Modeling of Diabatically Forced Tropical Circulations

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Introduction and Motivation

- Over recent decades, interest in tropical meteorology has increased due to its impact in extratropical weather forecasting.
- Also, as more forecasting centers used global models for numerical weather prediction, it became apparent that the tropical atmosphere was difficult to simulate.
- A.E. Gill was one of the major contributors to tropical meteorology, as he studied the major features of tropical circulations, such as coastal lows, Walker circulation, and the Hadley circulation.

Steady State Heat-Induced Tropical Circulation

- Gill (1980) constructed a model to explain the basic features of the response of the tropical atmosphere to diabatic heating.
- Much of the heating in the tropics is found over Africa, South America, and Indonesian region, which raises questions on the result of heating a limited area centered near the equator on tropical circulation.
- If heating is applied to a steady-state atmosphere and is small enough to use linear theory, then the response can be modeled in terms of equatorially trapped waves.
- The heating would create eastward Kelvin waves and slowly moving westward planetary waves, leading to an east-west asymmetry in tropical circulation.

Geostrophic Adjustment in Heat Induced Tropical Circulation

- Heckley and Gill (1984) addressed the problem of geostrophic adjustment in the tropics. Given a sudden heat source, how does the circulation adjust to a steady state and how long does the adjustment take?
- The long time scale for the adjustment has implications in numerical weather prediction. If the adjustment time is long, considerable care must be taken to ensure that the initial data from which the forecast is made are in exact balance with the model's forcing.
- If the initial data for a global forecast model is not in balance with the model's forcing, the tropical flow may have significantly changed before balance can be established.

The Basic Model

- We study the response of the tropical atmosphere to a given distribution of heating using the linearized shallow water equations, including transient effects.
- With dissipative processes and using the long wave approximation, the equations take the form:

$$\frac{\partial u}{\partial t}-\frac{1}{2}yv=\frac{-\partial p}{\partial x}$$

$$\frac{\partial v}{\partial t}+\frac{1}{2}yu=\frac{-\partial p}{\partial y}$$

$$\frac{\partial p}{\partial t}+\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=Q$$

$$w=\frac{\partial p}{\partial t}+Q$$

$$q=p+u$$

$$r=p-u$$

$$(u,p)\rightarrow(q,r)$$

$$\frac{\partial}{\partial t}\rightarrow\epsilon+\frac{\partial}{\partial t}$$

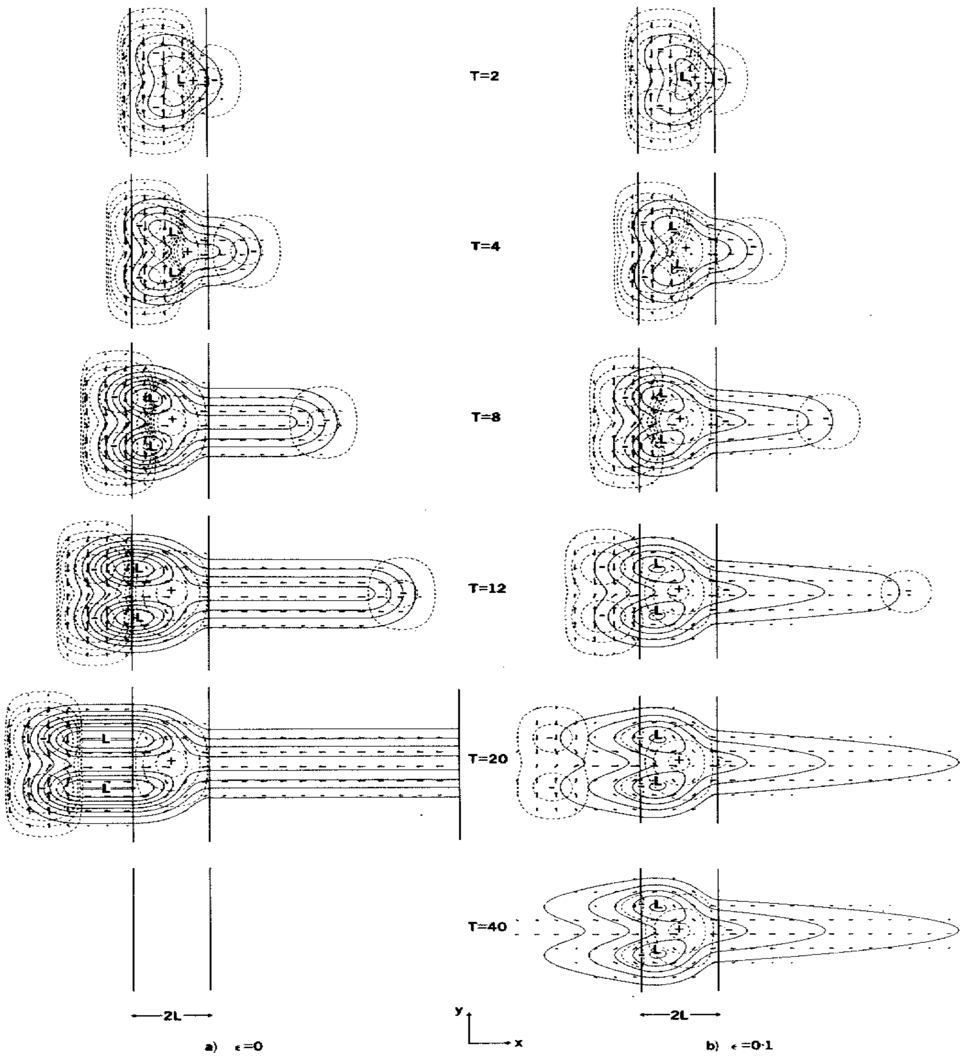
$$\frac{\partial q}{\partial t}+\epsilon q+\frac{\partial q}{\partial x}+\frac{\partial v}{\partial y}-\frac{1}{2}yv=-Q$$

