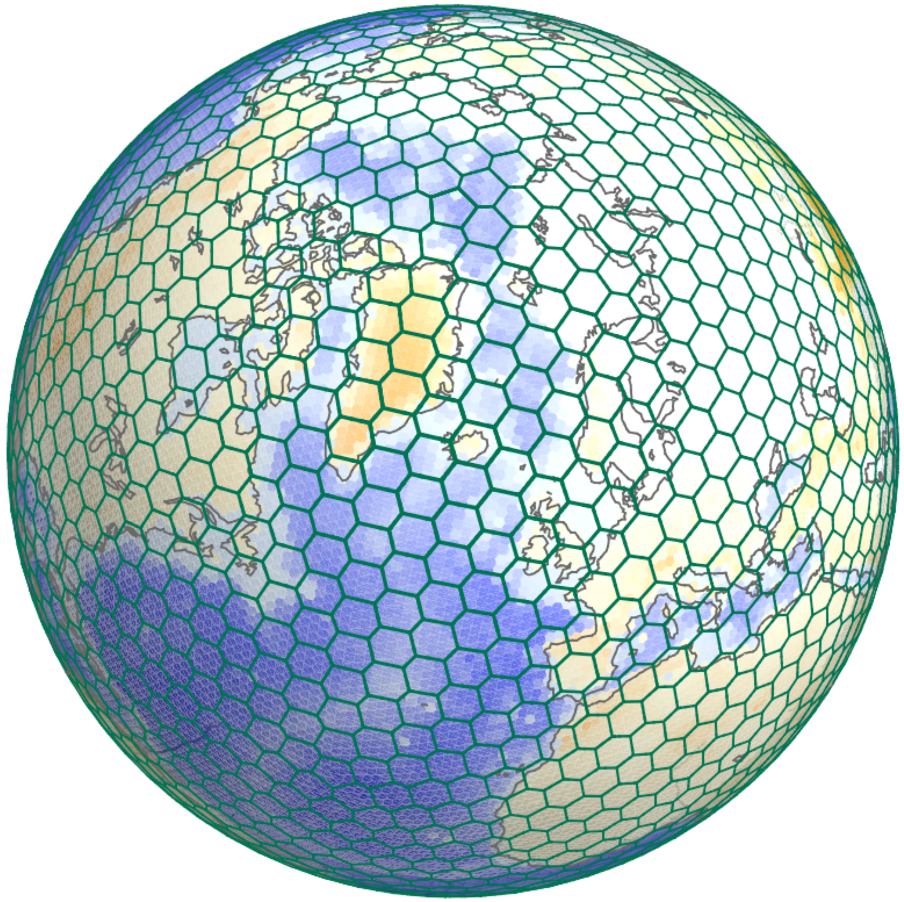
Icosahedral grid

- This grid provides approximately homogeneous and isotropic resolution over the sphere.
- There are 12 pentagons and N-12 hexagonal cells.
- Resolution as a function of cells:

(N) number of cells	global grid point spacing (km)
(6) 40,962	125.1
(7) 163,342	62.55
(8) 655,362	31.27
(9) 2,621,442	15.64
(10) 10,485,762	7.819
(11) 41,943,042	3.909

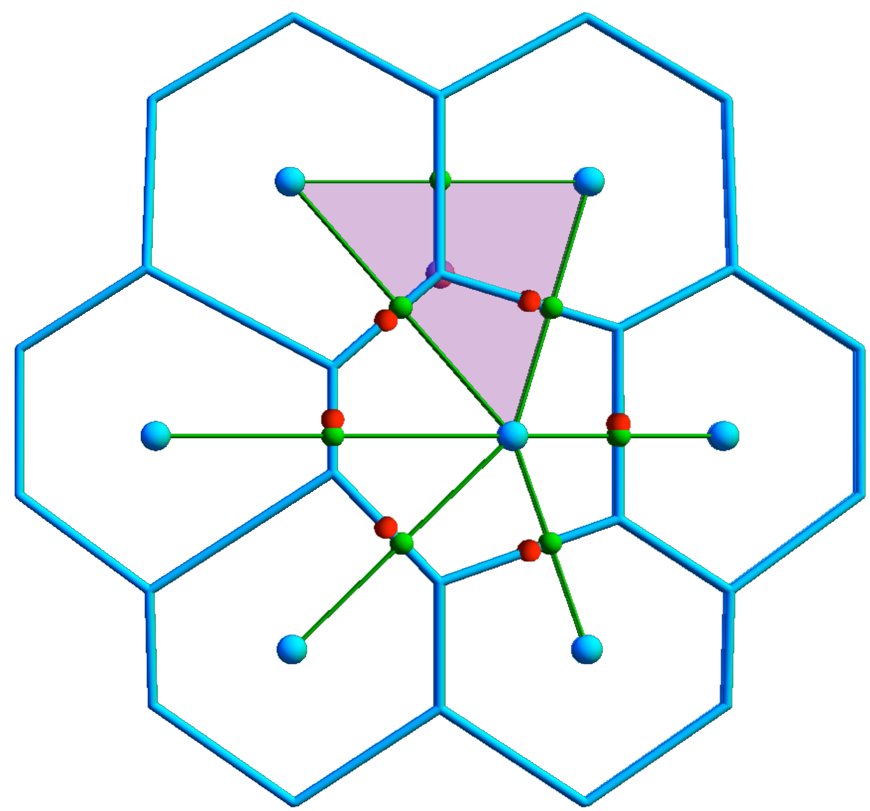


Resurrection of the tweaking algorithm

- The tweaking algorithm optimizes properties of the icosahedral grid which improve the finite-difference operators
- It lives again

