Design of a Nonhydrostatic Atmospheric Model Based on a Generalized Vertical Coordinate

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Introduction

The isentropic system of equations has particular advantages in numerical modeling of weather and climate. Among these is the elimination of the vertical velocity in adiabatic flow, which greatly reduces the numerical errors associated with vertical advection. Also, vertical resolution is enhanced in regions of high static stability which leads to better resolving of features such as the tropopause boundary. However, the extreme isentropic overturning that can occur in fine-scale atmospheric motion presents a challenge to nonhydrostatic modeling with the isentropic vertical coordinate.

We have built a nonhydrostatic model based on the hybrid vertical coordinate of Konor and Arakawa (1997) which is a terrain-following Eulerian σ coordinate near the surface and transitions to the quasi-Lagrangian θ coordinate with height. We incorporated adaptive grid techniques into the handling of the coordinate surfaces in order to allow isentropic overturning and negative static stabilities to occur while maintaining coordinate monotonicity and spatial smoothness of the model surfaces.

Here we present results from two-dimensional mountain wave experiments. The first is a small-amplitude wave simulation which illustrates the physically intuitive quasi-Lagrangian representation of vertical momentum transport with the isentropic vertical coordinate. The second is a simulation of the 11 January 1972 Boulder, Colorado downslope windstorm which features large-amplitude wave breaking. The results show the benefits of the isentropic coordinate in terms of representing layers with high static stability and reducing numerical dispersion error of vertical tracer transport.

The vertical coordinate

Following Konor and Arakawa (1997) we define the vertical coordinate as a prescribed function of height and potential temperature. Since our model is nonhydrostatic we use geometric height instead of pressure as the height metric.

The vertical coordinate (η) is defined as:

$$\eta = F(\theta, \sigma) = f(\sigma) + g(\sigma)\,\theta, \qquad (1)$$

where $\sigma = \dfrac{z - z_S}{z_T - z_S}$, z_S = surface height, and z_T = model top height.

The transition of η from terrain-following to potential temperature coordinates requires:

$$g(\sigma) \to 0 \quad \text{as } \sigma \to 0,$$

$$f(\sigma) \to 0 \text{ and } g(\sigma) \to 1 \quad \text{as } \sigma \to 1.$$

Vertical momentum transport:

σ versus hybrid vertical coordinates

Small-amplitude mountain wave experiment

Model initialization:
- Initially isothermal (T = 287 K)
- Uniform zonal flow (u = 20 m s^{-1})

Terrain characteristics:
- "Witch of agnesi" curve
- Mountain height = 10 m
- Mountain half-width = 2 km

Grid spacing:
- Δx = 200 m
- $\Delta z \cong$ 250 m

Zonally-averaged zonal momentum equation in η coordinates:

$$\frac{\partial}{\partial t}\overline{u} = \frac{1}{\overline{m}}\frac{\partial}{\partial \eta}\left[\overline{p'\frac{\partial z'}{\partial x}} - \overline{(m\dot{\eta})'u'}\right]$$