$$\frac{\partial r}{\partial t}+\epsilon r+\frac{\partial r}{\partial x}+\frac{\partial v}{\partial y}+\frac{1}{2}yv=-Q$$

$$\frac{\partial q}{\partial y}+\frac{1}{2}yq+\frac{\partial r}{\partial y}-\frac{1}{2}yv=-Q$$

$$w=\frac{\partial p}{\partial t}+\epsilon p+Q$$

Case 1: Symmetric Forcing

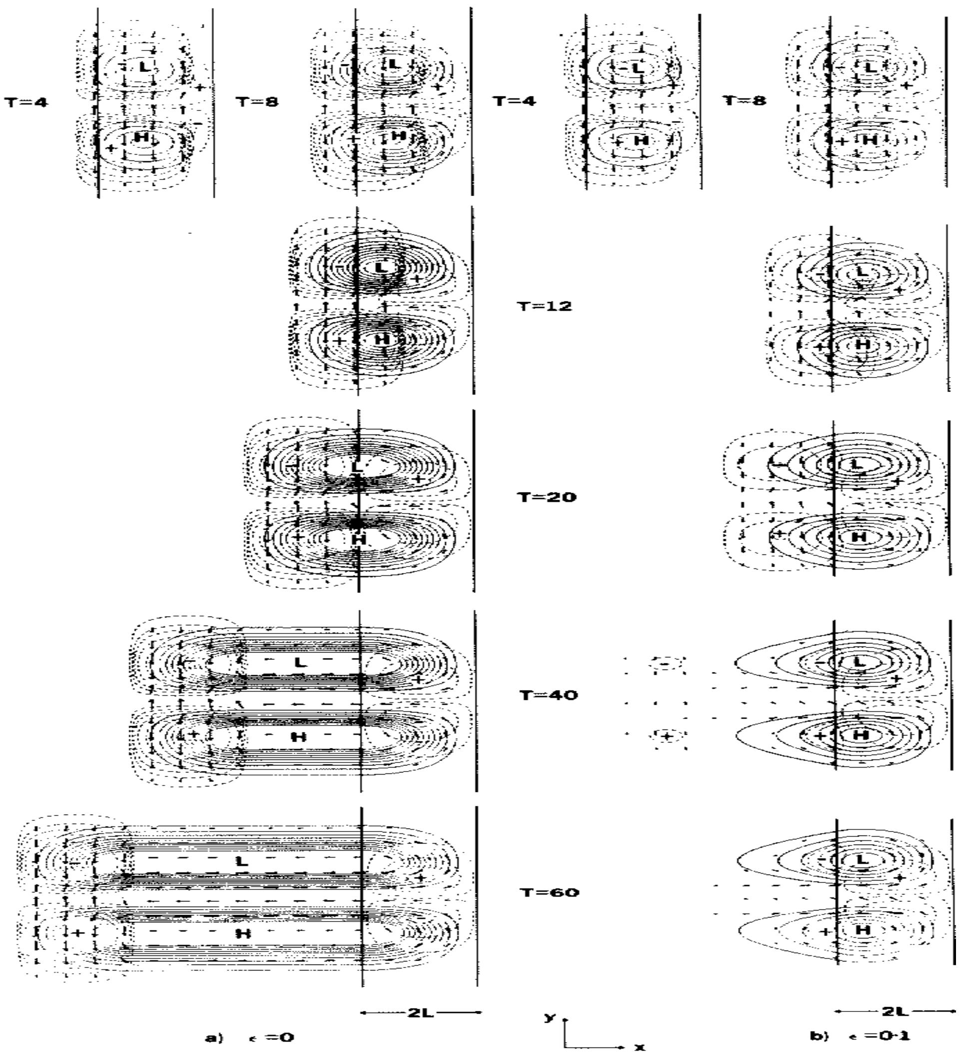


(a): Flow with ε=0 for 10 days (t=40)

(b): Flow with ε=0.1 for 10 days (t=40)

- The n=0 mode corresponds to the Kelvin wave and the n=1 mode corresponds to a planetary wave.
- The amplitude of the disturbance in the wave front decays exponentially in time for the Kelvin and planetary wave.
- The circulation consists of ascent within the heating region, descent to the east of the heating region.
- The dissipative effect determines the horizontal scale of the flow field, but has no effect on propagation speed of the waves

Case 2: Asymmetric Forcing



(a): Flow with ε=0 for 10 days (t=40)

(b): Flow with ε=0.1 for 10 days (t=40)

- The n=1 mode corresponds to the mixed planetary-gravity wave and the n=2 mode corresponds to the planetary wave.
- The flow is initially dominated by cross-equatorial circulation, but damping weakens this circulation until the two circulations are independent.
- The steady state takes a long time to become established (about 10 days).

