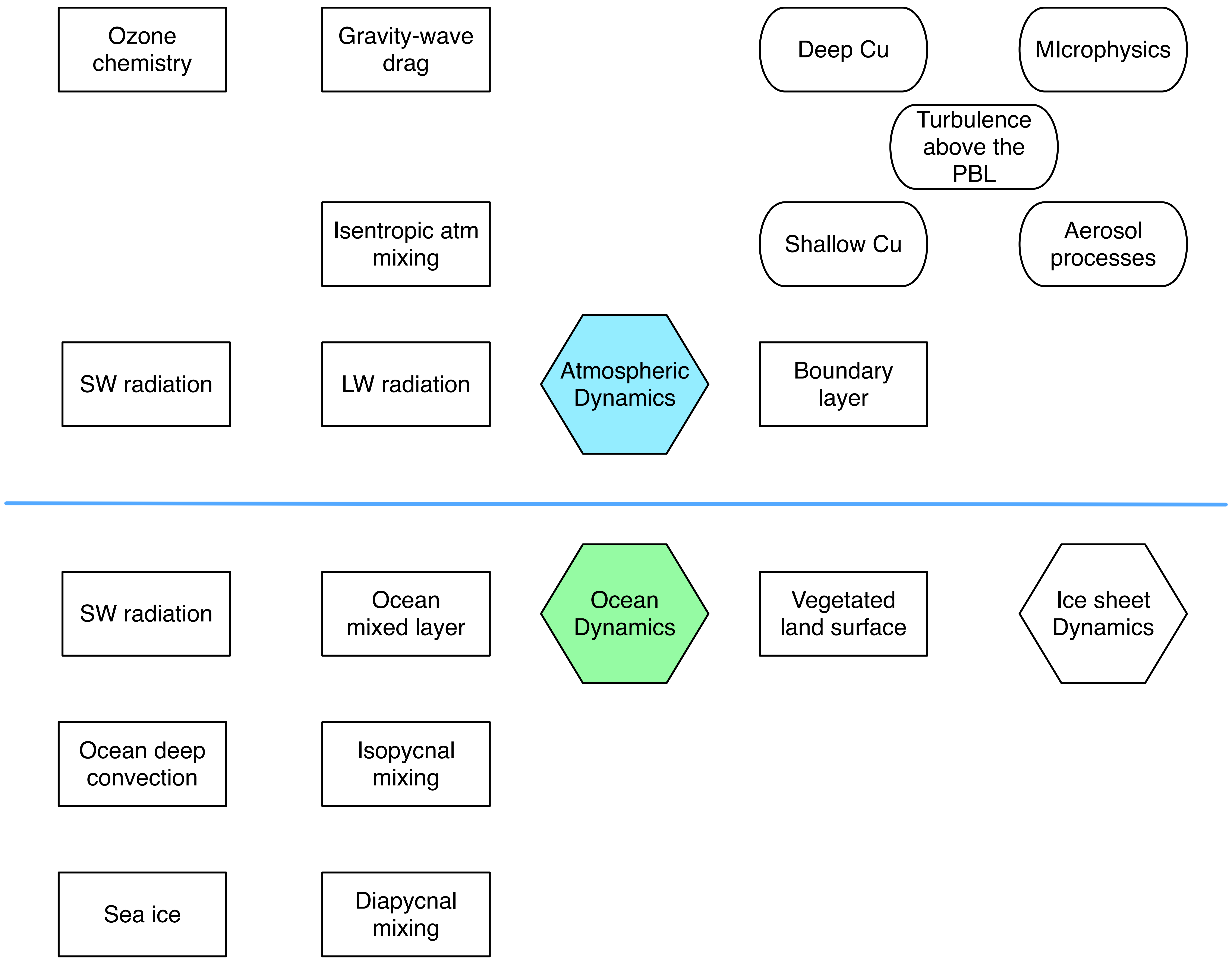
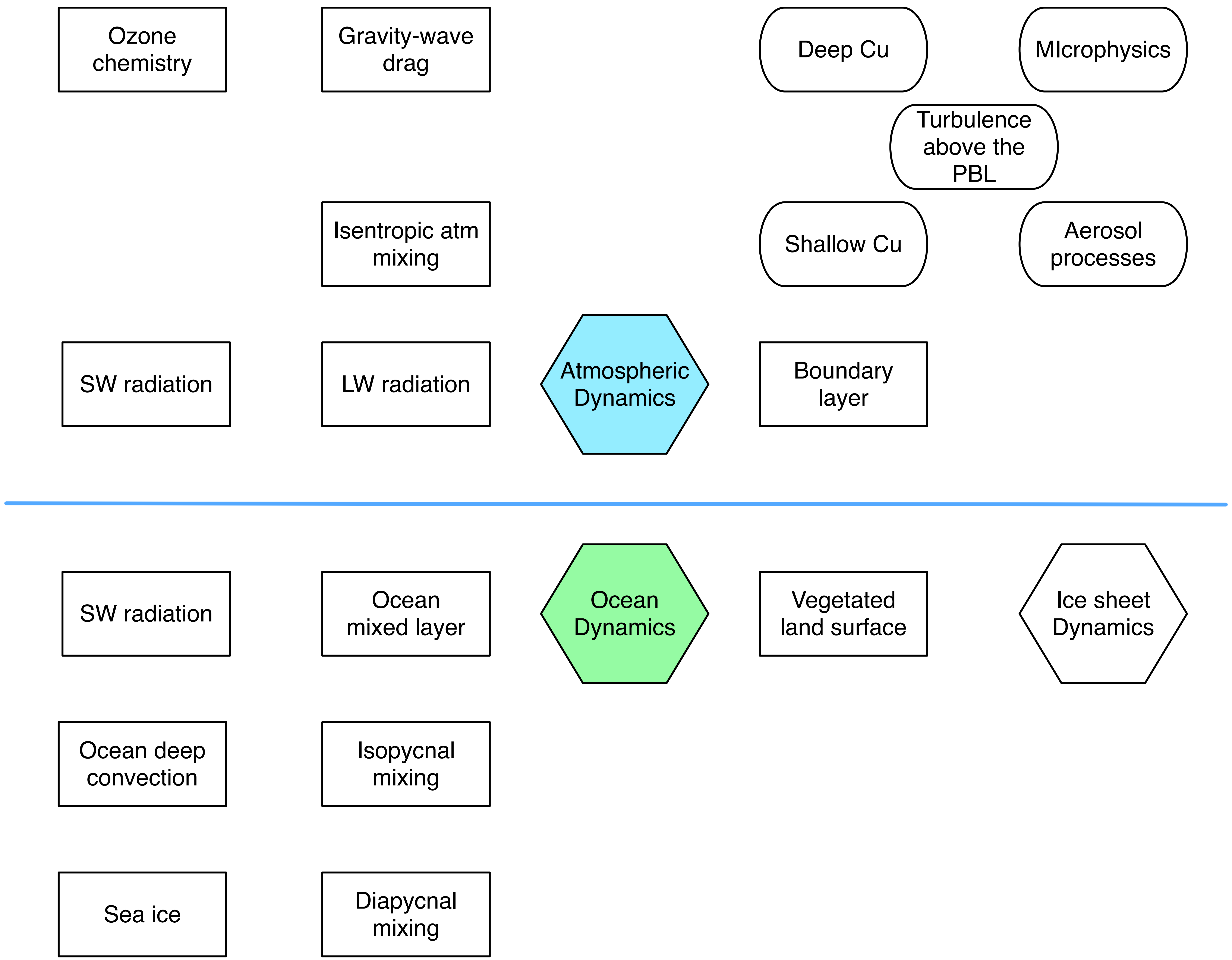
What is to be parameterized?



What is to be parameterized?



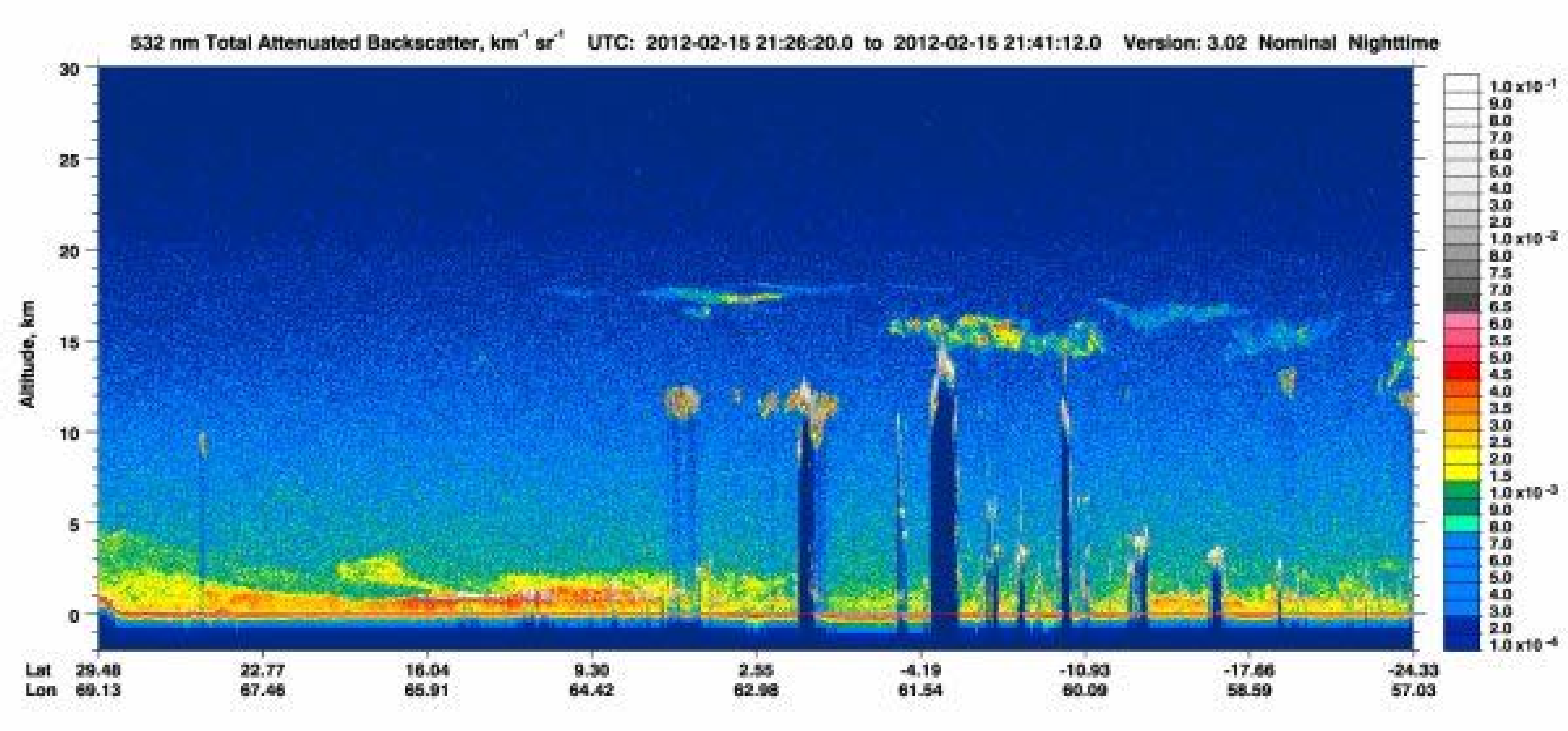
The most difficult issues come from the effects of small-scale dynamics, including turbulence and convection.

This is true even for radiation and microphysics.

Small-scale dynamics is closely tied to clouds.

Definition of the Atmospheric Boundary Layer

The atmospheric boundary layer (ABL), also known as the planetary boundary layer (PBL), is the portion of the lower atmosphere that is directly influenced by *turbulent* exchanges with the surface.



Turbulence and vorticity

Turbulence

Turbulence in deep convective clouds


Synoptic-scale turbulence

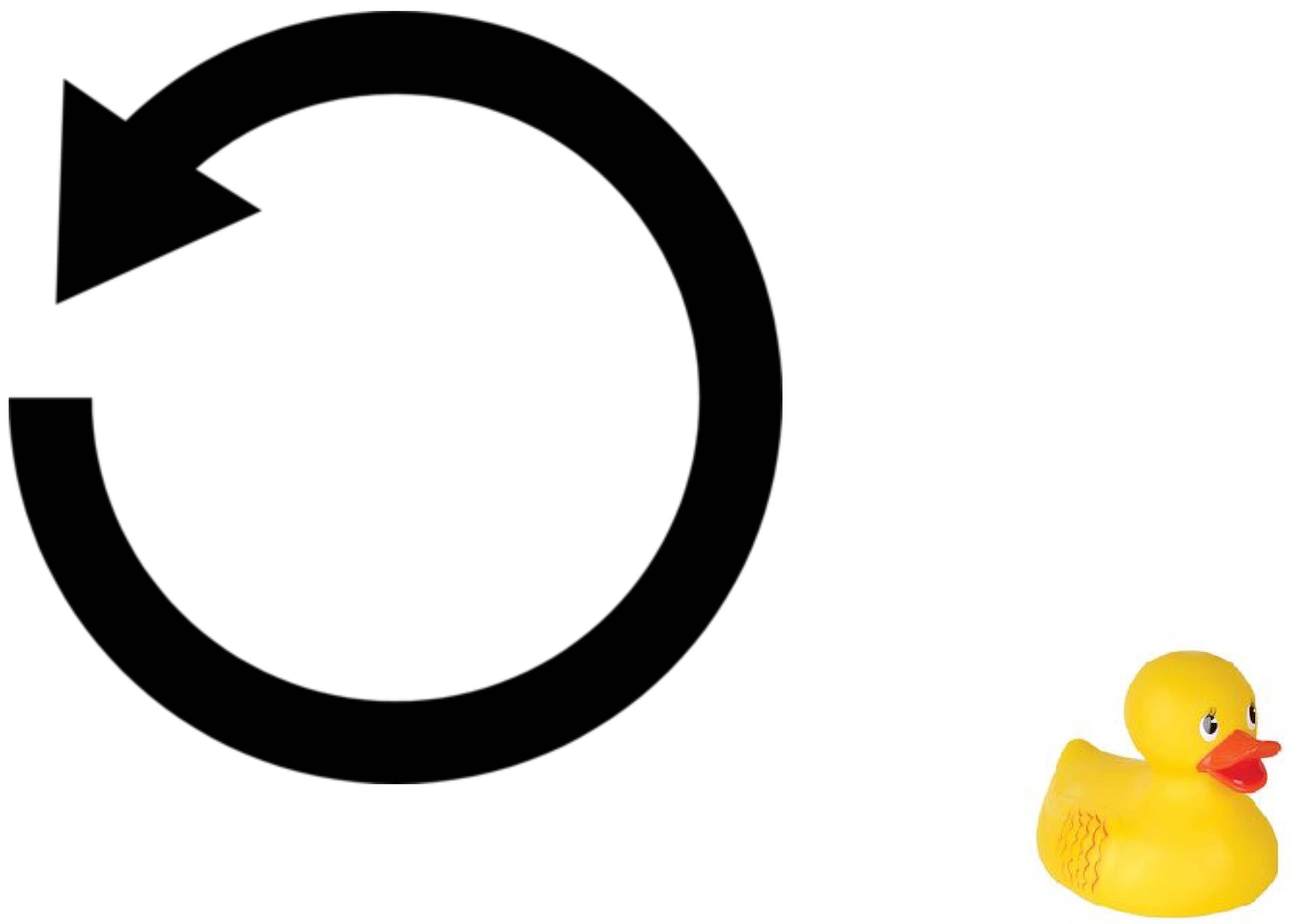
Turbulence in the Jovian atmosphere

Turbulence is made of vortices.

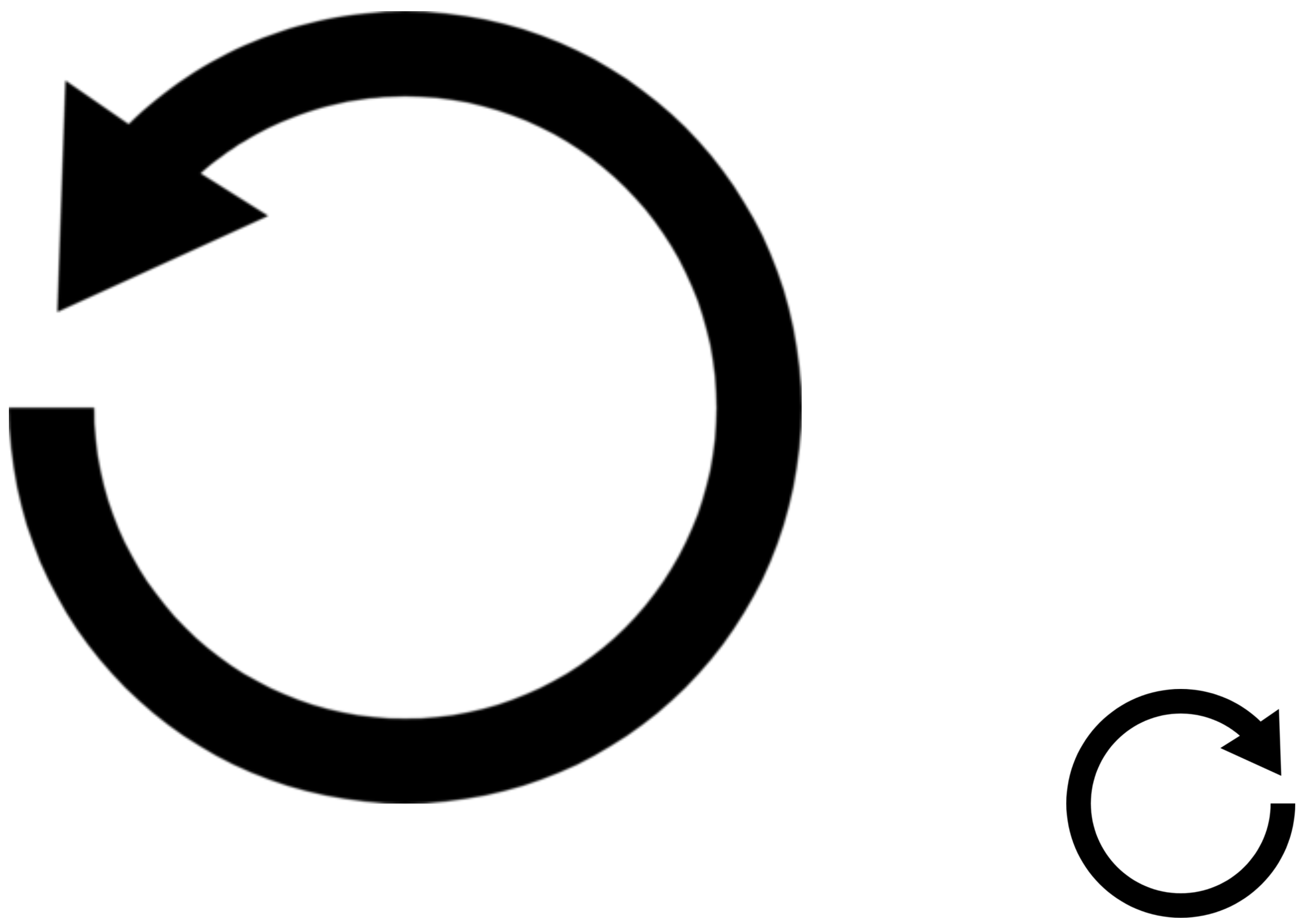
Vortices almost seem to be alive.

- They are identifiable “discrete objects” in an otherwise smooth fluid.
- They persist.
- They move.
- They dance with each other.

Vortices can advect things.

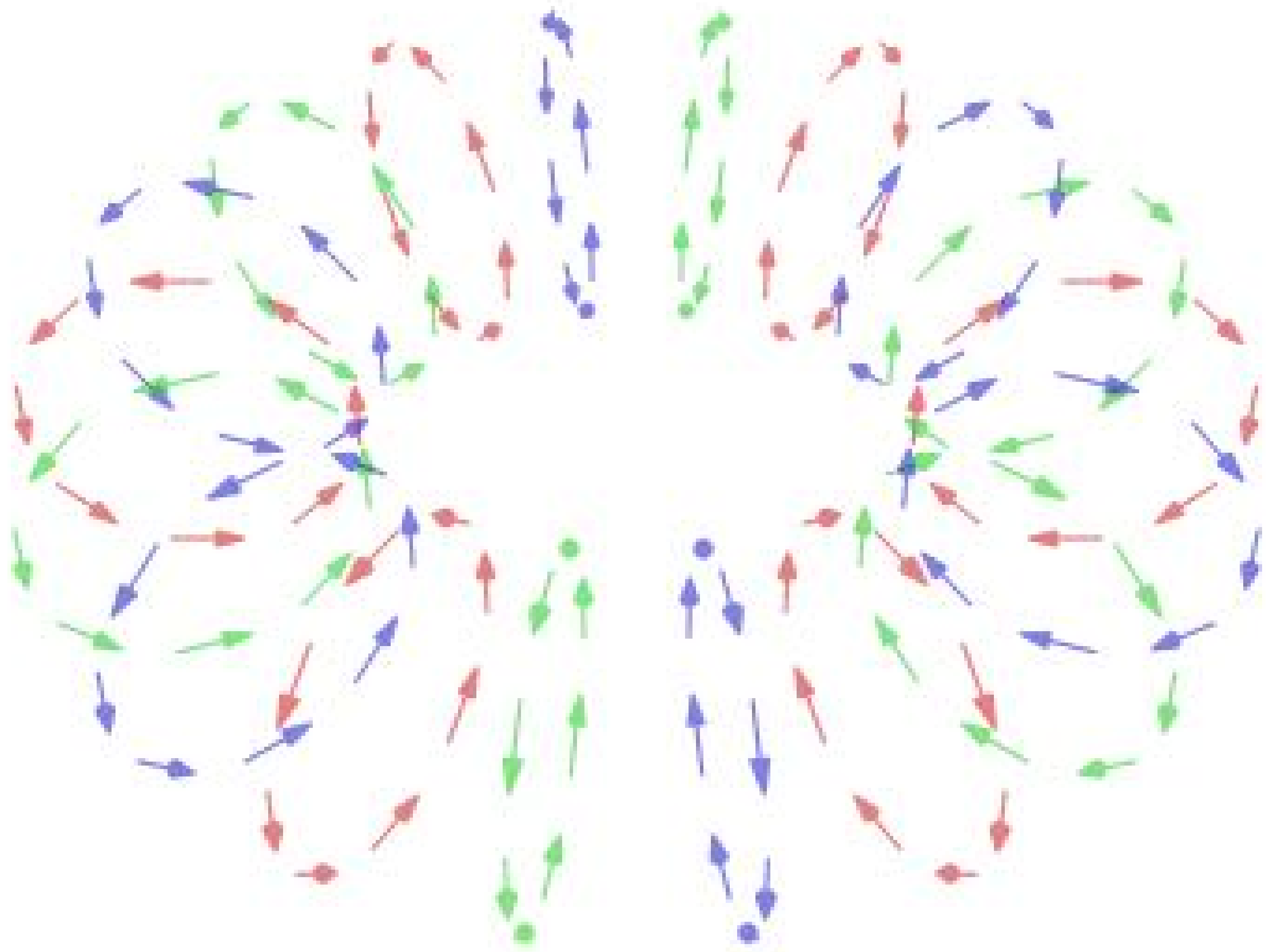


Vortices can advect each other.



Topology

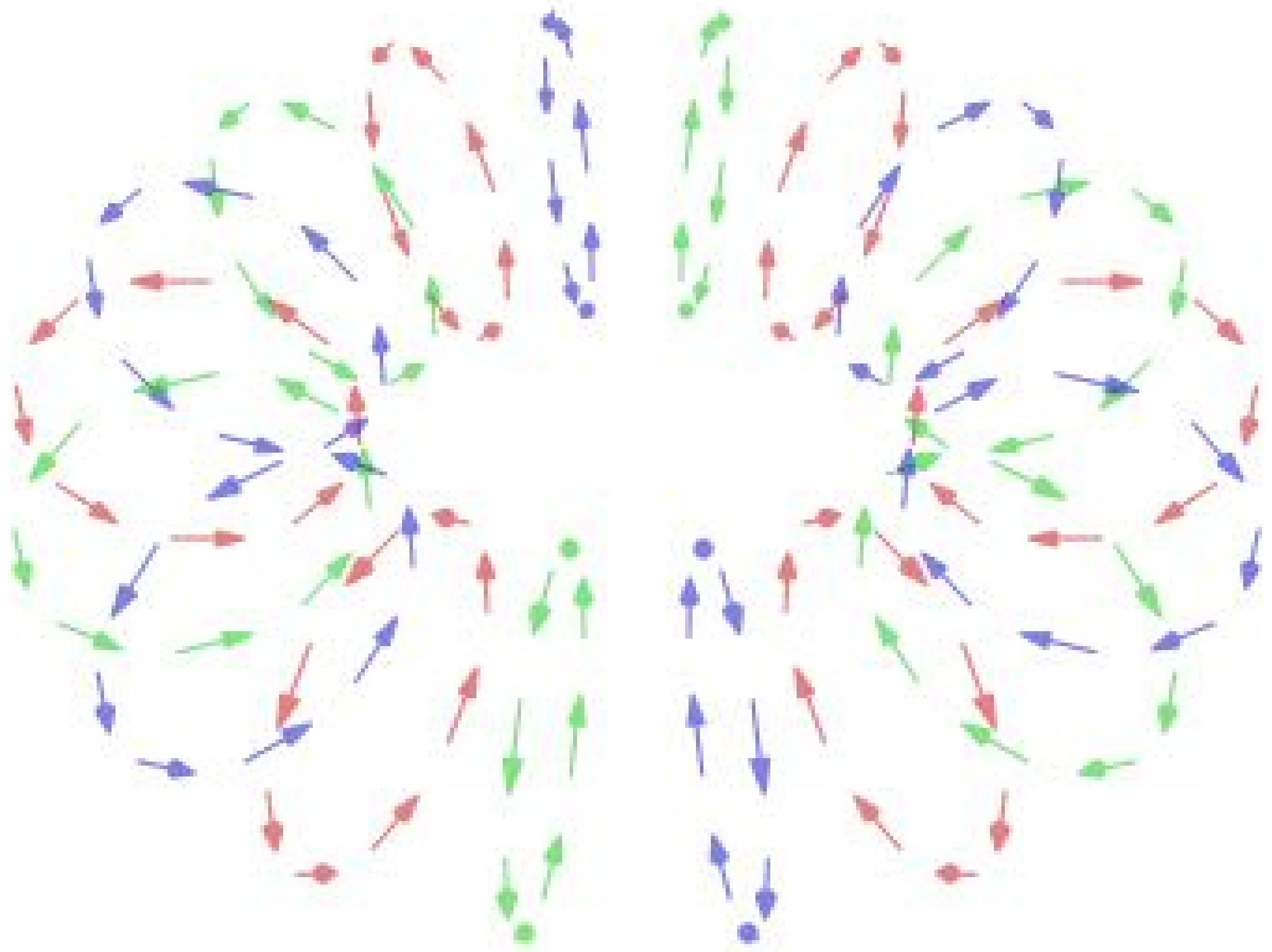
Vortex line



Vortex ring

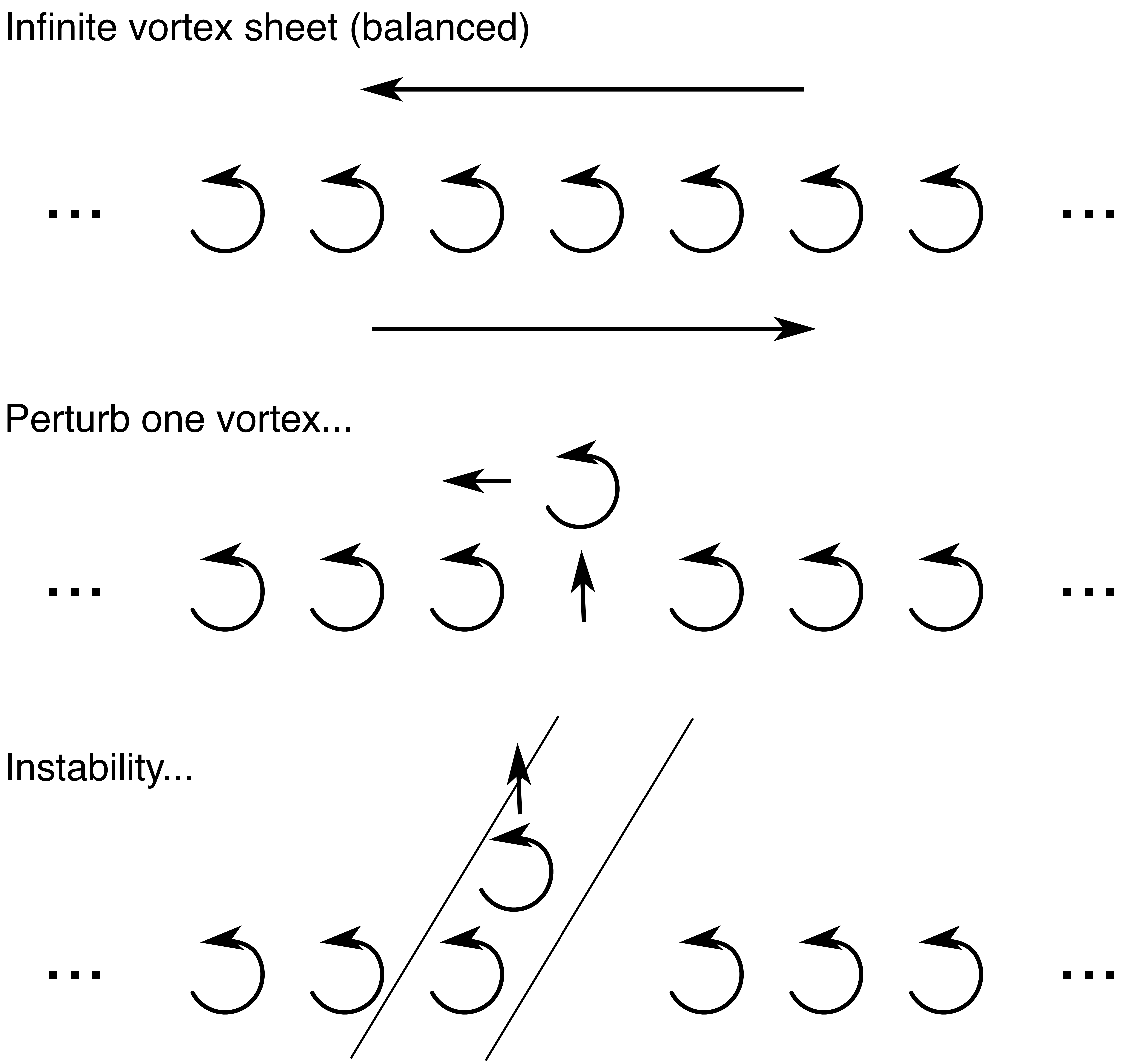
Topology

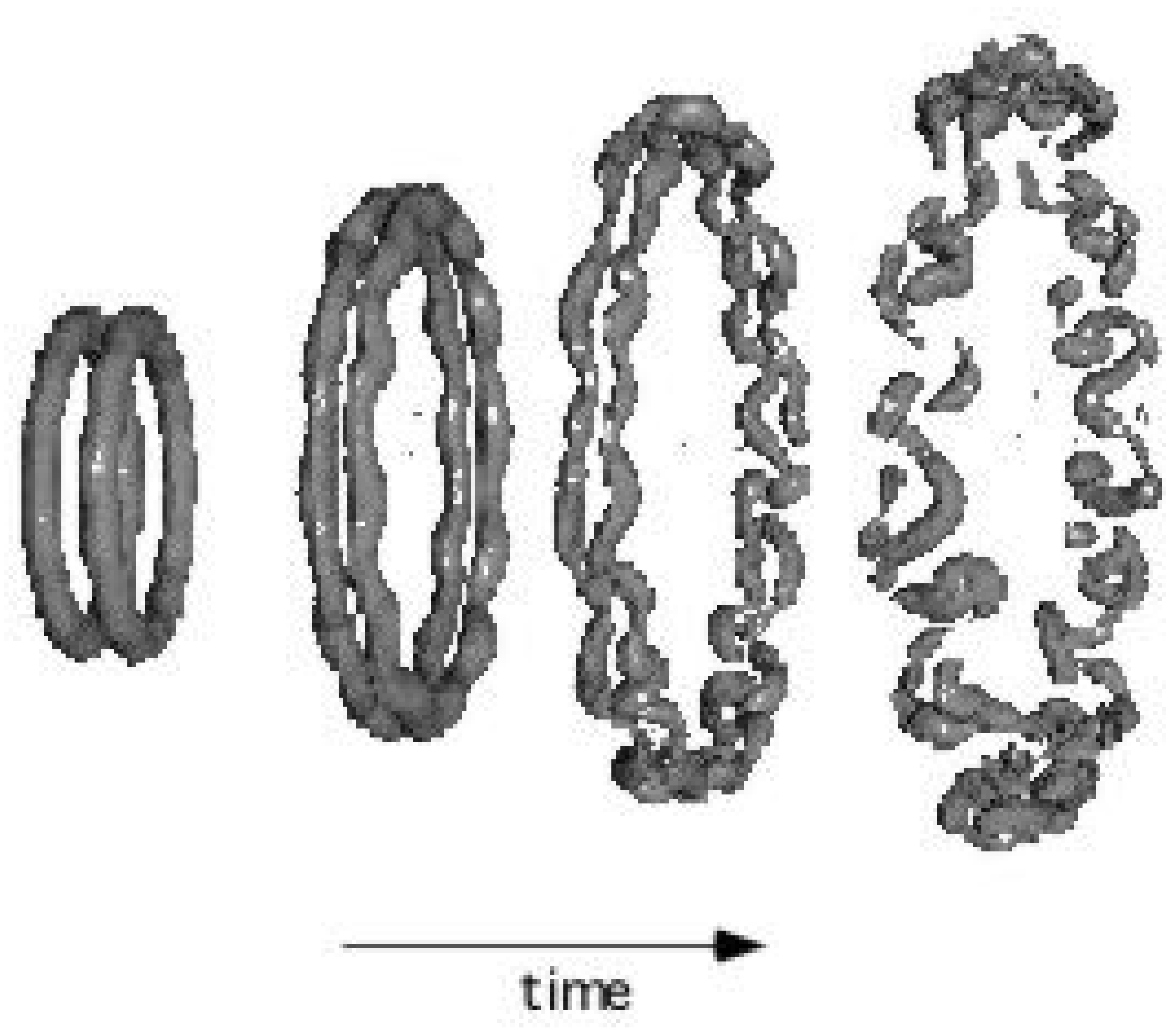
Vortex line



Vortex ring

Big vortices make little vortices.

