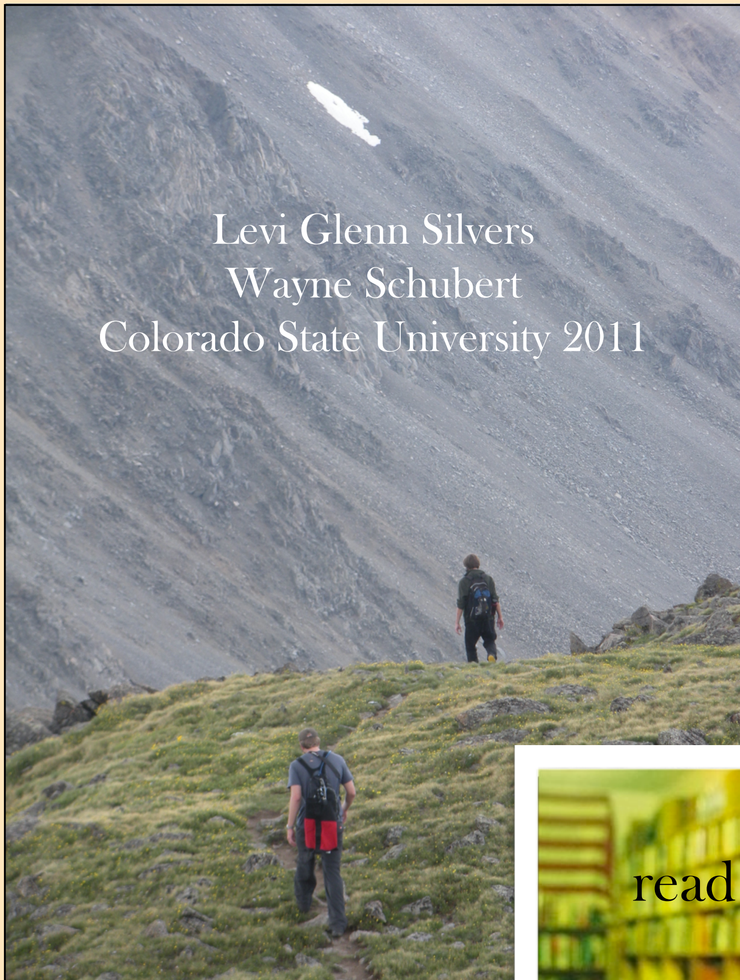
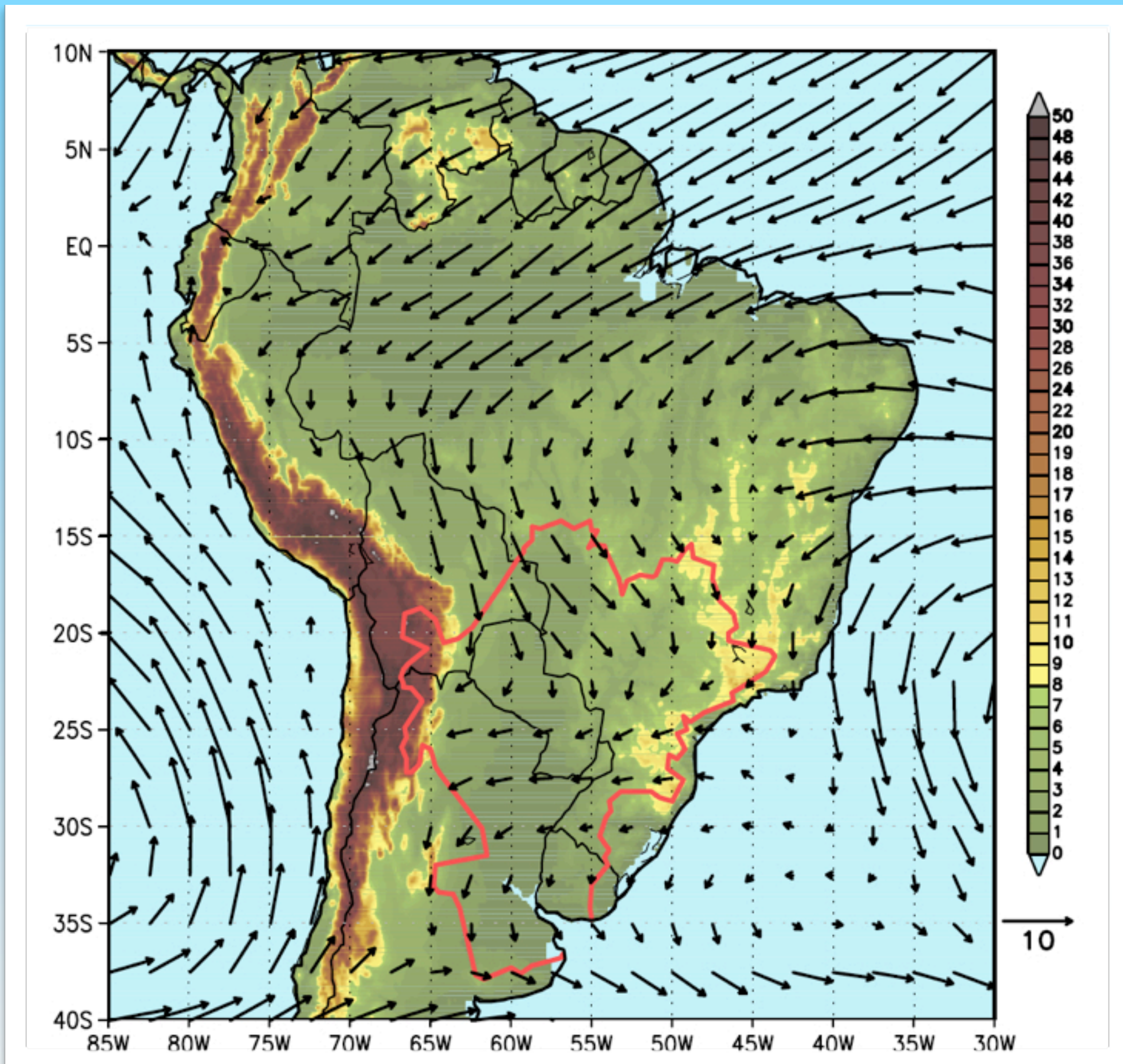
When I am not struggling to understand the Atmosphere
I like to....