Divergence of vertical momentum flux

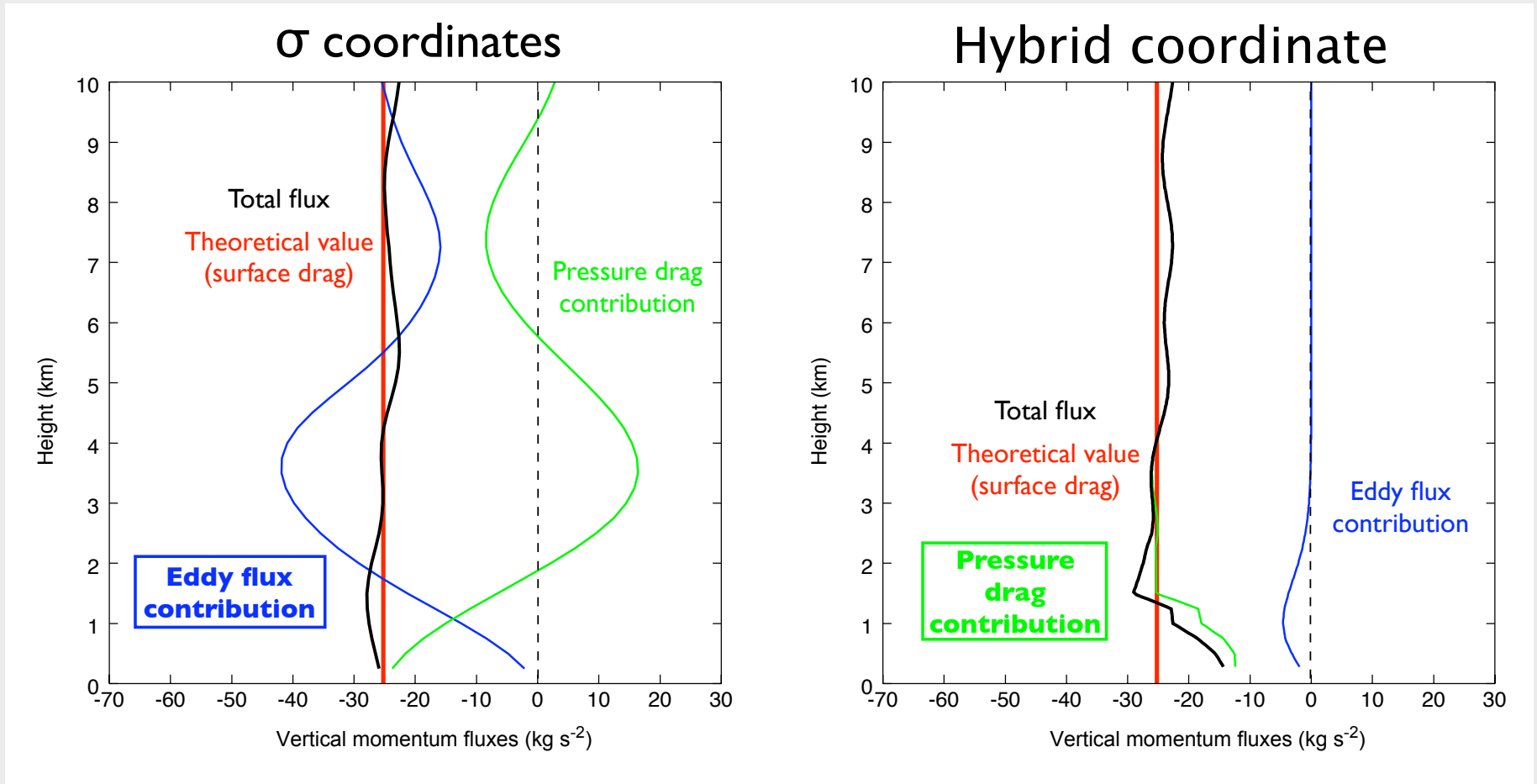


Figure 1: Vertical profiles of vertical momentum flux at steady-state (t = 1.11 hours).

Incorporation of an adaptive grid

In most of the regions of the model, the generalized vertical velocity $\dot{\eta}$ is diagnosed to maintain the functional relationship of σ and θ given by Equation 1 as in Konor and Arakawa (1997). We also incorporate adaptive grid techniques as in the nonhydrostatic models of Skamarock (1998), He (2002) and Zangl (2007). In regions of negative static stability and isentropic overturning, coordinate smoothing in the horizontal and vertical is applied and an alternative method of diagnosing the generalized vertical velocity is used as shown in Figure 2.