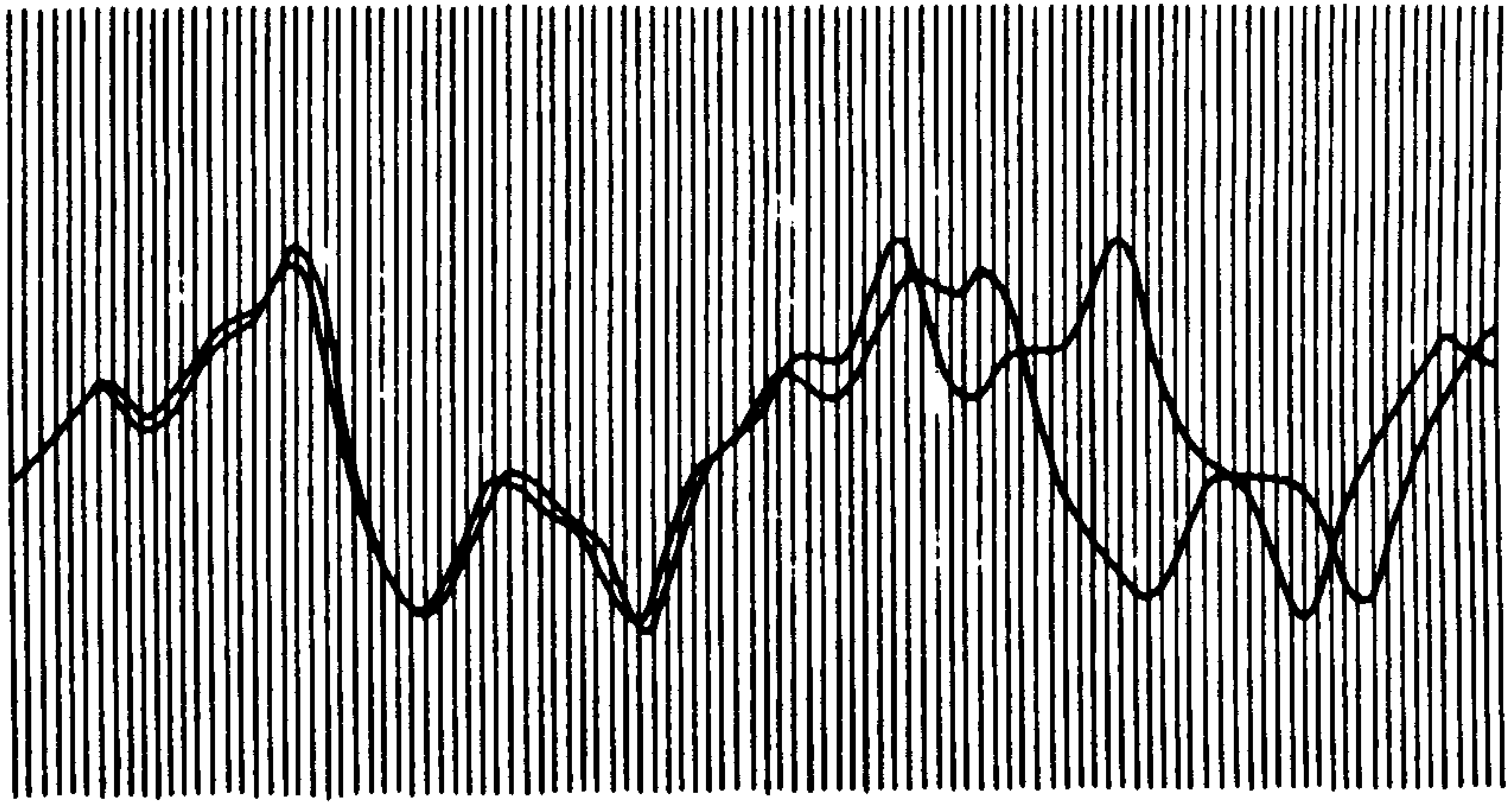




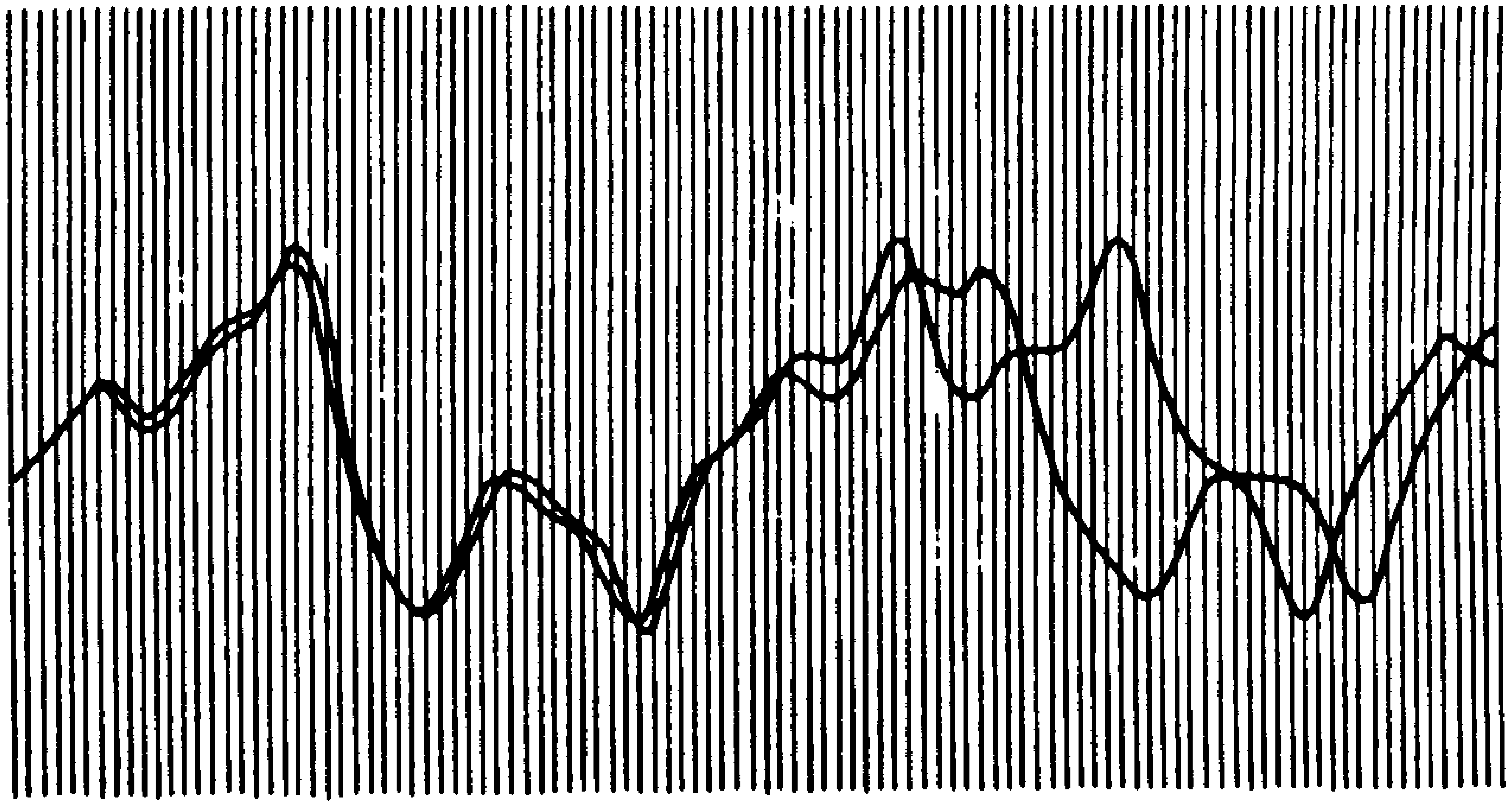
Big whirls have little whirls
That feed on their velocity.
Little whirls have lesser whirls
And so on to viscosity.

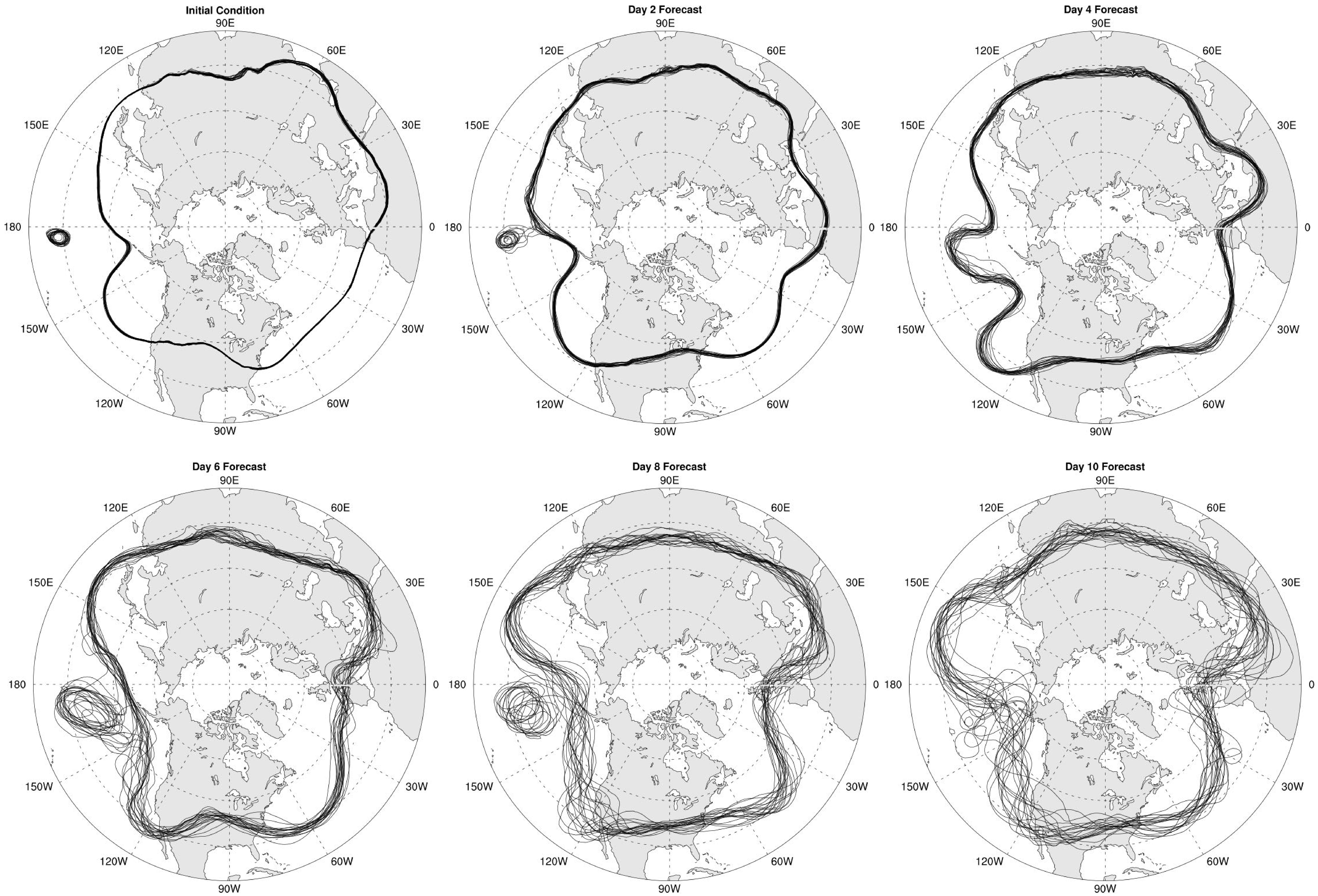
— Lewis Richardson

Turbulence is chaotic.



Turbulence is chaotic.





Initial Condition
Day 2 Forecast
Day 4 Forecast
Day 6 Forecast
Day 8 Forecast
Day 10 Forecast
90E
120E
60E
150E
30E
180
0
150W
30W
120W
60W
90W

Turbulence mixes.

The spinning motor makes a vortex.
The vortex makes baby vortices.
The vortices make turbulence.
The turbulence mixes.

Turbulence is hard.

According to an apocryphal story, Werner Heisenberg was asked what he would ask God, given the opportunity. His reply was: "When I meet God, I am going to ask him two questions: Why relativity? And why turbulence? I really believe he will have an answer for the first."

A similar witticism has been attributed to Horace Lamb in a speech to the British Association for the Advancement of Science: "I am an old man now, and when I die and go to heaven there are two matters on which I hope for enlightenment. One is quantum electrodynamics, and the other is the turbulent motion of fluids. And about the former I am rather optimistic."

https://en.wikipedia.org/wiki/Turbulence

Heisenberg

Lamb

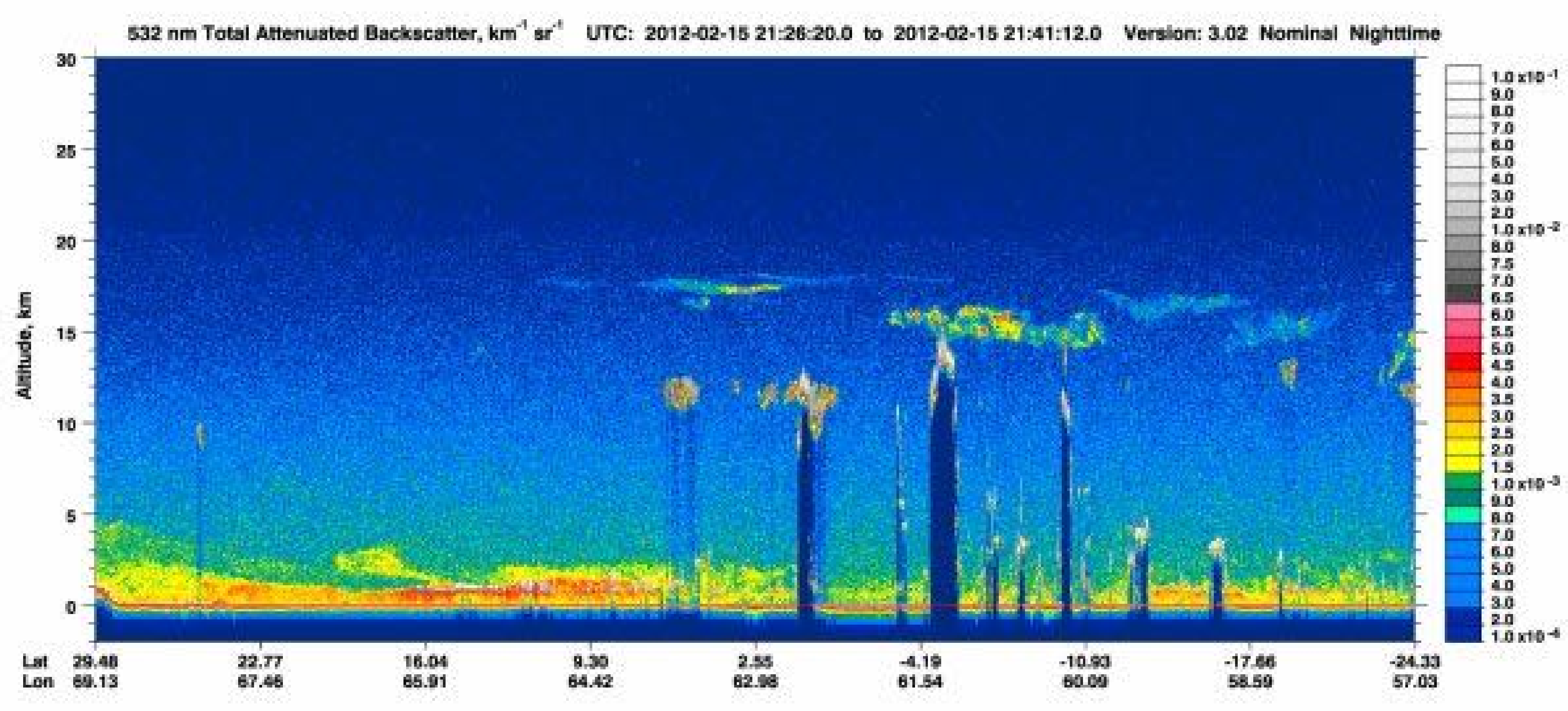
Vorticity across scales

Large-scale motions are controlled by the vertical component of the vorticity.

Small-scale motions are controlled by the horizontal vorticity vector.

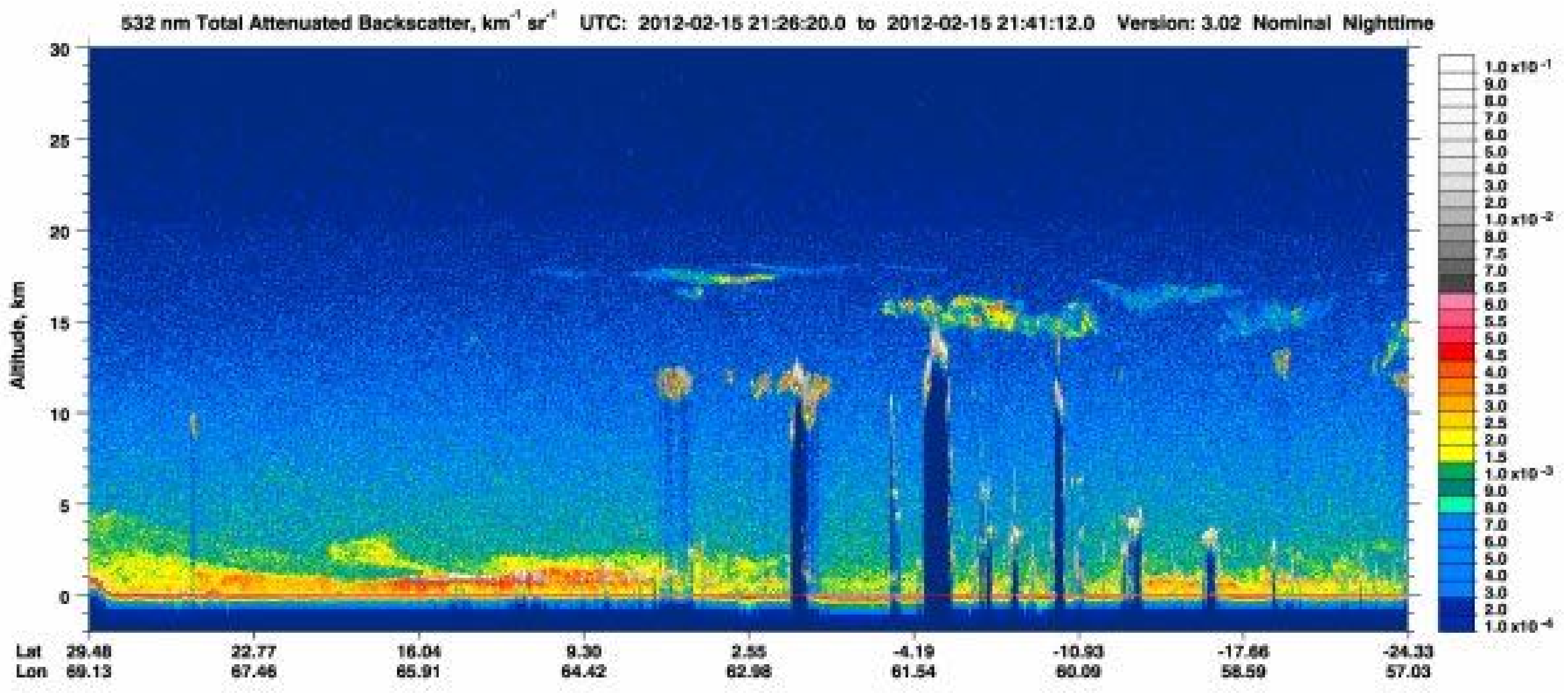
Some basic questions for boundary layer parameterization

- What determines the surface fluxes?
- What sets the depth of the boundary layer?
- What happens when clouds form inside the boundary layer?
- How does the boundary layer interact with cumulus convection?



Five approaches to boundary layer parameterization

- Similarity theories
- Eddy diffusion
- Mixed-layer models
- Higher-order closure
- Mass fluxes



We will discuss all of these, one by one.

Reynolds Averaging

It is neither feasible nor desirable to consider in detail all of the small-scale fluctuations that occur in the turbulent boundary layer. For this reason, we ``filter'' or "average'" or "smooth'" the data, and attempt to describe only the resulting *statistics* of the flow. Here we follow the approach of ``Reynolds Averaging,'' which takes its name from Osborne Reynolds, the famous aerodynamicist who invented it in the late 19th century.

Suppose that

$$\frac{\partial}{\partial t}(\rho q) + \nabla \cdot (\rho \mathbf{V} q) = S_q$$

where t is time and S_q is a source of q. The quantity inside the divergence operator is a flux of q due to an advecting mass flux $\rho\mathbf{V}$.

We now *decompose* each of the dependent variables as follows:

$$q = \overline{q} + q', \quad \mathbf{V} = \overline{\mathbf{V}} + \mathbf{V}', \quad S_q = \overline{S_q} + S_q' \ .$$

This is called the "`Reynolds decomposition." Here an overbar denotes an averaging operator that must be defined defined. Substitution gives

$$\frac{\partial}{\partial t}\left[\rho\left(\overline{q} + q'\right)\right] + \nabla \cdot \left[\rho\left(\overline{q} + q'\right)\left(\overline{\mathbf{V}} + \mathbf{V}'\right)\right] = \overline{S_q} + S_q' \ .$$

Here we have neglected additional terms that arise from variations of the density of the air. We want to *choose* the averaging operator in such a way that the average of this equation reduces to

$$\frac{\partial}{\partial t}(\rho\overline{q}) + \nabla \cdot \left[\rho\left(\overline{q}\ \overline{\mathbf{V}} + \overline{q'\mathbf{V}'}\right)\right] = \overline{S_q} \ .$$

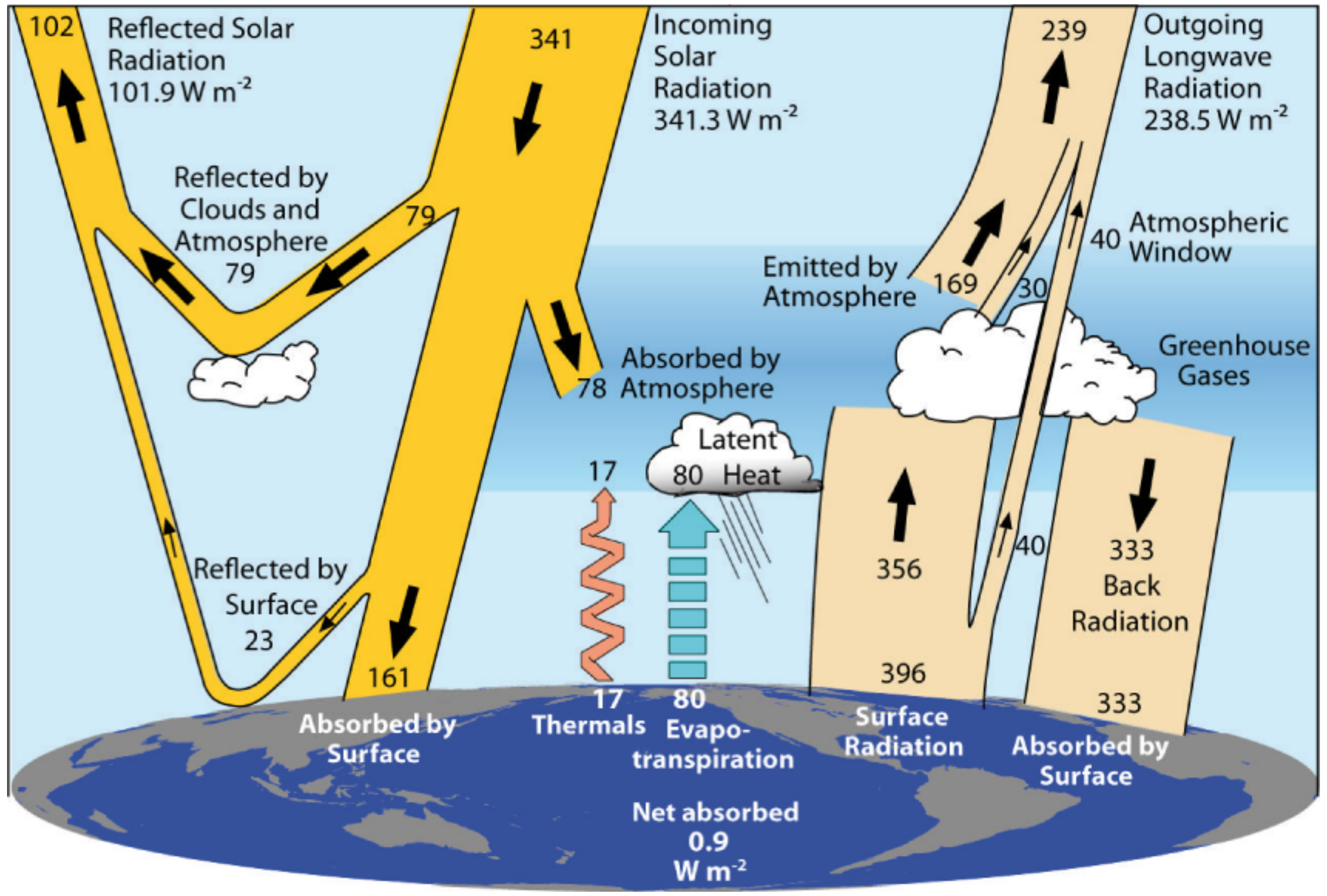
Here the flux divergence term has two parts. The first involves the product of two averages, and the second involves the average of the product of two primes. The quantity $\overline{q'\mathbf{V}'}$ is the flux due to the product of two fluctuations. It can be called the "turbulent flux of q," assuming that the fluctuations are associated with turbulence. Note, however, that fluctuations can also arise from other things, such as waves.

Surface fluxes

$$\rho\overline{w'T'} = \rho c_T \left|\overline{\mathbf{v}}\right|_S \left(T_g - T_a\right)$$

$$\rho\overline{w'q'} = \rho c_q \left|\overline{\mathbf{v}}\right|_S \left(q_g - q_a\right)$$

$$\rho\overline{w'\mathbf{v}'} = -\rho c_D \left|\overline{\mathbf{v}}\right|_S \overline{\mathbf{v}}$$



These are called "bulk aerodynamic formulas."

Monin-Obukhov similarity theory (MOST)

Andrei Sergeevich Monin and Alexander Mikhailovich Obukhov

MOST is used in all (not just most) Earth System Models.

Dimensional parameters

Monin and Obukhov (1954) hypothesized that, sufficiently near the surface, the statistical structure of the PBL turbulence is, to a good approximation, determined by the following seven parameters

$$z\ ,\frac{g}{\theta_{v0}}\ ,u_*\ ,\theta_*\ ,z_0\ ,\overline{u}\ ,\overline{\theta_v}$$

Here the "friction velocity" is given by

$$u_* \equiv \sqrt{\frac{\tau_S}{\rho_S}} \geqslant 0$$

and the "friction temperature," θ_*, satisfies

$$\kappa u_* \theta_* \equiv -\left(\overline{w'\theta_v'}\right)_S$$

A key parameter is the Monin-Obukhov length, which is defined by

$$L \equiv -\frac{u_*^3}{g\left(w'\overline{\theta_v'}\right)_S/\theta_{v0}}$$

Similarity assumptions

$$\frac{\partial \overline{u}}{\partial z} = \frac{u_*}{\kappa z}\phi_m(\zeta)$$

Nondimensional shear

$$\frac{\partial \overline{\theta_v}}{\partial z} = \frac{\theta_*}{z}\phi_h(\zeta)$$

Nondimensional stratification

Here θ_v is the virtual potential temperature, $\zeta \equiv z/L$, and the von Karman constant, κ, is defined so that $\phi_m(0) = 1$.

The boundary layer is said to be "unstable" for $\zeta < 0$ (upward surface flux of θ_v), "stable" for $\zeta > 0$ (downward surface flux of θ_v), and "neutral" for $\zeta = 0$ (no surface flux of θ_v).

For the neutral boundary layer, integration gives

$$\overline{u}(z) = \frac{u_*}{\kappa}\ln\frac{z}{z_0}$$

where z_0 is the "roughness length." This is the *definition* of z_0.

Observed similarity functions

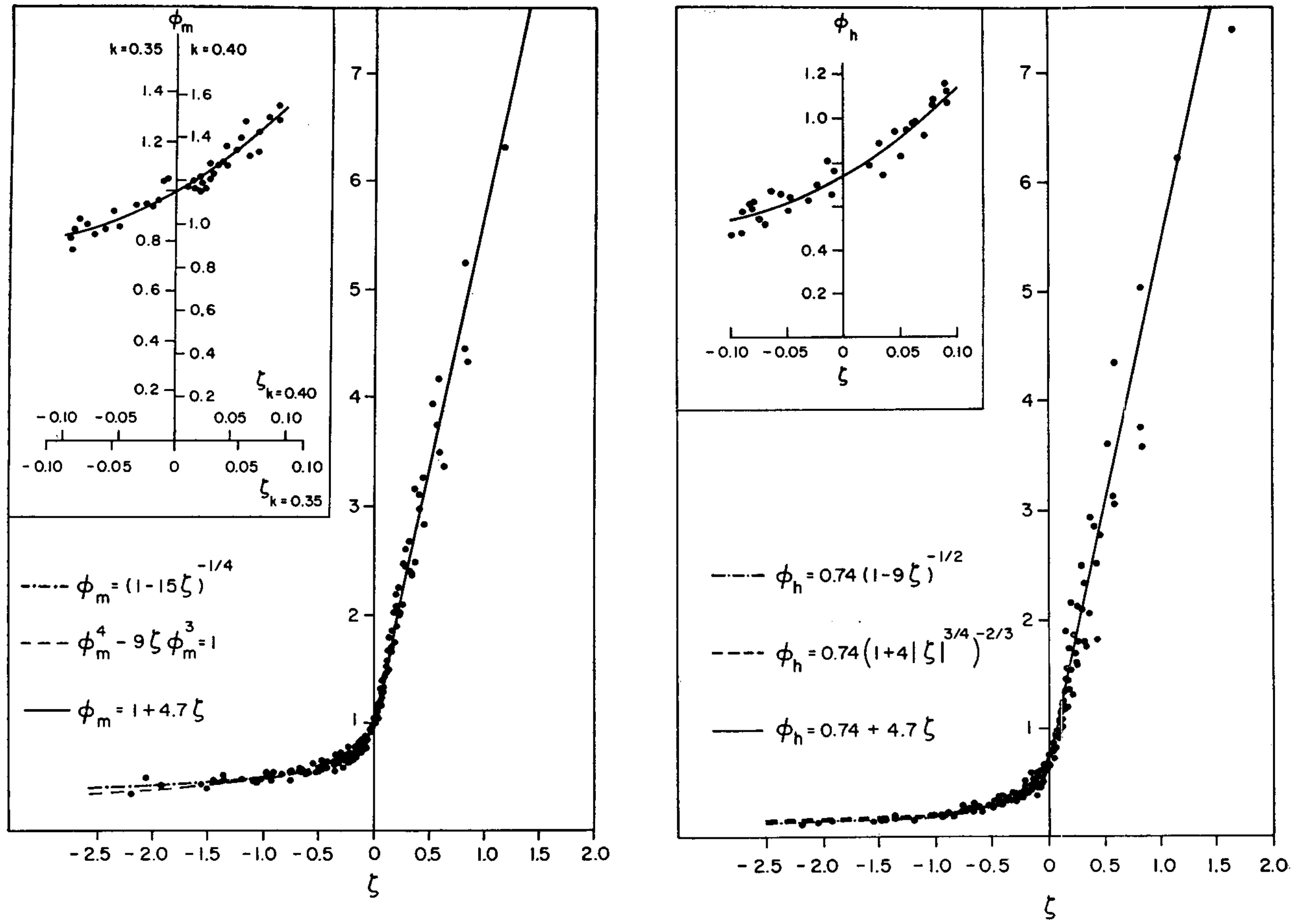


Figure 13.3: The observed forms of $\phi_m(\zeta)$ and $\phi_h(\zeta)$, from Businger et al. (1971).

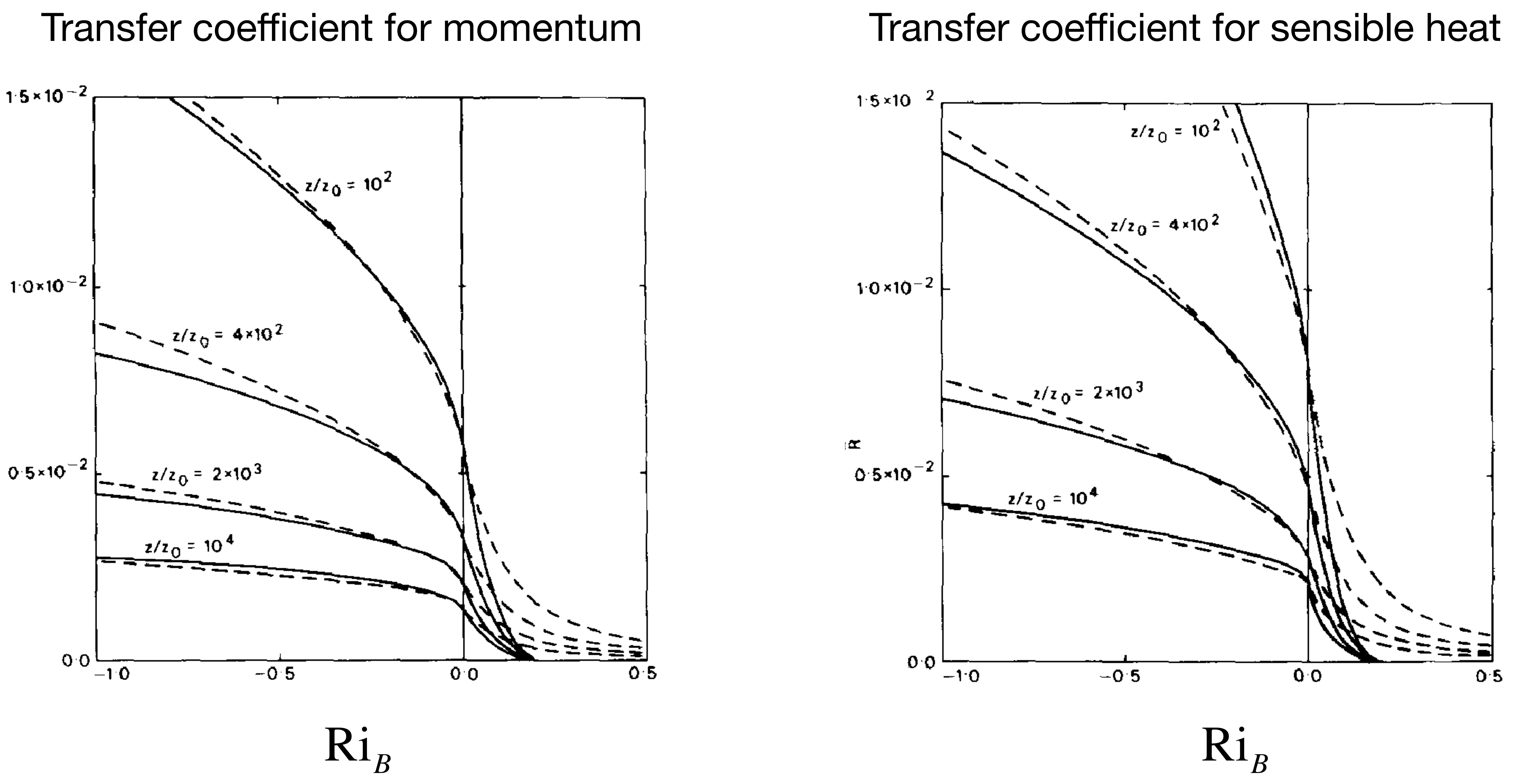


Figure 13.5: The transfer coefficients for momentum and sensible heat (vertical axes), as inferred plotted as functions of the bulk Richardson number (horizontal axes) and z/z_0. From Louis (1979).

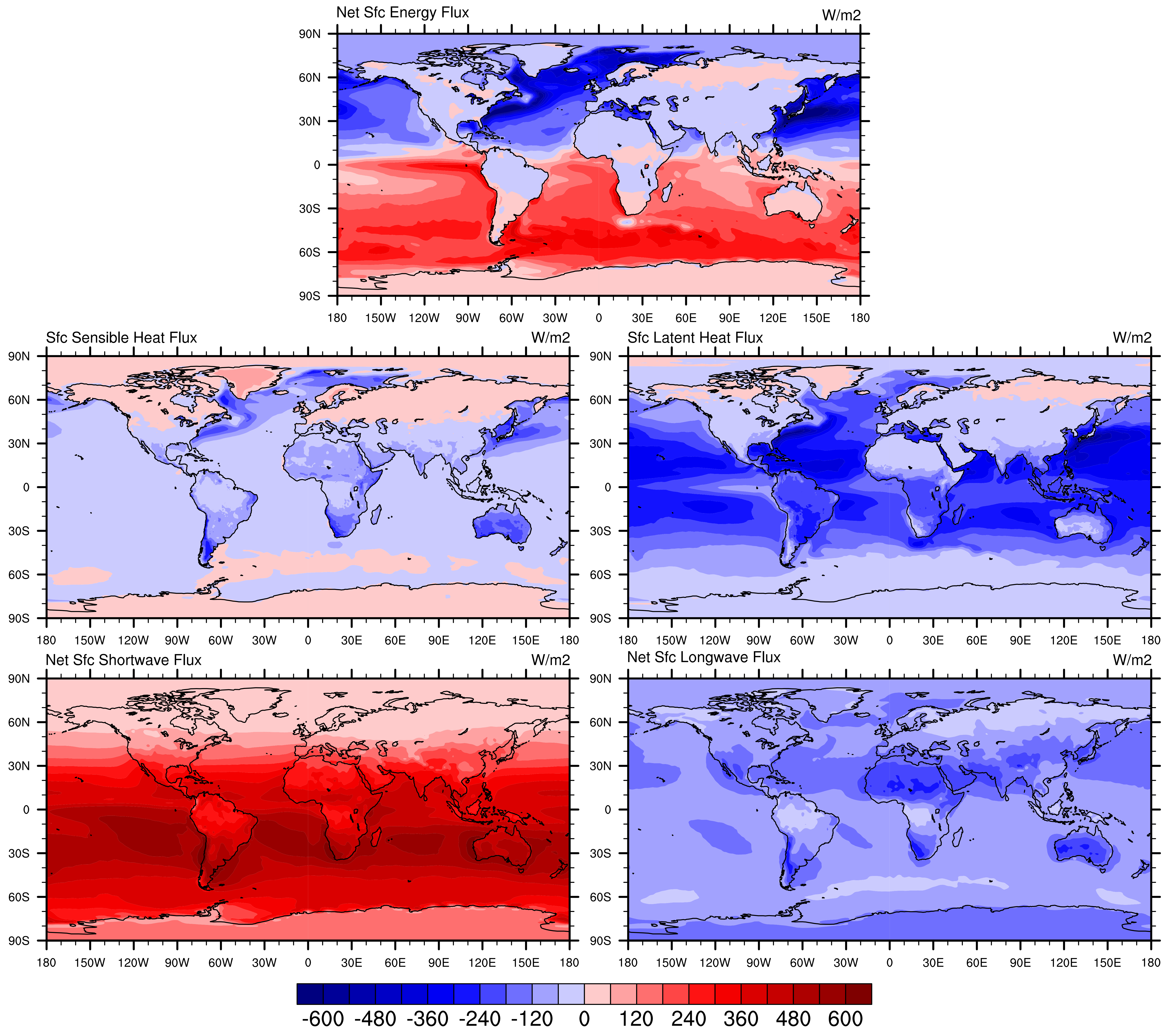
$$\mathrm{Ri}_B \equiv \frac{g}{\overline{\theta_v}} z \left[\frac{\overline{\theta_v}(z) - \overline{\theta_v}(z_0)}{\overline{u}^2(z)} \right]$$

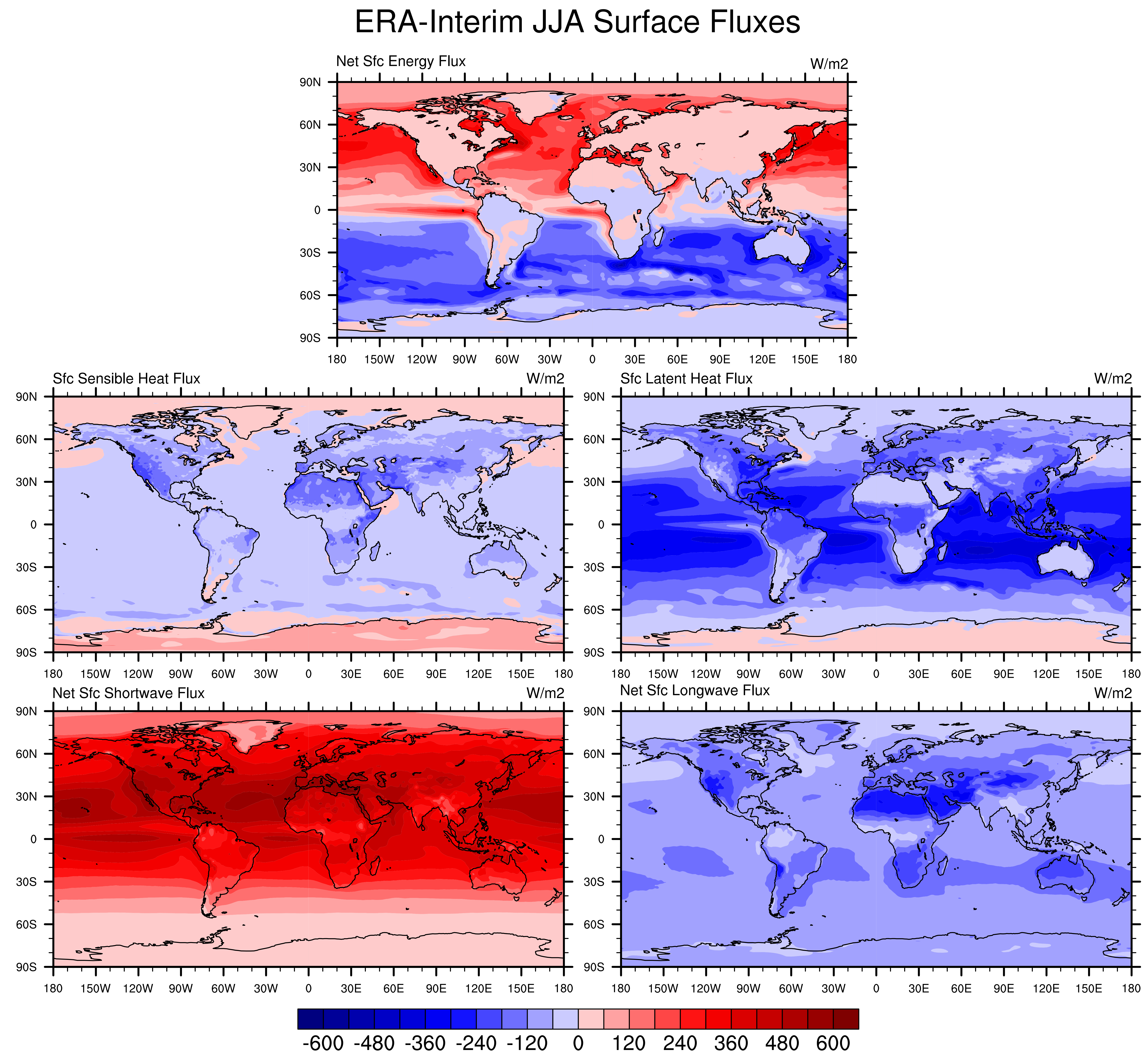
MOST is really only expected to work over flat surfaces in the presence of a mean horizontal wind.

