A New Parameterization of Momentum Transport by Organized Cumulus Clouds

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I. Introduction

An important effect of the convective clouds on its environment is the vertical redistributions of momentum. However, most General Circulation Models (GCMs) do not have such a parameterization.

The convective clouds influence the environmental wind through the detrainment, the subsidence induced by the cumulus clouds and the perturbation pressure gradient force. The perturbation pressure gradient force is the most challenging part of the parameterization of the momentum transport since it is highly dependent on the organization of the convective clouds.

II. A new parameterization of perturbation pressure gradient force

With the Coriolis term and the friction term neglected, a diagnostic pressure equation can be derived under the anelastic approximation:

$$-\nabla^2 p' = f \tag{1}$$

Here $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$, and f is a function of velocity, buoyancy and density.

There are two methods to solve this equation. One is the Green's function method:

$$p'(x, y, z) = \left[\int_v G(\xi, \eta, \zeta; x, y, z) f(\xi, \eta, \zeta) dv - \int_s p'_s \frac{\partial G}{\partial n} ds\right] / (4\pi), \tag{2}$$

where $G(\xi, \eta, \zeta; x, y, z) = \frac{1}{\sqrt{(\xi - x)^2 + (\eta - y)^2 + (\zeta - z)^2}}$ is the Green's function and p'_s is the perturbation pressure at the boundary of a convective system. The second term shows the effects of the boundary condition. We can only obtain a numerical solution using the Green's function method. However, this method does not have truncation errors. The other is the series expansion method.

Assuming $v_r = -\frac{r}{a}e^{-r/a}\cos(nz)\tilde{w}$, $w = \tilde{w}e^{-r/a}\sin(nz)$, and $B = \tilde{B}\sin(nz)e^{-r/a}$ for a nonlinear mesoscale convective system (MCS), where a is the radius of the MCS, we have

$$-\sigma\left(\frac{\partial p'}{\partial x}\right)_\sigma = \gamma_k \frac{\partial \bar{u}}{\partial z} M_c, \tag{3}$$

and

$$-\sigma\left(\frac{\partial p'}{\partial y}\right)_\sigma = \gamma_l \frac{\partial \bar{v}}{\partial z} M_c, \tag{4}$$

where σ is the area of a MCS. $\gamma_k = \gamma_l = 0.46$ can be derived for nonlinear MCSs in (3) and (4). A linear MCS can be regarded to consist of a series of small nonlinear MCSs. In the line-perpendicular direction (assumed to be the *x* direction for simplicity), the perturbation pressure gradient force is the same as (3). In the line-parallel direction (assumed to be the *y* direction),

$$-\sigma\left(\frac{\partial p'}{\partial y}\right)_\sigma = \gamma_l \frac{L_x}{L_y} \frac{\partial \bar{v}}{\partial z} M_c, \tag{5}$$

where L_x and L_y are the size of the linear MCS in the *x* and *y* directions, respectively.

Wu and Yanai (1994) assumed that

$$w = \tilde{w}\cos(kx)\cos(ly)\sin(nz), \quad -\frac{\pi}{2} < kx, ly < \frac{\pi}{2}, \tag{6}$$

where k, l and n is the wavenumber in *x, y* and *z* direction, respectively. They derived similar formulae as (3) and (5) for MCSs except with $\gamma_k = \frac{2k^2}{k^2 + l^2 + n^2}$ and $\gamma_l = \frac{2l^2}{k^2 + l^2 + n^2}$. In the line-perpendicular direction, their $1 \le \gamma_k \le 2$, which is much larger than the new parameterization. However, the mean γ_k from Zhang's and Wu's (2003) 2D cloud resolving model simulations of a 30-day subperiod of the TOGA COARE IOP is slightly larger than 0.4. Grubisic and Moncrieff's (2000) simulations of midlatitude MCSs produced a γ_k near 0.5 when a horizontal spacing of 794 m was used, and near 0.75 when a horizontal spacing of 394 m was used.

The perturbation pressure gradient force in the vertical direction can also be derived using a similar method. Some results are shown in Section IV.

