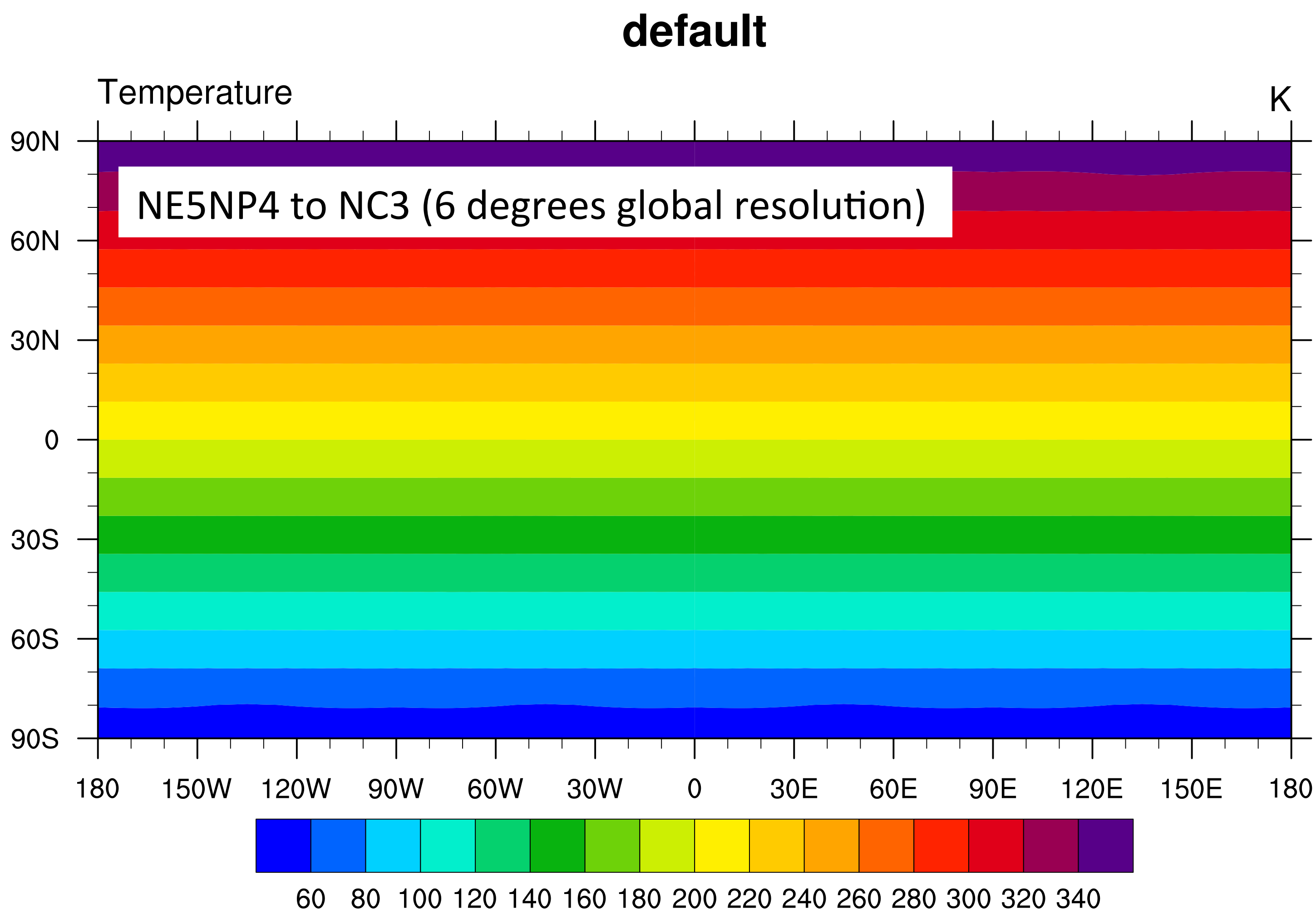
Smooth field ("spherical harmonic")





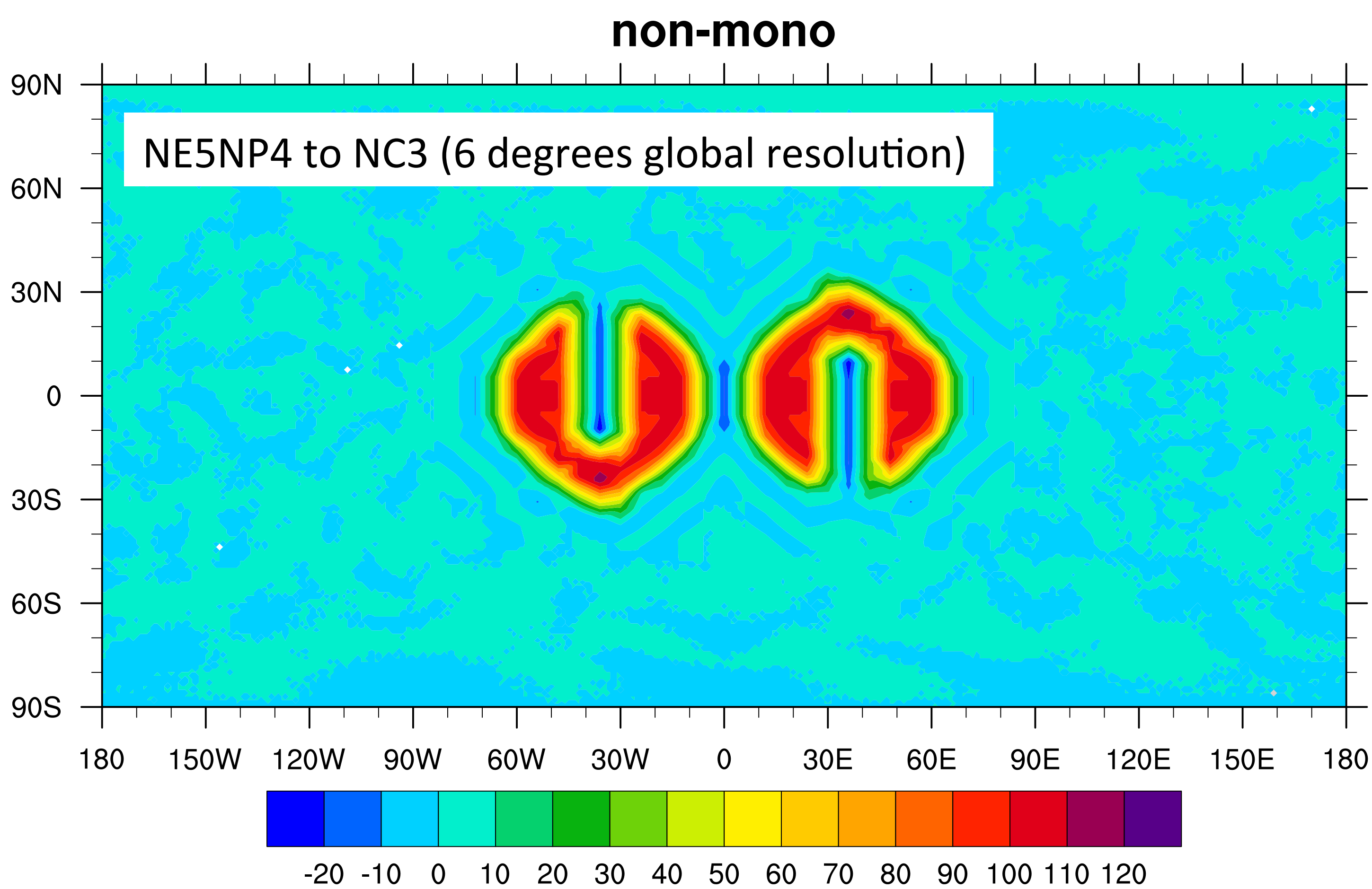
1st order monotone map (not flow dependent): see grid

Smooth field ("spherical harmonic")



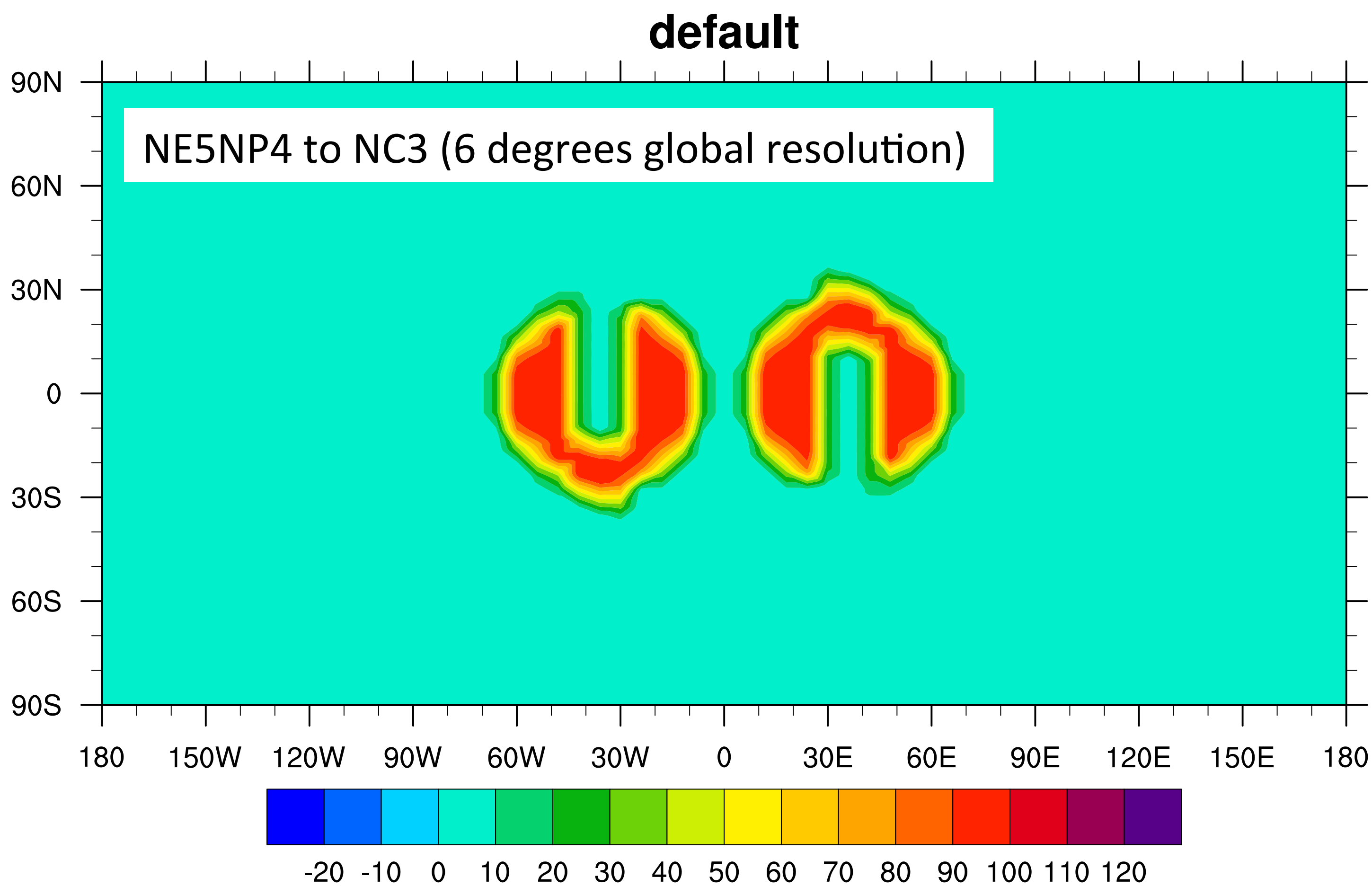
Optimally blend conservative and monotone map

Rough field ("slotted cylinder")



Non-monotone conservative

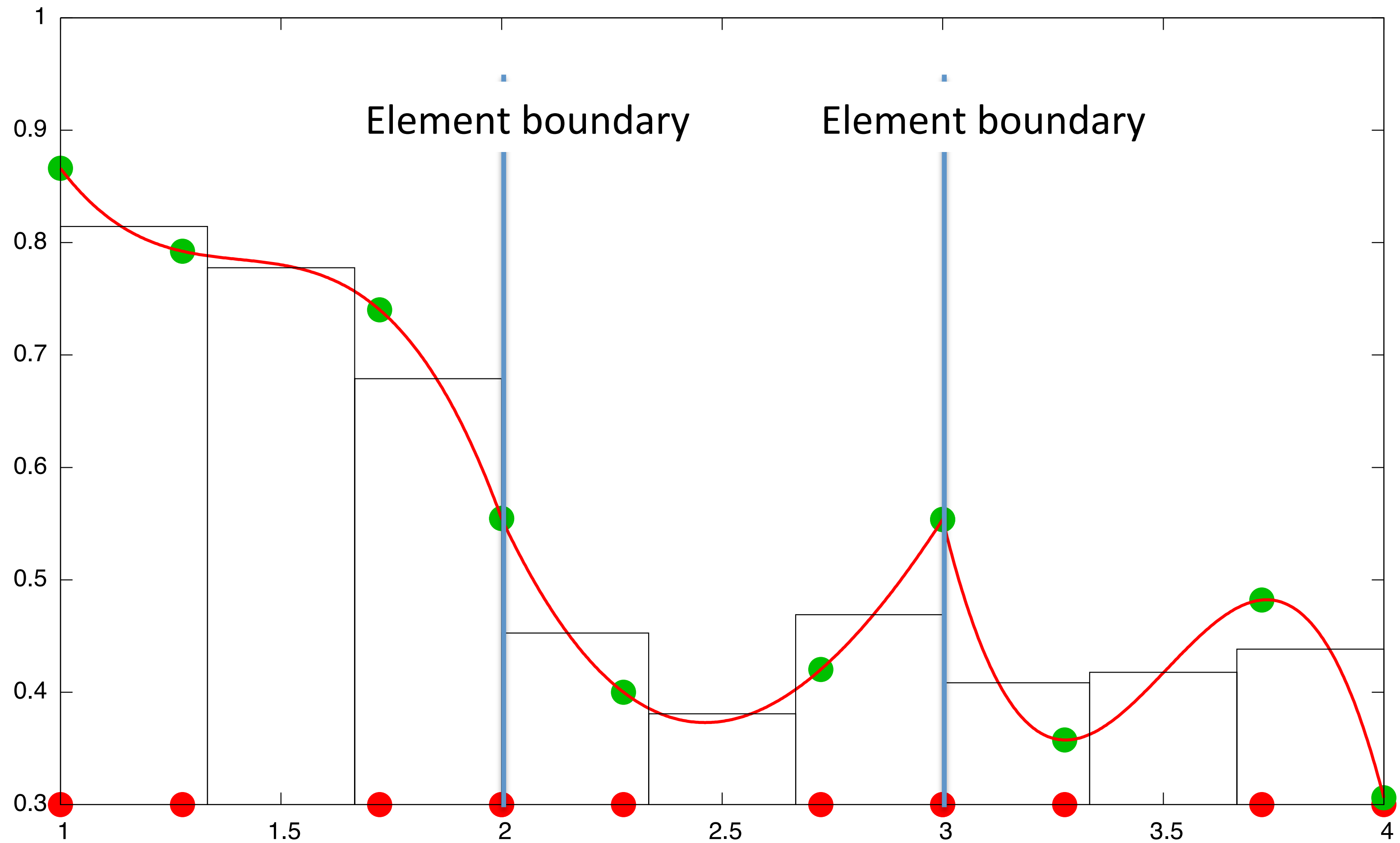
Rough field ("slotted cylinder")



Optimally blend conservative and monotone map

Passing tendencies (fv,fT,fq,…) to dynamics:

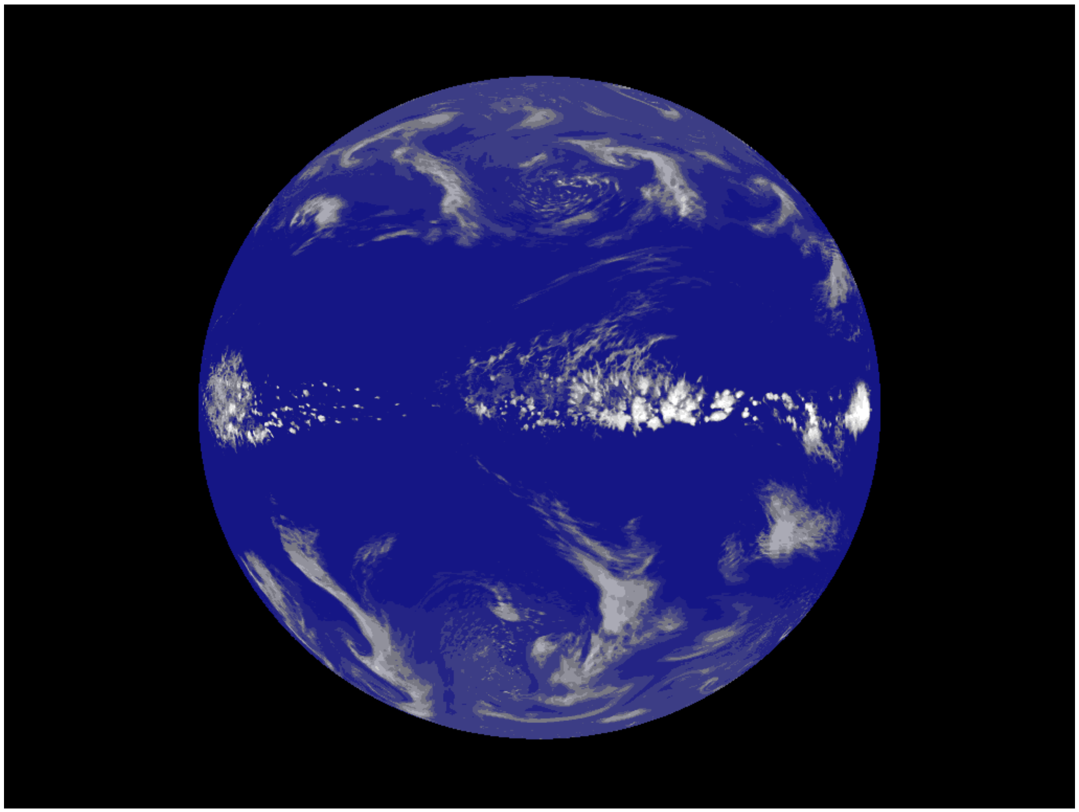
Use a 1st-order, shape-preserving, conservative linear map



CAM4 forcing: Aqua-planet

Atmospheric model with complete parameterization suite
Idealized surface: no land (or mountains), no sea ice
specified global sea surface temperatures everywhere

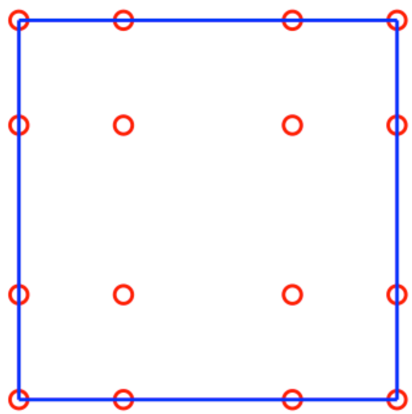
=> Free motions, no forced component



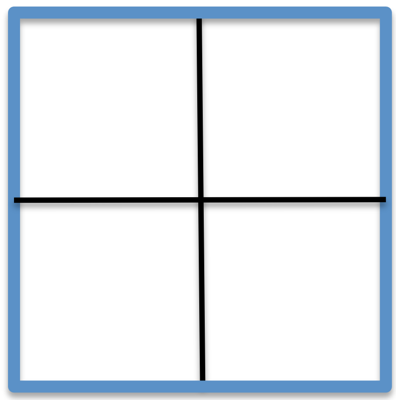
Why CAM4? More resolution sensitivity than CAM5 (and it is cheaper!)

