Implementation of Chin-Hoh's mixed SGS model and 5th-order WENO-Z momentum advection scheme into SAM

Takanobu Yamaguchi[1,2]

[1] Cooperative Institute for Research in Environmental Sciences, University of Colorado
[2] NOAA Earth System Research Laboratory, Boulder, CO

Brief history

I came to know a **4th-order momentum advection scheme, which conserves kinetic energy**.

≈

2015/3 I decided to test the scheme with an intern student to see if the scheme is better than the **2nd-order center scheme** used in SAM.

Dave told me that the project may help Chin-Hoh.

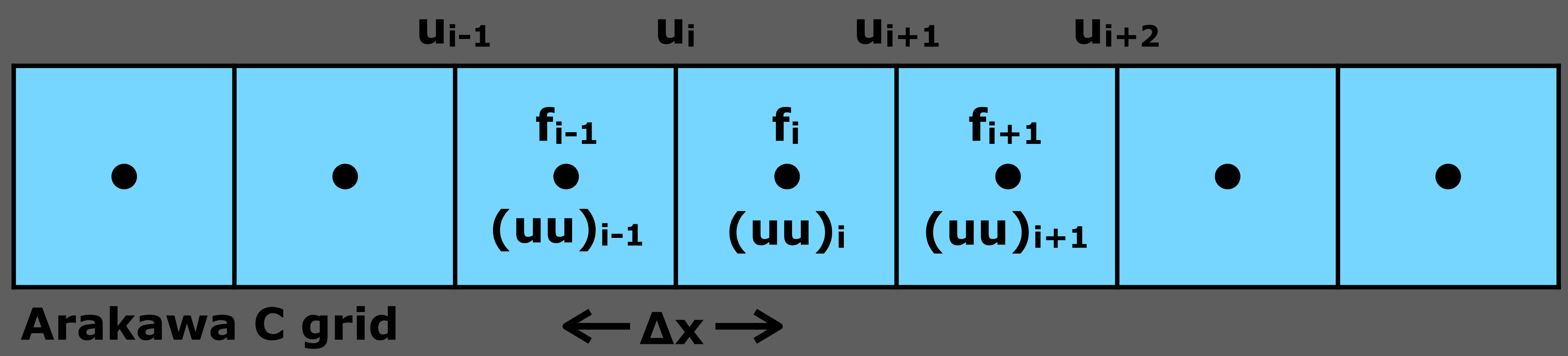
Chin-Hoh had been struggling with **noisy momentum field** of SAM, which did not work with her **mixed subgrid scale (SGS) model**.

2015/5 I decided to test an **odd-order scheme** instead of an **even order scheme**.

2015/6-8 My intern student worked on **5th-order WENO-Z scheme**.

2015/12 Implementation of WENO-Z as well as Chin-Hoh's mixed scheme into SAM were completed and test simulations have been performed.

Momentum advection



Flux-form momentum advection equation: $\partial u/\partial t = - \partial(uu)/\partial x$

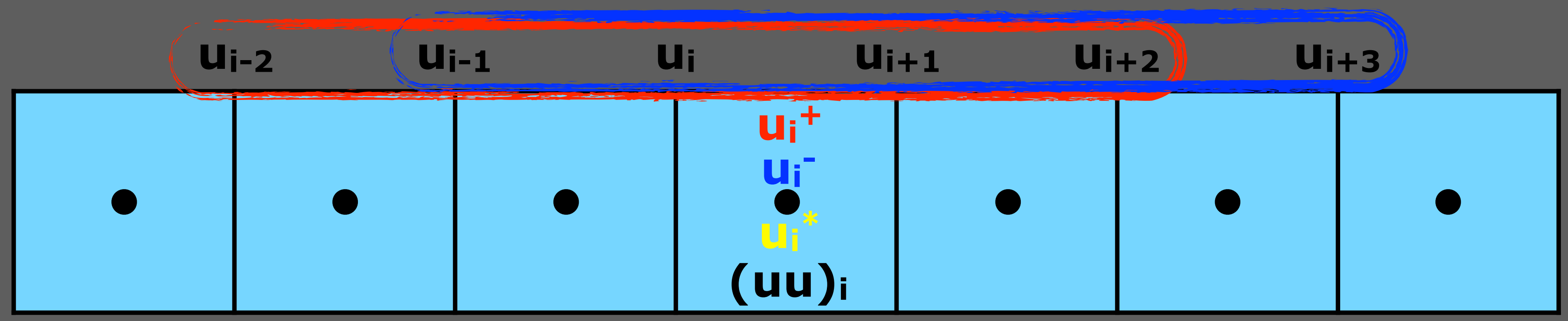
$$(\partial u/\partial t)_i = [\ (uu)_i - (uu)_{i-1}\ ] /\Delta x$$

2nd-order center scheme: $(uu)_i = [\ 0.5\ (u_{i+1} + u_i)\ ]\ [\ 0.5\ (u_{i+1} + u_i)\ ]$

"advecting" "advected"

- 2nd-order center scheme conserves total kinetic energy, i.e., $\sum u^2/2$.
- Even order scheme is dispersive.
- Models use even order scheme for momentum: e.g., SAM, UCLA-LES

5th-order WENO-Z scheme



Weighted Essentially Non-Oscillatory scheme (e.g., Jiang and Shu 1996 JCP)

- "Essentially" monotonic interpolation with odd number of points
- Reduced order of accuracy at critical point

WENO-Z (Borges et al. 2008 JCP):
Improved 5th-order WENO scheme without increasing computational cost

$$u_i^+ = \mathrm{WENOZ}(\ u_{i-2},\ u_{i-1},\ u_i,\ u_{i+1},\ u_{i+2}\ )$$
$$u_i^- = \mathrm{WENOZ}(\ u_{i+3},\ u_{i+2},\ u_{i+1},\ u_i,\ u_{i-1}\ )$$

$$u_i^* = 0.5\ (\ u_i^+ + u_i^-\ )$$

"advecting" "advecting"

$$(uu)_i = [\ u_i^* + |\ u_i^*\ |\ ]\ u_i^+ + [\ u_i^* - |\ u_i^*\ |\ ]\ u_i^+$$

"advected" "advected"