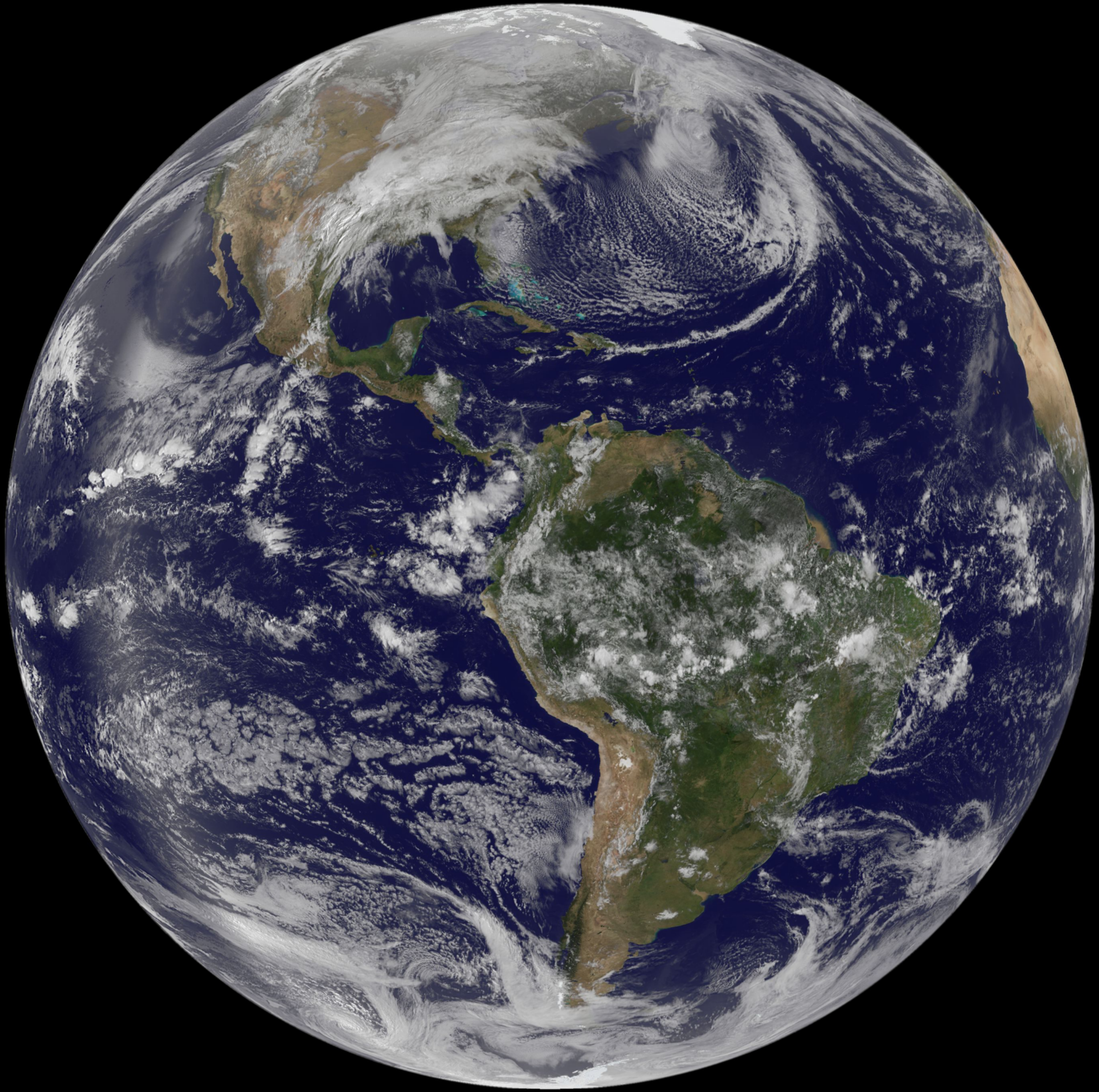
Something better is needed. Looking for a project?

Climatology of the surface fluxes of sensible heat, latent heat, and momentum

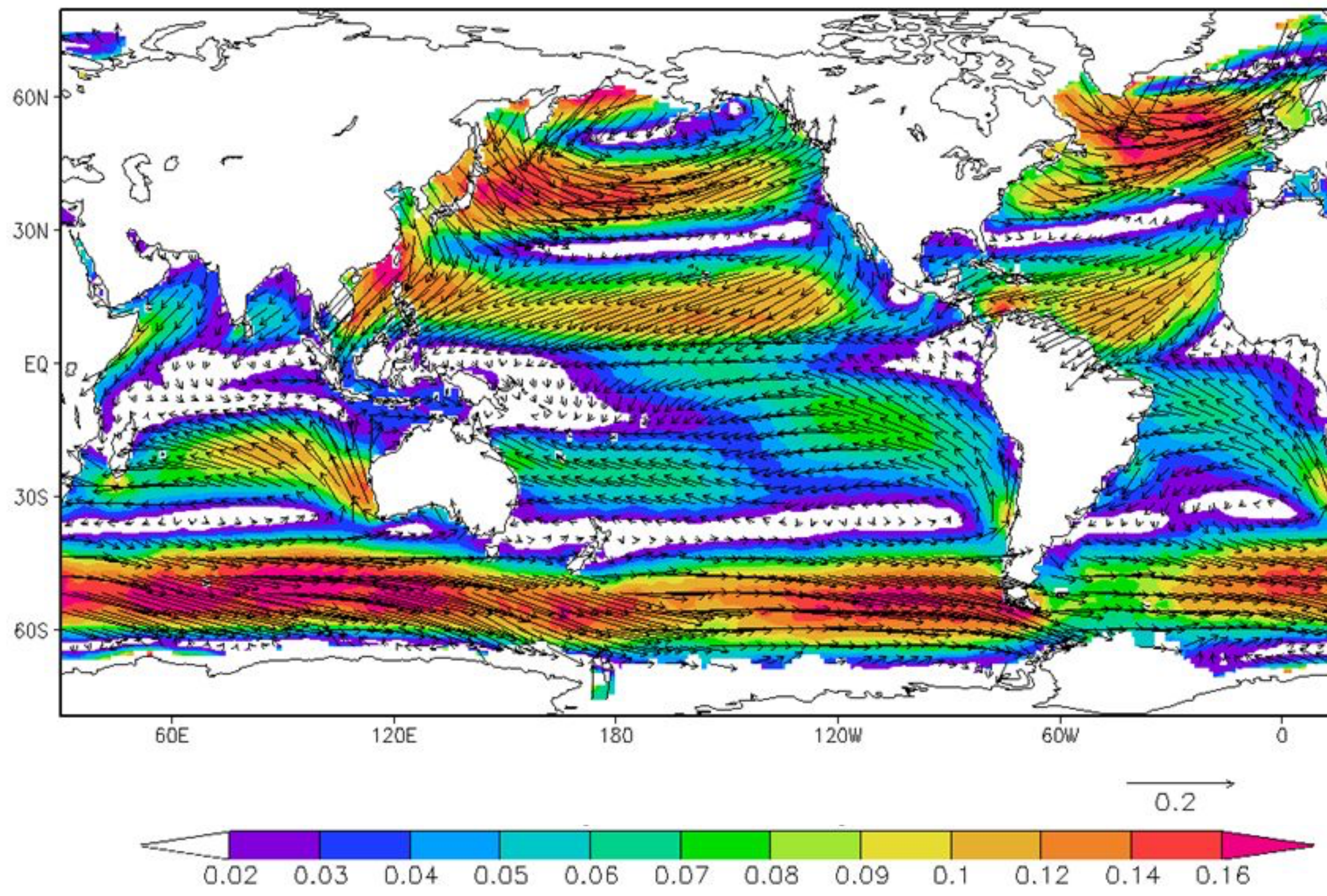
ERA-Interim DJF Surface Fluxes




ERA-Interim JJA Surface Fluxes
Net Sfc Energy Flux
W/m2
Sfc Sensible Heat Flux
W/m2
Sfc Latent Heat Flux
W/m2
Net Sfc Shortwave Flux
W/m2
Net Sfc Longwave Flux
W/m2
90N 60N 30N 0 30S 60S 90S
180 150W 120W 90W 60W 30W 0 30E 60E 90E 120E 150E 180
-600 -480 -360 -240 -120 0 120 240 360 480 600



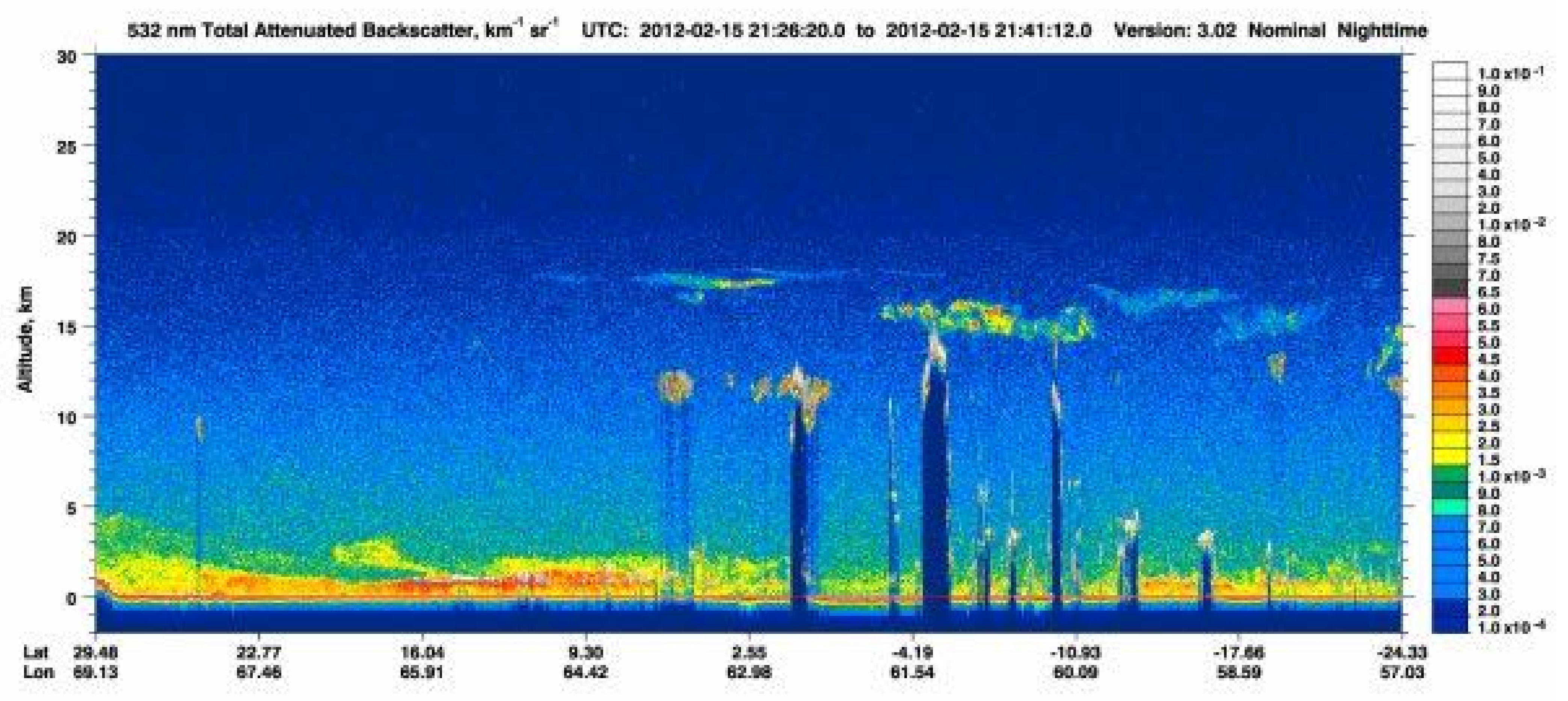
DJF mean wind stress from ERS1 and 2



N m^{-2} = Pa

Five approaches to boundary layer parameterization

- Similarity theories
- Eddy diffusion
- Mixed-layer models
- Higher-order closure
- Mass fluxes

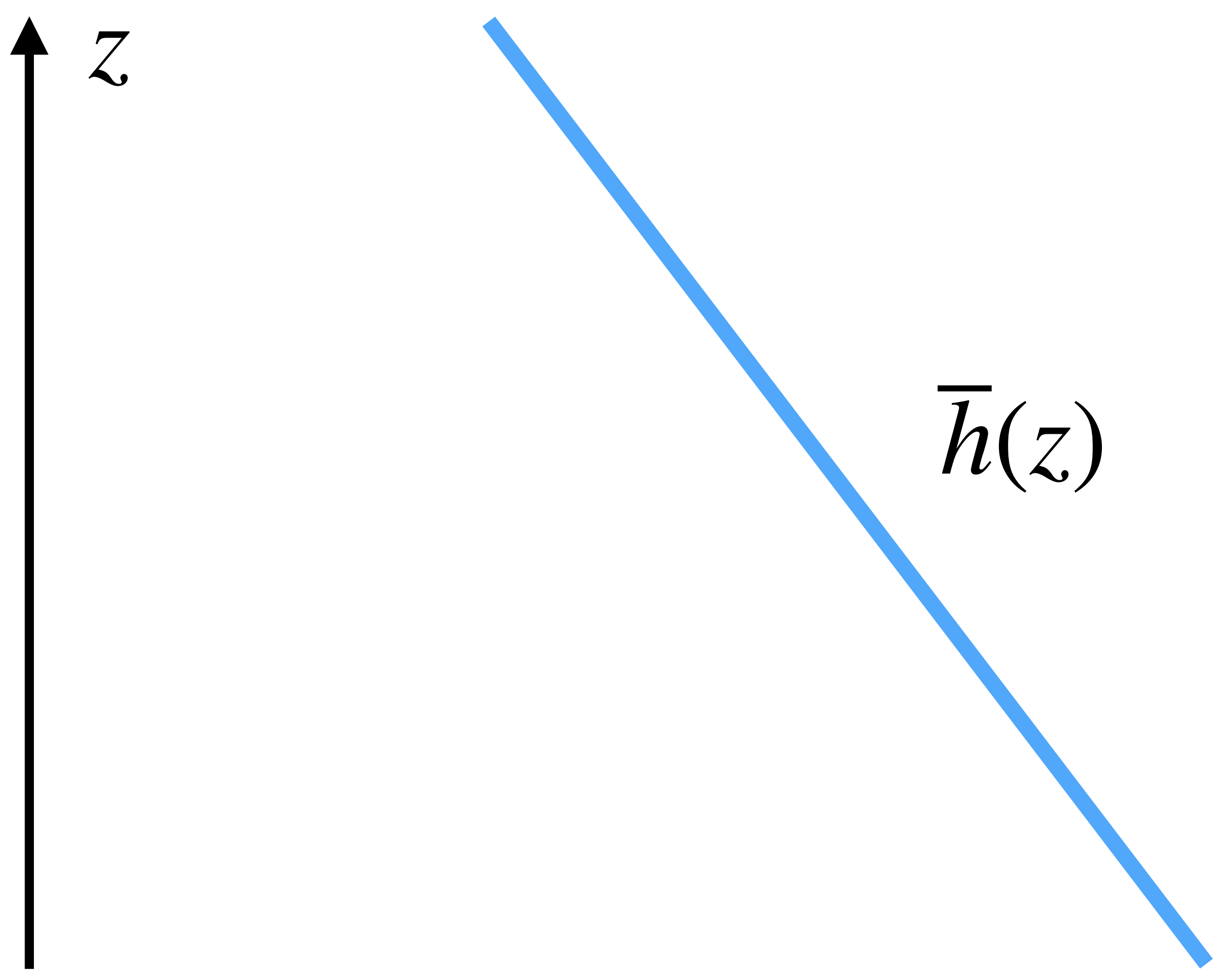


Eddy diffusion

$$\rho\overline{w'h'} = -\rho K \frac{\partial \overline{h}}{\partial z}$$

The diffusion coefficient K has dimensions of length squared divided by time.

For $K > 0$, the flux is "down the gradient."



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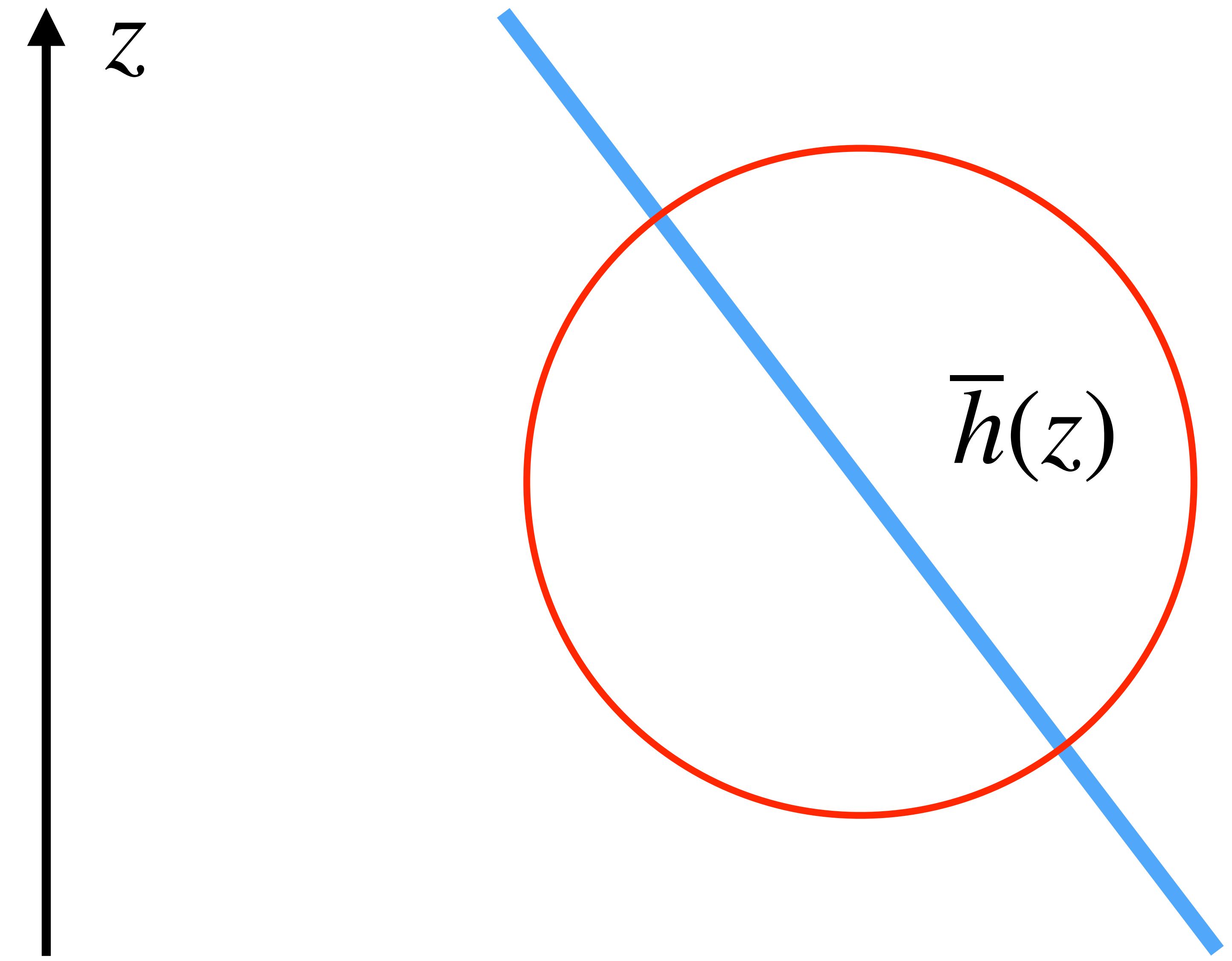

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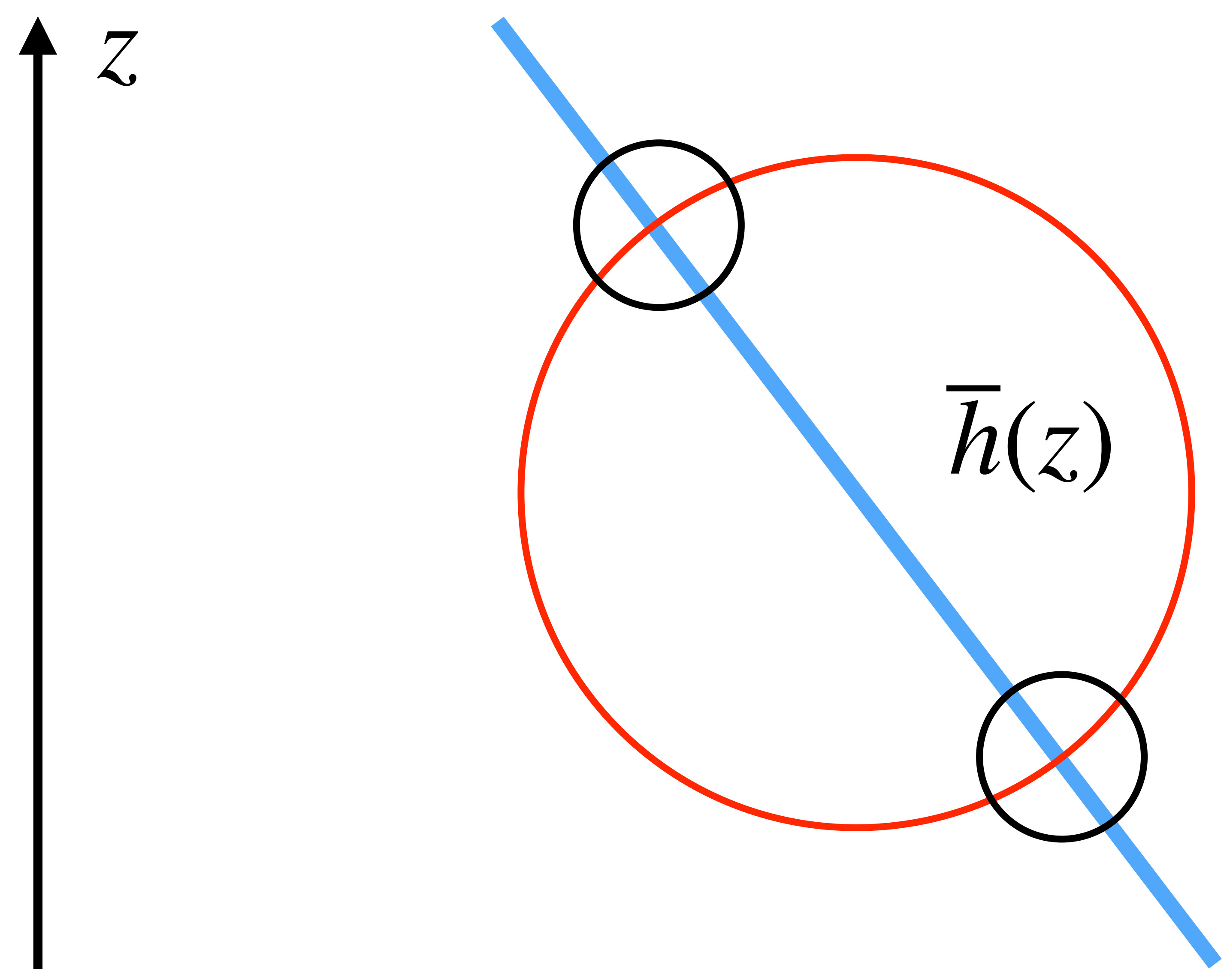

Five approaches to boundary layer parameterization

- Similarity theories
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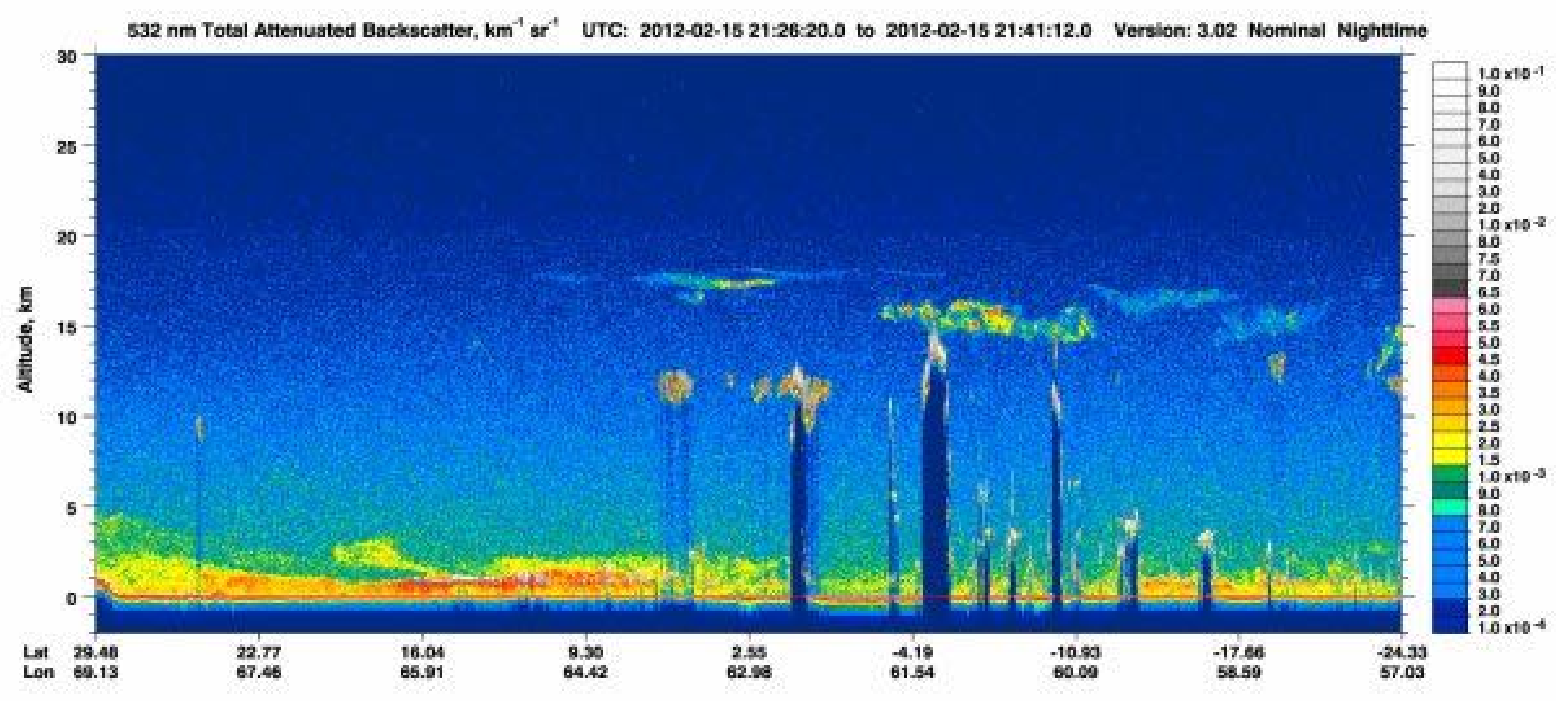
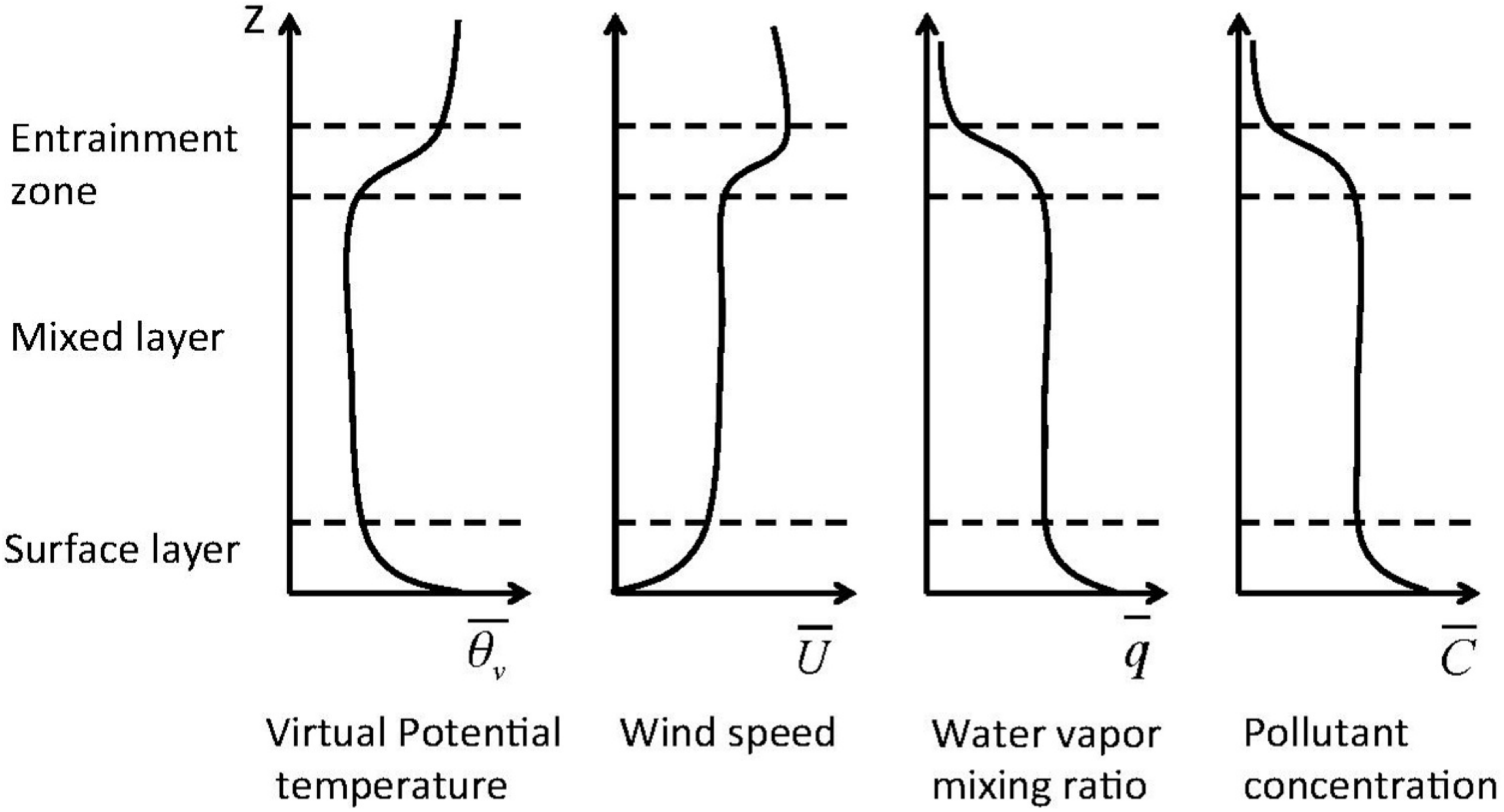


Mixed layers

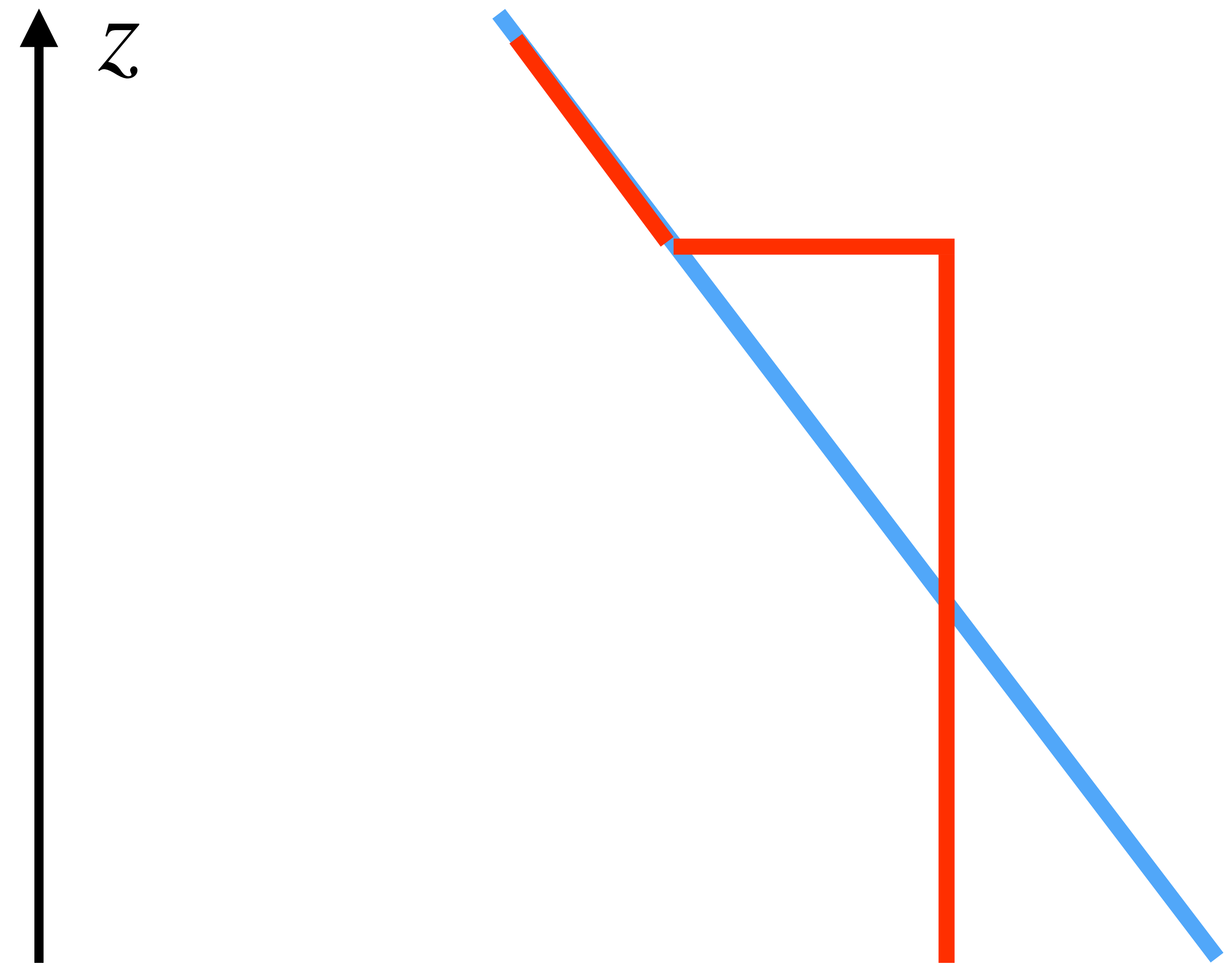


What can be mixed?

