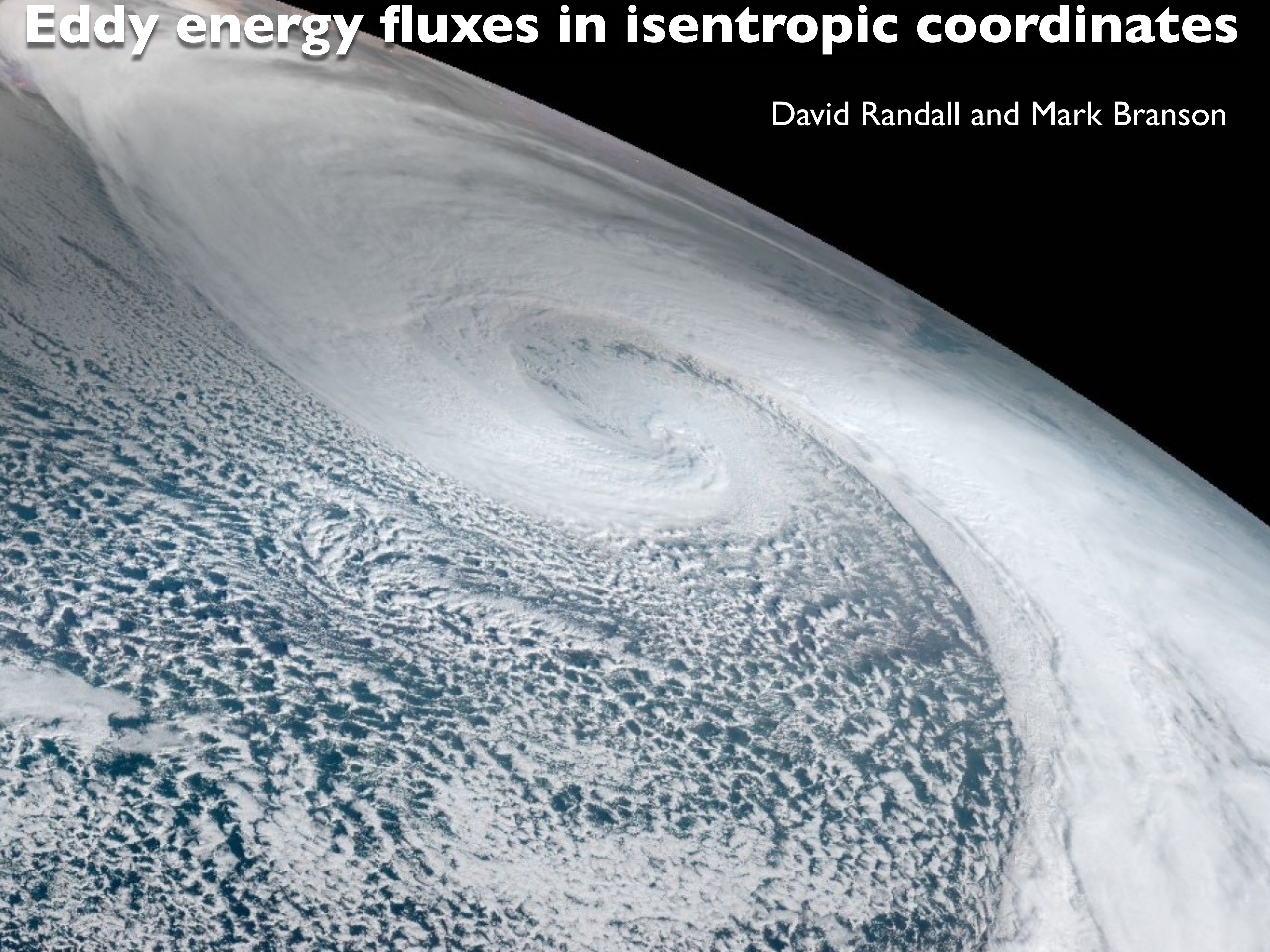
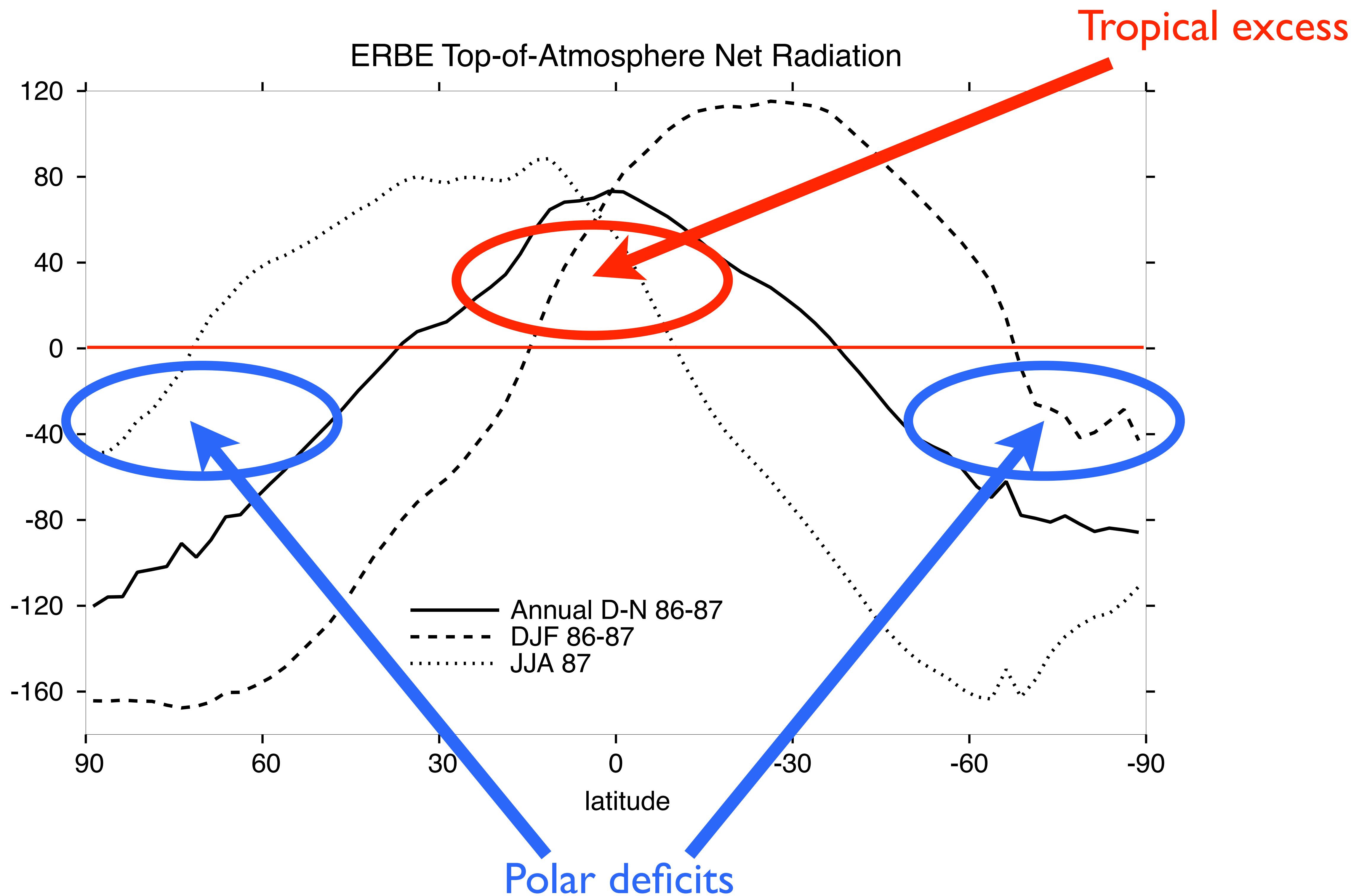
Eddy energy fluxes in isentropic coordinates

