A new 3D cloud model based on the vorticity equation

Joon-Hee Jung
Department of Atmospheric Science, Colorado State University, Colorado, USA

Akio Arakawa
Department of Atmospheric and Oceanic Sciences, UCLA, Los Angeles, California, USA

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Nonhydrostatic 3-D Models

- **Anelastic models**:

  The sound-wave mode is filtered out but, when the momentum equation is used, extra effort is required to obtain a pressure field that is consistent with the use of the anelastic continuity equation.

  e.g. *Wilhelmson, R. B., 1974; Schlesinger, R. E., 1975; Clark, T. L., 1979; Takahashi, T., 1981; Lipps, F. B. and R. S. Hemler, 1986; Redelsperger, J.-L. and G. Sommeria, 1986; Tao, W.-K. and S.-T. Soong, 1986; Kogan, Y. L., 1991; Shutts, G. J. and M. E. B. Gray, 1994; Khairoutdinov, M. F. and Y. L. Kogan, 1999*

  e.g. *Steiner, J. T., 1973 (Boussinesq approximation)*

- **Fully compressible models**:

  The sound-wave mode exists and it must be properly handled numerically.

  e.g. *Cotton, W. R. and G. J. Tripoli, 1978; Klemp, J. B. and R. B. Wilhelmson, 1978; Donner et al., 1999; Xue et al., 2000*

Mostly, the development work is directed towards the sophistication of existing models in cloud physics parameterization.

Anelastic 3-D Model

- **Usual approach:**

momentum equation: $\frac{d\mathbf{v}}{dt} = -\nabla\left(c_p\theta_o\pi\right) + \mathbf{F}\ ;\ \frac{d}{dt} \equiv \frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla,\ \pi = \left(\frac{P}{P_{oo}}\right)^{R/c_p}$

continuity equation: $\nabla\cdot\left(\rho_o\mathbf{v}\right) = 0$

pressure equation: $\nabla^2\left(c_p\theta_o\pi\right) = \text{source terms}$

- **Our approach:**

Since the pressure field is passive to the field of motion if it is anelastic, the vorticity equation, in which the pressure gradient force is eliminated, expresses relevant dynamical processes more explicitly than the momentum equation.

When the vorticity equation is used, computational design and interpretation of results can be more straightforward.

The use of the vorticity equation is common in 2-D anelastic models but not in 3D models. Our approach is unique in using the 3-D vorticity equation instead of the 3-D momentum equation.

Dynamical Framework of the Model

- **Continuity equation:** $$\frac{\partial}{\partial x}(\rho_0 u)+\frac{\partial}{\partial y}(\rho_0 v)+\frac{\partial}{\partial z}(\rho_0 w)=0$$
- **Definition of vorticity:** $$\rho_0\xi \equiv \frac{\partial w}{\partial y}-\frac{\partial v}{\partial z},\quad \rho_0\eta \equiv \frac{\partial u}{\partial z}-\frac{\partial w}{\partial x},\quad \rho_0\zeta \equiv \frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}$$
- **Vorticity equations:**

stretching effect twisting effect

$$\rho_0\frac{\partial \xi}{\partial t}=-\left[\frac{\partial}{\partial x}(\rho_0 u\xi)+\frac{\partial}{\partial y}(\rho_0 v\xi)+\frac{\partial}{\partial z}(\rho_0 w\xi)\right]+\underline{\rho_0\xi\frac{\partial u}{\partial x}}+\underline{\rho_0\eta\frac{\partial u}{\partial y}+\rho_0\zeta\frac{\partial u}{\partial z}}+f\frac{\partial u}{\partial z}+\frac{\partial F_w}{\partial y}-\frac{\partial F_v}{\partial z}$$

$$\rho_0\frac{\partial \eta}{\partial t}=-\left[\frac{\partial}{\partial x}(\rho_0 u\eta)+\frac{\partial}{\partial y}(\rho_0 v\eta)+\frac{\partial}{\partial z}(\rho_0 w\eta)\right]+\underline{\rho_0\eta\frac{\partial v}{\partial y}}+\underline{\rho_0\xi\frac{\partial v}{\partial x}+\rho_0\zeta\frac{\partial v}{\partial z}}+f\frac{\partial v}{\partial z}-\frac{\partial F_w}{\partial x}+\frac{\partial F_u}{\partial z}$$

$$\rho_0\frac{\partial \zeta}{\partial t}=-\left[\frac{\partial}{\partial x}(\rho_0 u\zeta)+\frac{\partial}{\partial y}(\rho_0 v\zeta)+\frac{\partial}{\partial z}(\rho_0 w\zeta)\right]+\underline{\rho_0\zeta\frac{\partial w}{\partial z}}+\underline{\rho_0\xi\frac{\partial w}{\partial x}+\rho_0\eta\frac{\partial w}{\partial y}}-f\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+\frac{\partial F_v}{\partial x}-\frac{\partial F_u}{\partial y}$$

This equation is applied only at $z=z_T$ to satisfy

$$\frac{\partial}{\partial x}(\rho_0\xi)+\frac{\partial}{\partial y}(\rho_0\eta)+\frac{\partial}{\partial z}(\rho_0\zeta)=0$$

$$(\rho_0\zeta)_z=-\int_{z_T}^{z}\left[\frac{\partial(\rho_0\xi)}{\partial x}+\frac{\partial(\rho_0\eta)}{\partial y}\right]dz+(\rho_0\zeta)_{z_T}$$

Dynamical Framework of the Model (Continued)

- **Updating the vertical component of velocity:**

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)w+\frac{\partial}{\partial z}\left[\frac{1}{\rho_0}\frac{\partial}{\partial z}(\rho_0 w)\right]=-\rho_0\frac{\partial \eta}{\partial x}+\rho_0\frac{\partial \xi}{\partial y}$$

- **Updating the horizontal components of velocity:**

- at the upper boundary: $u=-\frac{\partial \psi}{\partial y}+\frac{\partial \chi}{\partial x}$ and $v=\frac{\partial \psi}{\partial x}+\frac{\partial \chi}{\partial y}$

ψ: stream function
χ: velocity potential

$$\frac{\partial^2 \psi}{\partial x^2}+\frac{\partial^2 \psi}{\partial y^2}=\rho_0\zeta,\quad \left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\chi=-\frac{1}{\rho_0}\frac{\partial}{\partial z}(\rho_0 w)$$

$$\frac{\partial \overline{u}^{xy}}{\partial t}=-\frac{1}{\rho_0}\frac{\partial}{\partial z}\left(\rho_0\overline{uw}^{xy}\right)+\overline{fv}^{xy}+\overline{F_u}^{xy},\quad \frac{\partial \overline{v}^{xy}}{\partial t}=-\frac{1}{\rho_0}\frac{\partial}{\partial z}\left(\rho_0\overline{vw}^{xy}\right)-\overline{fu}^{xy}+\overline{F_v}^{xy}$$

- below the upper boundary:

$$u=\int_{z_T}^{z}\left(\frac{\partial w}{\partial x}+\rho_0\eta\right)dz+u_T(x,y,t)\qquad v=\int_{z_T}^{z}\left(\frac{\partial w}{\partial y}-\rho_0\xi\right)dz+v_T(x,y,t)$$

Dynamical Framework of the Model (Continued)

- **Advection of vorticity components and scalar variables:**

The model has a flexibility to choose from a family of scheme by changing a single parameter, including a quadratically-conserving 2nd-order scheme, a quadratically-bounded partially 3rd-order scheme, and schemes in-between.

