Balanced Vortices Generated by Ice in a Rotating Fluid: Theory and Laboratory Experiments

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Motivation

Balanced vortices are found in nature in the form of Meddies, Gulf Stream rings, and Southern Ocean eddies (Figure 1). These phenomena have been observed to play a fundamental role in the transport of warm water, salt, planktonic creatures, and even sound (e.g. Batten and Crawford 2005; Jian et al 2009). Understanding balanced vortices from a theoretical point of view, and being able to reproduce them experimentally, will provide a greater insight into what other effects they might have on the ocean and atmosphere.

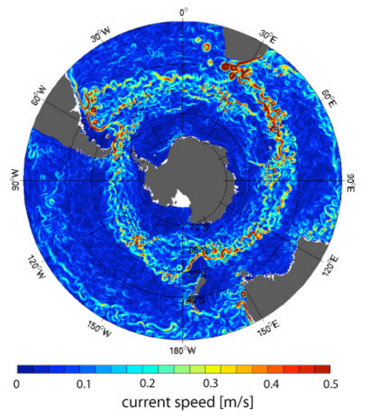
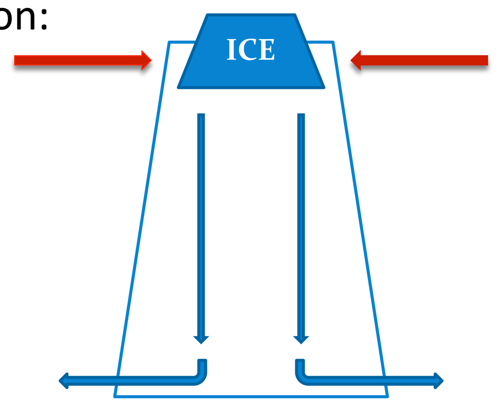


Figure 1. Surface current speed in the Southern Ocean calculated by the Southern Ocean State Estimate (Mazloff et al., submitted).

Theory

The life of a vortex consists of three major parts after it is introduced into a fluid in solid body rotation:

1. Spin up during ice melt
2. Balanced state after ice melt
3. Decay and spin down



The equation that governs during spin up is:

$$\frac{D}{Dt}\zeta \approx -f\nabla\cdot\vec{u}$$

where ζ is vorticity, f is the Coriolis parameter, and u is velocity.

The spin of the vortex is driven by the sinking and divergence of water cooled by the ice and then the warmer water converging toward the ice to replace the cold water (as shown above).

A vortex is considered balanced when the dominant balance is between the pressure gradient force and the Coriolis force. The equation for geostrophic balance governs this:

$$\vec{f}\times\vec{u} \approx -\frac{1}{\rho}\nabla p$$

Friction occurs at the bottom and side walls of the tank, but for this experiment it was considered that the effect of friction is relatively insignificant for the spin up and balanced stages of the vortex, though it does eventually dissipate the vortex.

Objectives

To develop a coherency between mathematical models of a balanced vortex and what is observed in ice-generated vorticies from tank experiments.

To determine the reproducible nature of the tank experiments represented in this study and to evaluate the strengths and weaknesses of the current setup.

Figure 2. Rotating tank with bottom dimensions of 16x16 inches.

Setup

For each experiment the tank was filled to a depth of 10cm using 20˚C tap water.

It was then left rotating until it achieved solid body rotation as determined using a paper dot and a particle tracker.

The ice was prepared using 1g of salt and 50mL of water in a cup with a 5cm radius.

Methodology

Visualization

To visualize the vortex in the water food coloring was mixed in with the ice before it was frozen.

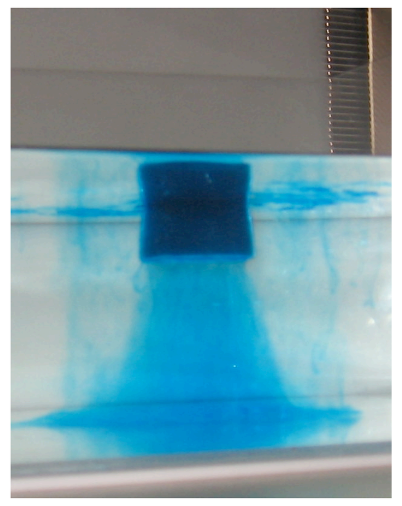


Figure 3. Example of vortex visualization using blue food coloring. Dark colors are best for qualitative observations while light colors reduce interference with the particle tracking software.

