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LAMONT-DOHERTY EARTH OBSERVATORY

Radiation in earth system models

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https://crew.ldeo.columbia.edu

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"Researchers at the Observatory work to understand the dynamics of the Earth's chemical, physical and biological systems, from the core to the upper atmosphere, including Earth-human interactions. Our scientists lead research in the fields of ocean, atmospheric and climate systems; cryospheric dynamics; paleoclimate; biogeoscience; and solutions to air, water or soil contamination impacting communities."

Lamont has institutional fellowships (open now!)

We also host NOAA and NSF fellows

What is needed from a radiation parameterization?

To advance the physical model in time:
- fluxes at the surface
- heating rates through the atmosphere

For interpretative purposes
- fluxes at the top-of-atmosphere
- clear-sky fluxes
- …

In advance other model components in time:
- e.g. PAR at the surface
- photolysis rates in the atmosphere
- …

These require two integrals
- over angle to get fluxes from intensities
- over wavelength to account for spectral dependence

Building a radiation parameterization: theory

From the equations describing radiation in the atmosphere

$$\Omega \cdot \nabla I_\nu(\Omega, \mathbf{x}) = -\beta_\nu(\mathbf{x}) I_\nu(\Omega, \mathbf{x}) + S_\nu(\mathbf{x})$$

$$F^{\pm}(\mathbf{x}) = \int_0^\infty \int^{2\pi} \hat{n} \cdot I_\nu(\Omega, \mathbf{x}) d\Omega d\nu$$

parameterizations of radiation in earth system model make 3+1 approximations

Form
Angular integration
Spectral integration
(Variability)

Each approximation converges to the underlying equations in some limit

Radiation is fundamentally different than say, the parameterization of convection or microphysics

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parameterizations of radiation in earth system model make three approximations

plane-parallel, homogeneous *ansatz*: $u \dfrac{dI_\nu(\tau_\nu, u, \phi)}{d\tau_\nu} = -I_\nu(\tau_\nu) + S_\nu(\tau_\nu)$

Context (i)

The atmosphere is thin but strongly vertically stratified

Practical and theoretical considerations lead us to solve the one-dimensional radiative transfer equation

$$u\frac{dI_\nu(u,\phi)}{d\tau} = -I_\nu + (1-\omega_0)B_\nu(T(\tau)) + \frac{\omega_0}{4\pi}\int_0^{2\pi}\int_{-1}^{1} I_\nu(u',\phi')P(u',\phi' \rightarrow u,\phi)du'd\phi'$$

Planck source

Scattering source

Implication: each horizontal location is independent

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$$\boldsymbol{\Omega} \cdot \nabla I_\nu(\boldsymbol{\Omega}, \mathbf{x}) = -\beta_\nu(\mathbf{x}) I_\nu(\boldsymbol{\Omega}, \mathbf{x}) + S_\nu(\mathbf{x})$$

$$F^{\pm}(\mathbf{x}) = \int_0^\infty \int^{2\pi} \hat{n} \cdot I_\nu(\boldsymbol{\Omega}, \mathbf{x}) d\boldsymbol{\Omega} d\nu$$

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plane-parallel, homogeneous *ansatz*: $u \frac{dI_\nu(\tau_\nu, u, \phi)}{d\tau_\nu} = -I_\nu(\tau_\nu) + S_\nu(\tau_\nu)$

analytic angular integration i.e. two-stream: $\frac{dF_\nu^+}{d\tau} = \gamma_1 F_\nu^+ - \gamma_2 F_\nu^- + S_\nu(\tau_\nu)$

Context (ii)

Solar and terrestrial radiation are spectrally disjoint

Terrestrial radiation is dominated by absorption and emission;
solar radiation by absorption and scattering

$$\mu \frac{dI_\nu(\mu, \phi)}{d\tau} = -I_\nu + (1 - \omega_0) B_\nu(T(\tau)) + \frac{\omega_0}{4\pi} \int_0^{2\pi} \int_{-1}^{1} I_\nu(\mu', \phi') P(\mu', \phi' \to \mu, \phi) d\mu' d\phi'$$

This motivates taking different approaches for angular integration
in the longwave and shortwave

Computing longwave spectral intensities: each layer

Neglecting scattering yields Schwartzchild equations uncoupled in direction

$$u\frac{dI_\nu}{d\tau_\nu} = -I_\nu + B_\nu(T(\tau_\nu))$$

The equation has an analytic solution for layers of finite $\Delta\tau$ if the source function $S = f(B(T(\tau)))$ is known

Given a discrete vertical coordinate ξ
$I^u_\nu(\xi^{i+i}) = I^u_\nu(\xi^i)e^{-\Delta\tau^i_\nu/u} + S^i_\nu$

S^i_ν can be computed analytically for many forms of $B(T(\tau))$
e.g. if the source is "linear in tau" with the layer

Computing longwave spectral intensities: multiple layers

To compute profiles

Determine downwelling $I_\nu^{\mu_i}(\xi^0)$ from boundary conditions (typically zero)

Determine $I_\nu^{\mu}(\xi^{i+i})$ from top-of-atmosphere to surface

Determine $I_\nu^{-\mu}(\xi^I)$ from surface reflection, emission

Determine $I_\nu^{-\mu}(\xi^{i-1})$ from top-of-atmosphere to surface

Computing longwave spectral fluxes

Fluxes may be computed from intensities using numerical quadrature, e.g.

$$F_\nu^- = \int_0^1 u I_\nu du \approx \sum_u w^u I_\nu^u$$

The integral is on [0,1] so u^i and w^i are usually determined with Gaussian quadrature

A common further approximation is to choose a single "diffusivity angle" $\bar{u}$ at which to compute intensities.