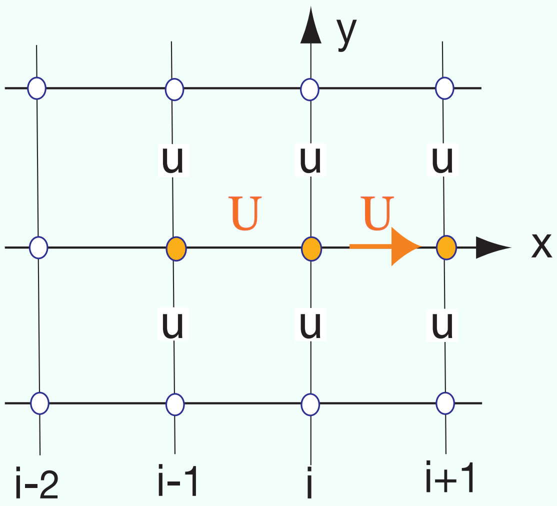
Assume U > 0 everywhere.

$\alpha = 0$ $\alpha = 1$

$$\frac{\partial q_i}{\partial t} = -\frac{1}{2d}\left[\left\{U_{i+1/2}(q_{i+1}+q_i) - \frac{2\alpha}{7}\left[U_{i+1/2}(q_{i+1}-q_i) - \sqrt{U_{i+1/2}}\sqrt{U_{i-1/2}}(q_i - q_{i-1})\right]\right\} - \left\{U_{i-1/2}(q_i+q_{i-1}) - \frac{2\alpha}{7}\left[U_{i-1/2}(q_i-q_{i-1}) - \sqrt{U_{i-1/2}}\sqrt{U_{i-3/2}}(q_{i-1} - q_{i-2})\right]\right\}\right]$$

In the equation, U is obtained from the velocity component u following J6 which is a second-order enstrophy conserving (but formally not energy conserving) Jacobian.

Thus, even when α =1, the scheme is partially third order for non-uniform flow.



Model Physics

- **Column physics:**

  No modification from the 2-D model (UCLA/CSU/University of Utah CRM).

  - bulk ice-phase microphysical parameterization
    (Lord et al., 1984; Krueger et al., 1995)

  - solar and infrared radiation parameterization
    (Fu et al., 1995)

- **Surface fluxes:** Flux-profile relationships (Deardorff, 1972)

- **Turbulence:**

  First-order closure (deformation and stability dependent K; Shutts and Gray, 1994)

Experiments

The emphasis of this presentation is to demonstrate the computational performance of the model as well as to gain understanding of basic convective processes from the point of view of vorticity dynamics.

Exp 1 : Simulation of a rising bubble using the 2nd-order scheme with a rigid upper boundary at 15 km, no buoyancy, no shear and no friction.

Exp 2 : Same as Exp 1 but using the partially 3rd-order scheme.

Exp 3 : Same as Exp 2 but with buoyancy in a neutral environment.

Exp 3S : Same as Exp 3 but with a uniform vertical shear of basic current.

Exp 4 : Same as Exp 3 but with a rigid upper boundary at 30 km with isothermal stratification between 15 km and 30 km.

Exp 4S : Same as Exp 4 but with a shear up to 15 km as in Exp 3S.

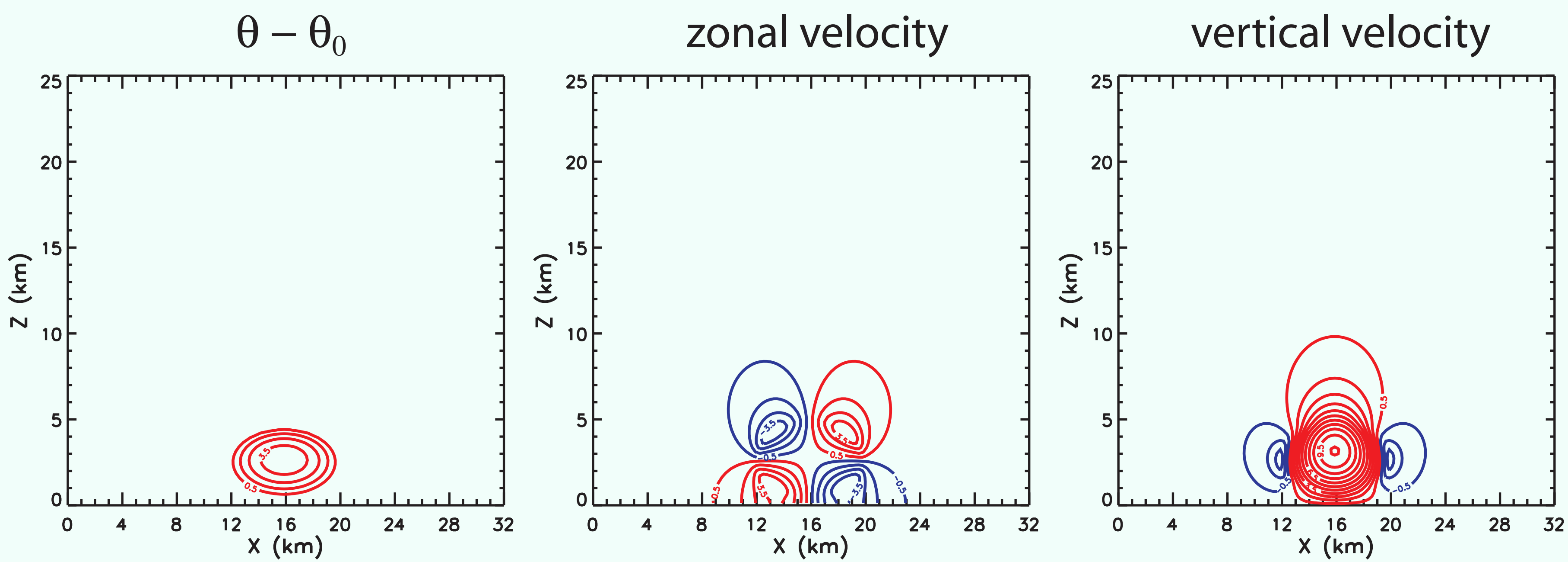
Exp 5 : Same as Exp 4 but with full physics.

Exp 5S : Same as Exp 5 but with a shear as in Exp 4S.

Exp 1 : Simulation of a rising bubble using the 2nd-order scheme with a rigid upper boundary at 15 km, no buoyancy, no shear and no friction.

Exp 2 : Same as Exp 1 but using the partially 3rd-order scheme.

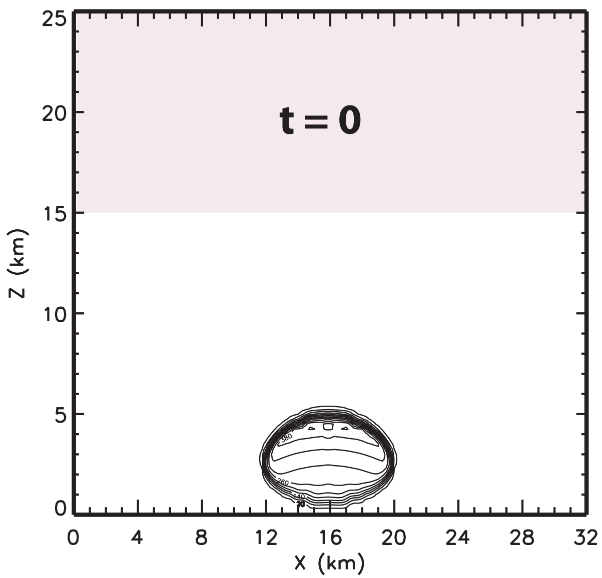
Initial condition: Generated by an ellipsoidal buoyant bubble