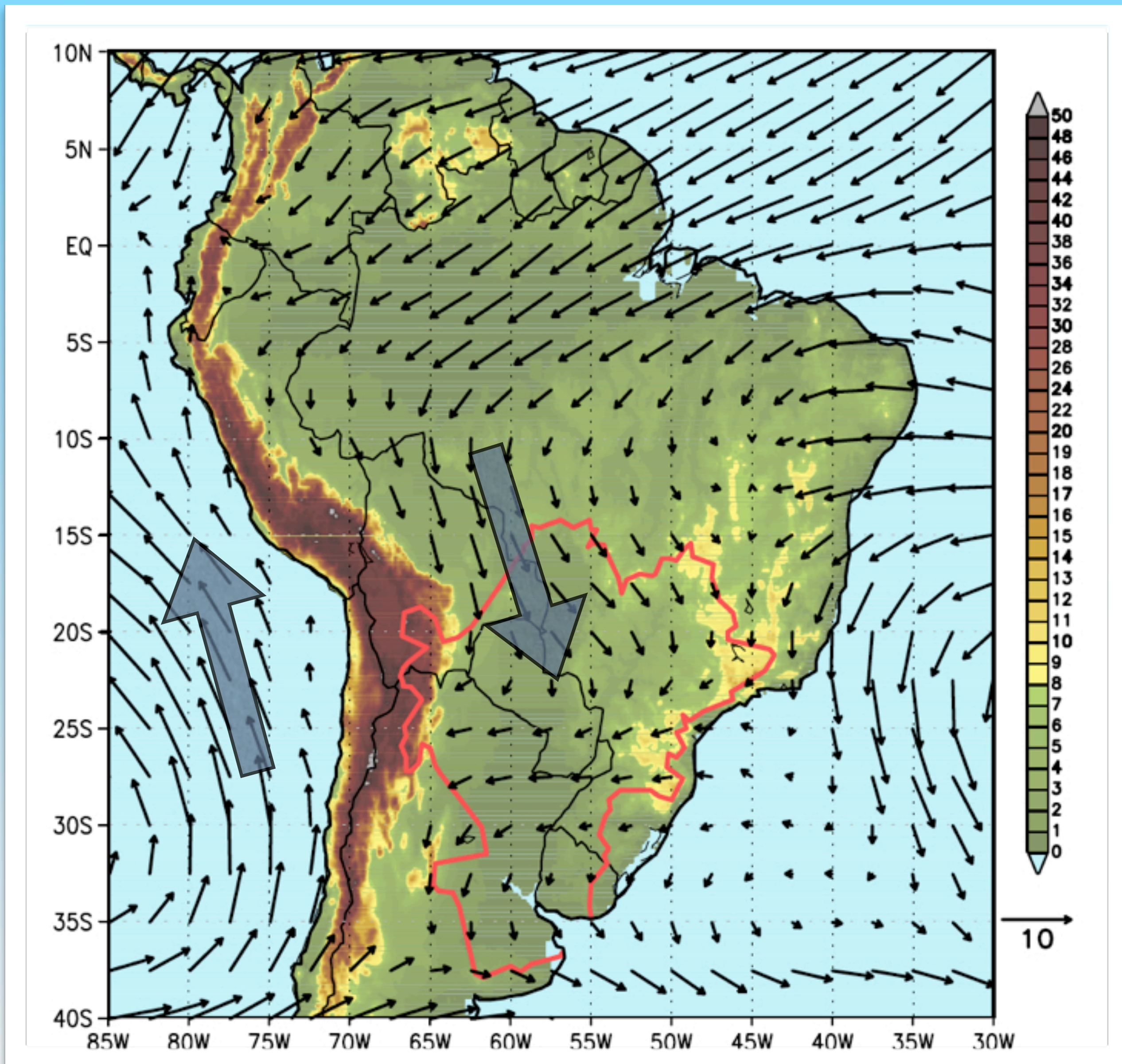


Topographically Bound Balanced Motions and Atmospheric Low-Level Jets

Jan 2003 Mean 925 hPa Wind Over South America



- NCEP-DOE Reanalysis II Data
- T62 Resolution
- Shading indicates topography
- Red outline marks the La Plata river basin
- Adapted from Tarasova et al. (2006) and Noguès-Paegle et al. (2001) thanks to B. McNoldy

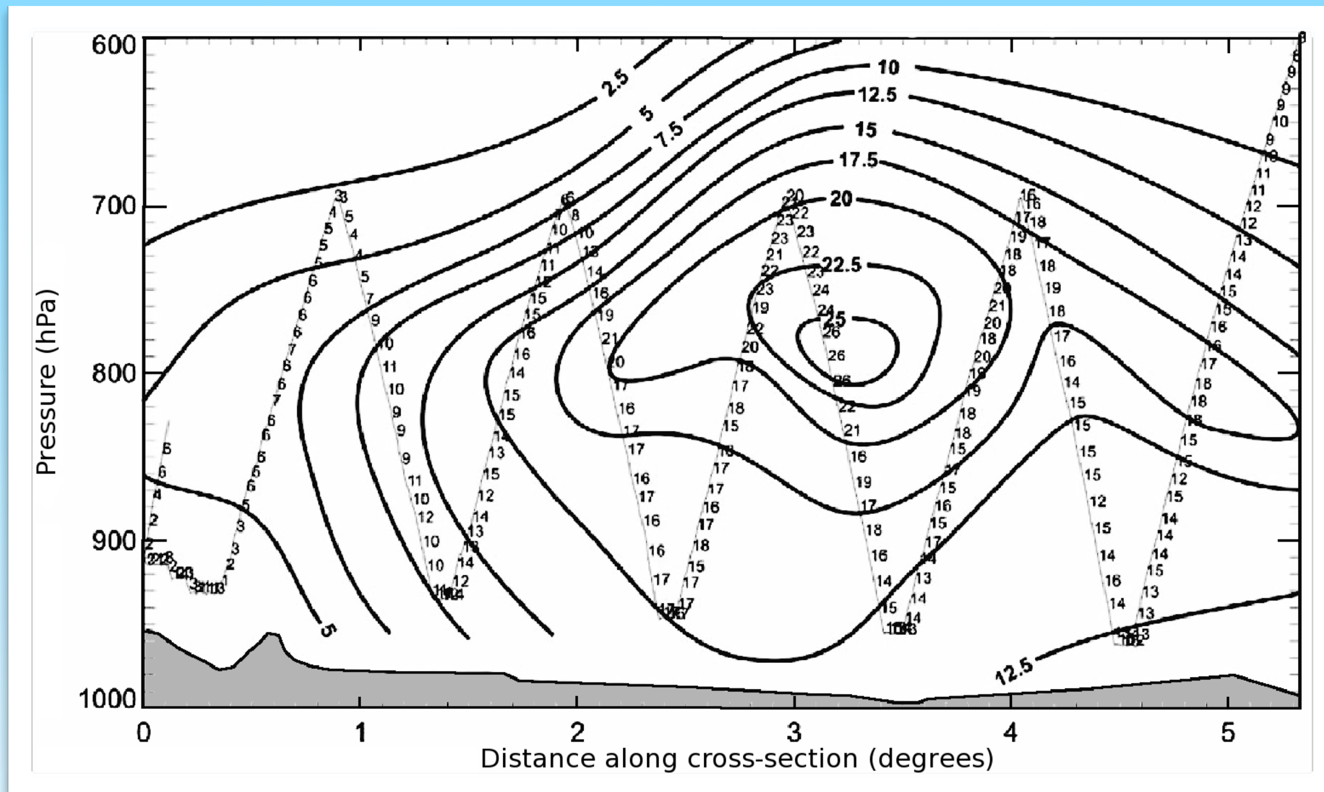


Jan 2003 Mean 925 hPa Wind Over South America

- Note the strong cyclonic flow centered over the Andes
- Involves 2 low level jets:
  - South American Low-Level Jet (SALLJ)
  - Coastal LLJ

What is an Atmospheric Low-Level Jet?