Computing shortwave spectral fluxes: formulation

The equation governing shortwave radiative transfer is integro-differential

$$u\frac{dI_\nu(u,\phi)}{d\tau} = -I_\nu + \frac{\omega_0}{4\pi}\int_0^{2\pi}\int_{-1}^{1} I_\nu(u',\phi')P(u',\phi' \rightarrow u,\phi)du'd\phi'$$

It can be solved by numerically integrating the scattering integral
e.g with discrete ordinates:
expanding azimuth in Fourier modes, polar angle in Legendre polynomials

But that's a lot of work

Computing shortwave spectral fluxes: formulation

Analytic integration give us the two-stream approximation
a set of first-order coupled ODEs

$$\frac{dF_\nu^+}{d\tau_\nu} = \gamma_1 F_\nu^+ - \gamma_2 F_\nu^- - \gamma_3 S_\nu(\tau_\nu)$$

$$\frac{dF_\nu^-}{d\tau_\nu} = -\gamma_1 F_\nu^- + \gamma_2 F_\nu^+ + (1-\gamma_3) S_\nu(\tau_\nu)$$

Coupling coefficients are chosen based on assumptions about azimuthally-averaged
$I_\nu(u)$ and $I_\nu(u)P(u' \rightarrow u)$
$\gamma_{1,2} = f(\omega_0, g), \gamma_3 = f(\omega_0, g, u_0)$

(But see doi:10.22541/essoar.171867251.13739862/v1)

Computing shortwave fluxes: each layer

It's convenient to separate solar radiation into
the direct, unscattered beam (subject to the Beer-Lambert-Bouguer law)
the diffuse radiation field

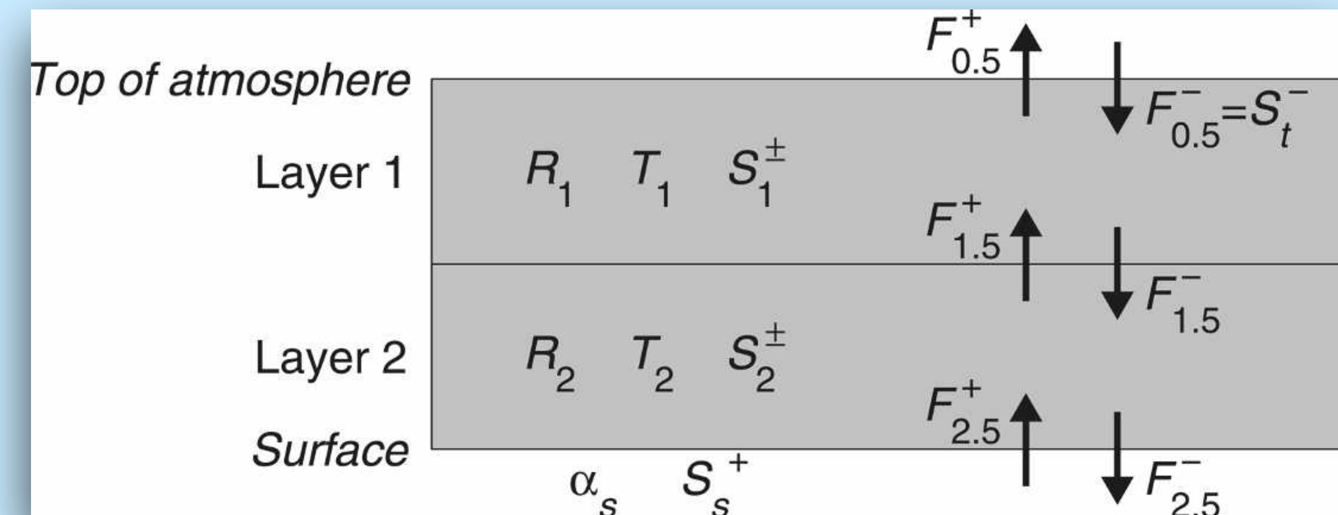
Then the two-stream equations describe the diffuse part and
the source is the contribution of the direct beam to the diffuse field
$S_\nu(\tau) = \pi \omega_0 \mu_0 S_\odot e^{-\tau/\mu_0}$

The two-stream equations can be integrated to determine
the reflectance and transmittance of a homogeneous layer
for diffuse radiation $\mathscr{T}_\nu^{\text{dif}}(\omega_0, g, \Delta\tau), \mathscr{R}_\nu^{\text{dif}}(\omega_0, g, \Delta\tau)$
and direct radiation $\mathscr{T}_\nu^{\text{dir}}(\omega_0, g, u_0, \Delta\tau), \mathscr{R}_\nu^{\text{dir}}(\omega_0, g, u_0, \Delta\tau)$

$\mathscr{T}_\nu^{\text{dir}}, \mathscr{R}_\nu^{\text{dir}}$ describe the contribution of the direct beam to the diffuse beam
including multiple scattering within the layer