Conclusions

- The circulation consists of ascent within the heating region, descent to the east of the heating region decaying exponentially, low-level easterlies, and upper-level westerlies.
- For symmetric forcing, the effect of damping causes descending air to spread over the entire region between the wave front and the heating region and causes the descent in the wave front to decay exponentially.
- For asymmetric forcing, flow at low-levels within the wave front is characterized by a non-divergent southerly cross-equatorial flow, northerlies to the north of the equator and southerlies to the south.
- The long time scale for geostrophic adjustment may explain the presence of large errors in the planetary-scale waves in early global forecast models.

The Effects on Heat-Induced Tropical Circulation in a Mean Wind

- In studying the basic features of heat-induced tropical circulation, the effects of the zonal mean wind was not considered.
- The effect of the barotropic part of the mean flow is important in understanding heat-induced flows since they will have shear in the horizontal and vertical.
- Using linear theory, Phlips and Gill (1987) studied the response of the tropical atmosphere to heating distribution in the presence of a zonal mean wind.
- Since the role and modeling of damping processes was poorly understood, they also examined the circulation for a range of damping time constants.

The Model

- Starting from Gill (1980), the shallow-water equations are linearized about a uniform mean zonal wind Δ, giving:

$$\left(\frac{\partial}{\partial t}+\Delta\frac{\partial}{\partial x}+\epsilon\right)q+\frac{\partial q}{\partial x}+\left(\frac{\partial}{\partial y}-\frac{1}{2}y\right)v=-Q$$

$$\left(\frac{\partial}{\partial t}+\Delta\frac{\partial}{\partial x}+\epsilon\right)r-\frac{\partial r}{\partial x}+\left(\frac{\partial}{\partial y}+\frac{1}{2}y\right)v=-Q \qquad \Delta=\frac{U}{NH}$$

$$2\left(\frac{\partial}{\partial t}+\Delta\frac{\partial}{\partial x}+\epsilon\right)v+\left(\frac{\partial}{\partial y}+\frac{1}{2}y\right)q+\left(\frac{\partial}{\partial y}-\frac{1}{2}y\right)r=0$$

- This system of equations produces a Kelvin mode (n = -1), mixed mode (n = 0), and planetary modes (n ≥ 1).
- This model produces 5 parameters: the length K^{-1}, width α^{-1}, and latitude y_0 of the heating region, the damping time constant ε, and the magnitude of the mean wind Δ.

Case 1: Changes in Dissipation Constant ε

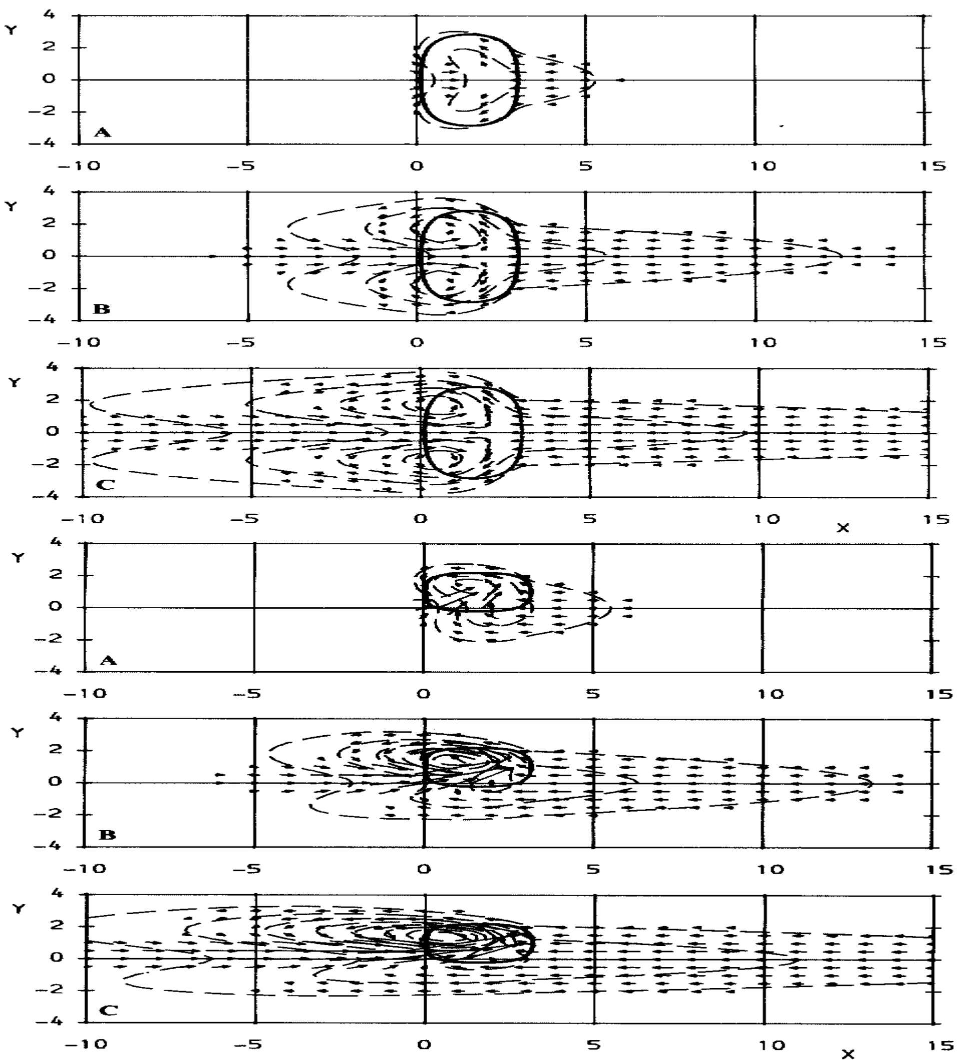


Figure 1: Tropical response with y_0=0; ε = (0.3, 0.1, 0.05) for (A, B, C)