- Large Vertical and horizontal shear
- Lower troposphere
- Wind speed often maximizes at night



Data from SALLJEX: Vera et al. (2006)

- Cross Section Through the SALLJ on 6 Feb 2003
- Low-Level Jets play a major role in the global hydrologic cycle

Research Hypothesis

- Heating of the lower surface is an important mechanism that contributes to the life-cycle of LLJs
- Forcing through heated orography strengthens the basic cyclonic flow around the Rockies and Andes
- Jets on opposite sides of the mountains are part of a single response to potential vorticity forcing that results from heating

Potential Vorticity

$$P = \frac{1}{\rho}\left(2\Omega + \nabla \times \mathbf{u}\right) \cdot \nabla\theta$$

- Why is PV a useful variable?
  - It carries all the necessary dynamical information on the balanced wind and mass fields
- The usefulness of PV rests on two fundamental components
  - Conservation of PV (Rossby 1939, 1940; Ertel 1942)
  - The invertibility principle (Eliassen and Kleinschmidt 1957)
- The evolution of potential vorticity on isentropic surfaces leads to insight into
  - Cutoff cyclones
  - Blocking anticyclones
  - Rossby wave propagation
- See Hoskins, McIntyre, and Robertson, 1985 for more discussion

Deriving a PV Invertibility Principle

- Modeling assumptions:
  - compressible, stratified fluid on an f-plane
  - inviscid, hydrostatic, y-independent motions
  - Geostrophic balance
- Potential vorticity (PV):

$$\frac{DP}{Dt} = 0$$

where

$$P = \left(f + \frac{\partial v}{\partial x}\right)\left(-\frac{1}{g}\frac{\partial p}{\partial \theta}\right)^{-1}$$

- Far-field flow vanishes:

$$p(x,\theta) \longrightarrow \tilde{p}(\theta) \quad \text{and} \quad P(x,\theta) \longrightarrow \tilde{P}(\theta)$$

- Far-field PV:

$$\tilde{P} = f\left(-\frac{1}{g}\frac{\partial \tilde{p}}{\partial \theta}\right)^{-1}$$

Deriving a PV Invertibility Principle

- Define the buoyancy frequency:

$$N^2(\theta) = \frac{g^2}{\theta^2}\left(-\frac{d\tilde{\Pi}}{d\theta}\right)^{-1}$$

- The PV ratio then leads to:

$$\frac{\partial v}{\partial x} + \left(\frac{f\theta^2 N^2 P}{g^2 \tilde{P}}\right)\frac{\partial \Pi'}{\partial \theta} = f\left(\frac{P}{\tilde{P}} - 1\right)$$

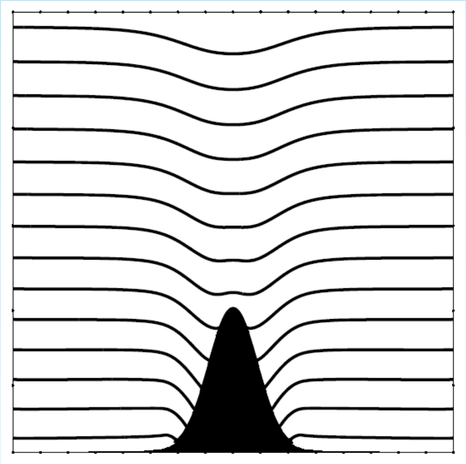
where the Exner function is given by

$$\Pi = c_p (p/p_0)^{R/c_p} \quad \text{and} \quad \tilde{\Pi} = c_p(\tilde{p}/p_0)^{R/c_p}$$

$$\Pi'(x,\theta) = \Pi(x,\theta) - \tilde{\Pi}(\theta)$$

Deriving a PV Invertibility Principle

- Including topography:
  - geopotential along topography: $\phi_S(x)$
  - potential temperature along topography: $\theta_S(x)$
  - boundary of massless layer on a theta surface: $x_S(\theta)$



- Massless layer: $\theta_B < \theta < \theta_S(x)$

- Additional assumptions:
  - symmetry of $\phi_S(x)$ and $\theta_S(x)$ about $x = 0$
  - geostrophic and hydrostatic balance
  - PV is uniform on each isentropic surface above the massless layer
  - Density is everywhere equal to the far field density

PV Invertibility Principle

Elliptic problem:

$$\frac{\partial v}{\partial x} + \left(\frac{f\theta^2 N^2}{g^2}\right)\frac{\partial \Pi'}{\partial \theta} = 0 \quad \text{for} \quad x_S(\theta) < x < \infty,$$

$$\left(\frac{f\theta^2 N^2}{g^2}\right)\frac{\partial \Pi'}{\partial \theta} = f \quad \text{for} \quad 0 \le x < x_S(\theta),$$

$$f\frac{\partial v}{\partial \theta} - \frac{\partial \Pi'}{\partial x} = 0 \quad \text{for} \quad 0 \le x < \infty, \quad \theta_B \le \theta \le \theta_T$$

Horizontal BC's:

$$\left.\begin{array}{r} v \to 0 \\ \Pi' \to 0 \end{array}\right\} \quad \text{as} \quad x \to \infty$$

Vertical BC's:

$$\Pi' = 0 \qquad \text{at} \quad \theta = \theta_T,$$

$$f\left(v - \theta\frac{\partial v}{\partial \theta}\right) = \frac{d\phi_S(x)}{dx} \qquad \text{at} \quad \theta = \theta_B$$

Simple Analytical Solutions

- Topography:
  - mountain of height H and width a

$$\phi_S(x) = gHe^{-x^2/a^2} \quad \Longrightarrow \quad \hat{\phi}_S(k) = \frac{gHa}{\sqrt{2}} e^{-a^2k^2/4}$$

- Simple reference state profile:
  - Buoyancy frequency is inversely proportional to θ

$$N(\theta) = N_1 \theta_B / \theta$$

Analytic Solutions and Special Cases

- Balanced wind and mass fields are forced in two ways

$$\hat{v}(k,\theta) = \frac{g}{f\theta_B N_1}\int_{\theta_B}^{\theta_T} F(k,\theta')\mathcal{G}(k,\theta,\theta')\,d\theta' - \frac{N_1}{g}\hat{\phi}_S(k)\left(\frac{\cosh[\kappa(\theta_T-\theta)]}{\cosh[\kappa(\theta_T-\theta_B)]+\kappa\theta_B\sinh[\kappa(\theta_T-\theta_B)]}\right)$$

$$\hat{\Pi}'(k,\theta) = -\frac{g^2}{f\theta_B^2 N_1^2}\int_{\theta_B}^{\theta_T} F(k,\theta')\frac{\partial\mathcal{G}(k,\theta,\theta')}{\partial\theta}\,d\theta' - \hat{\phi}_S(k)\left(\frac{\kappa\sinh[\kappa(\theta_T-\theta)]}{\cosh[\kappa(\theta_T-\theta_B)]+\kappa\theta_B\sinh[\kappa(\theta_T-\theta_B)]}\right)$$