- Odd-order scheme is dissipative, i.e., total kinetic energy does not conserve.
- Odd-order scheme dumps computational dispersion, i.e., smoother field.
- Models use odd-order scheme for momentum: e.g., Met Office LEM, WRF

nano SAM

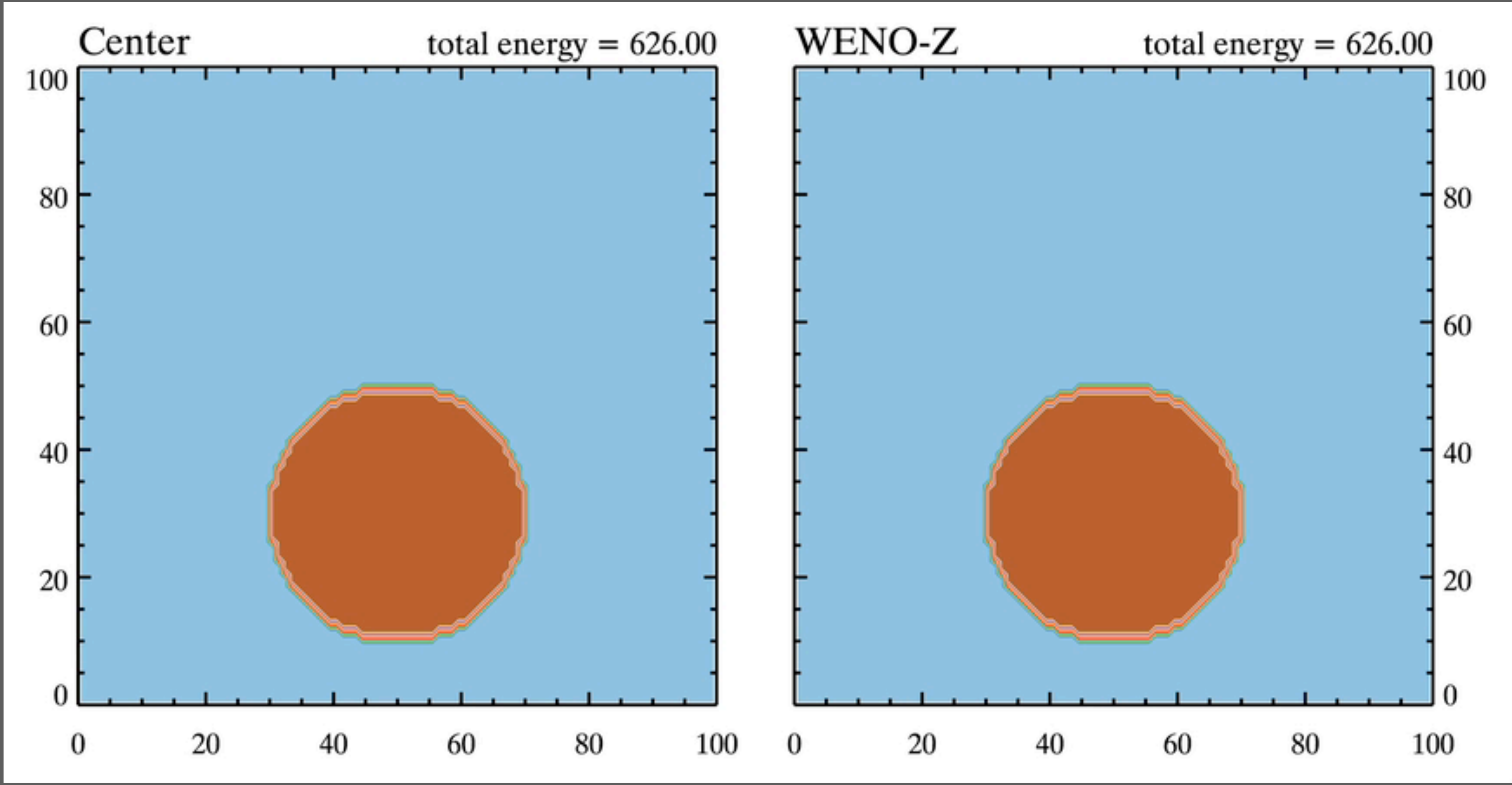
Very simplified SAM:
incompressible (ρ=1), 2D, u and w equation

$$\partial u/\partial t = - \partial(uu)/\partial x - \partial(wu)/\partial z - \partial p/\partial x - \partial \tau_{uu}/\partial x - \partial \tau_{uw}/\partial z$$
$$\partial w/\partial t = - \partial(uw)/\partial x - \partial(ww)/\partial z - \partial p/\partial z - \partial \tau_{uw}/\partial x - \partial \tau_{ww}/\partial z$$

advection | pressure gradient | diffusion

Experimental setup:
$u = 0$ m s^{-1} everywhere,
$w = 1$ m s^{-1} inside of bubble and $w = 0$ m s^{-1} outside,
$\Delta x = \Delta z = 1$ m, $\Delta t = 0.4$ s
$n_x = n_z = 100$
Periodic boundary in x
Rigid top and bottom in z

Advection only



SGS flux based on K-theory

$(\partial w/\partial t)_{diffusion} = - \partial \tau_{uw}/\partial x - \partial \tau_{ww}/\partial z$

SGS flux (τ_{uw}, τ_{ww}) is parameterized by

$\tau_{uw} = - K (\partial u/\partial z + \partial w/\partial x)$
$\tau_{ww} = - K (\partial w/\partial z + \partial w/\partial z)$

where K is eddy viscosity (positive value).

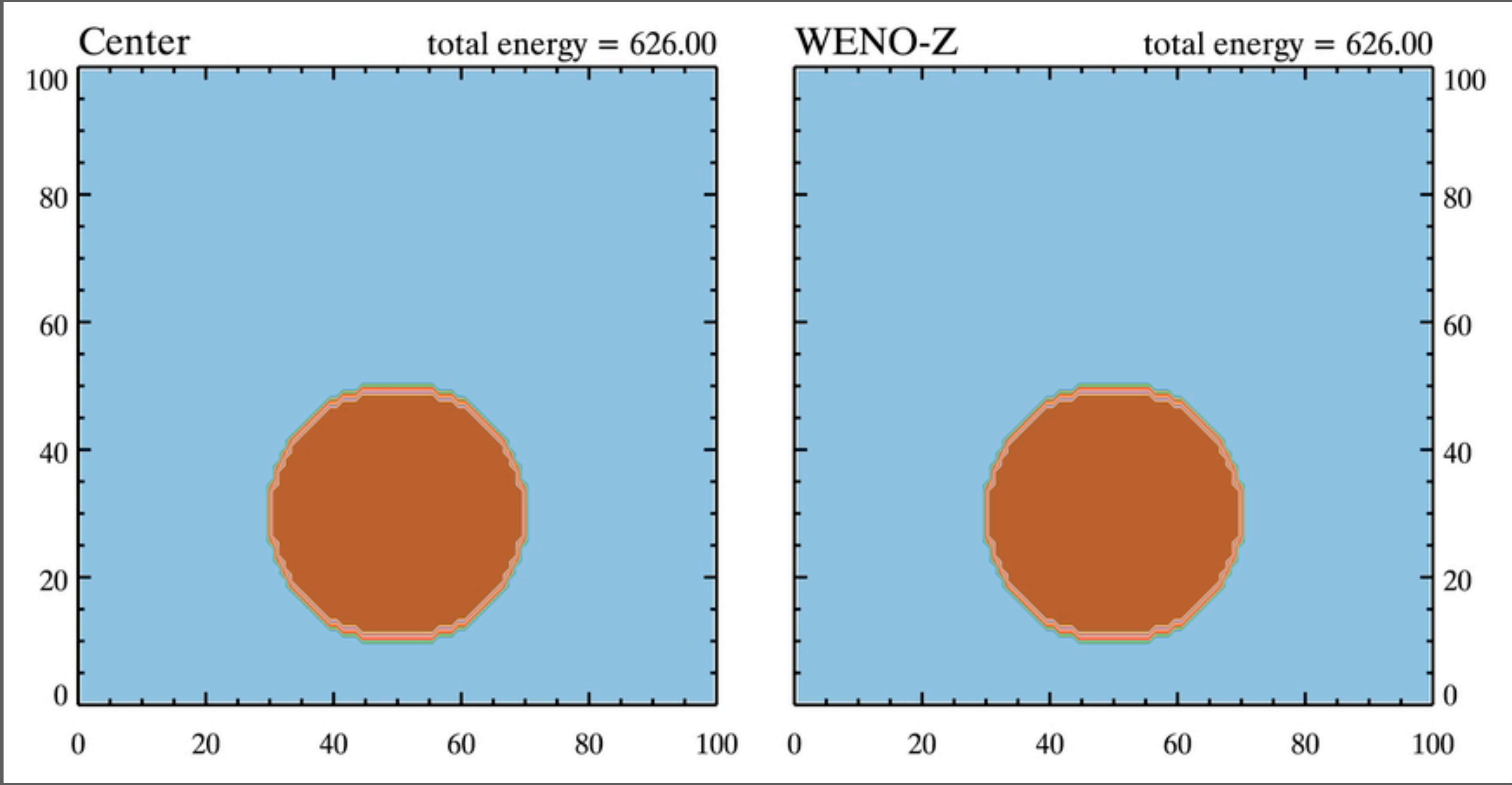
K is parameterized by SGS model.