Computing shortwave spectral fluxes: multiple layers

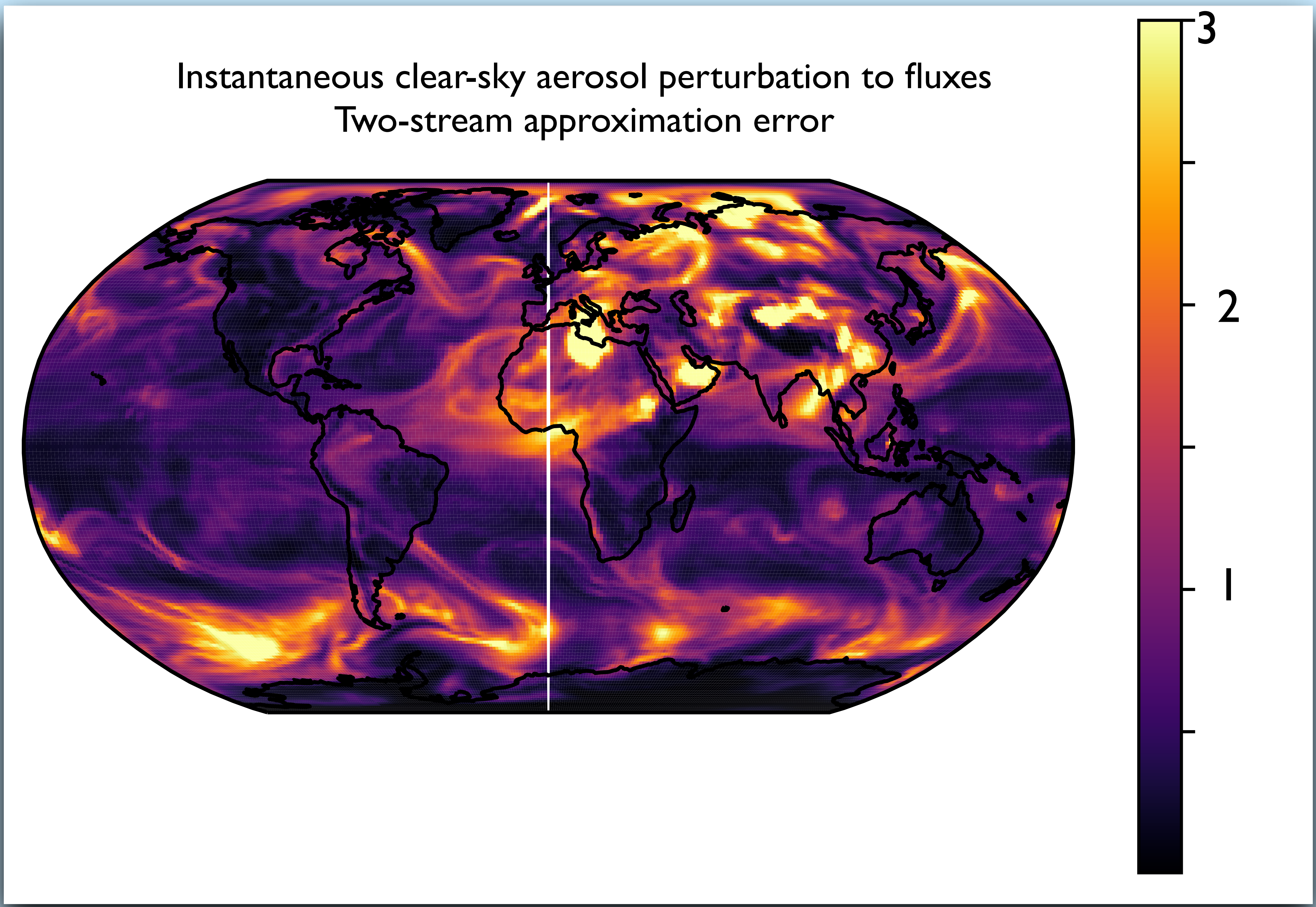
Profiles of fluxes are computed by solving the coupled set of equations for upward and downward flux at each layer



$$\begin{pmatrix} & 1 & & & & \\ 1 & -R_1 & -T_1 & & & \\ & -T_1 & -R_1 & 1 & & \\ & & 1 & -R_2 & -T_2 & \\ & & & -T_2 & -R_2 & 1 \\ & & & & 1 & -\alpha_s \end{pmatrix} \times \begin{pmatrix} F_{0.5}^{+} \\ F_{0.5}^{-} \\ F_{1.5}^{+} \\ F_{1.5}^{-} \\ F_{2.5}^{+} \\ F_{2.5}^{-} \end{pmatrix} = \begin{pmatrix} S_t^{-} \\ S_1^{+} \\ S_1^{-} \\ S_2^{+} \\ S_2^{-} \\ S_s^{+} \end{pmatrix},$$

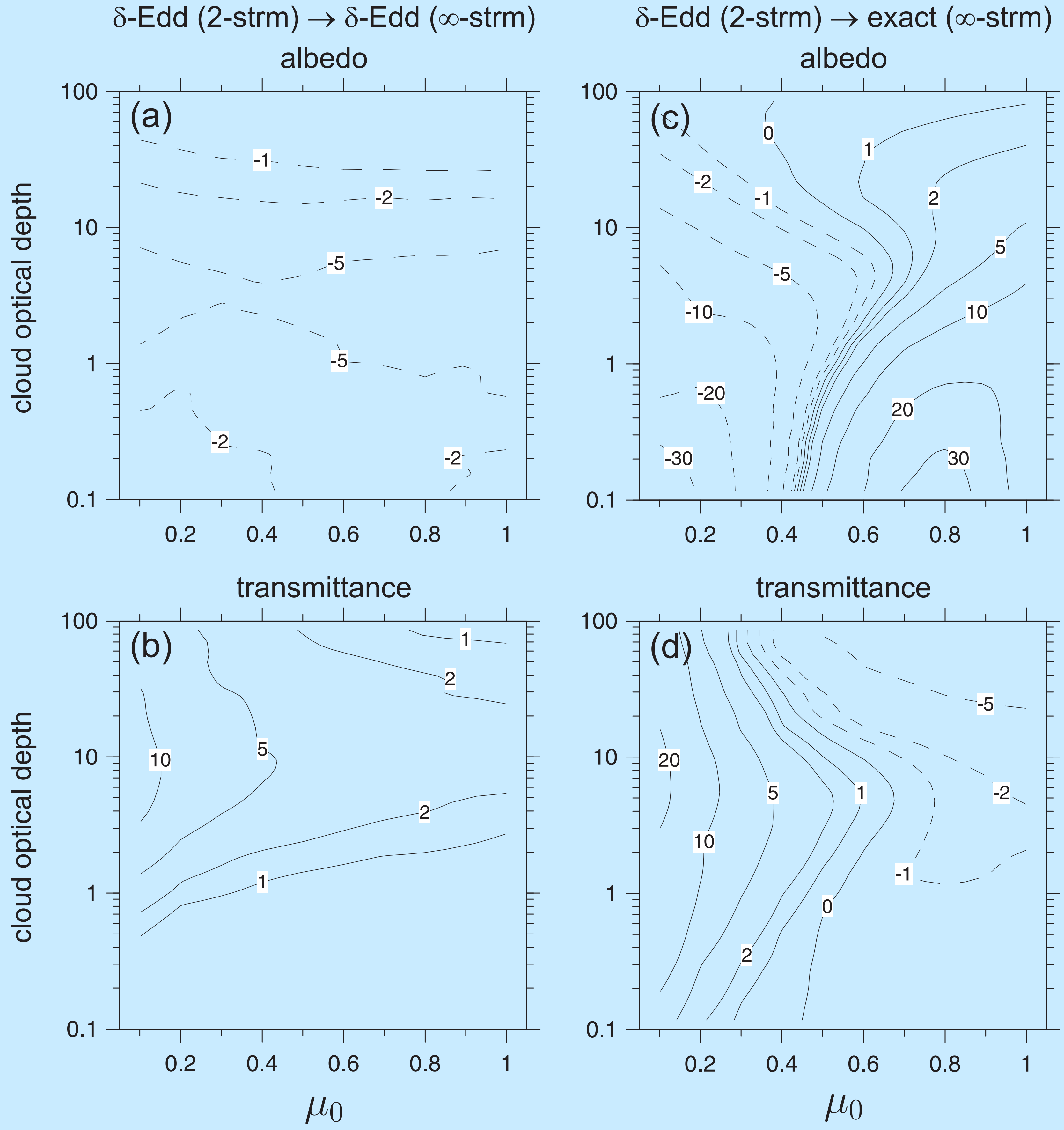
Illustration from Shonk and Hogan 2008, 10.1175/2007JCLI1940.1