Special Case	$F(k,\theta)$	$\hat{\phi}_S(k)$
• Flat topography • No variation of θ along bottom boundary	$=0$	$=0$
• Flat topography • Variation of θ along bottom boundary	$\neq 0$	$=0$
• Nonzero topography (Eliassen 1980) • No variation of θ along bottom boundary	$=0$	$\neq 0$
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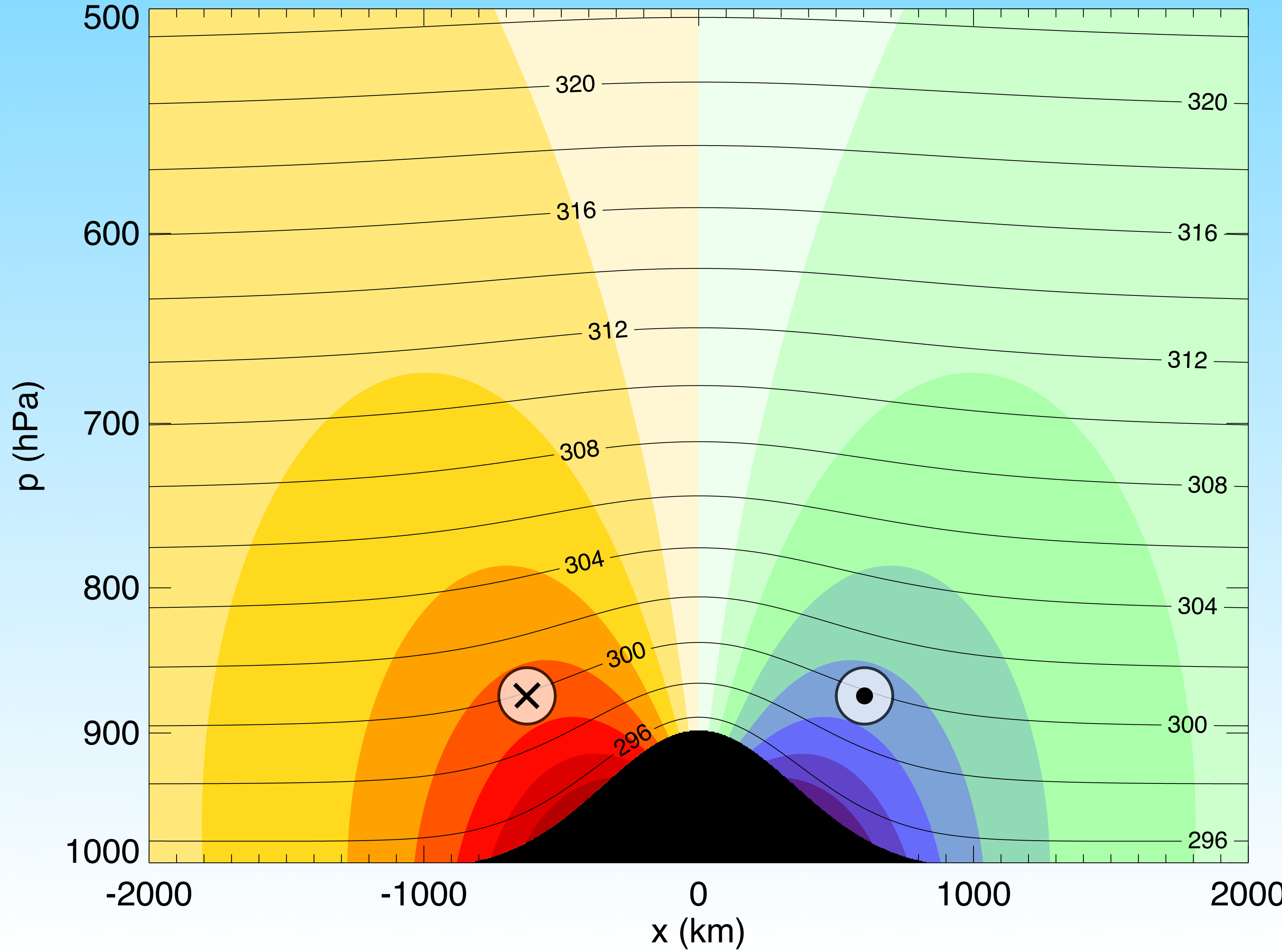
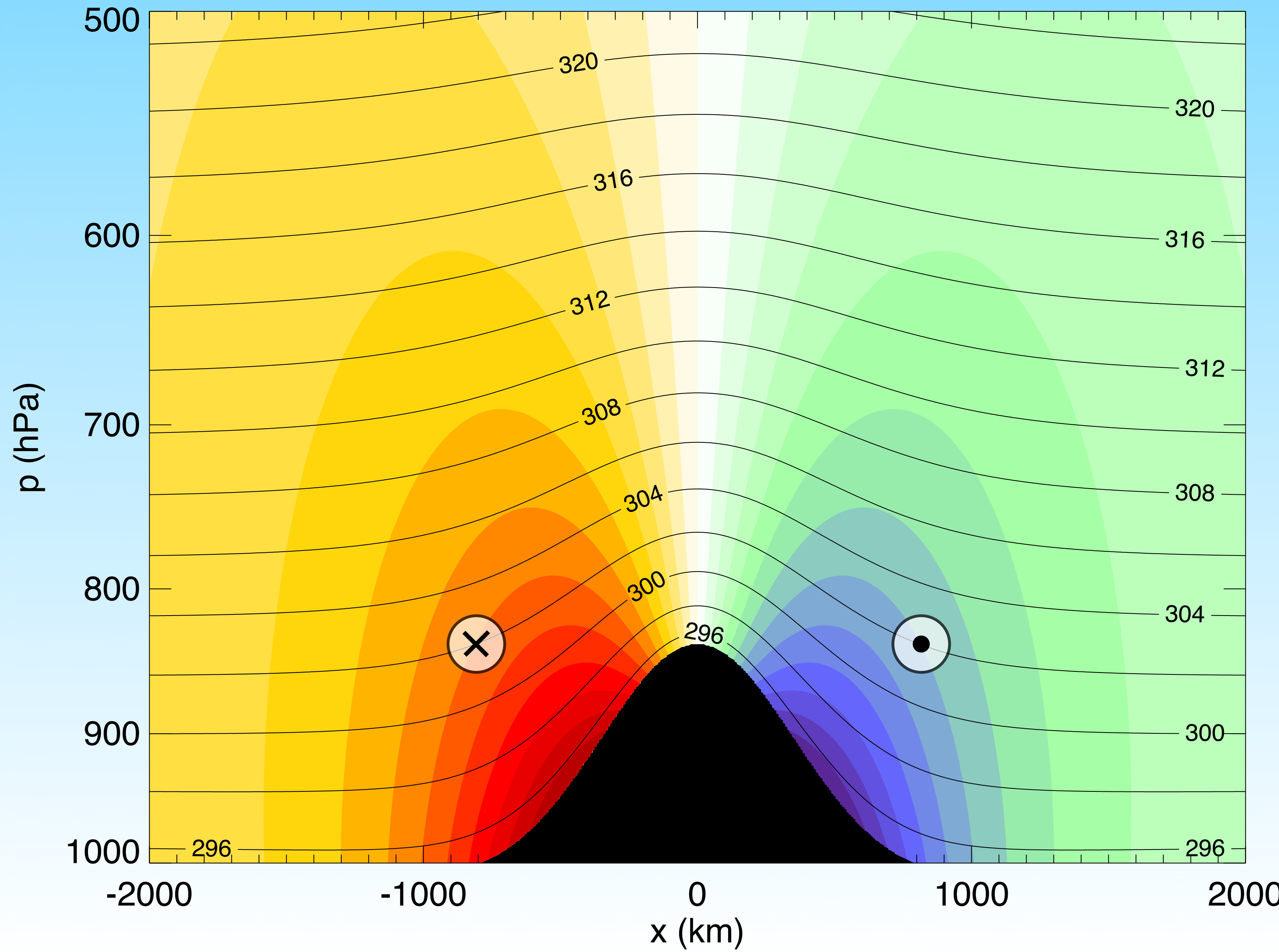
Isentropic Mountain with H = 1000 m