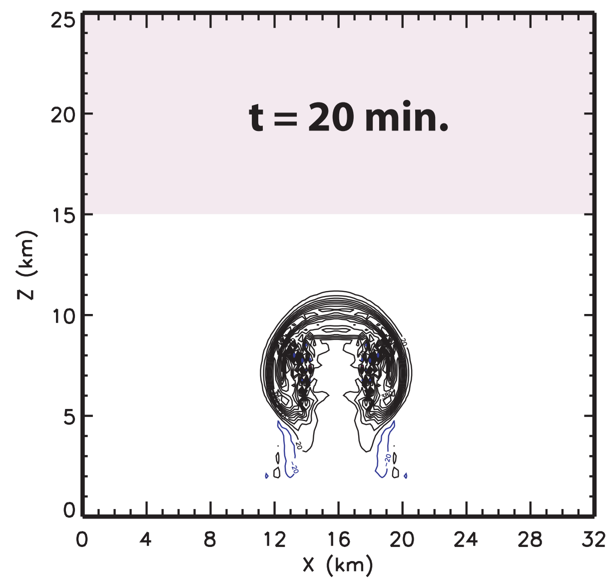


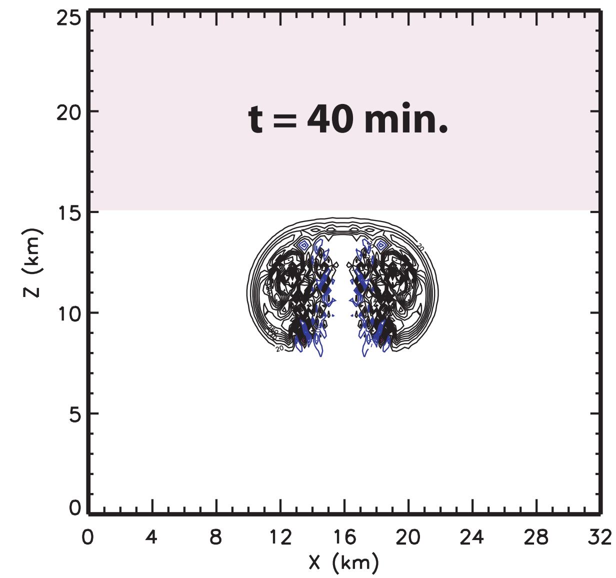
Grid size: 250 m
Time differencing: Second-order Adams-Bashforth

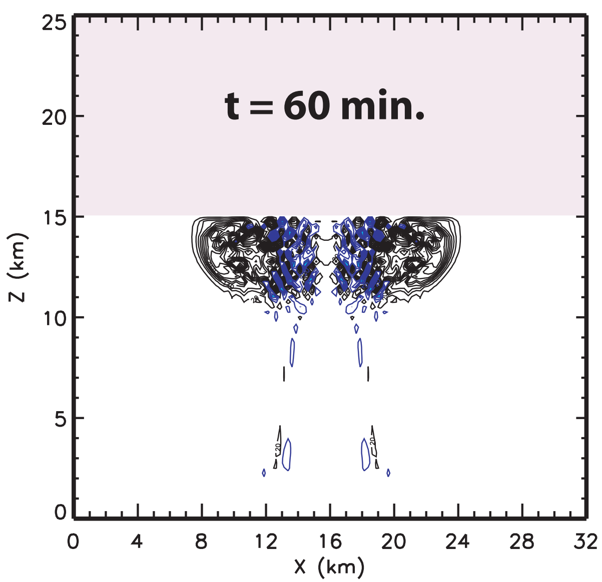
Potential Vorticity

EXP 1 (2nd-order vorticity advection)