Figure 2: Same as Figure 1 except y_0=1

- For heating centered on the equator
  - Westerly inflow is stronger and narrower than easterly inflow
  - Lowest pressure at the equator on the eastern side
- For heating centered off the equator
  - Strongest inflow is from the southwest and is north of the equator
  - Lowest pressure is on the eastern side and poleward of the heating region

Case 2: Zonal and Meridional Changes in Heating Region

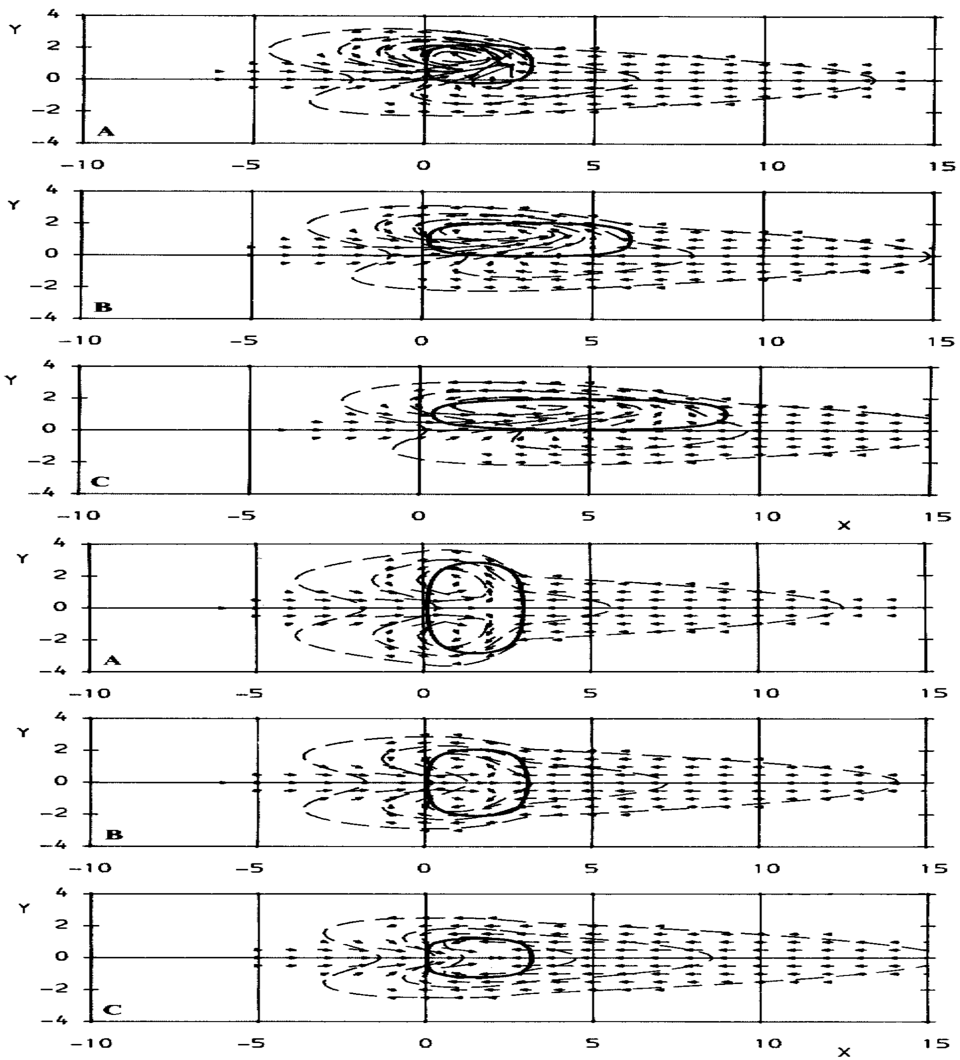


Figure 1: Tropical response with $y_0=1$; $\alpha = (1, 1.5, 3)$ for (A, B, C)

Figure 2: Tropical response with $y_0=0$; $K = (1, 0.5, 0.33)$ for (A, B, C)

- For zonal changes in the heating region
  - The length of the flow region does not change significantly
  - The meridional velocities decrease rapidly as the length of the heating region increases
- For meridional changes in the heating region
  - As the width of the heating region is reduced, the poleward flow into the region decreases