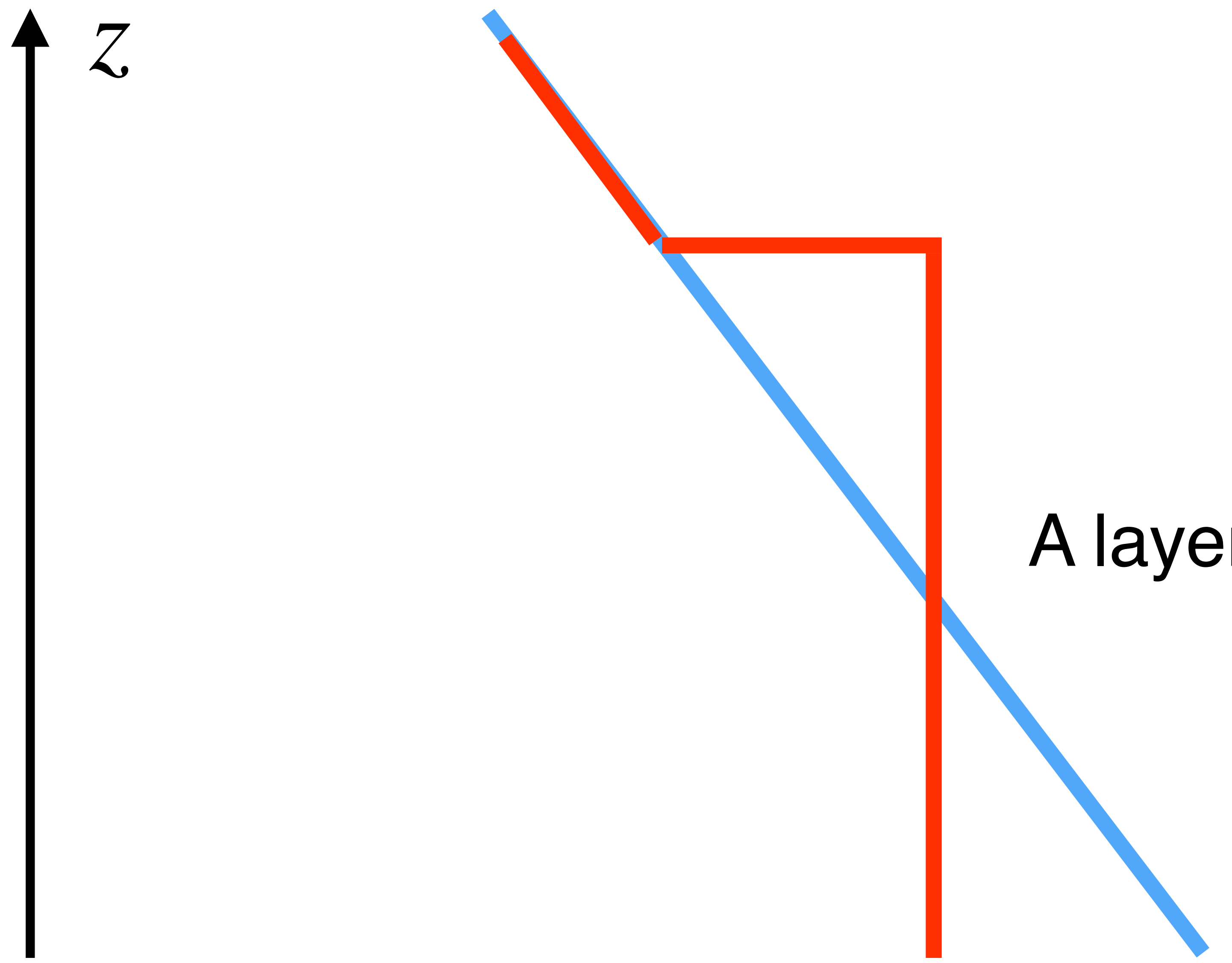
- Conservative quantities don’t change as the particles they are associated move around.
- *Only conservative quantities can be mixed.*
  - Potential temperature
  - Specific humidity
- Non-conservative quantities cannot be mixed.
  - Ordinary temperature
  - Relative humidity

One way to make a mixed layer

One way to make a mixed layer

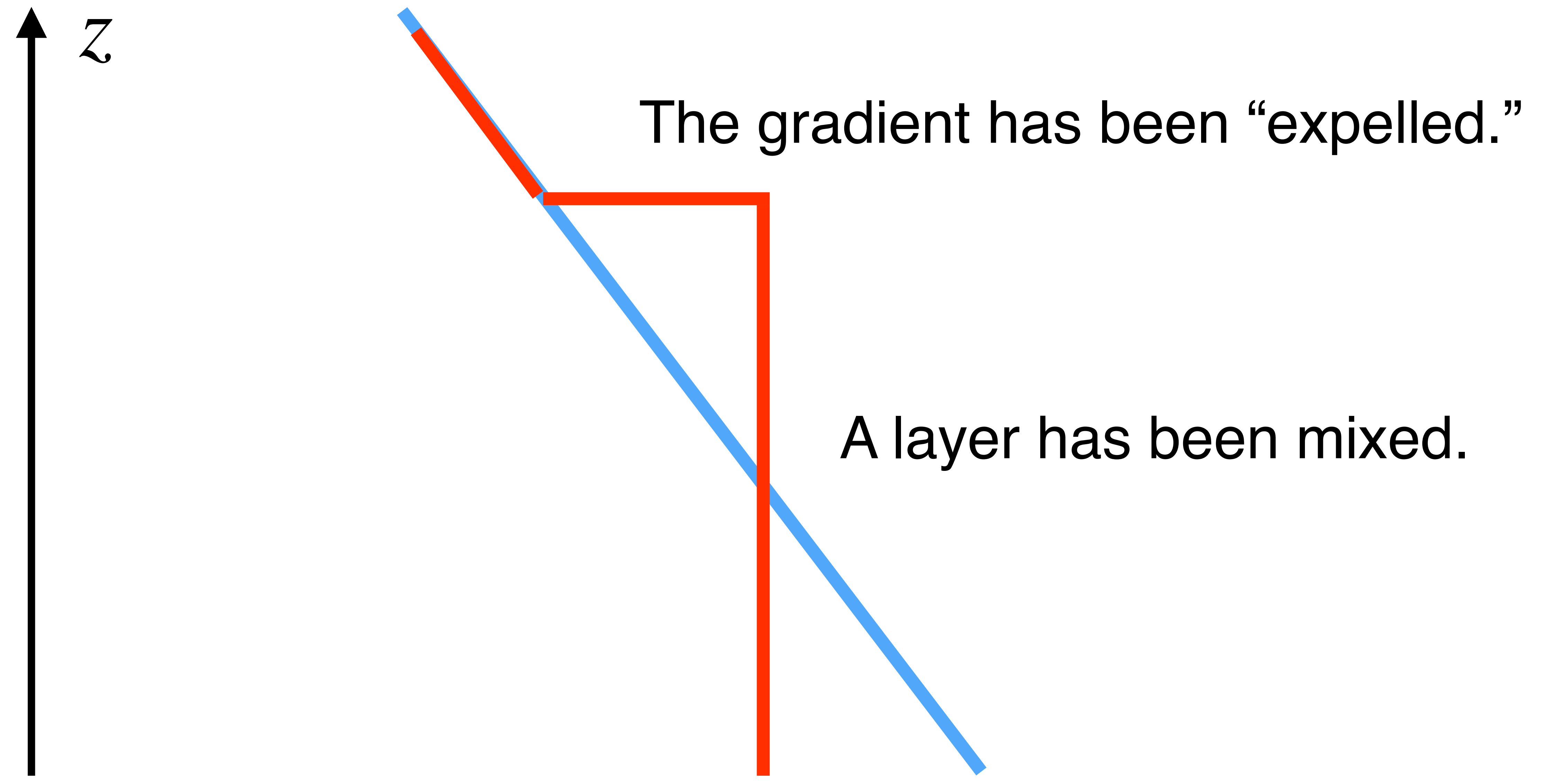


One way to make a mixed layer



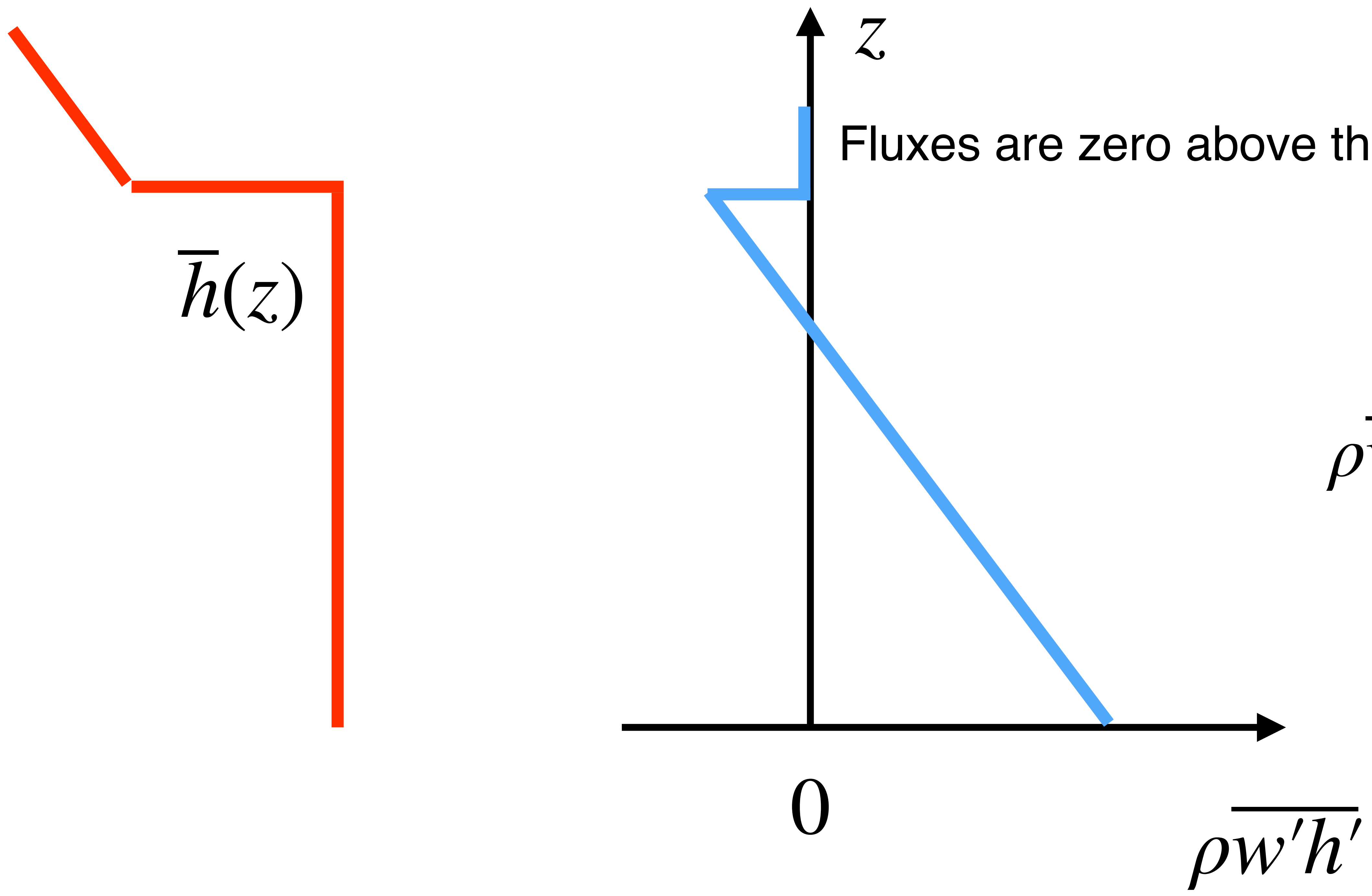
A layer has been mixed.

One way to make a mixed layer



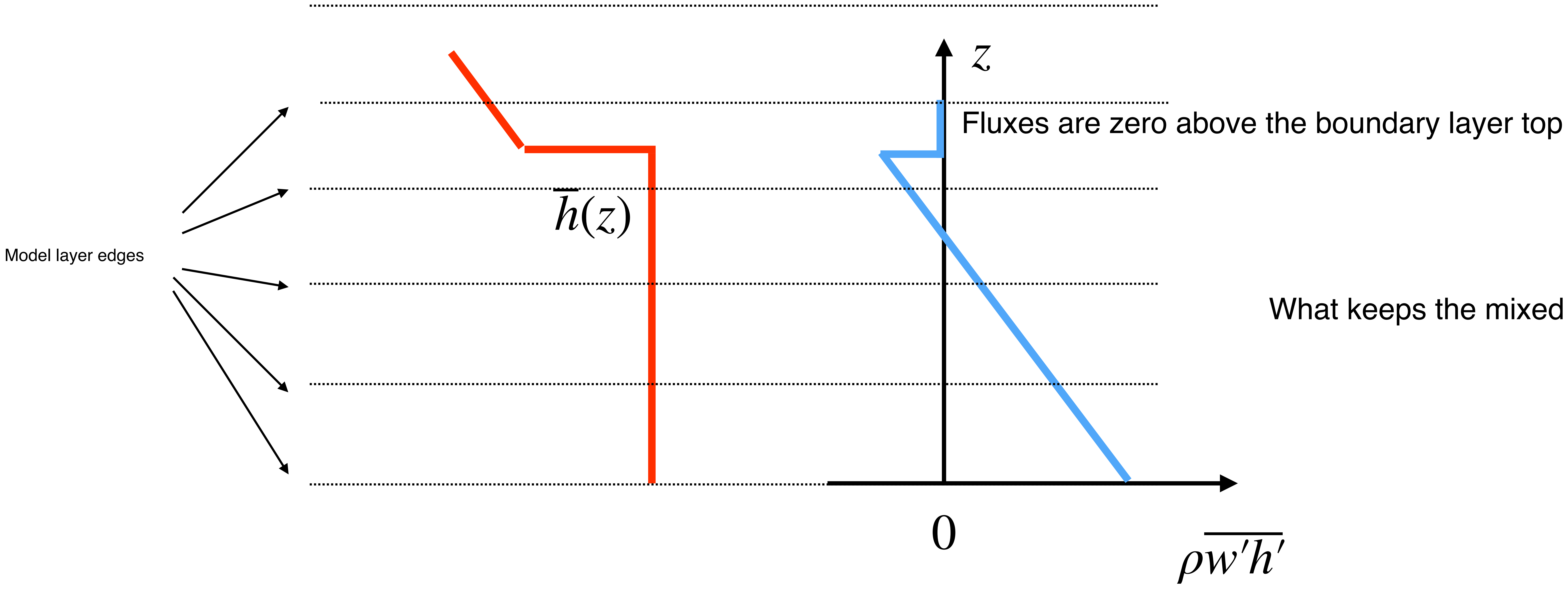
Fluxes are linear with height in a mixed layer.

$$\frac{\partial \overline{h}}{\partial t} = -\frac{1}{\rho}\frac{\partial}{\partial z}\rho\overline{w'h'}$$



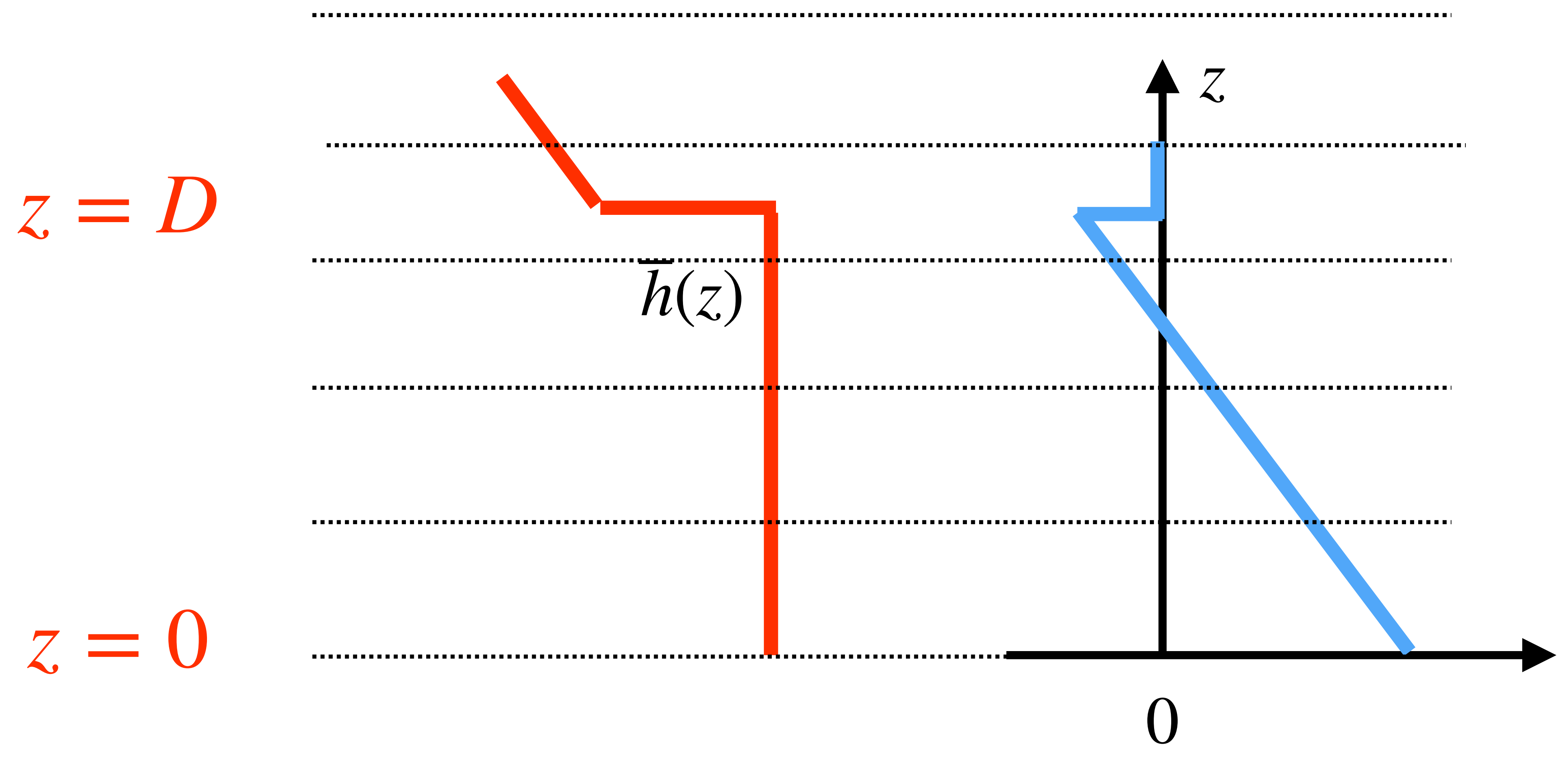
$$\rho\overline{w'h'} = \left[\left(-E\Delta\overline{h} + \Delta R\right)\frac{z}{D} + \rho c_T|\mathbf{V_S}|\left(h_g - \overline{h}_a\right)\left(1 - \frac{z}{D}\right)\right]$$

What keeps the mixed layer mixed?



What keeps the mixed layer mixed?

What keeps the mixed layer mixed?



What keeps the mixed layer mixed?

$$\rho\overline{w'h'} = \left[\left(-E\Delta\overline{h}\right)\frac{z}{D} + \rho c_T|\mathbf{V_S}|\left(h_g - \overline{h}_a\right)\left(1 - \frac{z}{D}\right)\right] - K\frac{\partial\overline{h}}{\partial z}$$

$$K = K_0\frac{z}{D}\left(1 - \frac{z}{D}\right)$$

" K profile parameterization," or KPP

VOLUME 6 JOURNAL OF CLIMATE OCTOBER 1993

Local Versus Nonlocal Boundary-Layer Diffusion in a Global Climate Model

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B. A. BOVILLE

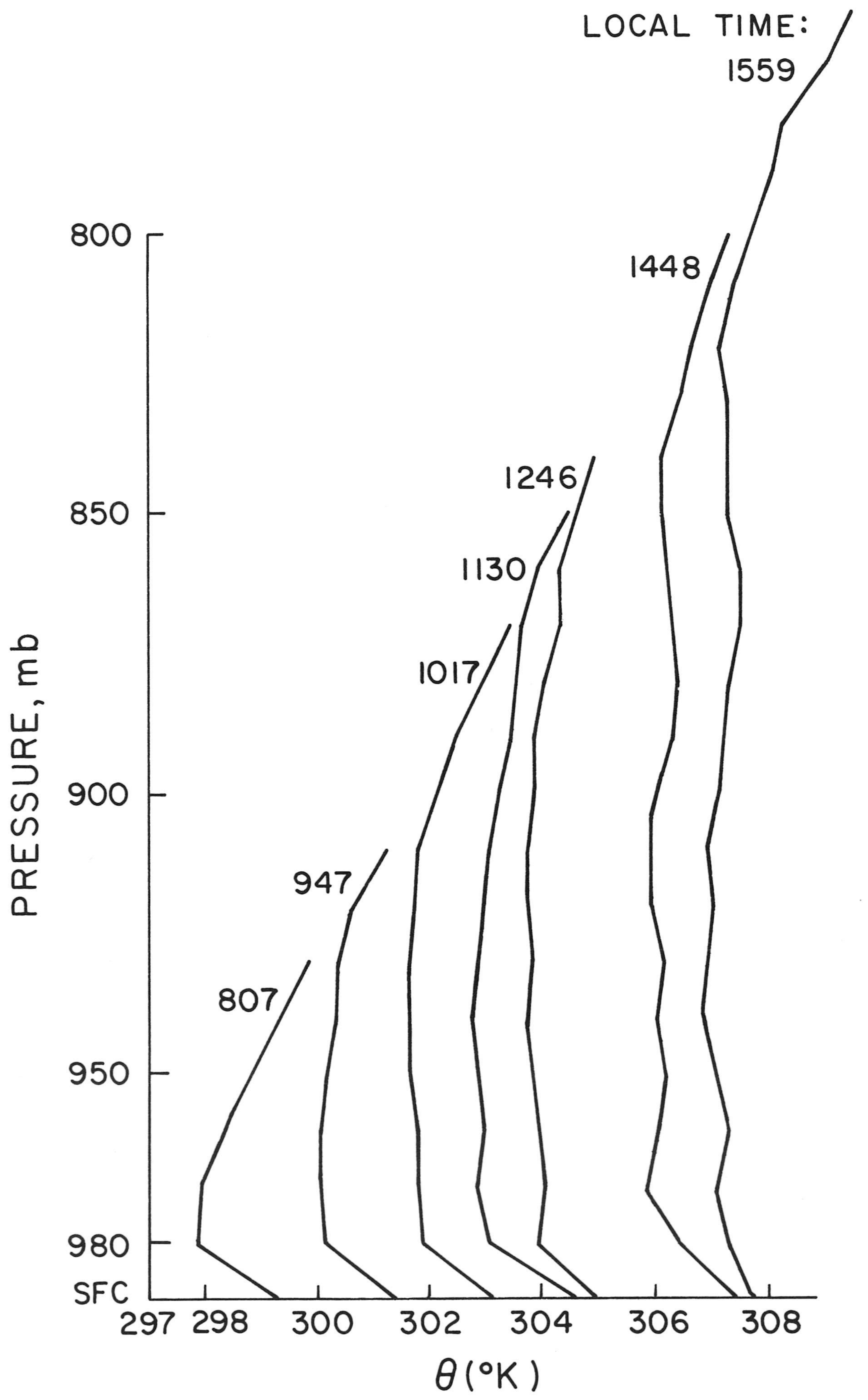
National Center for Atmospheric Research, Boulder, Colorado*

OCEANIC VERTICAL MIXING: A REVIEW AND A MODEL WITH A NONLOCAL BOUNDARY LAYER PARAMETERIZATION

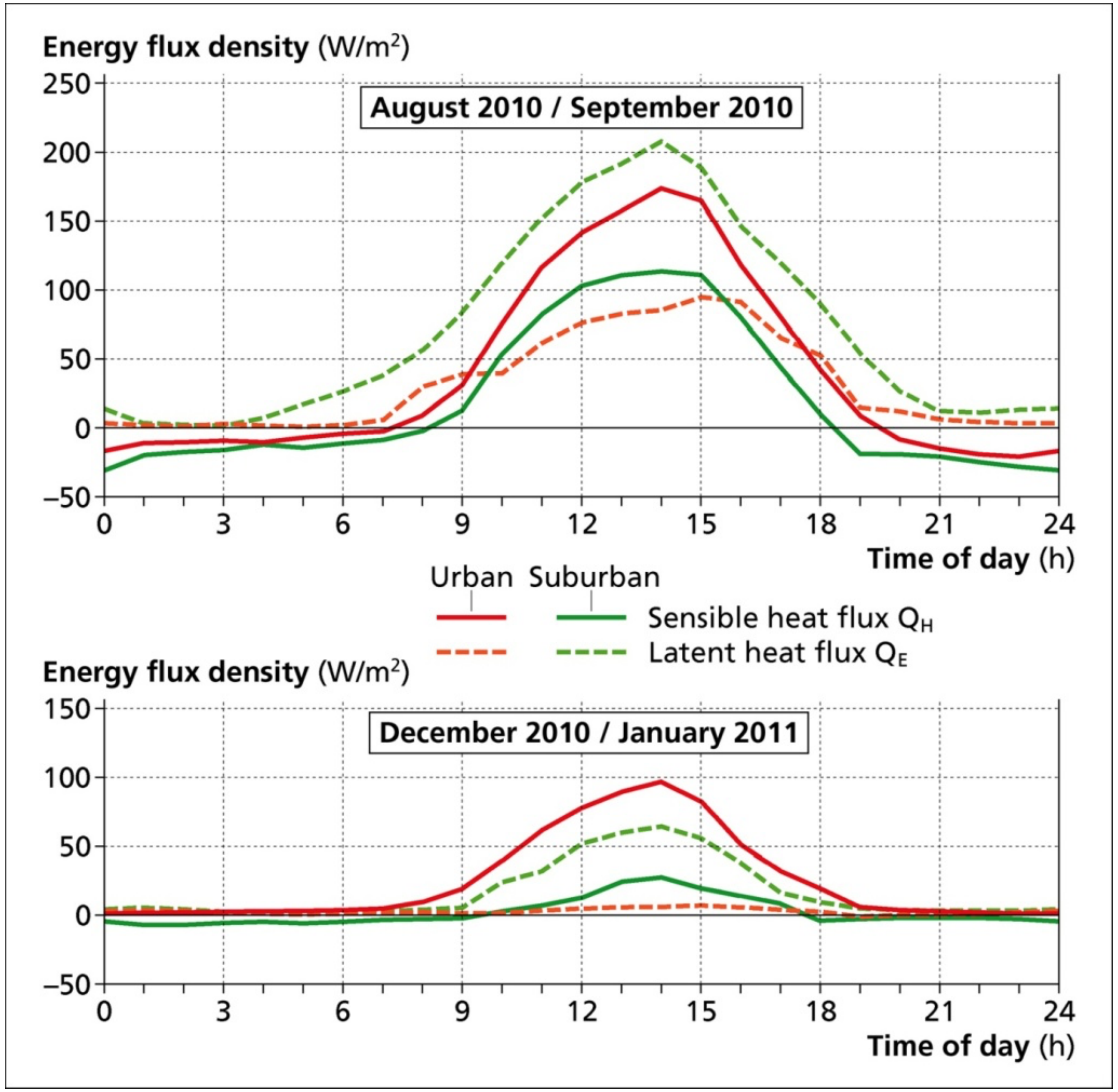
W. G. Large
J. C. McWilliams
S. C. Doney
National Center for Atmospheric Research
Boulder, Colorado

Reviews of Geophysics

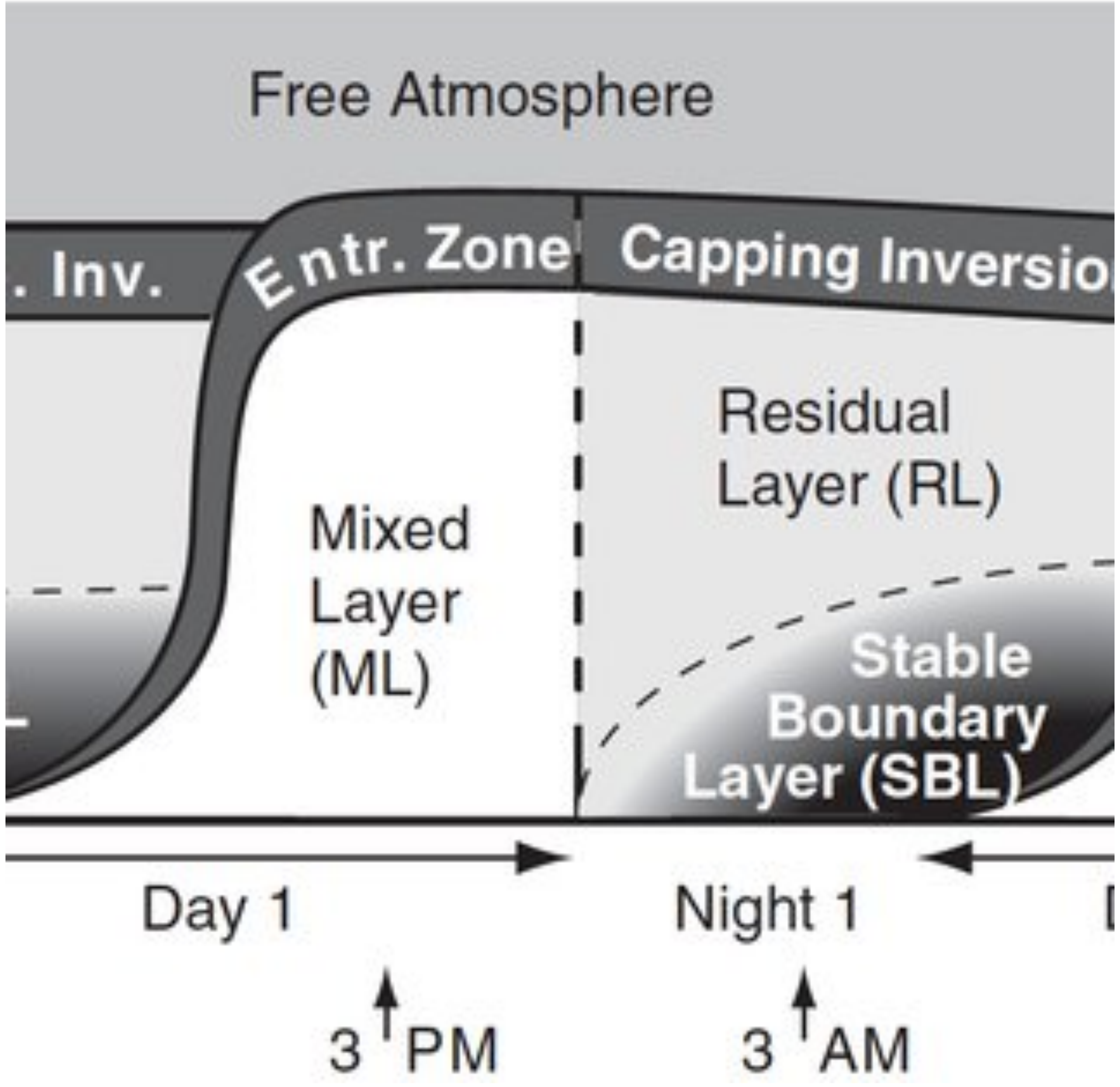
Mixed layers can deepen with time.



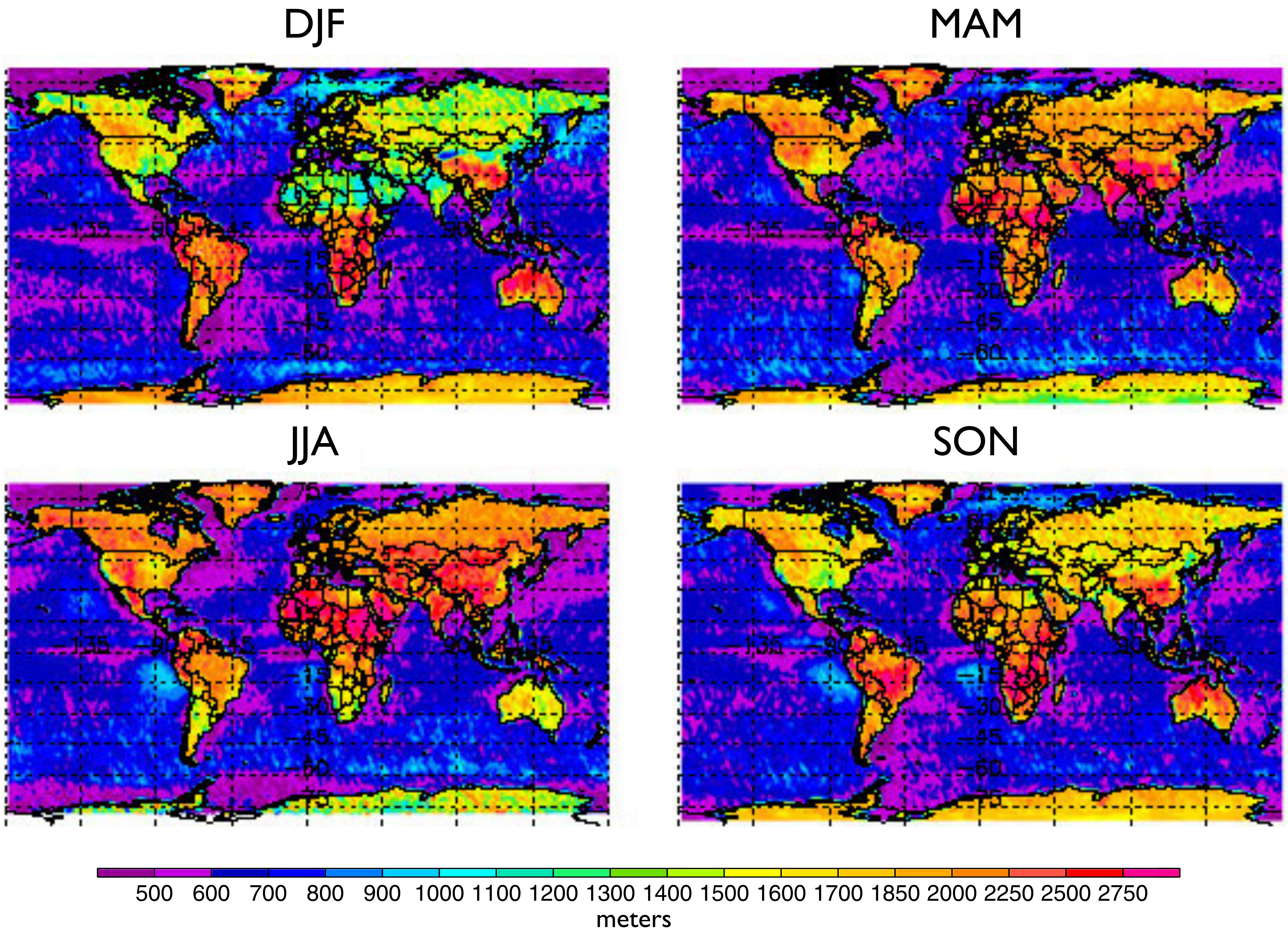
The diurnal cycle over land



“Stable” & “unstable” boundary layers



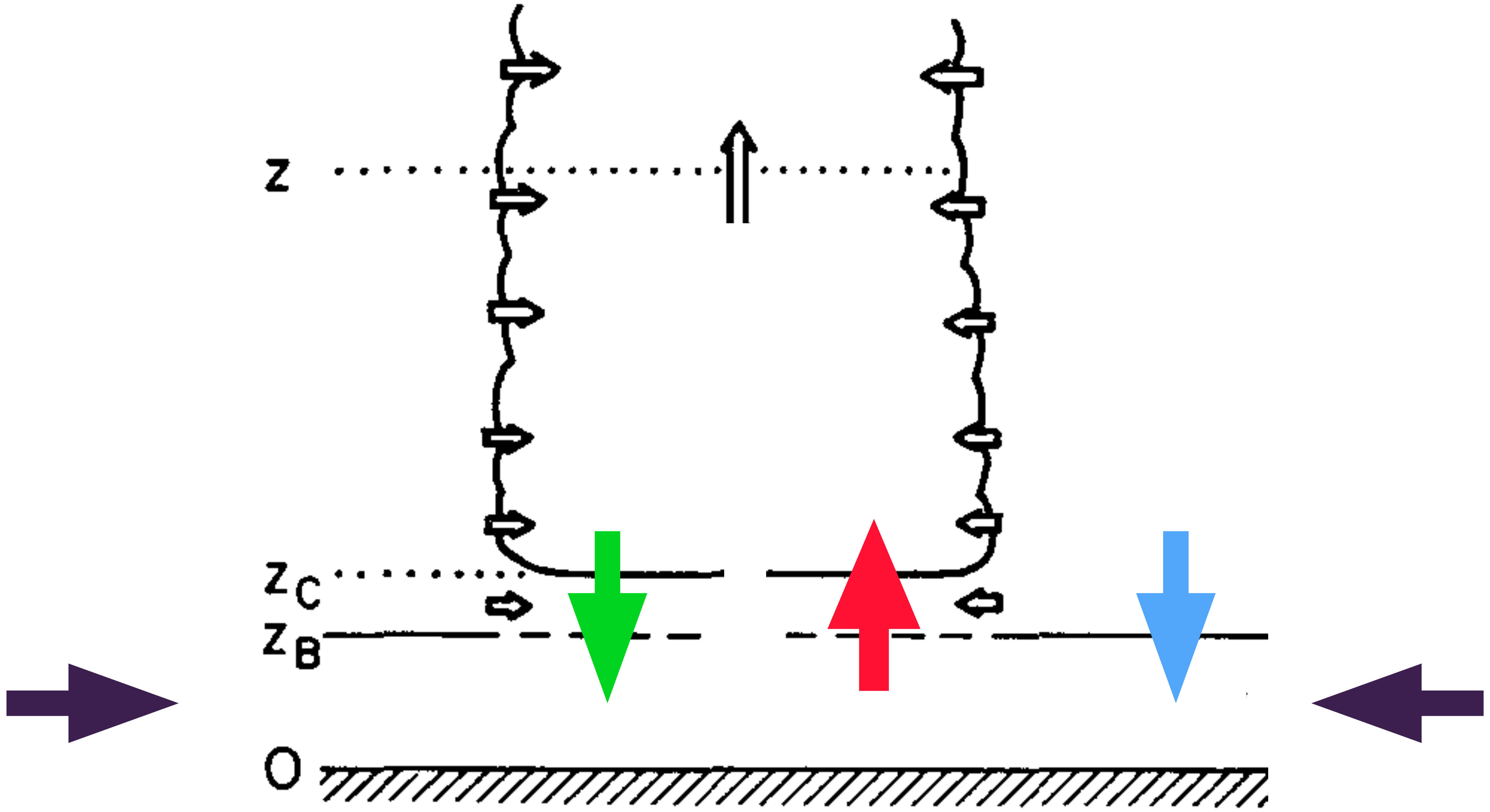
Obsevations of the PBL depth



Mean PBL depth retrieved by the CALIPSO satellite gridded to 1.25 1.25 . Data are from June 2006 to December 2011 **between local noon and 3 pm** for conditions without optically thick clouds.