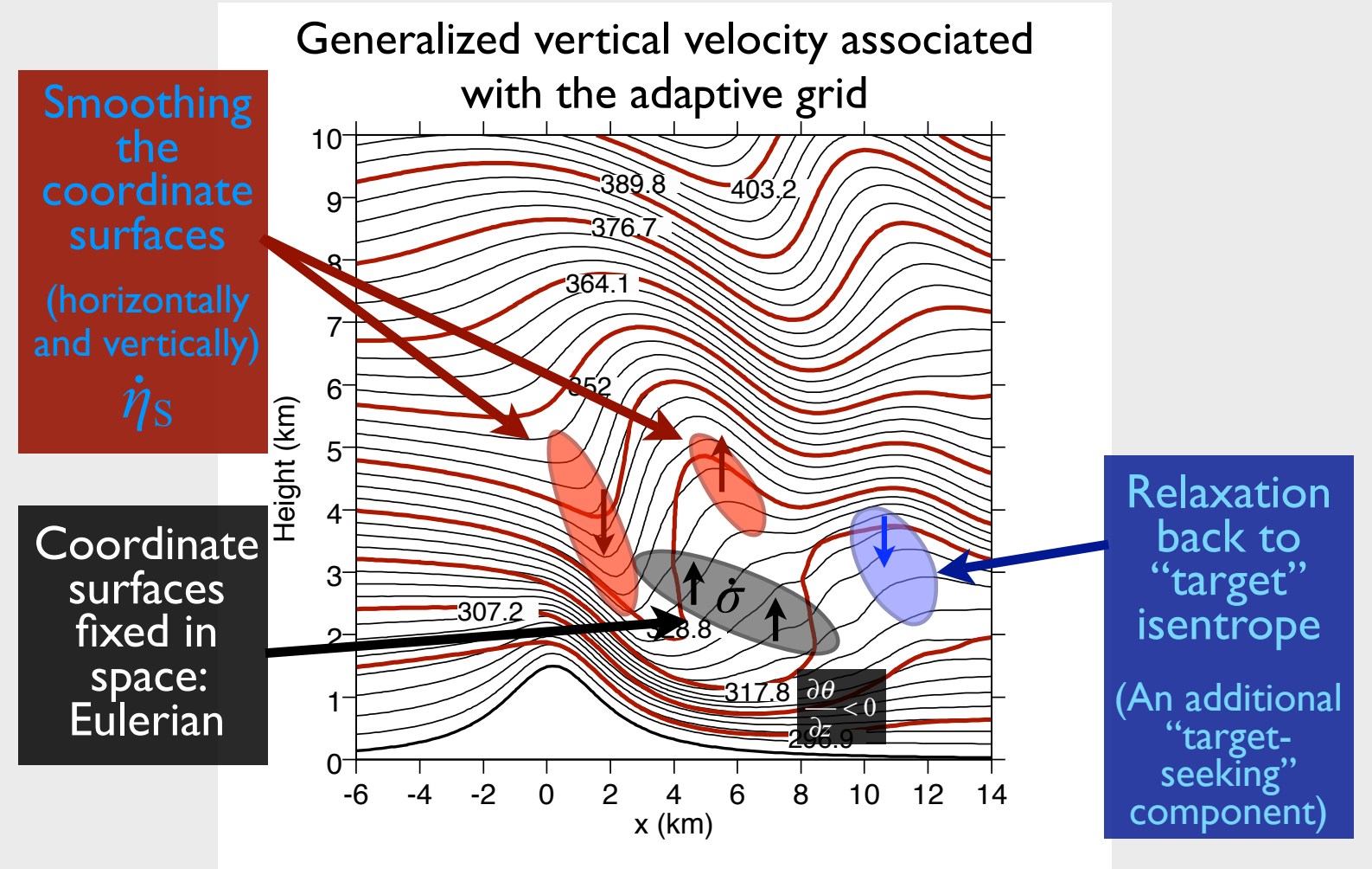


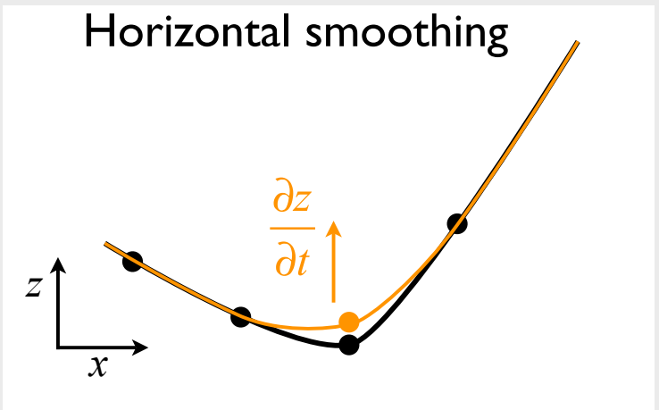
Figure 2: Regions where adaptive grid techniques are applied (ovals). Isentropic surfaces shown by black curves, coordinate surfaces by red curves.

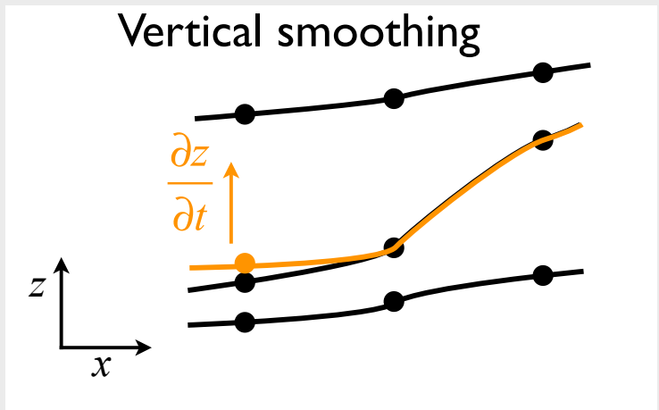
Starting point for diagnosing generalized vertical velocity:

$$\left(\frac{\partial}{\partial t}\right)_{\eta} F(\theta,\sigma) = \underbrace{\frac{\eta - F(\theta,\sigma)}{\tau}}_{\substack{\text{Relaxation of}\\ F(\theta,\sigma)\text{ back to}\\ \text{target }\eta}} - \underbrace{\dot{\eta}_S \frac{\partial F}{\partial \eta}}_{\substack{\text{Coordinate}\\ \text{smoothing}\\ \text{term}}}$$

"Smoothing" component of the generalized vertical velocity:

$$\dot{\eta}_S = -\left[\left(\frac{\partial z}{\partial t}\right)_{\text{smoothing},h} + \left(\frac{\partial z}{\partial t}\right)_{\text{smoothing},v}\right]\frac{\partial \eta}{\partial z}$$





- Horizontal smoothing

$$\left(\frac{\partial z}{\partial t}\right)_{\text{smoothing},h} \propto \text{diffusion term proportional to } \nabla^4 z$$

- Vertical smoothing

$$\left(\frac{\partial z}{\partial t}\right)_{\text{smoothing},v} \propto \text{diffusion term proportional to } \partial^2 z/\partial \eta^2$$

Simulation of the 11 January 1972 Boulder, Colorado Downslope Windstorm

A much-studied meteorological event is the 11 January 1972 Boulder, Colorado windstorm which produced surfaces winds in excess of 100 m.p.h. and caused extensive property damage. Such windstorms involve mountain wave amplification and wave breaking which vertically redistribute zonal momentum. The following plots compare three model simulations of the windstorm -- the "SIGMA500" case is a high vertical resolution σ-coordinate run with 500 levels in the lowest 25 km; the "SIGMA125" case is a "baseline" vertical resolution σ-coordinate run with 125 levels in the lowest 25 km; and the "HYBRID125" case is a hybrid vertical coordinate run, also with 125 levels in the lowest 25 km. It can be seen that the hybrid-coordinate model is able to capture the wave breaking, and produces improved results over the σ coordinate with the same number of levels, in terms of resolving layers with high static stability. The hybrid coordinate also results in less dispersion error associated with tracer transport compared to both the "baseline" and high vertical resolution σ coordinate model runs. The experimental setup follows Doyle et al (2000). All plots are at 3 hours simulation time unless otherwise noted.