III. Mechanisms of upgradient transport of momentum by cumulus clouds

Assuming all of the shear is in the *x* direction and the linear MCS is parallel to the *y* direction, the effects of the convective systems on the environmental wind can be expressed by

$$\frac{d\bar{u}}{dt} = D_c(u_c - \bar{u}) + \frac{M_c}{\rho}\frac{\partial \bar{u}}{\partial z} + \frac{1}{\rho}\sigma\left(\frac{\partial p'}{\partial x}\right)_\sigma, \tag{7}$$

where D_c, M_c, and u_c are the detrainment rate, the mass flux and the velocity of the convective system, respectively. Using (3) and (5), we have

$$\frac{d\bar{u}}{dt} = D_c(u_c - \bar{u}) + \sigma w\frac{\partial \bar{u}}{\partial z} - 0.46\sigma w\frac{\partial \bar{u}}{\partial z}. \tag{8}$$

From (8), we can see that convective clouds influence the environmental wind through the detrainment, the subsidence induced by the cumulus clouds and perturbation pressure gradient force. Because the perturbation pressure gradient force is always smaller than the subsidence term, the net effects of the three terms on the right hand side of (8) cannot be determined by the perturbation pressure gradient force. So the upgradient transport of the horizontal momentum may not be due to this term. There is no doubt that the overall effects of the subsidence and detrainment are to redistribution the momentum of the environment and the mean effects are downgradient. The question is whether subsidence and detrainment can cause local upgradient momentum transports. Observations indicate that these transports occur near the top and the base of the convective clouds. The following are two simple examples.