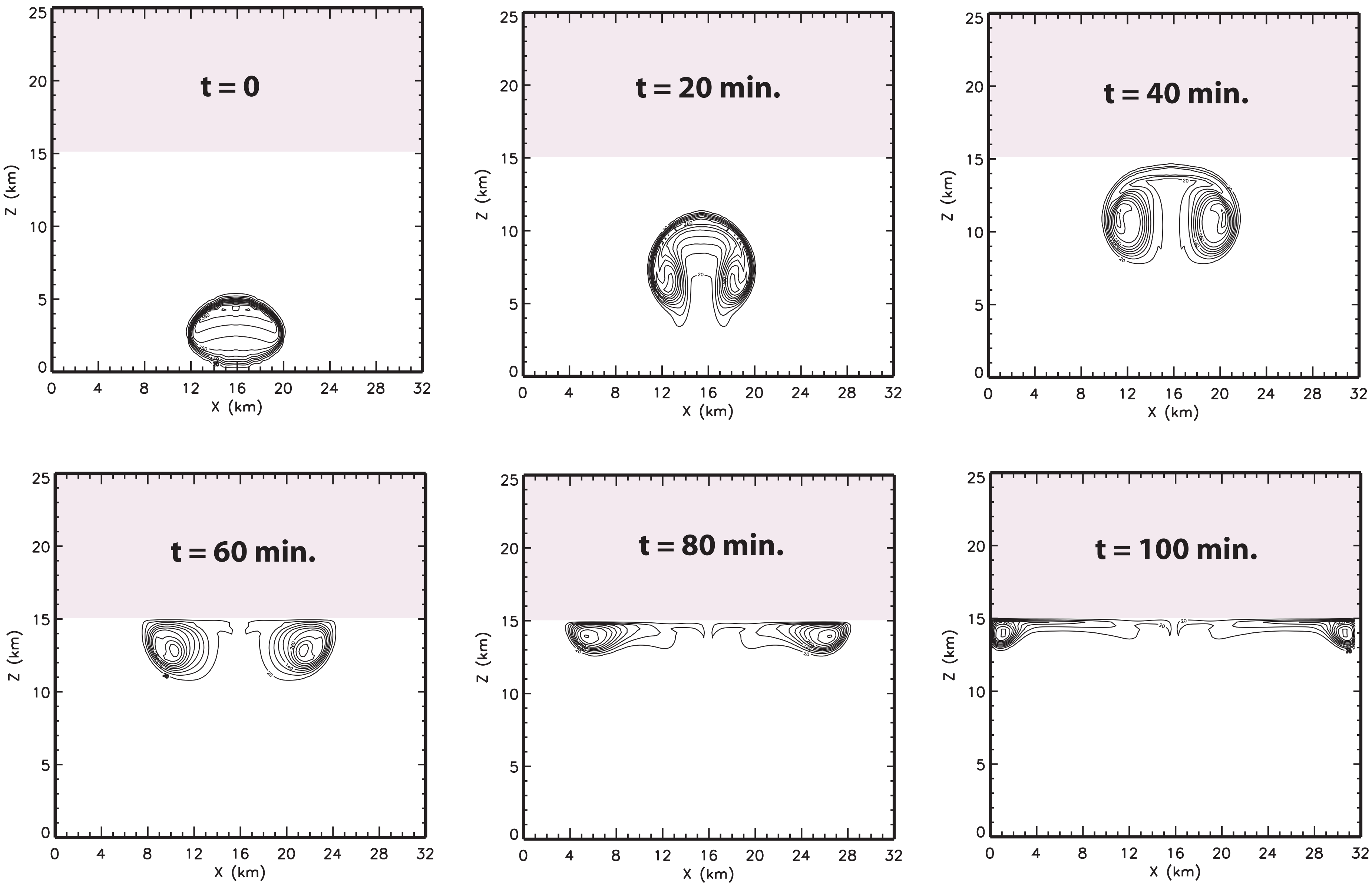


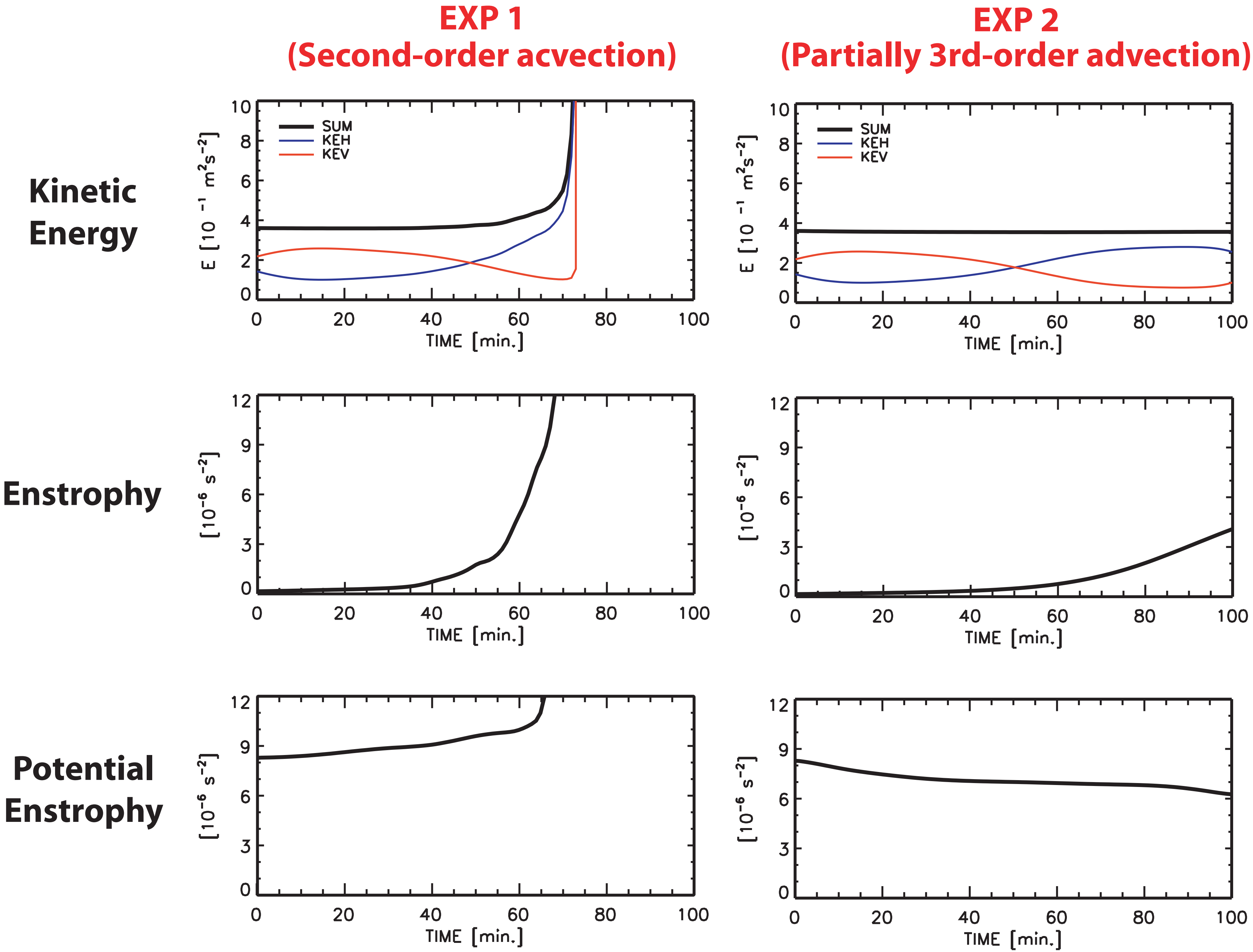
potential vorticity: The azimuthal component of vorticity divided by the distance from the axis of symmetry and density.

This quantity is supposed to be conserved for an axisymmetric flow when time is continuous.

Potential Vorticity

EXP 2 (partially 3rd-order vorticity advection)





EXP 1
(Second-order acvection)
EXP 2
(Partially 3rd-order advection)
Kinetic Energy
Enstrophy
Potential Enstrophy
SUM
KEH
KEV
E [10 $^{-1}$ m^{2}s^{-2}]
[10^{-6} s^{-2}]
TIME [min.]

Exp 3 : Same as Exp 2 but with buoyancy in a neutral environment.

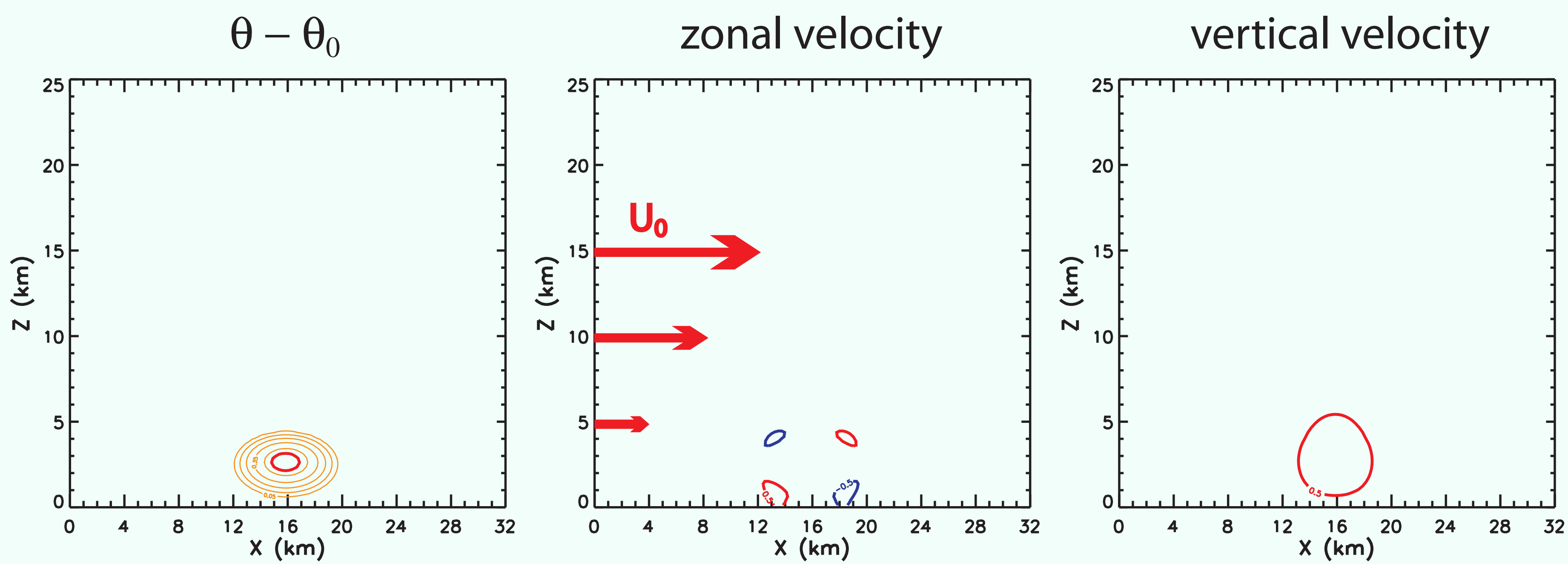
Exp 3S : Same as Exp 3 but with a uniform vertical shear of basic current.

Exp 4 : Same as Exp 3 but with a rigid upper boundary at 30 km with isothermal stratification between 15 km and 30 km.

Exp 4S : Same as Exp 4 but with a shear up to 15 km as in Exp 3S.