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Acknowledgments

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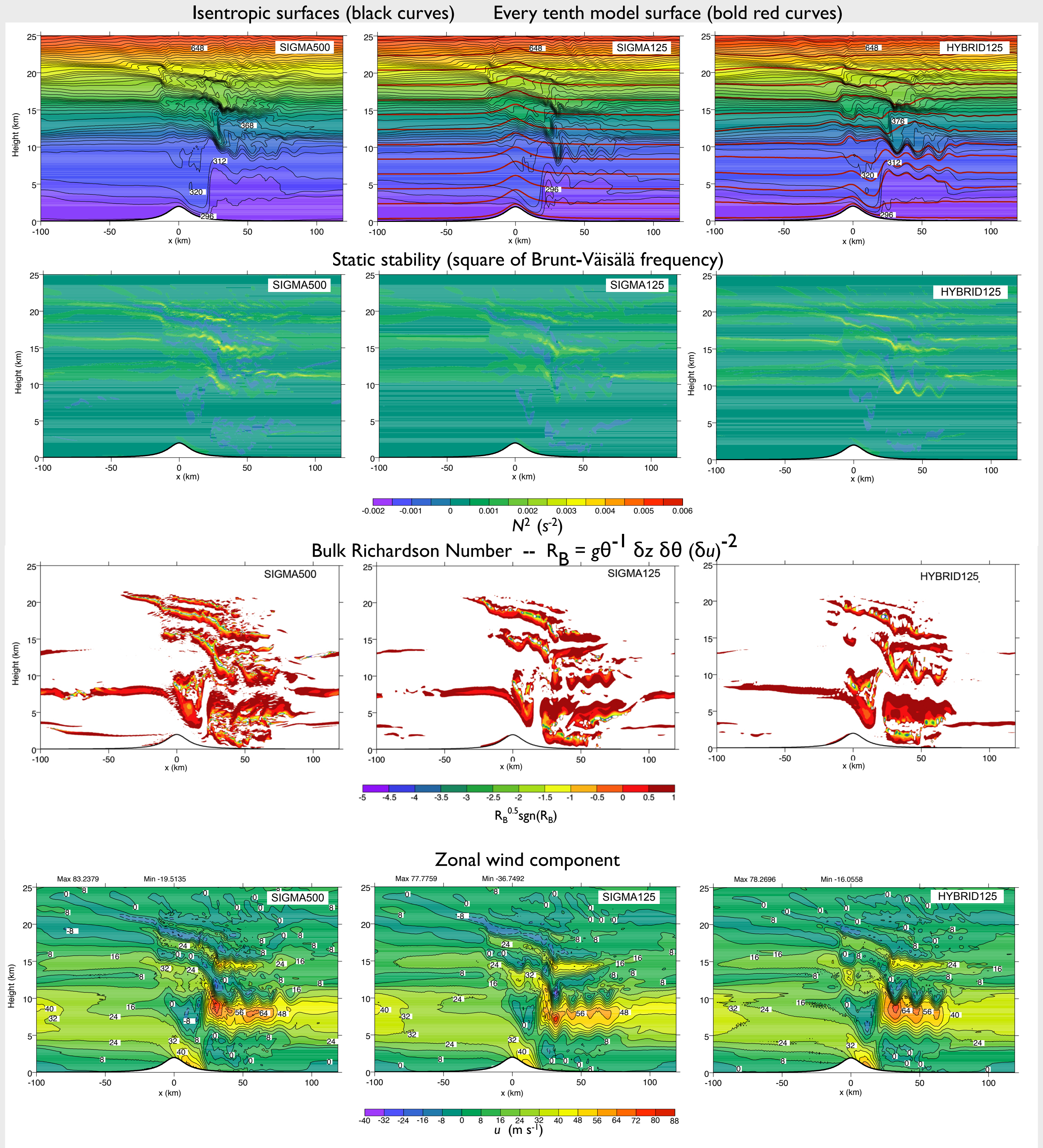