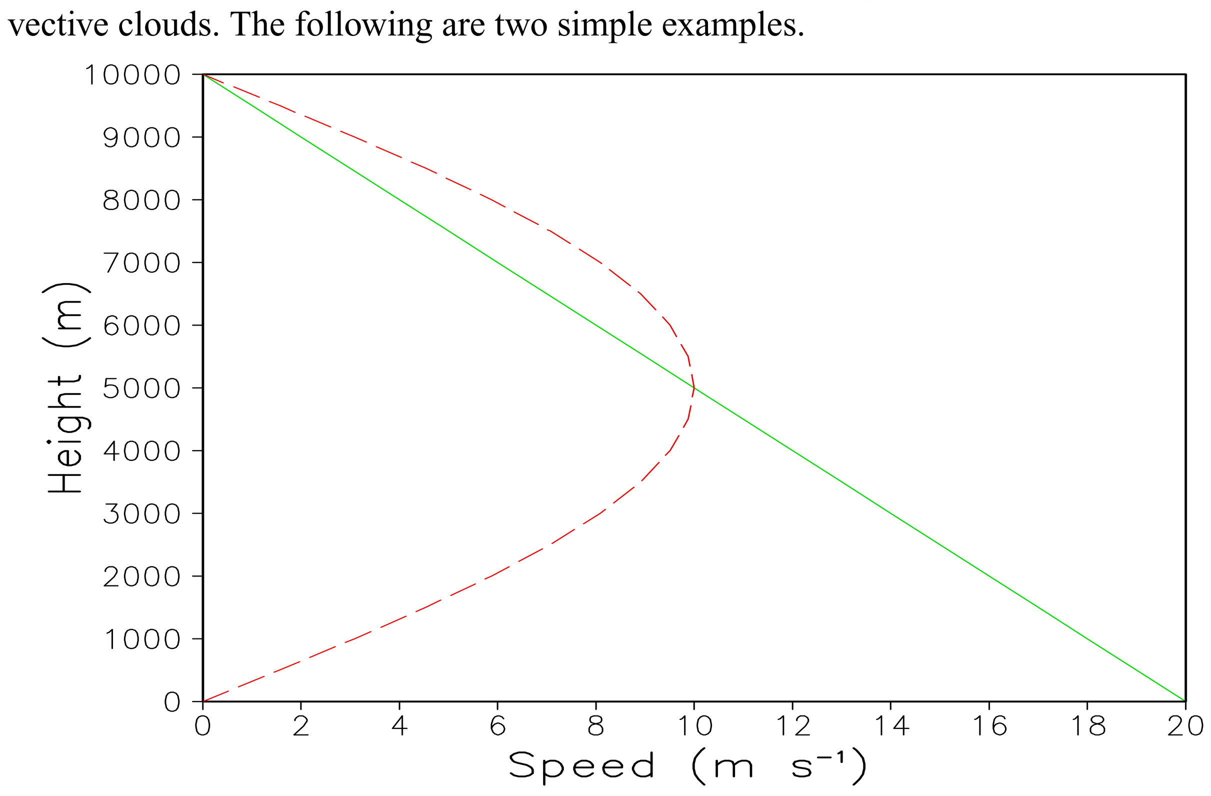

Figure 1: The vertical velocity (red line) and environmental wind in the *x* direction (green line) for Case I (local upgradient transport by subsidence term).

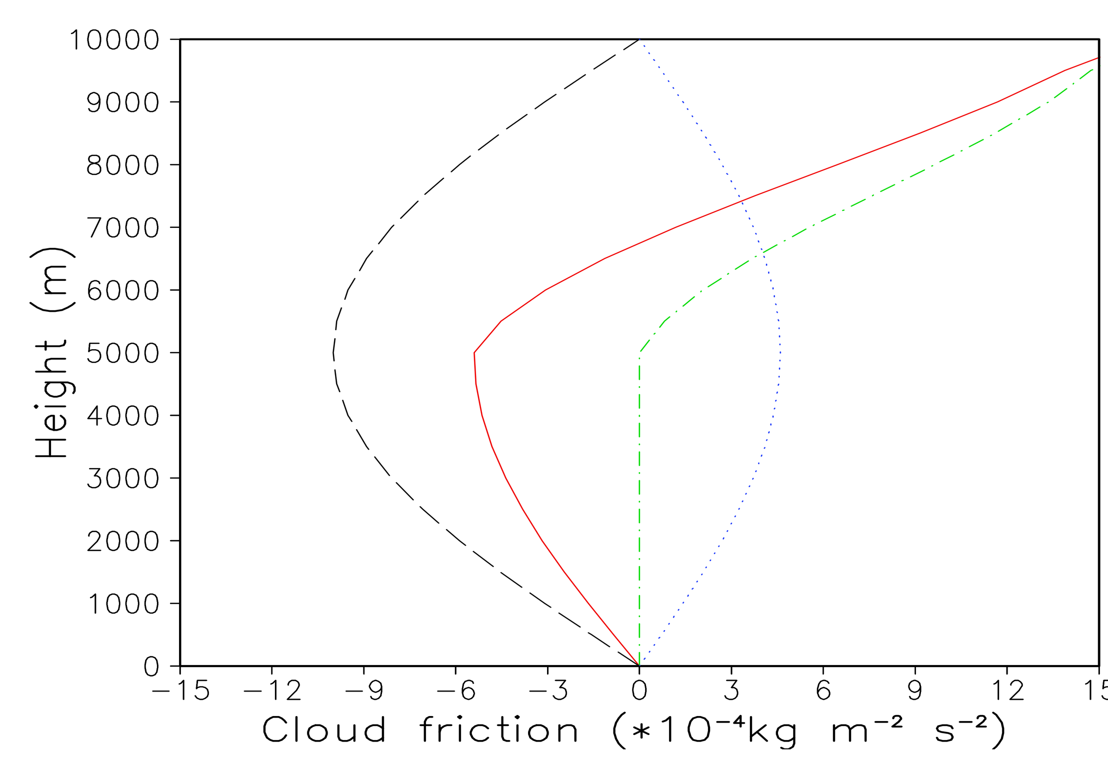

Figure 2: The cloud friction ($d\bar{u}/dz$) by the subsidence term (black line), by the detrainment term (green line), by the perturbation pressure gradient term (blue line) and by all three terms (red line).