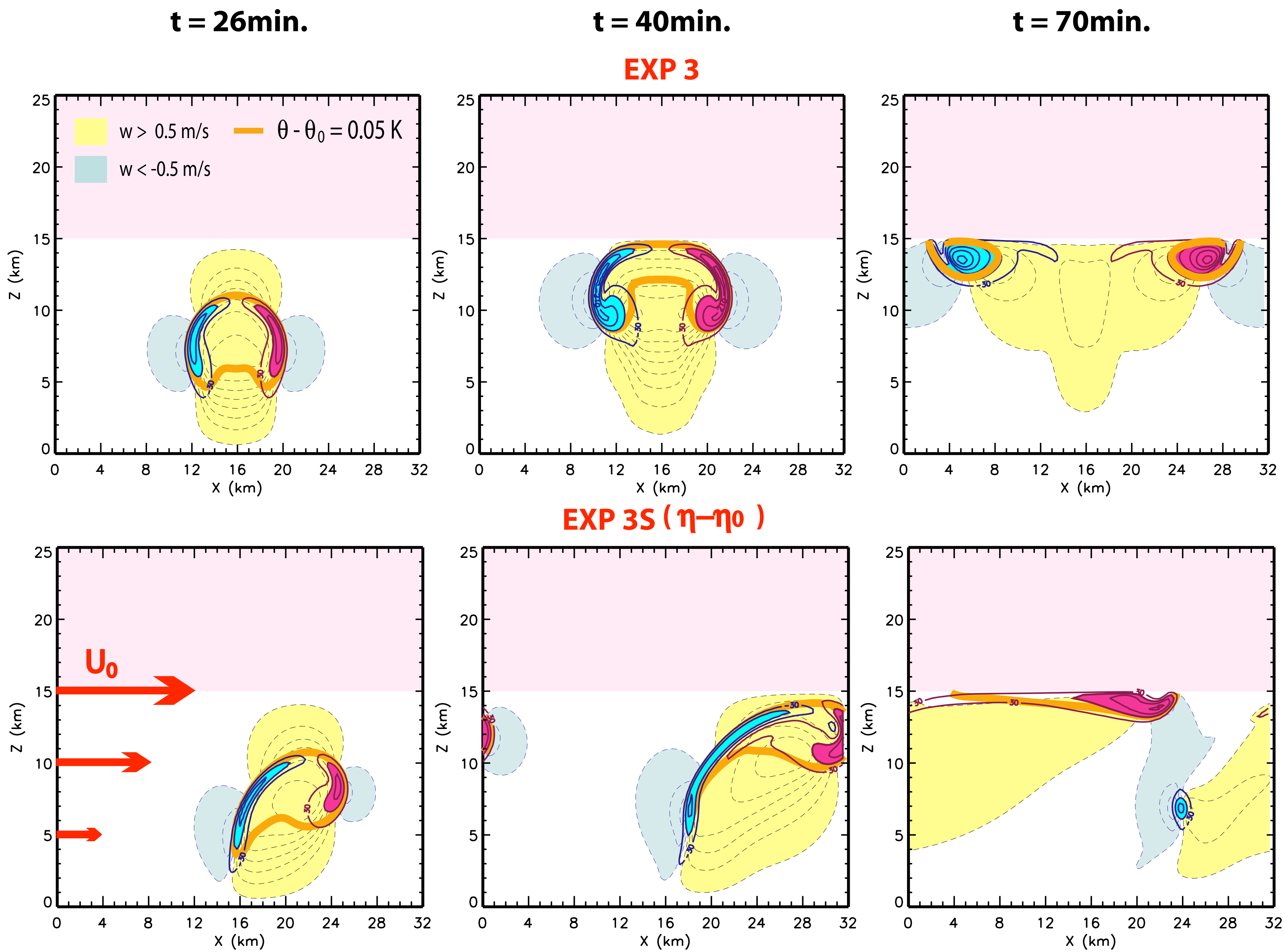
EXP 3 & 4

Initial condition: Generated by an ellipsoidal buoyant bubble



(at the center of y-domain)

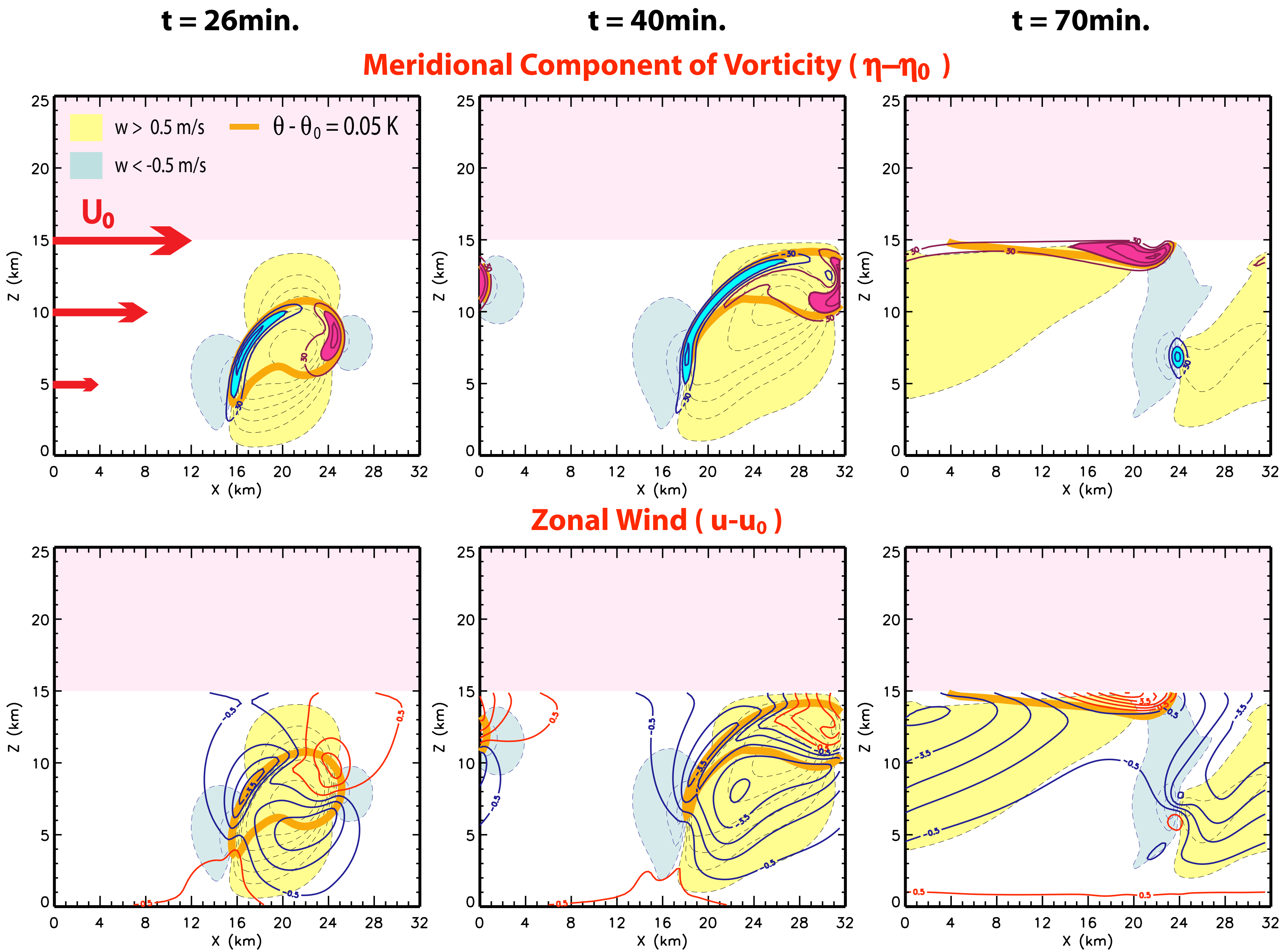
Meridional Component of Vorticity (η)



Development of asymmetry between positive and negative vortices in EXP 3S.