Approximation error is well-characterized…



After Freidenreich et al 2021, 10.1029/2019JD032323

droplets: r_e = 15 μm; $v_e = 0.1$; $\lambda \in$ [0.25 - 0.69 μm]



Barker et al. 2015, 10.1175/JAS-D-15-0033.1

Building a radiation parameterization: theory

From the equations describing radiation in the atmosphere

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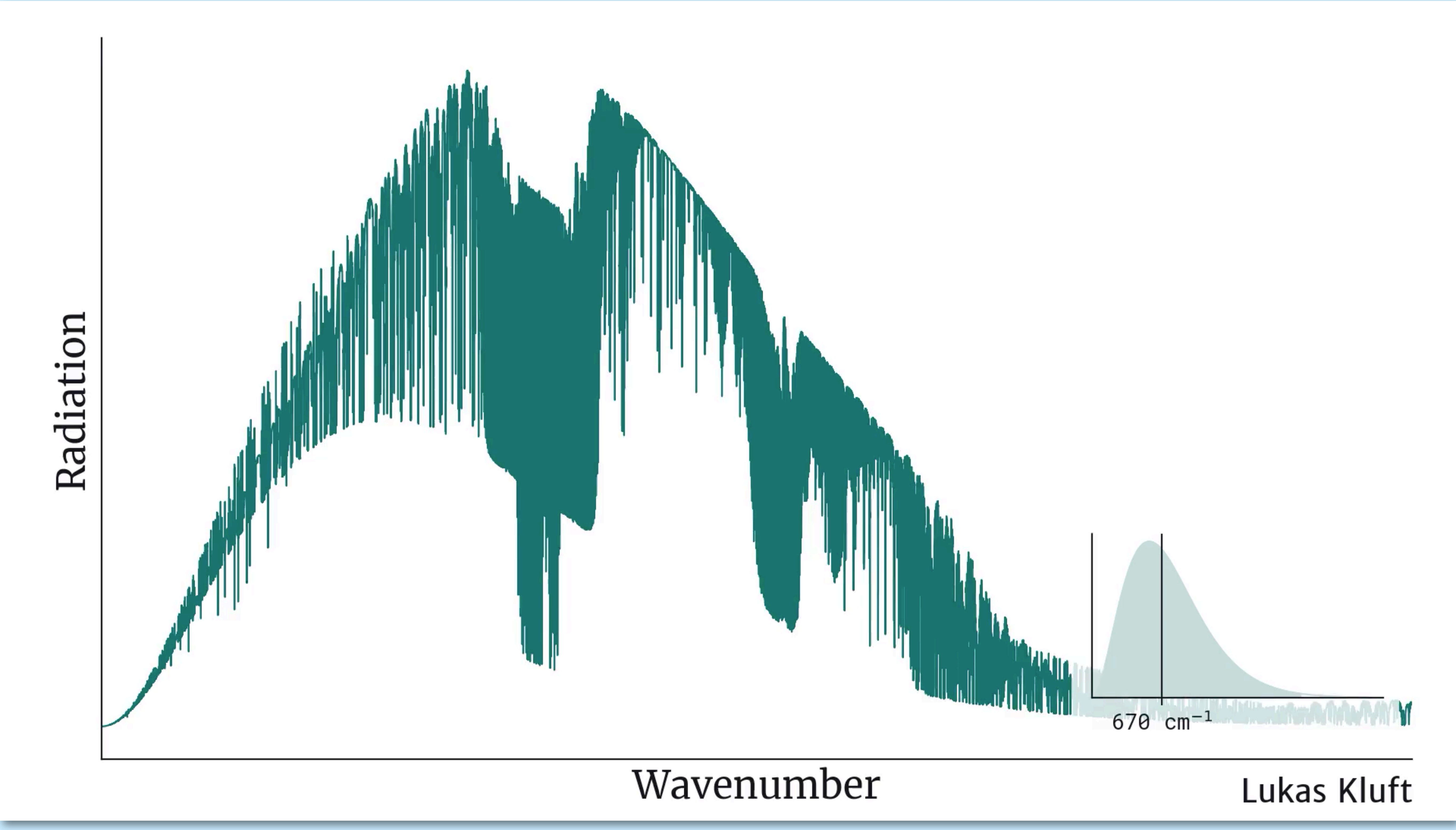
analytic angular integration i.e. two-stream: $\frac{dF_\nu^+}{d\tau} = \gamma_1 F_\nu^+ - \gamma_2 F_\nu^- + S_\nu(\tau_\nu)$

spectral integration: $\int_0^\infty F_\nu^+ d\nu \approx \sum_g^G F_g$

The electromagnetic spectrum is *complicated*



What kind of spectral detail do we need?