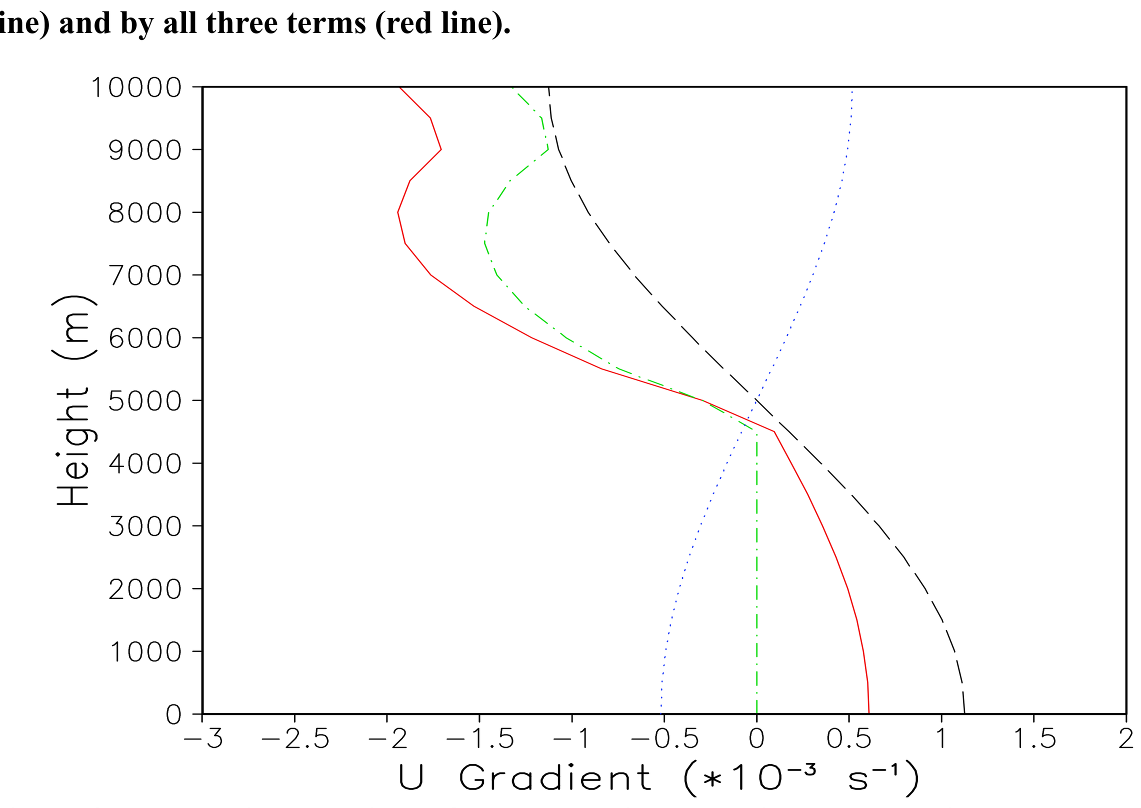

Figure 3: The vertical gradient of the change in the environmental wind produced by the subsidence term (black line), by the detrainment term (green line), by the perturbation pressure gradient term (blue line) and by all three terms (red line) in one hour.