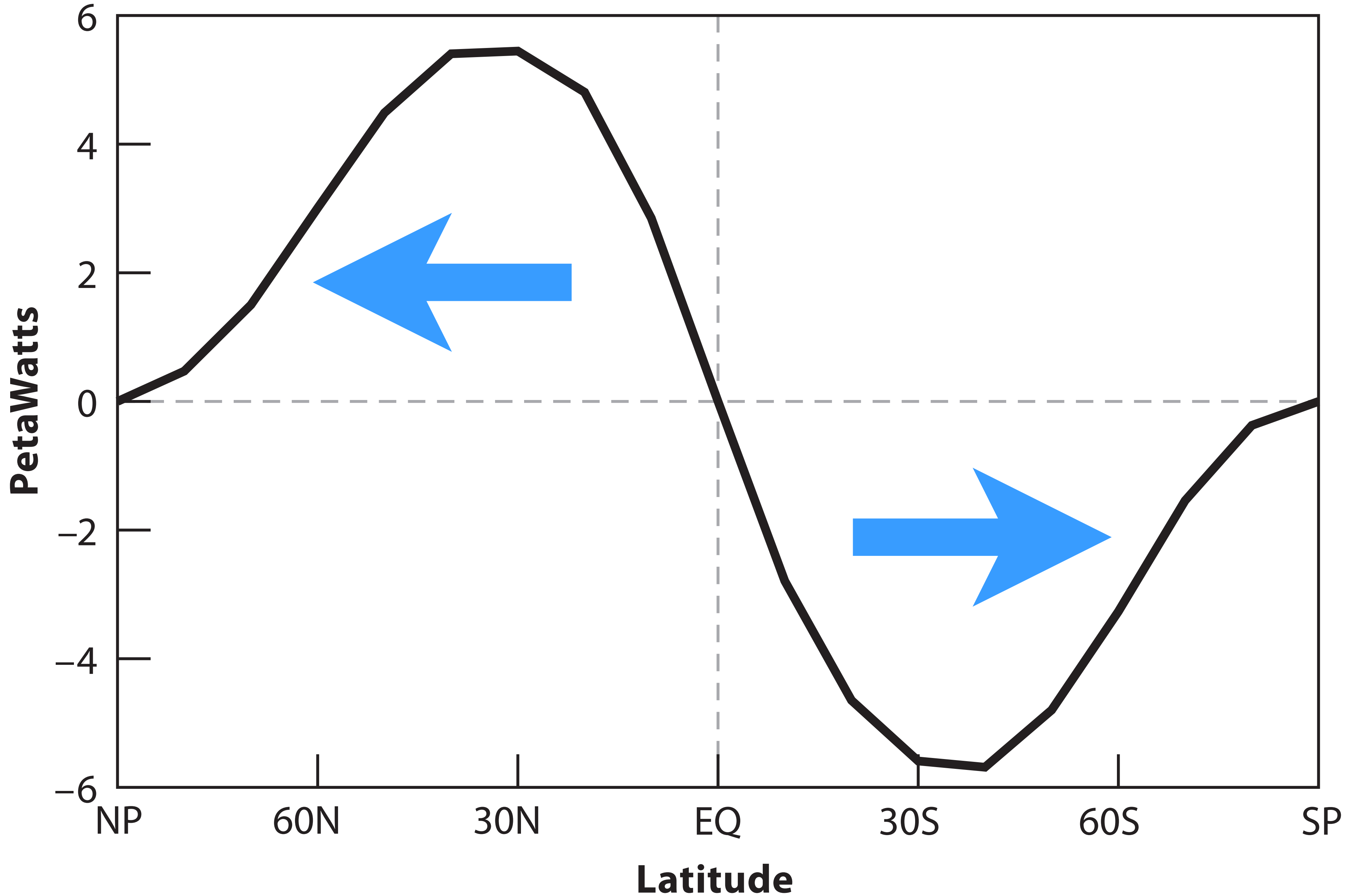
David Randall and Mark Branson

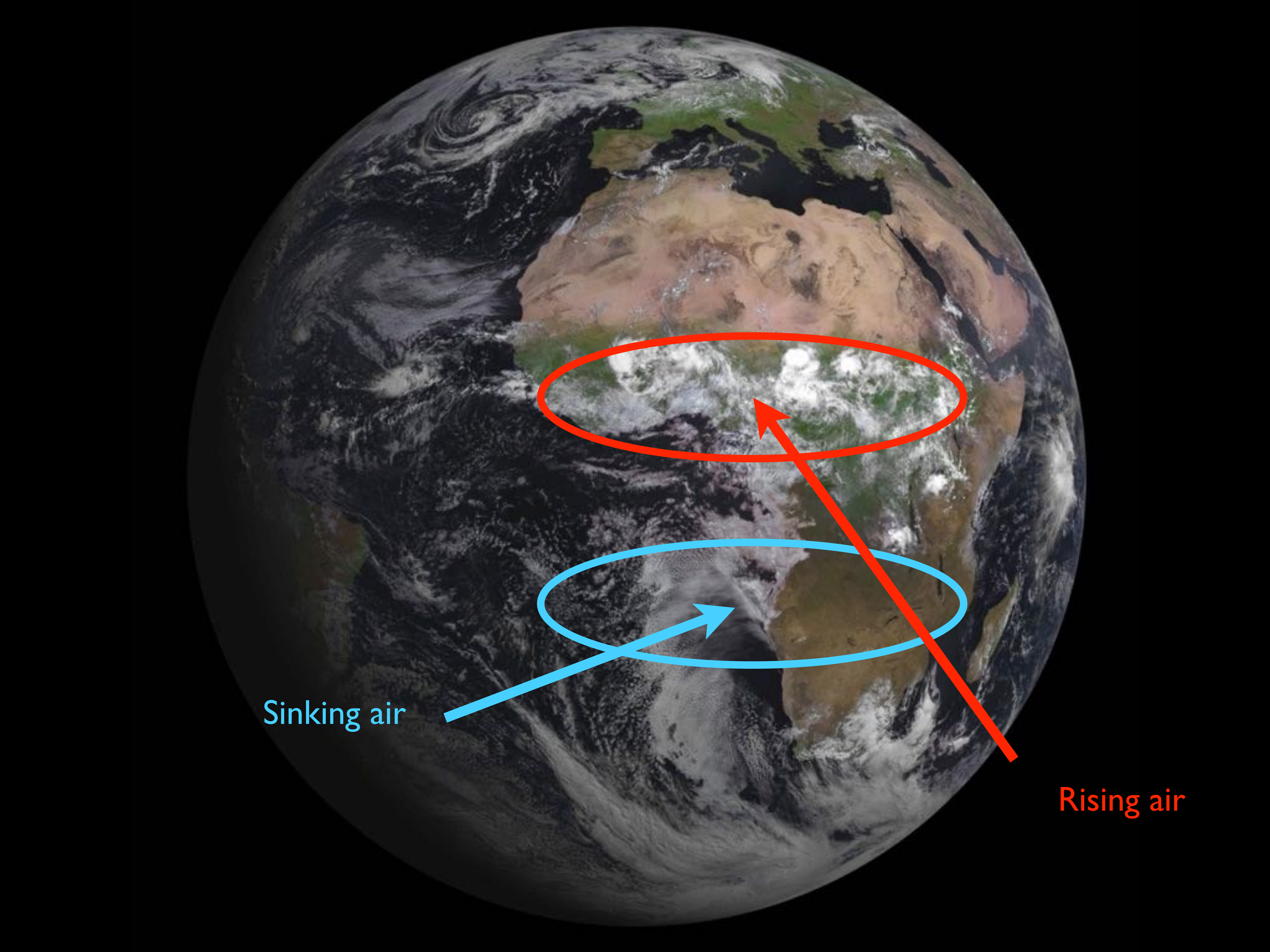


Net energy in and out

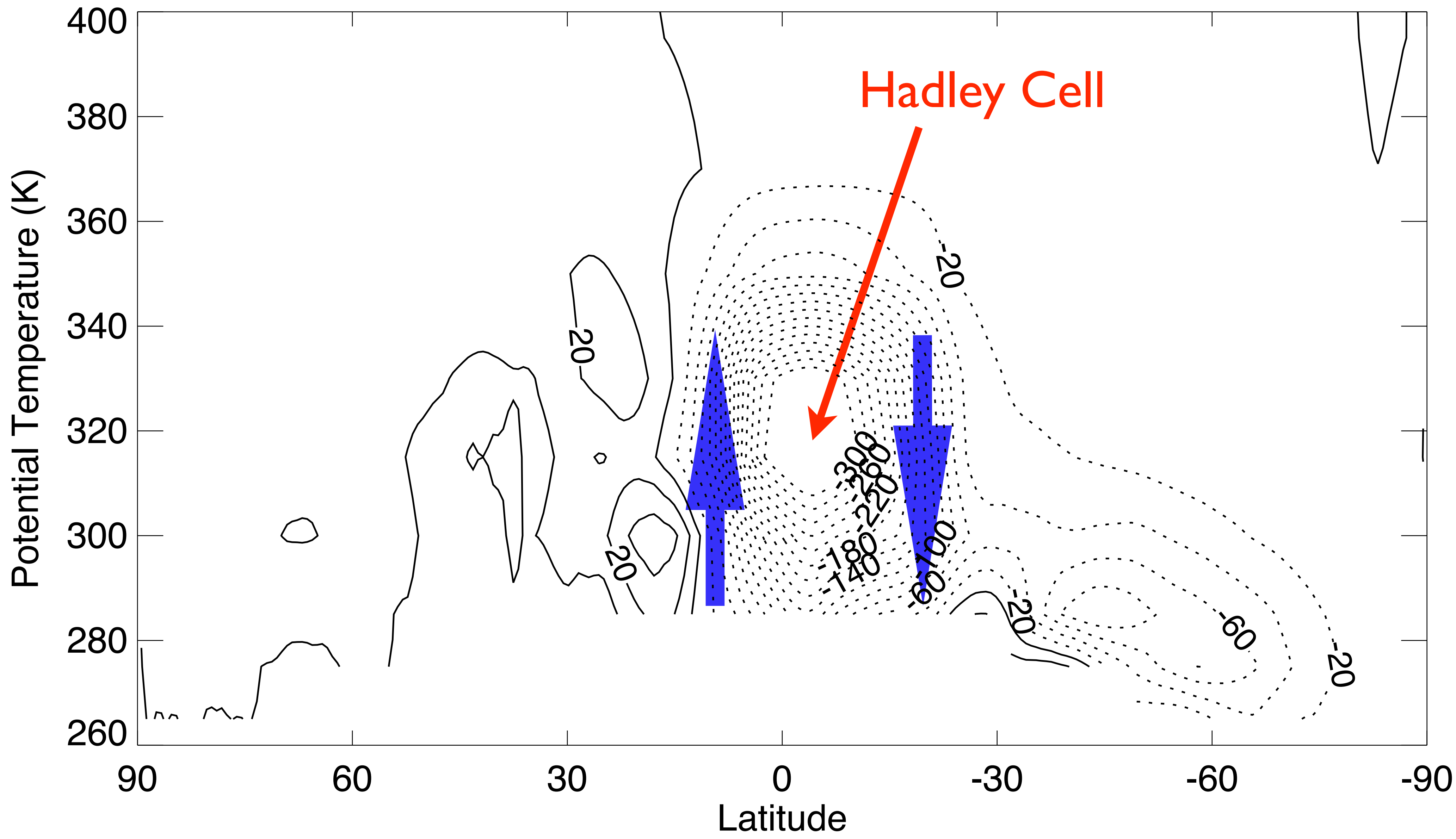


Energy has to be carried poleward.




Sinking air
Rising air

The tropical Hadley Cell carries energy upward and poleward.



Midlatitude “eddies” also carry energy upward and poleward.

Energy fluxes by kind

Lq_v	Latent energy
	+
$s \equiv c_p T + gz$	Dry static energy
	=
$h \equiv s + Lq_v$	Moist static energy