Configurations

NE30NP4

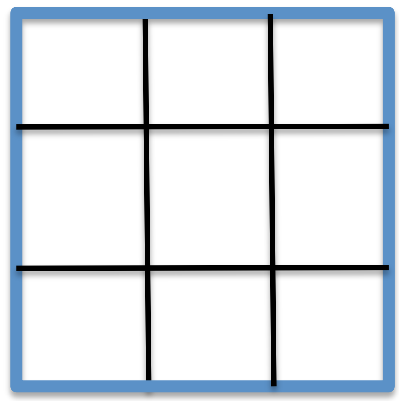


NE30NP4NC2



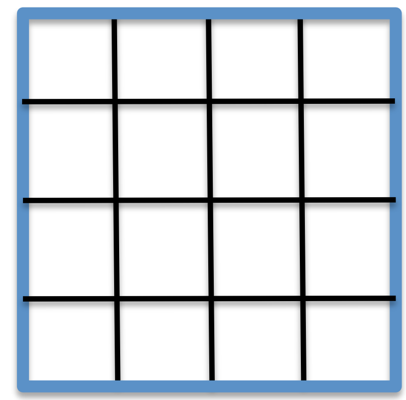
1.5° physgrid

NE30NP4NC3



1.0° physgrid

NE30NP4NC4

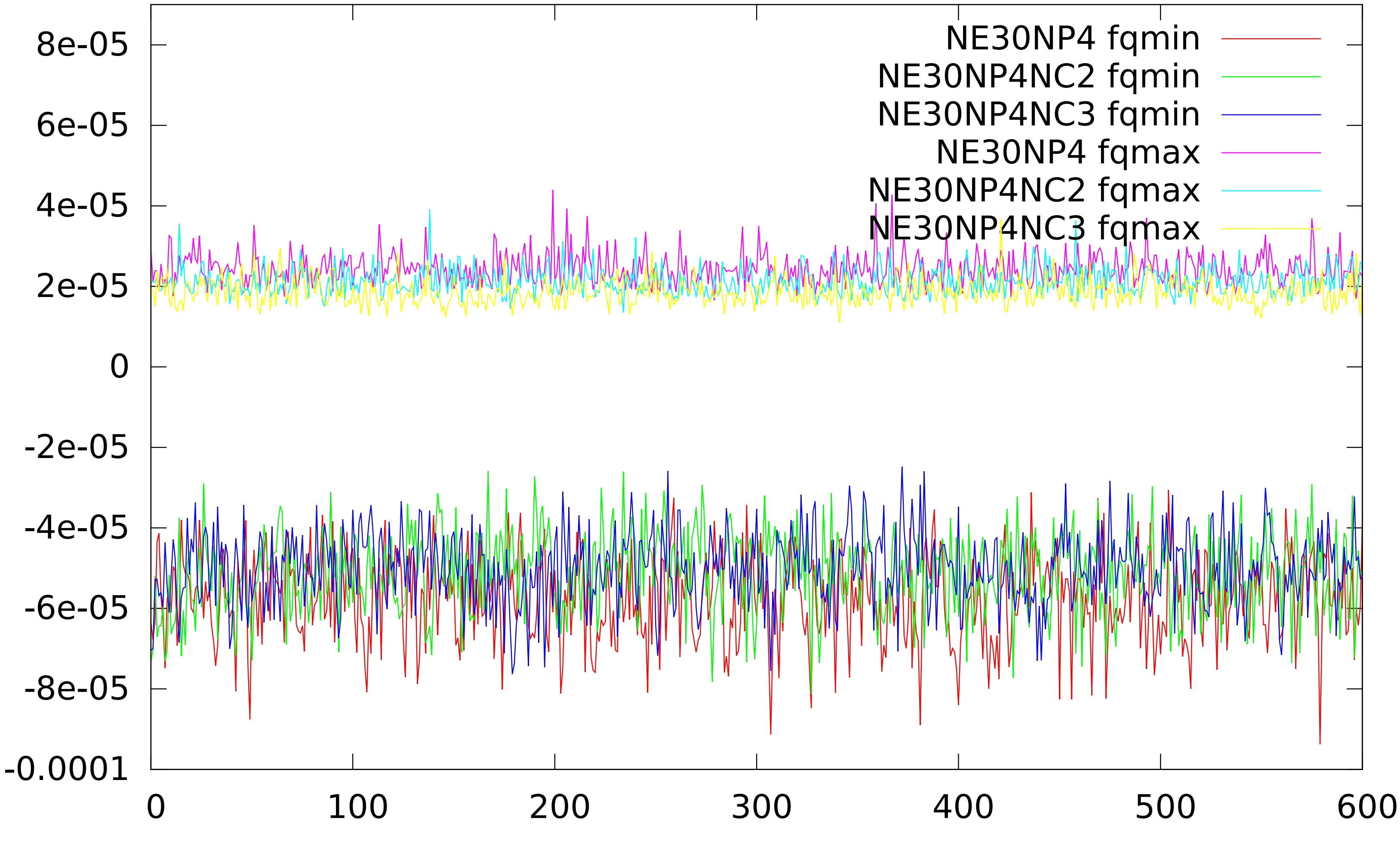


0.75° physgrid

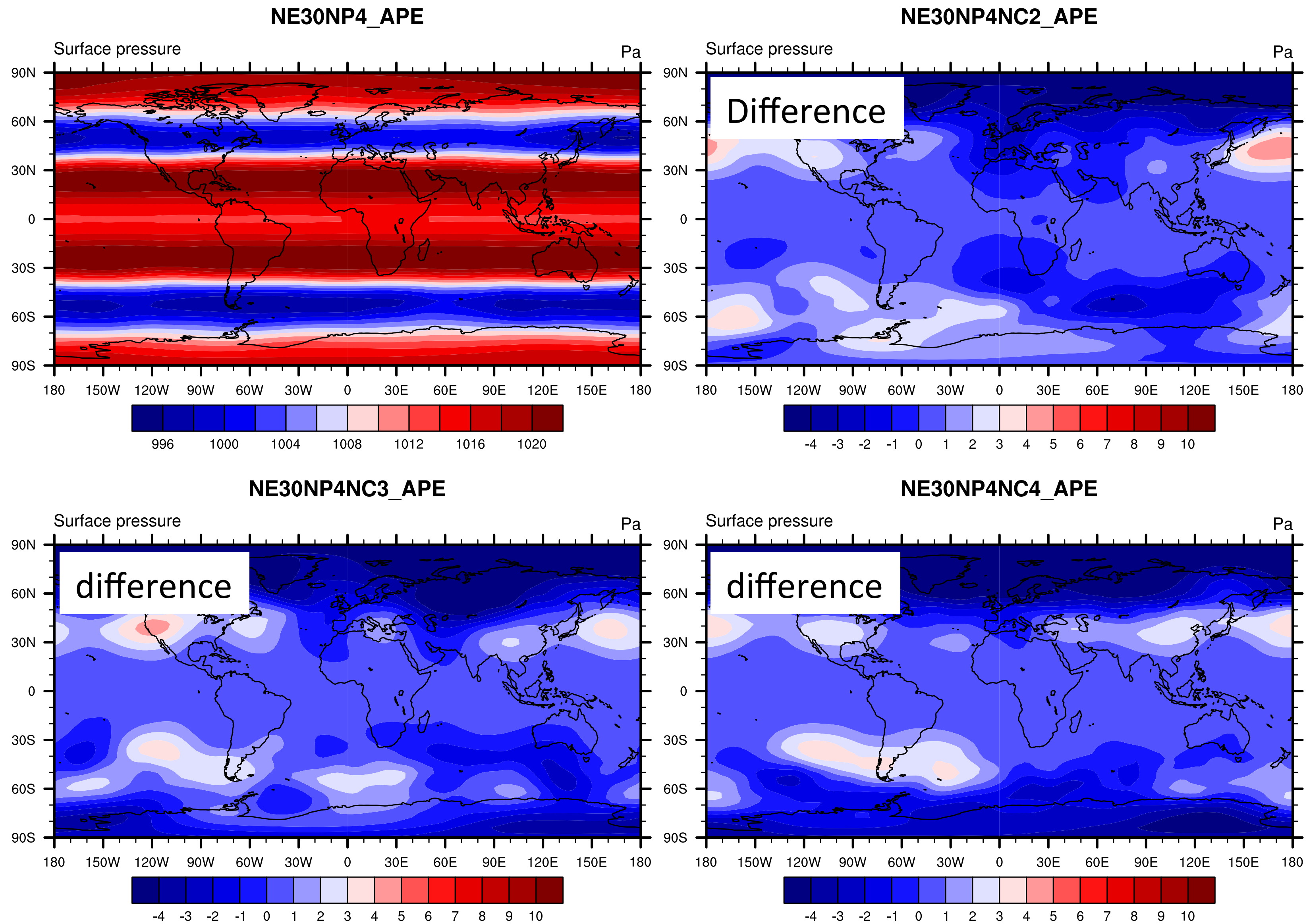
Data mapped to 3° lat-lon grid for analysis