What the tweaking algorithm does

- The Voronoi corner is defined as the point equidistant from surrounding grid points.
  - The cell wall bisects the line connecting grid points
  - The cell wall is perpendicular to the line connecting grid points



- The line connecting grid points does not necessarily bisect the cell wall.
- The tweaking algorithm positions all grid points so that red points are coincident with green points (or at least it does the best it can)

What the tweaking algorithm does

- The *goodness* of particular configuration of points can be expressed as a cost function:

$$F = \sum_{n=1}^{all\ cells} \sum_{i=1}^{cell\ walls} \left(goodness\ of\ wall \right)_{n,i}^{2}$$

- Solved using quasi-Newton methods:

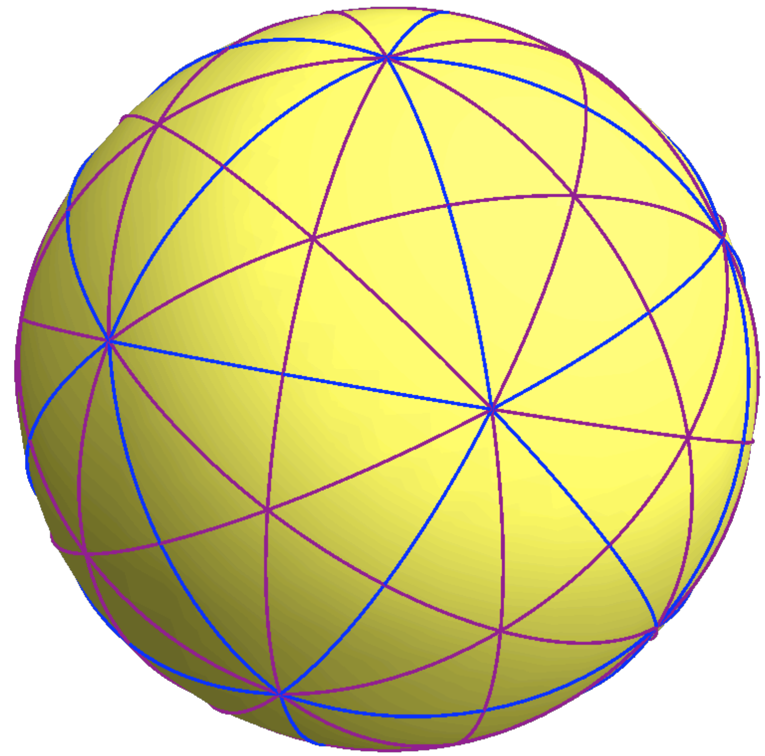
$$\underset{\mathbf{x}}{\text{minimize}}\, F\left(x_1, x_2, \ldots, x_m\right)$$

- Other grid properties can be optimized using different cost functions

Reduce the problem size

- The number of independent variables can be reduced by imposing symmetries intrinsic to the grid
- Great circles can partition the sphere into 120 unique triangular subdomains
- Number of independent variables:

grid resolution	number of independent variables
(8) 655,362 (31.27km)	8,192
(9) 2,621,442 (15.64km)	32,768
(10) 10,485,762 (7.819km)	131,072
(11) 41,943,042 (3.909km)	524,288
(12) 167,772,162 (1.955km)	2,097,152



The VVM-sphere. What has been done.

- Many key components of the VVM-sphere have been constructed and tested off-line.
- Now they are ready to be assembled into a model.

The list of components so far...

- ☑ *3D-elliptic solver.* Solve for vertical velocity w using $\boldsymbol{\eta}$.
- ☑ *Advection of $\boldsymbol{\eta}$.* Predict the horizontal component of vorticity $\boldsymbol{\eta}$.
- ☑ *Advection of ζ_T defined at cell corners.*. Predict the vertical component of vorticity ζ_T in the top layer.
- ☑ *2D-elliptic solver defined at cell corners.* Diagnose horizontal wind $\mathbf{v}_T$ in the top layer using ζ_T and δ_T.
- ☑ *Advection of θ defined at cell centers.* Predict potential temperature θ.