K-theory **always** removes small variability (including noise).

Popular SGS scheme:

- Smagorinsky scheme (diagnostic SGS TKE scheme)
- 1.5-order TKE scheme (prognostic SGS TKE scheme)

Advection with Smagorinsky scheme



Chin-Hoh's mixed SGS model

Moeng et al. (2010 JAS): For CSRM (dx ~ 1-4 km),

- Spectral peak of vertical velocity is around km scale waves.
- SGS flux = Leonard + cross + Reynolds ≈ 2 x Leonard + Reynolds
- Leonard term contains only grid scale value, thus represents largest SGS eddies (resolvable scale, RS, eddies).

Chin-Hoh parameterizes the SGS flux as, for instance,

$$\tau_{uw} = -K\left(\partial u/\partial z + \partial w/\partial x \right) + A\,\Delta_f^2 \left(\partial u/\partial x\, \partial w/\partial x + \partial u/\partial y\, \partial w/\partial y \right)$$

K-theory (partially Reynolds) — 2 x Leonard, RS flux

where A = 0.167 (Moeng et al. 2010 JAS), 0.3-0.5 (Moeng 2014 MWR).

2 x Leonard + Reynolds

RS flux represents energy backscattering.

Chin-Hoh's mixed SGS model

- Spectr
- SGS fl ... ynolds
- Leona ... largest SGS e

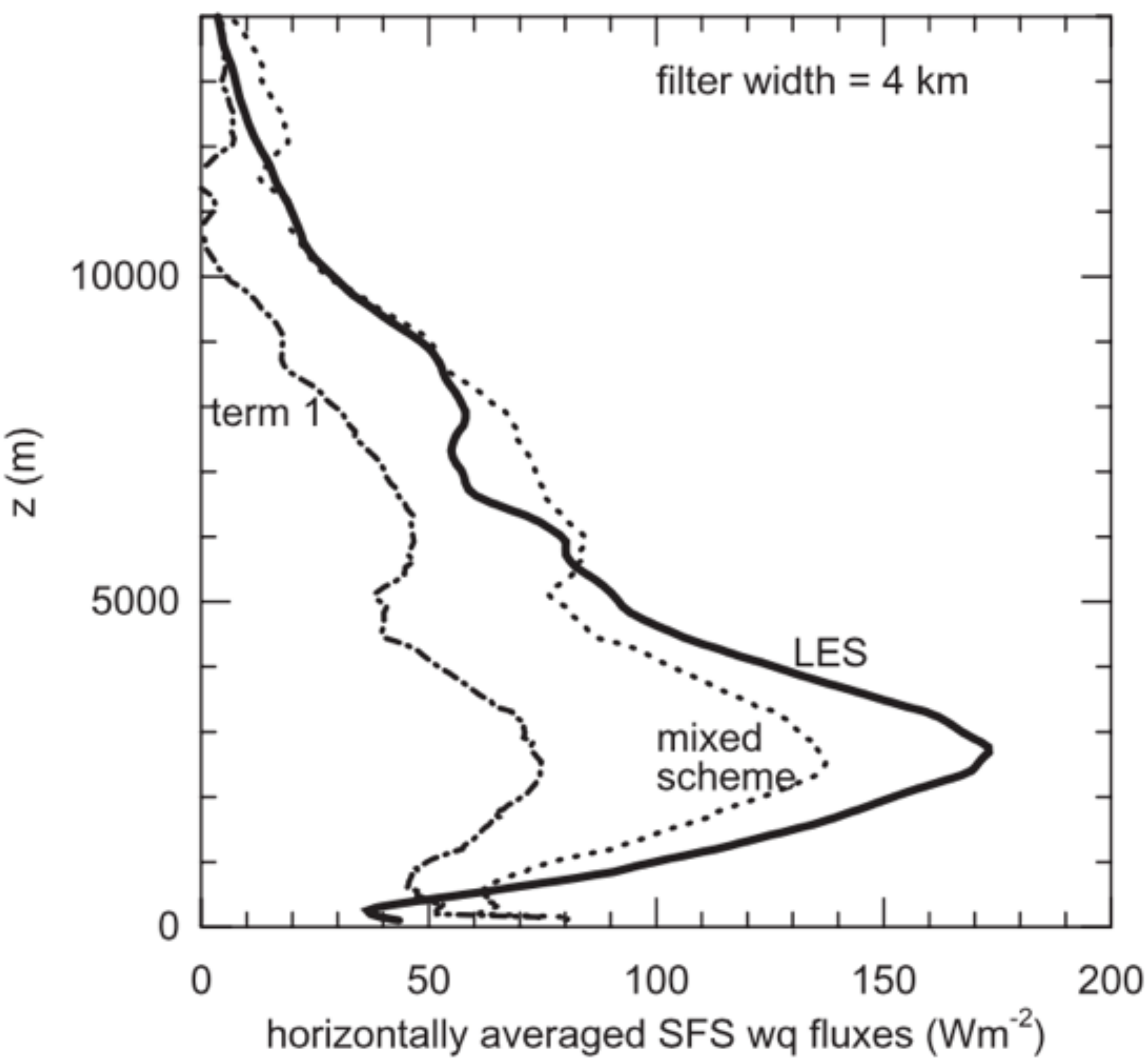


FIG. 10. Vertical distributions of the horizontally averaged τ_{wq} from Eq. (13) in comparison with the LES-retrieved $\langle \tau_{wq} \rangle$ with $\Delta_f = 4$ km.