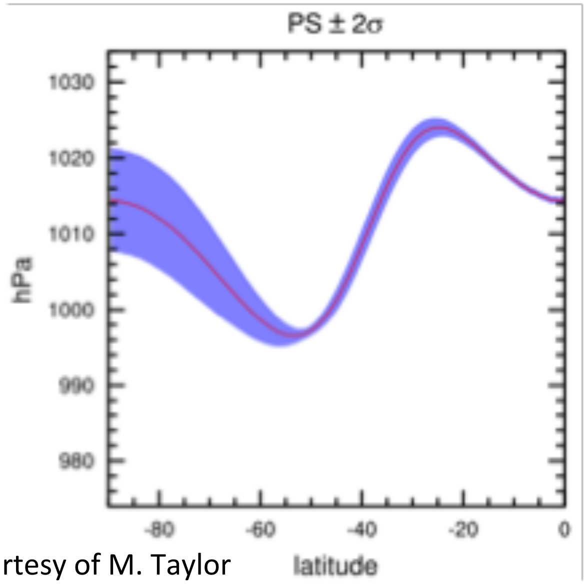
Length of simulations: 30 months

Min/max moisture forcing

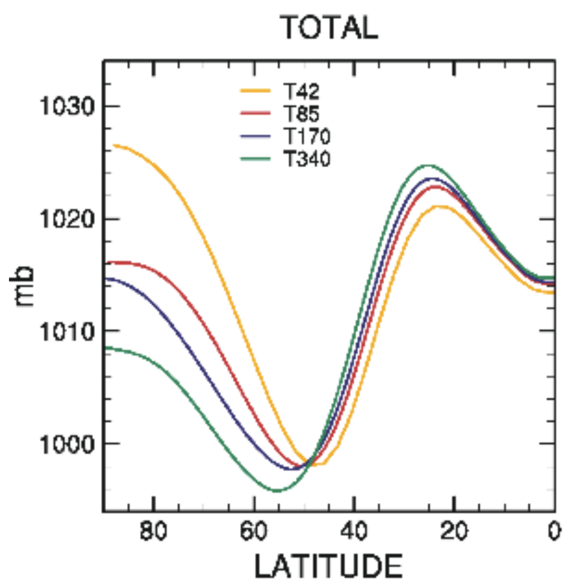


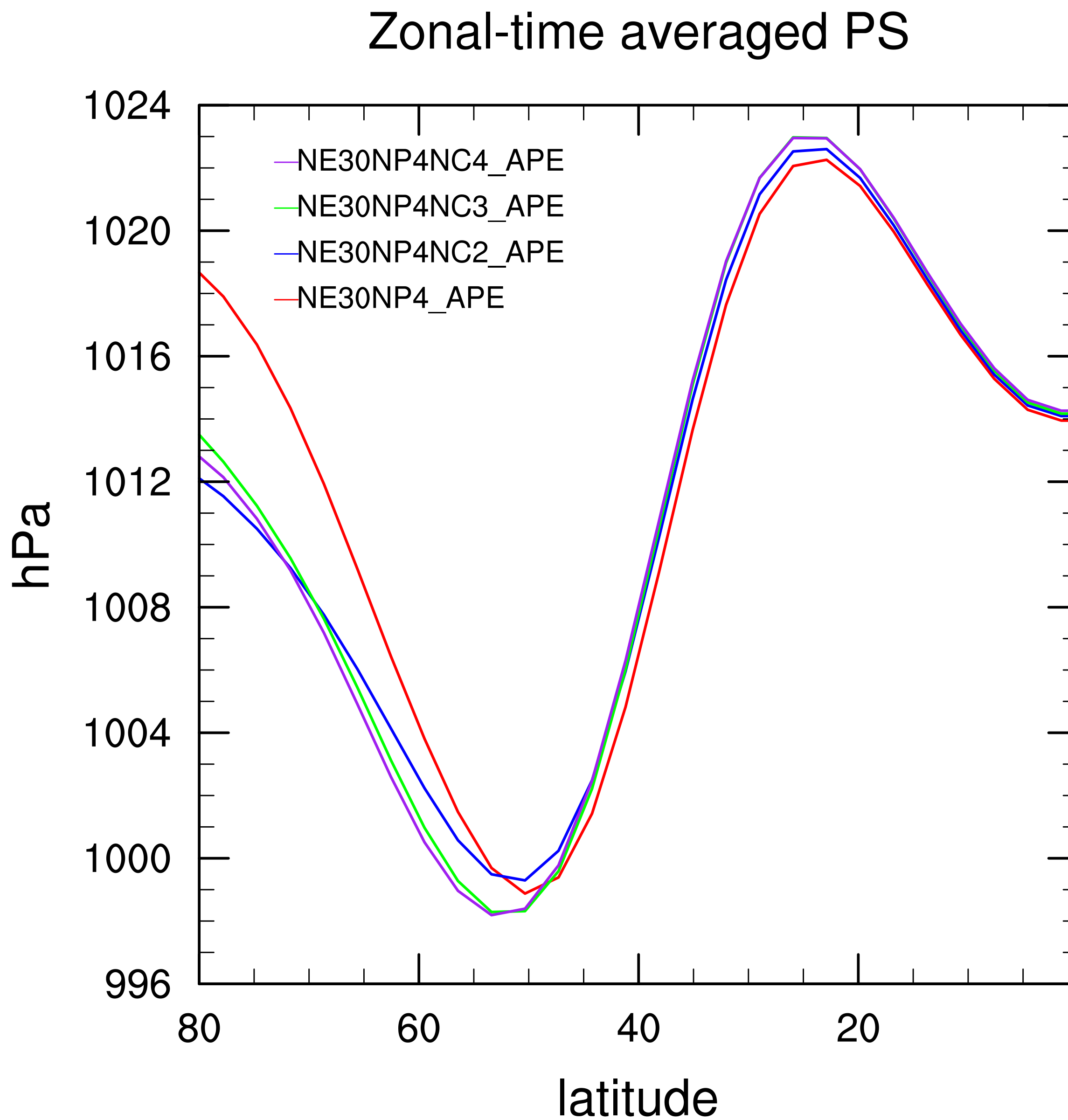
Time averaged PS



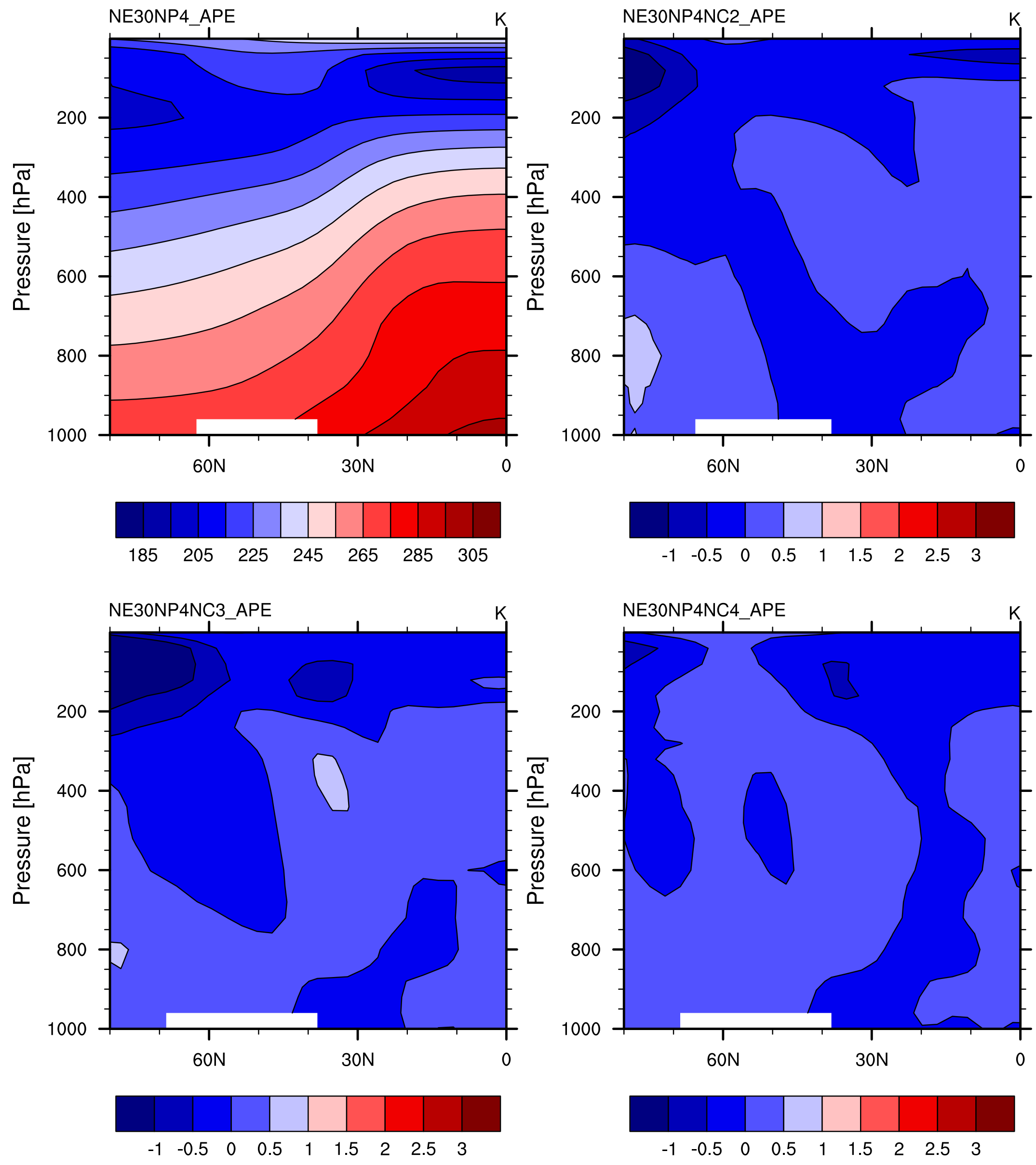


Courtesy of M. Taylor



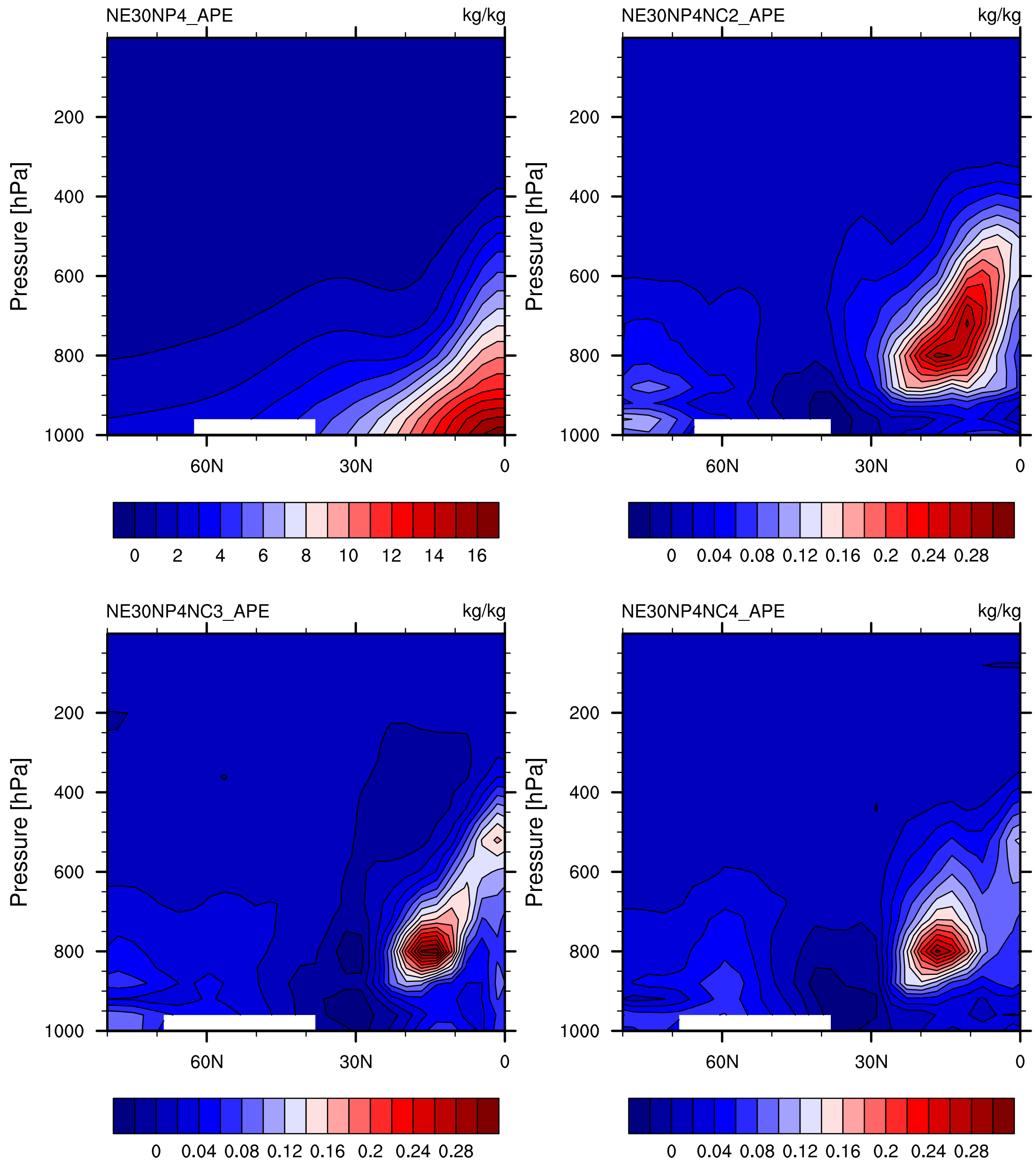


Zonal-time averaged T

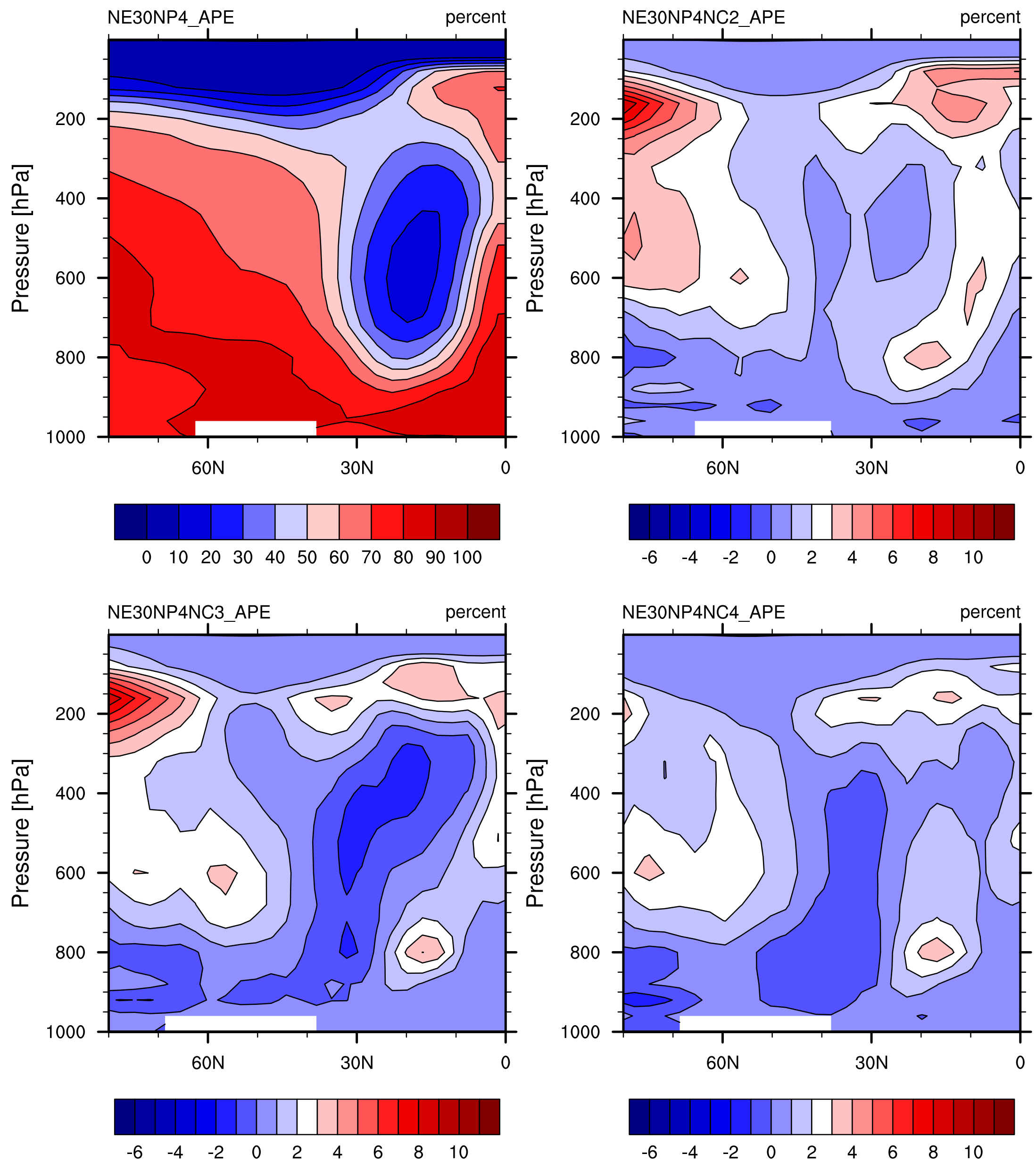


Zonal-time averaged Q

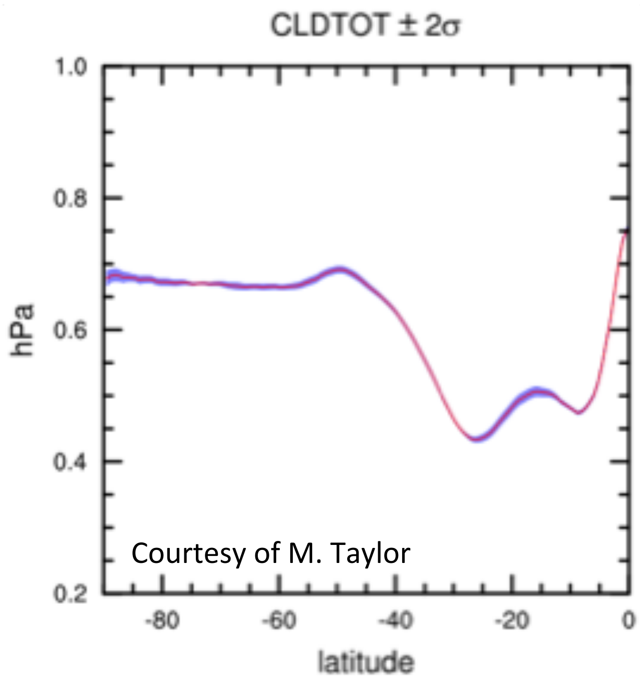
Q = Specific humidity

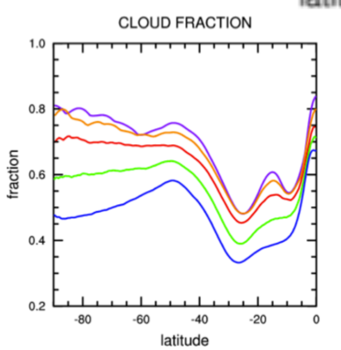


Zonal-time averaged RELHUM



RELHUM = Relative humidity

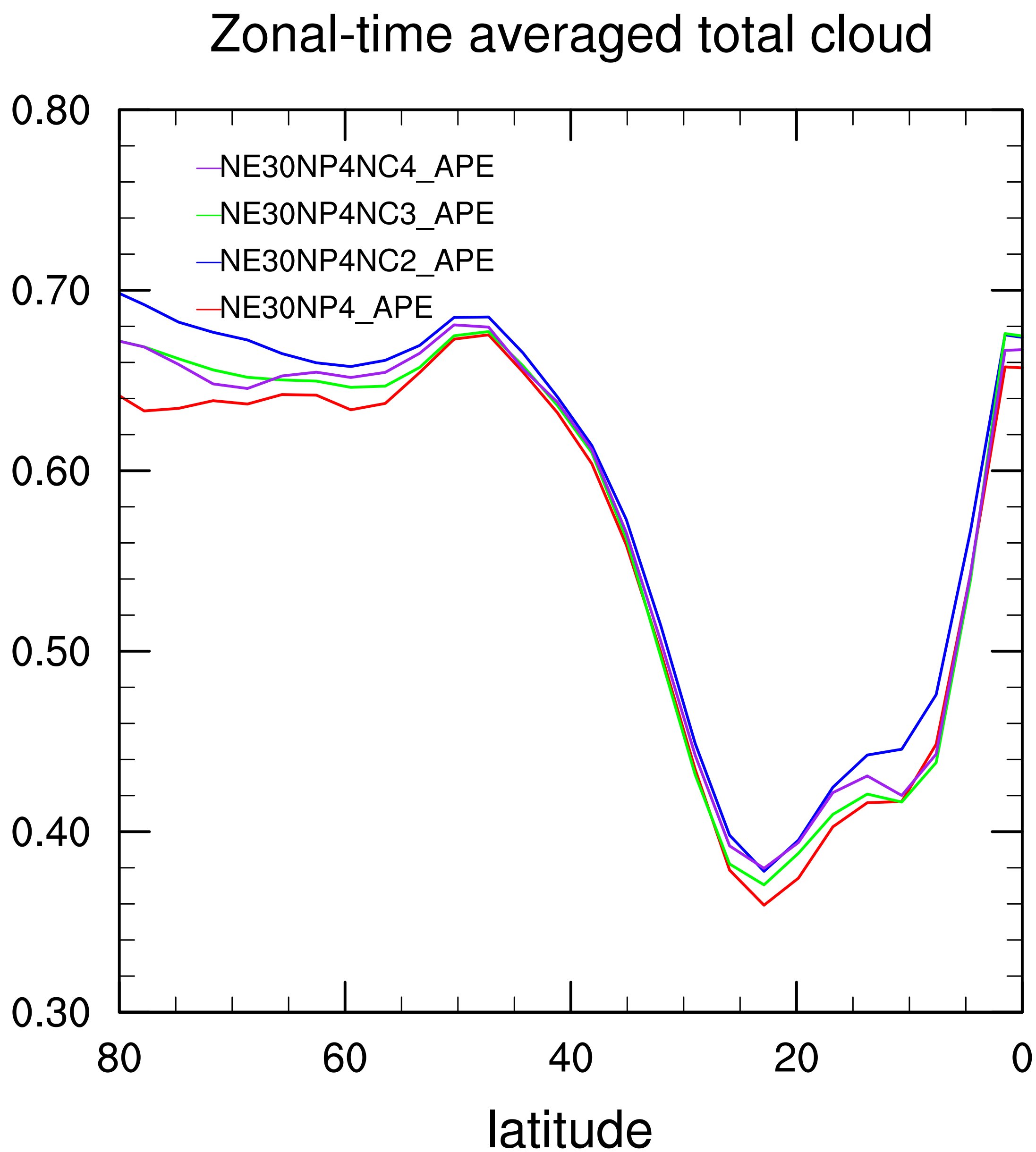


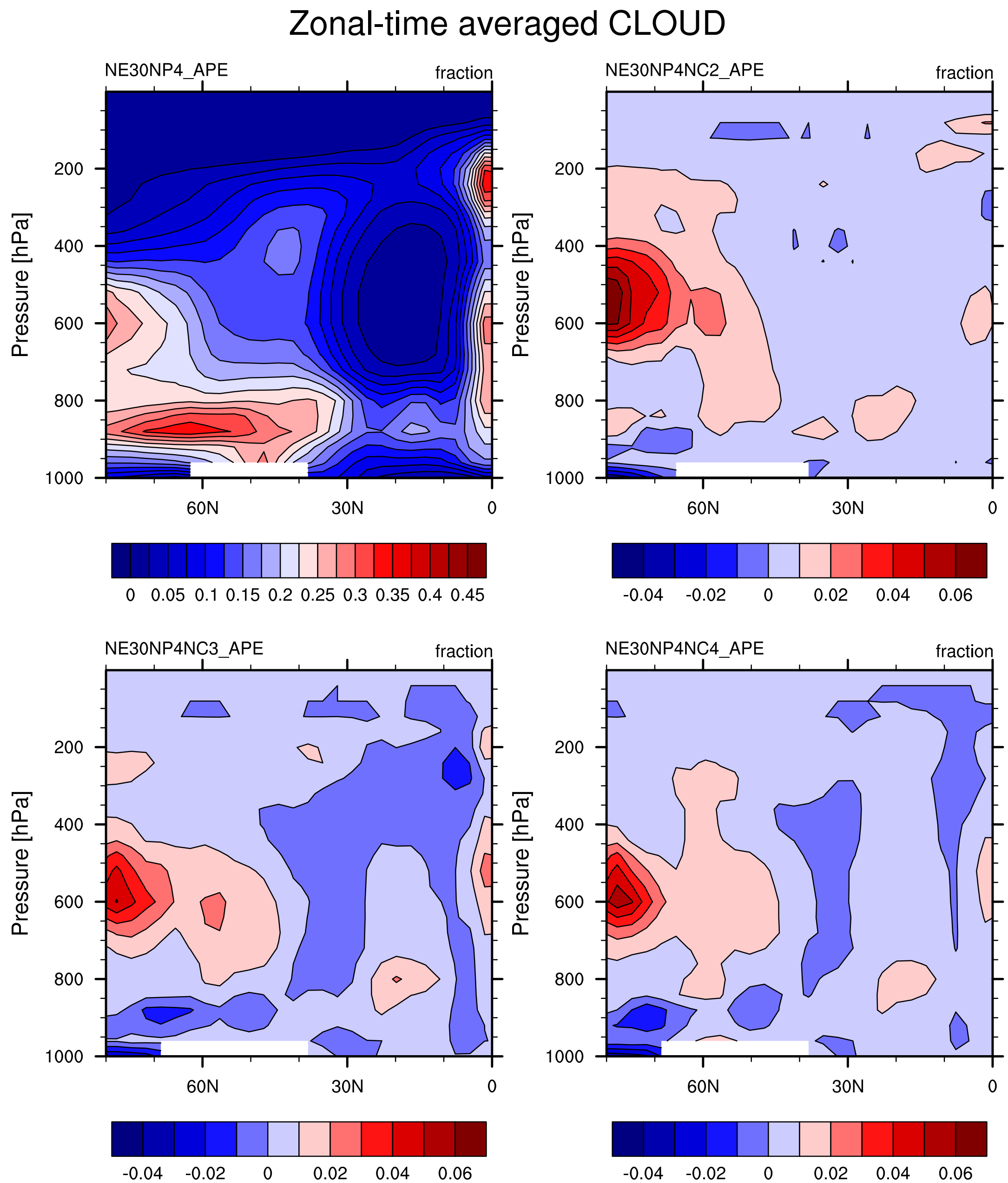


Courtesy of M. Taylor

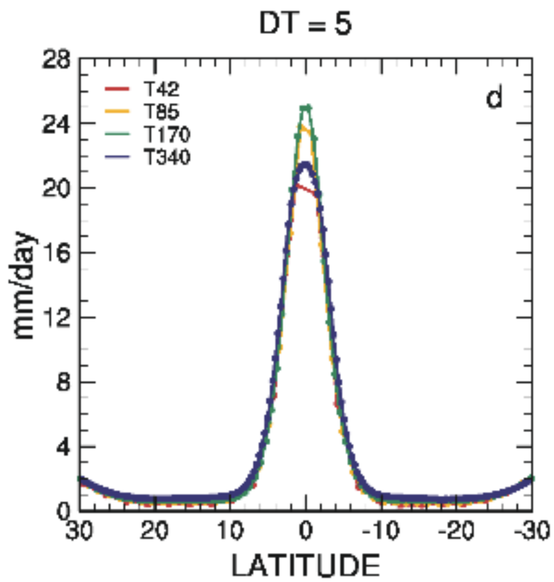
CAM-SE with resolutions:
2.7° 1.9° 1.0° 0.5° 0.25°

•Monotonic decrease in LWCF also seen in CAM 3.1 Aqua Planet Experiments (Williamson, Tellus 2008)
• Cloud Fraction monotonically decreases with resolution
• CAM-SE simulations (shown) are similar.

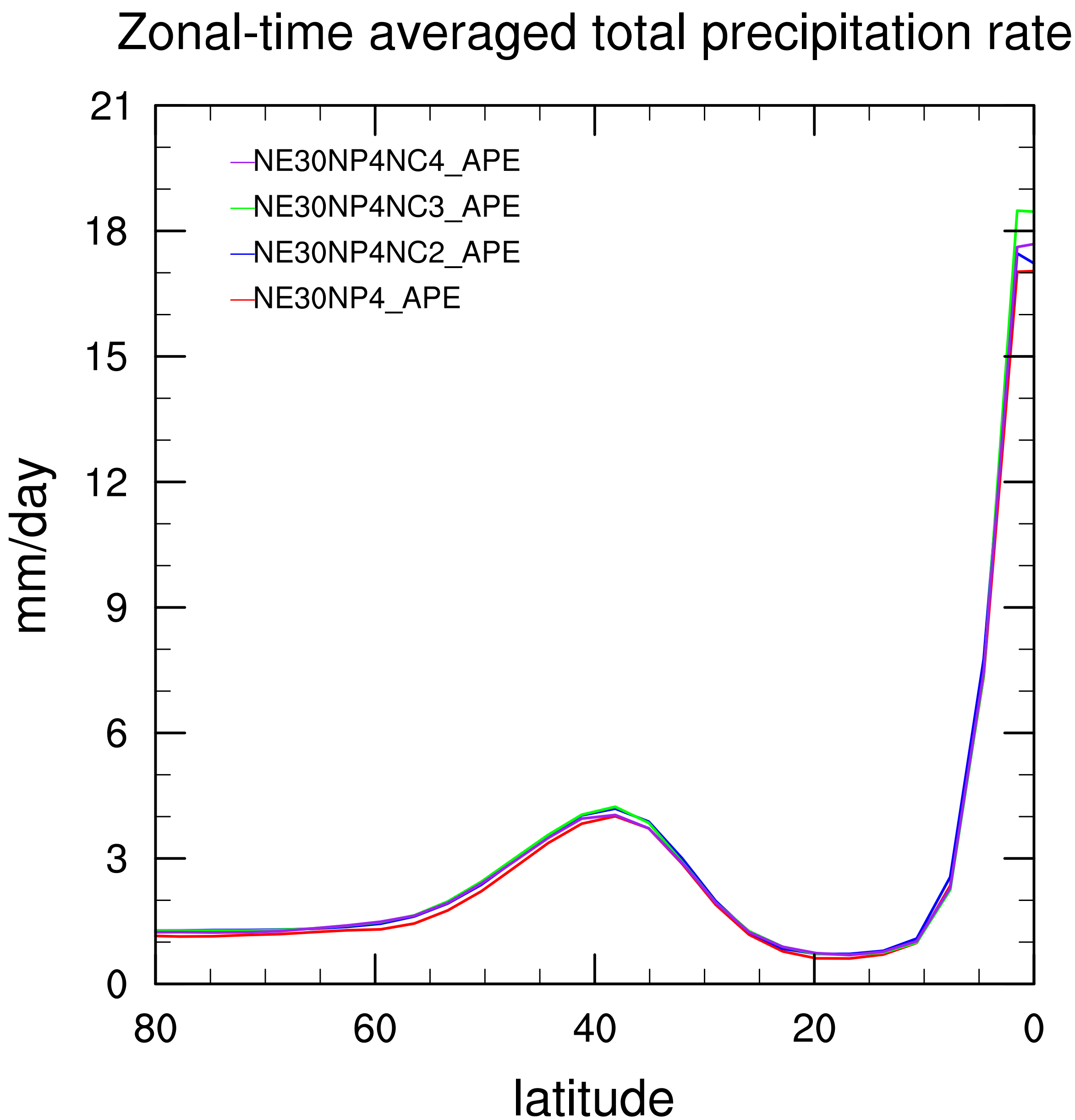




Zonal-time averaged CLOUD
NE30NP4_APE
fraction
NE30NP4NC2_APE
fraction
NE30NP4NC3_APE
fraction
NE30NP4NC4_APE
fraction
Pressure [hPa]
200
400
600
800
1000
60N
30N
0
0 0.05 0.1 0.15 0.2 0.25 0.3 0.35 0.4 0.45
-0.04 -0.02 0 0.02 0.04 0.06

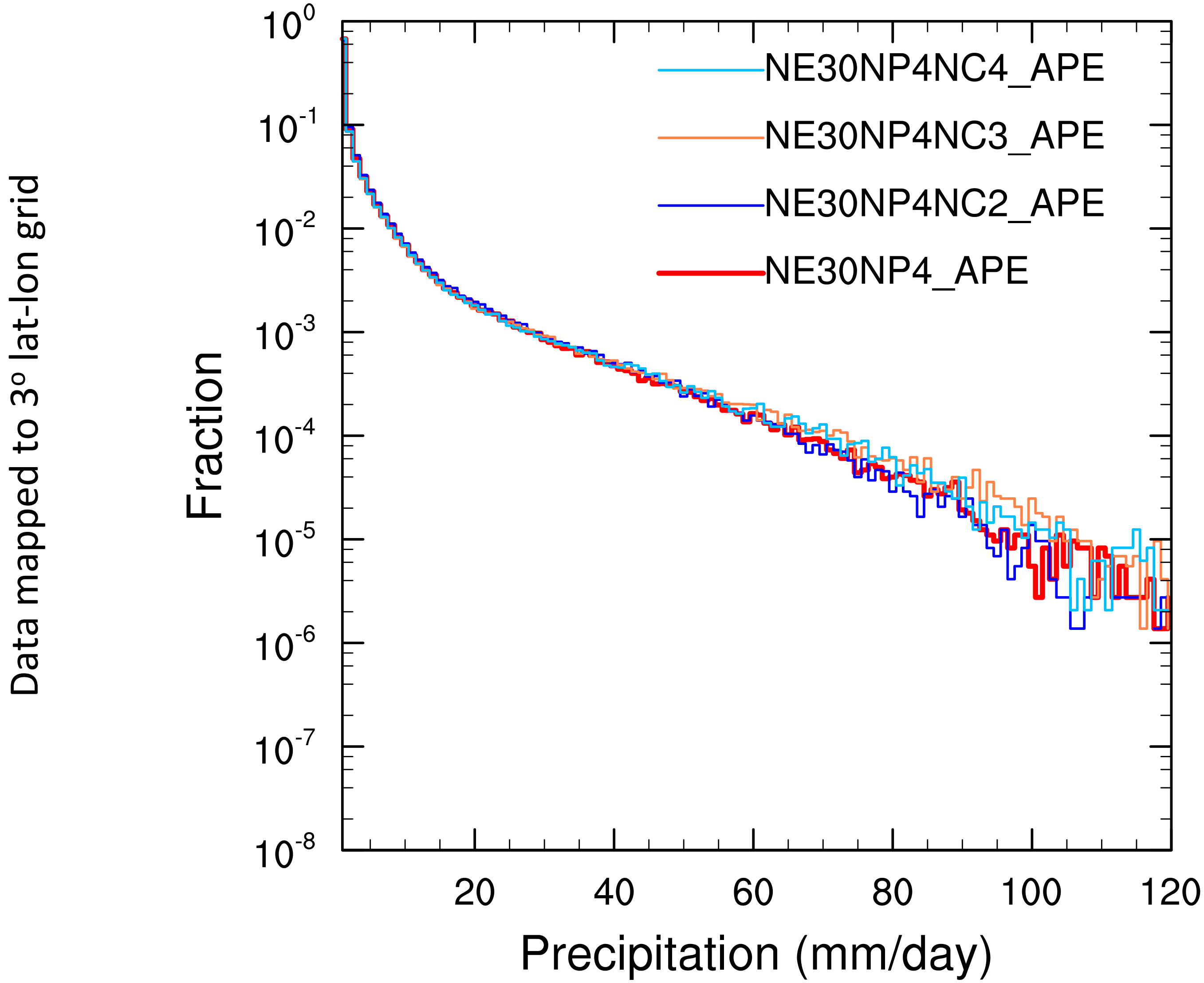


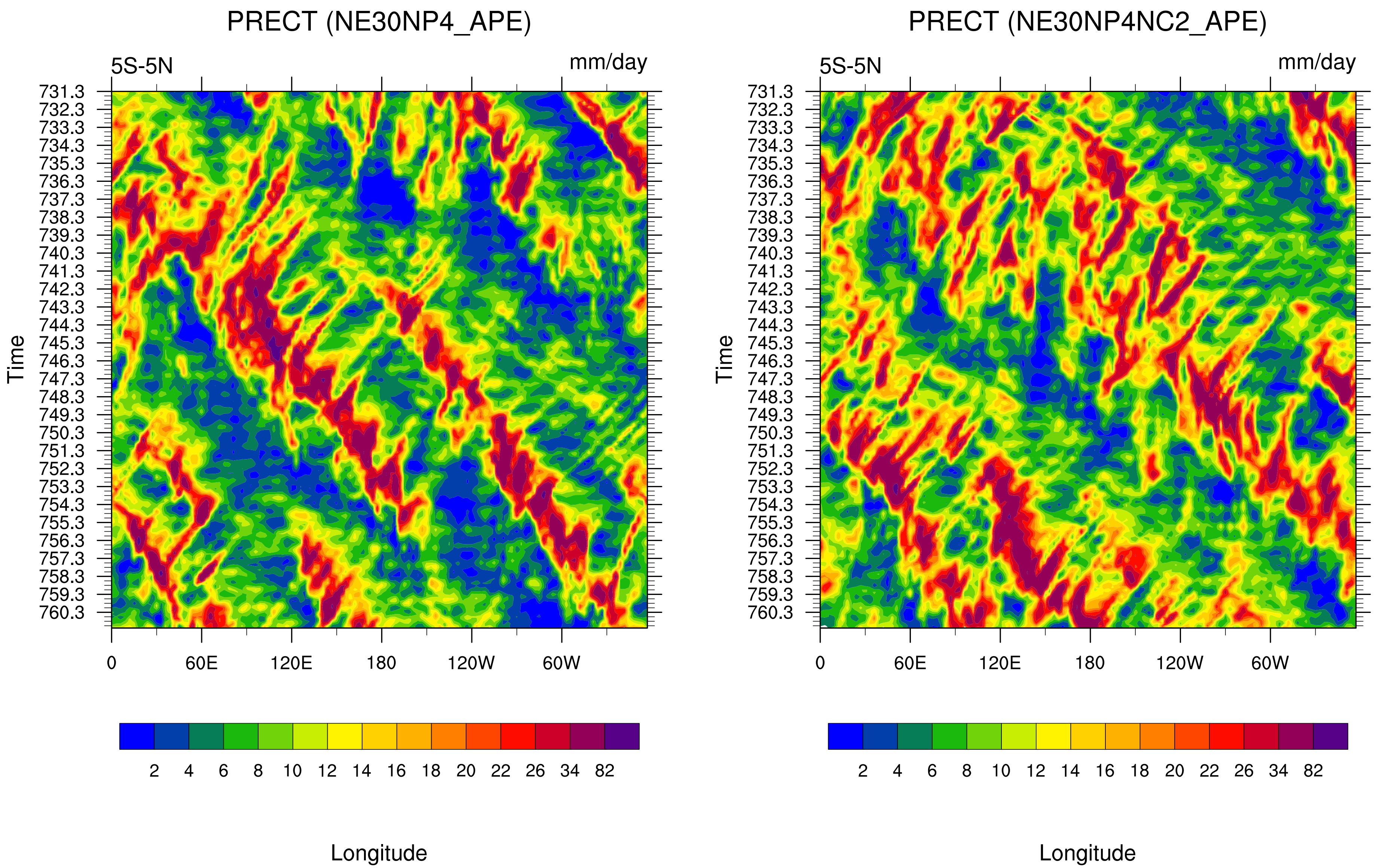
Williamson (2004)