A simple test for the 3D multigrid

- Prescribed analytic potential temperature perturbation

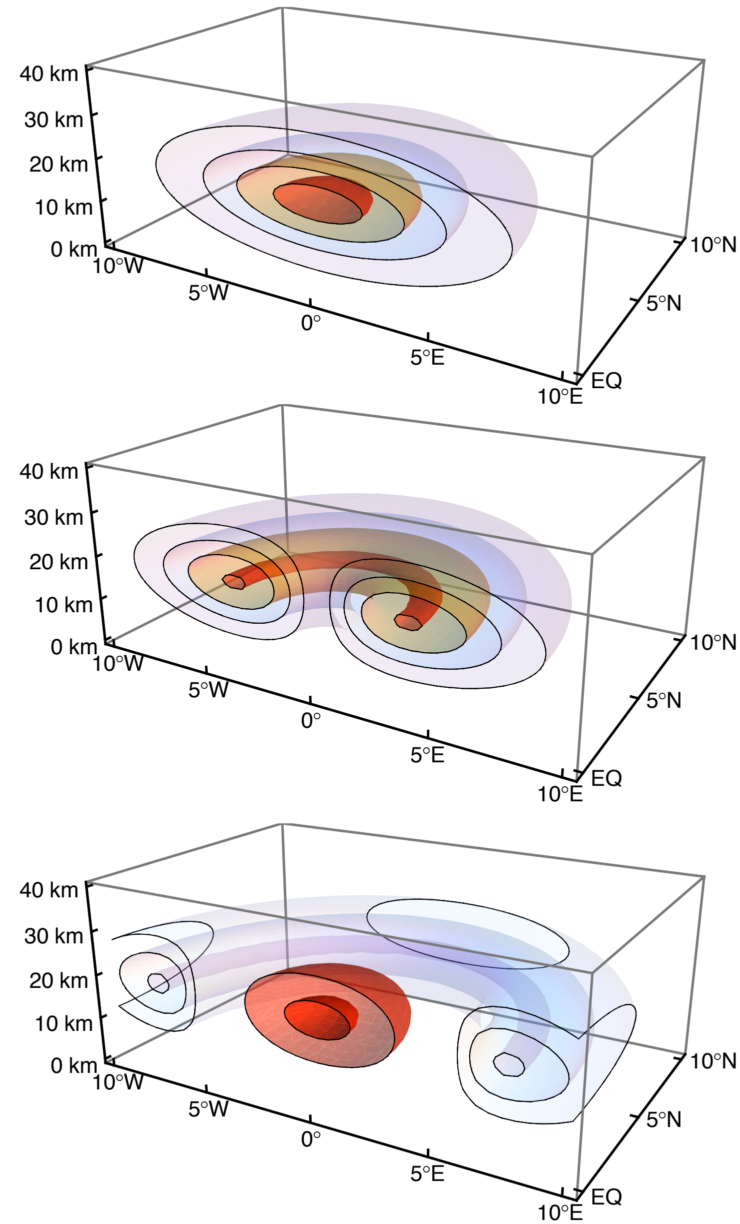
$$B = g\,\frac{\theta'}{\theta_0\,(=300\,K)}$$

- Implied tendency in the horizontal vorticity equation

$$\boldsymbol{\eta} = -\Delta t\;\mathbf{k} \times \nabla_H B$$

- Taking the curl forms the right-hand-side of the w equation.

$$\nabla^2 w + \frac{\partial}{\partial z}\left[\frac{1}{\rho}\frac{\partial}{\partial z}(\rho w)\right] = -\mathbf{k}\cdot\nabla_H \times \boldsymbol{\eta}$$



Scaling test of 3D-multigrid on Jaguar

- The **NCCS Cray XT5** with 181,00 cores
- 20 V-cycles
- 80 layers

Time (s)		Number of cores			
		5120	10240	20480	40960
Grid resolution	41,943,042 **(11)** (3.909km)	8.652	4.535	3.071	2.377
	167,772,162 **(12)** (1.955km)	35.567	18.071	8.885	5.646
	671,088,642 **(13)** (0.977km)	insufficient memory	79.85	36.137	18.903

Convergence of 3D-multigrid

- The following sequence of equations can be used to test convergence of the 3D-multigrid

- Solve for the vertical velocity. The *goodness* of the solution is determined by the number of multigrid cycles.

$$\nabla_H^2 w + \frac{\partial}{\partial z}\left[\frac{1}{\rho_0}\frac{\partial}{\partial z}\left(\rho_0 w\right)\right] = -\mathbf{k}\bullet\nabla_H \times \boldsymbol{\eta}$$

- Solve for the stream function in the top layer. Set the horizontal velocity in the top layer.

$$\mathbf{v}_T \equiv \mathbf{k} \times \nabla_T \chi_T \quad \text{where} \quad \nabla_H^2 \chi_T = -\left[\frac{1}{\rho_0}\frac{\partial}{\partial z}\left(\rho_0 w\right)\right]$$

- Vertically integrate to get horizontal velocity throughout the remainder of the column.

$$\mathbf{v} \equiv \mathbf{v}_T + \int_{z_T}^{z}\left[\nabla_H w - \mathbf{k} \times \boldsymbol{\eta}\right] dz$$

- With the resulting w and $\mathbf{v}$, the anelastic continuity equation should be zero.