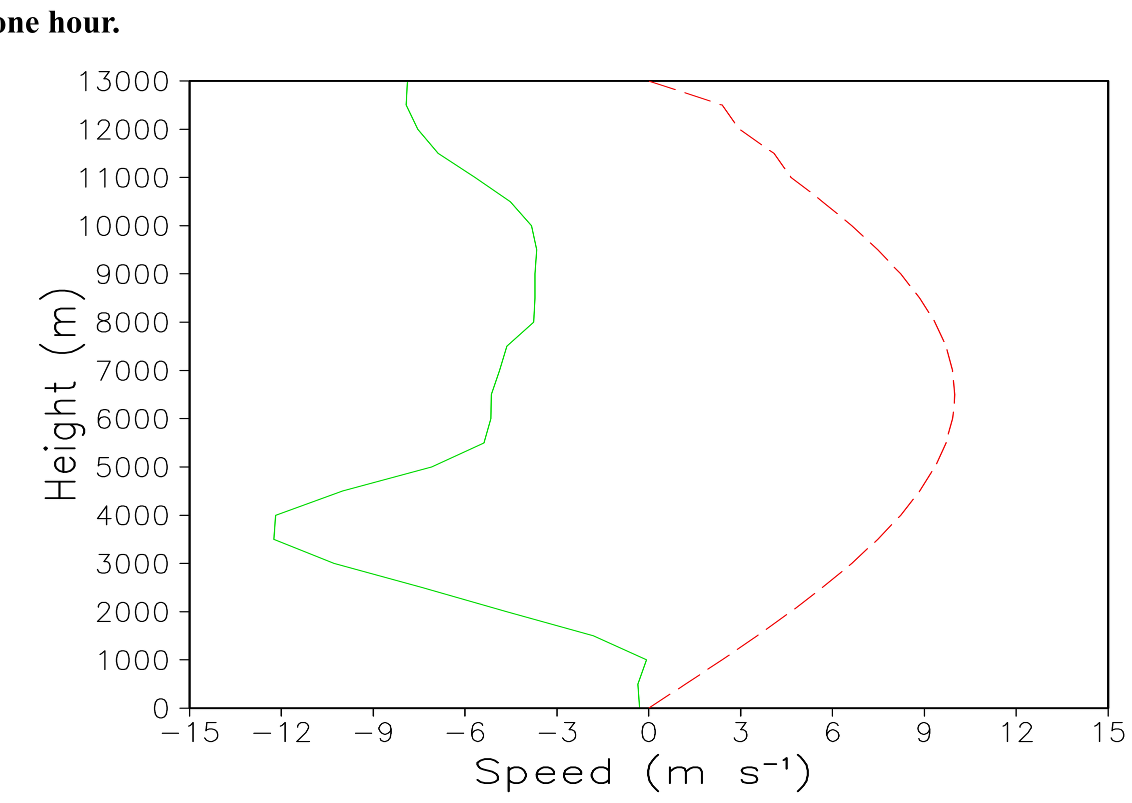

Figure 4: Same as Fig. 1 except for Case II (cloud top upgradient momentum transport by the detrainment and the horizontal perturbation pressure gradient force terms).

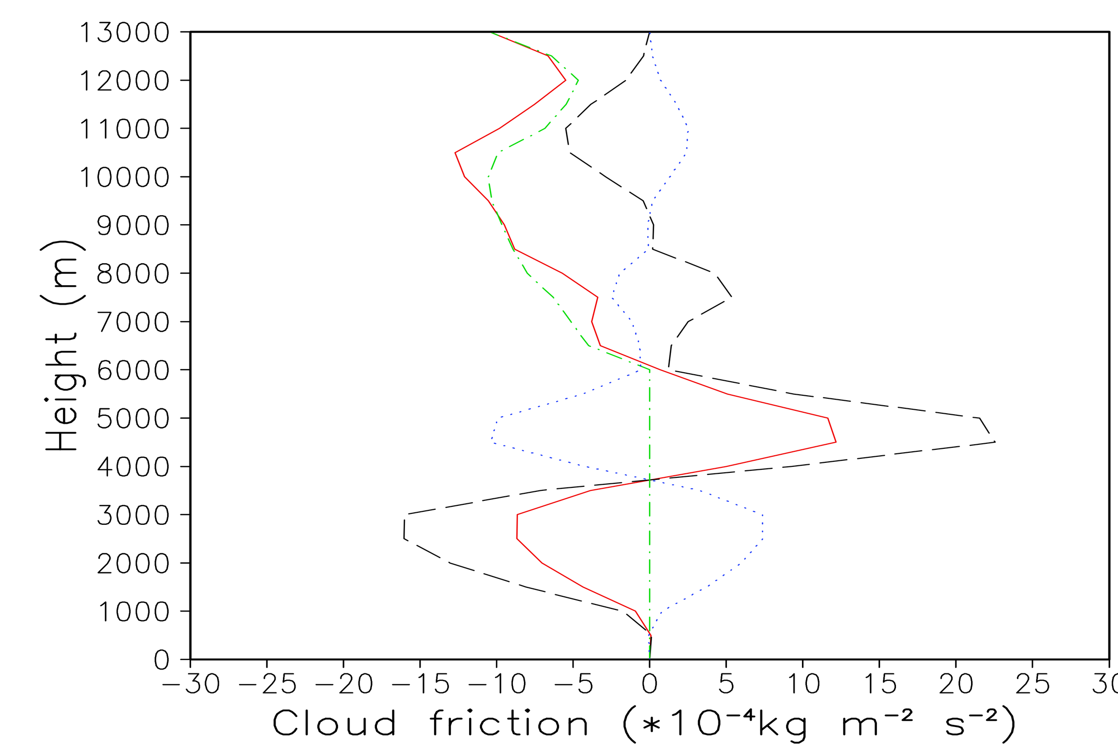

Figure 5: Same as Fig. 2 except for Case II.