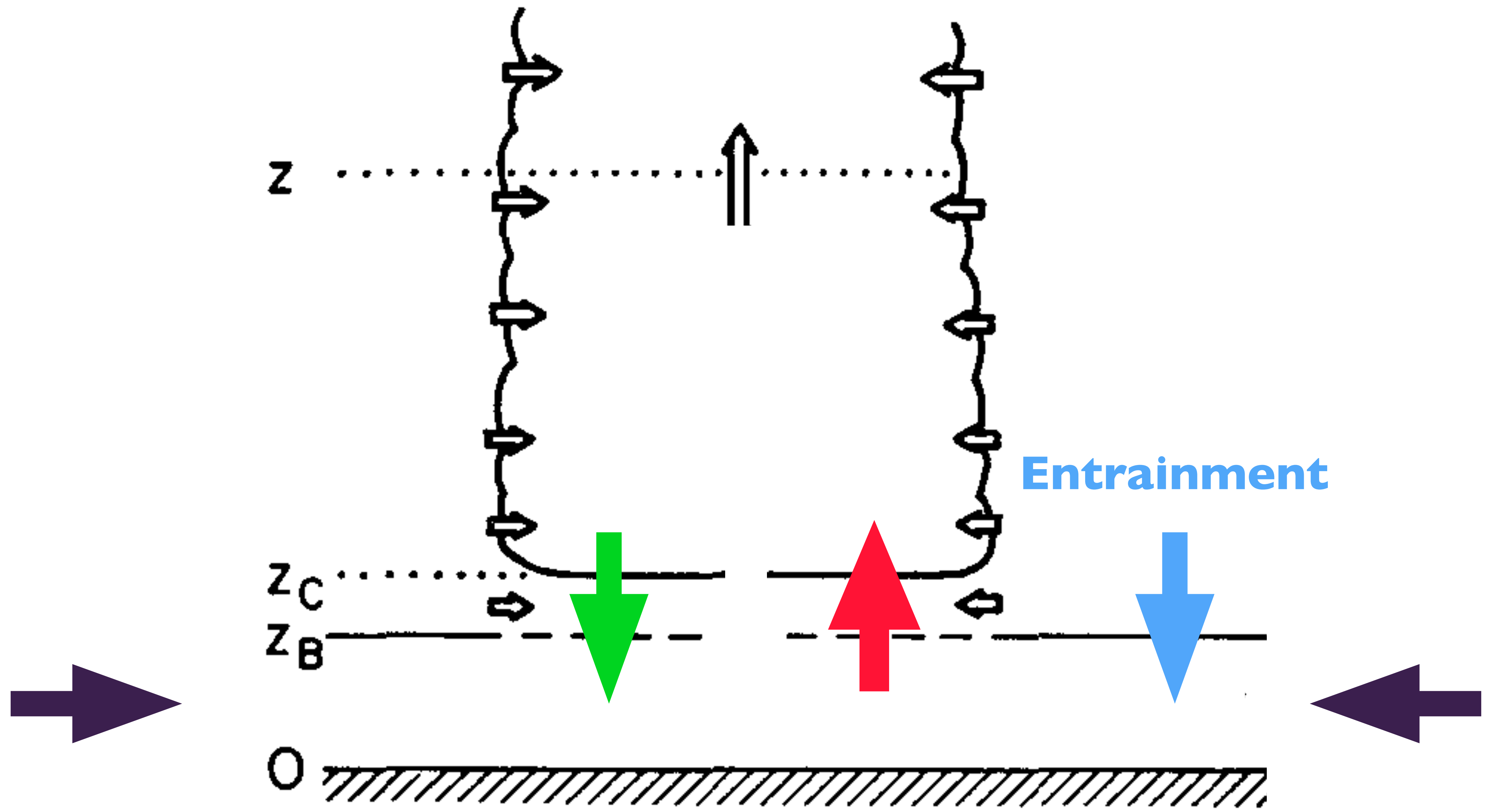
McGrath-Spangler, E. L., and A. S. Denning, 2013

PBL mass budget



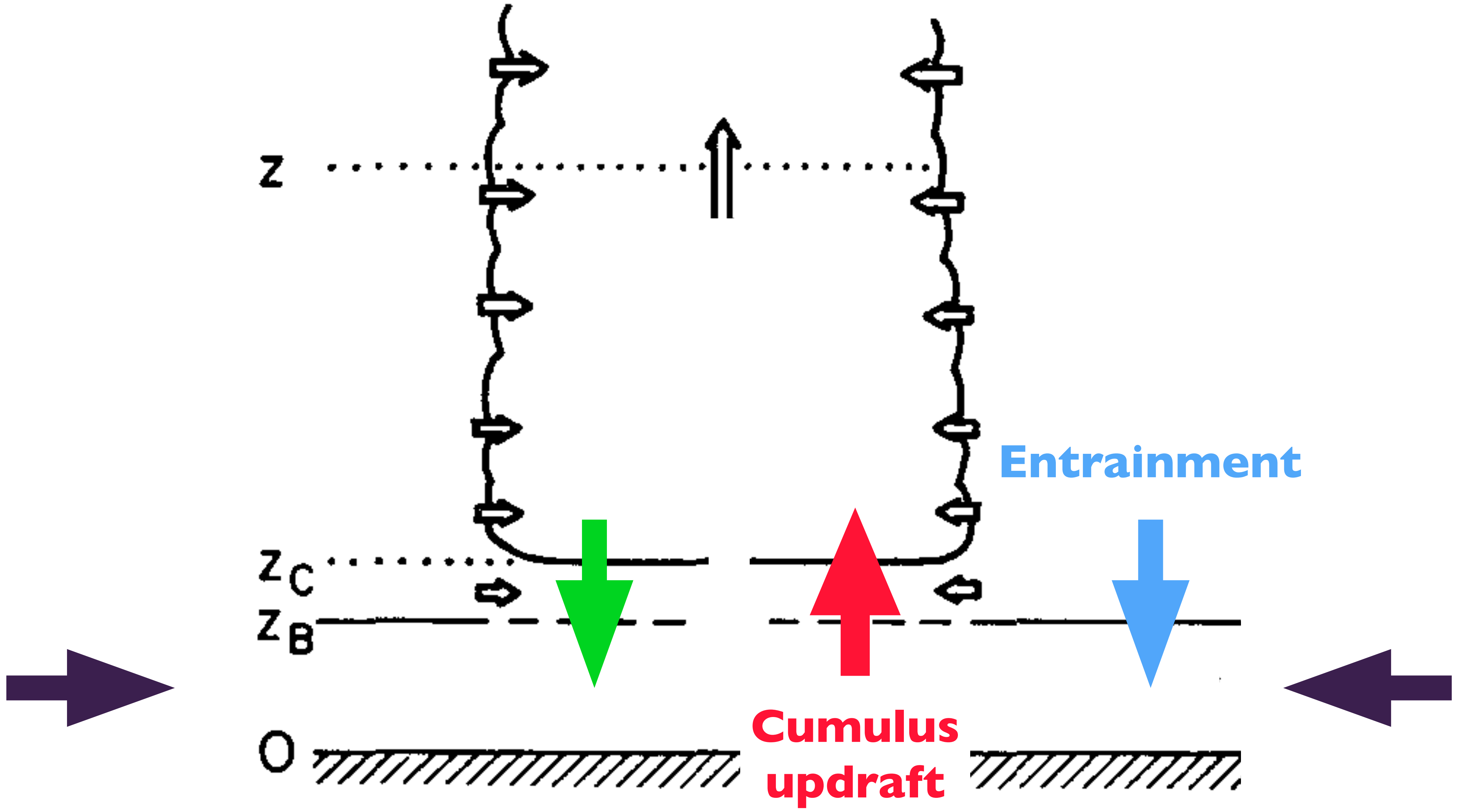
$$\frac{\partial}{\partial t}\int_{z_S}^{z_B} \rho \, dz = -\nabla \cdot \int_{z_S}^{z_B} \left(\rho \mathbf{v}\right) \, dz + E - M_u + M_d$$

PBL mass budget



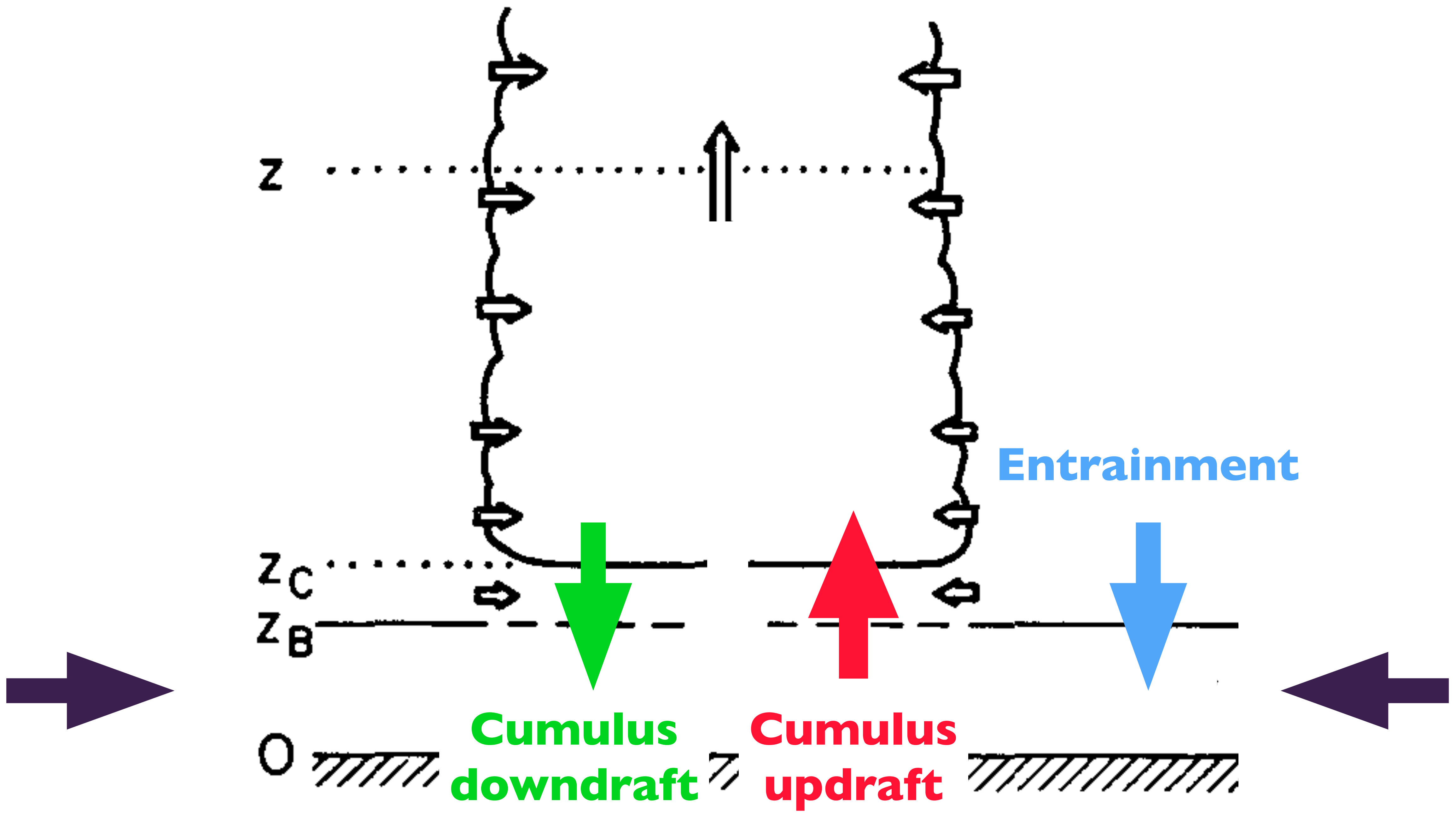
$$\frac{\partial}{\partial t}\int_{z_S}^{z_B} \rho \, dz = -\nabla \cdot \int_{z_S}^{z_B} \left(\rho \mathbf{v}\right) \, dz + E - M_u + M_d$$

PBL mass budget



$$\frac{\partial}{\partial t}\int_{z_S}^{z_B} \rho\, dz = -\nabla \cdot \int_{z_S}^{z_B} \left(\rho \mathbf{v}\right)\, dz + E - M_u + M_d$$

PBL mass budget

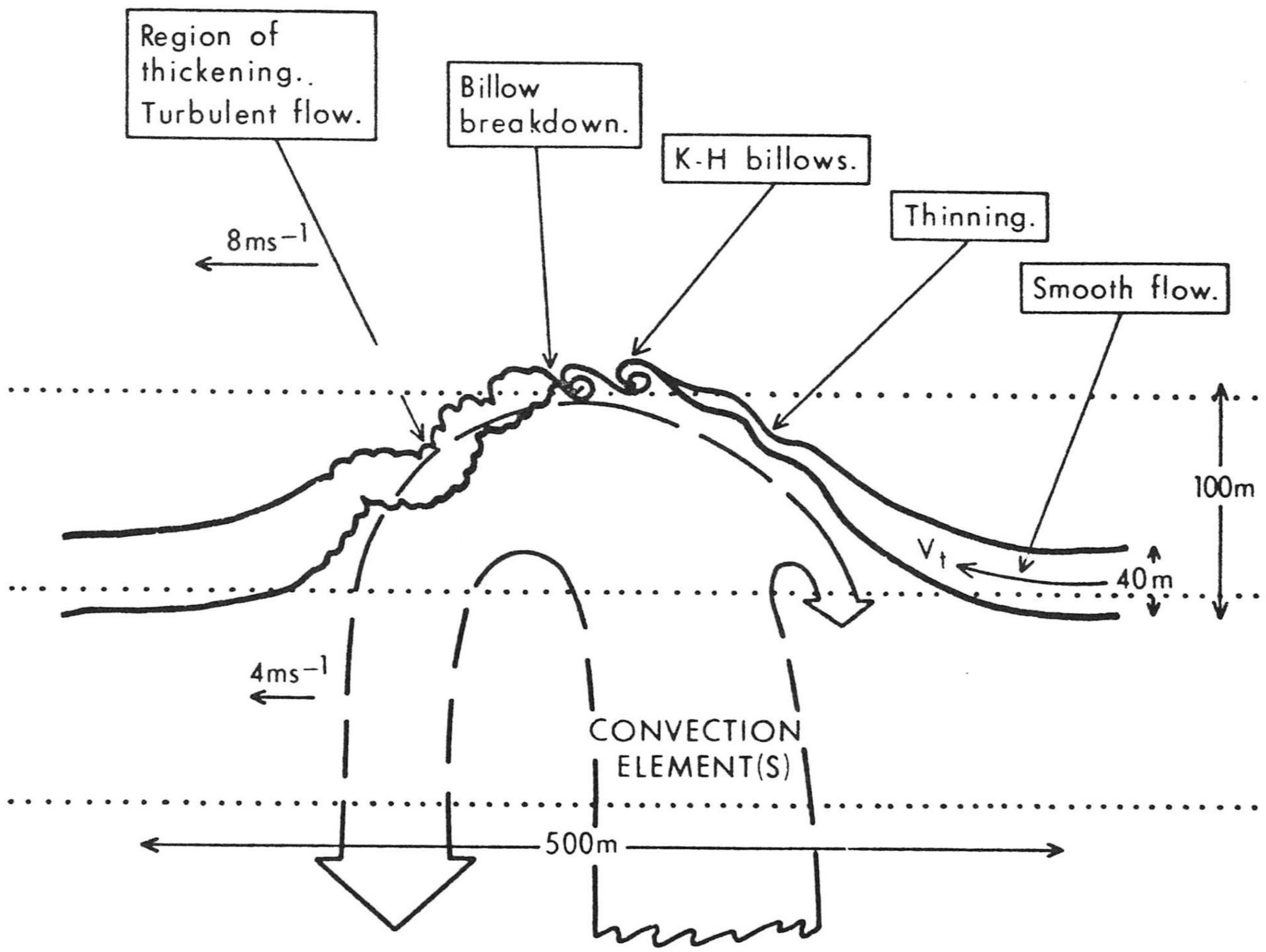


$$\frac{\partial}{\partial t}\int_{z_S}^{z_B} \rho \, dz = -\nabla \cdot \int_{z_S}^{z_B} \left(\rho \mathbf{v}\right) \, dz + E - M_u + M_d$$

How entrainment works

Entrainment is the active annexation of quiet fluid by turbulence.

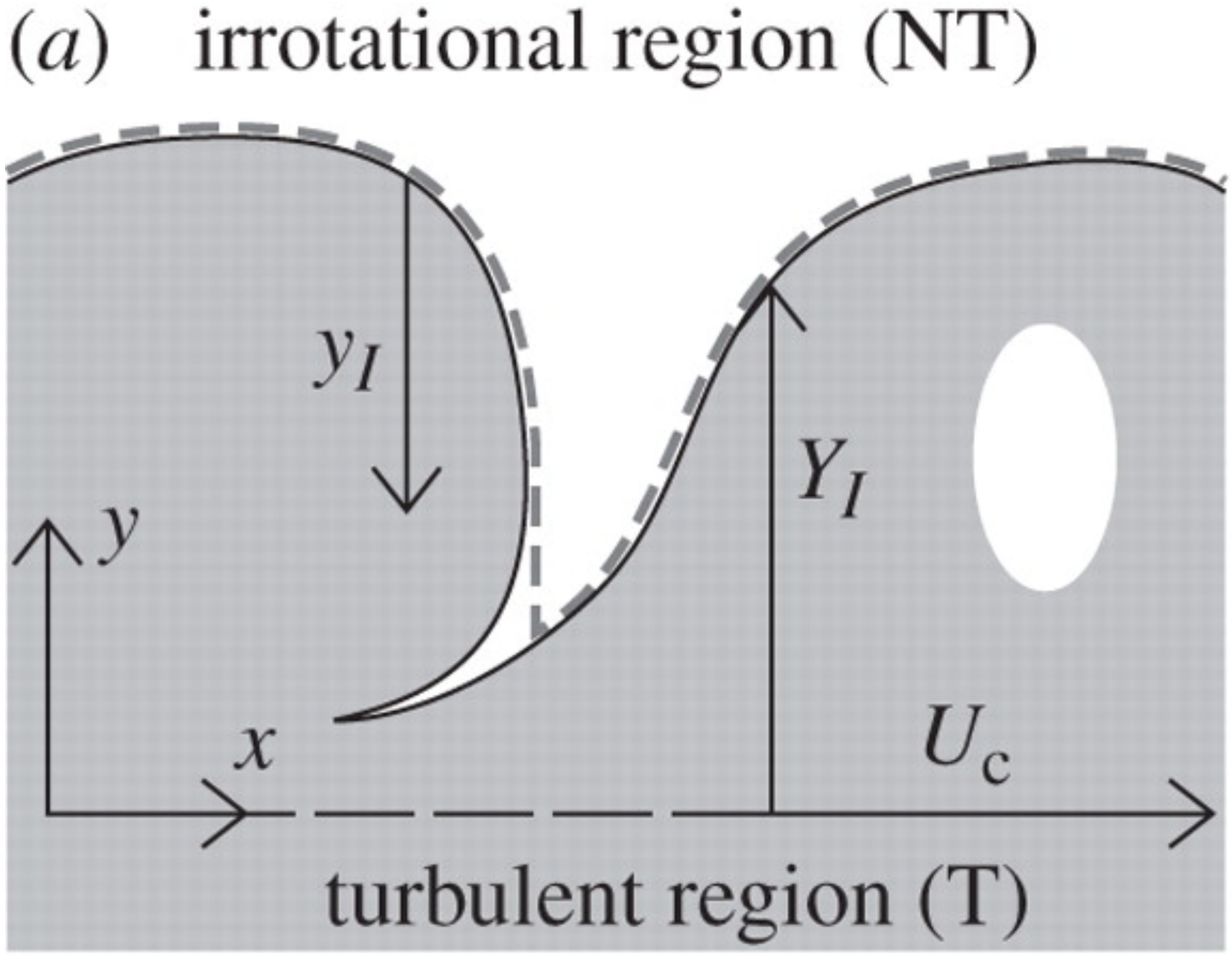
Entrainment is *not* the same as mixing.



Rayment and Readings 1974

Kelvin-Helmhotz instability

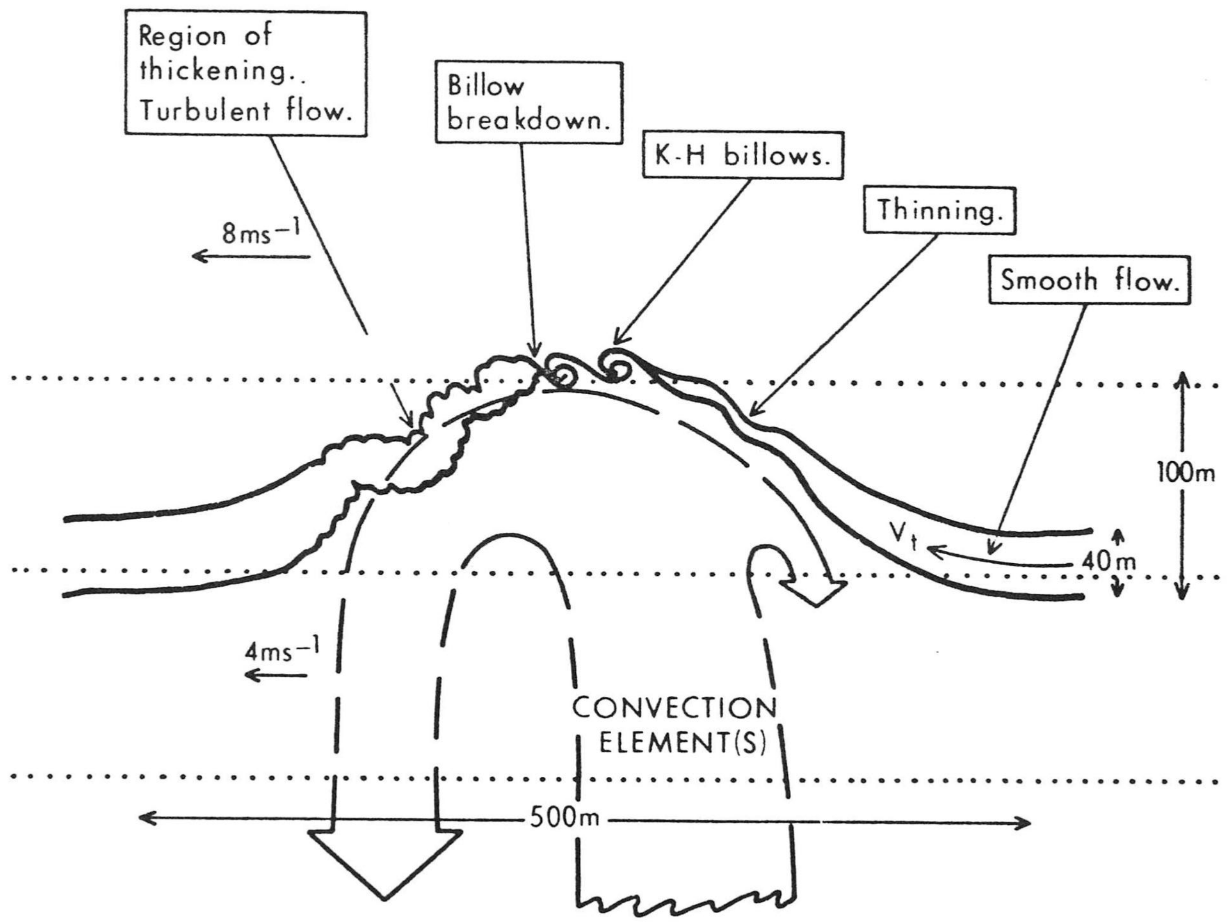
Entrainment across the boundary layer top



Fluxes due to entrainment

$$\rho\overline{w'h'} = -E\Delta\overline{h}$$

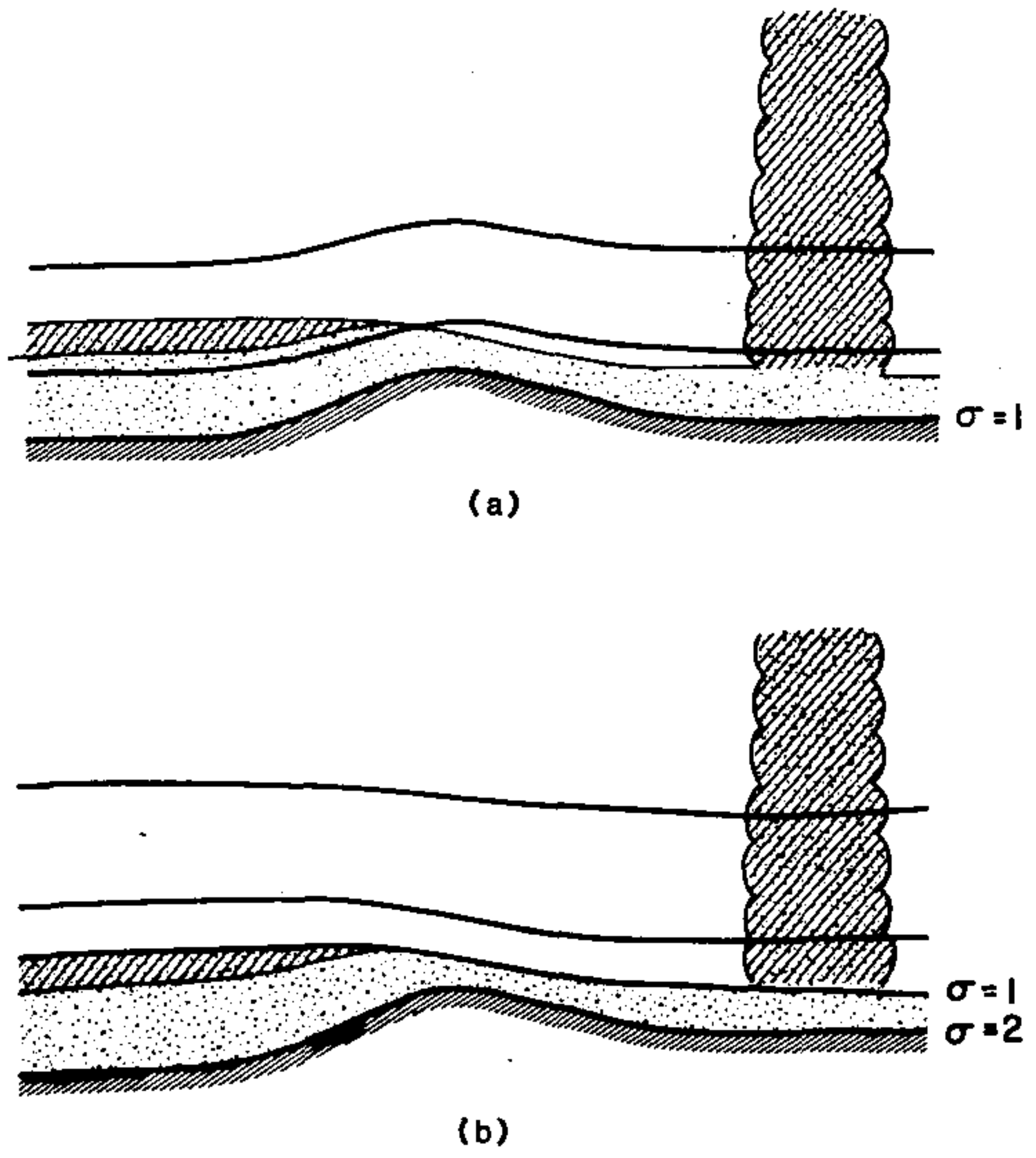
$$\rho\overline{w'q_t'} = -E\Delta\overline{q_t}$$

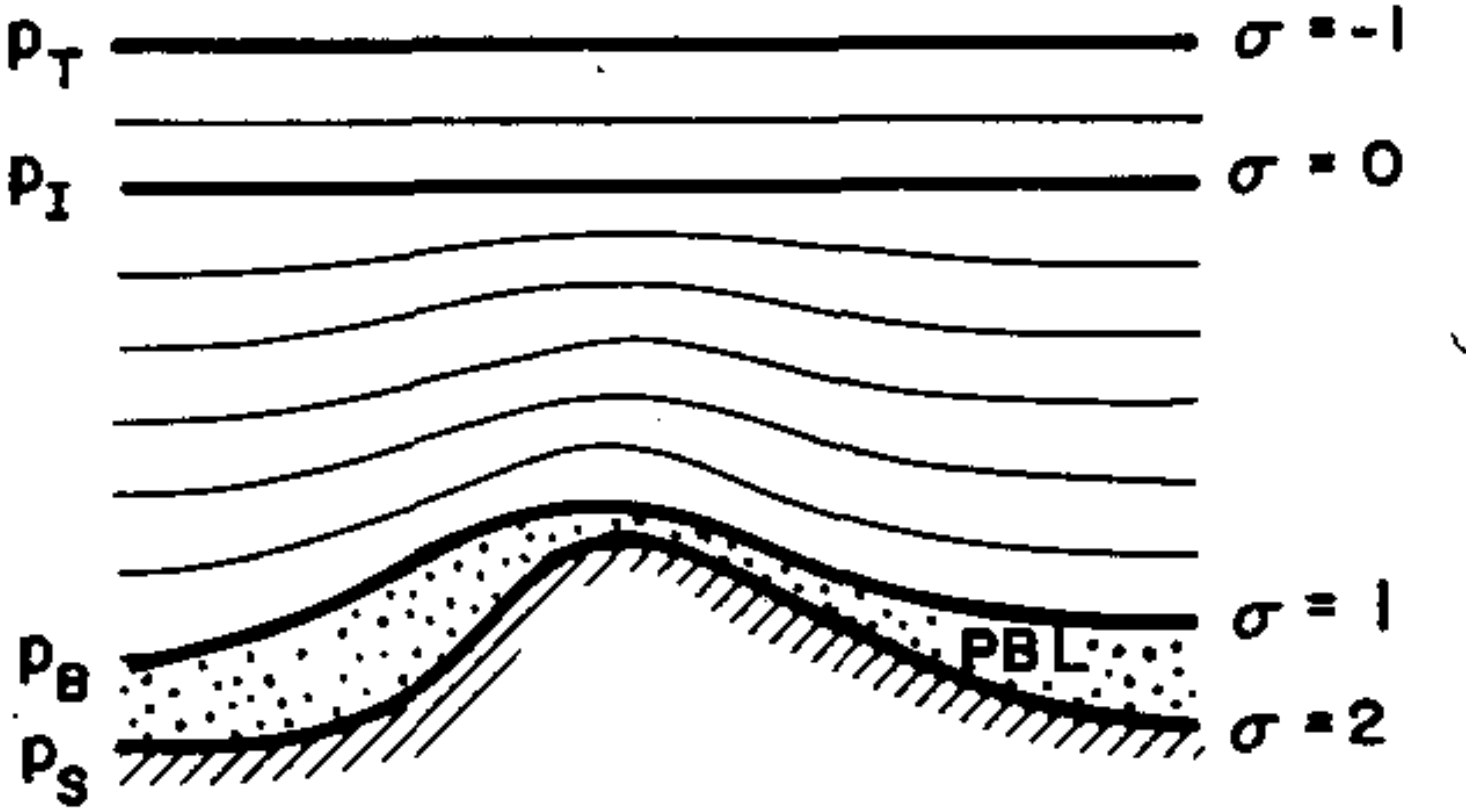


$$E = \frac{\rho e_M^{3/2}}{gh\Delta T_v/T_0}$$

Stronger turbulence leads to faster entrainment.
A stronger inversion leads to slower entrainment.

Embedding mixed layer models in GCMs





Suarez, M.J., Arakawa, A. and Randall, D.A., 1983. The parameterization of the planetary boundary layer in the UCLA general circulation model: Formulation and results. *Monthly Weather Review*, *111*(11), pp.2224-2243.

When clouds form inside the boundary layer

Subcloud and cloud layers

Joanne (Starr Malkus) Simpson

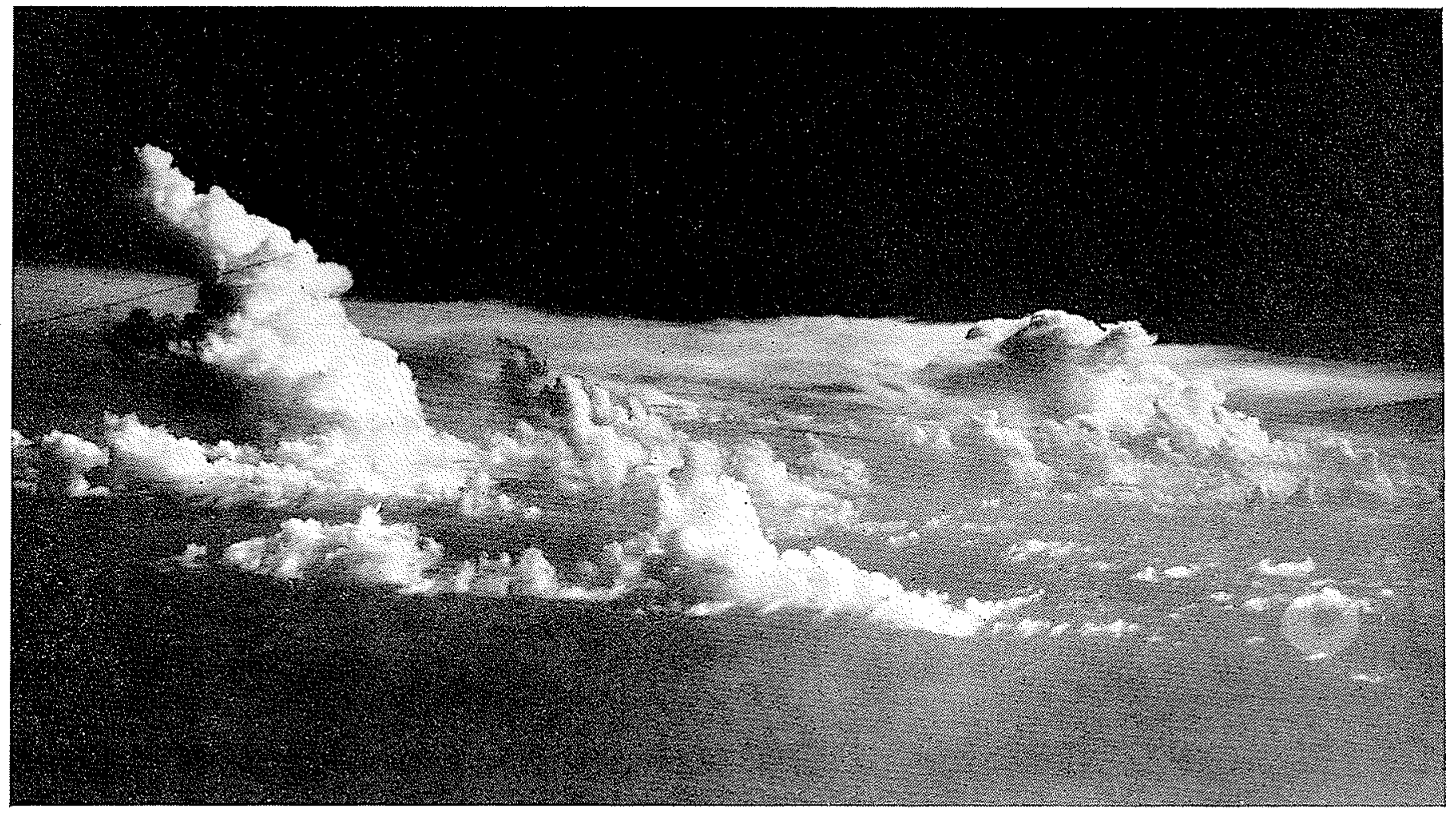


FIGURE 2. Typical aerial photograph of a trade cumulus group over the ocean near Puerto Rico, showing small cloudlets, larger towers with pronounced backslant (wind blows from left) and thin stratus sheet formed by cumulus spreading just below inversion base.