$$\nabla \cdot \left(\rho_0 \mathbf{v}\right) + \frac{\partial}{\partial z}\left(\rho_0 w\right) = 0$$

Convergence of 3D-multigrid

- With 40962 cells and 40 layers

iterations of 3D-multigrid	inf-norm of the continuity equation
2	1.09E-09
4	2.81E-15
8	2.26E-17
20	2.27E-17

Horizontal component of vorticity

Predict the horizontal component of vorticity $\boldsymbol{\eta}$.

$$\frac{\partial \tilde{\boldsymbol{\eta}}}{\partial t} = -\overbrace{\nabla_H \cdot (\tilde{\boldsymbol{\eta}}\mathbf{v})}^{\textit{horizontal advection}} - \overbrace{\frac{\partial}{\partial z}(\tilde{\boldsymbol{\eta}} w)}^{\textit{vertical advection}} + \overbrace{\tilde{\boldsymbol{\eta}} \cdot \nabla_H \mathbf{v}}^{\textit{stretching}} + \overbrace{\tilde{\xi} \frac{\partial \mathbf{v}}{\partial z}}^{\textit{tilting}} + \mathbf{v}\nabla_H \cdot (2\boldsymbol{\Omega}_H) - \overbrace{\mathbf{k} \times \nabla_H B}^{\textit{buoyancy}} + \overbrace{\mathbf{k} \times \left(\frac{\partial \mathbf{F}_\mathbf{v}}{\partial z} - \nabla_H F_w \right)}^{\textit{turbulent-drag force}}$$

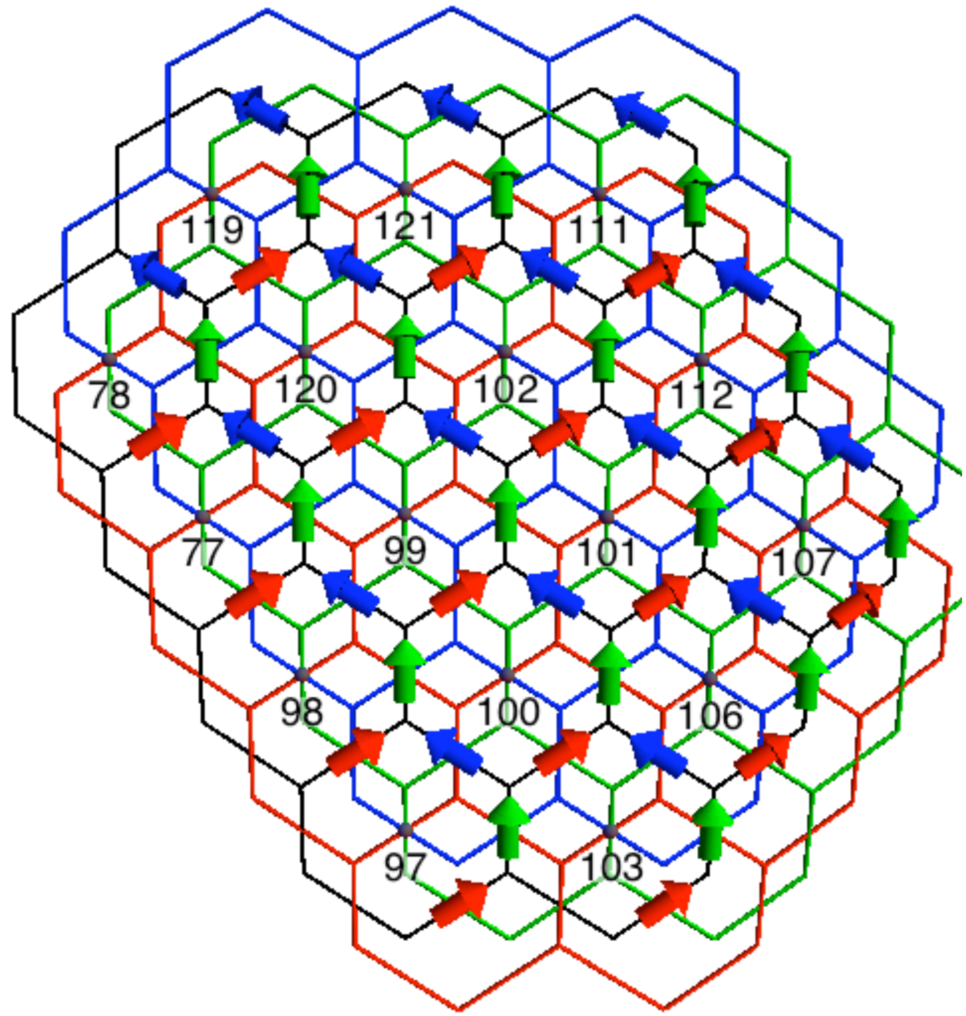
- The **horizontal advection** term is technically complicated. The horizontal component of vorticity is defined at cell edges and requires a *control volume grid defined* at cell edges.
- The remaining terms are less complicated and have been thoroughly described by Celal. Vertical advection, stretching and tilting have been coded but not tested.