τ_{uw} = ... w/∂y)

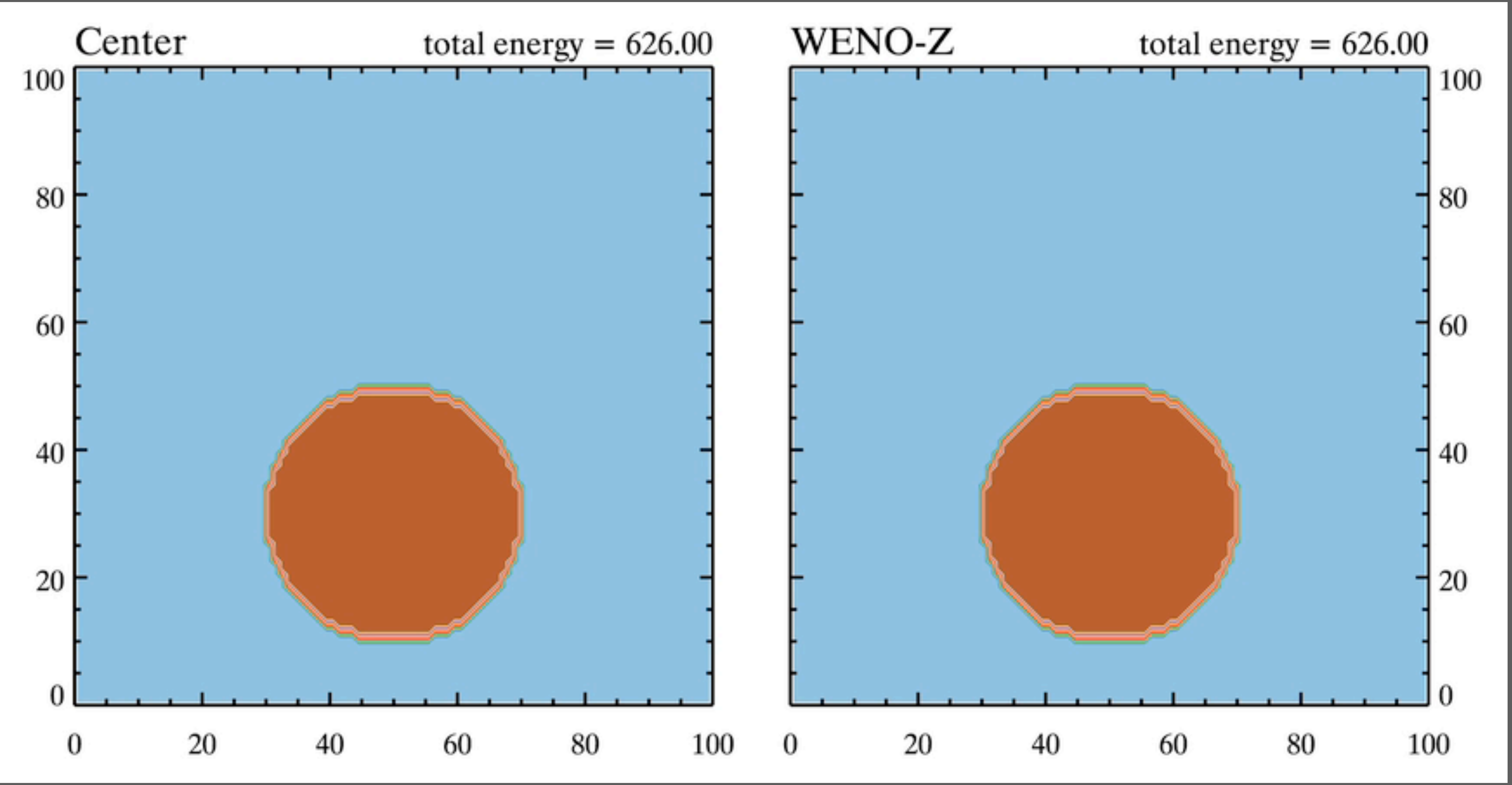
K-t

where A ... 4 MWR).