Isentropic Mountain with H = 1700 m



Numerical Solutions

- Purely analytic solutions are in closed form only for the case of an isentropic mountain
- For cases including a heated lower boundary, solutions can only be found with an iterative procedure
- Allows for more general cases: complex orography and heating, unapproximated density

$$\hat{v}(k,\theta) = \frac{g}{f\theta_B N_1}\int_{\theta_B}^{\theta_T} F(k,\theta')\mathcal{G}(k,\theta,\theta')\,d\theta' - \frac{N_1}{g}\hat{\phi}_S(k)\left(\frac{\cosh[\kappa(\theta_T-\theta)]}{\cosh[\kappa(\theta_T-\theta_B)]+\kappa\theta_B\sinh[\kappa(\theta_T-\theta_B)]}\right)$$

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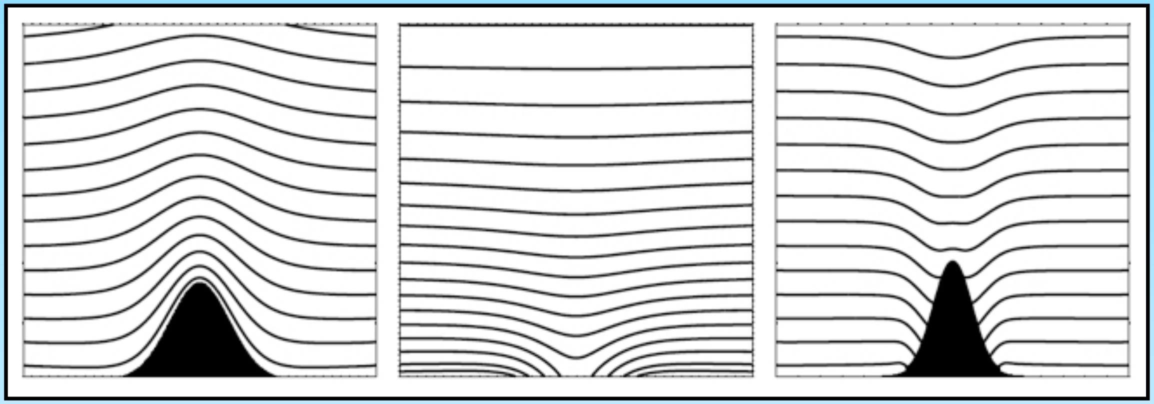
$$F(k,\theta) = (2/\pi)^{1/2}\int_0^{x_S(\theta)}\left(f+\frac{\partial v}{\partial x}\right)\cos(kx)\,dx$$

Numerical Results for Three Simple Cases

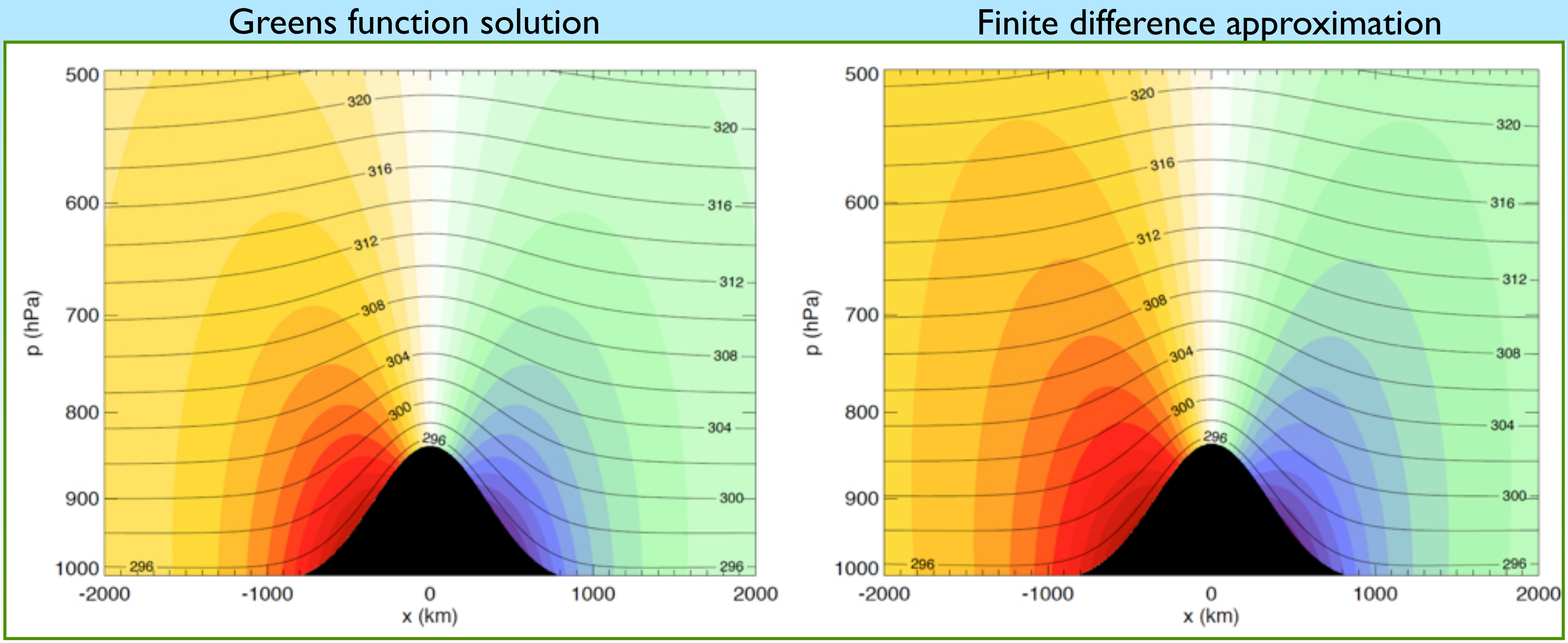


Isentropic Ridge Flat Heated Lower Surface Heated Ridge

- For cases including a heated lower boundary, solutions can only be found with an iterative procedure
- Solutions are computed for cases with PV anomalies in the massless layer
- These cases are representative of:
  - ~ gently sloping orography
  - ~ temperature gradients along flat boundaries
  - ~ large heated orography