Advection of the horizontal component of vorticity

✦ Follow Celal's algorithm as much as possible.

✦ The construction of advection control volumes goes as follows:

- Three *flavors* of vorticity are associated with each cell
- Identify all the edges of a particular type
- construct a grid of Voronoi cells for each flavor

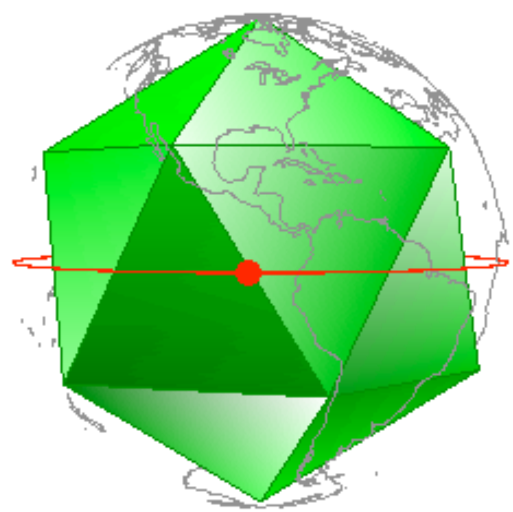


✦ Some small modifications are required near the pentagons.

✦ Since auxiliary grids are similar to the original grid. We can re-use the advection algorithms developed for the original grid.

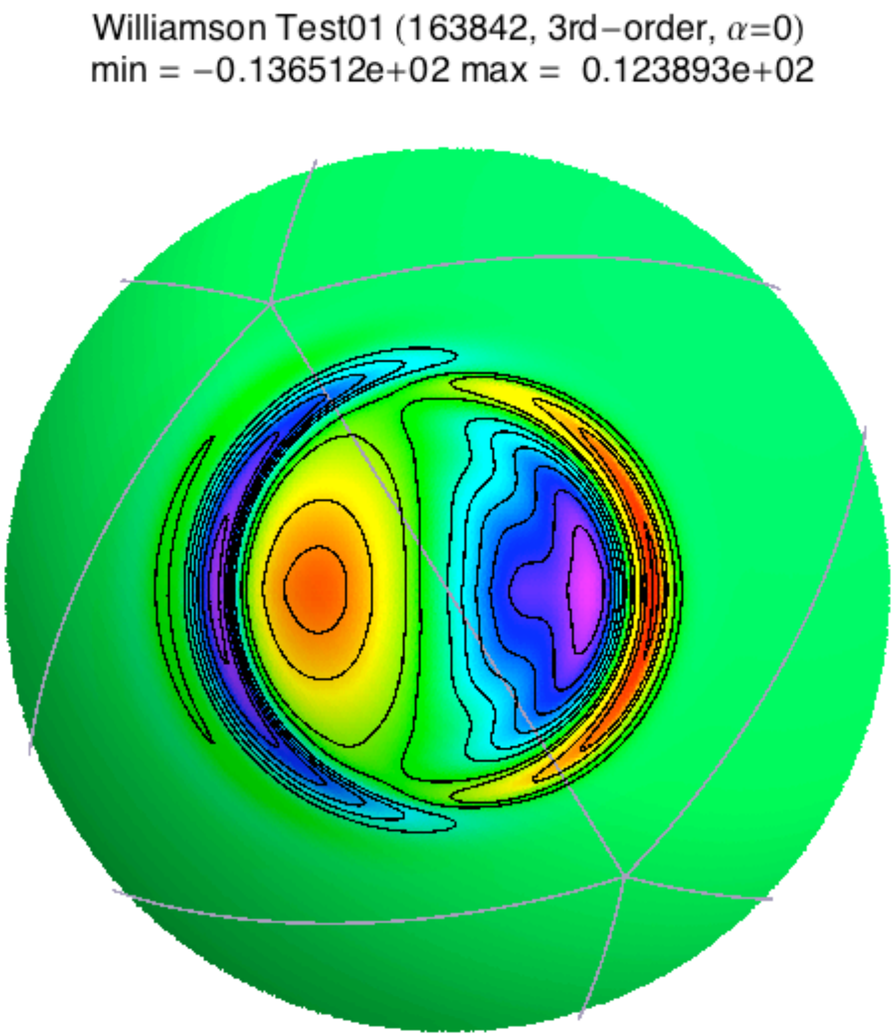
Scalar Advection Test

- ✦ Williamson *et al.* (1992)
  - A Cosine bell is advected once around the sphere in 12 days.
  - The initial bell is 1000 m.
  - Prescribed winds are approximately 38 m/s.
  - The initial pattern should remain unchanged.
- ✦ The path of advection is along a great circle. Regions near pentagons are the most anisotropic. Choose two paths:
  - **Path 1**. Along the equator. No pentagons.
  - **Path 2.** A path chosen to encounter four pentagons.