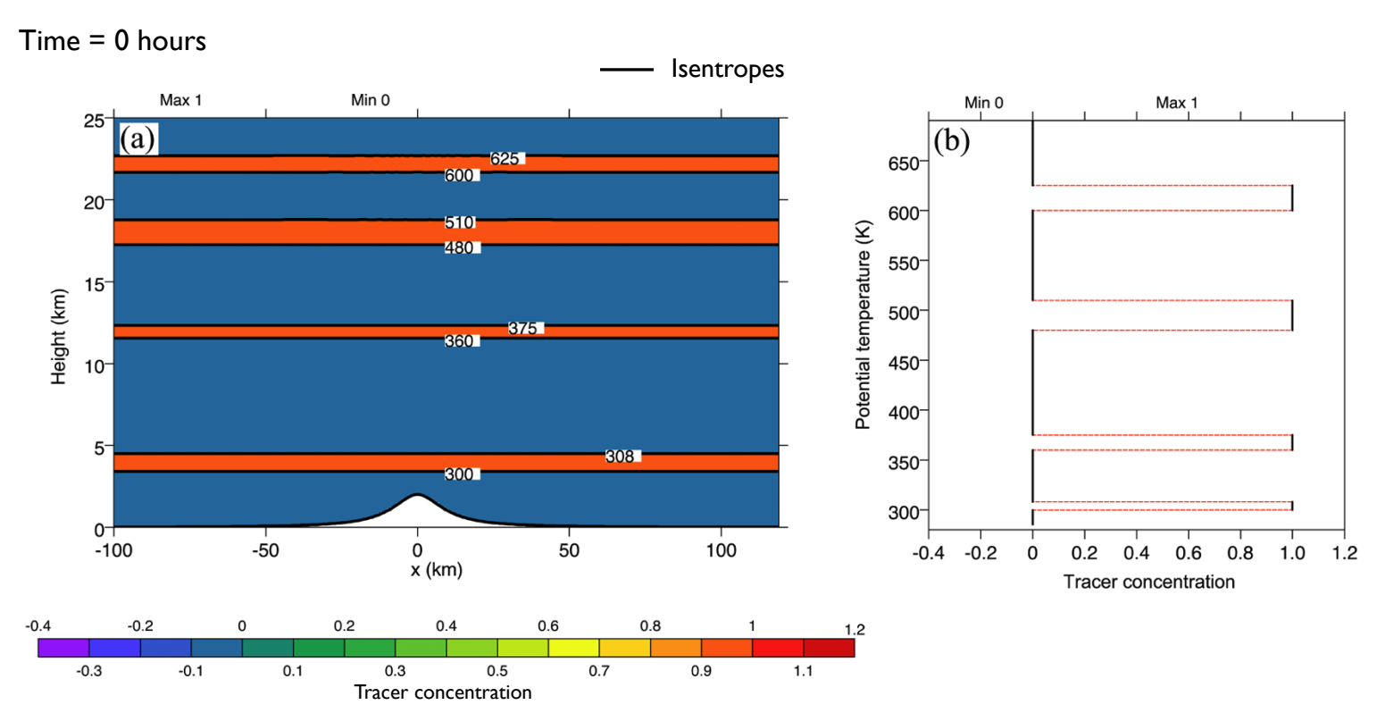

Passive tracer initialization:
Spatial distribution shown on left, scatter plot on right (red curves represent continuous solution for all times).