PAPERS IN PHYSICAL OCEANOGRAPHY AND METEOROLOGY

PUBLISHED BY

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

AND

WOODS HOLE OCEANOGRAPHIC INSTITUTION

VOL. XIII, No. 2

ON THE STRUCTURE OF THE TRADE WIND MOIST LAYER

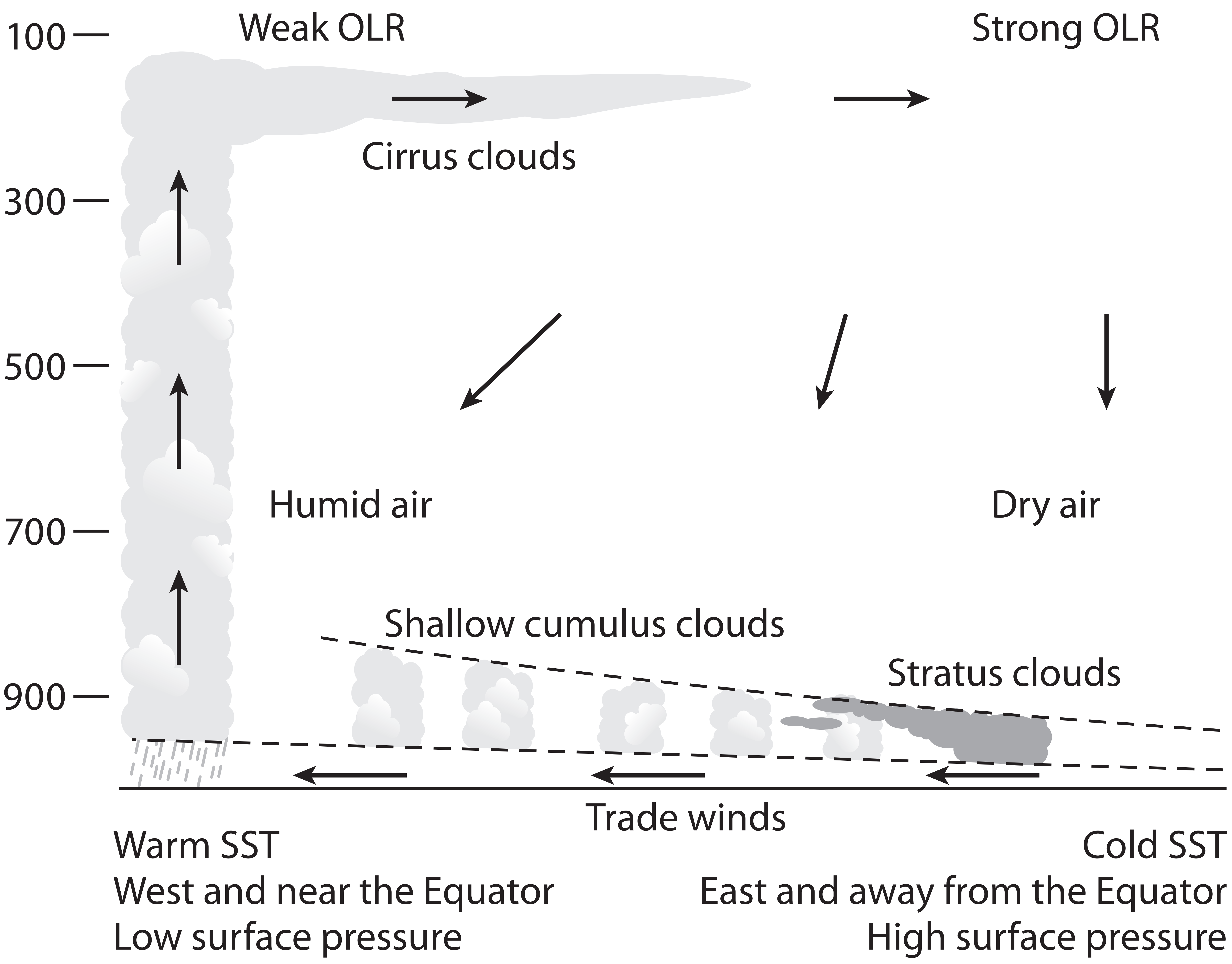
BY

JOANNE STARR MALKUS

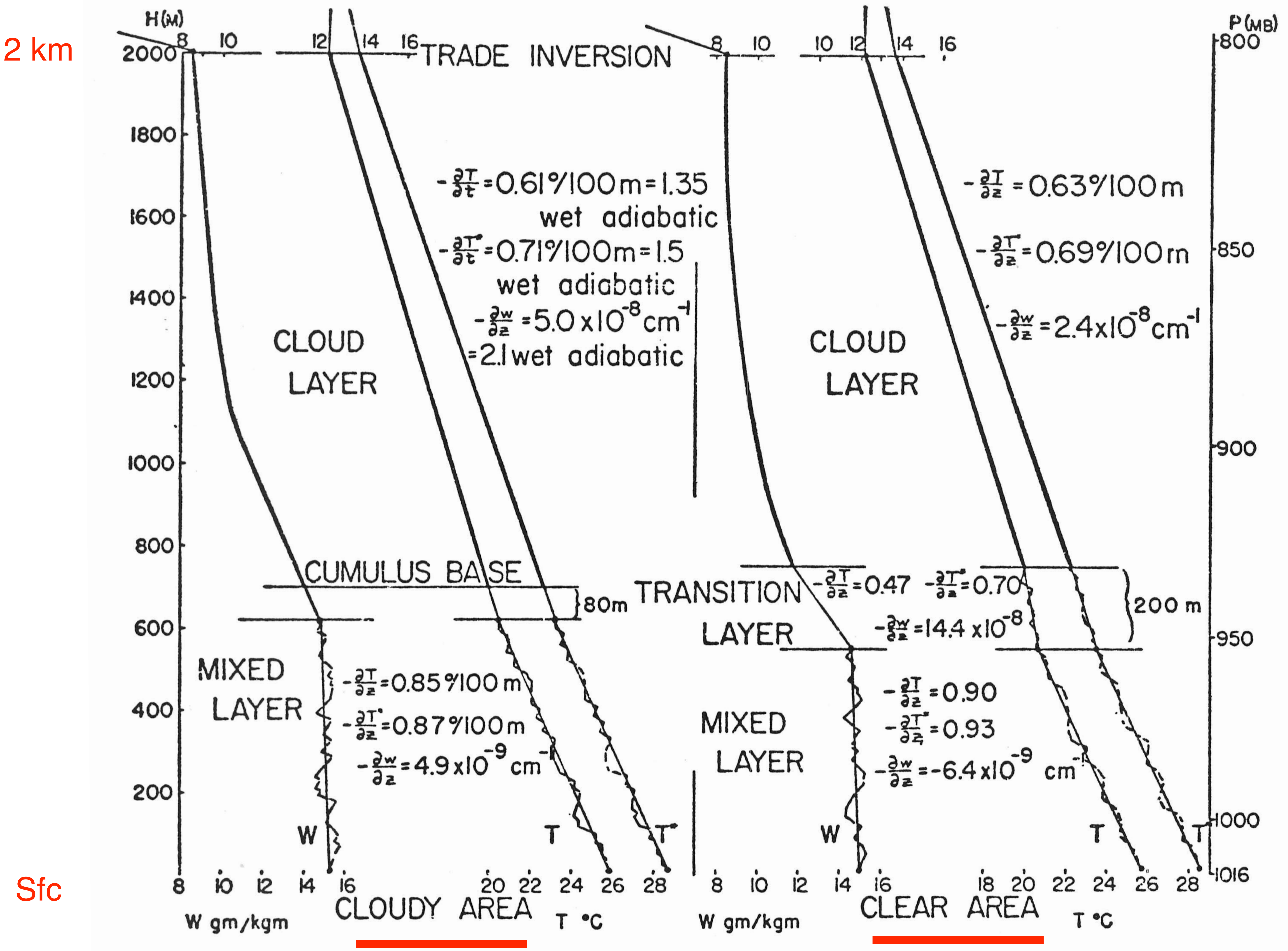
Contribution No. 969 from the Woods Hole Oceanographic Institution

CAMBRIDGE AND WOODS HOLE, MASSACHUSETTS

AUGUST, 1958


100
300
500
700
900
Weak OLR
Strong OLR
Cirrus clouds
Humid air
Dry air
Shallow cumulus clouds
Stratus clouds
Trade winds
Warm SST
West and near the Equator
Low surface pressure
Cold SST
East and away from the Equator
High surface pressure

Subcloud and cloud layers



Marine stratocumulus clouds

Mix of stratus and cumulus


Mix of stratus and cumulus

Off the California coast

Near cloud top

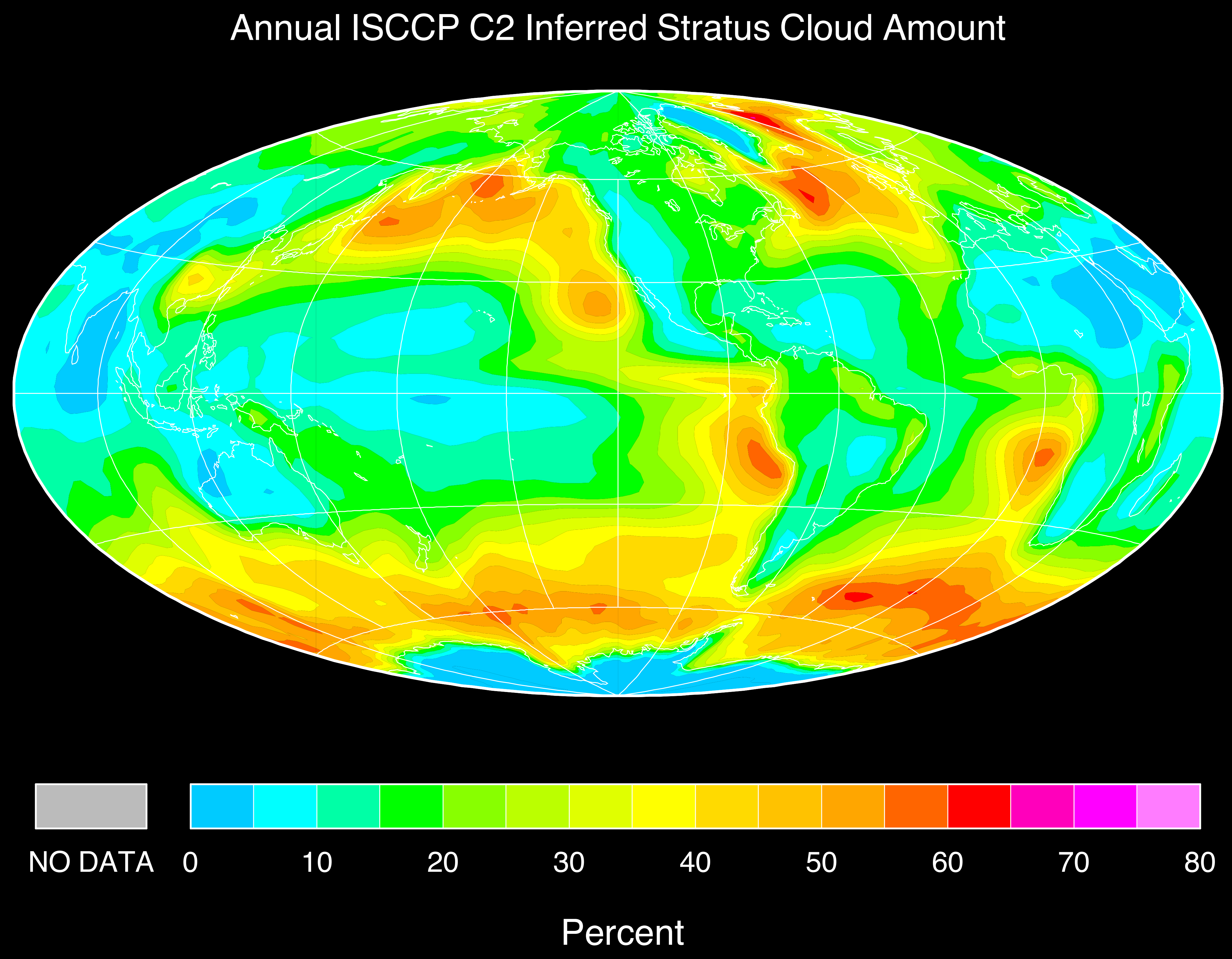
Cumulus under stratus

Coastal California

Wimpy but important

- Marine stratocumulus cloud layers are just a few hundred meters deep.
- They are capped by a strong inversion that is even thinner.
- The in-cloud turbulence is driven mainly by very strong radiative cooling confined to an extremely thin layer.
- It is virtually impossible to explicitly resolve these features in a large-scale model.
- They are very important for climate.

Impact on the Earth's radiation budget



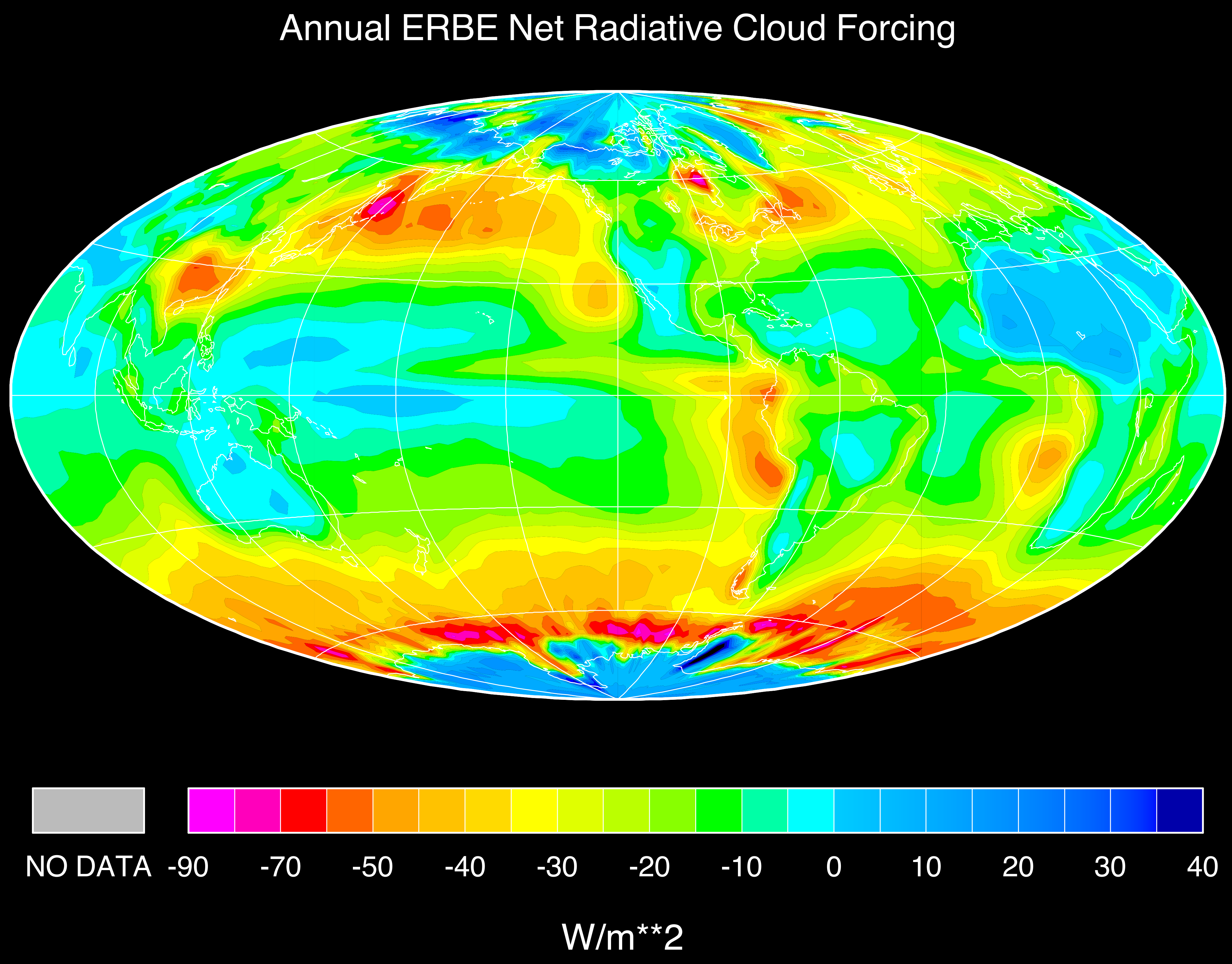
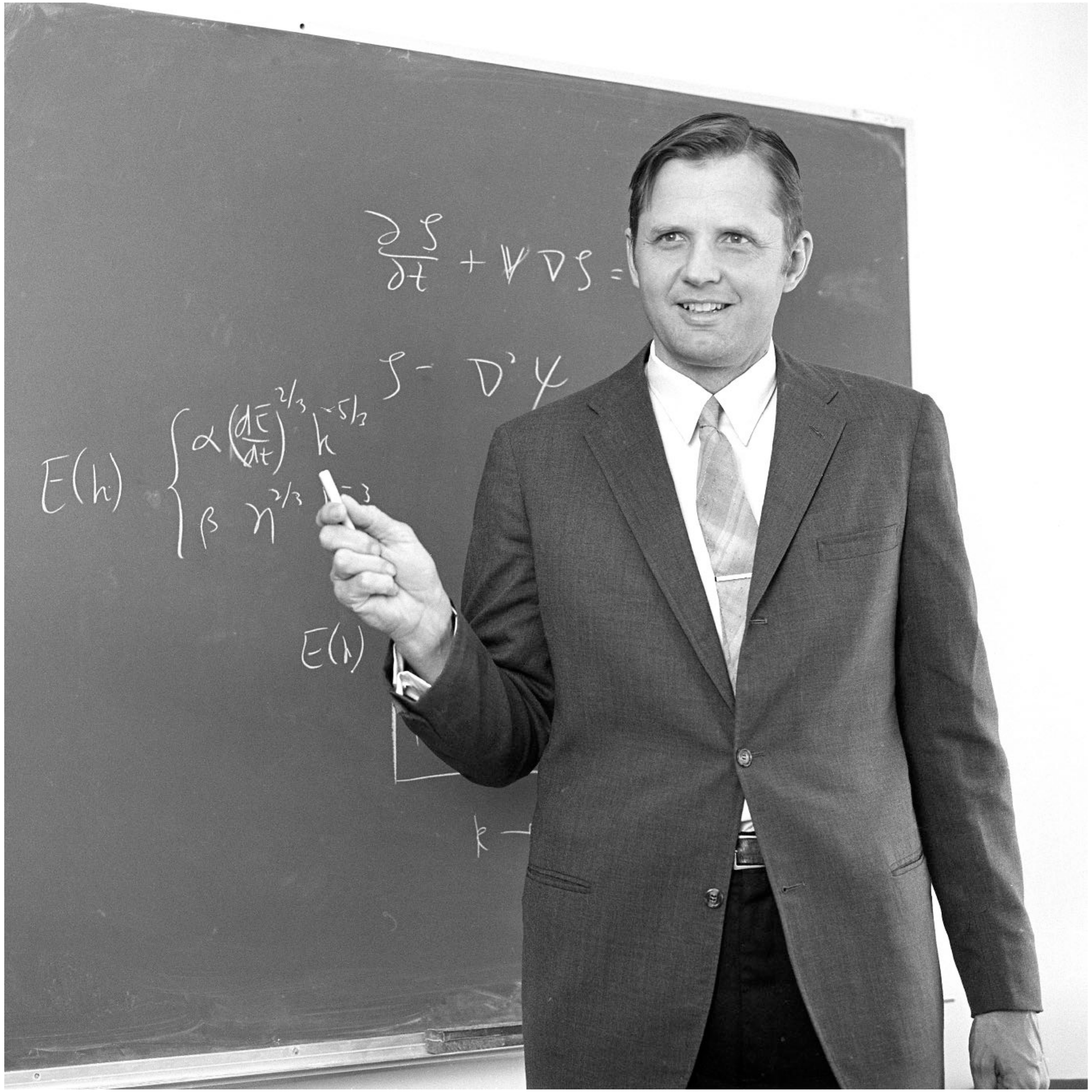


Figure from Norris & Leovy



Doug Lilly

551.511 : 551.576.11

Models of cloud-topped mixed layers under a strong inversion

By D. K. LILLY

National Center for Atmospheric Research, Boulder, Colorado

(Manuscript received 3 July 1967; in revised form 29 January 1968)

SUMMARY

Theoretical models are constructed with the aim of relating, explaining and predicting features of a radiatively active turbulent cloud layer over the sea and under a strong subsidence inversion. Both dry aerosol clouds (no phase change) and wet clouds (with a phase change and latent heat exchanges) are considered. For the wet cloud case an important element of the theory is the requirement that the wet-bulb potential temperature must increase upwards in the inversion. For both cases entrainment of the upper warm air is hypothesized to lie between upper and lower limits determined from the turbulent energy budget. The dry cloud case is solved for both steady state and transient results, with only the transient behaviour depending on the entrainment hypothesis. Only steady state solutions are presented for the more complex wet cloud case and these differ somewhat for the maximum and minimum entrainment limits. Observational data from Oakland, California are used for comparison with those steady state solutions, with results indicating the essential validity of the approach. Detailed comparisons, especially for determination of the most correct entrainment rate, are hampered both by inadequate measurement of the inversion properties and by uncertainties in the net radiation flux leaving the cloud top. Computations of the latter suggest that several presently used radiation models are still in serious disagreement, at least for application to downward flux under an inversion. It is suggested that the present theory provides a partial explanation of the origin of the trade wind inversion.

1. PHYSICAL FRAMEWORK AND MOTIVATION

A shallow turbulent moist stratum is characteristic of Equatorward flow just above the middle-latitude and tropical oceans, especially at the eastern ends of sub-tropical high pressure areas of the eastern North and South Pacific and South Atlantic oceans. This moist stratum, or marine layer, is typified by a solid or broken cloud cover, topped by a strong temperature inversion with warm dry air aloft. The extreme sharpness of the inversion, observed strikingly in kite soundings from the Meteor expedition (von Ficker 1936, examples presented by Riehl 1954), is normally partly concealed by the instrumental lag in balloon-borne radiosondes. It has often been revealed further, however, by instrumented airplane flights (James 1959; Edinger 1963, 1966) and slow-ascent radiosondes (Williams and De Mandel 1966).

Qualitative explanations for this régime, offered for many years (Petterssen 1938; Riehl 1954, Chapter 2; Neiburger 1944), were based on the following arguments. In conditions of strong or moderate subsidence the potential temperature of the lower troposphere may be substantially higher than that of the ocean surface. Since the ocean has effectively an almost infinite heat capacity, a temperature inversion or stable layer must form somewhere. The turbulence generated by surface shear could be expected to mix the lower layer and maintain a sharp, somewhat elevated inversion, while radiation from the top of a cloud would exert an additional cooling effect in the mixed layer.

In recent years the effect of stratification on the surface boundary layer has been clarified somewhat by observation and theoretical analysis. In particular, the length scale introduced by Monin and Obukhov is shown to act as an upper bound on the influence of surface shear-generated turbulence. Since this scale is not often greater than a few tens of meters, it seems that we must deny the importance of surface shear in the maintenance of a mixed layer 500 to 1,000 meters or more in thickness.

In the present theory we therefore disregard the effects of the surface shear-generated turbulence but consider radiation off the cloud tops as an essential element. In this respect we follow Petterssen, who particularly emphasized the radiative heat loss. This theory may not be valid for the occasionally observed cases of extremely shallow (< 100 m)

marine layers without condensation, but is intended to hold for the fog-, stratus- or stratocumulus-containing layer typically observed near the coast of California in the summer, and near northern Chile, southern Peru and South West Africa most of the year.

A further essential foundation of the present theory is the stability of the cloud top against penetration by the very dry upper air mass. If a parcel of the upper air is introduced into the cloud layer and mixed by turbulence, evaporation of cloud droplets into the dry parcel will reduce its temperature. If the mixed parcel reaches saturation at a colder temperature than that of the cloud top it will be negatively buoyant and can then penetrate freely into the cloud mass. In such a case the evaporation and penetration process will occur spontaneously and increase unstably until the cloud is evaporated. The condition for no change in temperature upon evaporative mixing is that the wet-bulb potential temperatures of the wet and dry layers be equal. We assume, therefore, that for stability of a cloud layer the inversion at its top must be sufficiently strong that the wet-bulb potential temperature remains constant or increases upward at the cloud top. For a difference of 5-10 g/kg in mixing ratio between mixed layer and upper dry layer this requires ~ 13-26°C increase in potential temperature across the inversion. Such inversion strengths are commonly observed in west coast soundings and it appears, on the basis of somewhat inadequate data, that the theoretical requirement is usually satisfied.

We further assume that the large-scale vertical motion and the upper air (above the inversion) temperature and humidity structure are known, as well as the near-surface wind, surface temperature, and (saturated) surface humidity. Certain simplified formulations are used for calculating surface heat and moisture fluxes and radiative heat-flux. Precipitation is neglected, which probably restricts the application to rather thin and not too cold cloud layers.

The interaction of large-scale atmospheric properties and thermal convection is a principal unsolved problem in the development of forecast and/or general circulation models. The phenomenon considered here represents one form, in some respects a relatively simple one, of this interaction. There is a considerable lack of highly definitive observational data on layered convection, in fact on most kinds of non-violent cloud convection. A principal motivation for this work was to sharpen the questions to be asked and to help avoid purely exploratory observations, which may already exist in sufficient abundance.

2. A SIMPLIFIED DRY 'CLOUD' LAYER

As a simplified illustration of the mechanism which will be explored more realistically in the next section, we consider a shallow radiation-absorbing cloud of dust, smoke, or other inert matter occupying the lower part of a horizontally homogeneous air mass subject to a specified large-scale vertical motion field $w(z) \leqslant 0$. Radiative cooling is occurring in the air above the cloud at a rate which will maintain a stable potential temperature profile $\theta_U(z)$, i.e., $\partial\theta_U/\partial z > 0$. The upper layer above the cloud is assumed to be non-turbulent. The lower boundary, the Earth's surface, is maintained at a potential temperature θ_S. Turbulent heat-flux from the surface is assumed proportional to the near-surface wind and to the surface-air temperature difference through a heat transfer coefficient C_T, that is

$$(\overline{w'\theta'})_0 = C_T V_0 (\theta_S - \theta_0), \quad . \quad . \quad . \quad . \quad (1)$$

where θ_0 is the potential temperature of the air at a small height above the surface. If this heat transfer is positive it is assumed that the entire cloud layer, from the surface to $z = H$, is well mixed at a very high Rayleigh number, and is therefore nearly isentropic with potential temperature θ_0 except very close to the boundaries. At the top of the cloud layer a net outward radiative heat flux F_{UH} tends to cool the top, and therefore the entire cloud layer.

We now proceed to derive expressions for the time rates of change of θ_0 and H, as functions of their initial conditions and the values of w, F_{UH} and θ_U, considered as

“Models of cloud-topped mixed layers”

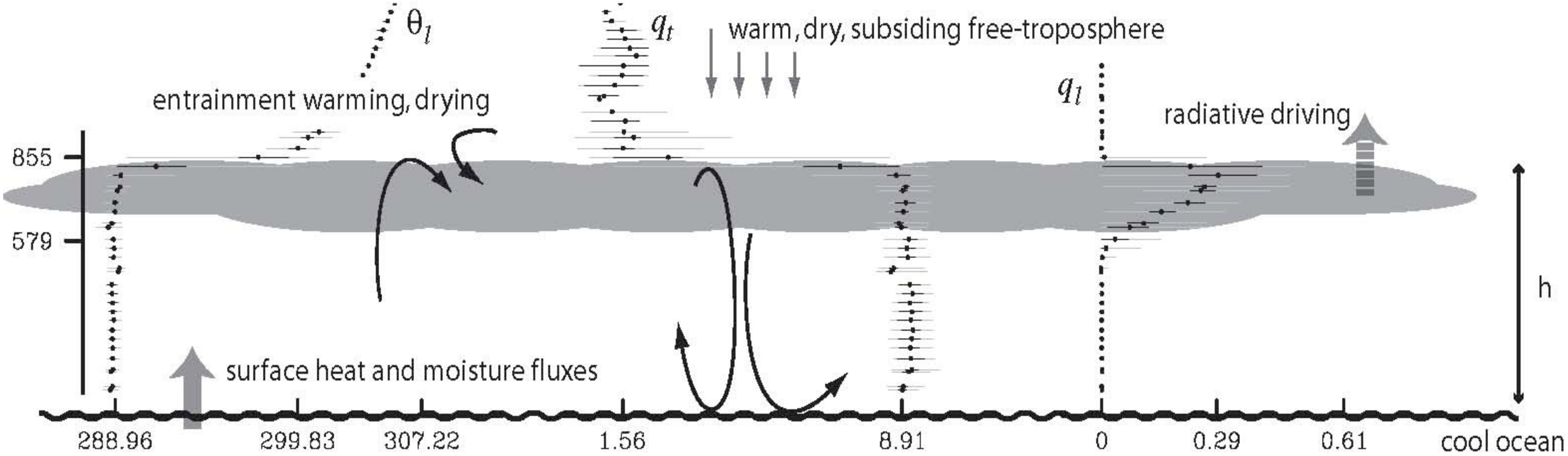


Figure from Bjorn Stevens

Concentrated radiative cooling

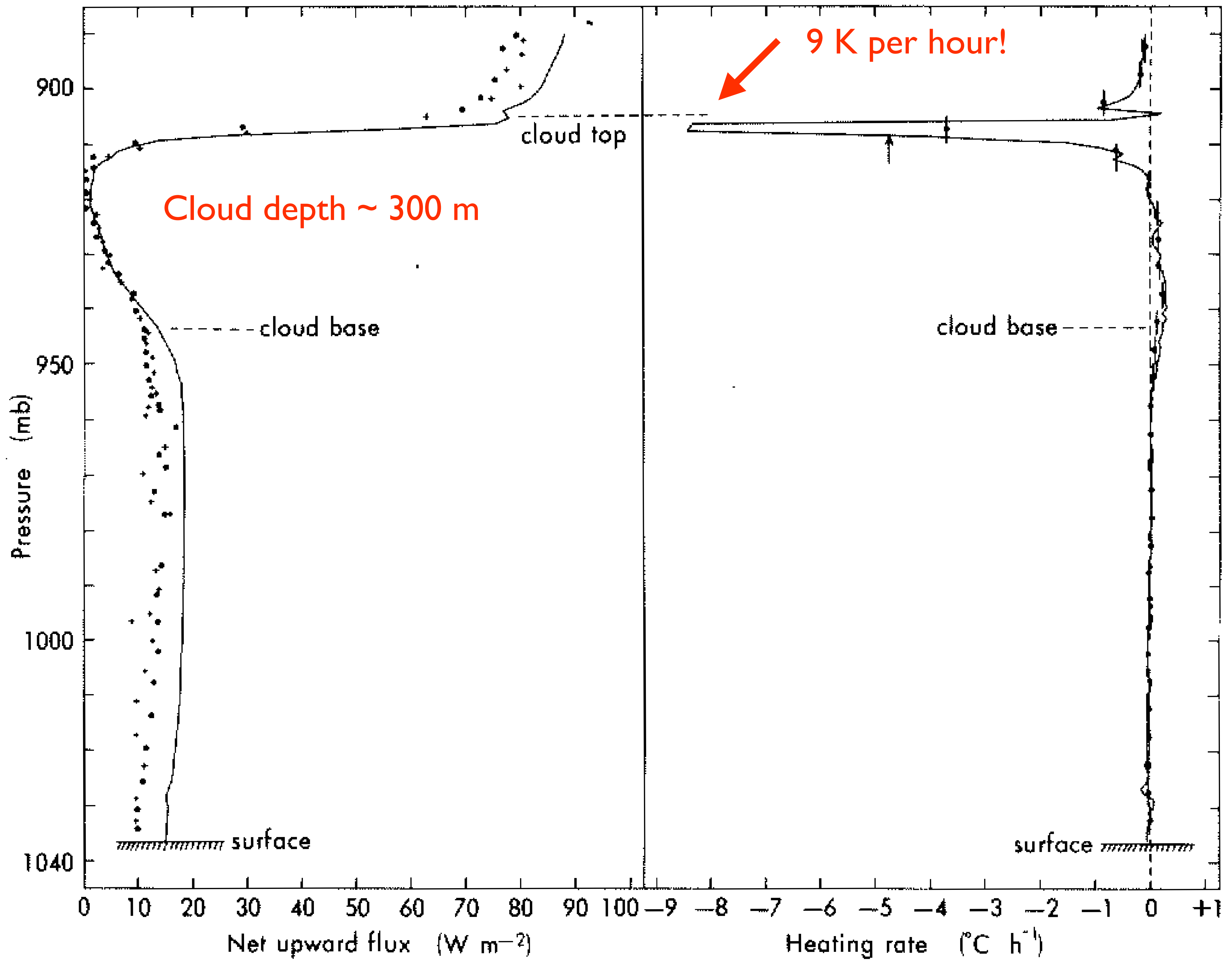


Figure 3. Comparison of the observed and theoretical net infrared fluxes and heating rates for profile 1, 19–20 November 1976. The continuous lines are from the radiation scheme, using the balloon temperatures and humidities and the scaled ASSP data. The dots and crosses represent the corrected fluxes from the upper and lower radiometers, respeetively. Only every fifth value has been plotted.

Slingo et al. (1982)

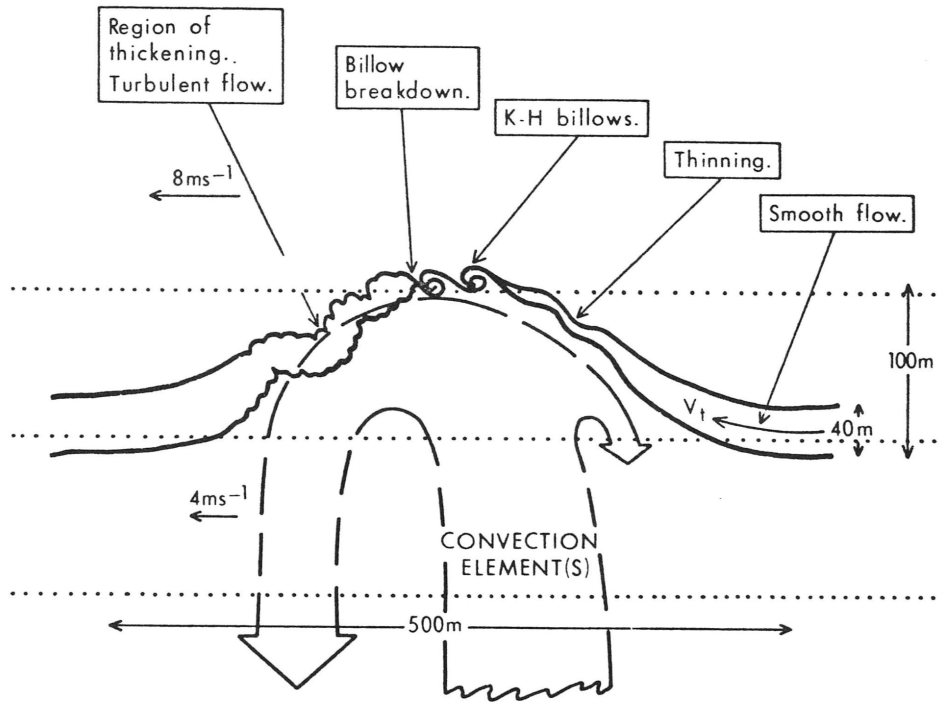
Entrainment across cloud top

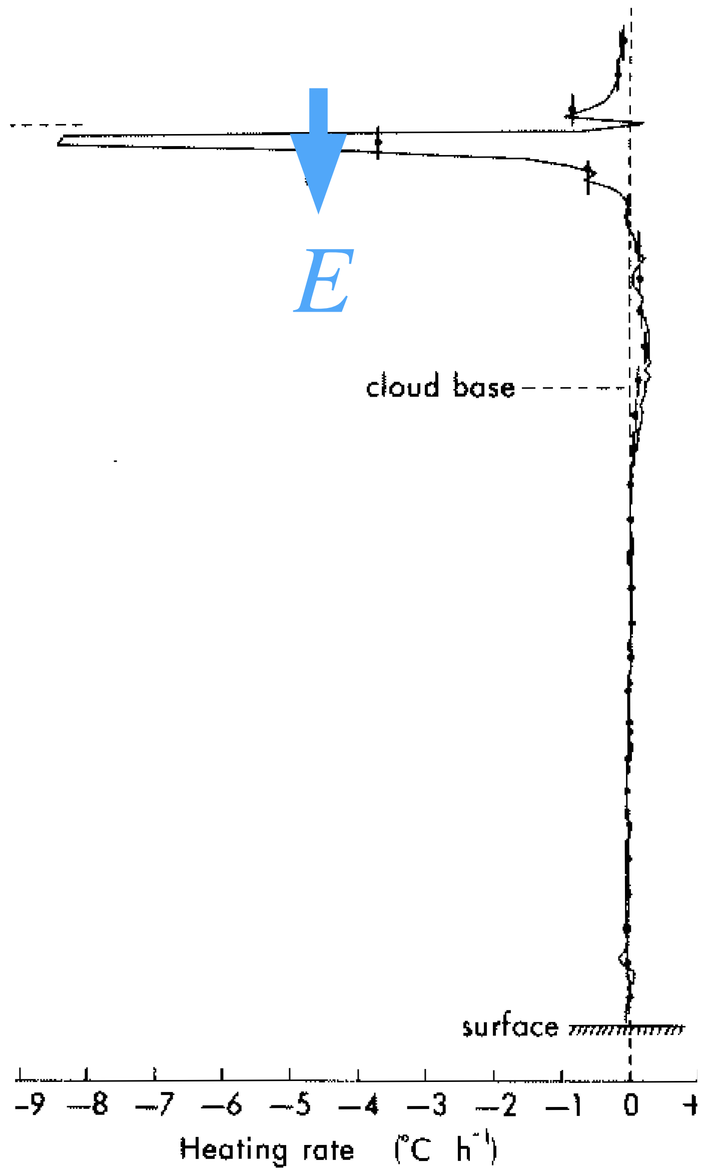
Fluxes due to entrainment

$$\rho \overline{w'h'} = - E \Delta \overline{h} + \Delta R$$

$$\rho \overline{w'q_t'} = - E \Delta \overline{q_t}$$

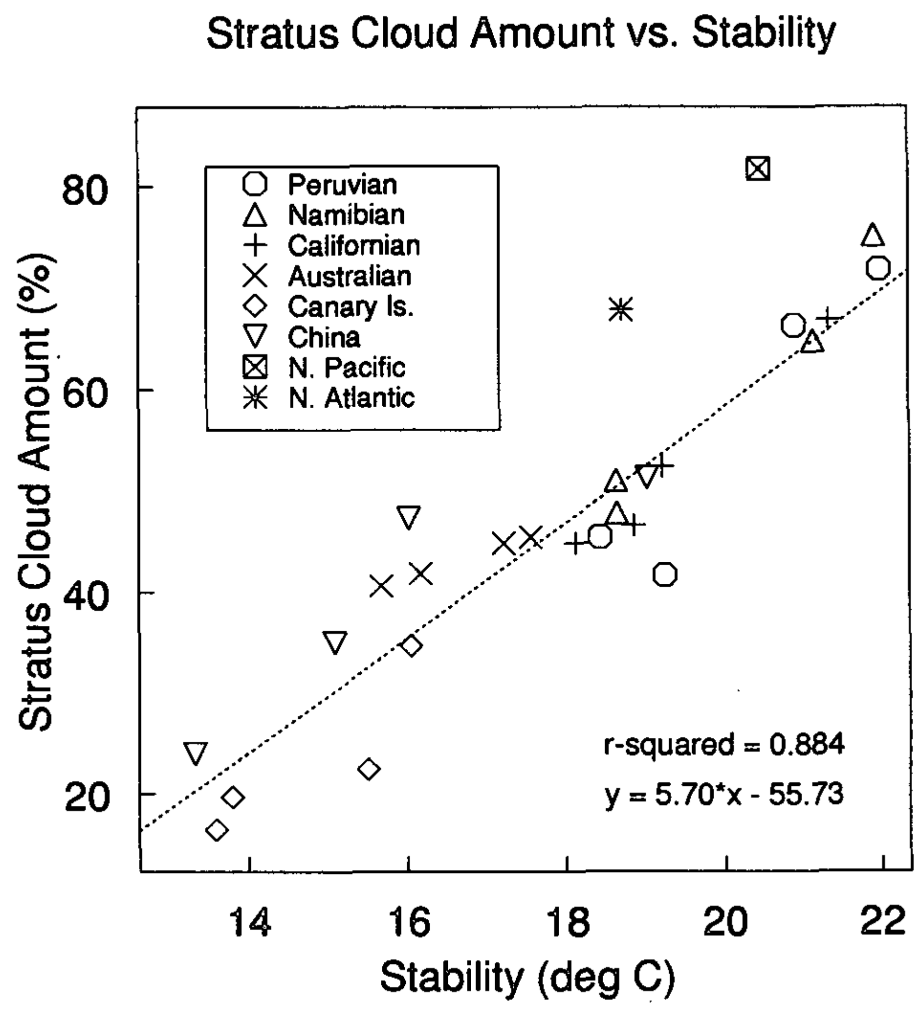
$$E = \frac{\rho e_M^{3/2} \theta_0}{g h \Delta \theta_v}$$





More TKE leads to faster entrainment.
A stronger inversion leads to slower entrainment.

Rapid entrainment of dry air can break up a stratus cloud.
This is why weak inversions lead to smaller cloud amounts.