Case 3: Changes in Zonal Mean Wind Δ-Centered on the Equator

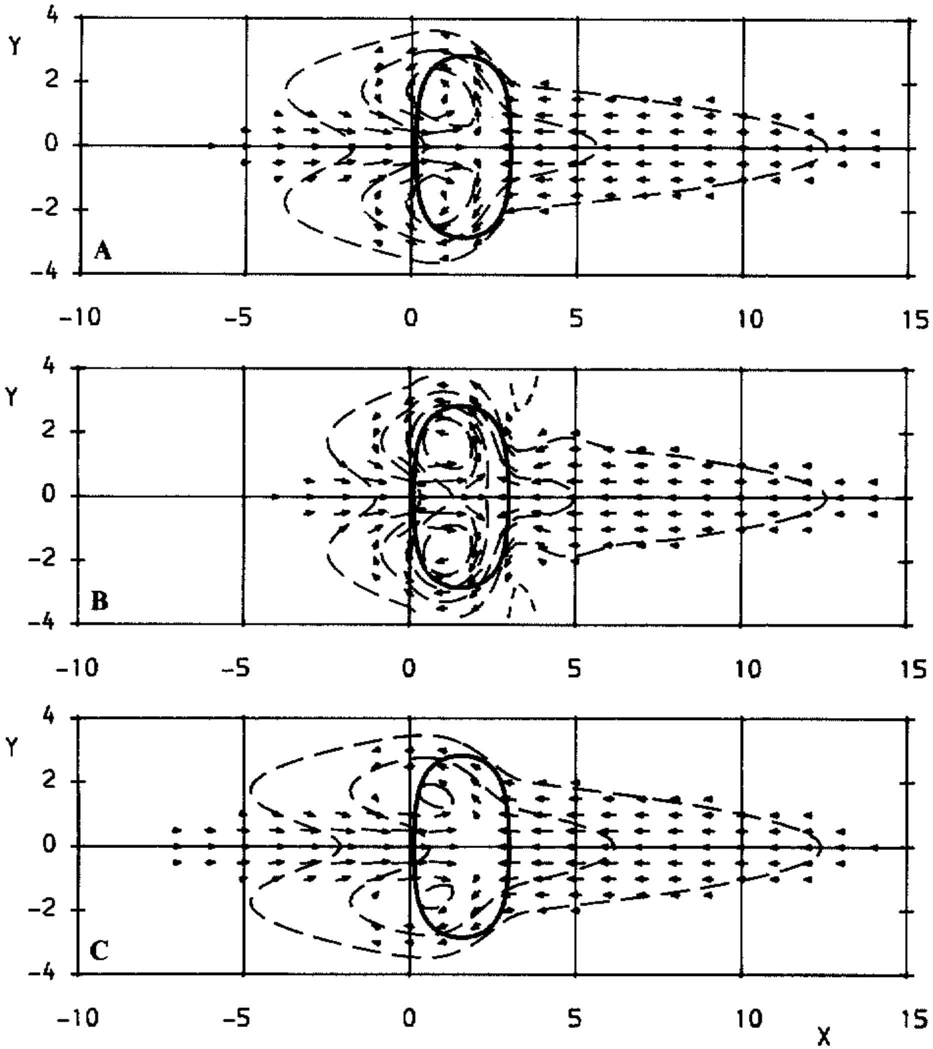
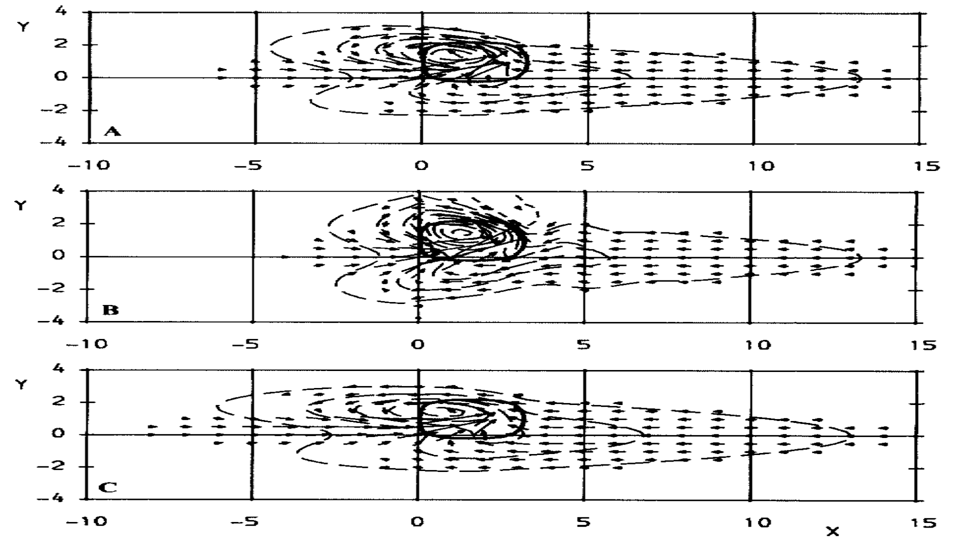


Figure: Tropical response with y_0=0; Δ = (0, 0.1, -0.1) for (A,B,C).