The moist static energy is not the sum of the internal, potential, and latent energies, but the advective flux of moist static energy is equal to the sum of the fluxes of the internal, potential, and latent energies due to advection and pressure-work.

ECMWF

EUROPEAN CENTRE FOR MEDIUM RANGE WEATHER FORECASTS

ERA-Interim is a global atmospheric reanalysis that starts in 1979.

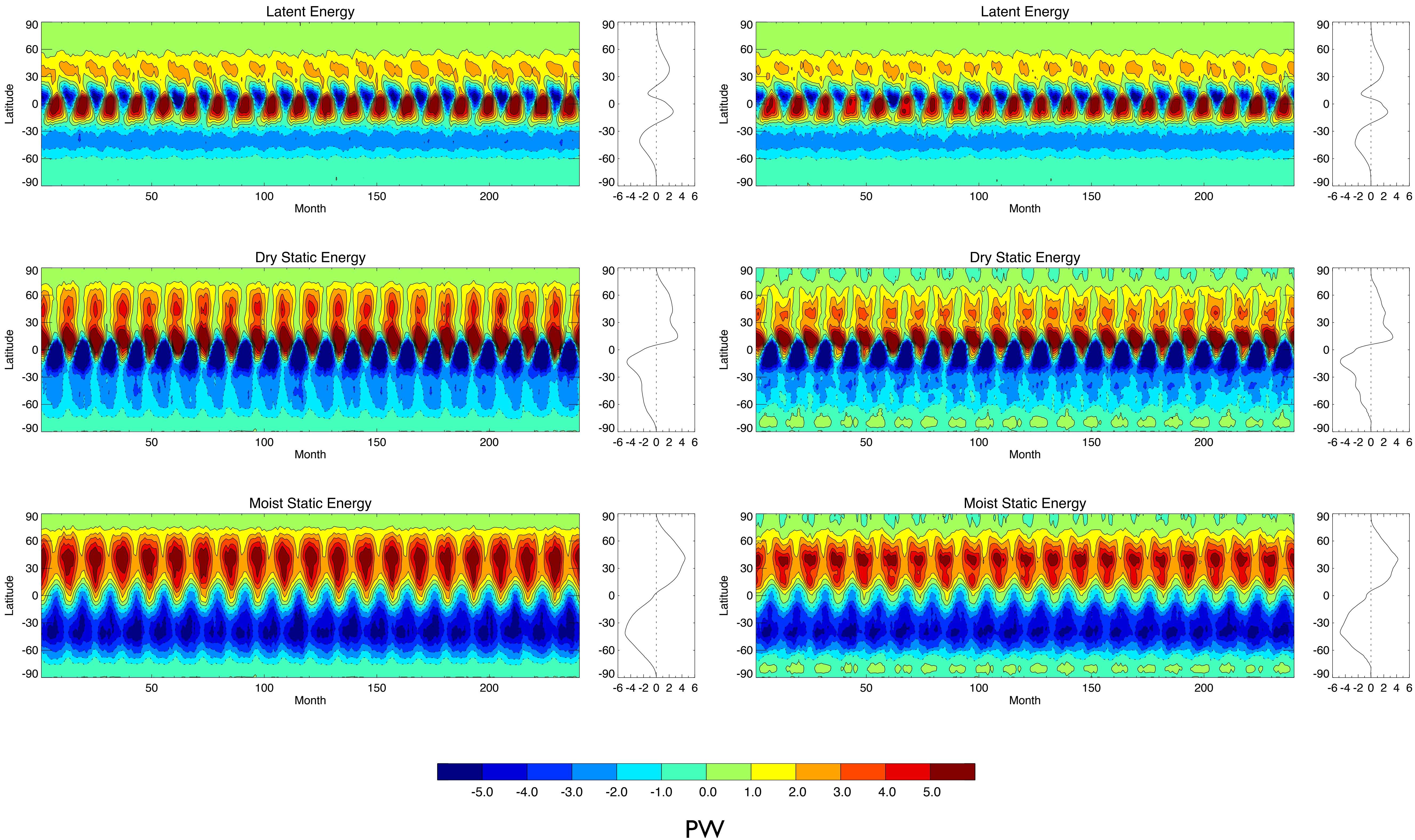
ERA-I includes both isobaric and isentropic analyses.