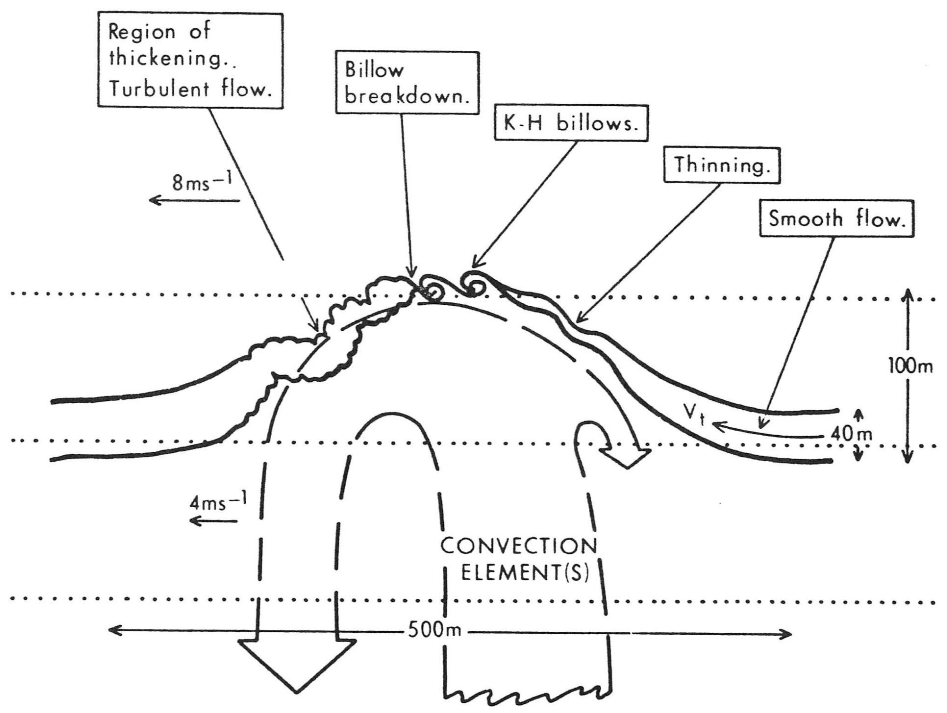
Klein & Hartmann (1993)

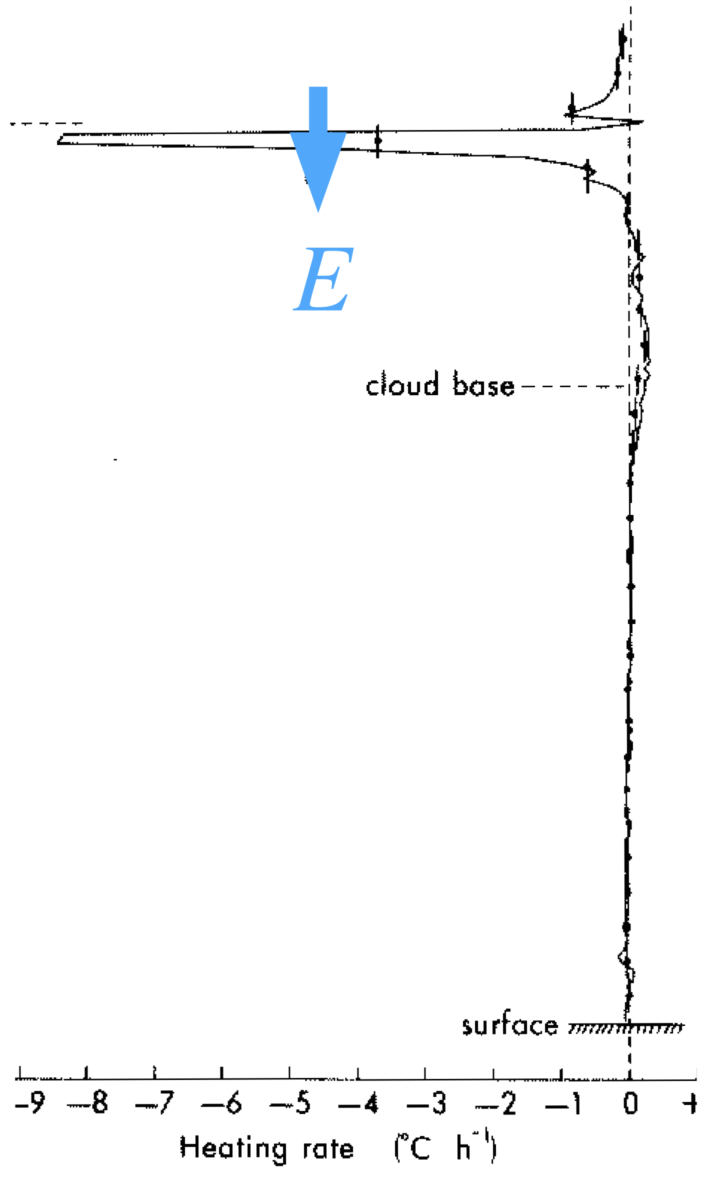
Fluxes due to entrainment

$$\rho\overline{w'h'} = -E\Delta\overline{h} + \Delta R$$

$$\rho\overline{w'q_t'} = -E\Delta\overline{q_t}$$

$$E = \frac{\rho e_M^{3/2}\theta_0}{gh\Delta\theta_v}$$





More TKE leads to faster entrainment.
A stronger inversion leads to slower entrainment.

Rapid entrainment of dry air can break up a stratus cloud.
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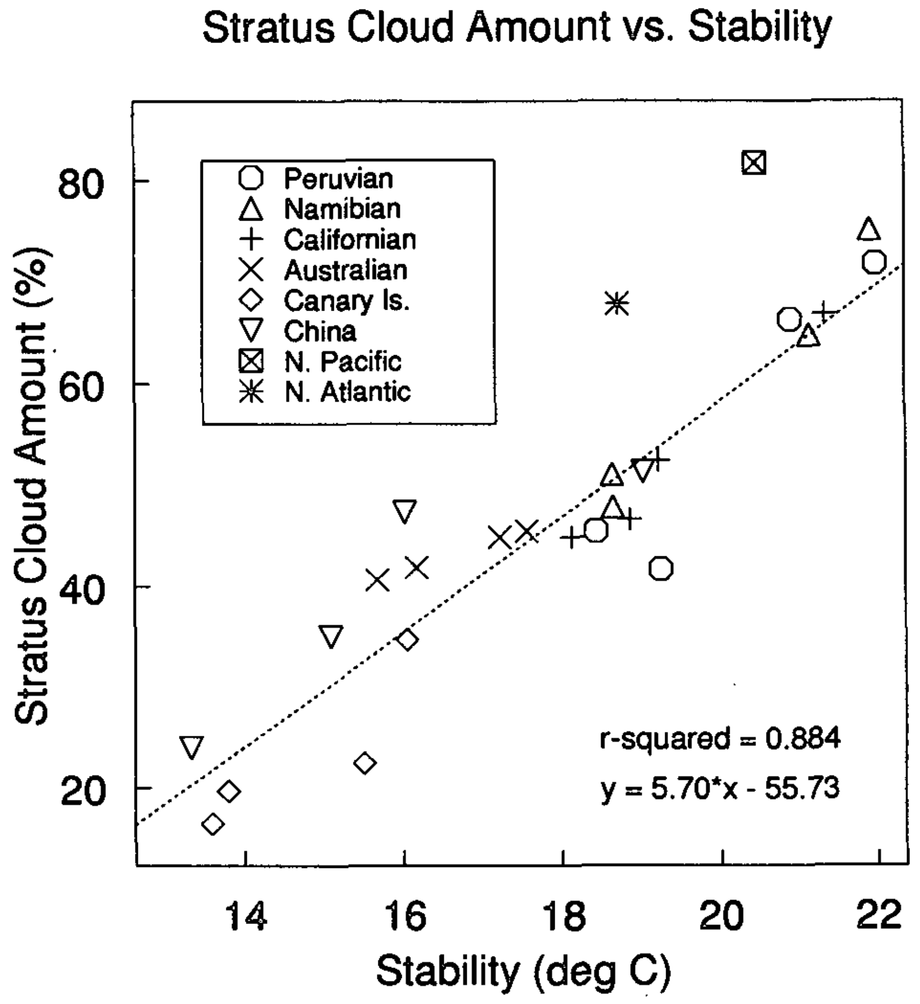
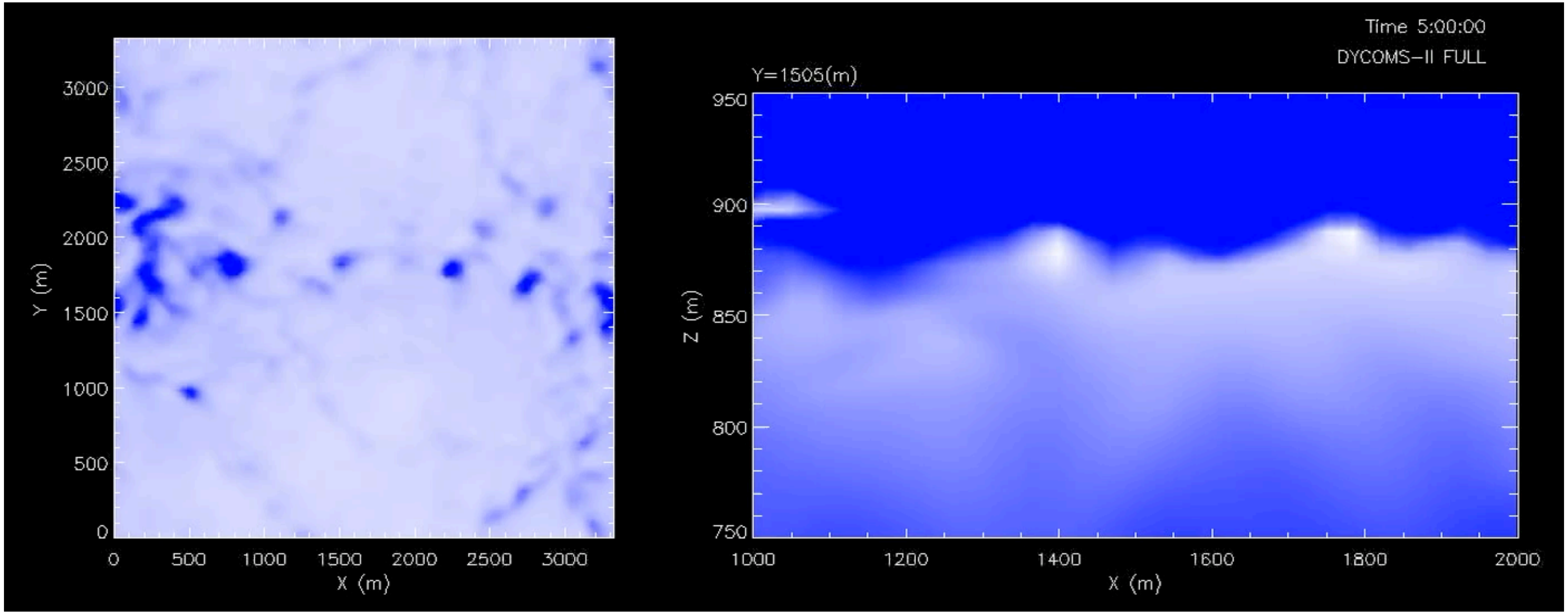


Klein & Hartmann (1993)

Clouds don’t entrain.

Turbulence entrains.

Clouds are turbulent.



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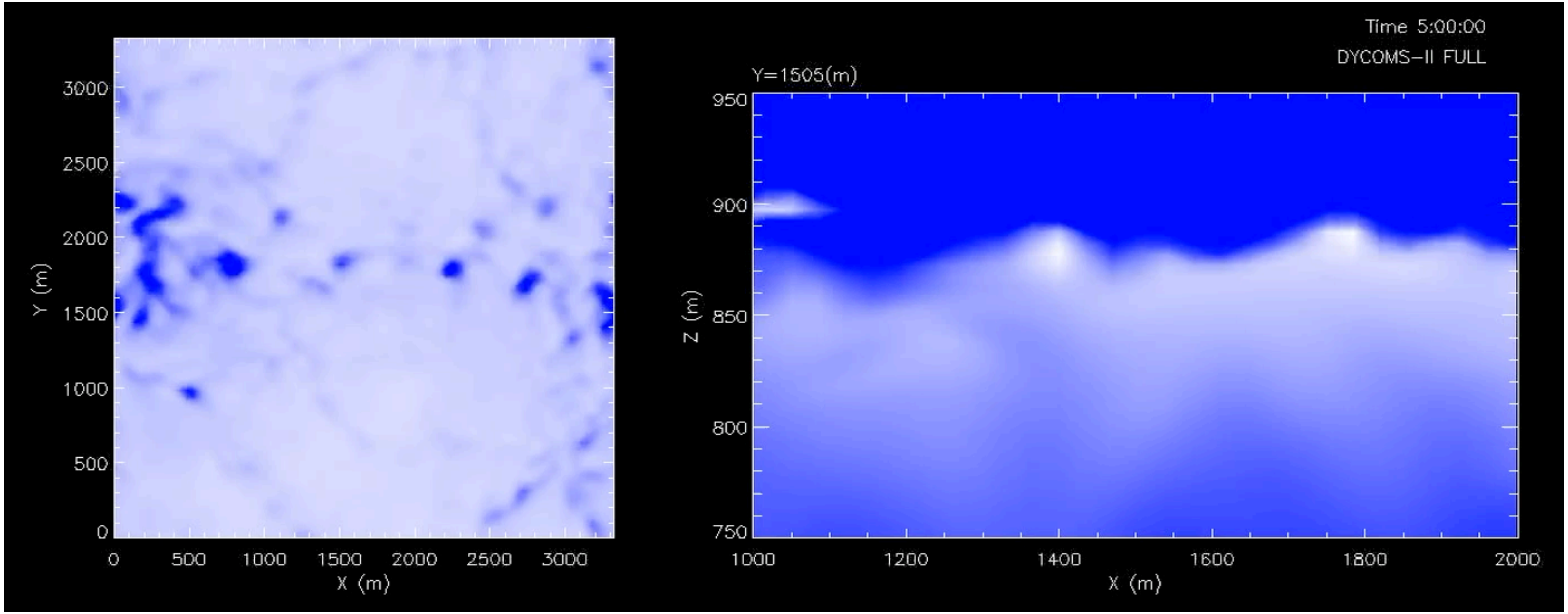


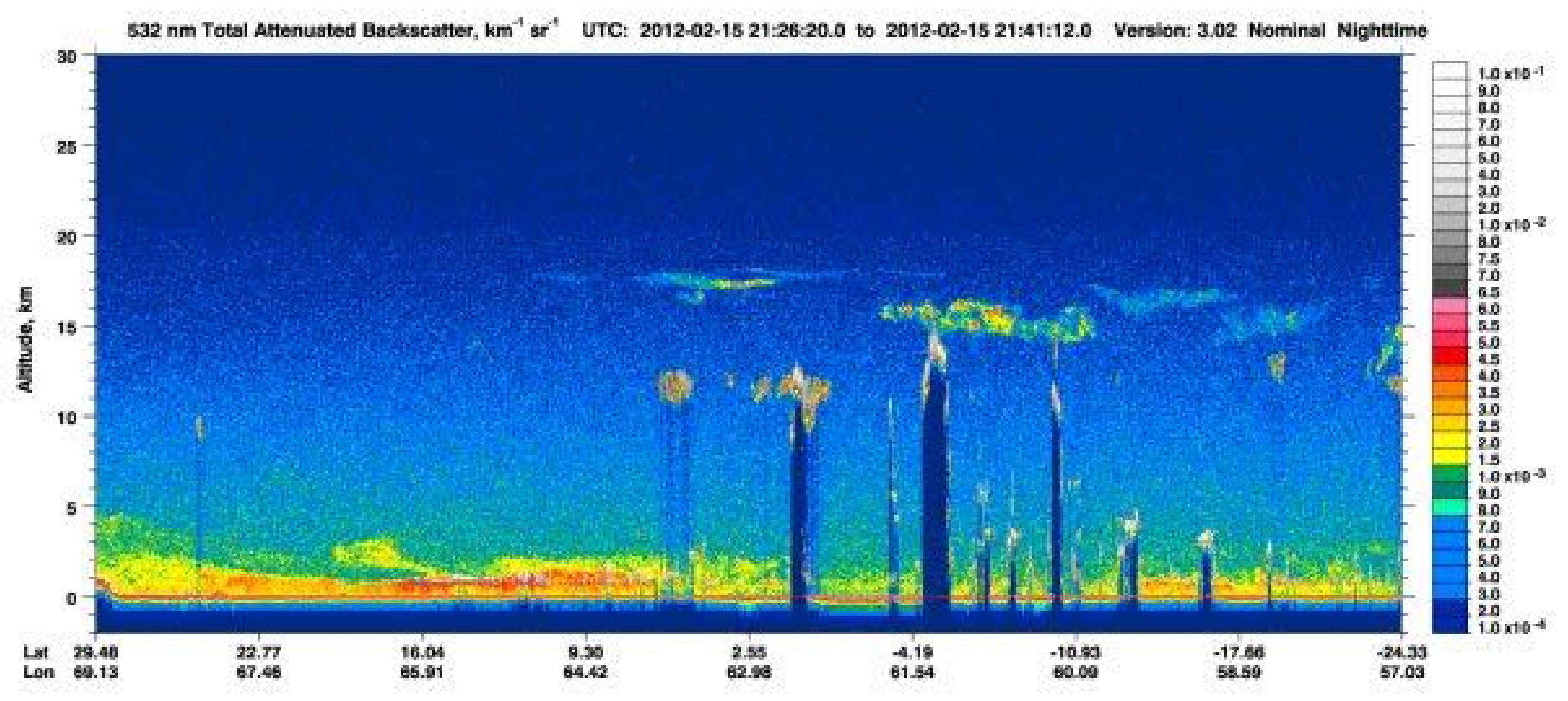
Stratocumulus clouds “lock in” over the eastern subtropical oceans

Stratocumulus clouds "lock in" over the eastern subtropical oceans

Stratocumulus clouds "lock in" over the eastern subtropical oceans

Five approaches to boundary layer parameterization

- Similarity theories
- Eddy diffusion
- Mixed-layer models
- Higher-order closure
- Mass fluxes



Higher-order closure

(HOC)

- A bar means a grid-area average, also called a "*first moment*." It is a statistic. $\overline{q}$
- A prime means a departure from a grid-area average. The average of a prime is zero. q'
- A "*prime prime bar*," which can be called a "*second moment*," is a statistic that arises from correlated variations on unresolved scales. $\overline{w'q'}$
- Some second moments are fluxes of first moments.
- A *third moment* has the form "prime prime prime bar." Some third moments are fluxes of second moments. $\overline{w'w'q'}$
- A model that predicts anything higher than first moments is called a "*higher-order closure*" model.

HOC started in the 1960s

- Obscure technical reports
- Engineering applications
- Immediate interest from atmospheric scientists

Glushko, G. S., 1965: Turbulent Boundary Layer on a Flat Plate in an Incompressible Fluid. *Bull. Acad. Sci. USSR*, Mech. Ser., no. 4, 13-23.

Bradshaw, P., D. H. Ferriss, and N. P. Atwell, 1967: Calculation of Boundary Layer Development Using the Turbulent Energy Equation. *J. Fluid Mech.*, **25,** 593-616.

Beckwith, I. E. and D. M. Bushnell, 1968: Detailed Description and Results of a Method for Computing Mean and Fluctuating Quantities in Turbulent Boundary Layers. *NASA TN D-4815*.

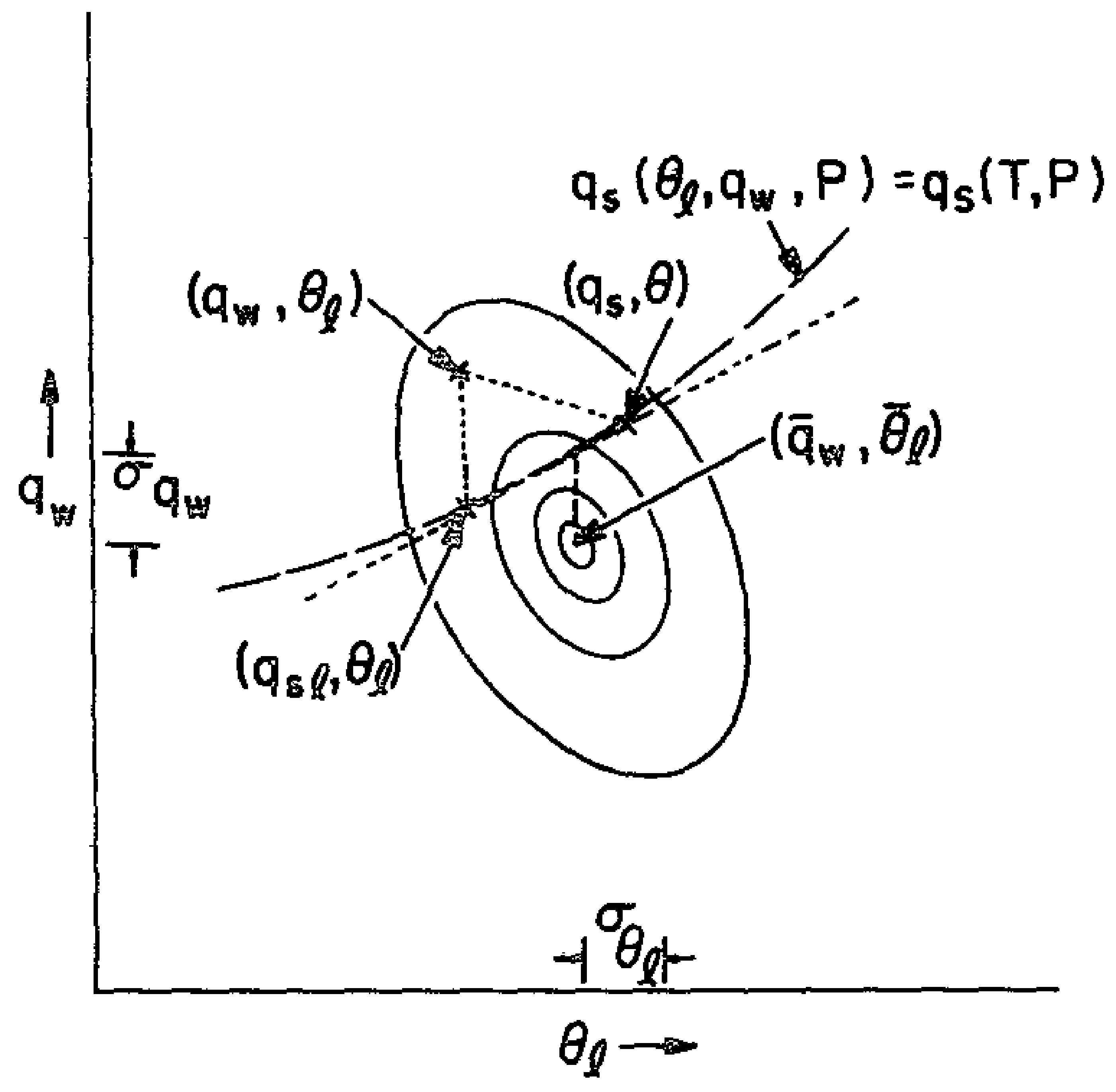
Donaldson, C. duP., and H. Rosenbaum, 1968: Calculation of the turbulent shear flows through closure of the Reynolds equations by invariant modeling. *NASA SP-216,* pp. 231-253.

HOC is being used in Earth System Models today.

CLUBB is an implementation of HOC that is being used in both CESM and E3SM.

There are other parameterizations based on HOC, including SHOC.

Fractional cloudiness



Five parameters:

Two means

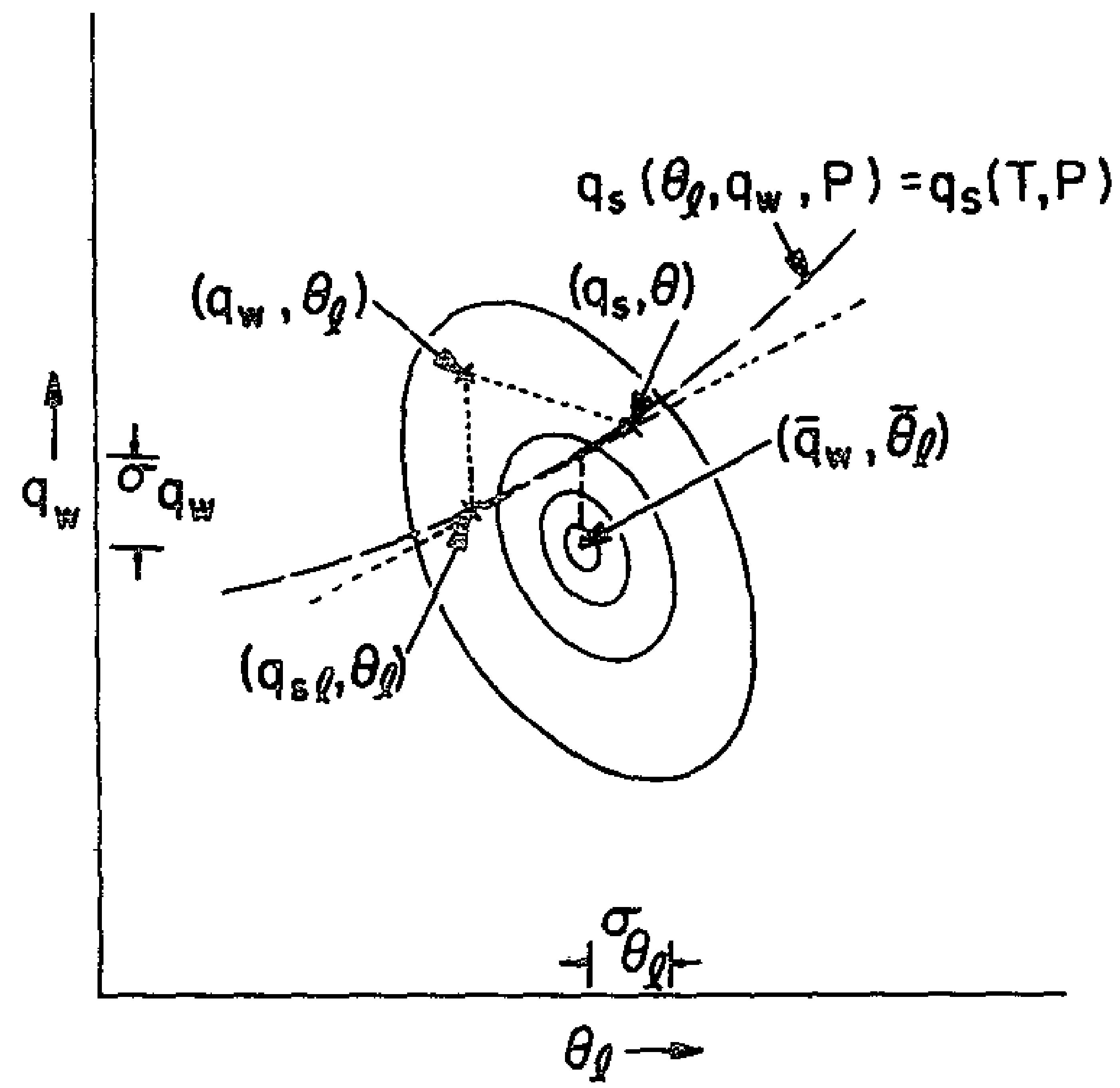
Two variances

One covariance

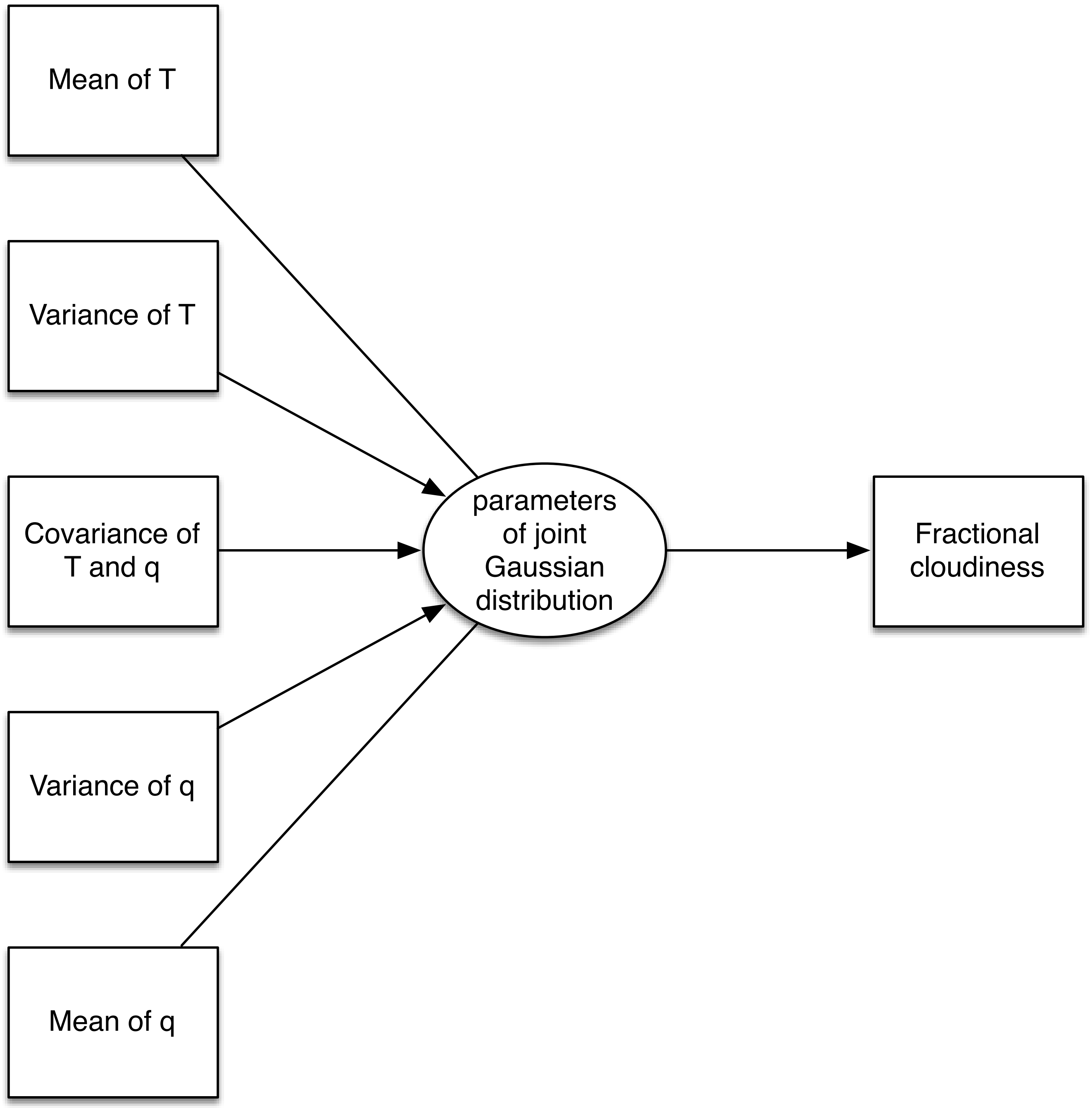
Sommeria & Deardorff (1977) and Mellor (1977)

Jim Deardorff

Their idea

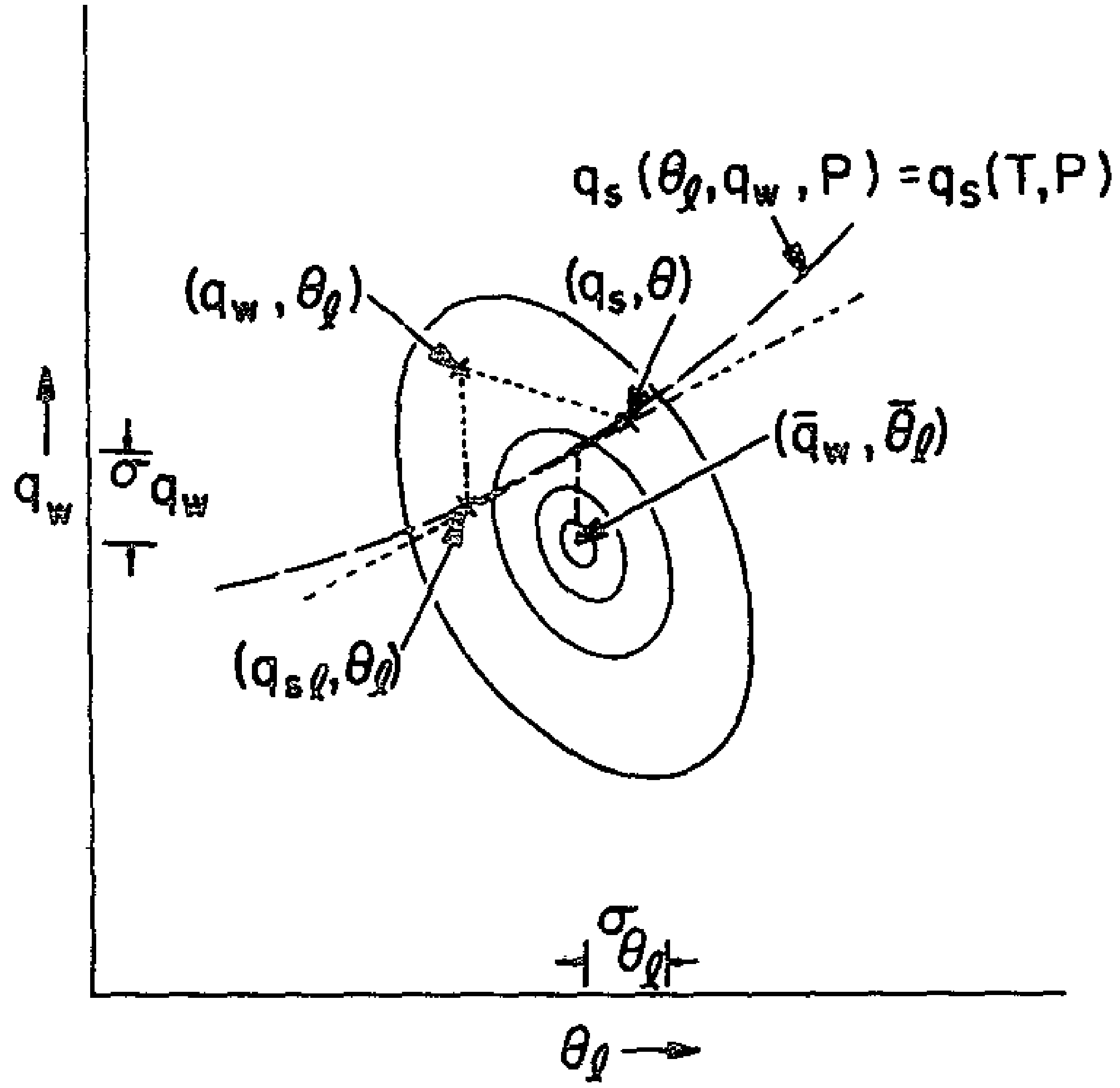


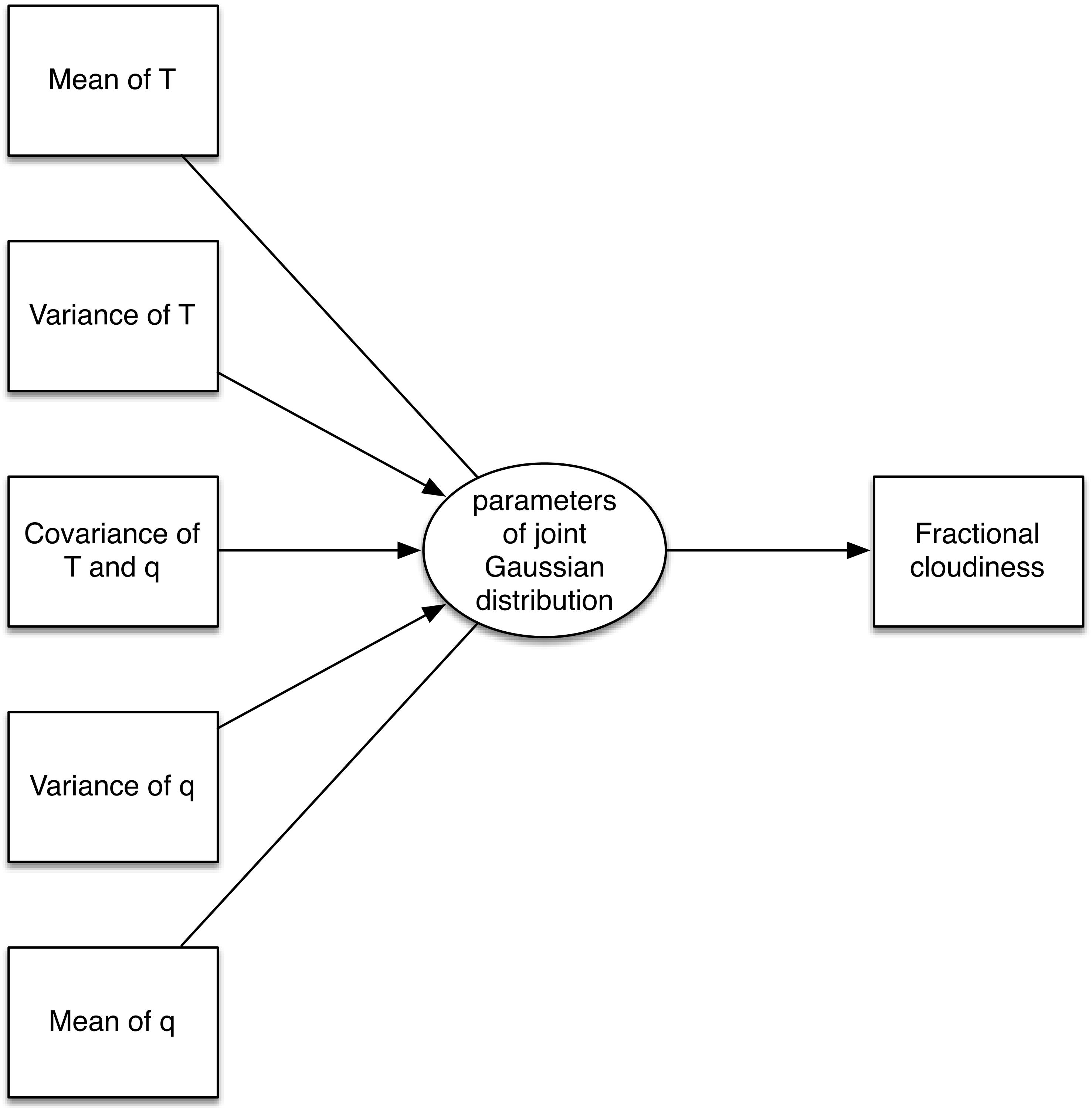
$q_s(\theta_\ell, q_w, P) = q_s(T,P)$
(q_w, θ_ℓ)
(q_s, θ)
$(\bar{q}_w, \bar{\theta}_\ell)$
$(q_{s\ell}, \theta_\ell)$
q_w
σ_{q_w}
σ_{θ_ℓ}
θ_ℓ



Mean of T
Variance of T
Covariance of T and q
Variance of q
Mean of q
parameters of joint Gaussian distribution
Fractional cloudiness

Their idea





Sommeria and Deardorff used an *assumed joint distribution* of temperature and moisture.