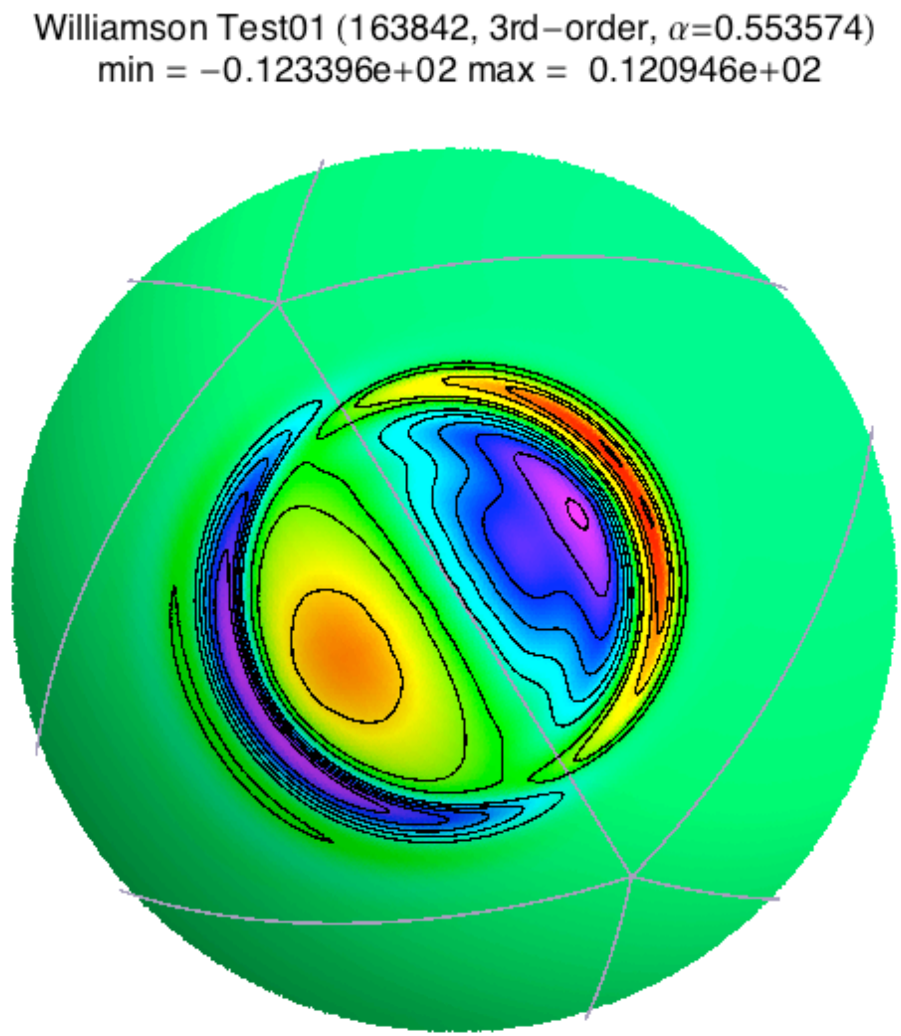


Scalar Advection Test

- Difference plots show numerical - analytic. (2 m contour interval)