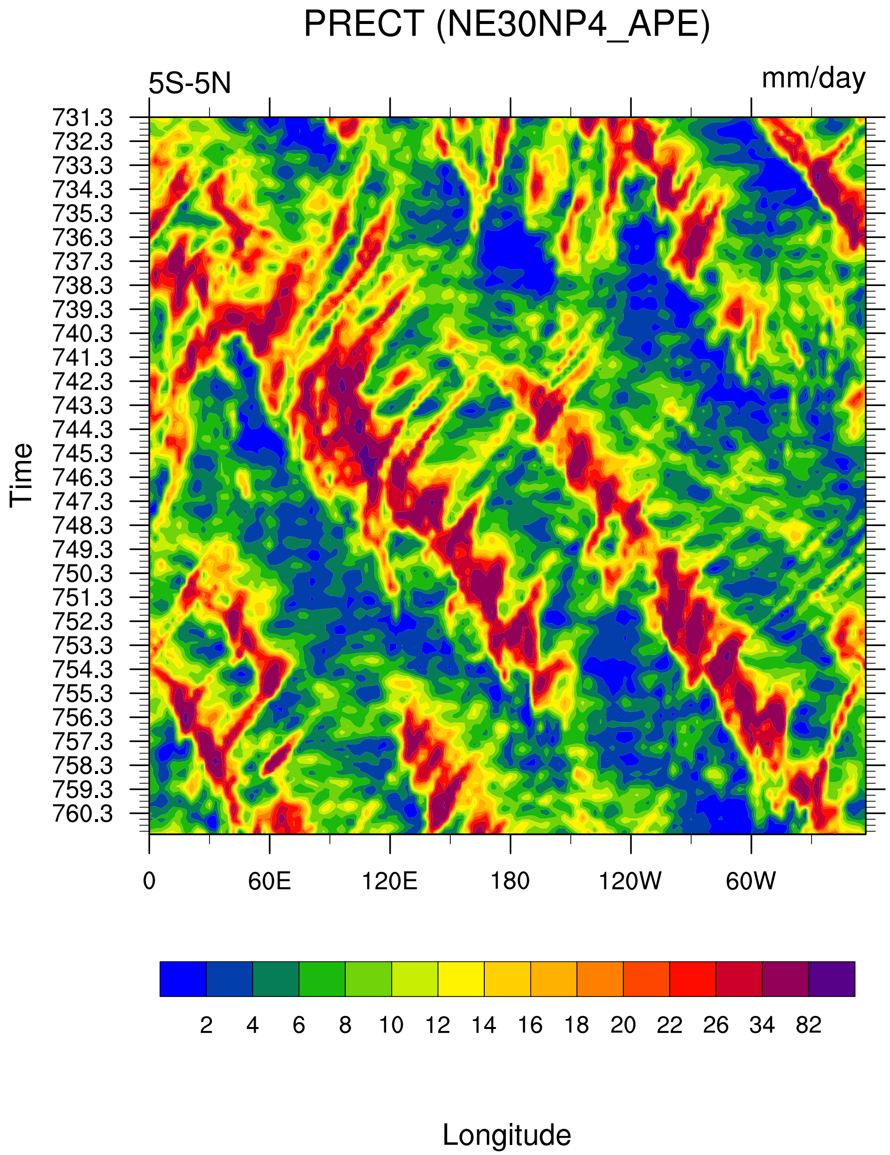
PDF

PRECT (30 month simulation - 6h data)

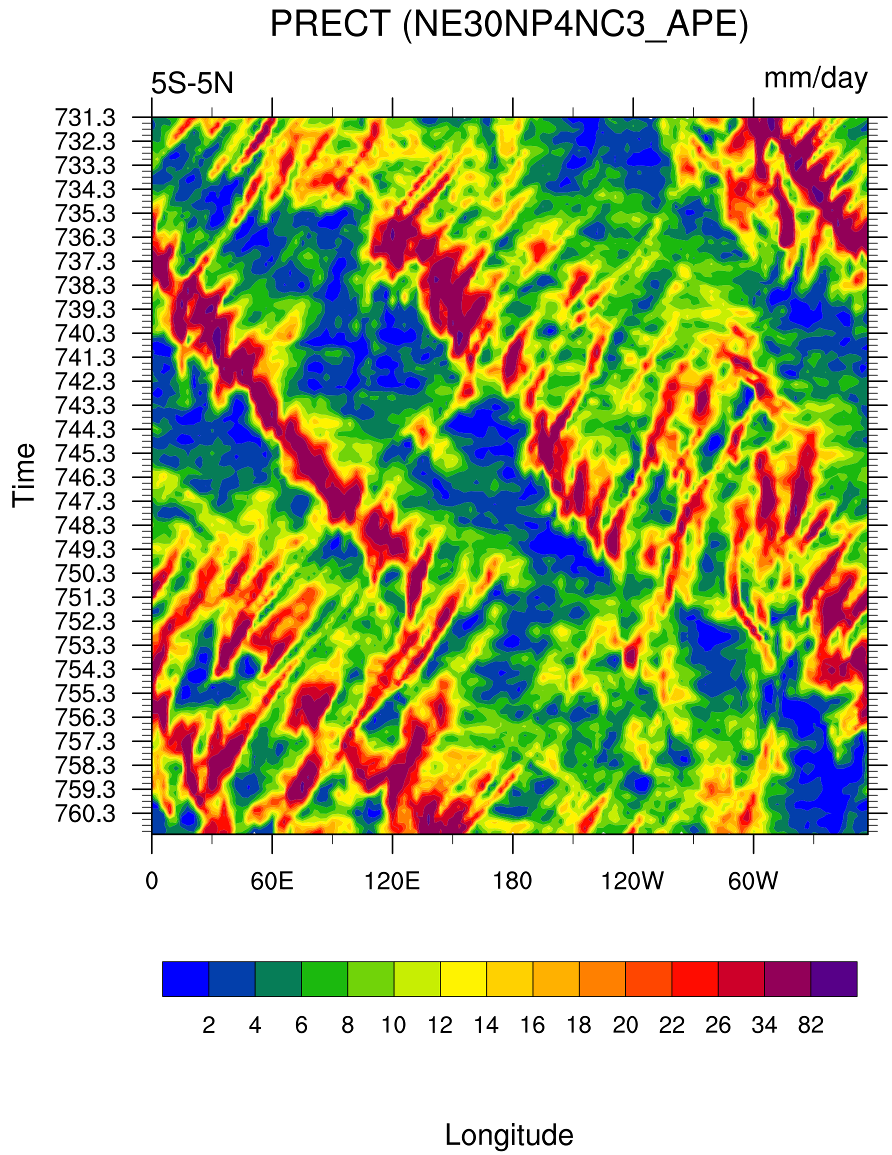




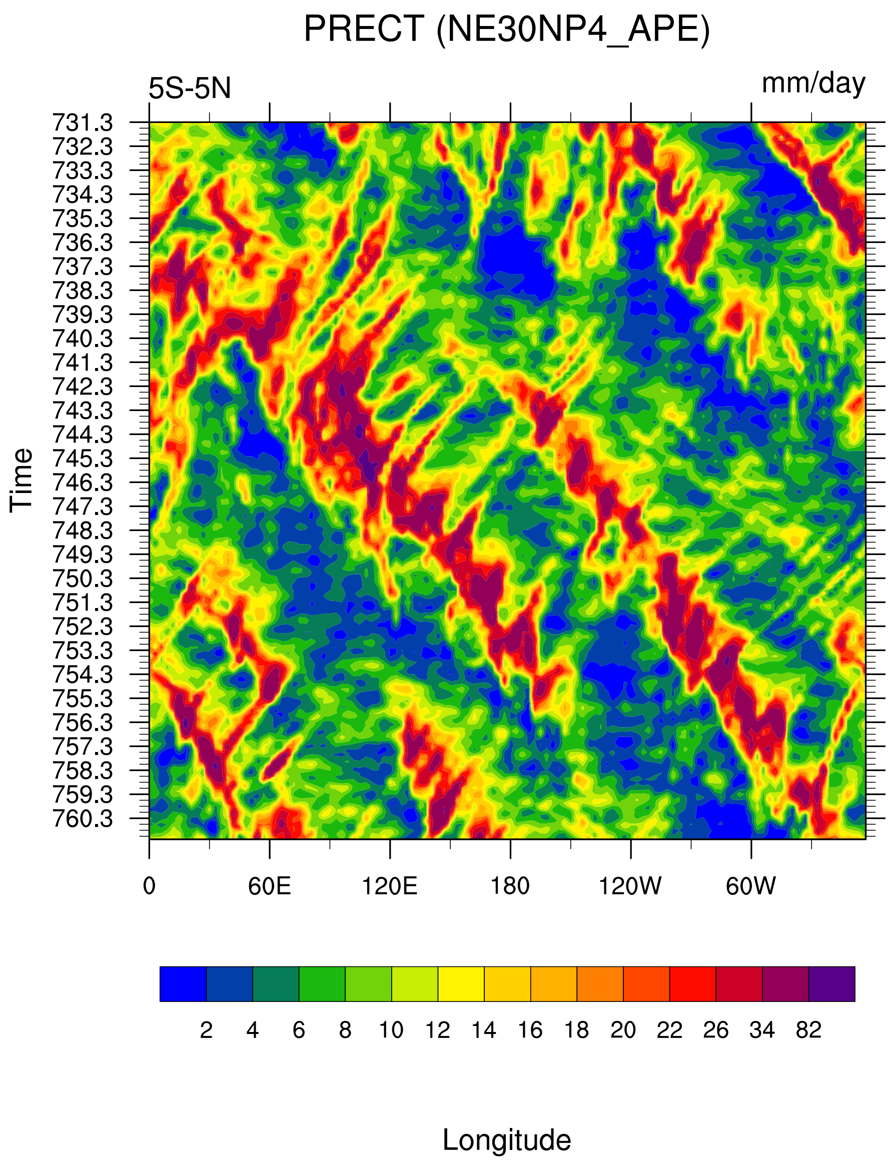
PRECT (NE30NP4_APE)
5S-5N
mm/day
Time
731.3
732.3
733.3
734.3
735.3
736.3
737.3
738.3
739.3
740.3
741.3
742.3
743.3
744.3
745.3
746.3
747.3
748.3
749.3
750.3
751.3
752.3
753.3
754.3
755.3
756.3
757.3
758.3
759.3
760.3
0
60E
120E
180
120W
60W
2
4
6
8
10
12
14
16
18
20
22
26
34
82
Longitude
PRECT (NE30NP4NC2_APE)
5S-5N
mm/day
Time
0
60E
120E
180
120W
60W
2
4
6
8
10
12
14
16
18
20
22
26
34
82
Longitude

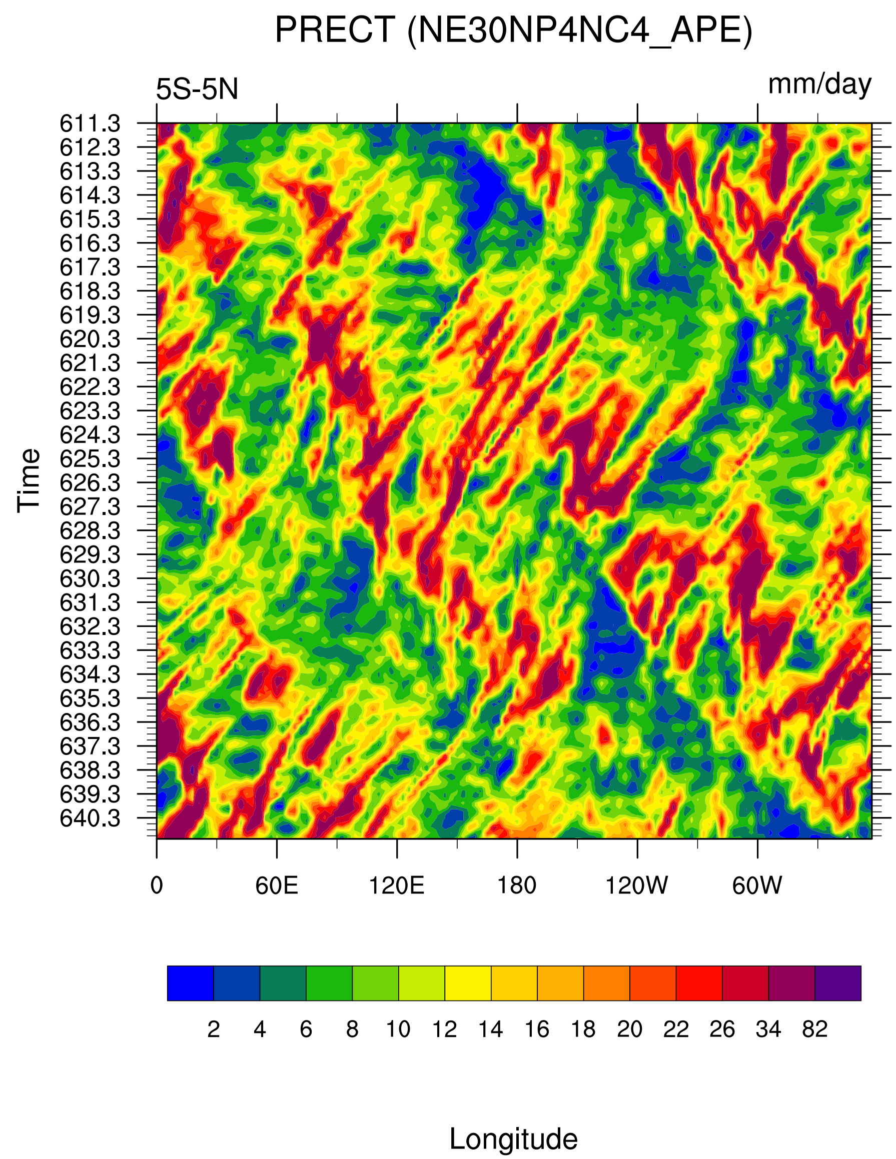


PRECT (NE30NP4_APE)
5S-5N
mm/day
Time
731.3
732.3
733.3
734.3
735.3
736.3
737.3
738.3
739.3
740.3
741.3
742.3
743.3
744.3
745.3
746.3
747.3
748.3
749.3
750.3
751.3
752.3
753.3
754.3
755.3
756.3
757.3
758.3
759.3
760.3
0
60E
120E
180
120W
60W
2 4 6 8 10 12 14 16 18 20 22 26 34 82
Longitude

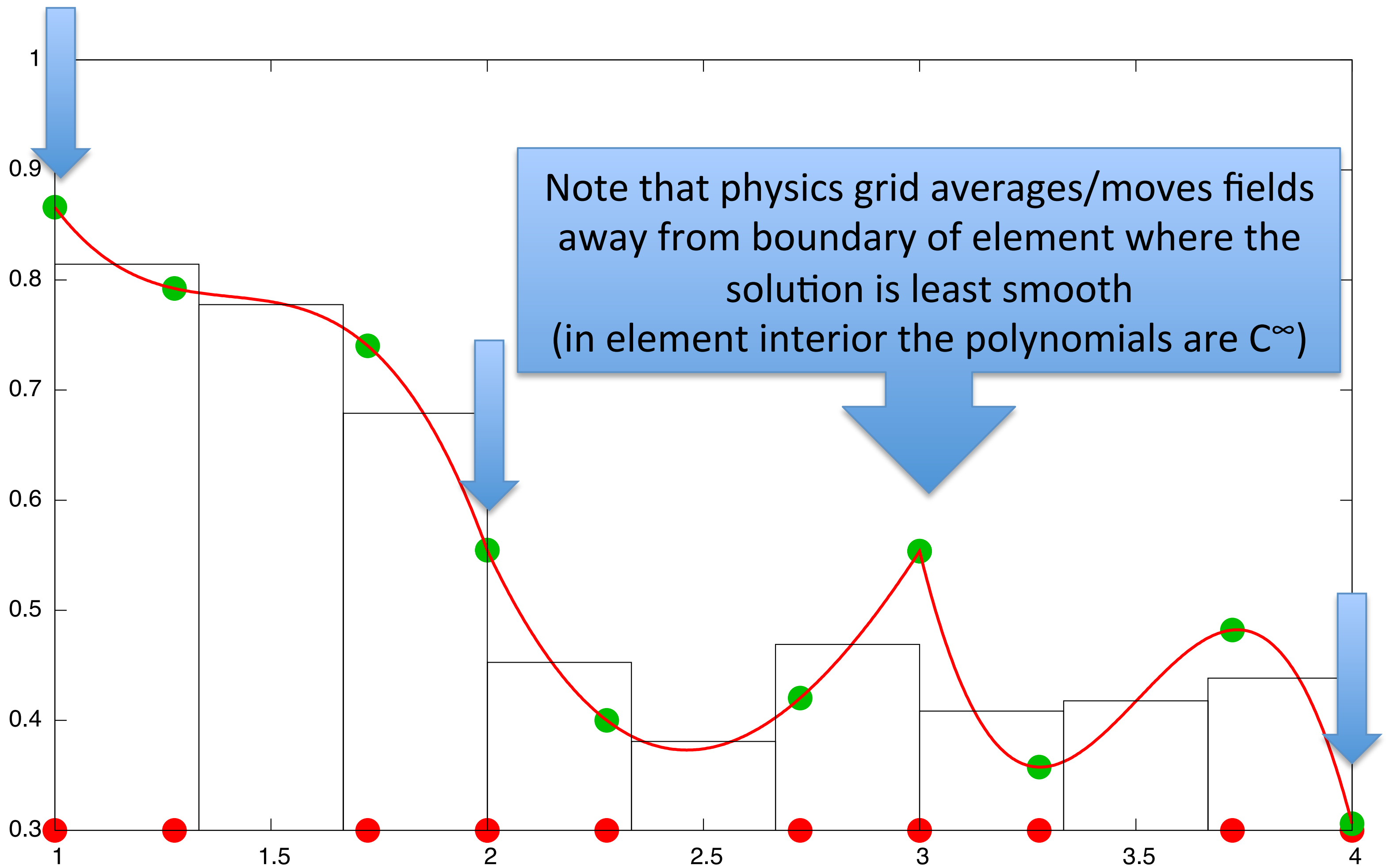


PRECT (NE30NP4NC3_APE)
5S-5N
mm/day
Time
731.3
732.3
733.3
734.3
735.3
736.3
737.3
738.3
739.3
740.3
741.3
742.3
743.3
744.3
745.3
746.3
747.3
748.3
749.3
750.3
751.3
752.3
753.3
754.3
755.3
756.3
757.3
758.3
759.3
760.3
0
60E
120E
180
120W
60W
2 4 6 8 10 12 14 16 18 20 22 26 34 82
Longitude


PRECT (NE30NP4_APE)
5S-5N
mm/day
Time
731.3
732.3
733.3
734.3
735.3
736.3
737.3
738.3
739.3
740.3
741.3
742.3
743.3
744.3
745.3
746.3
747.3
748.3
749.3
750.3
751.3
752.3
753.3
754.3
755.3
756.3
757.3
758.3
759.3
760.3
0
60E
120E
180
120W
60W
2 4 6 8 10 12 14 16 18 20 22 26 34 82
Longitude

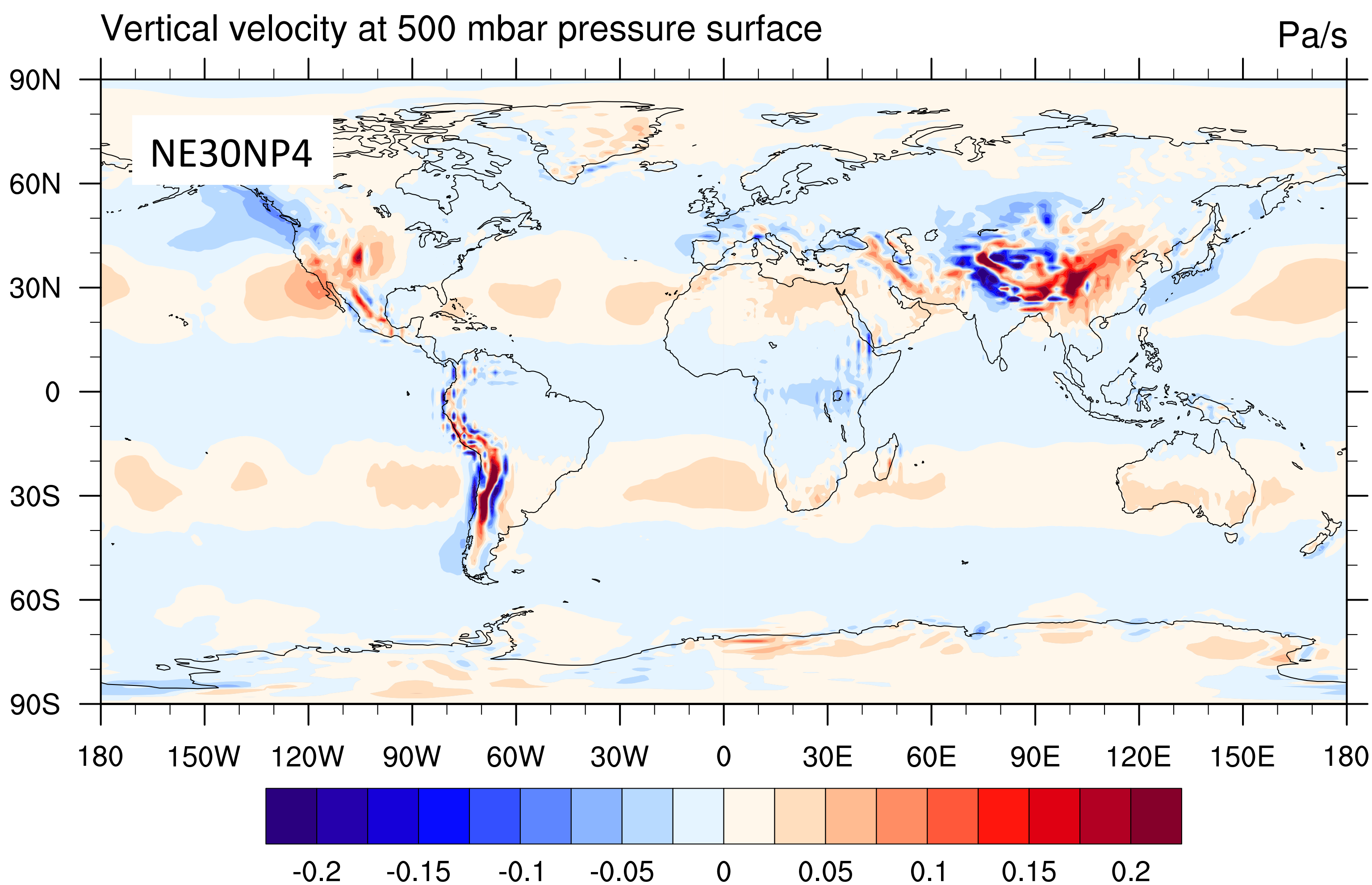

PRECT (NE30NP4NC4_APE)
5S-5N
mm/day
Time
611.3
612.3
613.3
614.3
615.3
616.3
617.3
618.3
619.3
620.3
621.3
622.3
623.3
624.3
625.3
626.3
627.3
628.3
629.3
630.3
631.3
632.3
633.3
634.3
635.3
636.3
637.3
638.3
639.3
640.3
0
60E
120E
180
120W
60W
2 4 6 8 10 12 14 16 18 20 22 26 34 82
Longitude