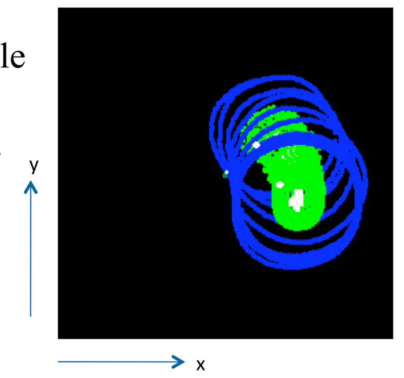


Figure 4. Sample data from the ptrack software. View from top.

Tracking

A particle tracking software called *ptrack* was used to track the motion of the vortex (Ravela 2009). A white paper disk the same size as the ice's surface was marked with two dots, one at the center and one on the edge. This allowed for random motion due to drift to be later subtracted from the data. The output from ptrack is saved as a text file of time and position and was imported into MatLab for analysis.

Experiment

Rotation Rates Used

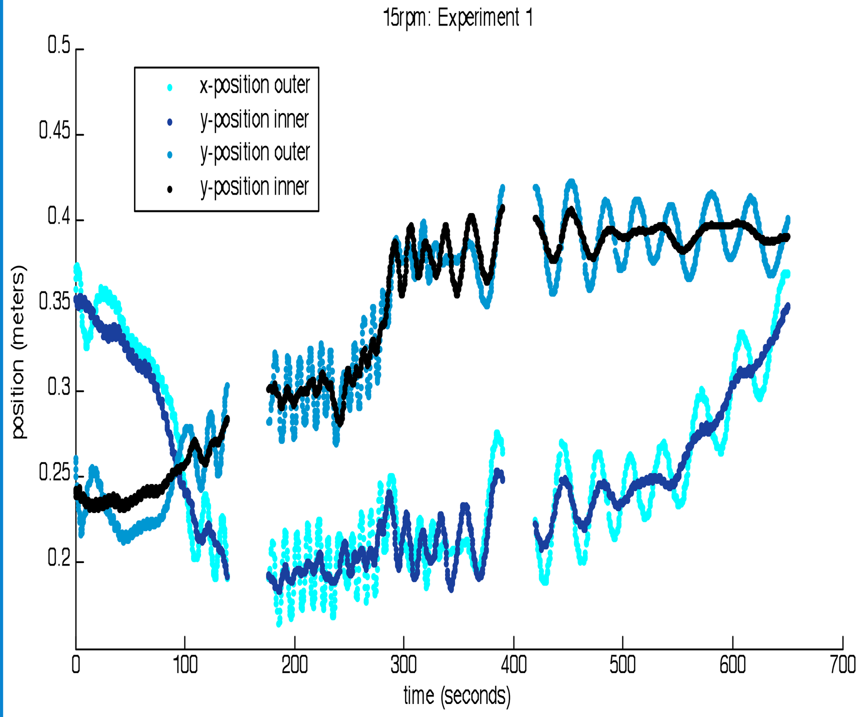
RPM	$\tau = \frac{60s}{RPM}$	$\Omega = \frac{2\pi}{\tau}$	$f = 2\Omega$
3rpm	20	~0.3	0.6
5rpm	12	~0.5	1
10rpm	6	~1	2
15rpm	4	~1.5	3

The experiment was conducted a total of fifteen times. Three experiments are repeated using different rotation rates. Due to difficulties with the particle tracker, the 10 rpm data was done six, four of them producing useable data.