The vertically integrated energy fluxes should be the same in the two analyses.

Vertically Integrated Energy Fluxes

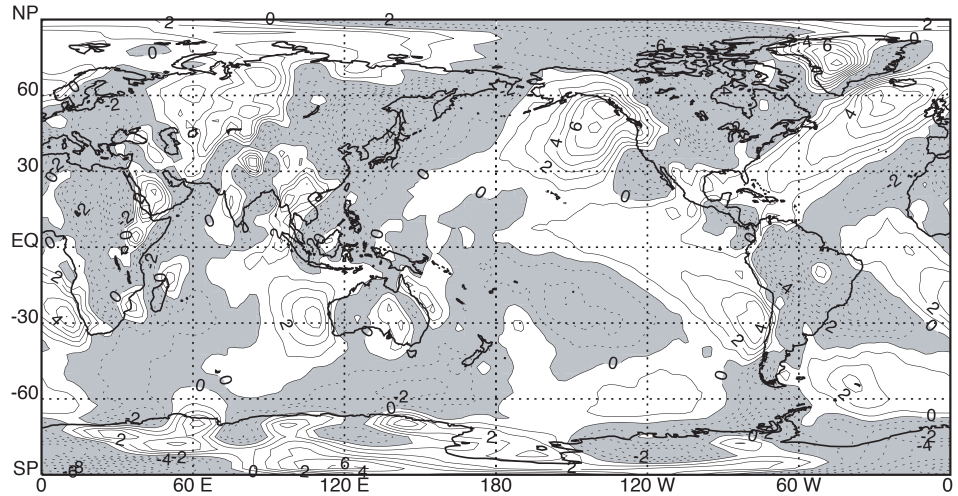
from ERA-I

Pressure   Theta

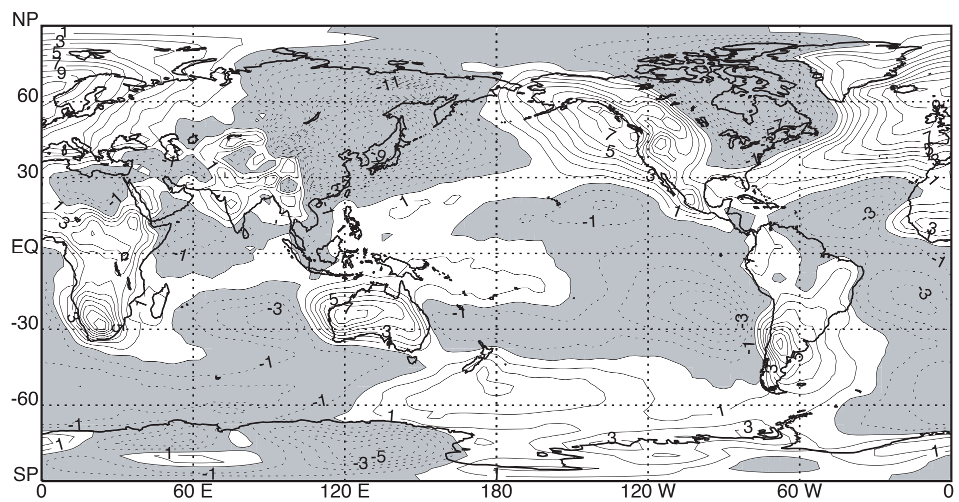


We are going to be discussing "eddy fluxes."

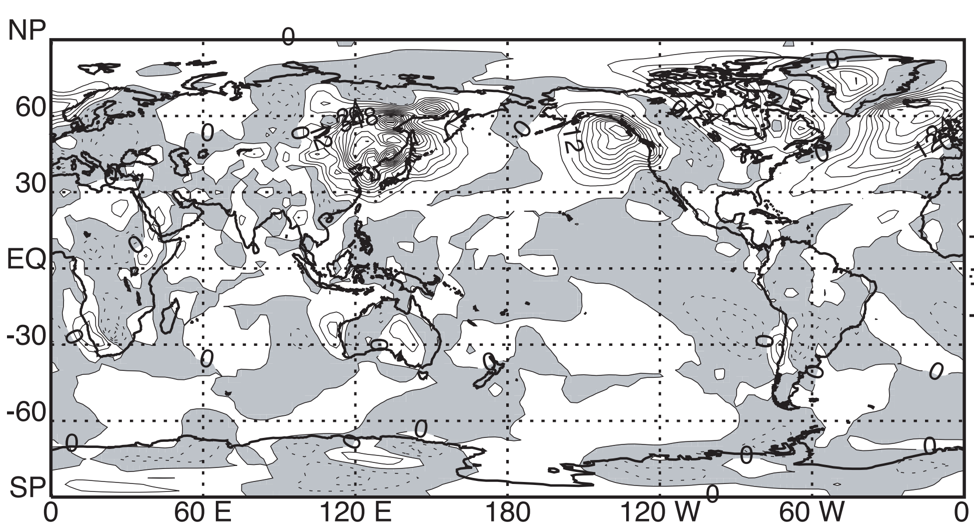
$\overline{v^*}^t$



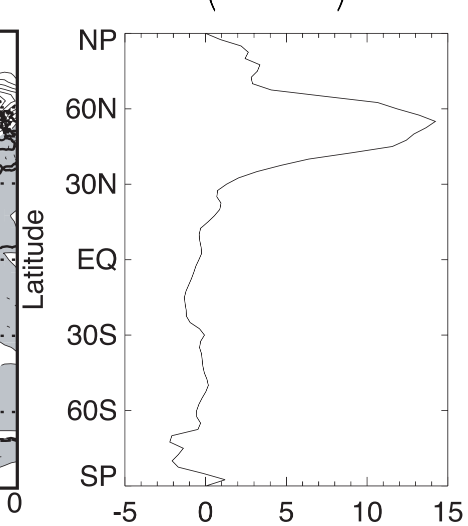
$\overline{T^*}^t$



$\overline{v^*}^t\,\overline{T^*}^t$



$\overline{\left(\overline{v^*}^t\,\overline{T^*}^t\right)}^\lambda$



Eddies are departures from the zonal mean.

The eddy fields shown here, as examples, are the January-mean meridional wind and temperature at 850 hPa.