(at the center of y-domain)

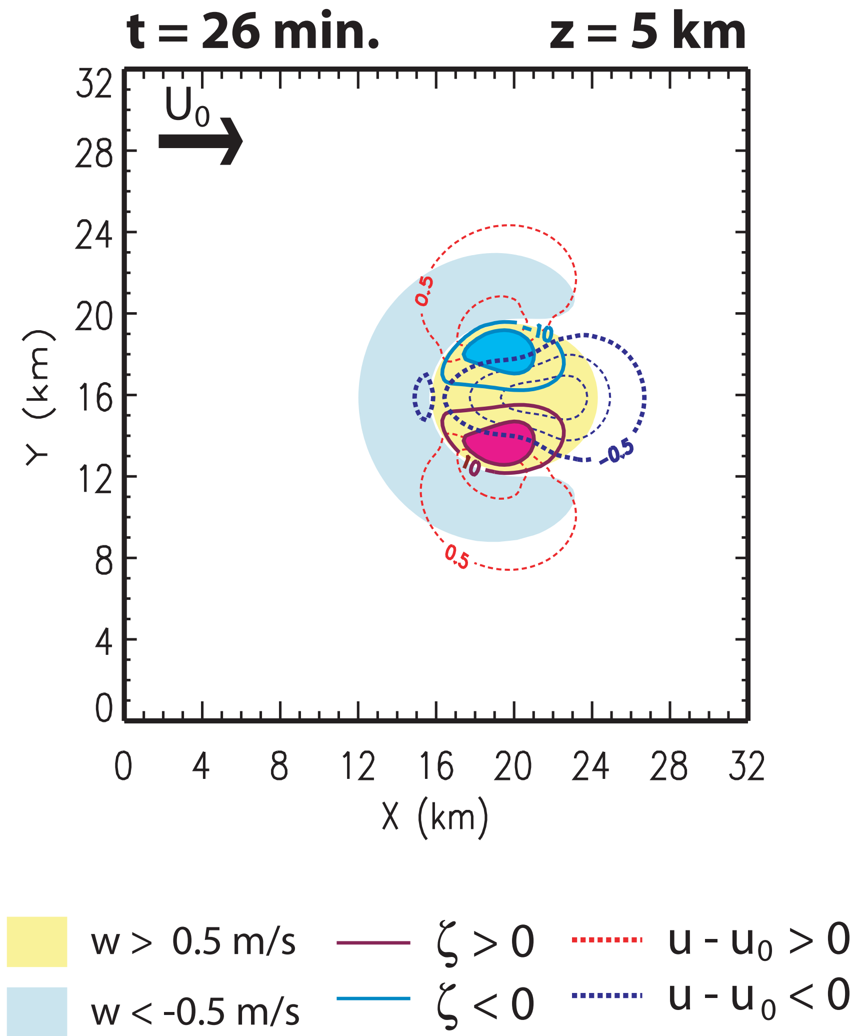
EXP 3S



Deceleration of zonal wind where convection is active.

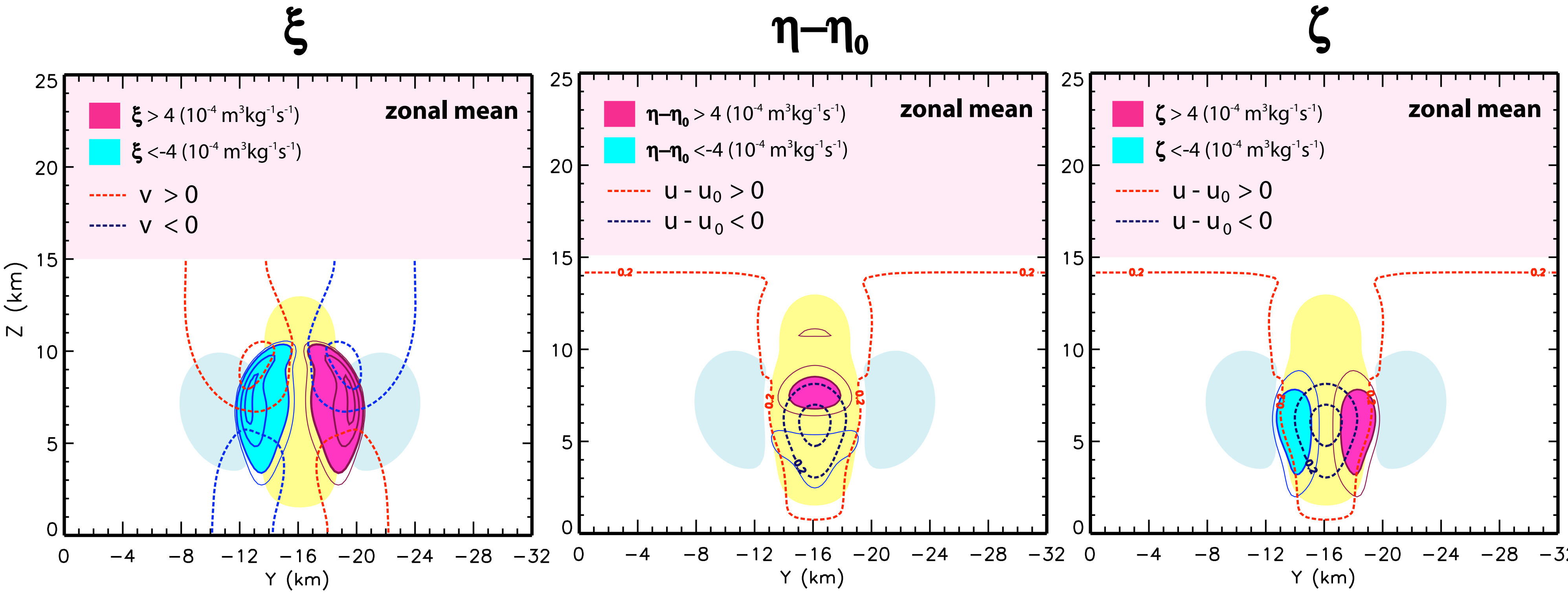
EXP 3S

Vertical Component of Vorticity (ζ) and Zonal Wind ($u-u_0$)



EXP 3S

t = 26 min.



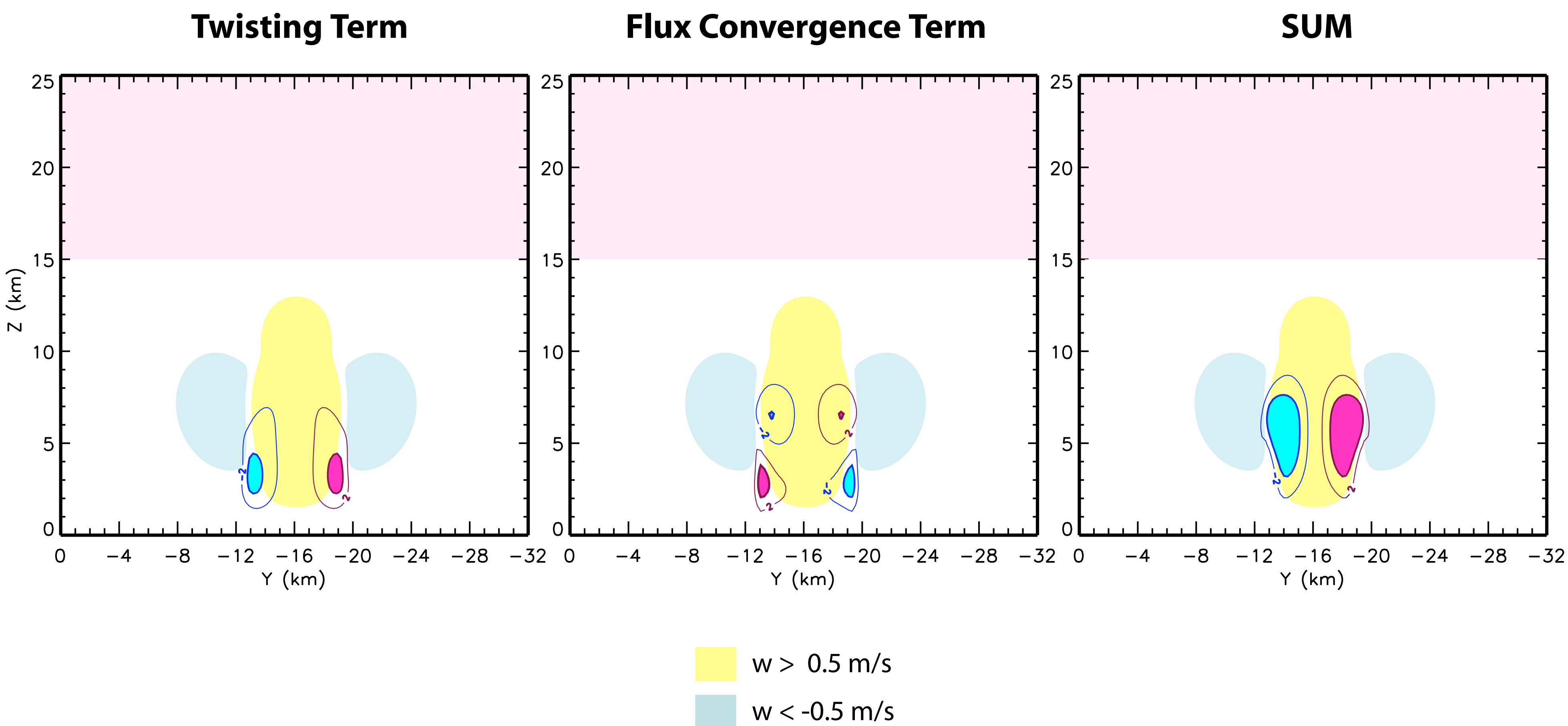
$$\bar{\xi} \equiv \frac{1}{\rho_0}\left(\frac{\partial \bar{w}}{\partial y} - \frac{\partial \bar{v}}{\partial z}\right) \qquad \bar{\eta} \equiv \frac{1}{\rho_0}\frac{\partial \bar{u}}{\partial z} \qquad \bar{\varsigma} \equiv -\frac{1}{\rho_0}\frac{\partial \bar{u}}{\partial y}$$