Stationary grid scale forcing

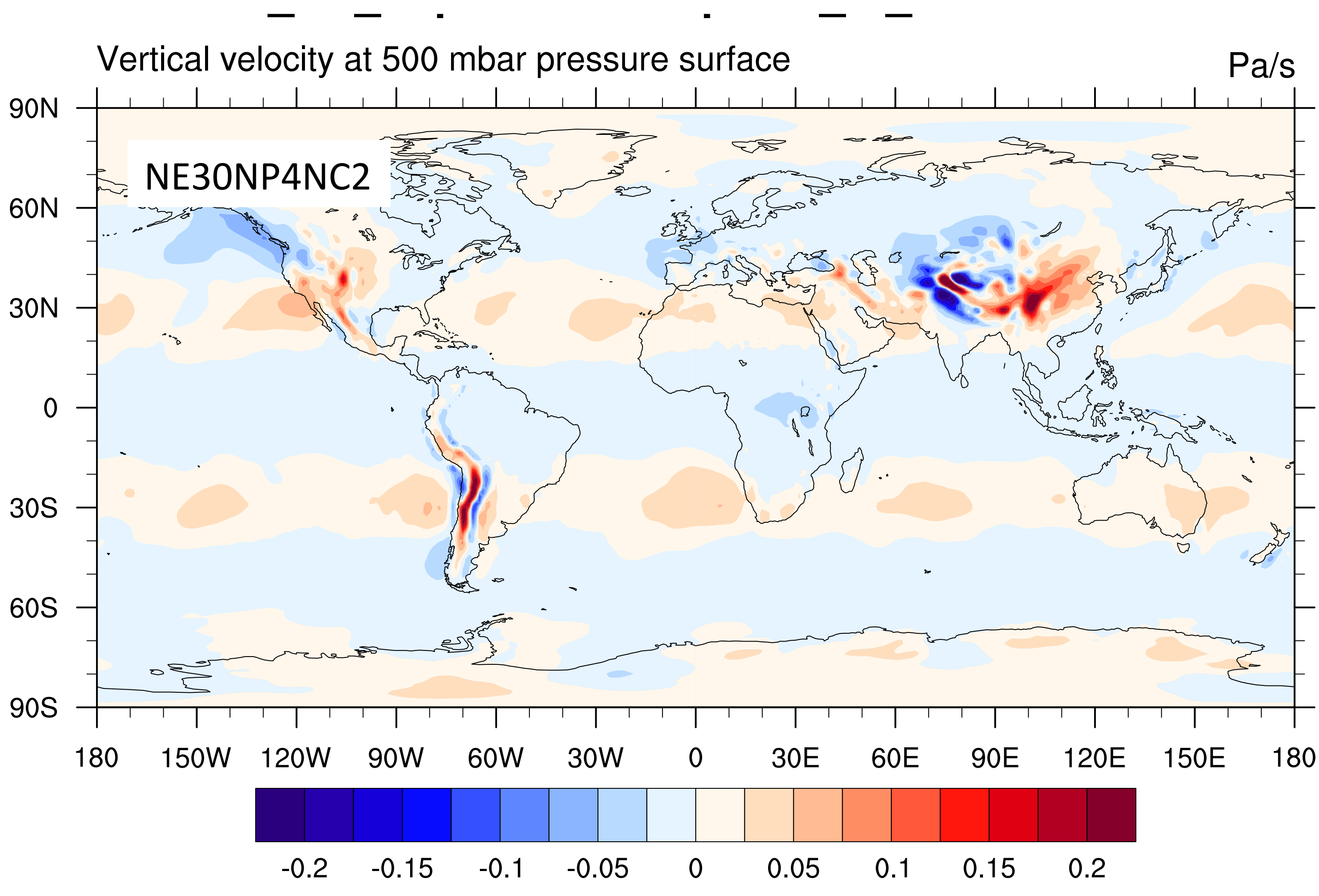


Note that physics grid averages/moves fields away from boundary of element where the solution is least smooth
(in element interior the polynomials are C∞)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
1
1.5
2
2.5
3
3.5
4

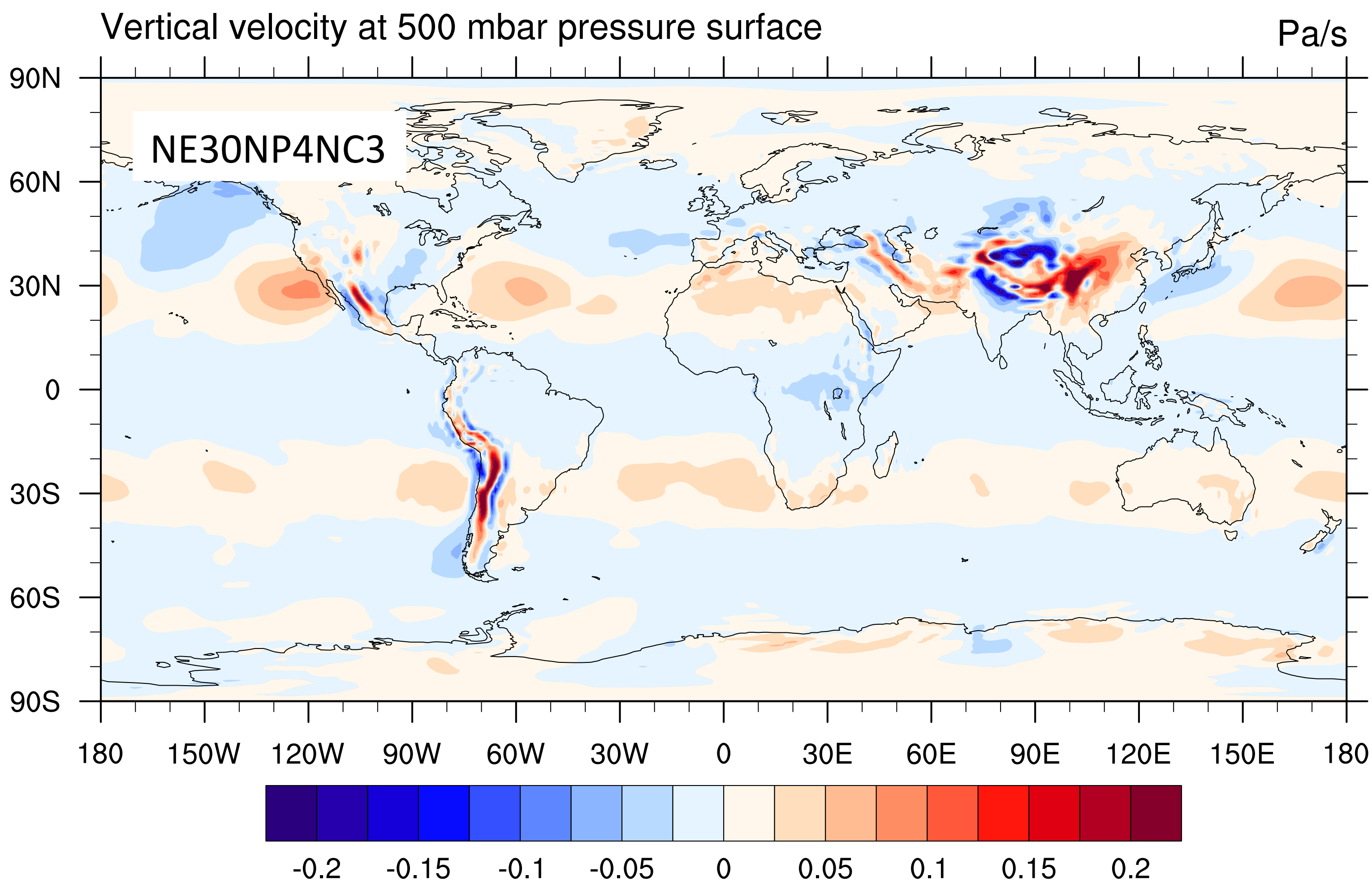
Held-Suarez with topography



Held-Suarez with topography



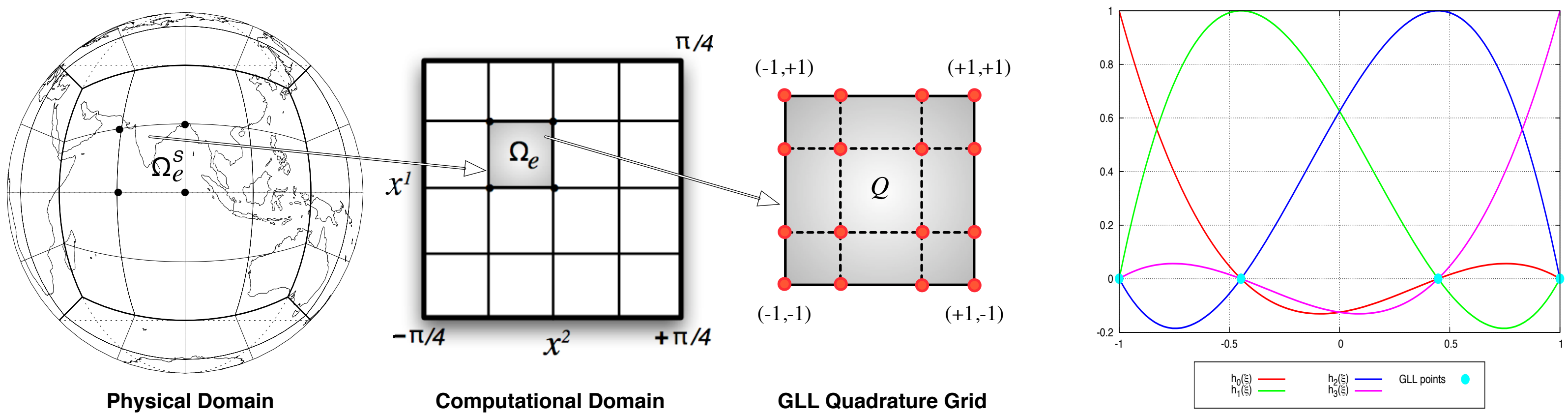
Held-Suarez with topography



CAM-SE: NCAR Community Atmosphere Model with Spectral Elements dynamical core

Continuous Galerkin finite-element method (Taylor et al., 1997) on a cubed-sphere:

Nair et al., 2011

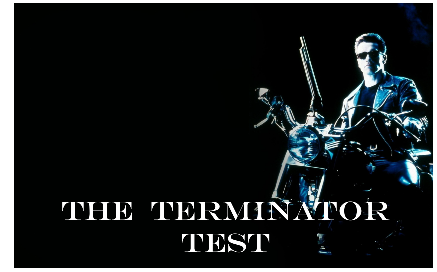


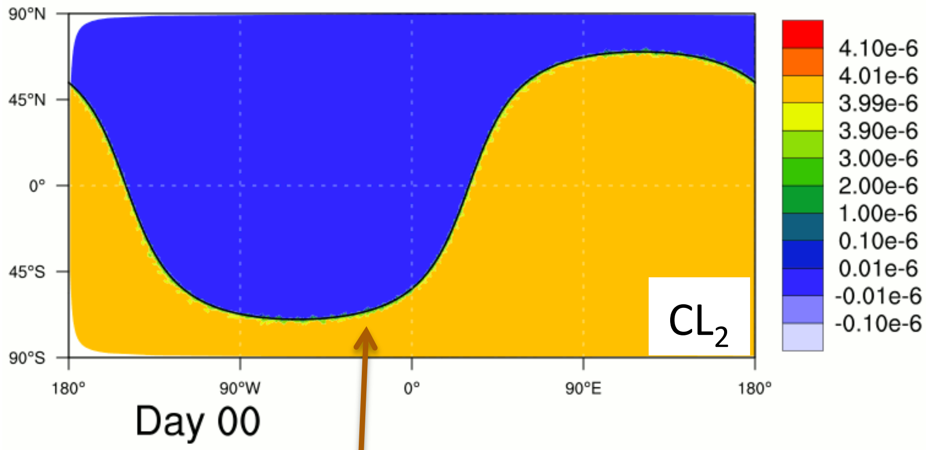
- Discretization is mimetic => mass-conservation & total energy conservation
- Conserves axial angular momentum very well (Lauritzen et al., 2014)
- Support static mesh-refinement and retains formal order of accuracy!
- Highly scalable to at least O(100K) processors (Dennis et al., 2012)
- Competitive "AMIP-climate" (Evans et al., 2012)
- **Lower computational throughput for many-tracer applications**
- **Tracer transport accuracy?**

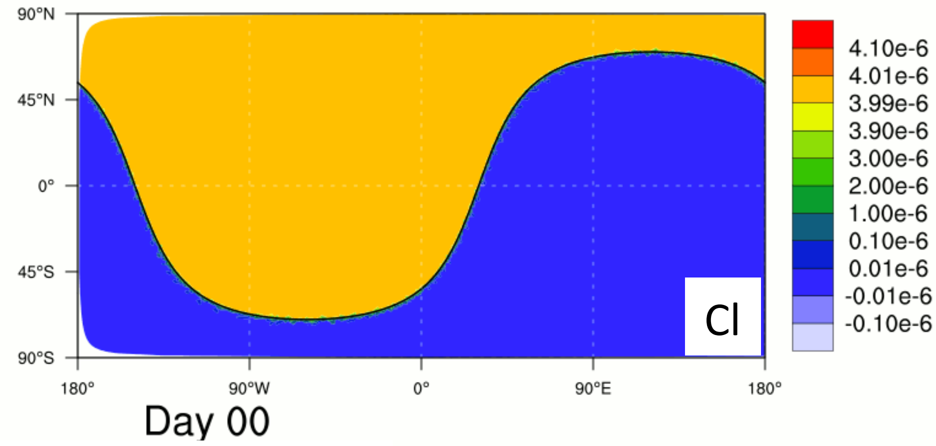
The terminator 'toy'-chemistry test: A simple tool to assess errors in transport schemes

(Lauritzen et al., 2015, GMD)

See: http://www.cgd.ucar.edu/cms/pel/terminator.html







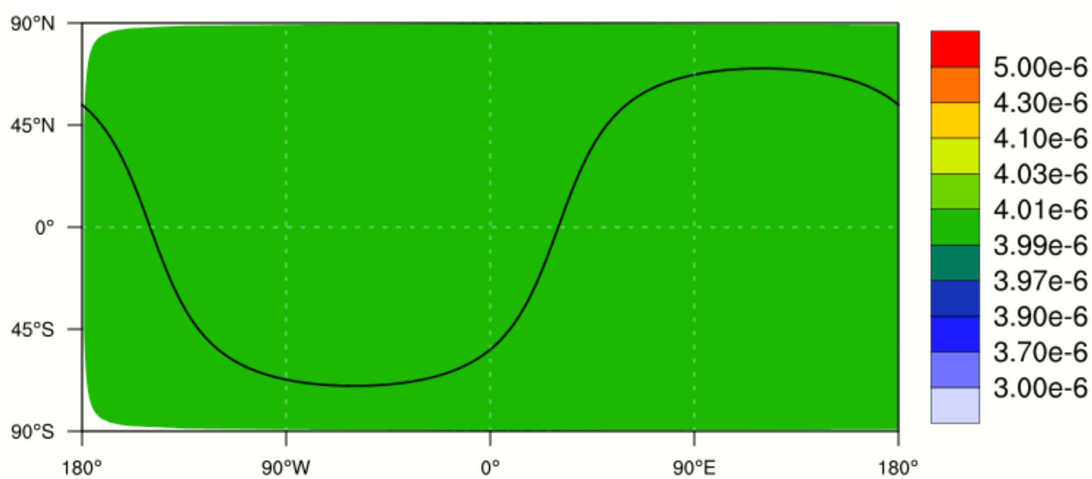
Non-linear Terminator 'toy' chemistry:

$$Cl_2 \rightarrow Cl + Cl : k_1$$

$$Cl + Cl \rightarrow Cl_2 : k_2$$

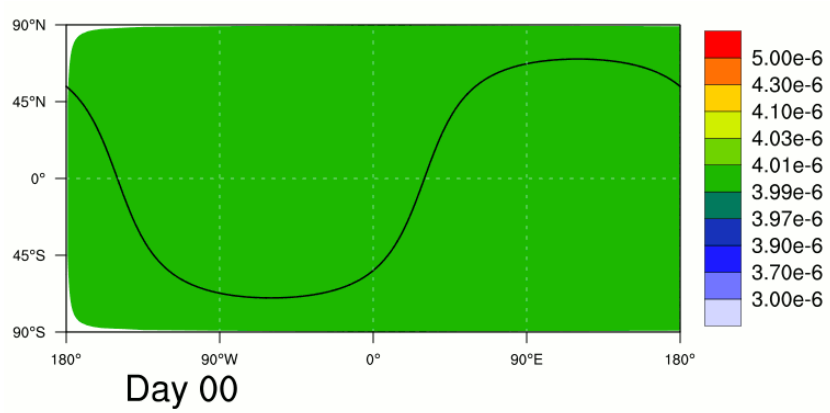
Exact solution:
$Cl+2*Cl_2$ = constant

$Cl+2*Cl_2$ = constant

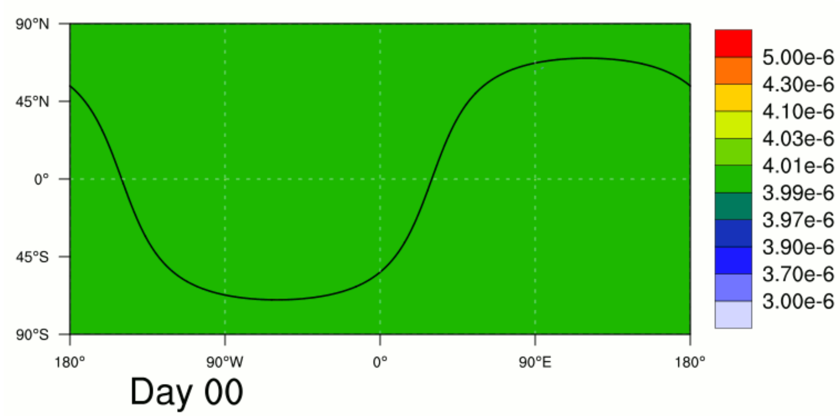


Wind field: Nair and Lauritzen deformational flow

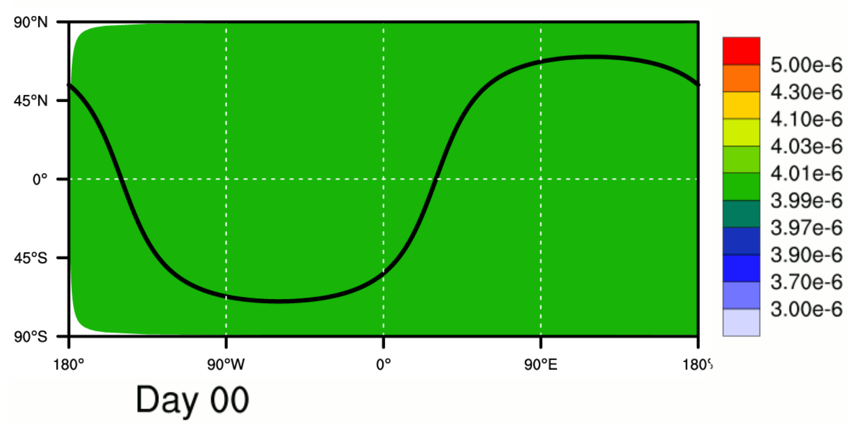
Errors are due to non-conservation of linear correlations usually caused by the limiter/filter and/or physics-dynamics coupling!



CAM-SE



CAM-FV

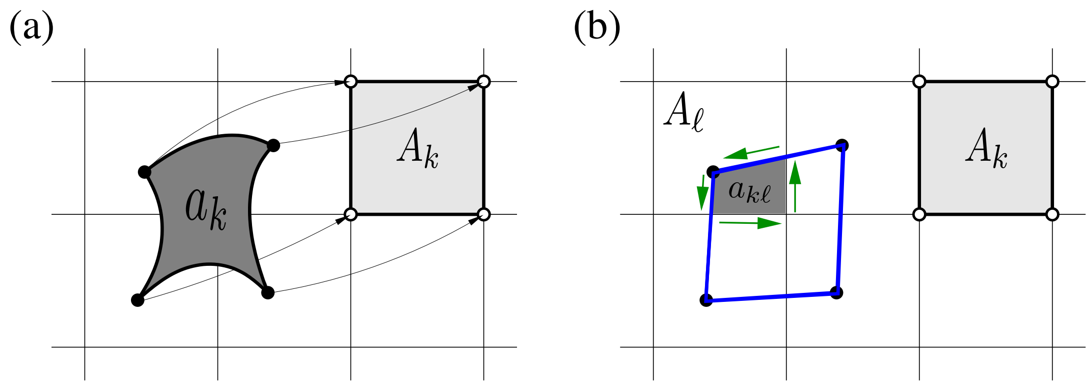


CSLAM

A way to accelerate tracer transport:

Basic formulation | Lauritzen et al. (2010), Erath et al. (2013), Erath et al. (2012)

Conservative Semi-LAgrangian Multi-tracer (CSLAM)



Finite-volume Lagrangian form of continuity equation for air (pressure level thickness, Δp), and tracer (mixing ratio, q):

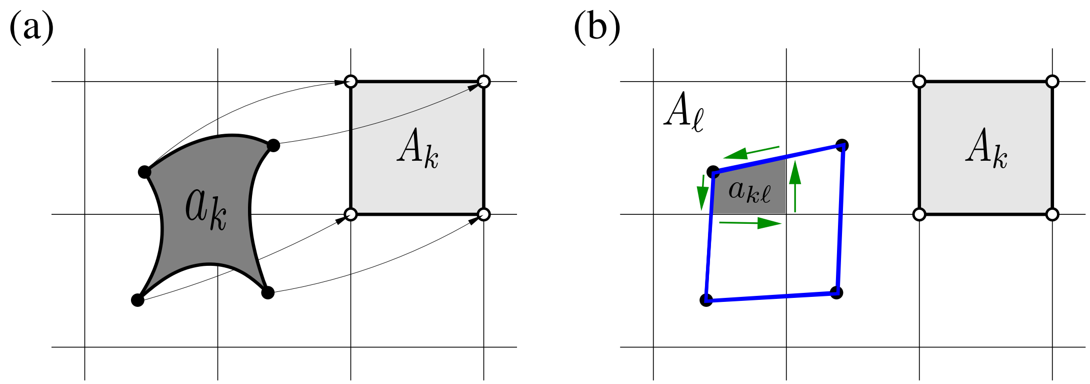
$$\int_{A_k} \psi_k^{n+1}\, dA = \int_{a_k} \psi_k^n\, dA \quad = \quad \sum_{\ell=1}^{L_k} \left[\sum_{\imath+\jmath\leq 2} c_\ell^{(\imath,\jmath)} w_{k\ell}^{(\imath,\jmath)} \right], \quad \psi = \Delta p,\, \Delta p\, q,$$

where n time-level, $a_{k\ell}$ overlap areas, L_k #overlap areas, $c^{(\imath,\jmath)}$ reconstruction coefficients for ψ_k^n, and $w_{k\ell}^{(\imath,\jmath)}$ weights.