Eddy fluxes

We can break any field into its zonal mean and "eddy" parts.

$$v = \overline{v}^{\lambda} + v^{*} \quad \text{and} \quad T = \overline{T}^{\lambda} + T^{*}$$

When we multiply two fields together and take the zonal average of the product, we get two contributions:

one from the product of the zonal means, and

the other from the product of the eddies:

$$\begin{aligned}
\overline{vT}^{\lambda} &= \overline{\left(\overline{v}^{\lambda} + v^{*}\right)\left(\overline{T}^{\lambda} + T^{*}\right)}^{\lambda} \\
&= \overline{\overline{v}^{\lambda}\,\overline{T}^{\lambda} + \overline{v}^{\lambda} T^{*} + v^{*}\overline{T}^{\lambda} + v^{*}T^{*}}^{\lambda} \\
&= \overline{v}^{\lambda}\,\overline{T}^{\lambda} + \overline{v^{*}T^{*}}^{\lambda}
\end{aligned}$$

eddy flux

Flux decomposition

In pressure coordinates, the DSE flux looks like this:

$$\overline{vs}^{\lambda} = \overline{v}^{\lambda}\,\overline{s}^{\lambda} + \overline{v^{*}s^{*}}^{\lambda}$$

In theta coordinates the flux involves the product of three things, so it's more complicated:

$$\overline{\rho_\theta v s}^{\lambda} = \overline{\rho_\theta v}^{\lambda}\,\overline{s}^{\lambda} + \overline{\left(\rho_\theta v\right)^{*} s^{*}}^{\lambda}$$

where

$$\overline{\rho_\theta v}^{\lambda} = \overline{\rho_\theta}^{\lambda}\,\overline{v}^{\lambda} + \overline{\rho_\theta^{*} v^{*}}^{\lambda} .$$

Bolus mass flux

Substitution gives

$$\overline{\rho_\theta v s}^{\lambda} = \left(\overline{\rho_\theta}^{\lambda}\,\overline{v}^{\lambda} + \overline{\rho_\theta^{*} v^{*}}^{\lambda}\right)\overline{s}^{\lambda} + \overline{\left(\rho_\theta v\right)^{*} s^{*}}^{\lambda}$$

Terminology

$$\overline{\rho_\theta v s}^\lambda = \left(\overline{\rho_\theta}^\lambda \overline{v}^\lambda + \overline{\rho_\theta^* v^*}^\lambda \right) \overline{s}^\lambda + \overline{\left(\rho_\theta v\right)^* s^*}^\lambda$$

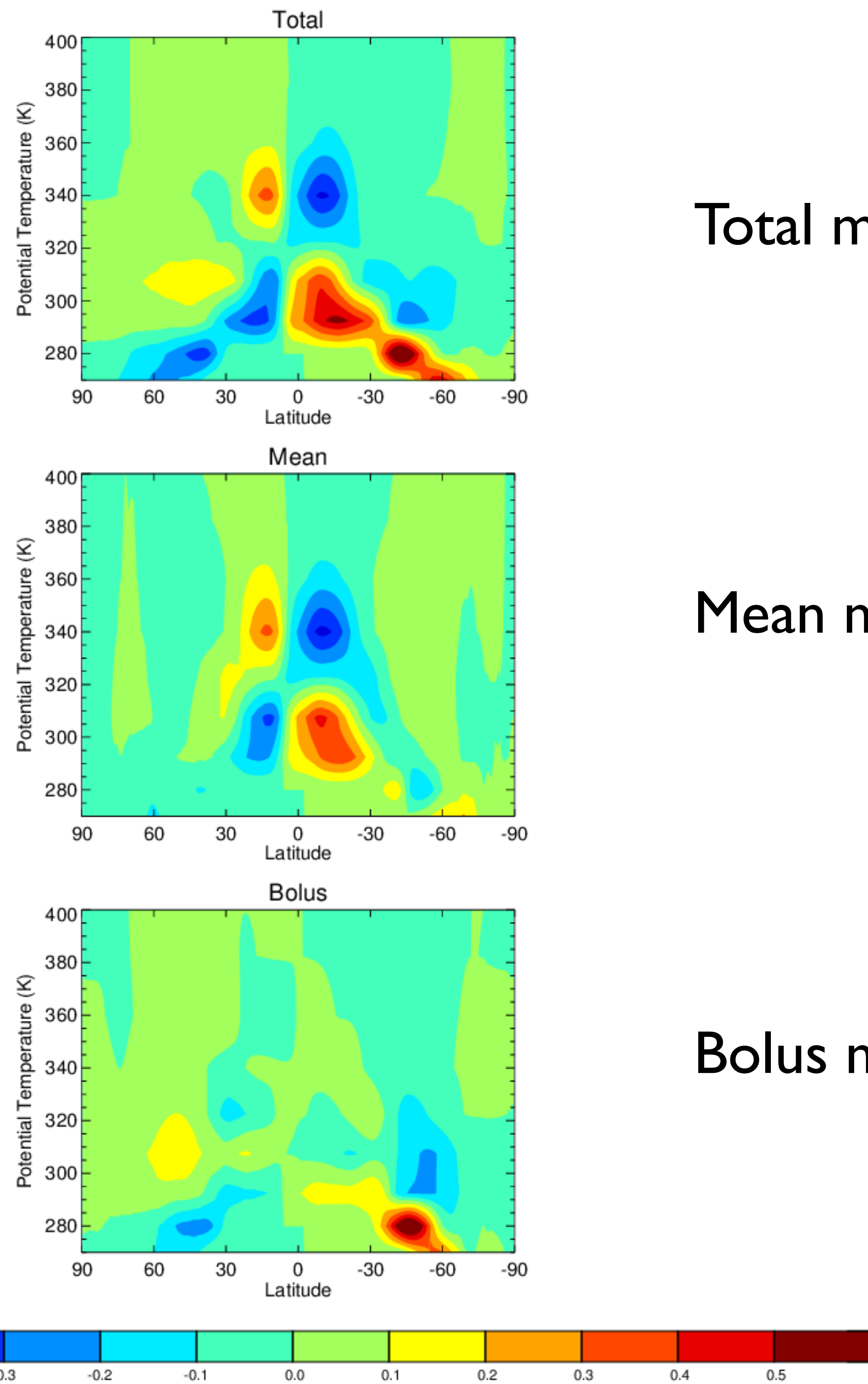
$\overline{\rho_\theta v s}^\lambda$ is the *total flux* of DSE,

$\overline{\rho_\theta}^\lambda \overline{v}^\lambda \overline{s}^\lambda$ is the *mean flux* of DSE,

$\left(\overline{\rho_\theta^* v^*}^\lambda \right) \overline{s}^\lambda$ is the *bolus flux* of DSE, and

$\overline{\left(\rho_\theta v\right)^* s^*}^\lambda$ is the *eddy flux* of DSE.