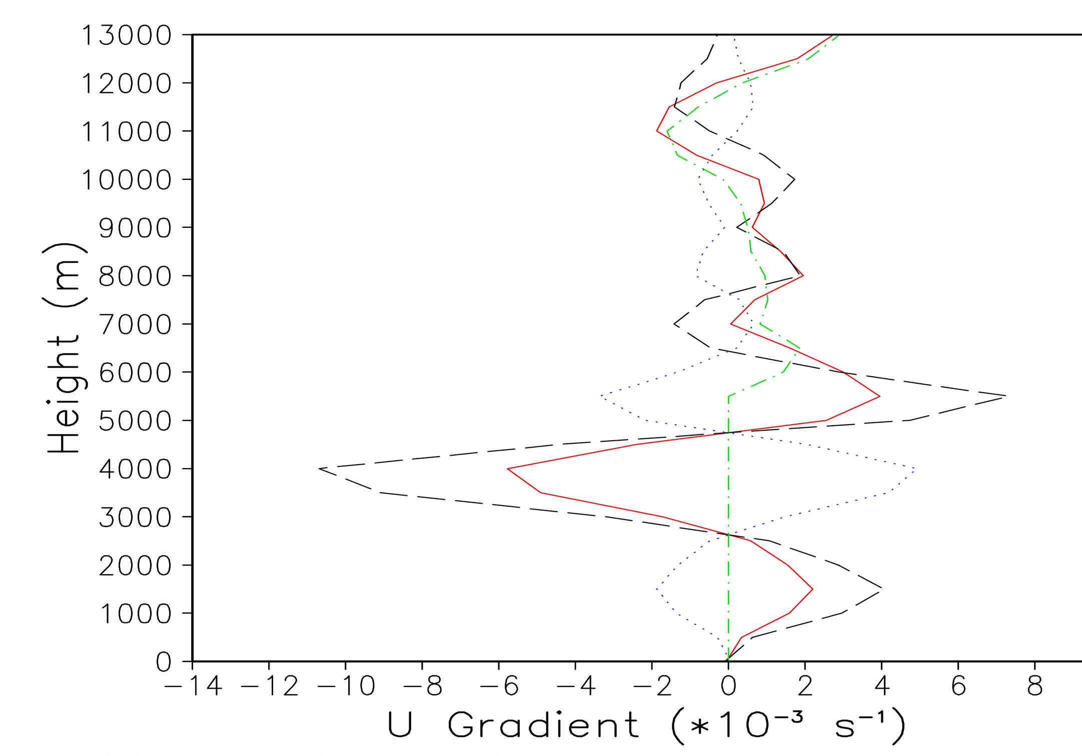

Figure 6: Same as Fig. 3 except for Case II

IV. Vertical perturbation pressure gradient force

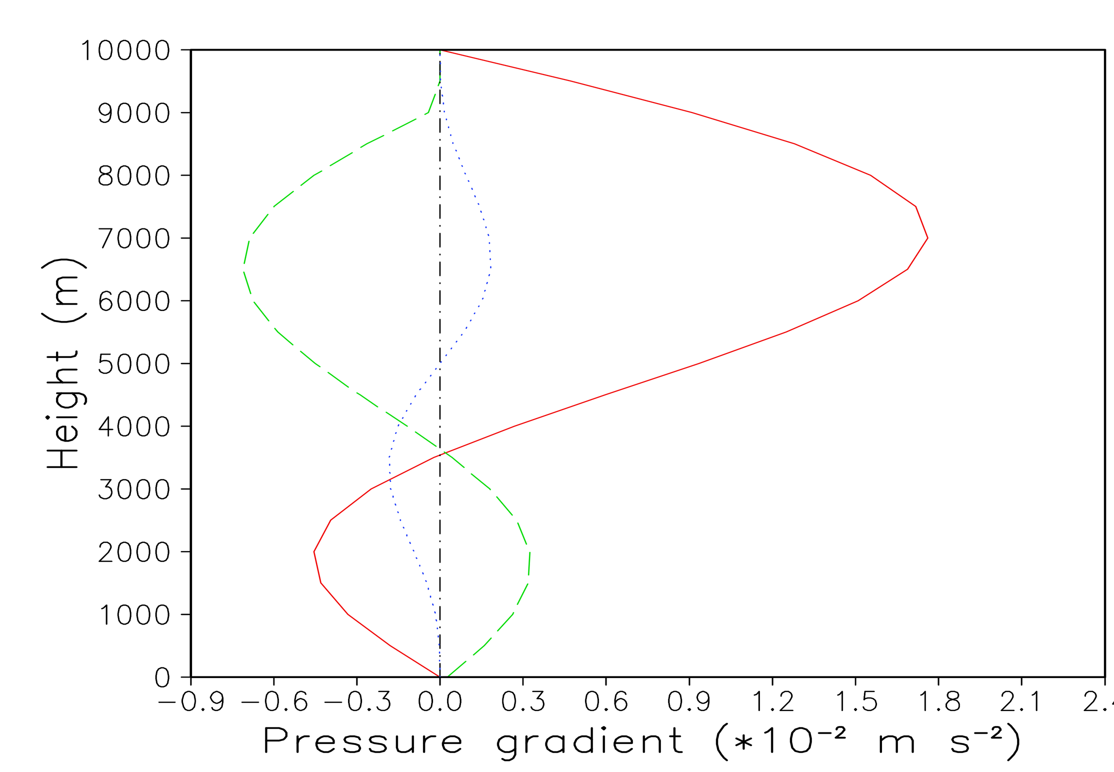

Figure 7: Vertical pressure gradient force produced by the buoyancy term (green line), by the nonlinear term (blue line) and by the density term (black line) for a nonlinear MCS. The buoyancy term (red line) is also shown.

V. Discussion and conclusions

In this study, a parameterization of cumulus momentum transport has been proposed. The diagnostic equation for the convective-scale perturbation pressure was solved analytically to obtain the horizontal and vertical components of convective-scale perturbation pressure gradient force. The horizontal perturbation pressure gradient force is proportional to the vertical wind shear and the convective mass flux. It is also proportional to the aspect ratio of a squall line in the line-parallel direction. The mixing coefficient for the horizontal pressure gradient force agrees well with those from cloud resolving model simulations of Grubisic and Moncrieff (2000) and Zhang and Wu (2003). The convective-scale vertical perturbation pressure gradient force produced by the buoyancy terms tends to counteract the negatively buoyant updraft below the cloud base and positively buoyant updraft above the cloud base. This pressure gradient can be a factor to trigger the convection.