A way to accelerate tracer transport:

Basic formulation | Lauritzen et al. (2010), Erath et al. (2013), Erath et al. (2012)

Conservative Semi-LAgrangian Multi-tracer (CSLAM)



CSLAM time-step restriction: flow deformation (upstream area must be simply connected)

Current implementation in CAM-SE: CN < 1, where Courant number.

Spectral-element advection: RK2 with CN<0.3

=> 3 times longer time-step with CSLAM compared to SE advection scheme

A way to accelerate tracer transport:

Basic formulation | Lauritzen et al. (2010), Erath et al. (2013), Erath et al. (2012)

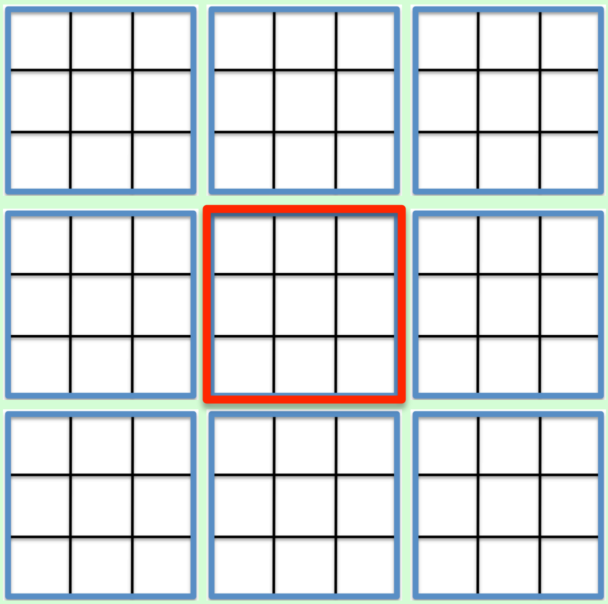
Conservative Semi-LAgrangian Multi-tracer (CSLAM)

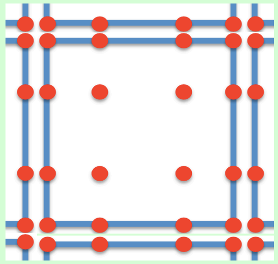
MPI communication

For every 30 minute physics time-step:

- SE performs 6 tracer time-steps with 2 Runga-Kutta stages => **12 MPI calls**
- CSLAM performs 2 tracer time-steps (CN<1) => **2 MPI calls**

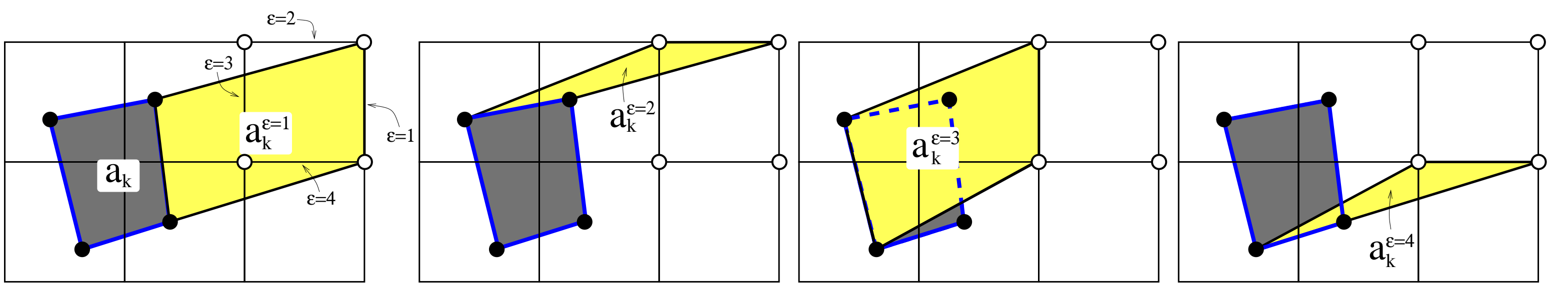
That said, CSLAM needs a much larger halo than SE.





A way to accelerate tracer transport:

Flux-form CSLAM ≡ Lagrangian CSLAM



$$\int_{A_k} \psi_k^{n+1}\, dA = \int_{A_k} \psi_k^{n}\, dA - \sum_{\epsilon=1}^{4} s_{k\ell}^{\epsilon} \int_{a_k^{\epsilon}} \psi\, dA, \quad \psi = \Delta p,\, \Delta p\, q.$$

where

- a_k^{ϵ} = 'flux-area' (yellow area) = area swept through face ϵ
- $s_{k\ell}^{\epsilon}$ = 1 for outflow and -1 for inflow.

Flux-form and Lagrangian forms of CSLAM are equivalent (Lauritzen et al., 2011).

Coupling finite-volume semi-Lagrangian transport with spectral element dynamics

4. Consistency

The continuity equations for air and tracers are coupled:

$$\int_{A_k} \Delta p_k^{n+1}\, dA = \int_{a_k} \delta p_k^n\, dA, \quad (2)$$

$$\int_{A_k} (\Delta p\, q)_k^{n+1}\, dA = \int_{a_k} (\delta p_k q)^n\, dA. \quad (3)$$

If $q = 1$ then (3) should reduce to (2).

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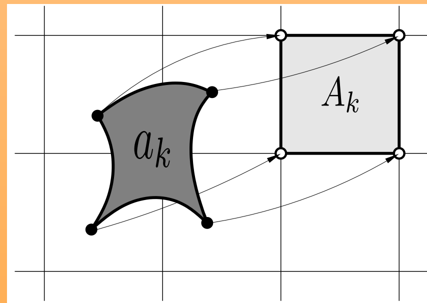
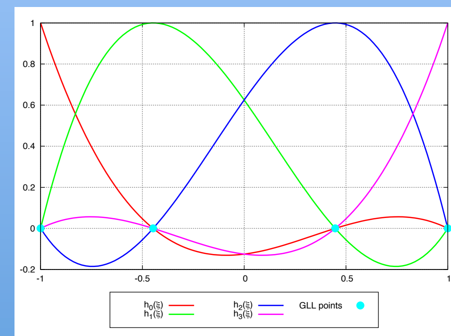
Coupling finite- ... an tra ... leme ...

Solved with semi-Lagrangian scheme (CSLAM)
(max Courant number < 1)



Solved with spectral-element Eulerian advection operator
(max Courant number < 0.3)



The continuity equ... for air and tracers ... coupled:

$$\int_{A_k} \Delta p_k^{n+1}\, dA = \int_{a_k} \delta p_k^n\, dA, \tag{2}$$

$$\int_{A_k} (\Delta p\, q)_k^{n+1}\, dA = \int_{a_k} (\delta p_k q)^n\, dA. \tag{3}$$

If $q = 1$ then (3) should reduce to (2).

We need to couple without violating mass-conservation, shape-preservation, and consistency

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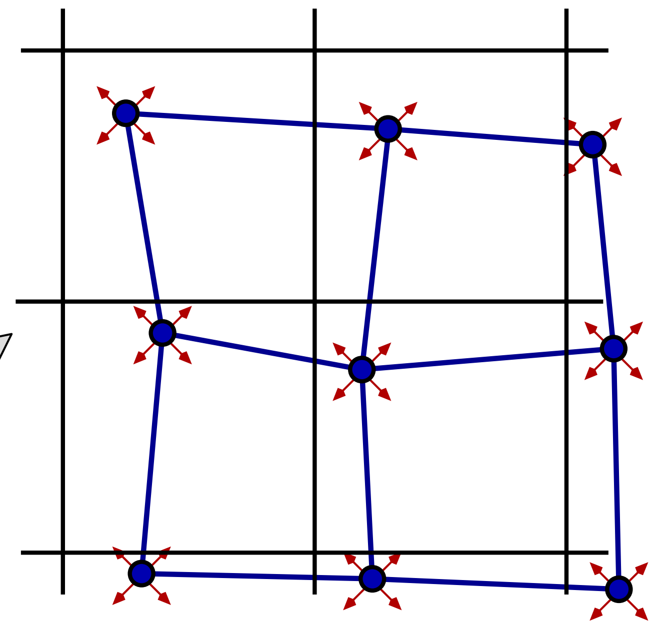
4. Consistency

Find upstream area, a_k, so that CSLAM predicted mass field is equal to CAM-SE predicted mass field:

$$\Delta p_k^{n+1}(\text{CAM-SE}) \quad = \quad \frac{1}{\Delta A}\int_{a_k} \delta p_k^n \, dA \; (\text{CSLAM}) \; , . \qquad (4)$$

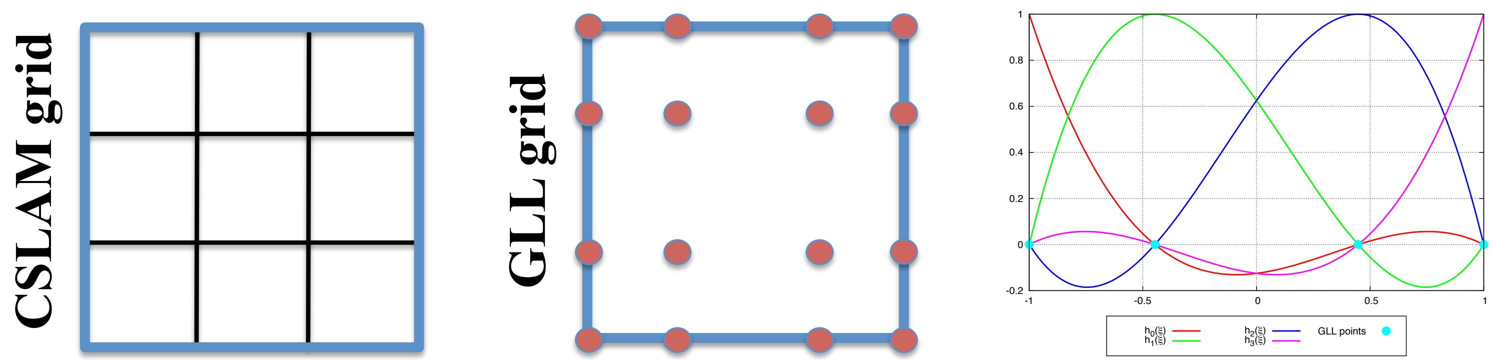
If we choose to move departure points around so that (4) is fulfilled a global iteration problem results!

(and I am not sure it is well-posed!)



Solution: Cast problem in flux-form

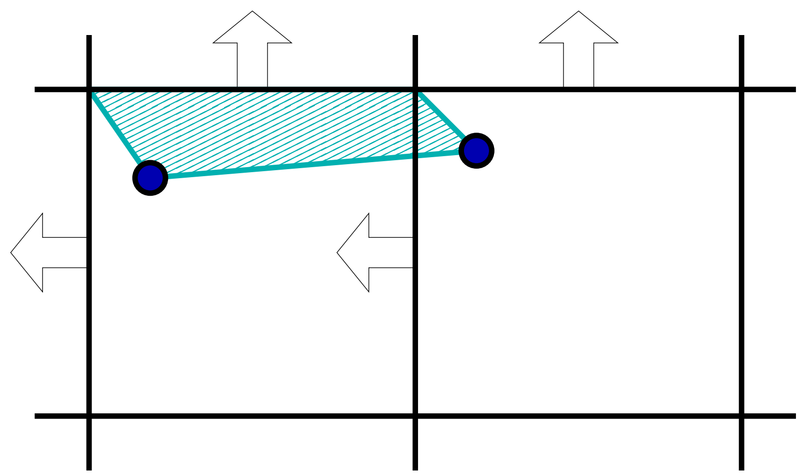
- Spectral-element method does not operate with fluxes: Taylor et al. have derived a method to compute fluxes, $\mathcal{F}^{(SE)}$, through the CSLAM control volume faces! presented at ICMS conference in March, 2015.



Solution: Cast problem in flux-form

Given $\mathcal{F}^{(SE)}$ find swept areas, $\delta\Omega$, so that:

1. $$\mathcal{F}^{(CSLAM)} = \int_{\delta\Omega} \Delta p(x,y)\, dA = \mathcal{F}^{(SE)} \qquad \forall\ \delta\Omega.$$
2. The sum of all the swept areas, $\delta\Omega$, span the domain without cracks or overlaps



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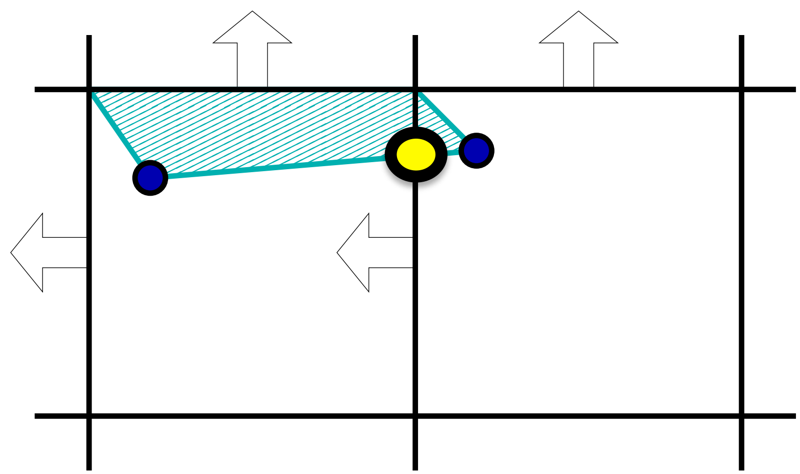
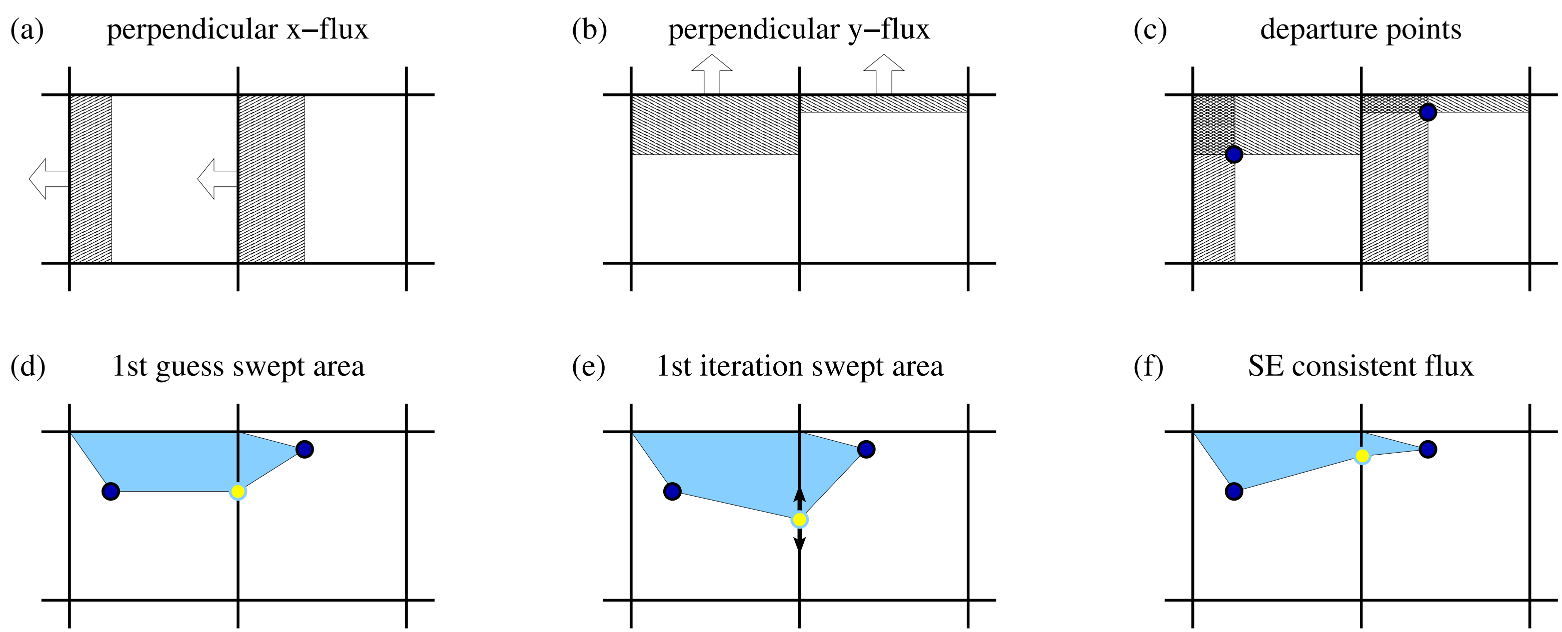


Solution: Cast problem in flux-form

Consistent SE-CSLAM algorithm: step-by-step example



Well-posed? As long as flow deformation $\left|\frac{\partial u}{\partial x}\right| \Delta t \lesssim 1$ (Lipschitz criterion)

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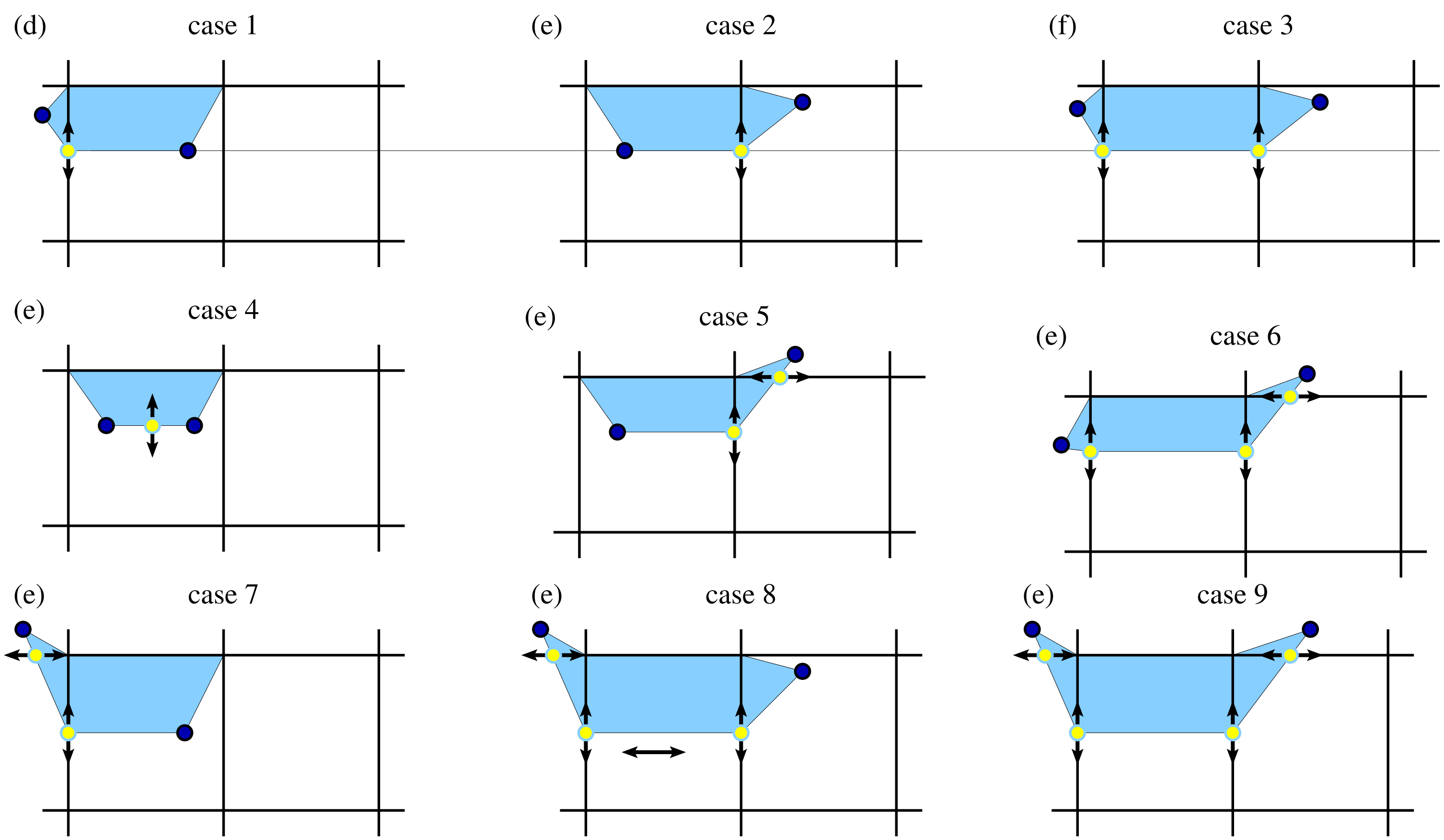
Solution: Cast problem in flux-form