All of three components of vorticity are involved so that the motion is purely 3D.

EXP 3S

ζ Generation Terms (10^{-4} m^3 kg^{-1} s^{-1})

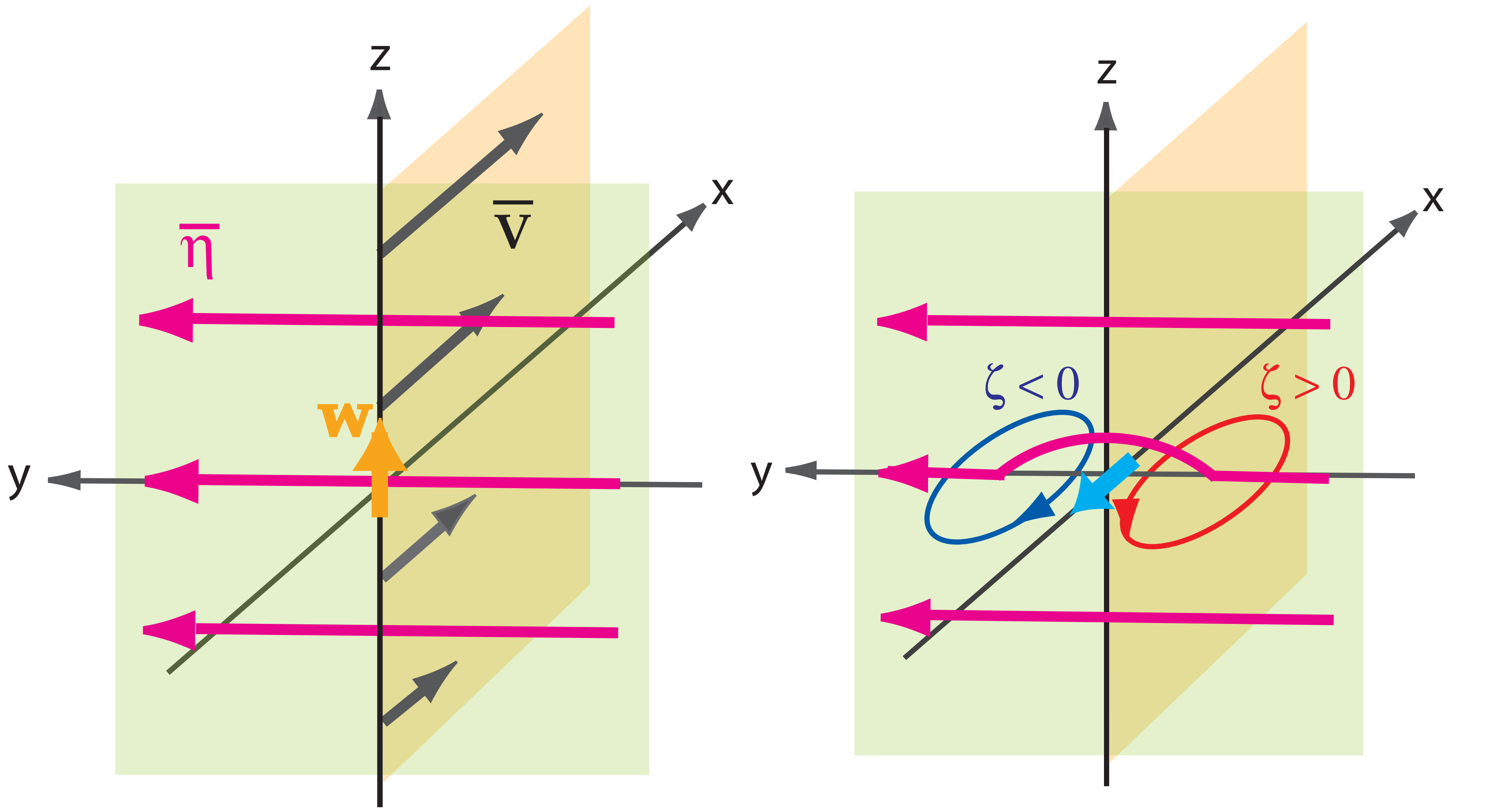
(accumulated effects up to t = 26 min. / zonal mean)



At the lower level, the flux convergence term almost compensates the twisting term, indicating that circulation carries the twisted effect upwards.

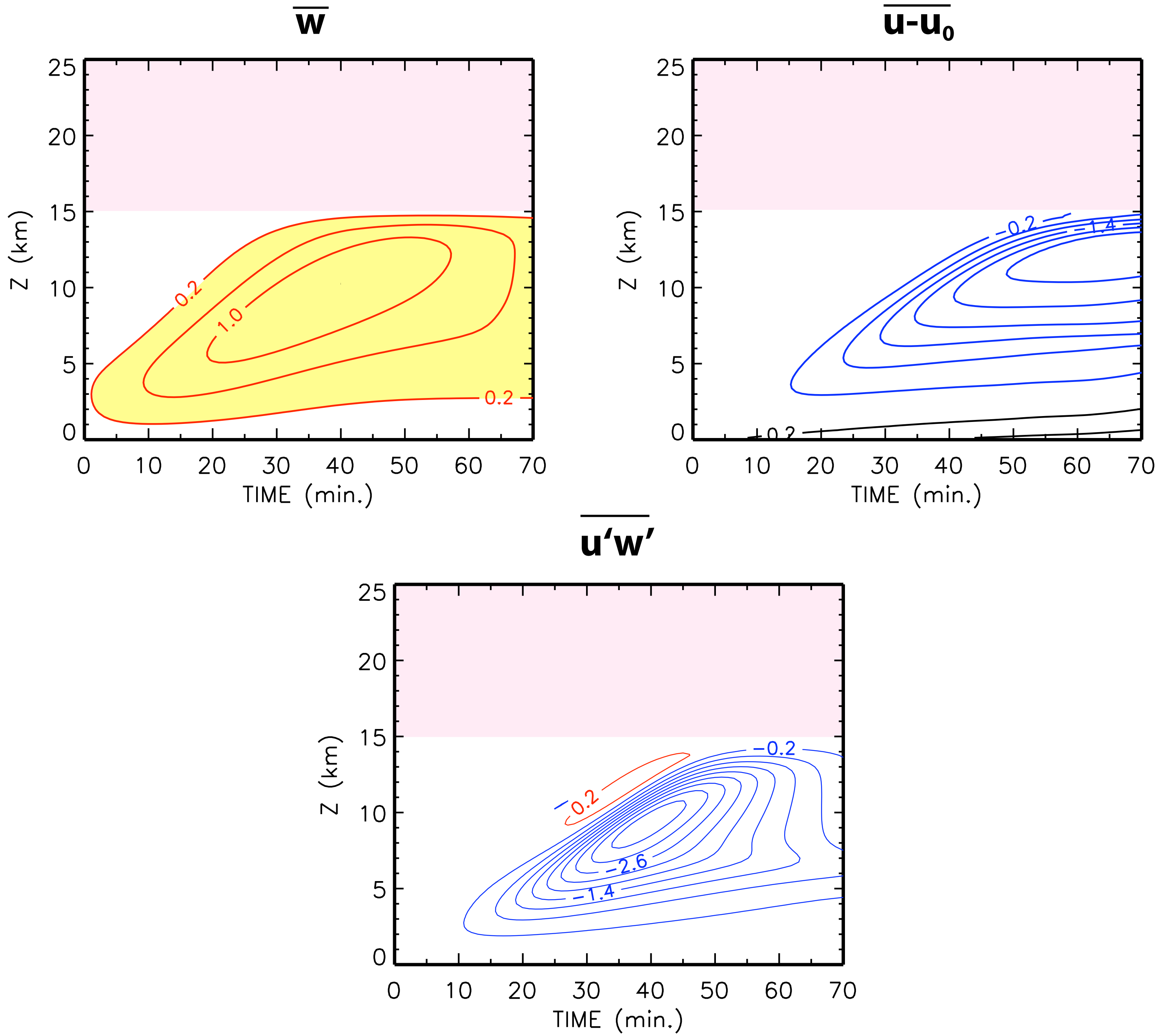
Initial zonal flow
and local vertical velocity