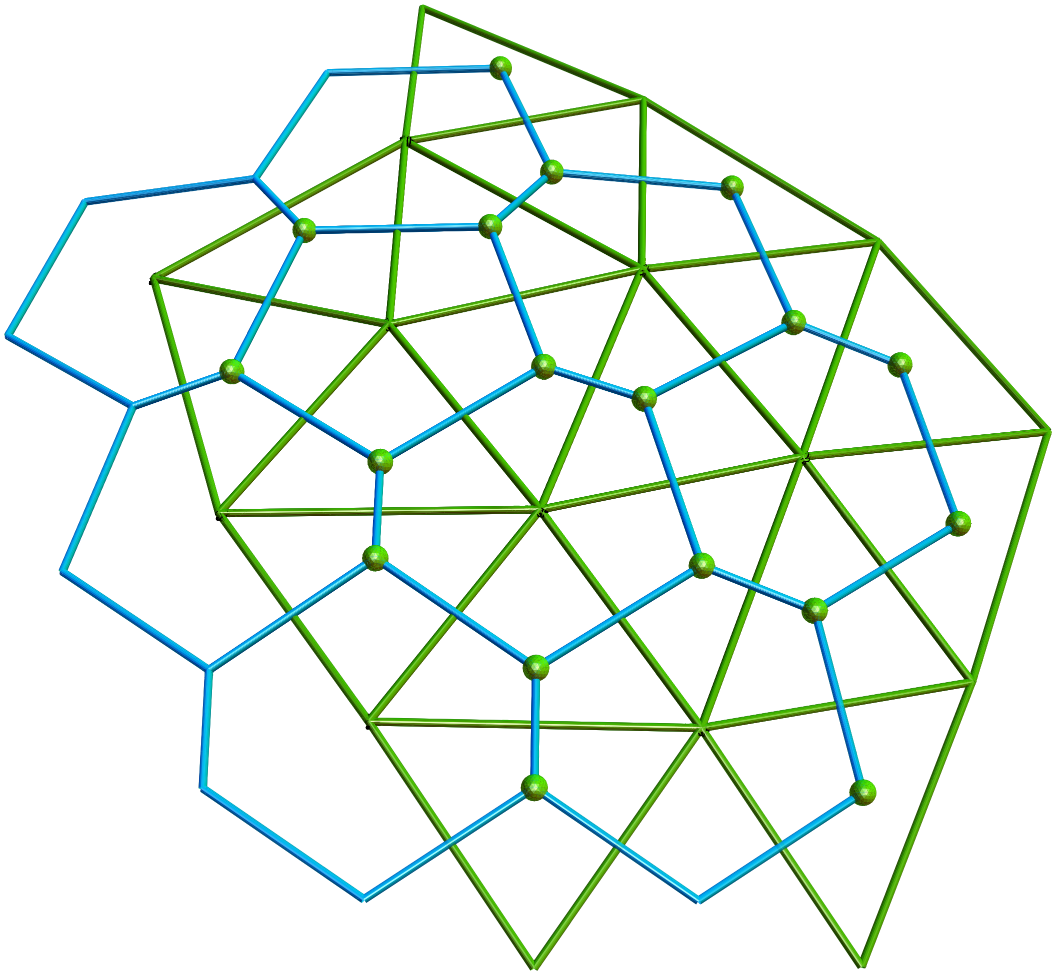


path 1



path 2

The dual of the hexagon-pentagon grid

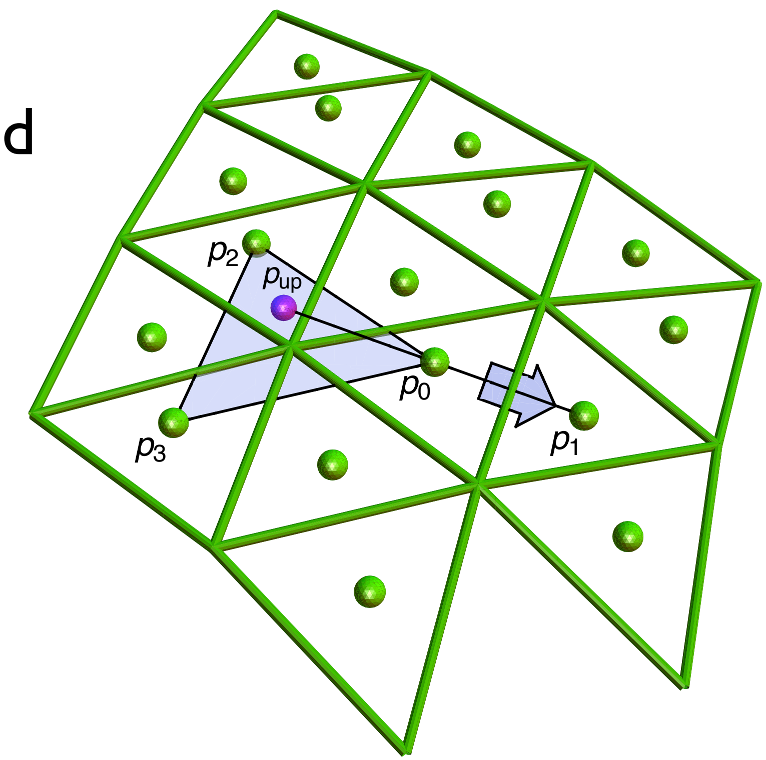


Advection of ζ

Predict the vertical component of vorticity ζ_T in the top layer.

$$\frac{\partial \tilde{\xi}_T}{\partial t} = -\overbrace{\nabla_H \cdot \left(\tilde{\xi}\mathbf{v}\right)_T}^{horizontal\ advection} - \overbrace{\left[\frac{\partial}{\partial z}\left(\tilde{\xi} w\right)\right]_T}^{vertical\ advection} + \overbrace{\left[\tilde{\boldsymbol{\eta}} \cdot \nabla_H w\right]_T}^{tilting} + \overbrace{\left[\tilde{\xi}\frac{\partial w}{\partial z}\right]_T}^{stretching} + \left[w \nabla_H \cdot \left(2\mathbf{\Omega}_H\right)\right]_T + \overbrace{\left[\mathbf{k} \cdot \nabla_H \times \mathbf{F}_\mathbf{v}\right]_T}^{turbulent-drag\ force}$$

- ✦ The 3rd-order upstream biased horizontal advection has been coded and tested.
- ✦ The remaining terms are pretty straightforward. However, they still need to be coded.



2D-elliptic solver defined at cell corners

Diagnose horizontal wind $\mathbf{v}_T$ in the top layer using ζ_T and δ_T. Integrate to get $\mathbf{v}$ in the column.