Comparison of Analytic and Finite Difference solutions for Isentropic mountain

- General agreement is encouraging
- Density is assumed to only depend on θ at left



Isentropic Mountain

- Identical wind field plotted two different ways
- All computations made in $(x, \theta)-$space, then interpolated to $(x, p)-$space
- Note absence of massless layer

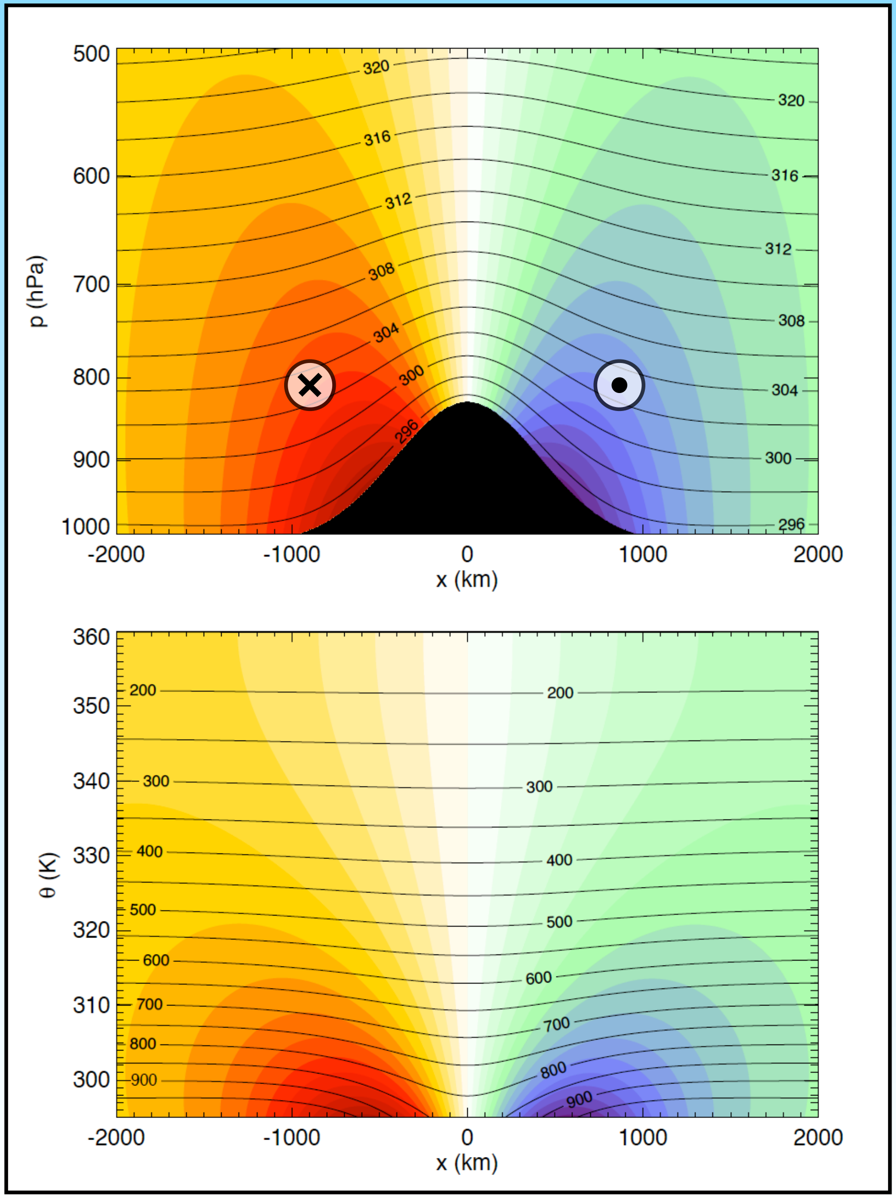
$$v_{max} = 15.4m/s$$

$$H = 1800m$$

- PV is conserved

$$P = \left(f + \frac{\partial v}{\partial x}\right)\left(-\frac{1}{g}\frac{\partial p}{\partial \theta}\right)^{-1}$$

Compressed isentropes require $\partial v/\partial x$ to decrease while stretched isentropes require $\partial v/\partial x$ to increase

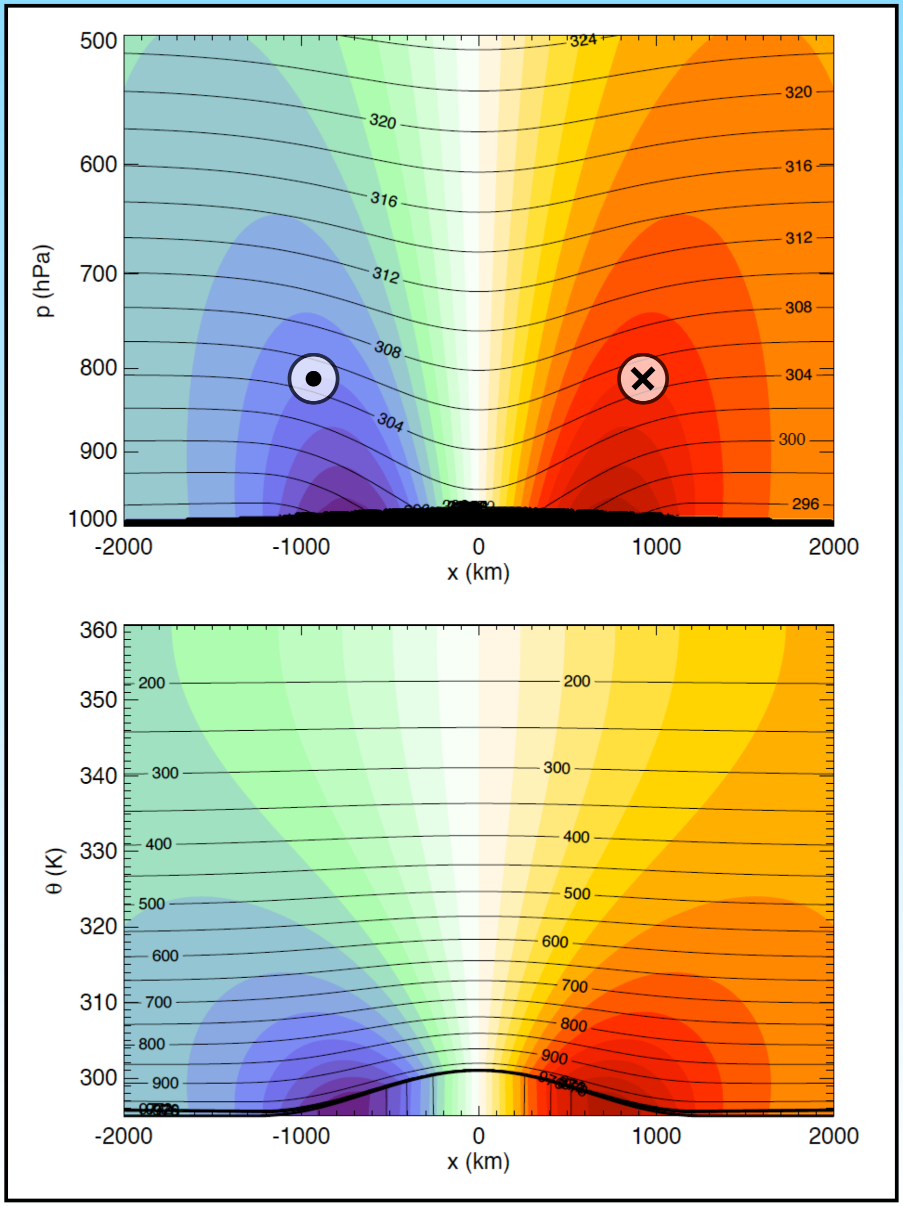


Heated Flat Lower Boundary

- Temperature anomaly along lower surface is 6 K
- Note presence of massless layer
- Cyclonic flow, massless layer is indicated by black object along lower surface (top) and thick line (bottom)