Focus on absorption by gases because spectral variation in condensate optical properties is much slower with wavenumber

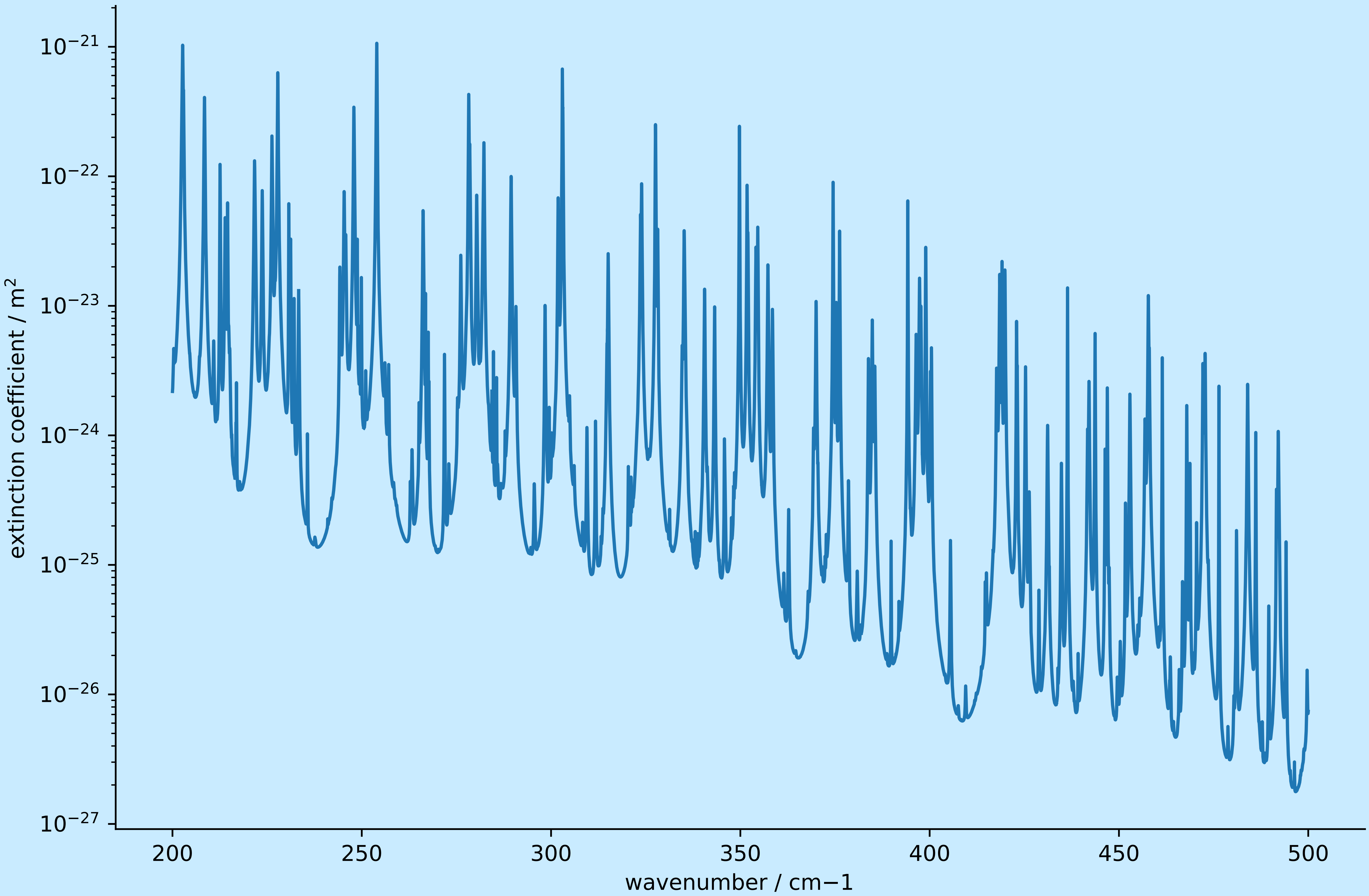
Heating rates dominated by water vapor (especially complicated because heteronuclear, polar, …)

Fluxes also influenced by carbon dioxide, ozone, and methane.

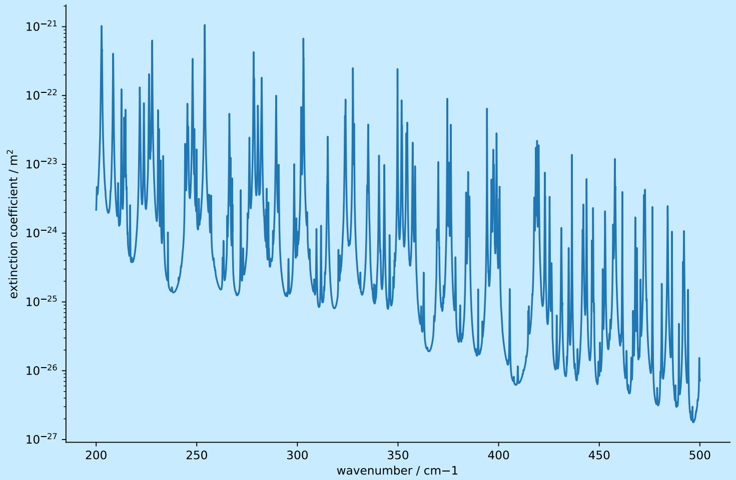
Radiative forcing of climate change further influenced by nitrous oxide, CFCs, carbon monoxide, …

So we need a fair amount of spectral detail, depending on the application
c.f Suki Manabe's work on radiative convective equilibrium