Annual-mean isentropic mass fluxes



10^{10} kg s^{-1} K^{-1}

Total mass flux

=

Mean mass flux

+

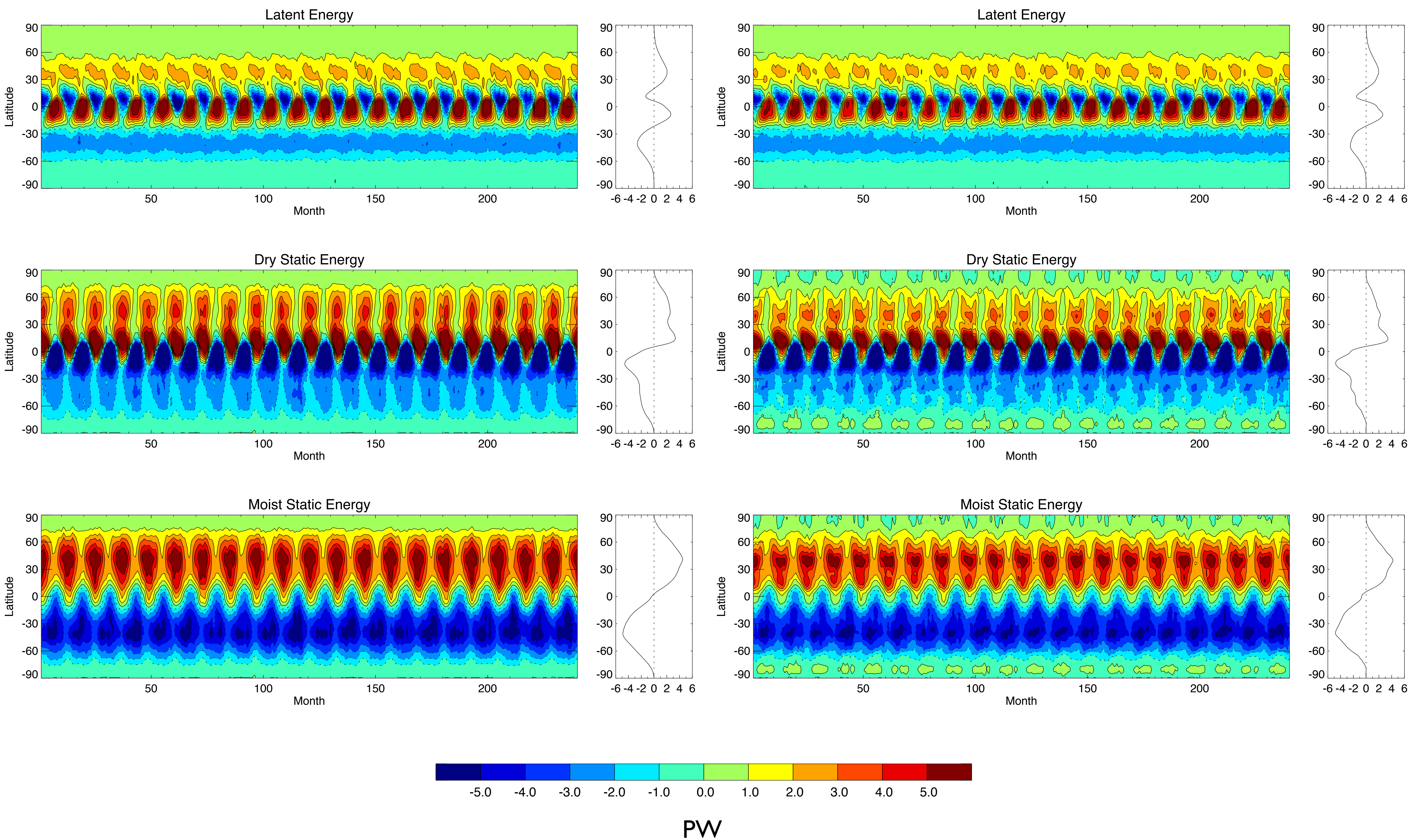
Bolus mass flux

Following Townsend and Johnson (1985)

First let's look at the vertically integrated total fluxes again.

Total Energy Fluxes from ERA-I

Pressure | Theta



Now the just eddy fluxes.

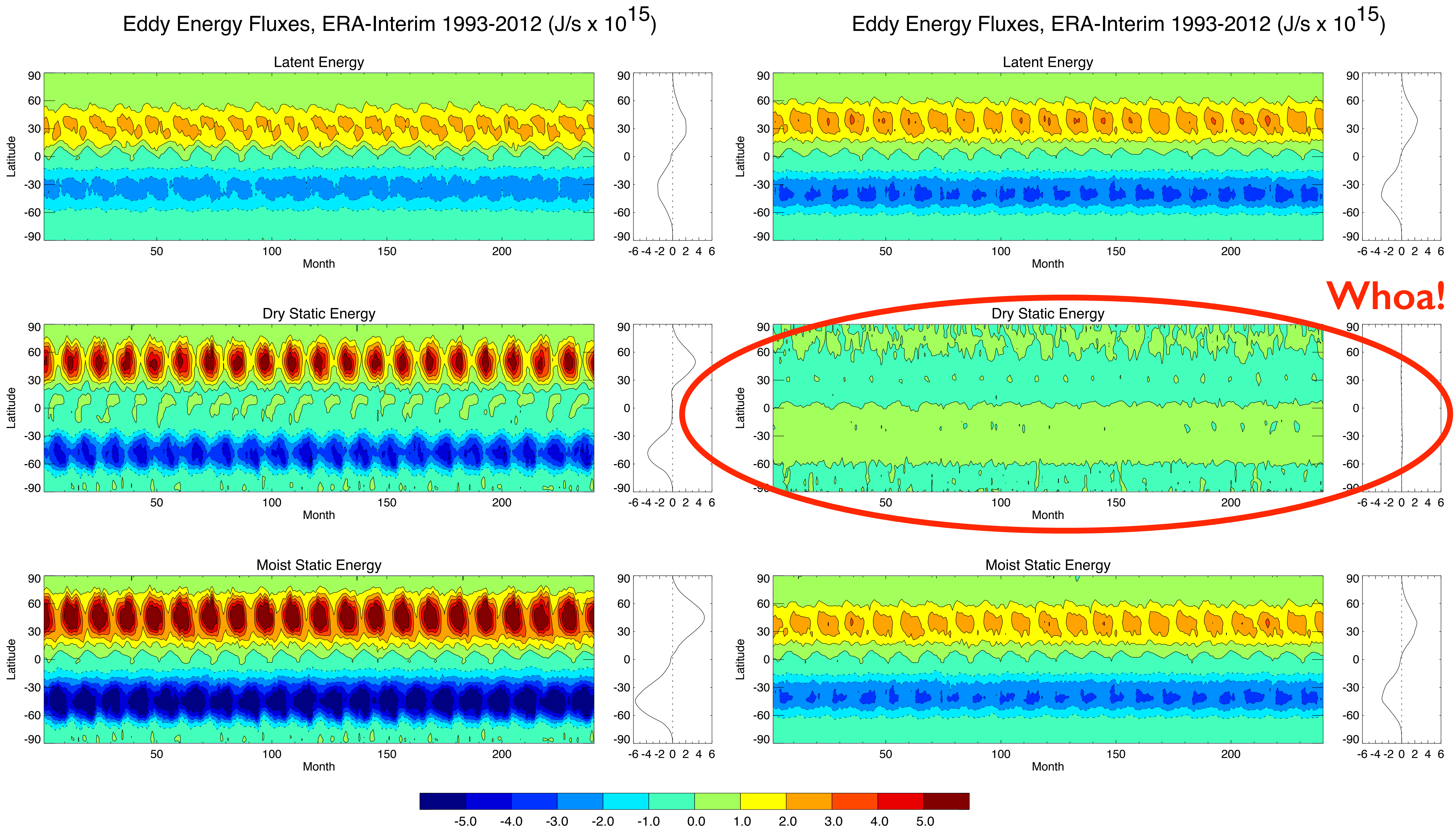
Note:

The vertically integrated eddy fluxes are not required or even expected to agree between the two vertical coordinate systems.