Consistent SE-CSLAM algorithm: flow cases



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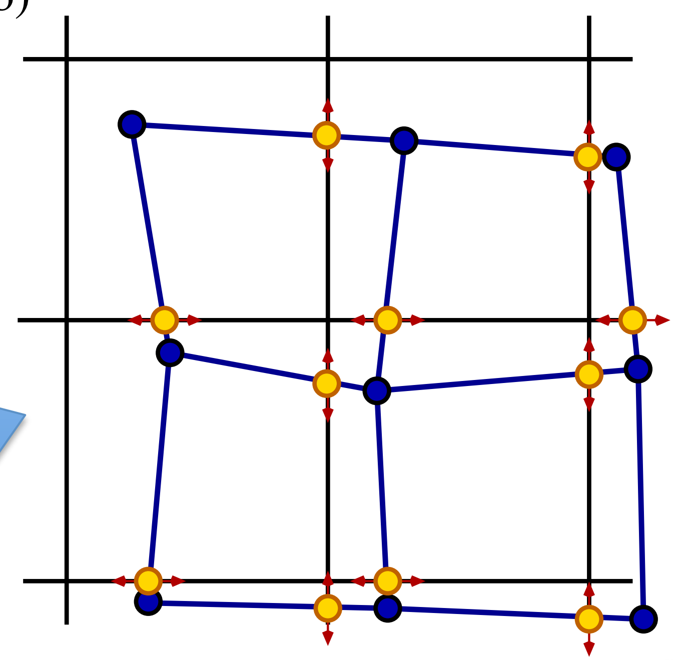
4. Consistency

Find upstream area, a_k, so that CSLAM predicted mass field is equal to CAM-SE predicted mass field:

$$\Delta p_k^{n+1}(\text{CAM-SE}) \quad = \quad \frac{1}{\Delta A}\int_{a_k} \delta p_k^n \, dA \; (\text{CSLAM}) \;, . \tag{4}$$

Local iteration problem to find equivalent upstream areas:

(b)



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Lauritzen, Taylor, Overfelt, Ullrich and Goldhaber (2016, IN PREP)

CAM-SE-CSLAM

A new model configuration based on CAM-SE:

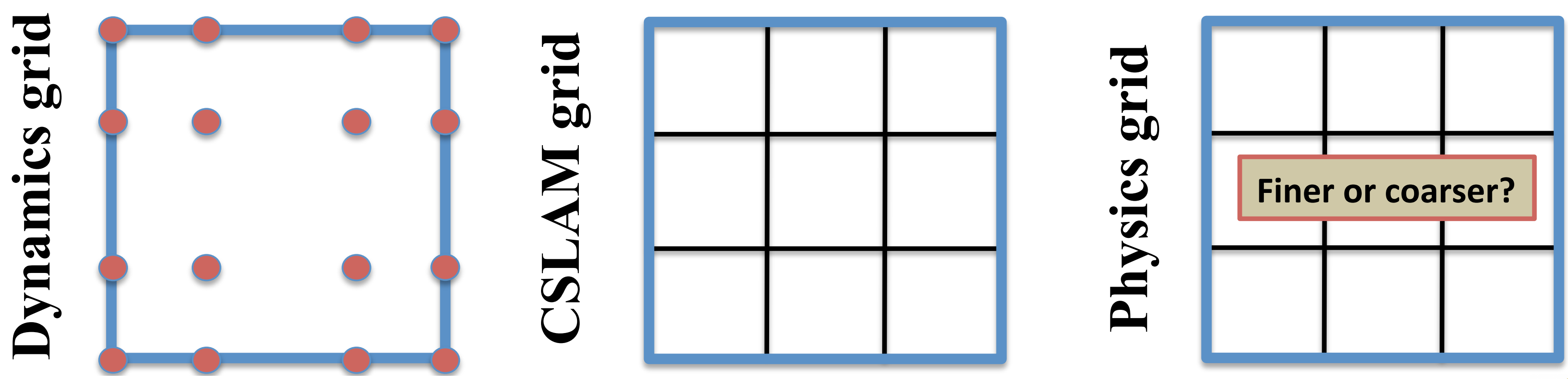
- **SE**: Spectral-element dynamical core solving for $\vec{v}$, T, p_s

  (Dennis et al., 2012; Evans et al., 2012; Taylor and Fournier, 2010; Taylor et al., 1997)

- **CSLAM**: Semi-Lagrangian finite-volume transport scheme for tracers

  (Lauritzen et al., 2010; Erath et al., 2013, 2012; Harris et al., 2010)

- **Phys-grid**: Separating physics and dynamics grids, i.e. ability to compute physics tendencies based on cell-averaged values within each element instead of quadrature points



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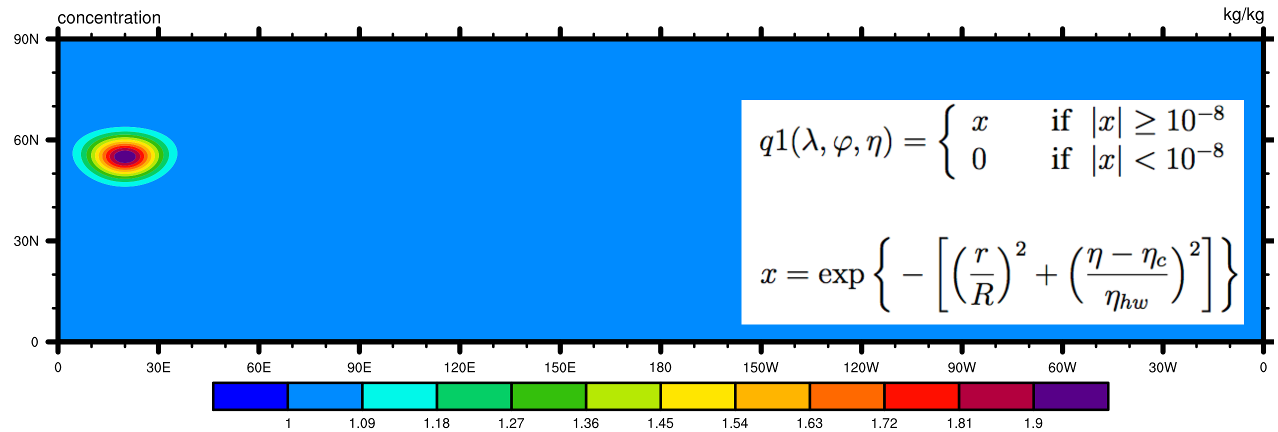
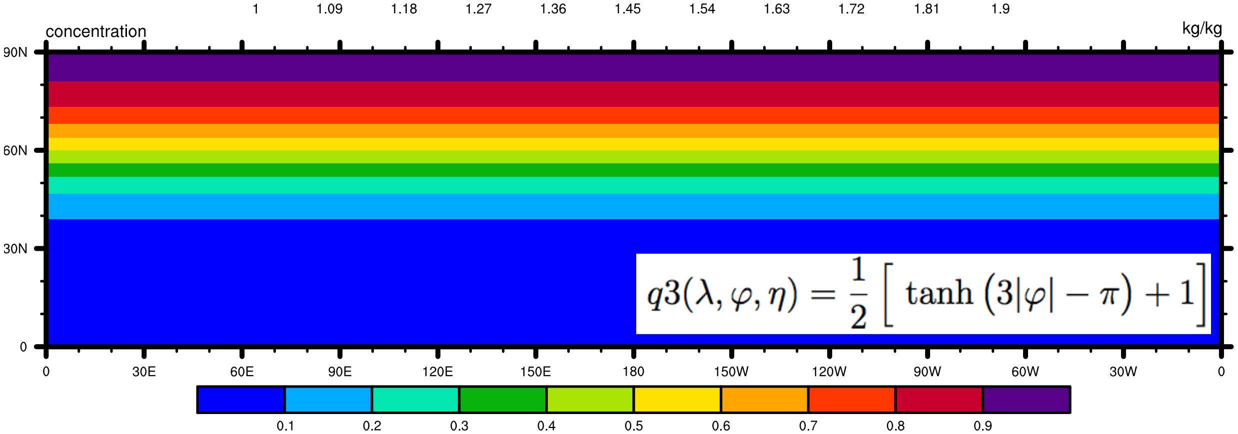
Lauritzen, Taylor, Overfelt, Ullrich and Goldhaber (2016, IN PREP)

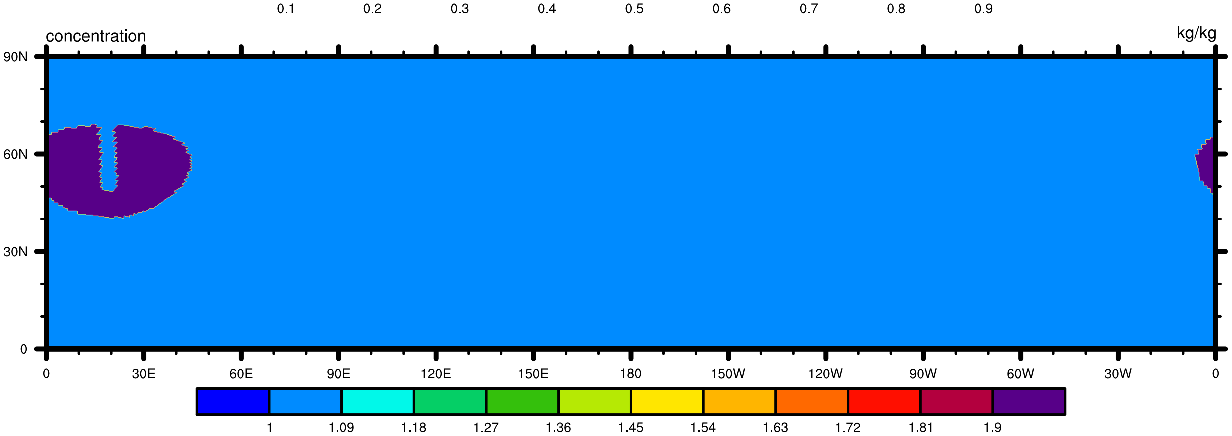
3 tracers: initial conditions



Gaussian "ball"



Zonally symmetric (smooth)



Slotted cylinder

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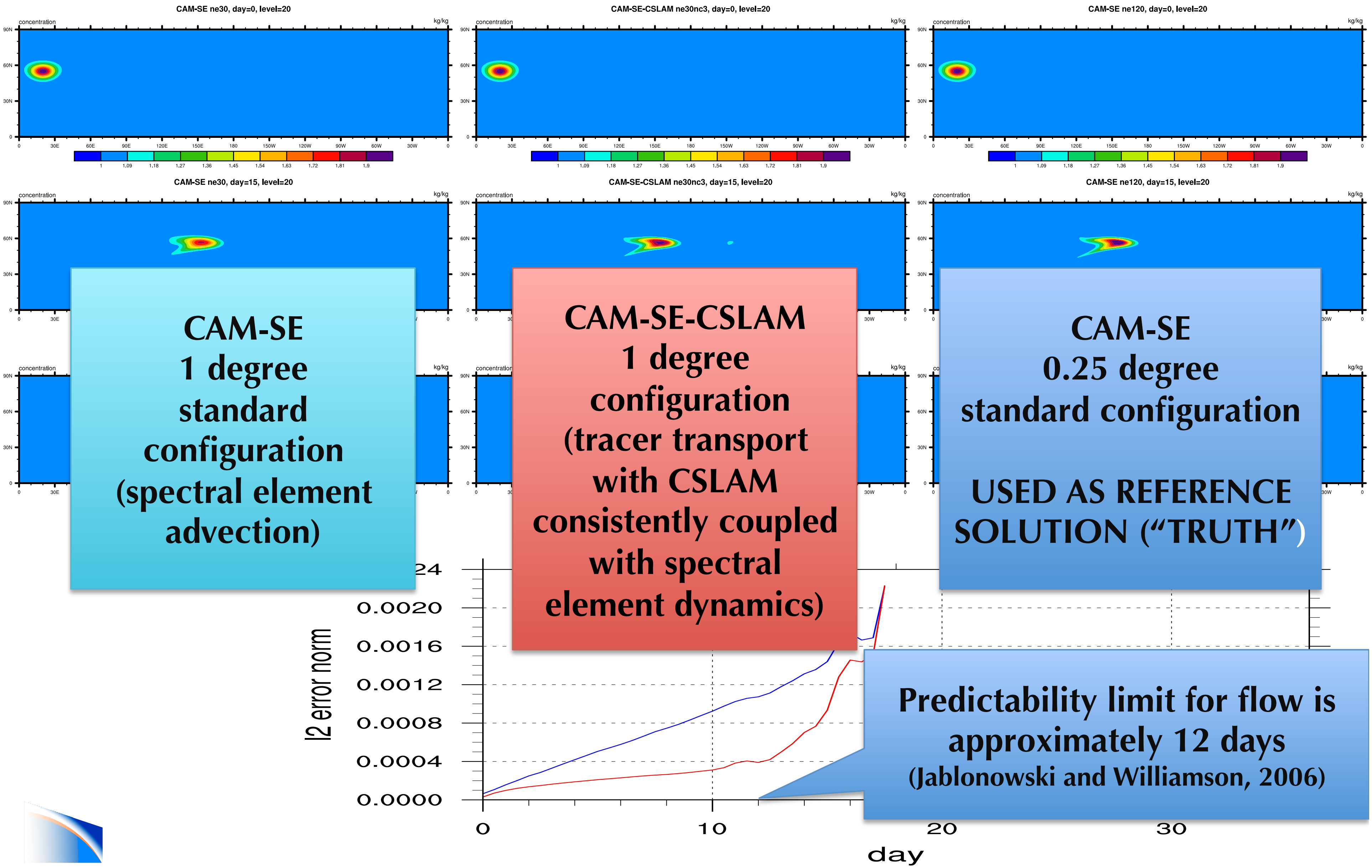
CAM-SE ne30, day=0, level=20
CAM-SE-CSLAM ne30nc3, day=0, level=20
CAM-SE ne120, day=0, level=20
CAM-SE ne30, day=15, level=20
CAM-SE-CSLAM ne30nc3, day=15, level=20
CAM-SE ne120, day=15, level=20
concentration
kg/kg
CAM-SE
1 degree
standard
configuration
(spectral element
advection)
CAM-SE-CSLAM
1 degree
configuration
(tracer transport
with CSLAM
consistently coupled
with spectral
element dynamics)
CAM-SE
0.25 degree
standard configuration
USED AS REFERENCE
SOLUTION ("TRUTH")
Predictability limit for flow is
approximately 12 days
(Jablonowski and Williamson, 2006)
l2 error norm
0.0020
0.0016
0.0012
0.0008
0.0004
0.0000
0
10
20
30
day

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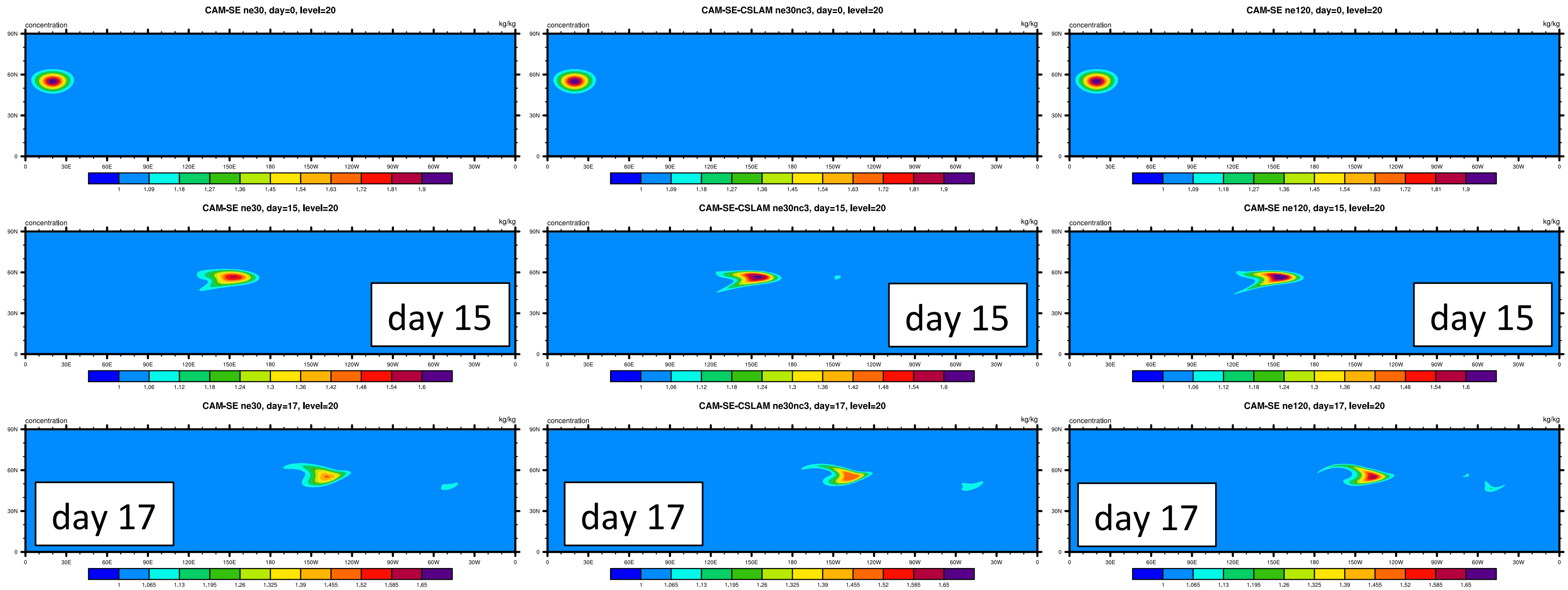
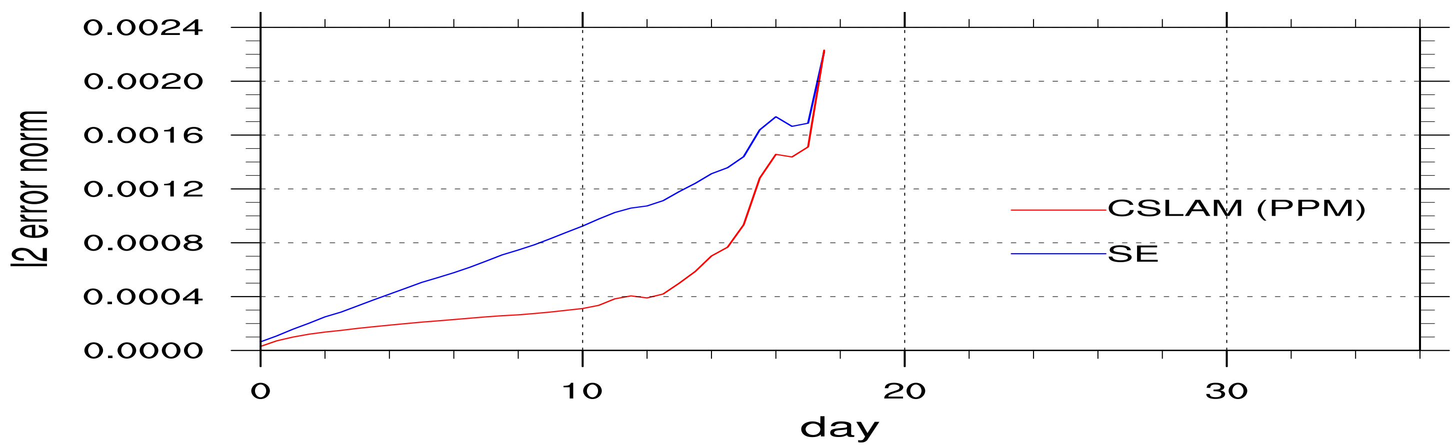
CAM-SE | CAM-SE-CSLAM | CAM-SE reference





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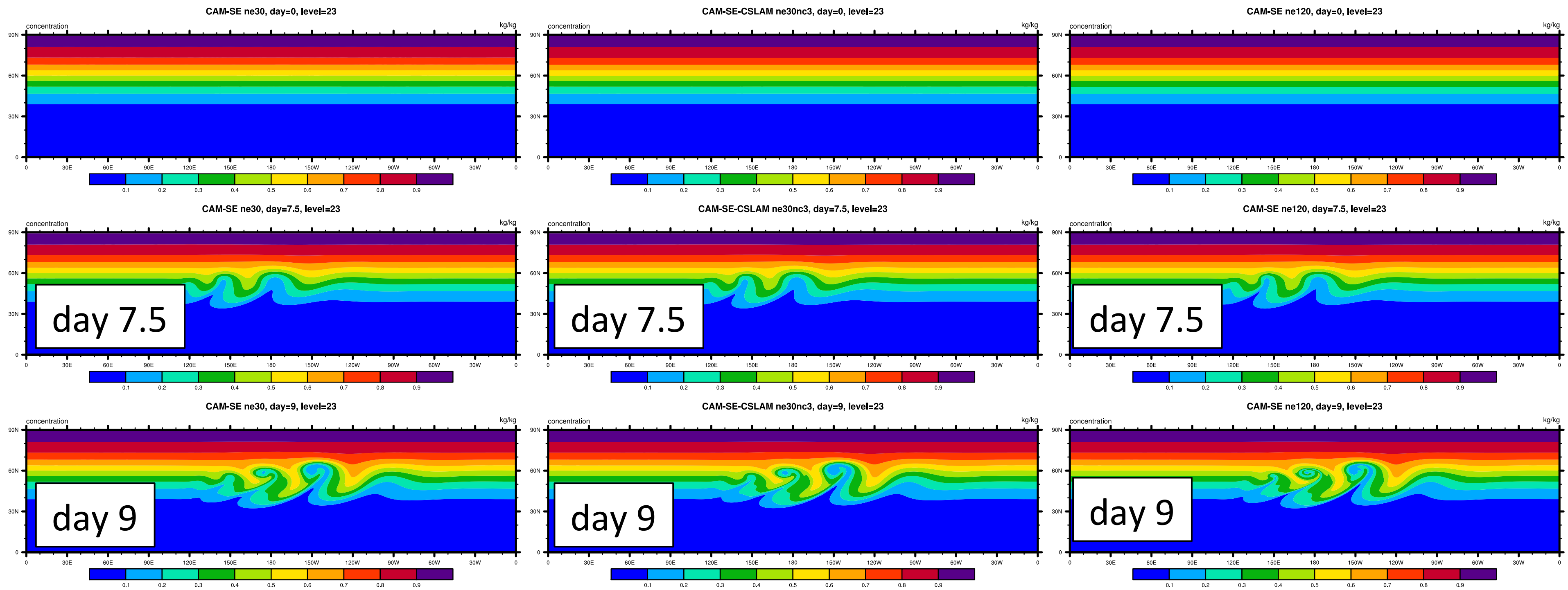
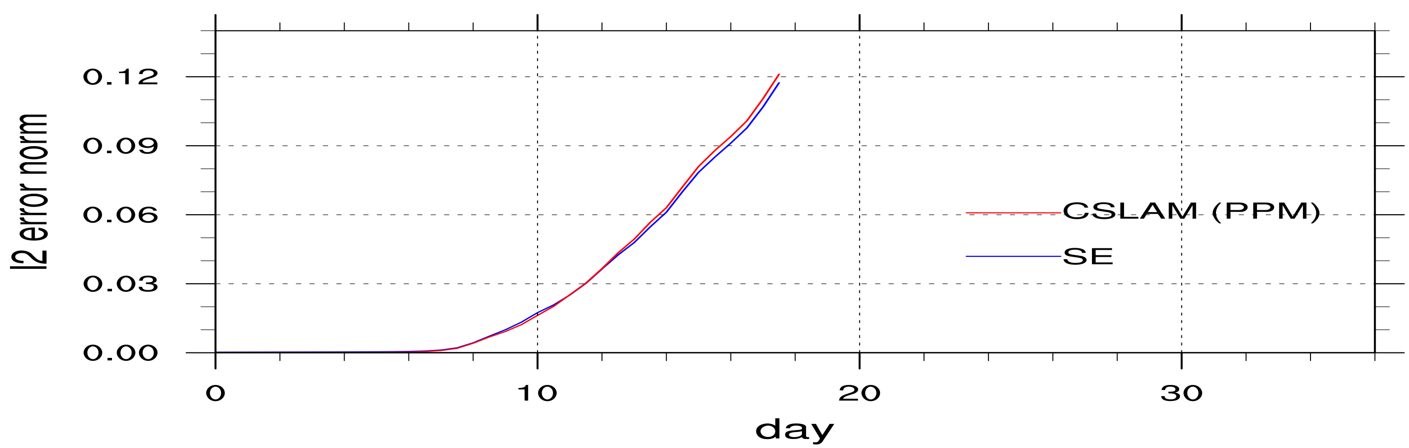
CAM-SE | CAM-SE-CSLAM | CAM-SE reference