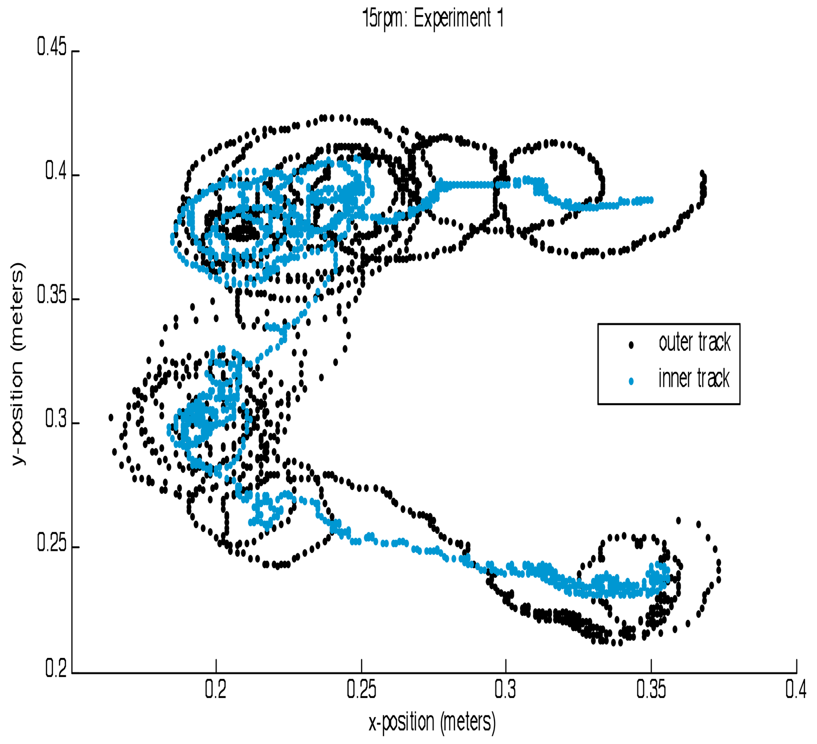
The lower bound of the rotation rates was chosen based on how slow the engine could rotate the tank consistently for the duration of the experiment and the upper bound was chosen based on when friction starts to create too much drift to produce a coherent vortex.

Figure 5. This picture was taken using the co-rotating camera that allowed the capturing of movies from within the frame of reference.

Data

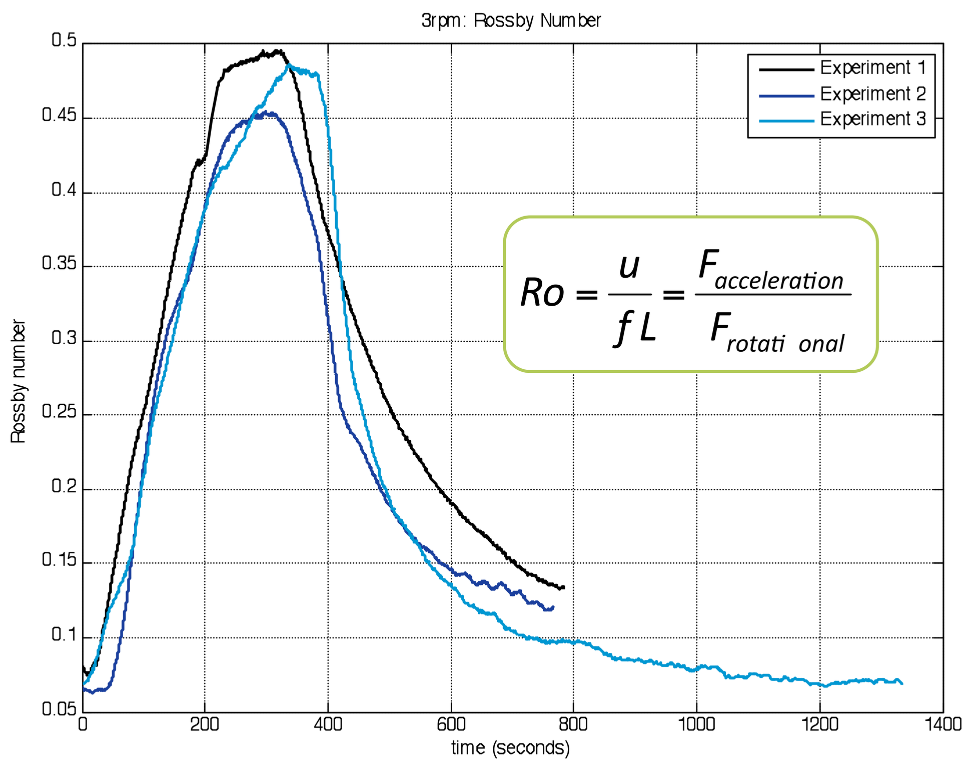


Graph 1. The time versus position graph shows the three stages of the vortex. The darker lines represent the random motion of the ice as the vortex drifts around the tank. The ice melt occurs around seven minutes.



