Why Are Cumulus Updrafts Narrow?

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Bjerknes' (1938) proposed a very simple explanation for the smallness of the fractional area covered by cumulus updrafts, denoted here by σ_c . Suppose that at a certain time the temperature is horizontally uniform, with lapse rate

$$\Gamma \equiv -\frac{\partial T}{\partial z} . \tag{1}$$

In a cloudy region, we have

$$\frac{\partial T_c}{\partial z} = w_c \left(\Gamma - \Gamma_m \right) . \tag{2}$$

In a neighboring clear region,

$$\frac{\partial}{\partial z} \tilde{T} = \tilde{w} \left(\Gamma - \Gamma_d \right) . \tag{3}$$

The mean vertical motion satisfies

$$\bar{w} = \left(1 - \sigma_c \right) \tilde{w} + \sigma_c w_c , \tag{4}$$

so we can write

$$w_c = \bar{w} + \left(1 - \sigma_c \right) \left(w_c - \tilde{w} \right) , \tag{5}$$

and similarly

$$\tilde{w} = \bar{w} - \sigma_c \left(w_c - \tilde{w} \right) . \tag{6}$$

Substituting into (2) and (3), we find that

$$\frac{\partial}{\partial t}\left(T_c - \tilde{T}\right) = w_c\left(\Gamma - \Gamma_m\right) - \tilde{w}\left(\Gamma - \Gamma_d\right)$$
$$= \overline{w}\left(\Gamma_d - \Gamma_m\right) + \left(w_c - \tilde{w}\right)\left[\left(1 - \sigma_c\right)\left(\Gamma - \Gamma_m\right) + \sigma_c\left(\Gamma - \Gamma_d\right)\right]. \tag{7}$$

If the sounding is conditionally unstable, then

$$\Gamma - \Gamma_m > 0 \text{ and } \Gamma - \Gamma_d < 0 . \tag{8}$$

Therefore $T_c - T$ will increase most rapidly if $\sigma_c \to 0$.

The physical interpretation is simple. With a conditionally unstable sounding, saturated rising motion is aided by positive buoyancy created through condensation, while unsaturated sinking motion must fight against the dry-stable stratification. The rate at which the updraft gains buoyancy is proportional to the updraft speed, while the rate at which the downdraft must fight against the stratification is proportional to the downdraft speed. Therefore, the convection is favored by rapid rising motion in the cloudy region, and slow sinking motion in the clear region, both of which can be achieved, for a given value of $w_c - \tilde{w}$, by making the updraft narrow, and the downdraft broad.

Reference

Bjerknes, J., 1938: Saturated-adiabatic ascent of air through dry-adiabatically descending environment. *Quart. J. Roy. Meteor. Soc.*, **64**, 325-330.