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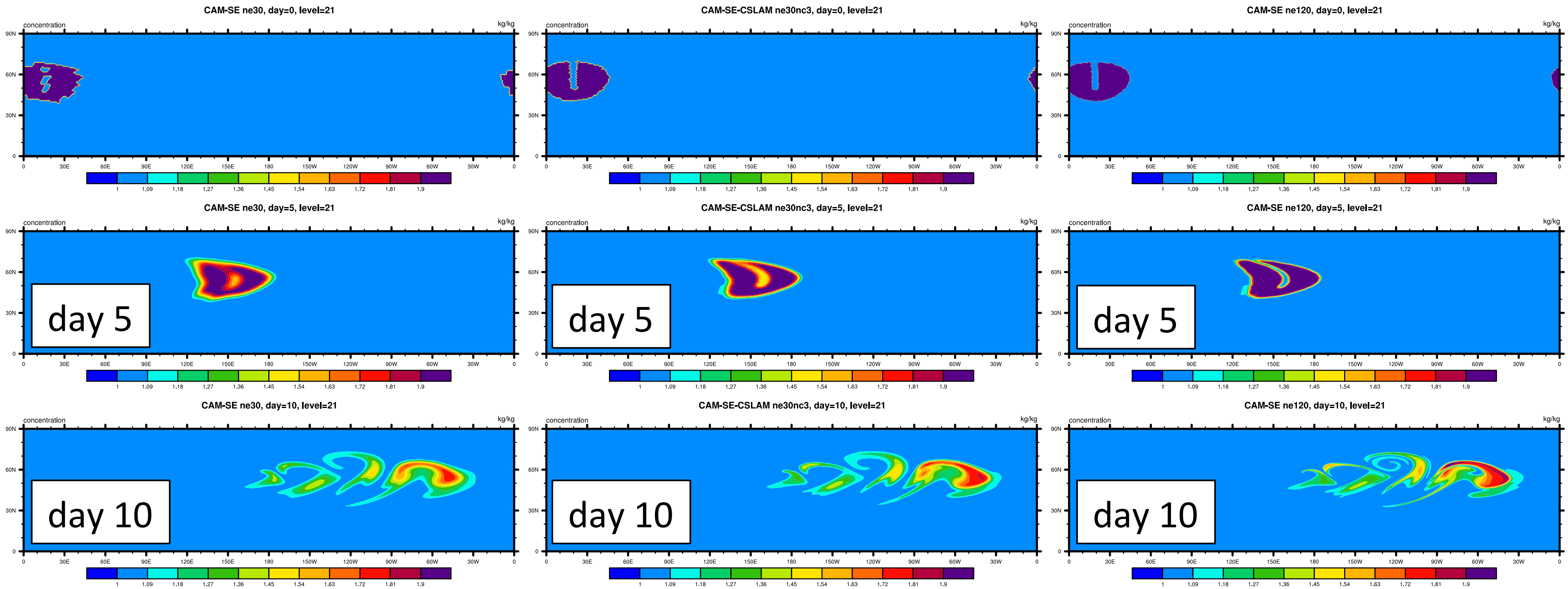
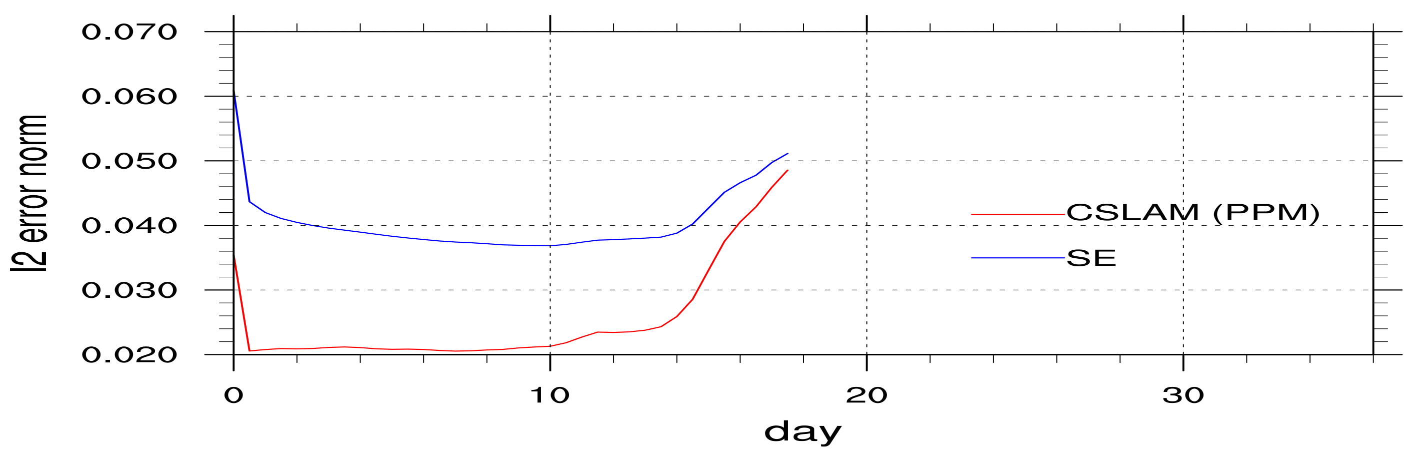
CAM-SE   CAM-SE-CSLAM   CAM-SE reference





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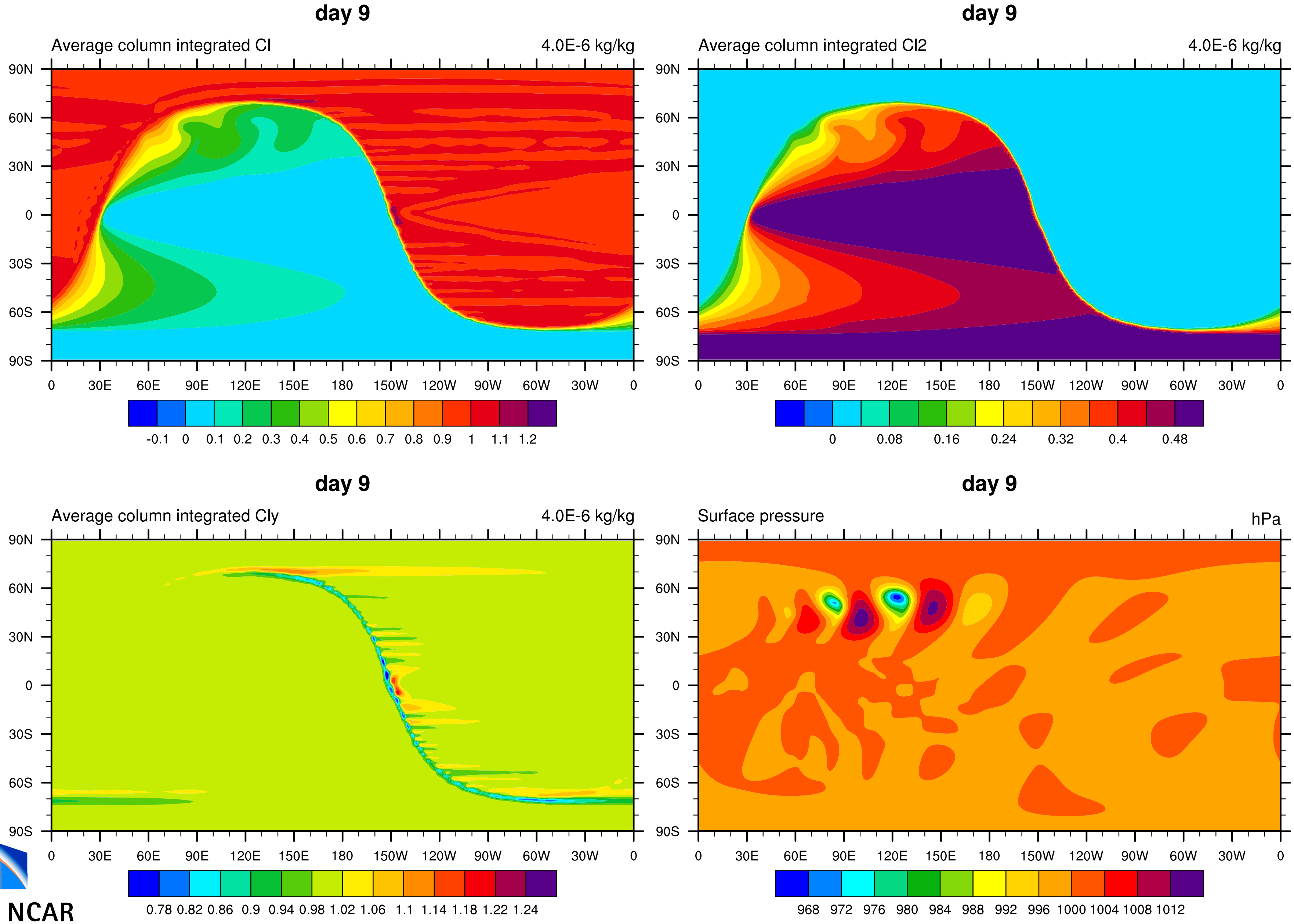
CAM-SE


day 9
Average column integrated Cl
4.0E-6 kg/kg
90N 60N 30N 0 30S 60S 90S
0 30E 60E 90E 120E 150E 180 150W 120W 90W 60W 30W 0
-0.1 0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1 1.1 1.2
day 9
Average column integrated Cl2
4.0E-6 kg/kg
90N 60N 30N 0 30S 60S 90S
0 30E 60E 90E 120E 150E 180 150W 120W 90W 60W 30W 0
0 0.08 0.16 0.24 0.32 0.4 0.48
day 9
Average column integrated Cly
4.0E-6 kg/kg
90N 60N 30N 0 30S 60S 90S
0 30E 60E 90E 120E 150E 180 150W 120W 90W 60W 30W 0
0.78 0.82 0.86 0.9 0.94 0.98 1.02 1.06 1.1 1.14 1.18 1.22 1.24
day 9
Surface pressure
hPa
90N 60N 30N 0 30S 60S 90S
0 30E 60E 90E 120E 150E 180 150W 120W 90W 60W 30W 0
968 972 976 980 984 988 992 996 1000 1004 1008 1012

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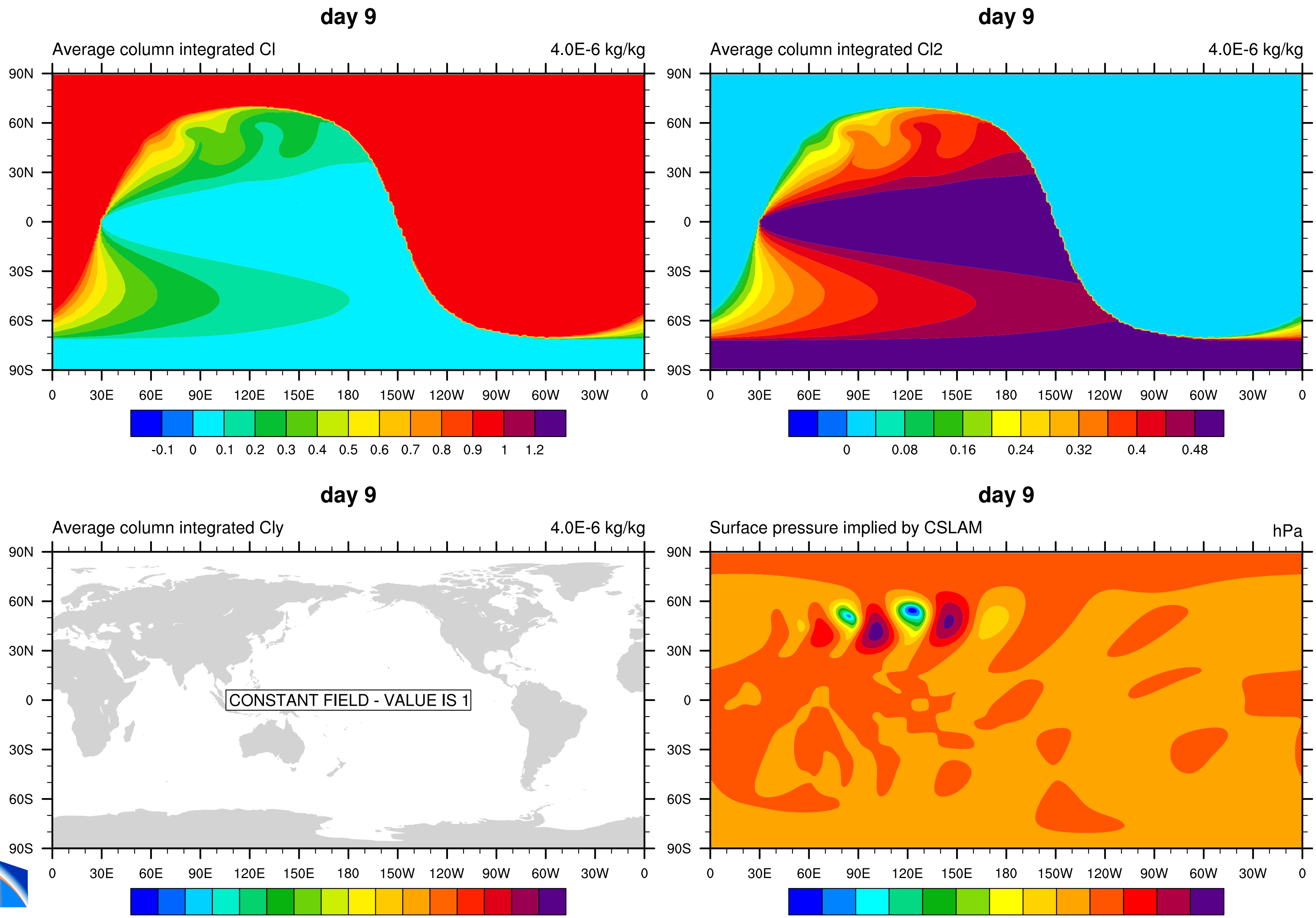
CAM-SE-CSLAM



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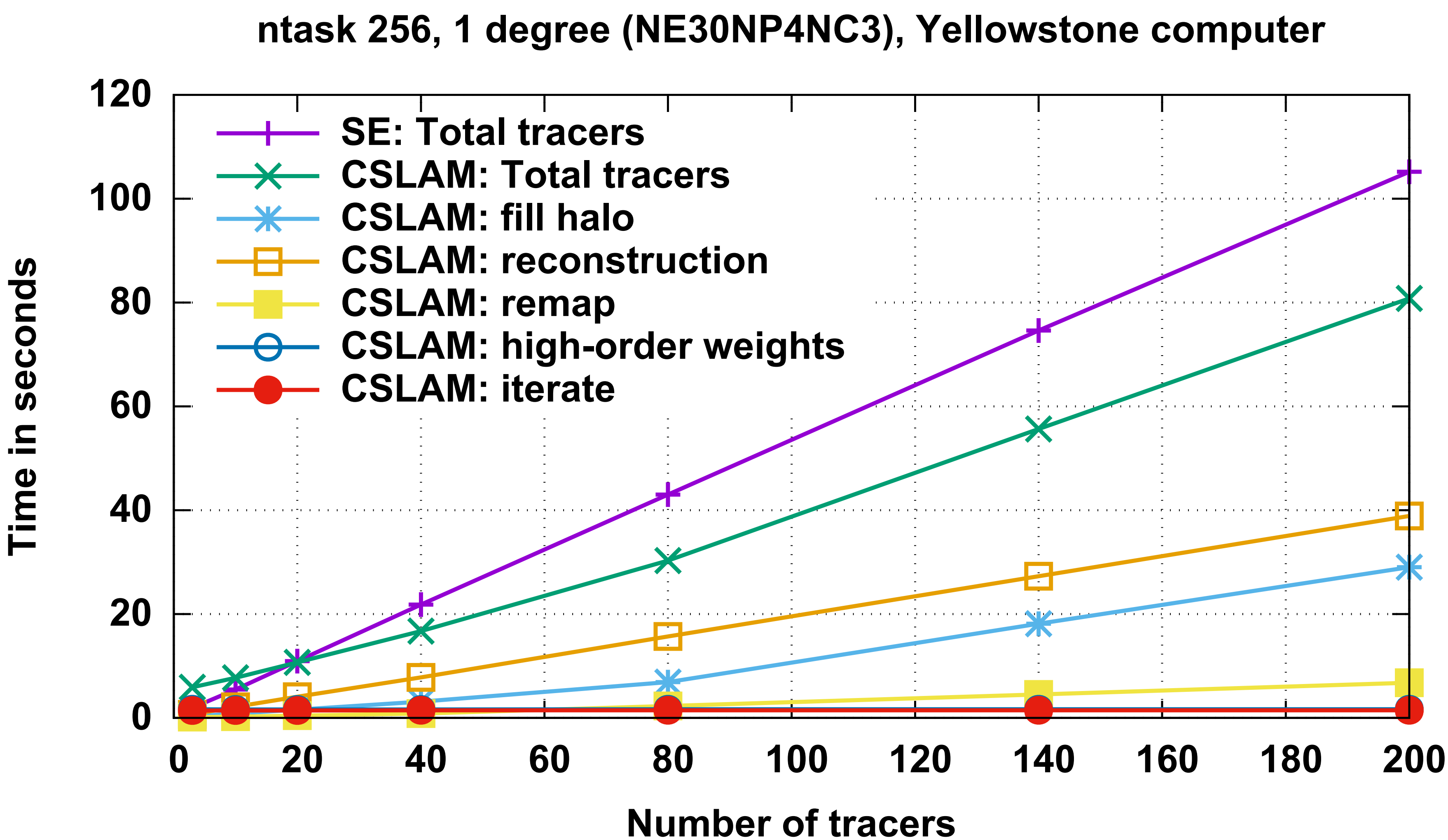
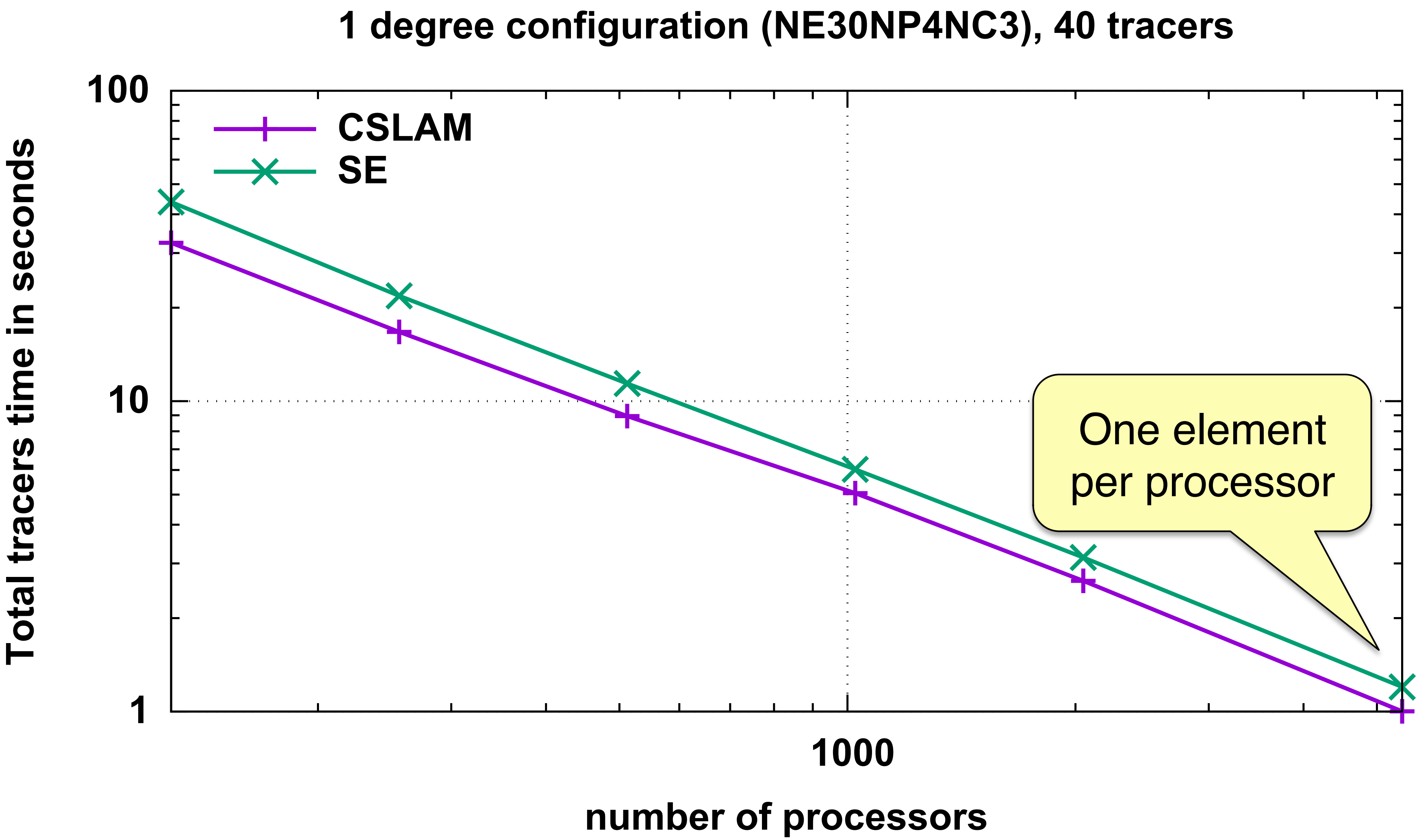
Performance



Performance



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