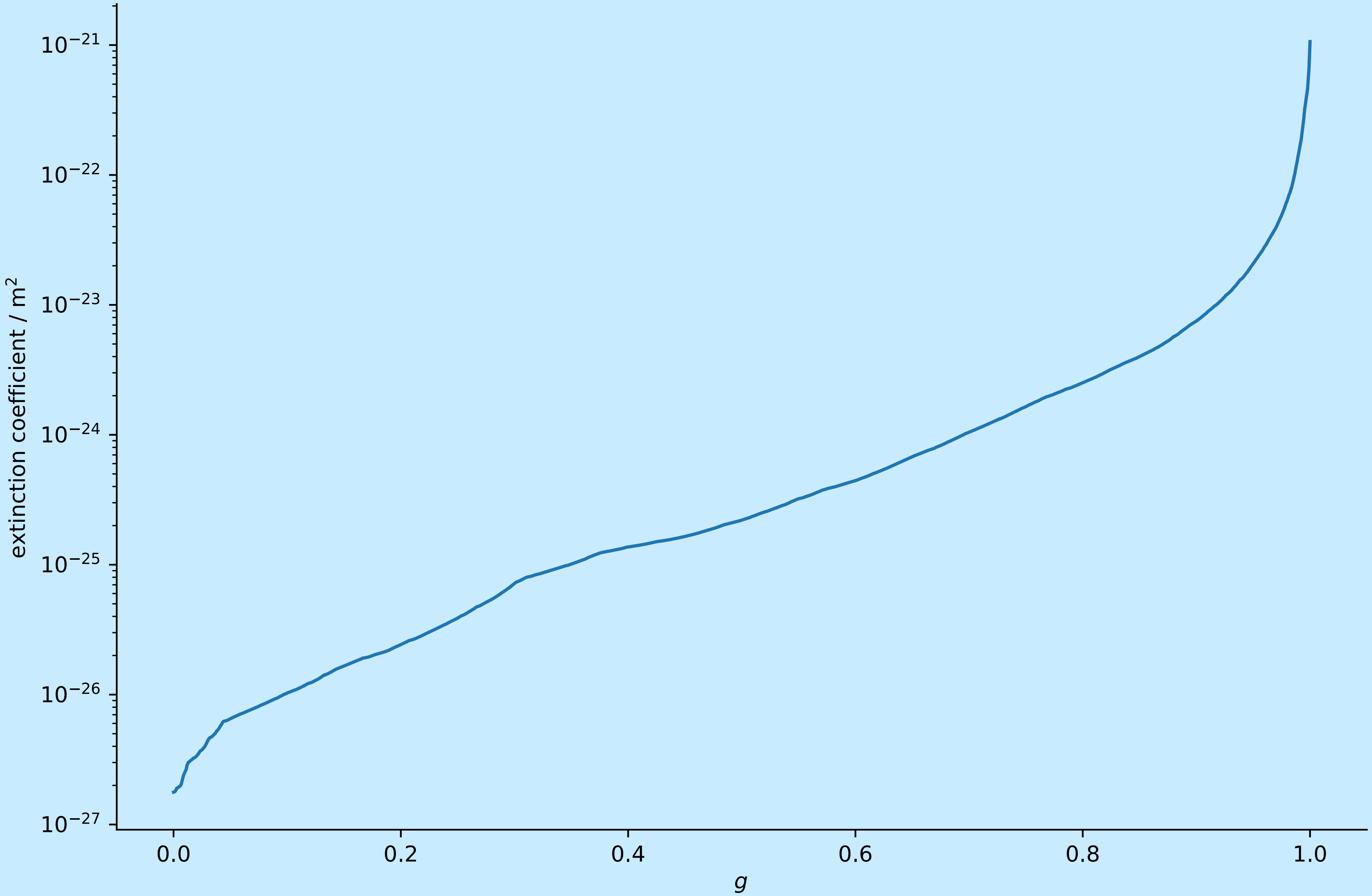
Accommodating spectral detail



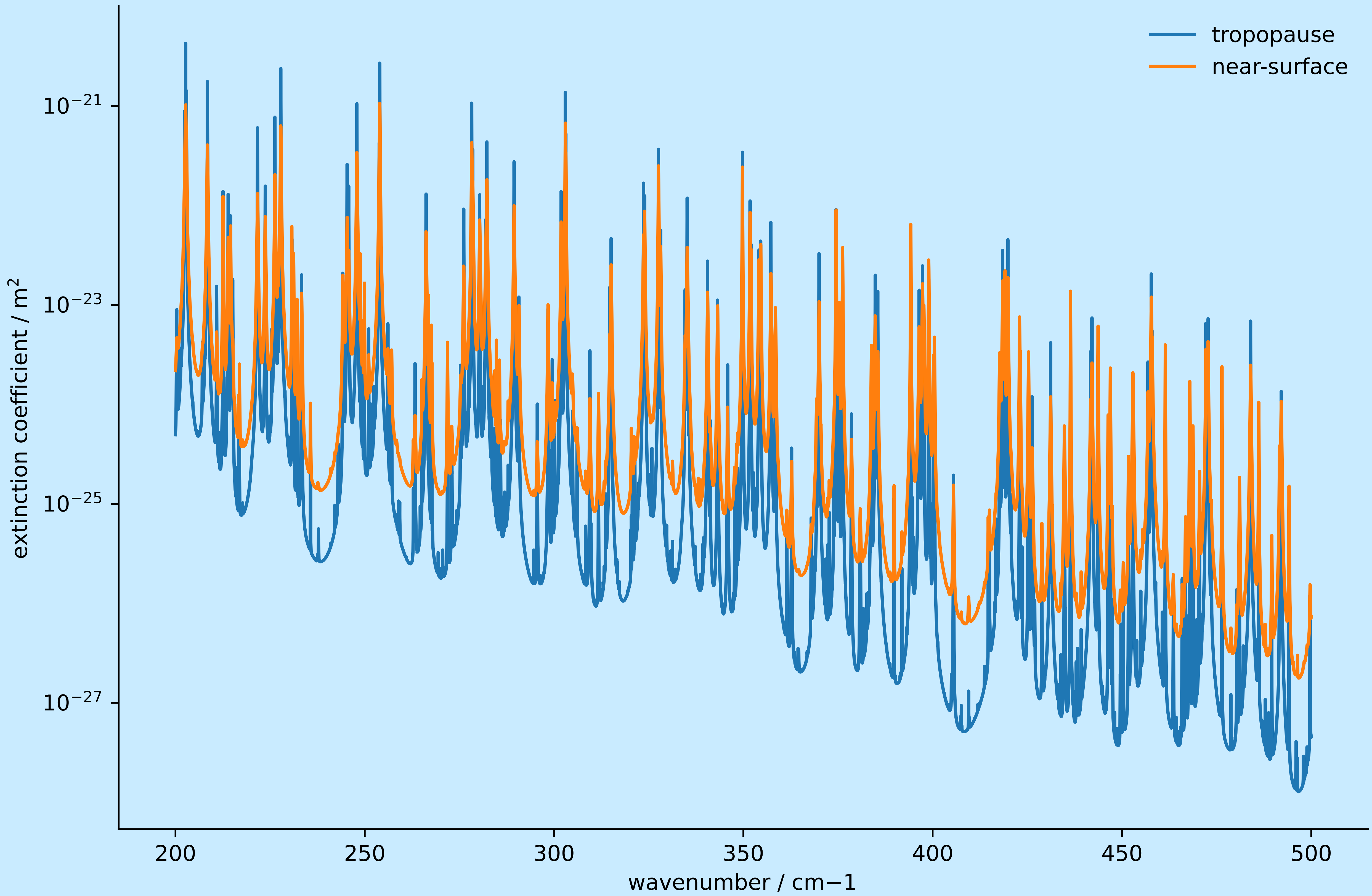
Accommodating spectral detail



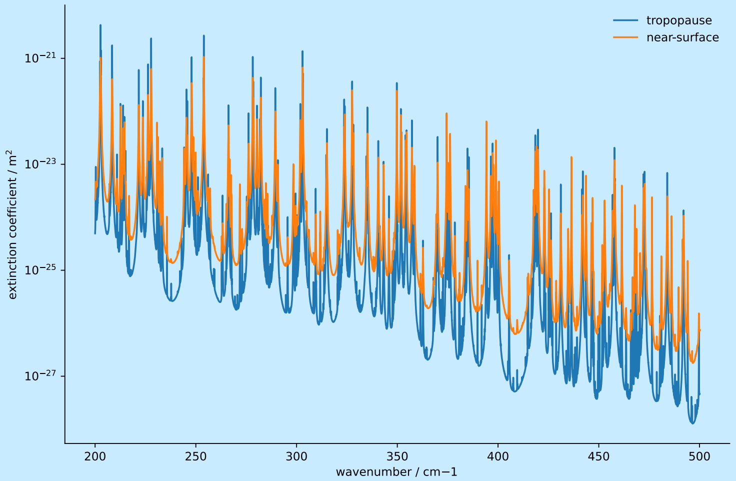
k-distributions make a map $\mathcal{M}(\nu \rightarrow g)$
and average $\beta(g)$

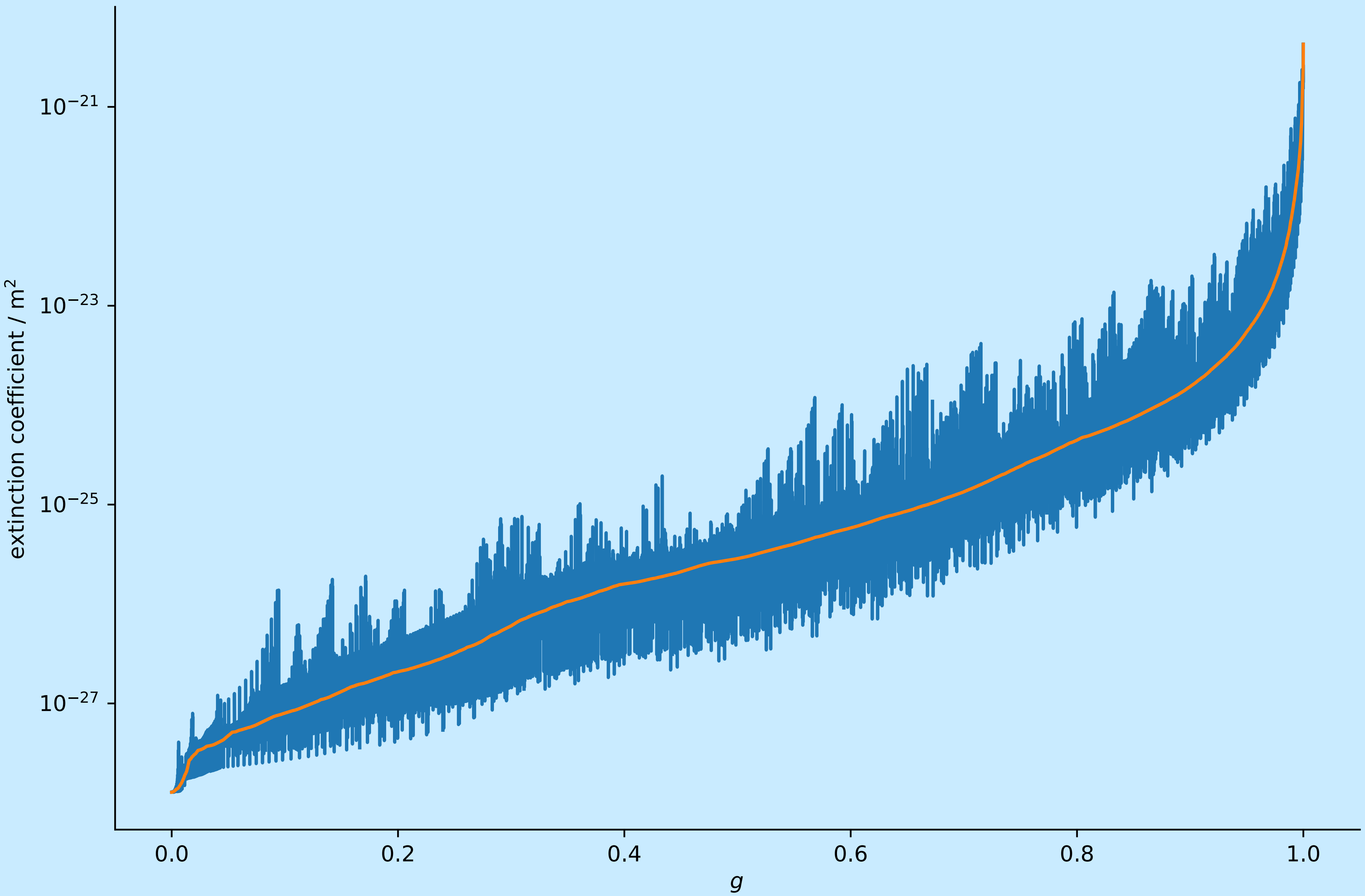


Correlated k-distributions for vertically-varying atmosphere



Correlated k-distributions for vertically-varying atmosphere





No theory = artisanal approaches to correlated-k

Constructing a correlated k-distributions requires many arbitrary choices including

Is $\mathscr{M}(\nu \rightarrow g)$ mapping constant with height, respecting monochromatic physics, or variable, ensuring smooth integrals driven by local extinction?