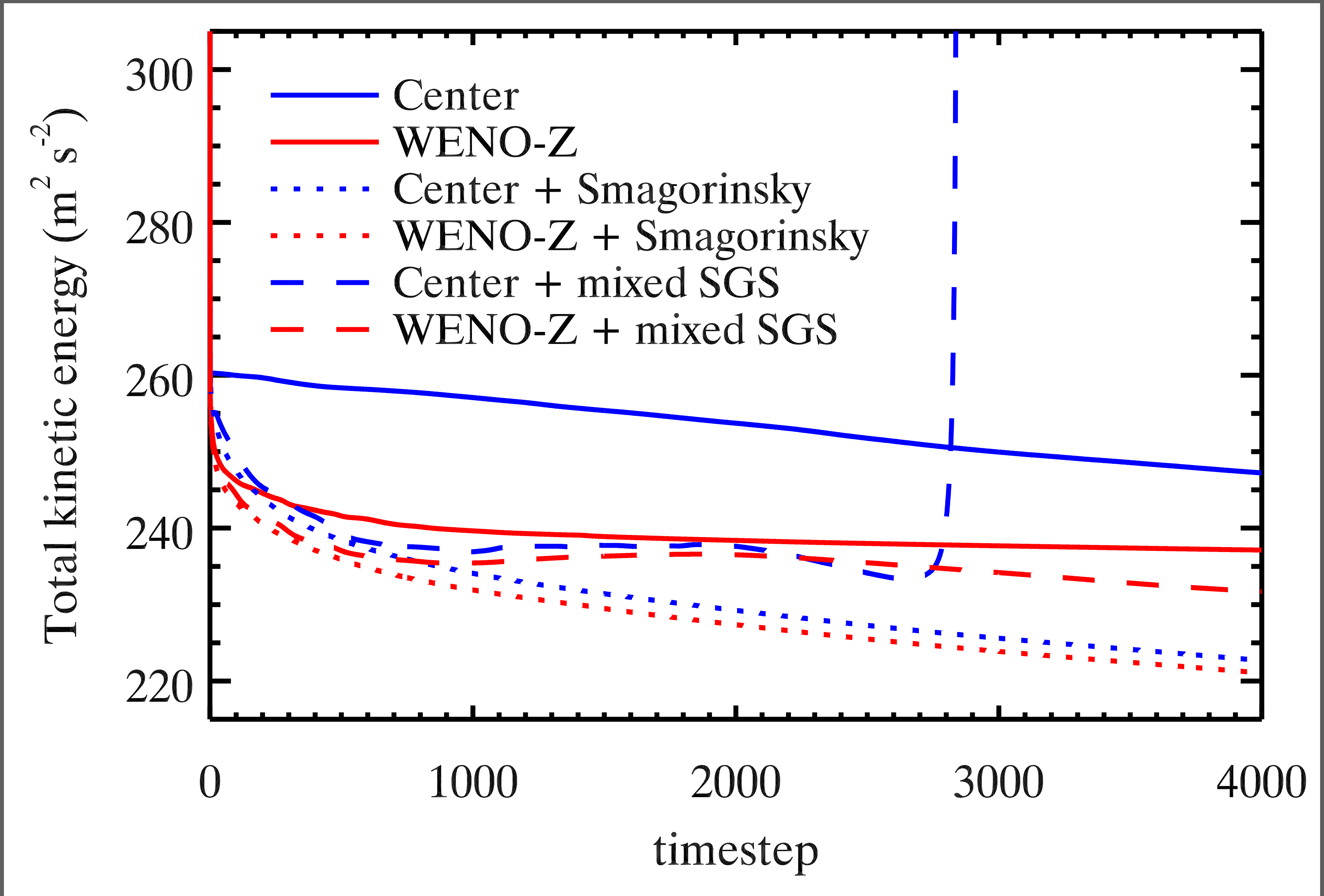
olds

Moeng et al. (2010 JAS)