Graph 2. The time versus position graph shows the inner track (light) and the outer track (dark). The goal is to have it look similar to the image the particle tracker produces. It helps to assess the accuracy of the data analysis.

These graphs provide a sample of the data collected from these experiments. For each run the data was analyzed in MatLab and graphs of the position in time, x versus y, velocity, and Rossby number were created.



Graph 3. The Rossby number versus time graph provides a more advanced analysis of the experiments. Note the consistency across runs.

Analysis

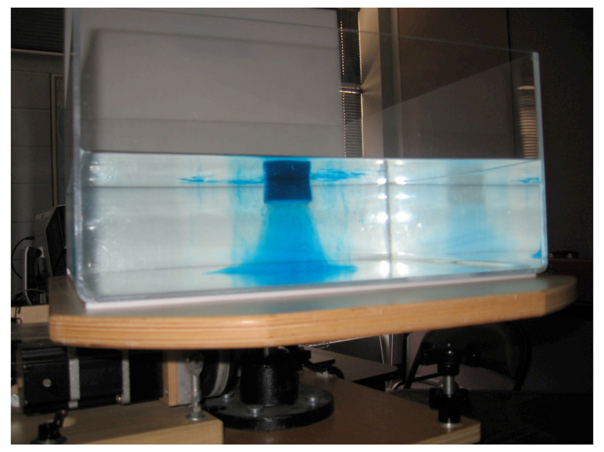
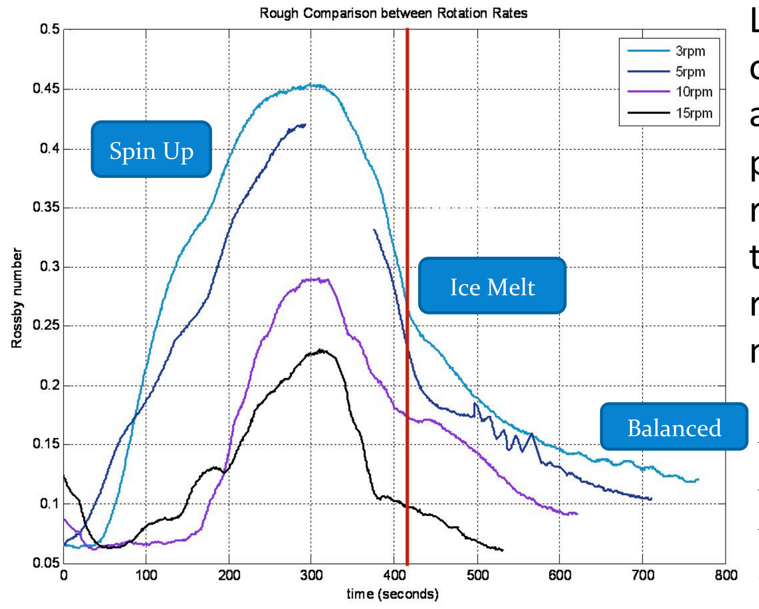


Figure 6. Cone associated with vortex distinct throughout experiment.

Visually, across all the experiments a distinct cone (like the one in Figure 6) was observed. It maintained a top width equal to the five centimeters of the ice until the ice melted when it formed a more dome-like structure with a wider upper and lower radius.

This balanced state lasted the longest in the 3rpm experiments because higher rotation rates start to reincorporate the significance of other factors like friction.

The data shows that the ice melt occurs consistently around seven minutes. This is a result of the temperature of the tank water and the amount of salted ice being held constant.



Low Rossby numbers were observed in this experiment and decreased inversely proportional to the rotation rate, which is expected since the rotation rate is inversely related to the Rossby number.

Graph 5. The Rossby line that best represented each rotation rate was used to create a simplified visual comparison. (Gap in 5rpm data due to the particle tracker stopping and then restarting tracking the point.)

These experiments did, in many ways, behave as mathematically predicted by the balanced vortex theory; however, similar experiments conducted by J. A. Whitehead (1990) produced a Rossby number closer to one. Further analysis is required to determine the significance of this difference. Reproducing these experiments consistently requires a lot of time learning the nuances of the particle tracking software, but the results are reproducible.

References

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