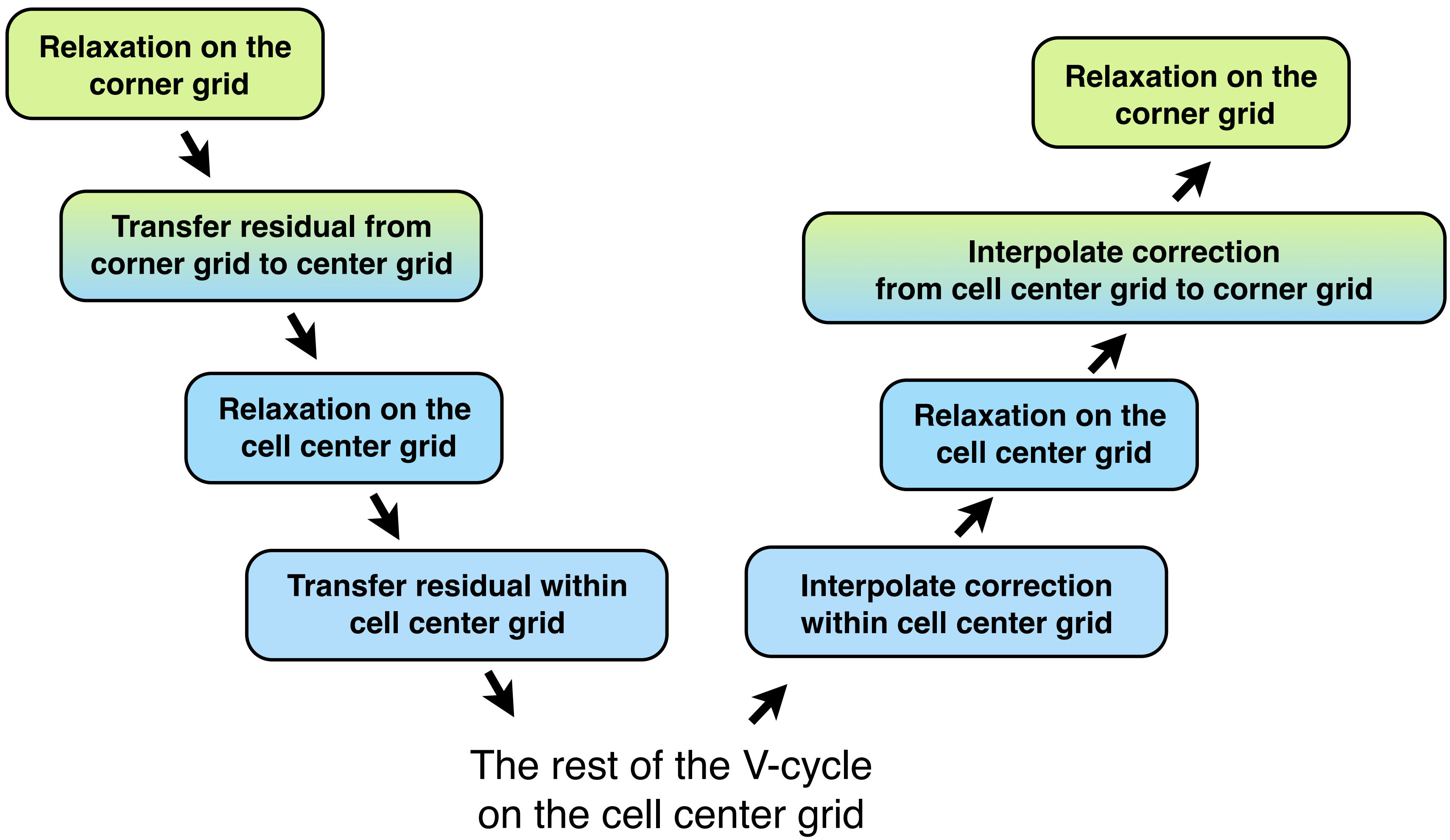
$$\mathbf{v}_T \equiv \mathbf{k} \times \nabla_H \psi_T + \nabla_H \chi_T$$

where $\nabla_H^2 \psi_T = \zeta_T$ and $\nabla_H^2 \chi_T = \delta_T = \nabla_H \cdot \mathbf{v}_T = -\left[\frac{1}{\rho_0} \frac{\partial}{\partial z} (\rho_0 w) \right]$

- ✦ This will require solving a pair of 2D elliptic equations where the horizontal vorticity is defined on the *corner* grid.

2D-elliptic solver defined at cell corners

- This can be done with a modification of the 2D-elliptic solver defined at cell centers.
- Schematically the algorithm looks like this:



Advection of θ defined at cell centers

Predict potential temperature θ.

$$\frac{\partial(\rho_0\theta)}{\partial t} = -\overbrace{\nabla_H \cdot (\rho_0\theta\mathbf{v})}^{\textit{horizontal advection}} - \overbrace{\frac{\partial}{\partial z}(\rho_0\theta w)}^{\textit{vertical advection}} + \overbrace{\frac{\rho_0 Q}{\Pi}}^{\textit{heating}}$$

- ✦ The horizontal component of wind is well positioned for the advection.
- ✦ The 3rd-order upstream biased horizontal advection based at cell centers is coded and has been throughly tested.