Eddy energy fluxes from ERA-I

Pressure

Theta

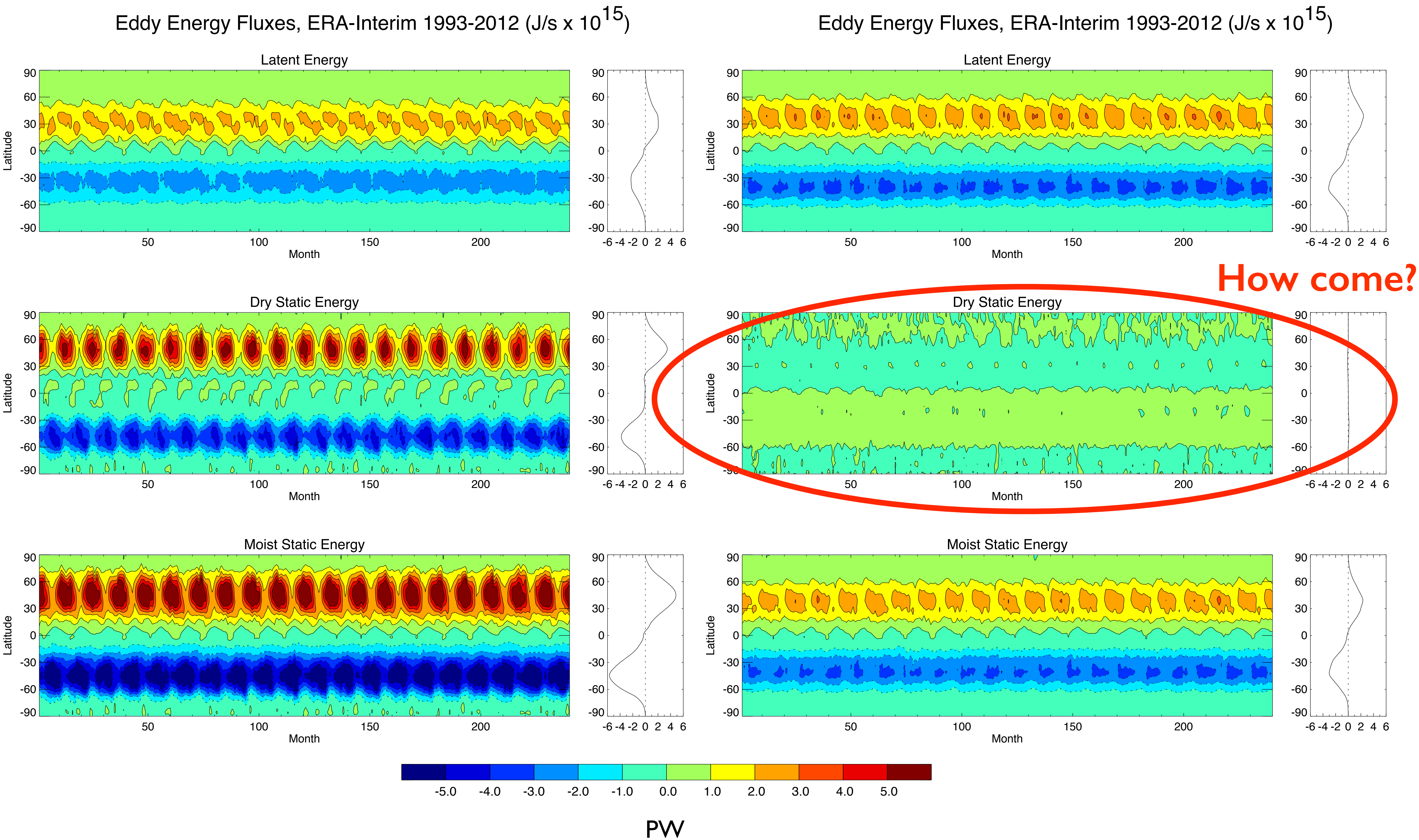


PW

Eddy energy fluxes from ERA-I

Pressure

Theta

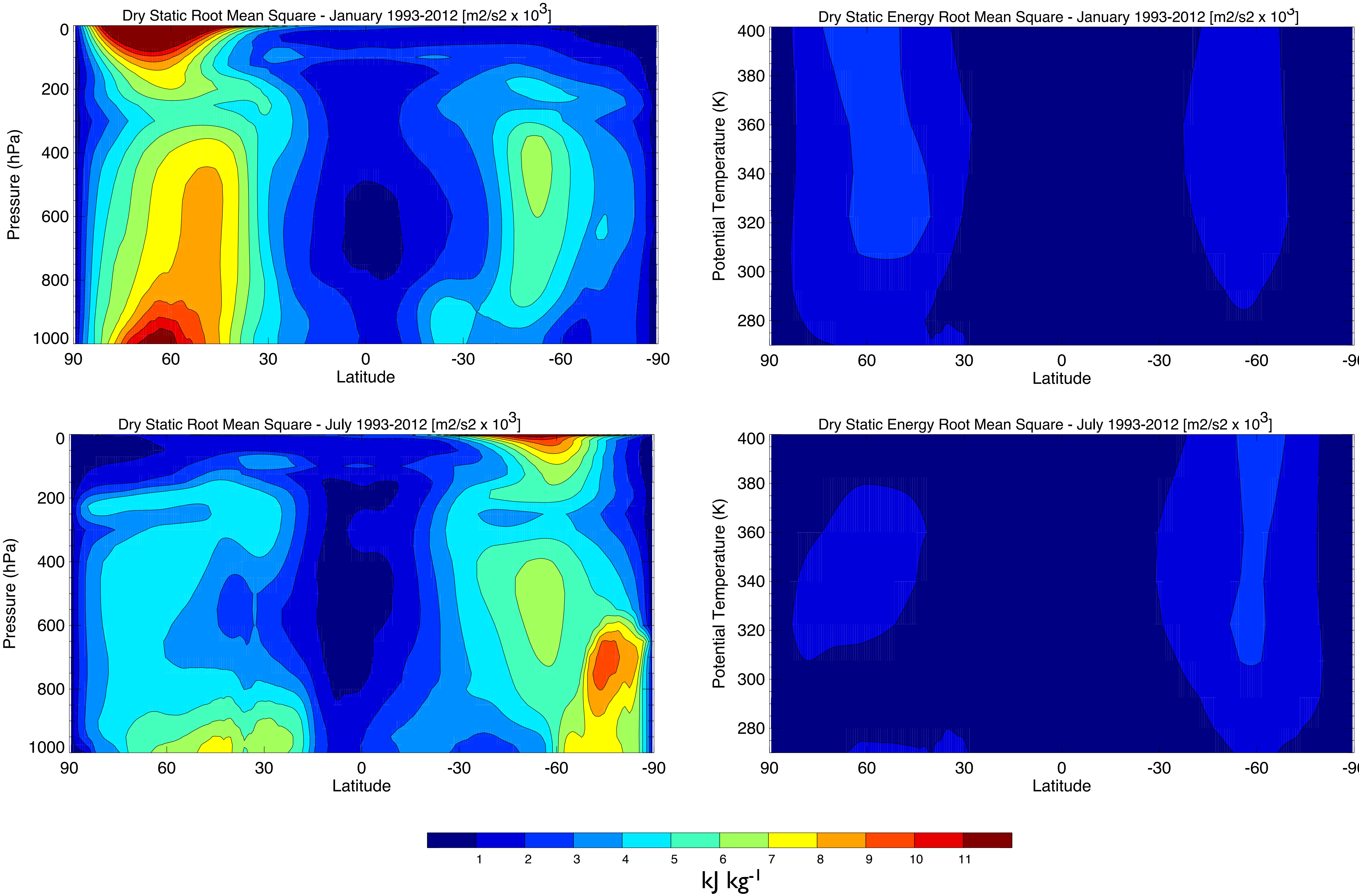


There are two reasons why the isentropic eddy meridional flux of DSE is much smaller than the isobaric eddy flux of DSE.