- In a westerly mean wind Δ = 0.1
  - The cyclonic circulation is stronger.
  - The planetary waves are stronger, but decay faster.
  - Stationary planetary waves are imposed on the Kelvin wave
- In an easterly mean wind Δ = -0.1
  - Amplitude and decay rate of planetary waves are reduced

Case 4: Changes in Zonal Mean Wind Δ-Centered off the Equator



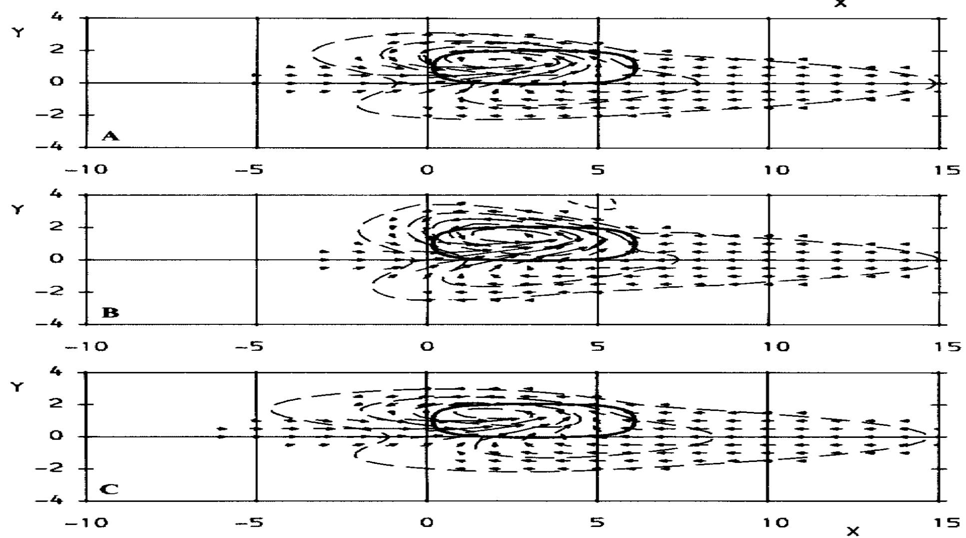
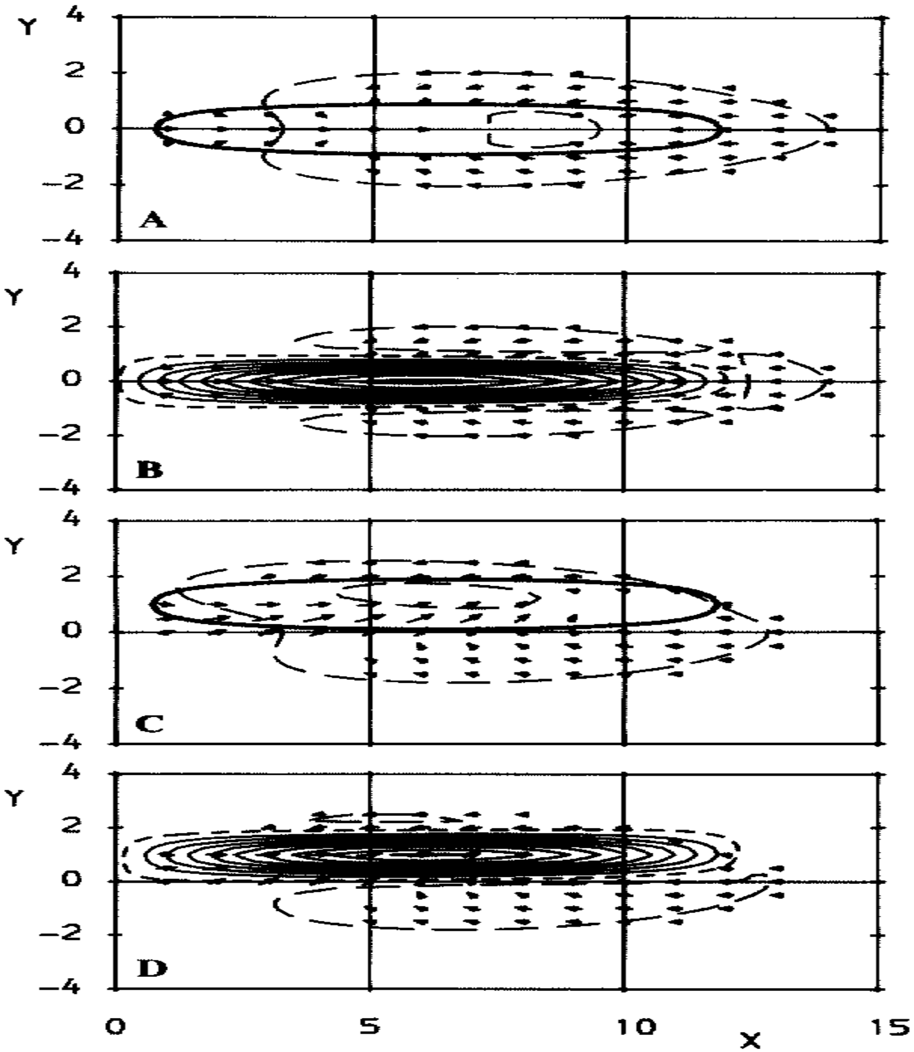


Figure 1: Tropical response with y_0=1 and K = 1; Δ = (0, 0.1, -0.1) for (A, B, C)

Figure 2: Same as Figure 1 except K = 0.5

- Amplitude and decay rate of the planetary modes are increased in a westerly mean wind.
- Strong flows are produced in Southern Hemisphere
- As the heating region increases, the effects of the westerly mean wind are milder, and short stationary waves are no longer visible.