$$v_{max} = 12.7 \quad m/s$$

- In the massless layer the pseudodensity $\partial p/\partial \theta$ vanishes



Summary and Conclusions

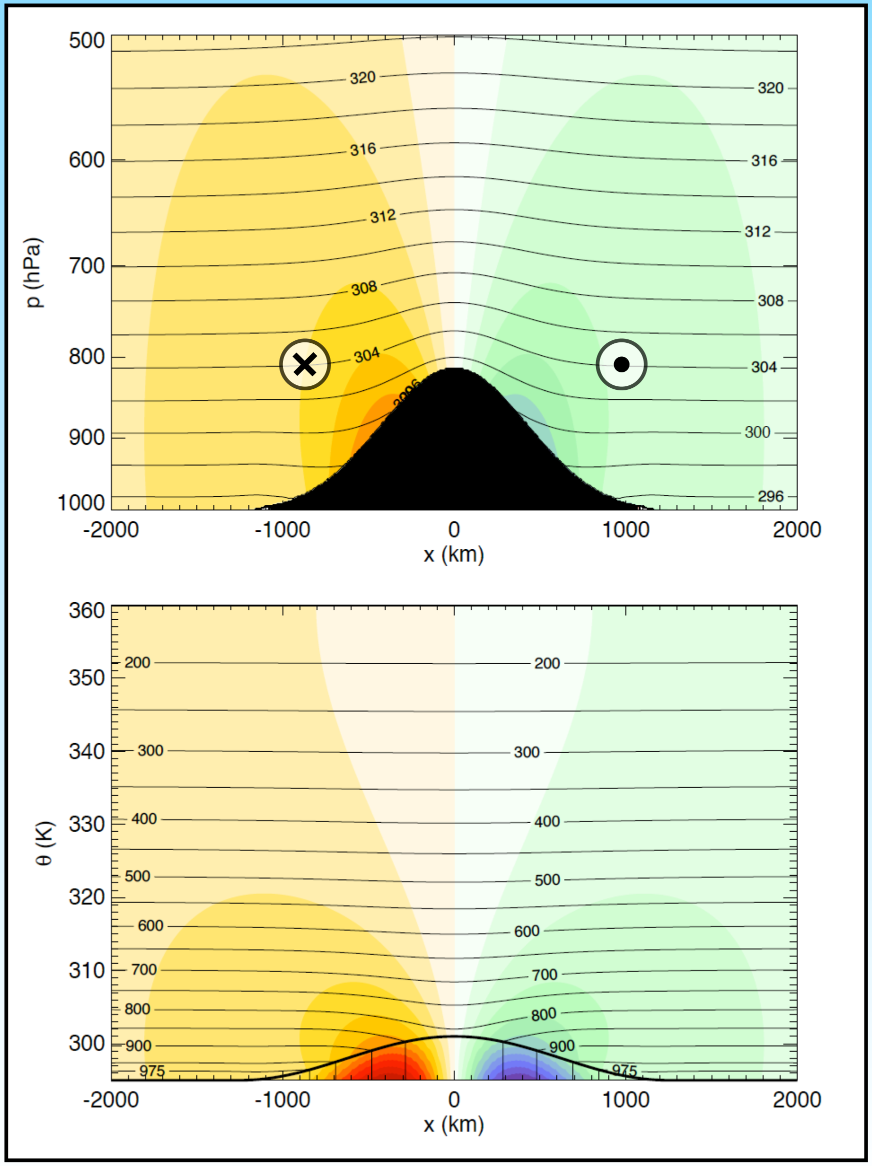
- The SALLJ and Coastal LLJ are not separate entities
- The PV invertibility principle has been solved in isentropic coordinates with a lower boundary that includes mountains
- We have shown that a balanced response to heated topography is an important contributing factor for LLJs
- For a sufficiently strong temperature anomaly, a cyclonic circulation will be generated
- Generalization of these results to the sphere and the use of more realistic topography will allow for more direct comparison with observations

Heated Ridge

- Temperature anomaly along lower surface is 6 K
- Anti-cyclonic flow, massless layer is indicated by black object along lower surface (top) and thick line (bottom)