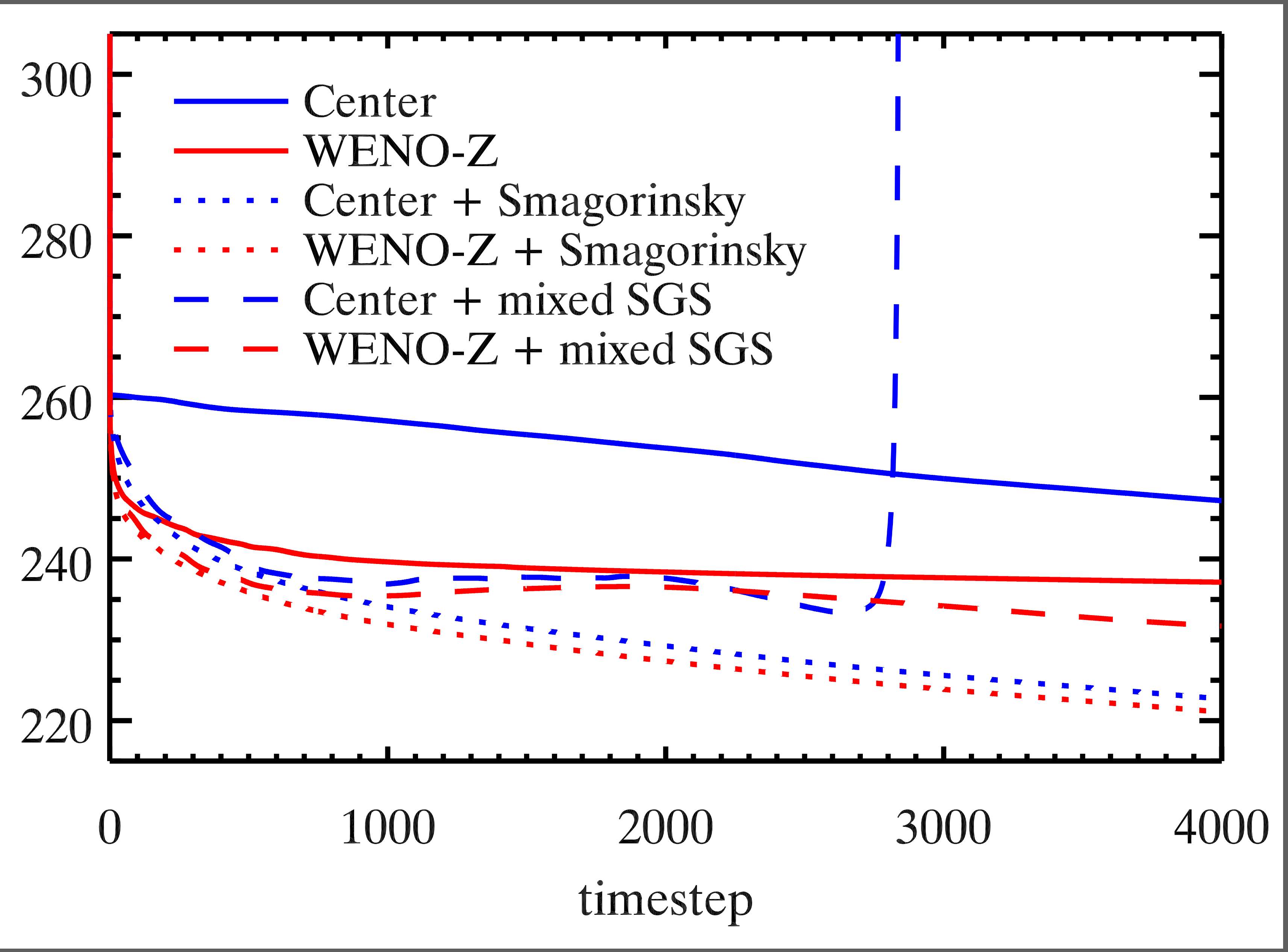
Advection with Chin-Hoh's mixed scheme



Total kinetic energy



Total kinetic energy

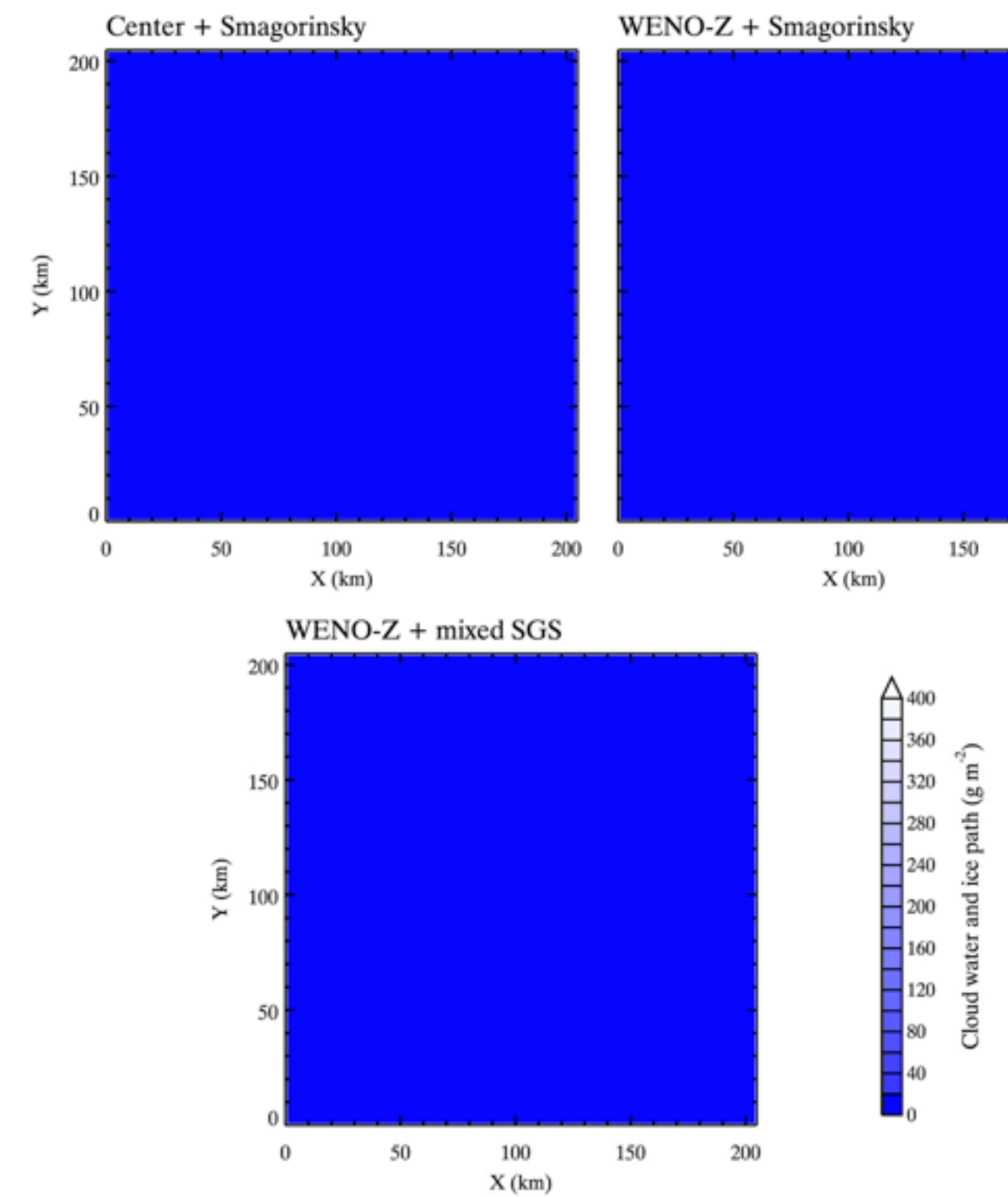


	Max Courant number
SAM's defoult	0.7
Center	
Center + Smagorinsky	
WENO-Z	
WENO-Z + Smagorinsky	0.6
WENO-Z + mixed scheme	0.55

CSRM test

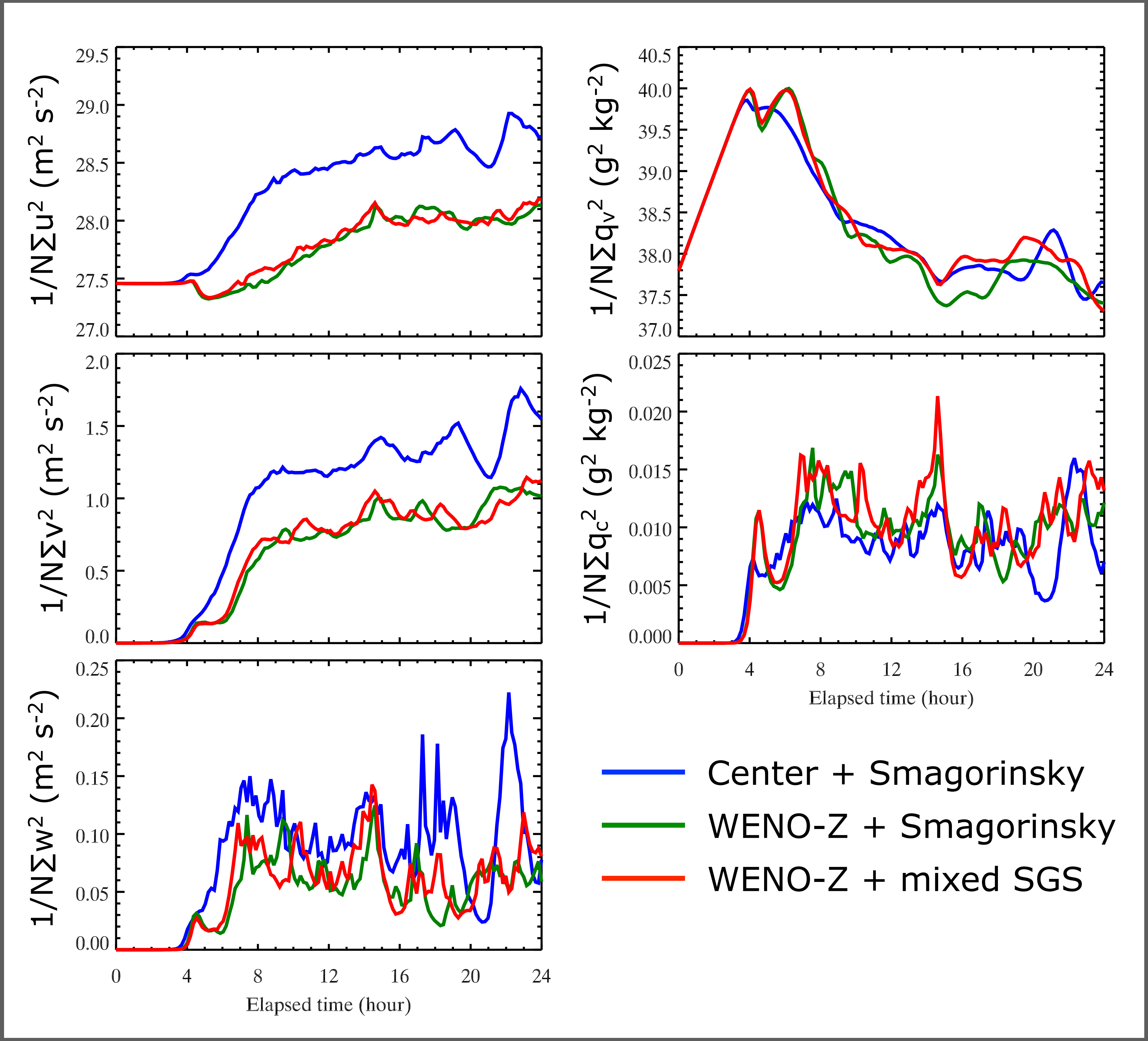
- Case: Giga-LES 1 (idealized GATE)
- 256 levels (same as Giga-LES 1)
- Physics (same as Giga-LES 1)
  - 1-moment microphysics
  - Radiative forcing
- 5th-order ULTIMATE-MACHO scalar advection scheme
- Δx = 3.2 km and 1.6 km
- A = 0.167 for the mixed SGS scheme