Case 5: Strong Damping in Long Narrow Heating Region



A: Surface winds and pressure with K = 0.25, α = 3, ε = 0.2, and y_0 = 0.

B: Vertical velocity with the same parameters as A.

C: Same as A except y_0 = 1.

D: Same as B except y_0 = 1.

- For strong damping in a long, narrow heating region, 2D Hadley circulation is produced.
  - NE and SE trade winds to the north and south
  - As heating is moved northwards, SE trade winds turn into SW.
  - Strongest flows are equatorwards, sinking across the equator.

Conclusions

- In the presence of a mean wind, the response to deep heating still consists of Kelvin and planetary waves to leading order. The effect of mean wind is to change the speed of the waves, with corresponding changes in their amplitudes and decay rates.
- In the presence of a westerly mean wind, there are short stationary mixed and planetary waves, but their amplitudes are small compared with the long waves.
- In the presence of an easterly mean wind, the short stationary waves are replaced by inertial boundary layers at the edges of the heating region.

Nonlinear Effects of Heat-Induced Tropical Circulation

- Linear models of heat-induced tropical circulations has been useful in understanding the primary observed features of the circulation.
- However, large wind anomalies at the surface and tropopause are not small compared with the propagation speed of the waves, indicating the importance of nonlinearity.
- Gill and Phlips (1986) used the linear model of Gill (1980) as a starting point and examined how the solution is distorted as the amplitude of the forcing is increased.
- The linear solution already contains the parameter ε, which measures the importance of friction, so for nonlinear effects, another parameter δ is introduced to measure the ratio between the particle velocity and wave velocity.

The Model and Forms of Solutions

- In this study, the amplitude parameter is made much larger than the frictional parameter so that nonlinear effects dominate. The small-friction limit is then used as the first term in an expansion in powers of δ.
- The nonlinear solutions involve a harmonic part and a height-independent part where the latter generates a barotropic nonlinear correction to the linear correction and the former a second baroclinic component:

$$\delta = \frac{u_0}{C}$$

$$u = u_L + \delta u_{NL} \qquad u_{NL} = u_0 + u_2 \cos(2z)$$

$$v = v_L + \delta v_{NL} \qquad v_{NL} = v_0 + v_2 \cos(2z)$$

$$w = w_L + \delta w_{NL} \qquad w_{NL} = w_2 \sin(2z)$$

$$\phi = \phi_L + \delta \phi_{NL} \qquad \phi_{NL} = \phi_0 + \phi_2 \cos(2z)$$

$$\theta = \theta_L + \delta \theta_{NL} \qquad \theta_{NL} = \theta_2 \sin(2z)$$

Results of Nonlinear Modifications, 1

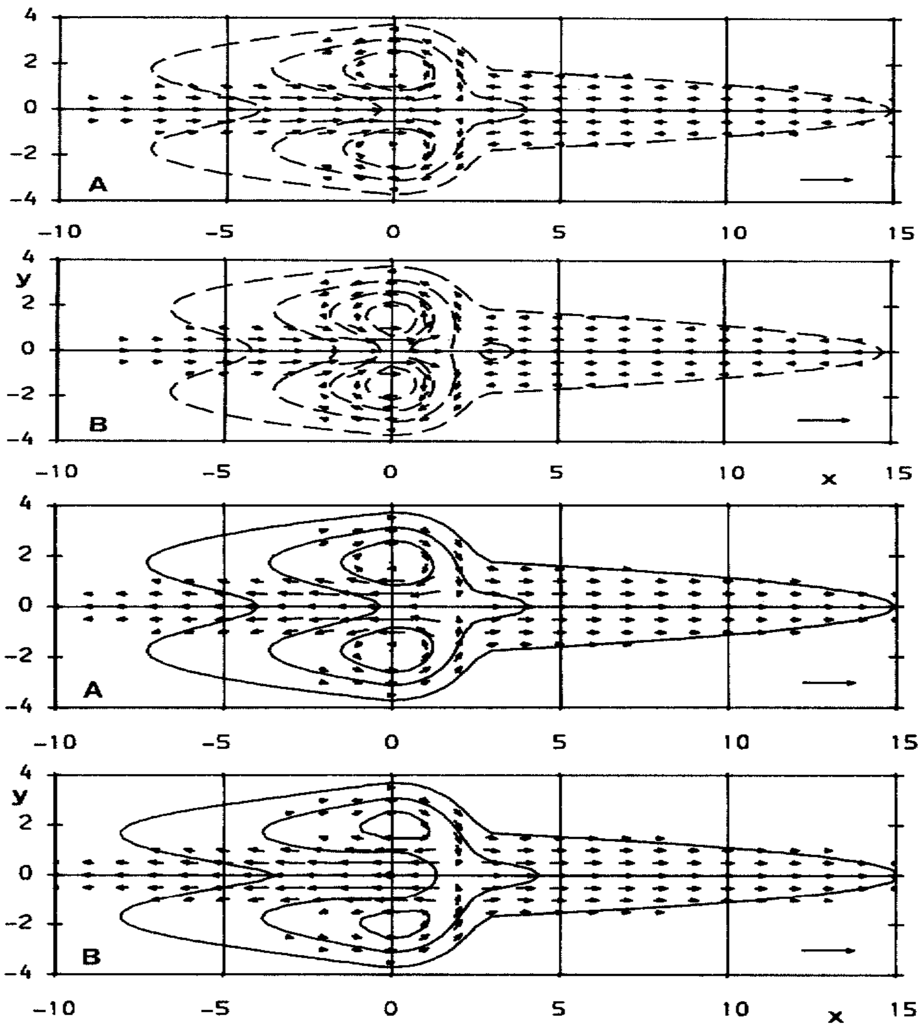
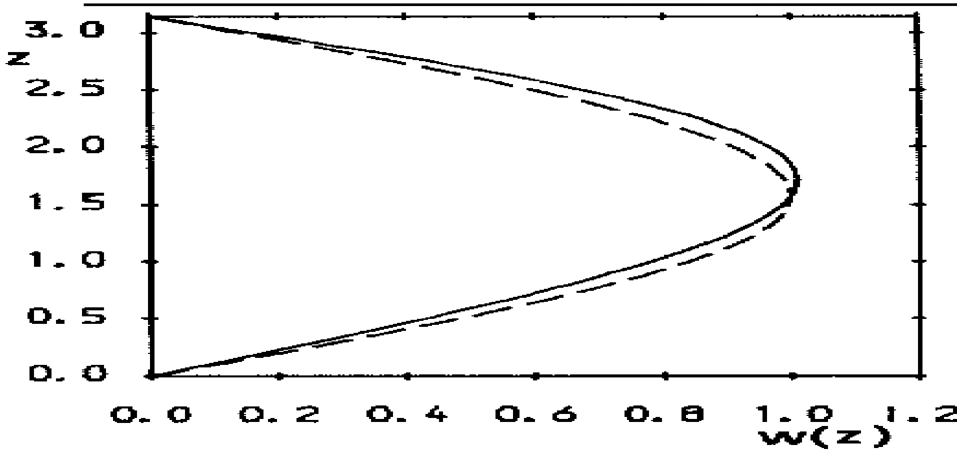