RMS of DSE

On pressure surfaces

On theta surfaces



The isentropic eddy flux of DSE is smaller than the isobaric eddy flux partly because the DSE changes less on isentropic surfaces than on isobaric surfaces.

Why is that true?

Cousins

s and θ are similar.

Both are conserved under dry adiabatic processes.

Both increase upward in a dry statically stable atmosphere.

They are not the same though. In particular,

$$\mathbf{HPGF} = -\nabla_{\theta} s$$

How s varies

In middle latitudes,

$$\frac{\left|s_\theta^*\right|}{L} \sim \left|\frac{1}{a\cos\varphi}\left(\frac{\partial s}{\partial\lambda}\right)_\theta\right| \cong |fv| \longrightarrow \left|s_\theta^*\right| \sim 10^3 \quad \mathrm{J\ kg^{-1}} \quad .$$

Using

$$s = \Pi\theta + \phi \quad \text{where} \quad \Pi \equiv c_p\left(\frac{p}{p_0}\right)^\kappa$$

we can write

$$\frac{\left|s_p^*\right|}{L} \sim \left|\frac{1}{a\cos\varphi}\left(\frac{\partial s}{\partial\lambda}\right)_p\right| = \left|\frac{1}{a\cos\varphi}\left[\Pi\left(\frac{\partial\theta}{\partial\lambda}\right)_p + \left(\frac{\partial\phi}{\partial\lambda}\right)_p\right]\right|$$

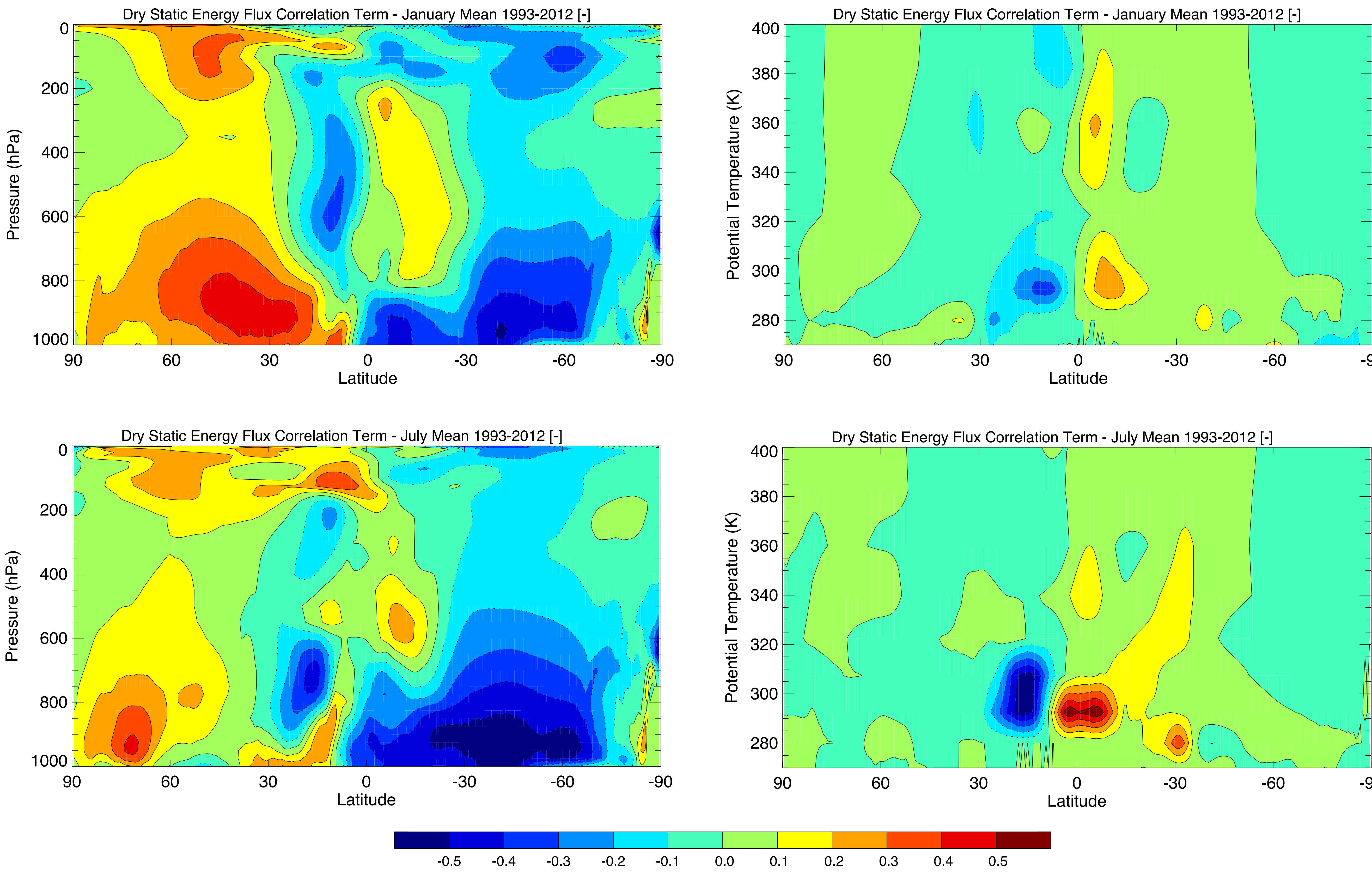
$$= \left|\Pi\frac{\left|\theta_p^*\right|}{L} + fv\right| \longrightarrow \left|s_p^*\right| \sim 5\times10^3\,\mathrm{J\ kg^{-1}} \quad .$$

Second reason…

Correlations

$\mathrm{Cor}\left(v^{*}, s^{*}\right)$ on pressure surfaces

$\mathrm{Cor}\left[\left(\rho_{\theta} v\right)^{*}, s^{*}\right]$ on theta surfaces



The isentropic eddy flux of DSE is smaller than the isobaric eddy flux partly because the DSE is poorly correlated with the eddy meridional mass flux on isentropic surfaces.

Why is that true?

Assuming geostrophic balance,

$$v^* s^* \cong \frac{1}{fa\cos\varphi}\left(\frac{\partial s^*}{\partial \lambda}\right)_\theta s^*$$
$$= \frac{1}{fa\cos\varphi}\left[\frac{\partial}{\partial \lambda}\left(\frac{s^{*2}}{2}\right)\right]_\theta \quad ,$$

which implies that

$$\overline{v^* s^*}^\lambda \cong 0 \; .$$

Using this result, it can be shown that

$$\overline{(\rho_\theta v)^* s^*}^\lambda \cong \overline{\rho_\theta^* v^* s^*}^\lambda \quad .$$

Conclusions

- The eddy meridional flux of DSE along isentropic surfaces is small compared to the meridional flux of latent energy.
- As a result, the eddy part of the poleward energy flux along isentropic surfaces is almost entirely an eddy flux of latent energy.
- Two contributing explanations, about equally important:
  - Small variations of the DSE along isentropic surfaces;
  - Weak correlations between the geostrophic meridional wind and the DSE, when averaged along isentropic surfaces.

So what?

It should be possible to build an idealized model of the zonally averaged global circulation in which the eddy flux of DSE is neglected.

$$\overline{\rho_\theta v s}^\lambda = \left(\overline{\rho_\theta}^\lambda \overline{v}^\lambda + \overline{\rho_\theta^* v^*}^\lambda \right) \overline{s}^\lambda + \overline{(\rho_\theta v)^* s^*}^\lambda$$