Δx = 1.6 km



Total variance ($1/N\sum f^2$)

Δx = 3.2 km



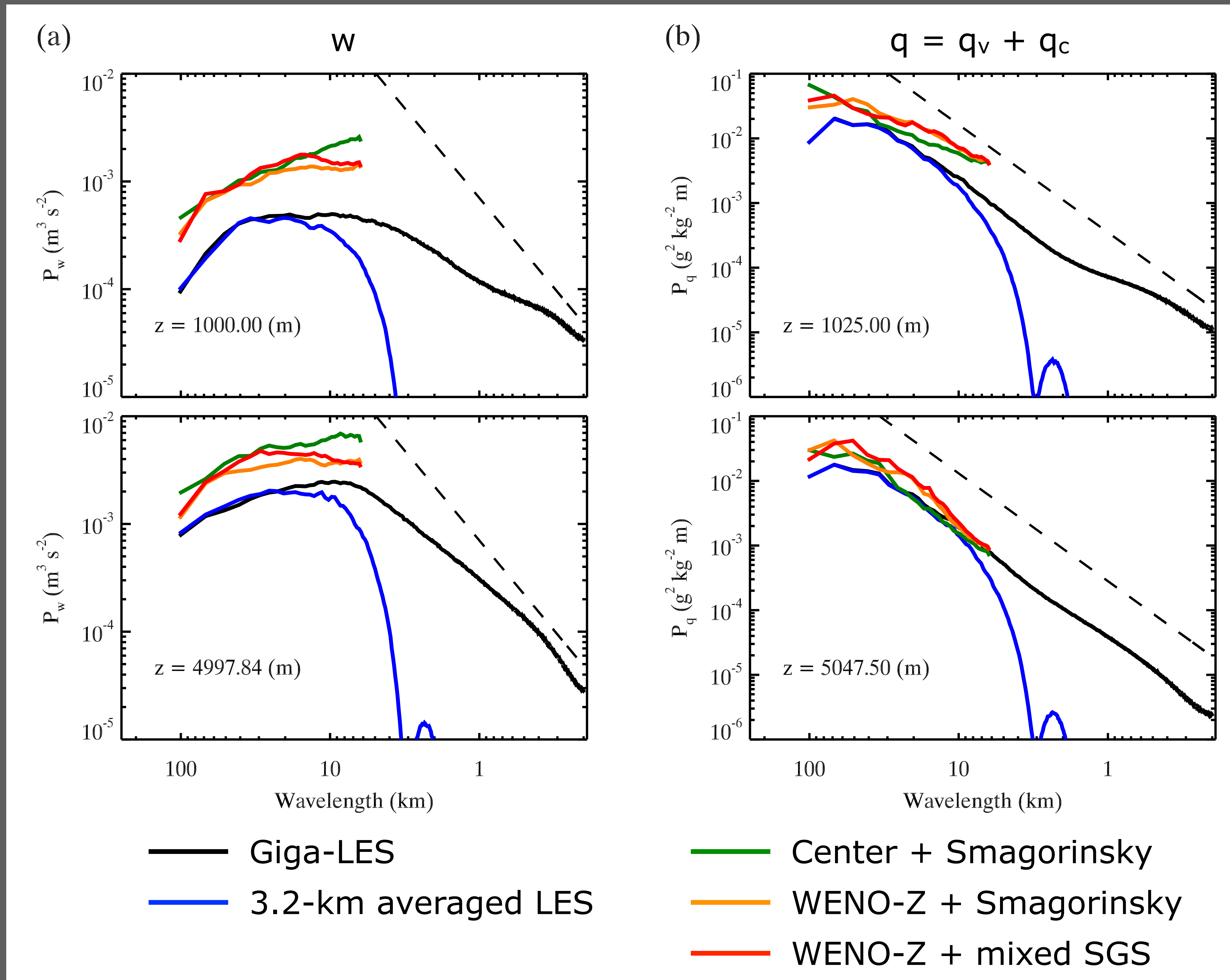
Power spectra

Δx = 3.2 km

last 6-hour mean

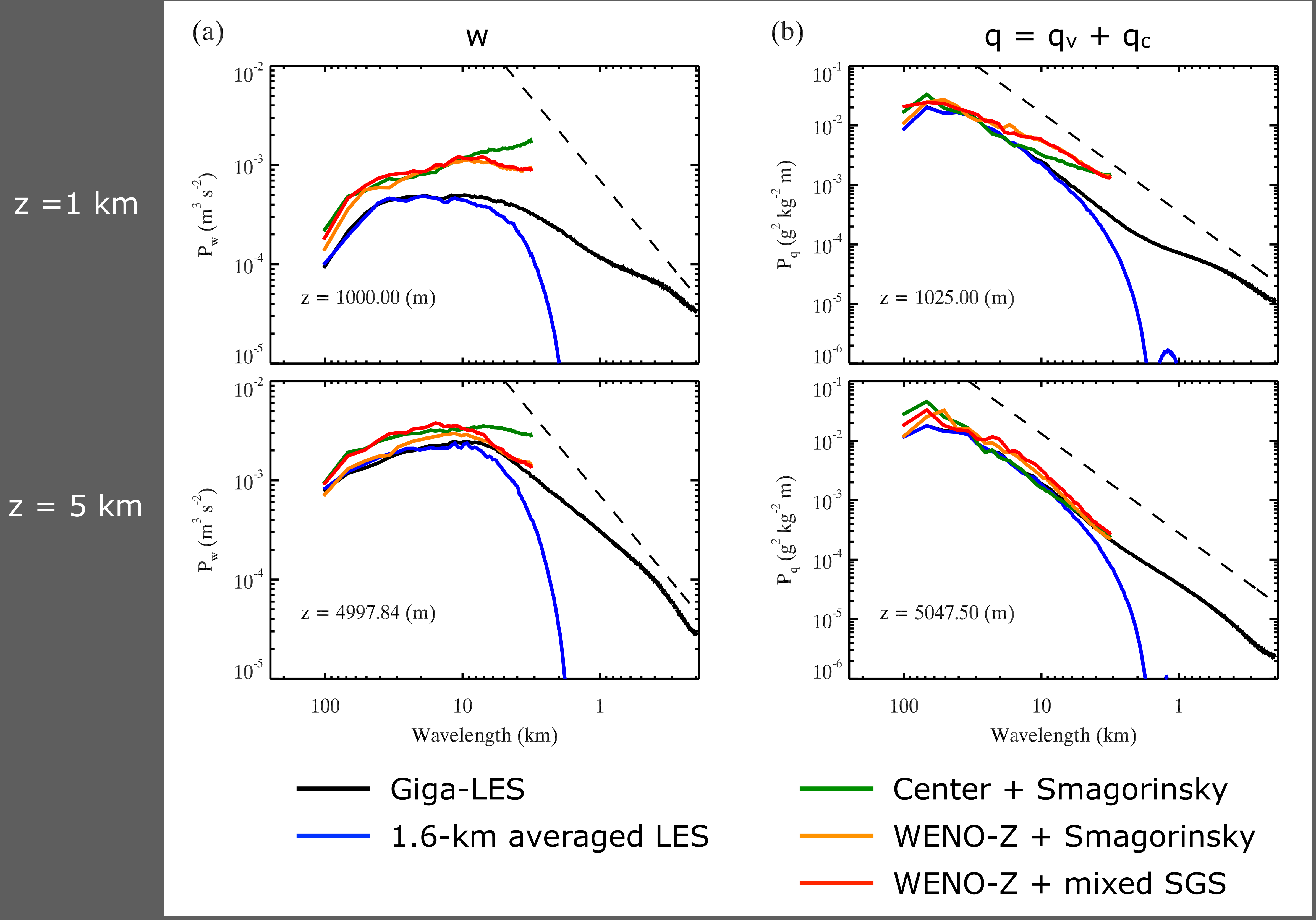
z =1 km

z = 5 km

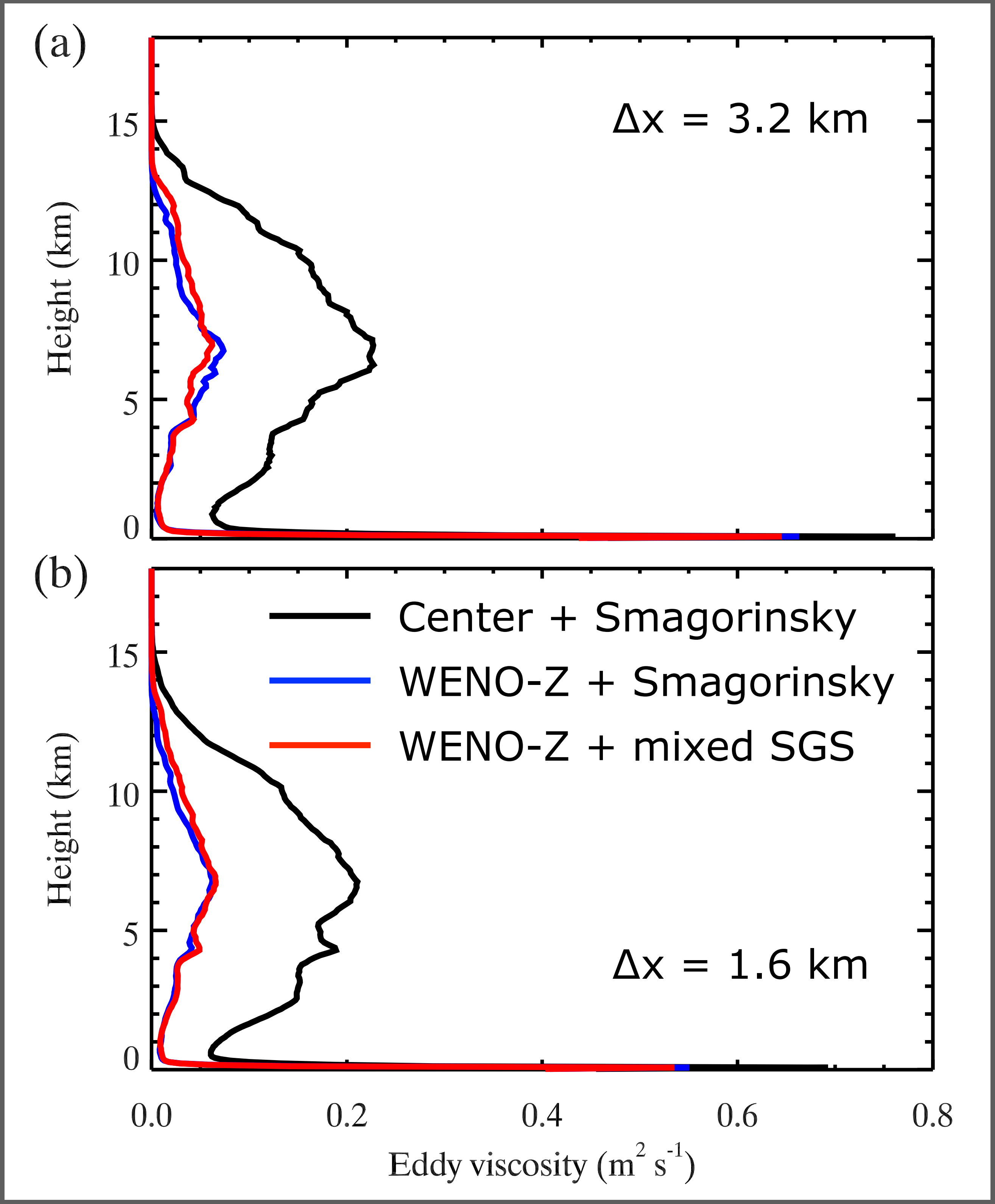


Power spectra

Δx = 1.6 km last 6-hour mean



Eddy viscosity



last 6-hour mean

- τ_{ww} = - K (∂w/∂z + ∂w/∂z)
- Larger K removes small variability strongly.
- SAM sets eddy viscosity = eddy diffusivity
- Large K means smoother scalar field.

Summary

Center + Smagor

- Overestimation of power in w
- Large eddy viscosity due to numerical dispersion
- Excess removal of power in q
- Closest to the filtered q spectra

WENO-Z + Smagor

- Overestimation of power in w
- Closest to filtered w spectra
- Overestimation of power in q

WENO-Z + mixed

- Overestimation of power in w
- Overestimation of power in q

The above results are obtained with a 5th-order monotonic scalar advection scheme.

Because eddy diffusivity = eddy viscosity in SAM,
WENO-Z+mixed may produce best result with appropriately chosen length scale.

Does using different mixing length for horizontal and vertical diffusion for highly anisotropic gird help?

Coefficient value proposed by Moeng (2014, MWR) backscatters more energy.

Summary

- At this point, center & mixed SGS scheme do not work.
- WENO-Z
  - Expensive (maybe less than interactive radiation, 2-moment microphysics)
  - Maximum Courant number ~ 0.5-0.6 (for AB3 with SGS models)
- Is the mixed scheme useful for stratocumulus CSRM?

	relative budget	relative cost increase		
	center+Smagor		WENO-Z +Smagor	WENO-Z +mixed
total	1	1	1.35	1.42
mom advection	0.016	1	18.24	18.24
diffusion (mom +scalar)	0.026	1	1	1.83

Summary

- At this point, center & mixed SGS scheme do not work.
- WENO-Z
  - Expensive (maybe less than interactive radiation, 2-moment microphysics)
  - Maximum Courant number ~ 0.5-0.6 (for AB3 with SGS models)
- Is the mixed scheme useful for stratocumulus CSRM?

DYCOMS-II RF01, 24-km domain LES & CSRM
PBL mean w spectral density