Figure 1: The flow at the surface with pressure contours (linear vs. nonlinear)

Figure 2: The flow at the tropopause with pressure contours (linear vs. nonlinear)

- Nonlinear corrections are significant only near the western end of the heating region and in the decay region immediately to the west.
- The twin cyclones at the ground are more intense and the westerly jet is stronger

Results of Nonlinear Modifications, 2



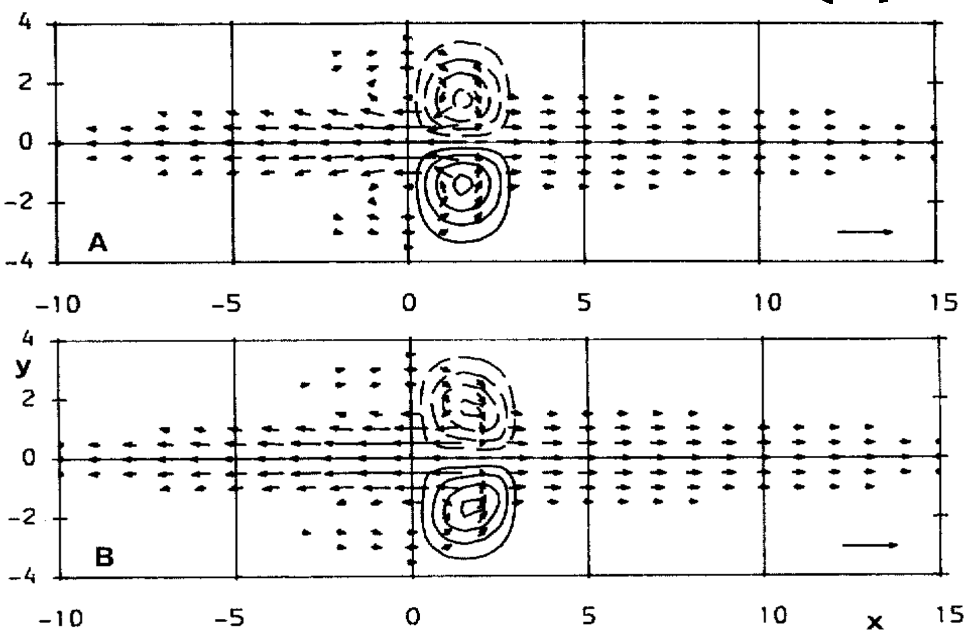


Figure 1: The vertical velocity in the heating region (linear vs. nonlinear)

Figure 2: Contours of absolute vorticity times divergence (linear vs. nonlinear)

- The divergence at upper levels is increased, and the low-level convergence is decreased.
- Largest changes in vorticity and divergence occur near the equator.

Conclusions

- The general effects of nonlinearity appear to be a modest distortion of the flow, even for large values of δ.
- The strongest nonlinear effect came through analysis of vorticity where the meridional gradient of absolute vorticity changed from its planetary value to zero at the western end of the heating region.
- For nonlinear modifications
  - there is more divergence at the upper boundary and less convergence at the lower boundary
  - the meridional velocity in the heating region shifted east at the upper boundary and west at the lower boundary
  - weaker easterlies and stronger westerlies at the upper boundary on the western side of the heating region
  - and low pressure at the upper boundary shifted to the east on the western side.

References

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