The local upgradient momentum transport can be produced by subsidence and detrainment, as shown by the two examples shown in this study.The horizontal perturbation pressure gradient force is always smaller than the subsidence term and cannot be the dominant factor for the upgradient momentum transport. The large low-level shear and low-level jet are needed for the detrainment to produce upgradient transport near cloud top so that the momentum of the low-level jet can be brought by the detrainment to the upper level. The nonlinear MCSs usually do not have strong low-level shear and a low-level jet. Therefore, the detrainment term cannot produce upgradient transport near the cloud top of nonlinear MCSs.

Further details of this study are given in Cheng et al. (2005).

VI. References

Cheng, A., and K.-M. Xu, 2005: A new parameterization of momentum transport by organized cumulus clouds. In preparation for *J. Atmos. Sci.*

Grubisic, V., and M. W. Moncrieff, 2000: Parameterization of convective momentum transport in highly baroclinic conditions. *J. Atmos. Sci.*, **57**, 3035-3049.

Wu X., and M., Yanai, 1993: Effects of vertical wind shear on the cumulus transport of momentum: observations and parameterization. *J. Atmos. Sci.*, **51**, 1640-1660.

Zhang G. J., and X. Wu, 2003: Convective momentum transport and perturbation pressure field from a cloud-resolving model simulation. *J. Atmos. Sci.*, **60**, 1120-1139.