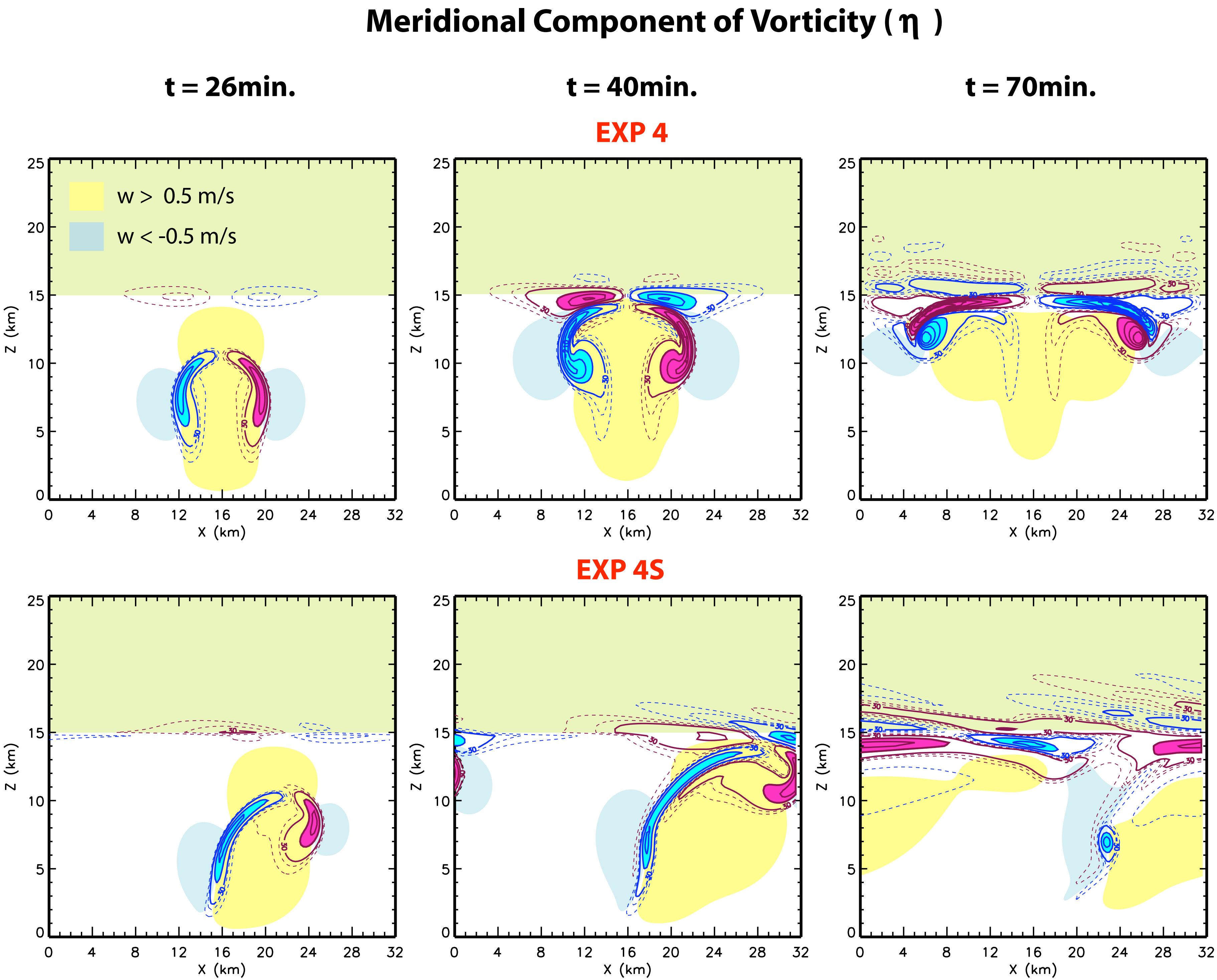
Generation of ζ through twisting
and local deceleration of zonal flow



(at the center of y-domain)

EXP 3S




Meridional Component of Vorticity (η)
t = 26min.
t = 40min.
t = 70min.
EXP 4
EXP 4S
w > 0.5 m/s
w < -0.5 m/s
Z (km)
X (km)
30

Summary and Conclusions

- A model for simulating convection using the 3-D vorticity equation is developed and has successfully been tested without and with physics.
- It is demonstrated that the partially 3rd-order quadratically-bounded advection scheme effectively controls computational noise by keeping dispersion errors small.
- Using the simulated results, the interactions between 3-D motions generated by the convection and basic shear flow are studied from the point of view of vorticity dynamics.
  - Without moist physics, it is shown that the rising bubble under basic shear flow induces local deceleration through the twisting effects on vorticity components.
  - With moist physics, the deceleration takes place stepwise, probably due to stepwise reorganization of updrafts and downdrafts.

Relaxed Anelastic Balance

- **Continuity equation:** $$\frac{\partial \rho'}{\partial t} + \frac{\partial}{\partial x}(\rho_0 u) + \frac{\partial}{\partial y}(\rho_0 v) + \frac{\partial}{\partial z}(\rho_0 w) = 0$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) w + \frac{\partial}{\partial z}\left[\frac{1}{\rho_0}\frac{\partial}{\partial z}(\rho_0 w)\right] + \frac{\partial}{\partial z}\left(\frac{\partial}{\partial t}\frac{\rho'}{\rho_0}\right) = -\rho_0 \frac{\partial \eta}{\partial x} + \rho_0 \frac{\partial \xi}{\partial y}$$

Approximately,

$$\frac{1}{c^2}\frac{\partial^2 w}{\partial t^2} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) w + \frac{\partial}{\partial z}\left[\frac{1}{\rho_0}\frac{\partial}{\partial z}(\rho_0 w)\right] + \rho_0 \frac{\partial \eta}{\partial x} - \rho_0 \frac{\partial \xi}{\partial y}$$

where c is the speed of sound wave.

Since our interest is in the near-equilibrium state, we can replace the wave equation by a diffusion equation that shares the same equilibrium solution,

$$\mu \frac{\partial w}{\partial t} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) w + \frac{\partial}{\partial z}\left[\frac{1}{\rho_0}\frac{\partial}{\partial z}(\rho_0 w)\right] + \rho_0 \frac{\partial \eta}{\partial x} - \rho_0 \frac{\partial \xi}{\partial y}$$

where μ defines the time scale for adjustment toward anelastic balance.