Separate fits for finite spectral intervals (bands)?

What gas(es) determine the mapping, and how are other gases incorporated?

What are the number and weights of the quadrature points in g?

How is the solar spectrum mapped?

How are the resulting coefficients to be tuned?

See Hogan and Matricardi 2022, doi:10.1029/2022MS003033 for the state-of-the-art

or Czarnecki et al. 2023 doi:10.1029/2023MS003819 for a nifty alternative

Spectral integration in practice

Absorption by gases determines the spectral discretization

Absorption coefficients are tabulated/summarized from reference data across a range of $T, p, (q)$ based on choices for $\chi^i(\xi)$

- Empirical fits to a finite set of conditions - extrapolation is risky

Optical properties for clouds, aerosols, surface are determined on the same spectral grid

- Coupling: requires information from host model and spectral integration method
- Properties are tabulated/summarized as a function of state e.g. r_{eff}, surface roughness, composition, …
- normally assumed constant within finite spectral bands

Allowing for small-scale variability

Many models of the atmosphere allow for sub-grid scale variability
especially in clouds

Radiative fluxes are non-local
Variability in more than one layer requires further information to connect them
(precipitation has the same issue)

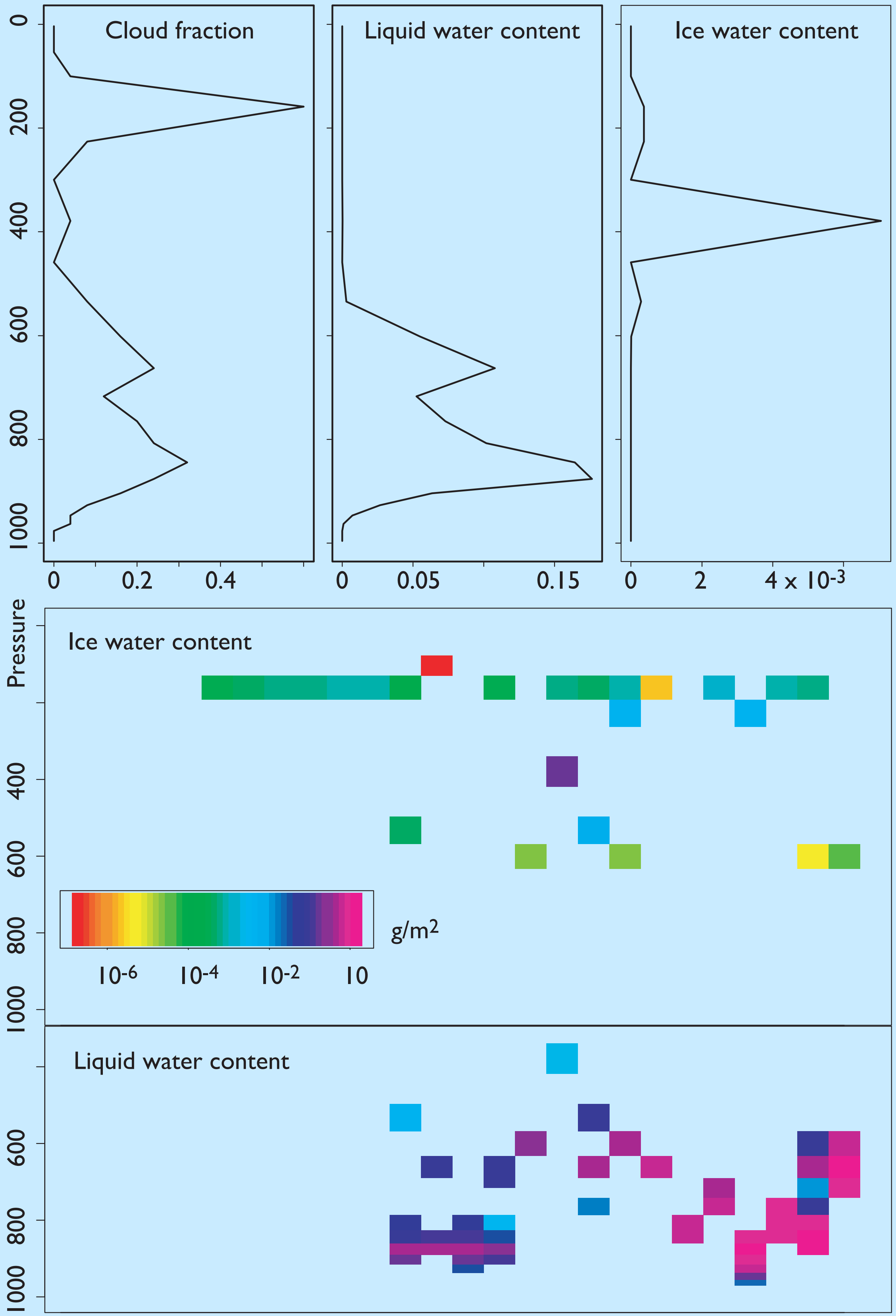
"Cloud overlap" describes how
fractional cloudiness is one layer is related to
fractional cloud in other layers above and below

More generally, computing radiative fluxes requires information about
relationships in the vertical
These relationships are often imposed/assumed for lack of alternatives

A narrow class of assumptions admits analytic treatment

But arbitrary variability within and between layers can be sampled

Pincus et al. 2006: 10.1175/MWR3257.1



Combining integrals

We might compute fluxes for many samples and average

$$\bar{F} = \frac{1}{M} \sum_m \sum_g F_g^m$$

or average samples in M' hopefully-representative subsets

Either approach increases computational cost nearly-linearly

More commonly, sampling over sub-grid state is combined with spectral integration

Each spectral point sees a different random sample: $\bar{F} \approx \sum_g F_g^{m(g)}$

Monte Carlo Independent Column Approximation

McICA introduces noise in estimates of cloud radiative effects

Sampling independently in space and time confines this noise to scales which don't impact resolved flow

Building a radiation parameterization: character and practice

Radiation parameterizations solve governing equations in certain limits
No cartoons - fundamentally different than parameterizations for unresolved flows or processes

The ideas are transparent but implementations
are hungry for empirical data
involved to build
require careful coupling with the host model