$$v_{max} = 11.13 \quad m/s$$

$$H = 1800 \quad m$$

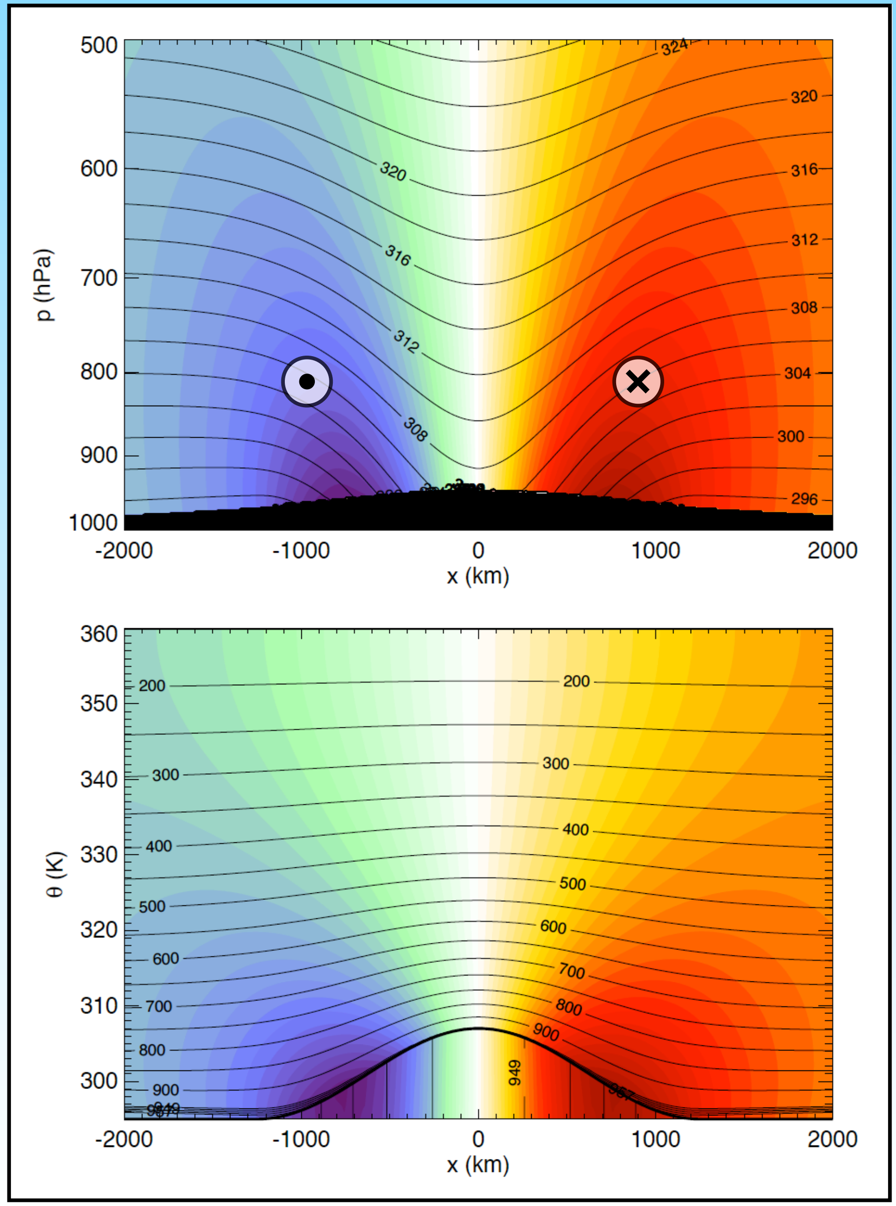


Heated Flat Lower Boundary

- Temperature anomaly along lower surface is 12 K

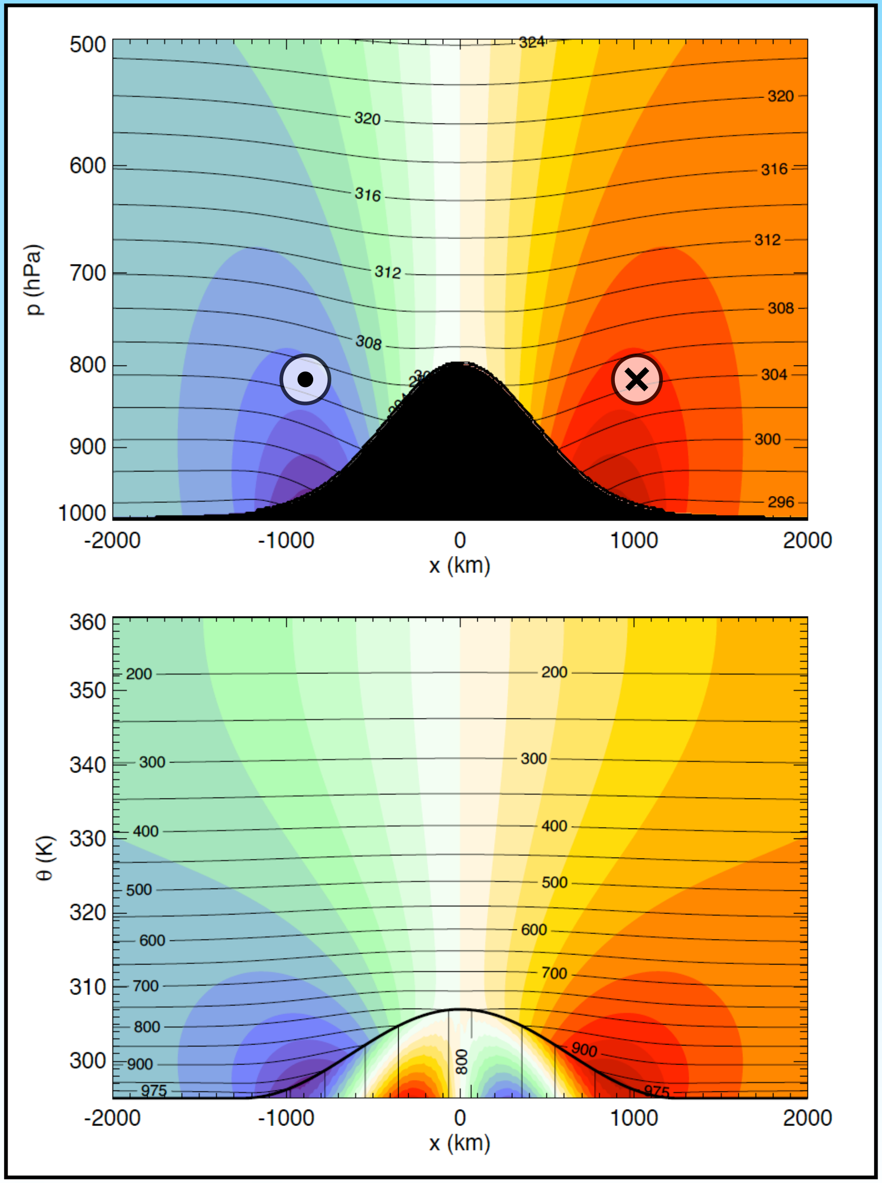
Anti-cyclonic flow, massless layer is indicated by black object along lower surface (top) and thick line (bottom)

$$v_{max} = 27.27 m/s$$



Heated Ridge

- Temperature anomaly along lower surface is 12 K



Cyclonic flow, massless layer is indicated by black object along lower surface (top) and thick line (bottom)

$$v_{max} = 10.3m/s \qquad H = 1800m$$

- Flow is opposite sign as that for the heated ridge with a temperature anomaly of 6 K

References

Rossby, Ertel, Eliassen, Eliassen and Kleinshmidt, Tarasova et al. 2006; Noguès-Paegle et al. 2001; Jiang et al. 2007; Vera et al. (2006); Hoskins et al. 1985.

Eliassen, A., 1980: Balanced motion of a stratified, rotating fluid induced by bottom topography. *Tellus*, **32**, 537–547.

Jiang, X., N.-C. Lau, I. M. Held, and J. J. Ploshay, 2007: Mechanisms of the Great Plains low-level jet as simulated in an a GCM. *J. Atmos. Sci.*, **64**(2), 532–547. doi:10.1175/JAS3847.1.

Nogués-Paegle, J., J. Paegle, and Coauthors, 2001: American low-level jets - a scientific prospectus and implementation plan. Available online at http://www.clivar.org/organization/vamos/Publications/alls.pdf.

Tarasova, T. A., J. P. R. Fernandez, I. A. Pisnichenko, J. A. Marengo, J. C. Ceballos, and M. J. Bottino, 2006: Impact of new solar radiation parameterization in the eta model on the simulation of summer climate over South America. *J. Appl. Meteor. Climatol.*, **45**, 318–333.

Vera, C., J. Baez, M. Douglas, C. Emmanuel, J. Marengo, J. Meitin, M. Nicolini, J. Nogues-Paegle, J. Paegle, O. Penalba, C. Saulo, M. S. Dias, P. S. Dias, and E. Zipser, 2006: The South American low-level jet experiment. *Bull. Amer. Meteor. Soc.*, **87**, 63–77.