They needed, as input, 2 first moments and 3 second moments to determine the parameters of the joint distribution. HOC can predict the 3 second moments.

Sample equations of HOC

Predict a first moment

$$\frac{\partial}{\partial t}\left(\rho_0\bar{\theta}\right)+\frac{\partial}{\partial x_j}\left(\rho_0\bar{u}_j\bar{\theta}+\rho_0\overline{u_j'\theta'}+\bar{H}_j\right)=\frac{\theta_0}{c_pT_0}\bar{Q}$$

Predict a second moment

$$\frac{\partial}{\partial t}\left(\rho_0\overline{u_i'\theta'}\right)+\frac{\partial}{\partial x_j}\left(\bar{u}_j\rho_0\overline{u_i'\theta'}+\overline{u_j'\rho_0u_i'\theta'}\right)$$

$$=-\rho_0\overline{u_i'u_j'}\frac{\partial\bar{\theta}}{\partial x_j}-\rho_0\overline{u_j'\theta'}\frac{\partial\bar{u}_i}{\partial x_j}+2\varepsilon_{i,j,k}\rho_0\overline{u_j'\theta'}\Omega_k$$

$$\overline{-\rho_0\theta'\frac{\partial}{\partial x_i}\left(\frac{\delta p'}{\rho_0}\right)}+\rho_0\frac{\overline{(\theta')^2}}{\theta_0}g_i+\theta'\frac{\overline{\partial\Im_{i,j}'}}{\partial x_j}+\frac{\theta_0}{T_0}\frac{\overline{u_i'Q'}}{c_p}-\overline{u_i'\frac{\partial H_j'}{\partial x_j}}$$

Predict a third moment

$$\frac{\partial}{\partial t}\overline{\theta'\theta'\theta'}+\bar{u}_j\frac{\partial}{\partial x_j}\overline{\theta'\theta'\theta'}+3\overline{\theta'\theta'u_j'}\frac{\partial\bar{\theta}}{\partial x_j}+\frac{1}{\rho_0}\frac{\partial}{\partial x_j}\left(\rho_0\overline{u_j'\theta'\theta'\theta'}\right)=\frac{3}{\rho_0}\overline{\theta'\theta'\frac{\partial H_{3,j}'}{\partial x_j}}-\frac{3\overline{\theta'\theta'}}{\rho_0}\frac{\partial}{\partial x_j}\left(\rho_0\overline{\theta'u_j'}\right)$$

Sample equations of HOC

Predict a first moment

$$\frac{\partial}{\partial t}\left(\rho_0\bar{\theta}\right)+\frac{\partial}{\partial x_j}\left(\rho_0\bar{u}_j\bar{\theta}+\rho_0\overline{u_j'\theta'}+\bar{H}_j\right)=\frac{\theta_0}{c_pT_0}\bar{Q}$$

Predict a second moment

$$\frac{\partial}{\partial t}\left(\rho_0\overline{u_i'\theta'}\right)+\frac{\partial}{\partial x_j}\left(\bar{u}_j\rho_0\overline{u_i'\theta'}+\overline{u_j'\rho_0u_i'\theta'}\right)$$

$$=-\rho_0\overline{u_i'u_j'}\frac{\partial\bar{\theta}}{\partial x_j}-\rho_0\overline{u_j'\theta'}\frac{\partial\bar{u}_i}{\partial x_j}+2\varepsilon_{i,j,k}\rho_0\overline{u_j'\theta'}\Omega_k$$

$$\overline{-\rho_0\theta'\frac{\partial}{\partial x_i}\left(\frac{\delta p'}{\rho_0}\right)}+\rho_0\frac{\overline{\left(\theta'\right)^2}}{\theta_0}g_i+\overline{\theta'\frac{\partial\mathfrak{I}_{i,j}'}{\partial x_j}}+\frac{\theta_0}{T_0}\frac{\overline{u_i'Q'}}{c_p}-\overline{u_i'\frac{\partial H_j'}{\partial x_j}}$$

Predict a third moment

$$\frac{\partial}{\partial t}\overline{\theta'\theta'\theta'}+\bar{u}_j\frac{\partial}{\partial x_j}\overline{\theta'\theta'\theta'}+3\overline{\theta'\theta'u_j'}\frac{\partial\bar{\theta}}{\partial x_j}+\frac{1}{\rho_0}\frac{\partial}{\partial x_j}\left(\rho_0\overline{u_j'\theta'\theta'\theta'}\right)=\frac{3}{\rho_0}\overline{\theta'\theta'\frac{\partial H_{3,j}'}{\partial x_j}}-\frac{3\overline{\theta'\theta'}}{\rho_0}\frac{\partial}{\partial x_j}\left(\rho_0\overline{\theta'u_j'}\right)$$

Sample equations of HOC

Predict a first moment

$$\frac{\partial}{\partial t}\left(\rho_0\bar{\theta}\right)+\frac{\partial}{\partial x_j}\left(\rho_0\bar{u}_j\bar{\theta}+\rho_0\overline{u_j'\theta'}+\bar{H}_j\right)=\frac{\theta_0}{c_pT_0}\bar{Q}$$

Predict a second moment

$$\frac{\partial}{\partial t}\left(\rho_0\overline{u_i'\theta'}\right)+\frac{\partial}{\partial x_j}\left(\bar{u}_j\rho_0\overline{u_i'\theta'}+\overline{u_j'\rho_0u_i'\theta'}\right)$$

$$=-\rho_0\overline{u_i'u_j'}\frac{\partial\bar{\theta}}{\partial x_j}-\rho_0\overline{u_j'\theta'}\frac{\partial\bar{u}_i}{\partial x_j}+2\varepsilon_{i,j,k}\rho_0\overline{u_j'\theta'}\Omega_k$$

$$\overline{-\rho_0\theta'\frac{\partial}{\partial x_i}\left(\frac{\delta p'}{\rho_0}\right)}+\rho_0\frac{\overline{(\theta')^2}}{\theta_0}g_i+\overline{\theta'\frac{\partial\mathfrak{I}_{i,j}'}{\partial x_j}}+\frac{\theta_0}{T_0}\frac{\overline{u_i'Q'}}{c_p}-\overline{u_i'\frac{\partial H_j'}{\partial x_j}}$$

Predict a third moment

$$\frac{\partial}{\partial t}\overline{\theta'\theta'\theta'}+\bar{u}_j\frac{\partial}{\partial x_j}\overline{\theta'\theta'\theta'}+3\overline{\theta'\theta'u_j'}\frac{\partial\bar{\theta}}{\partial x_j}+\frac{1}{\rho_0}\frac{\partial}{\partial x_j}\left(\rho_0\overline{u_j'\theta'\theta'\theta'}\right)=\frac{3}{\rho_0}\overline{\theta'\theta'\frac{\partial H_{3,j}'}{\partial x_j}}-\frac{3\overline{\theta'\theta'}}{\rho_0}\frac{\partial}{\partial x_j}\left(\rho_0\overline{\theta'u_j'}\right)$$

Sample equations of HOC

Predict a first moment

$$\frac{\partial}{\partial t}\left(\rho_0\bar{\theta}\right)+\frac{\partial}{\partial x_j}\left(\rho_0\bar{u}_j\bar{\theta}+\rho_0\overline{u_j'\theta'}+\bar{H}_j\right)=\frac{\theta_0}{c_pT_0}\bar{Q}$$

Predict a second moment

$$\frac{\partial}{\partial t}\left(\rho_0\overline{u_i'\theta'}\right)+\frac{\partial}{\partial x_j}\left(\bar{u}_j\rho_0\overline{u_i'\theta'}+\overline{u_j'\rho_0u_i'\theta'}\right)$$

$$=-\rho_0\overline{u_i'u_j'}\frac{\partial\bar{\theta}}{\partial x_j}-\rho_0\overline{u_j'\theta'}\frac{\partial\bar{u}_i}{\partial x_j}+2\varepsilon_{i,j,k}\rho_0\overline{u_j'\theta'}\Omega_k$$

$$-\overline{\rho_0\theta'\frac{\partial}{\partial x_i}\left(\frac{\delta p'}{\rho_0}\right)}+\rho_0\frac{\overline{(\theta')^2}}{\theta_0}g_i+\overline{\theta'\frac{\partial\mathfrak{I}'_{i,j}}{\partial x_j}}+\frac{\theta_0}{T_0}\frac{\overline{u_i'Q'}}{c_p}-\overline{u_i'\frac{\partial H_j'}{\partial x_j}}$$

Predict a third moment

$$\frac{\partial}{\partial t}\overline{\theta'\theta'\theta'}+\bar{u}_j\frac{\partial}{\partial x_j}\overline{\theta'\theta'\theta'}+3\overline{\theta'\theta'u_j'}\frac{\partial\bar{\theta}}{\partial x_j}+\frac{1}{\rho_0}\frac{\partial}{\partial x_j}\left(\rho_0\overline{u_j'\theta'\theta'\theta'}\right)=\frac{3}{\rho_0}\overline{\theta'\theta'\frac{\partial H'_{3,j}}{\partial x_j}}-\frac{3\overline{\theta'\theta'}}{\rho_0}\frac{\partial}{\partial x_j}\left(\rho_0\overline{\theta'u_j'}\right)$$

Sample equations of HOC

Predict a first moment

$$\frac{\partial}{\partial t}\left(\rho_0\bar{\theta}\right)+\frac{\partial}{\partial x_j}\left(\rho_0\bar{u}_j\bar{\theta}+\rho_0\overline{u_j'\theta'}+\bar{H}_j\right)=\frac{\theta_0}{c_pT_0}\bar{Q}$$

Predict a second moment

$$\frac{\partial}{\partial t}\left(\rho_0\overline{u_i'\theta'}\right)+\frac{\partial}{\partial x_j}\left(\bar{u}_j\rho_0\overline{u_i'\theta'}+\overline{u_j'\rho_0u_i'\theta'}\right)$$

$$=-\rho_0\overline{u_i'u_j'}\frac{\partial\bar{\theta}}{\partial x_j}-\rho_0\overline{u_j'\theta'}\frac{\partial\bar{u}_i}{\partial x_j}+2\varepsilon_{i,j,k}\rho_0\overline{u_j'\theta'}\Omega_k$$

$$-\overline{\rho_0\theta'\frac{\partial}{\partial x_i}\left(\frac{\delta p'}{\rho_0}\right)}+\rho_0\frac{\overline{(\theta')^2}}{\theta_0}g_i+\overline{\theta'\frac{\partial\mathfrak{I}_{i,j}'}{\partial x_j}}+\frac{\theta_0}{T_0}\frac{\overline{u_i'Q'}}{c_p}-\overline{u_i'\frac{\partial H_j'}{\partial x_j}}$$

Predict a third moment

$$\frac{\partial}{\partial t}\overline{\theta'\theta'\theta'}+\bar{u}_j\frac{\partial}{\partial x_j}\overline{\theta'\theta'\theta'}+3\overline{\theta'\theta'u_j'}\frac{\partial\bar{\theta}}{\partial x_j}+\frac{1}{\rho_0}\frac{\partial}{\partial x_j}\left(\rho_0\overline{u_j'\theta'\theta'\theta'}\right)=\frac{3}{\rho_0}\overline{\theta'\theta'\frac{\partial H_{3,j}'}{\partial x_j}}-\frac{3\overline{\theta'\theta'}}{\rho_0}\frac{\partial}{\partial x_j}\left(\rho_0\overline{\theta'u_j'}\right)$$

Closures Needed

1) Closures for the effects of higher moments that are not predicted, e.g., as mentioned above, the fourth moments in a third-order closure model.

2) Closures for moments involving the pressure, which occur in the equations for moments that involve velocity components.

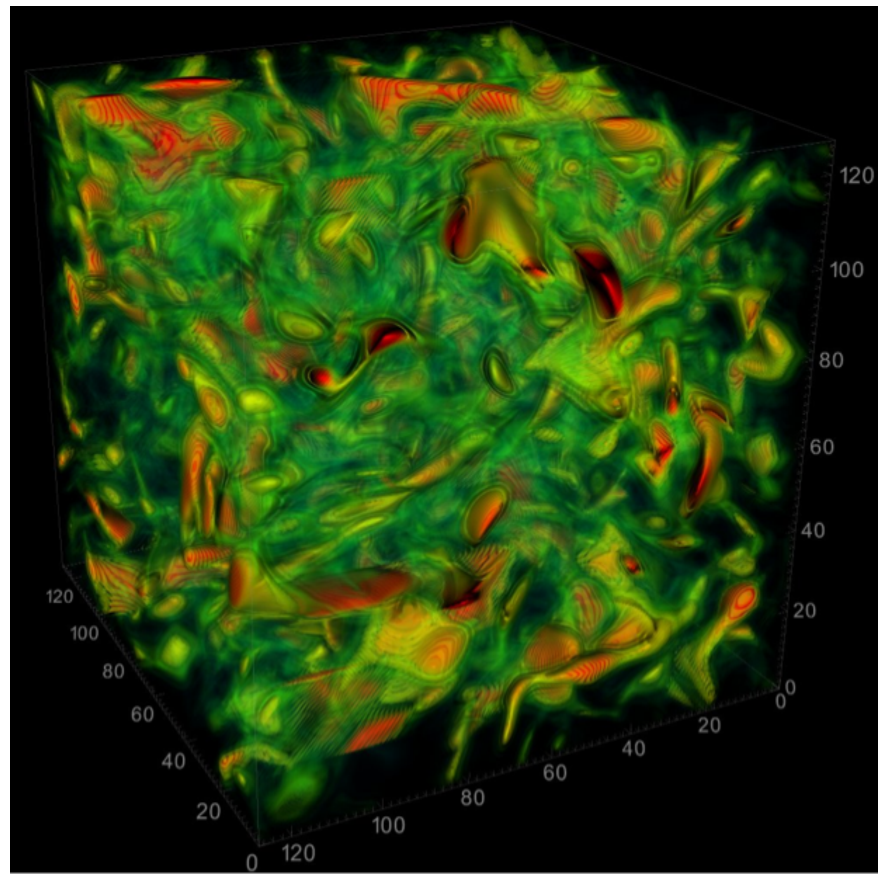
3) Closures for dissipation rates, which are especially important in the equations governing variances.

4) Closures to determine SGS phase changes (e.g., Sommeria and Deardorff, 1977; Mellor, 1977) and other microphysical processes (e.g., Larson et al., 2005), as well as radiative heating and cooling.

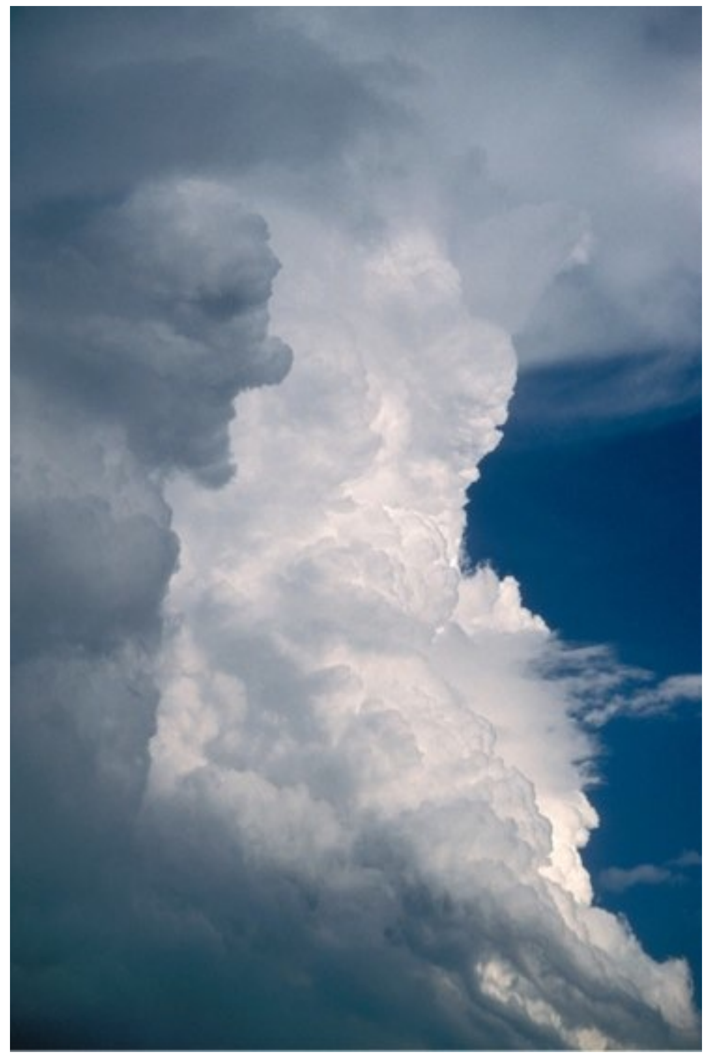
What is HOC good for?

- Flux and cloud fraction parameterizations in models
- Interpretation of observations and high-resolution simulations

Is HOC a "theory of everything?"



Turbulence



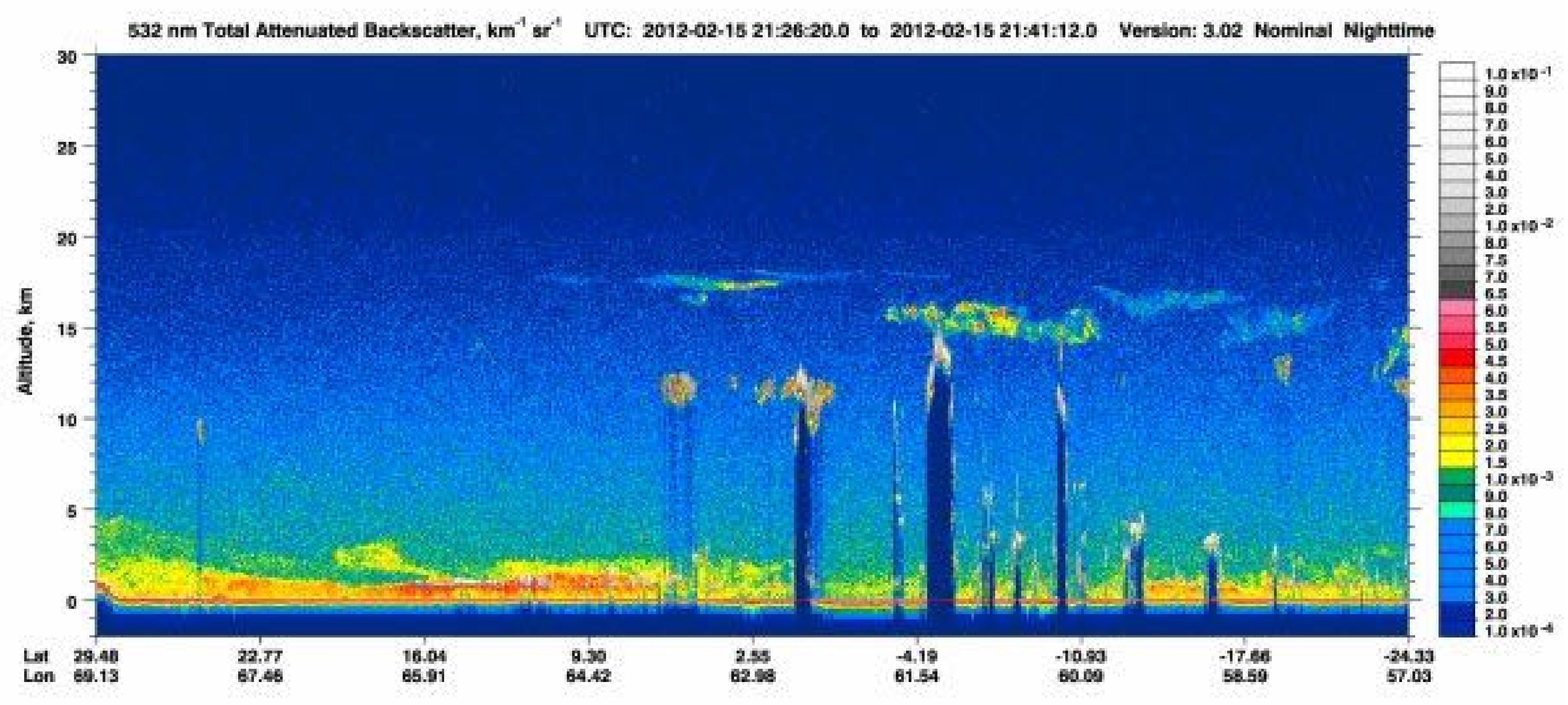
Deep convection

Gravity waves

To represent all of these things, a very general closure would be needed.

Five approaches to boundary layer parameterization

- Similarity theories
- Eddy diffusion
- Mixed-layer models
- Higher-order closure
- Mass fluxes



Mass fluxes

Reynolds averaging is exact (with grid-cell means) and completely general. We can write

$$\rho\overline{w'h'} = \rho\sum_{j=1}^{J}\left[\sigma_j\left(w_j - \overline{w}\right)\left(h_j - \overline{h}\right)\right] = \sum_{j=1}^{J}\left[M_j\left(h_j - \overline{h}\right)\right] \ ,$$

where

$$M_j \equiv \rho\sigma_j\left(w_j - \overline{w}\right)$$

is a mass flux. This demonstrates that mass fluxes arise purely from Reynolds averaging, and that they involve no approximation for $J \to \infty$. In practice, of course, J must be finite.

Sometimes we also make the approximation

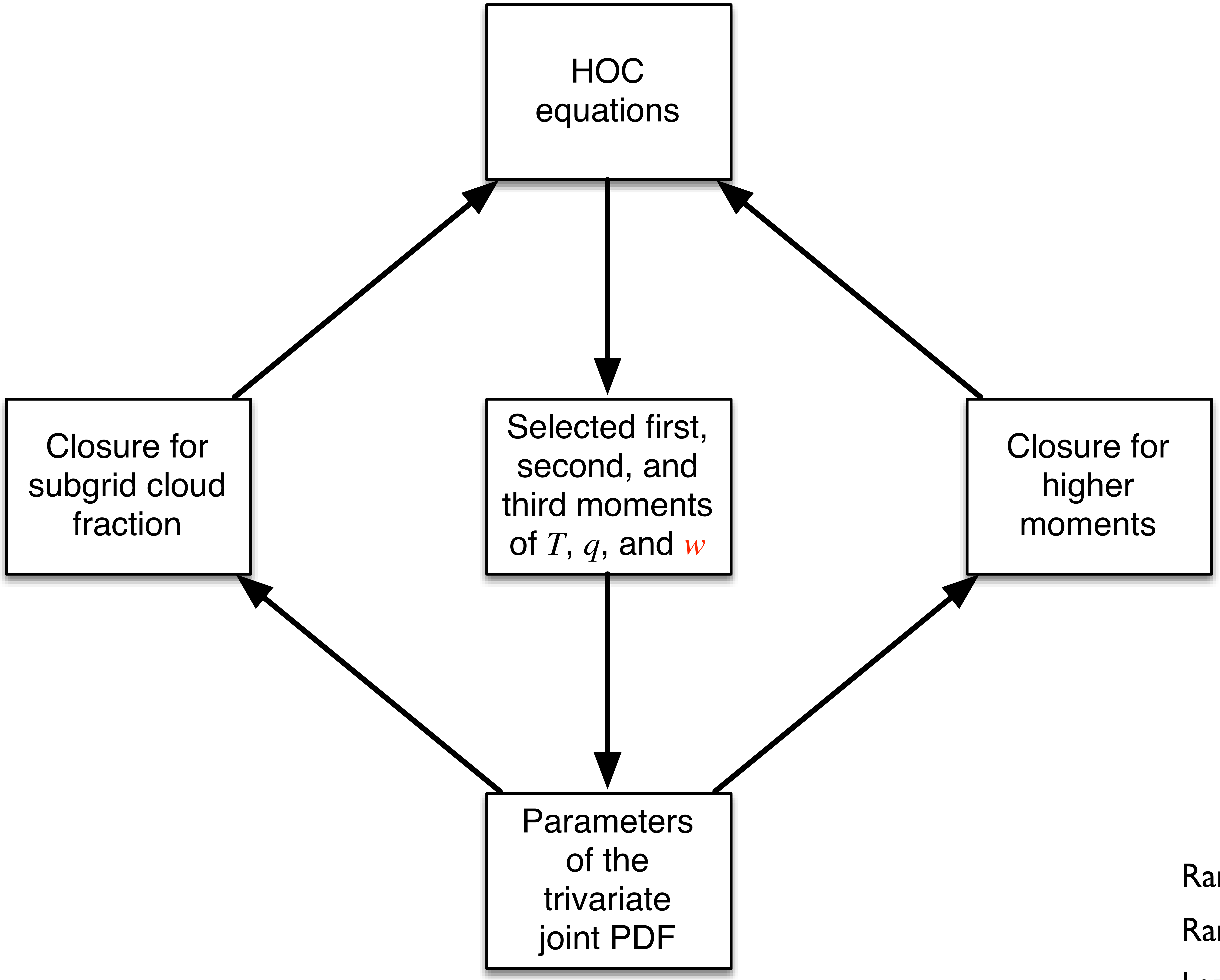
$$M_j \cong \rho\sigma_j w_j \ ,$$

which can be justified when $\overline{w}$ is sufficiently small, which it will be if the grid cell is sufficiently large. The approximate form is expected to fail at high resolution, when $\overline{w}$ can easily be comparable to or even larger than w_j.

We can calculate $\rho\overline{w'h'}$ if the M_j and h_j can be determined somehow. In principle we could define a mass flux for each square millimeter of a grid cell. This is not a good idea because so much detail is (presumably) unnecessary, and because it would be very impractical to deal with such a large number of mass fluxes. We need a way to reduce the number of mass fluxes to a manageable value, without losing too much accuracy.

Suppose that we have a very detailed numerical simulation of the convective turbulence inside a grid cell, and we want to use the *statistics* of the simulation to compute values of σ_j, w_j, and h_j that can be used ito obtain an accurate value of $\rho\overline{w'h'}$. Historically, plume models have been used to calculate the h_j, and “mass-flux closures” have been used to determine the M_j. As a result, there is a tendency to associate the mass-flux method with plume models, but we have shown above that *mass fluxes can be defined without any reference to plumes*.

Let's include w in the joint distribution.



Randall JAS 1987

Randall, Shao, and Moeng, JAS 1992

Lappen and Randall, JAS 2001

Closures Found

1) Closures for the effects of higher moments that are not predicted, e.g., as mentioned above, the fourth moments in a third-order closure model. ✔

2) Closures for moments involving the pressure, which occur in the equations for moments that involve velocity components.

3) Closures for dissipation rates, which are especially important in the equations governing variances.

4) Closures to determine SGS phase changes (e.g., Sommeria and Deardorff, 1977; Mellor, 1977) and other microphysical processes (e.g., Larson et al., 2005), as well as radiative heating and cooling. ✔

Simple case of two delta functions

Mean (first moment):

$$\overline{h} = \sigma h_u + (1-\sigma)\, h_d$$

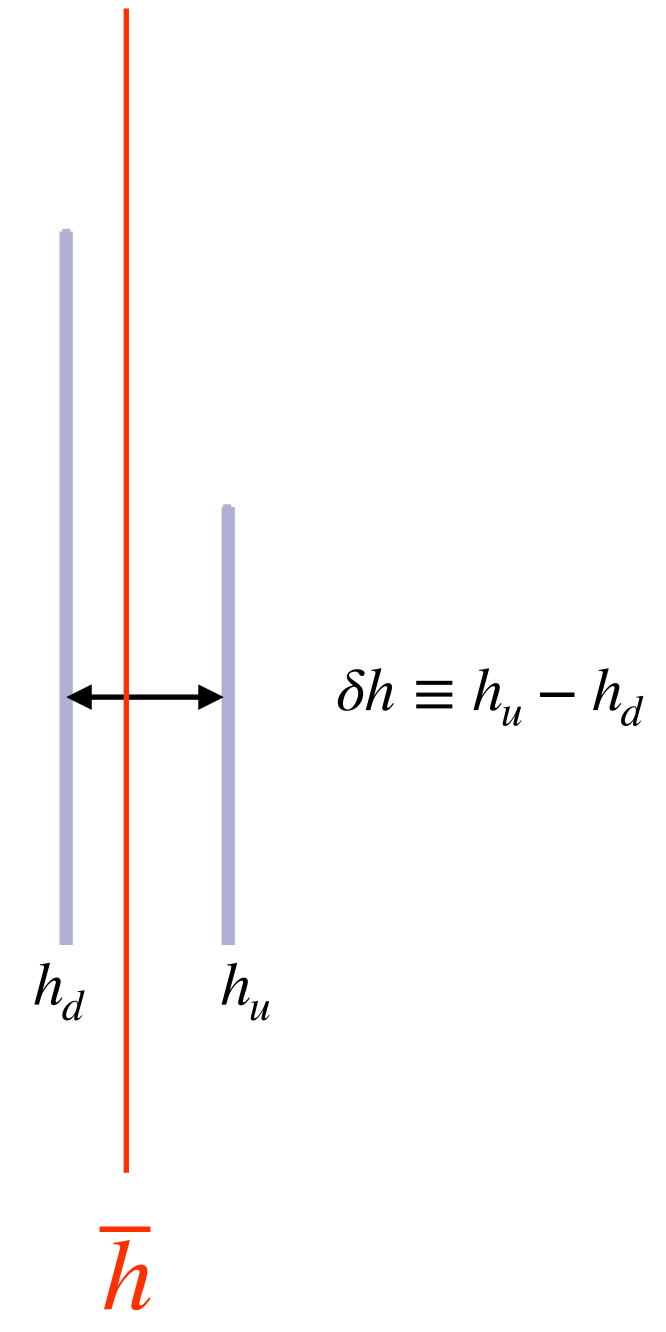
Variance (second moment about the mean):

$$\overline{h'^2} = \sigma(1-\sigma)\left(\delta h\right)^2 \text{ where } \delta h \equiv h_u - h_d$$

Third moment about the mean:

$$\overline{h'^3} = \sigma(1-\sigma)(1-2\sigma)\left(\delta h\right)^3$$

$$\text{Skewness } S_h \equiv \frac{\overline{h'^3}}{\left(\overline{h'^2}\right)^{3/2}} = \frac{1-2\sigma}{\sqrt{\sigma(1-\sigma)}}$$



Two delta functions

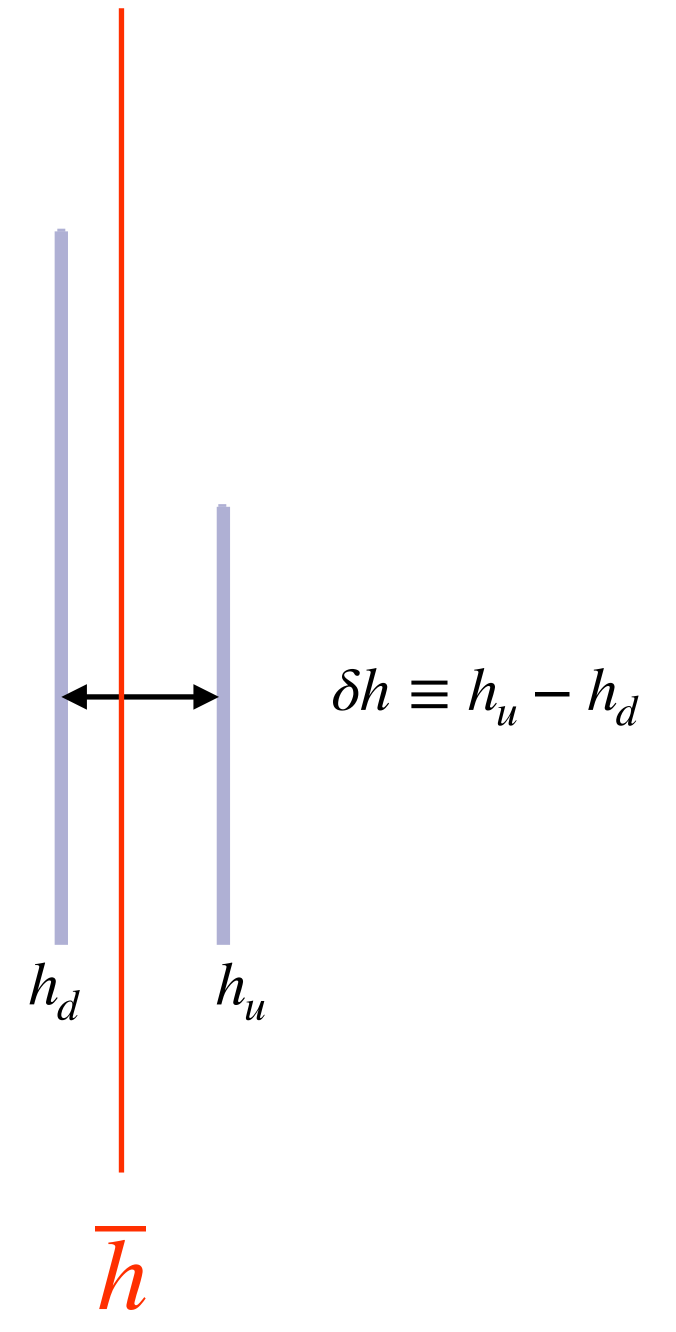
$$\delta h = \pm \sqrt{\frac{\overline{h'^2}}{\sigma(1-\sigma)}}$$

$$\overline{h'^3} = \sigma(1-\sigma)(1-2\sigma)\left[\frac{\overline{h'^2}}{\sigma(1-\sigma)}\right]^3$$

$$\text{Skewness } S_h \equiv \frac{\overline{h'^3}}{\left(\overline{h'^2}\right)^{3/2}} = \frac{1-2\sigma}{\sqrt{\sigma(1-\sigma)}}$$

$$\sigma = \frac{1}{2}\left(1 \pm \frac{S_h}{\sqrt{4+S_h^2}}\right)$$

$$\delta h = \mp \sqrt{\overline{h'^2}\left(4+S_h^2\right)}$$



If we know $\overline{h'^2}$ and $\overline{h'^3}$, we can calculate σ and δh.

From σ, δh, and $\overline{h}$, we can calculate h_u and h_d.

In this way, the predicted moments determine the distribution.

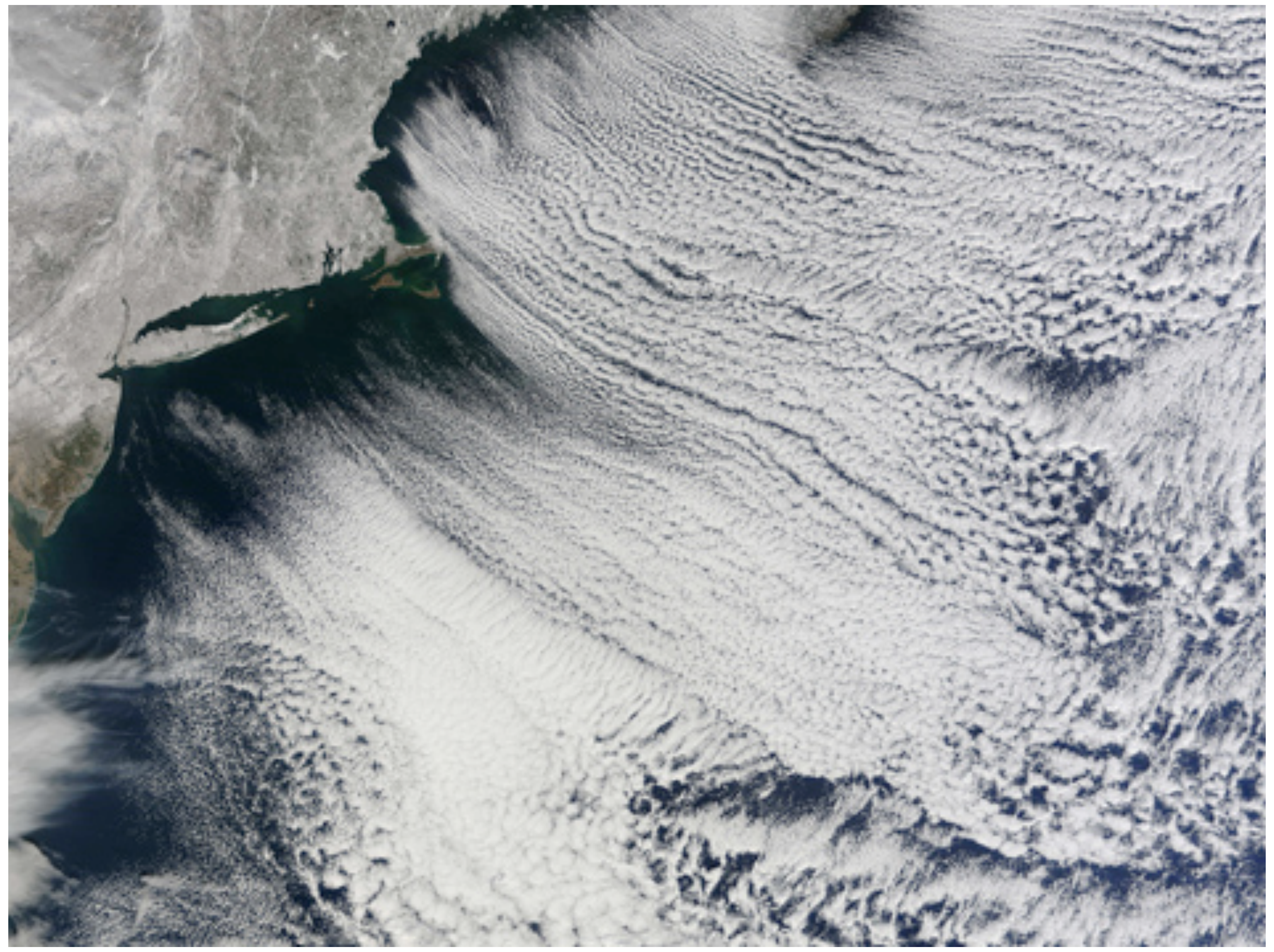
Phenomenology

One delta function represents the clouds, and the other represents the environment.

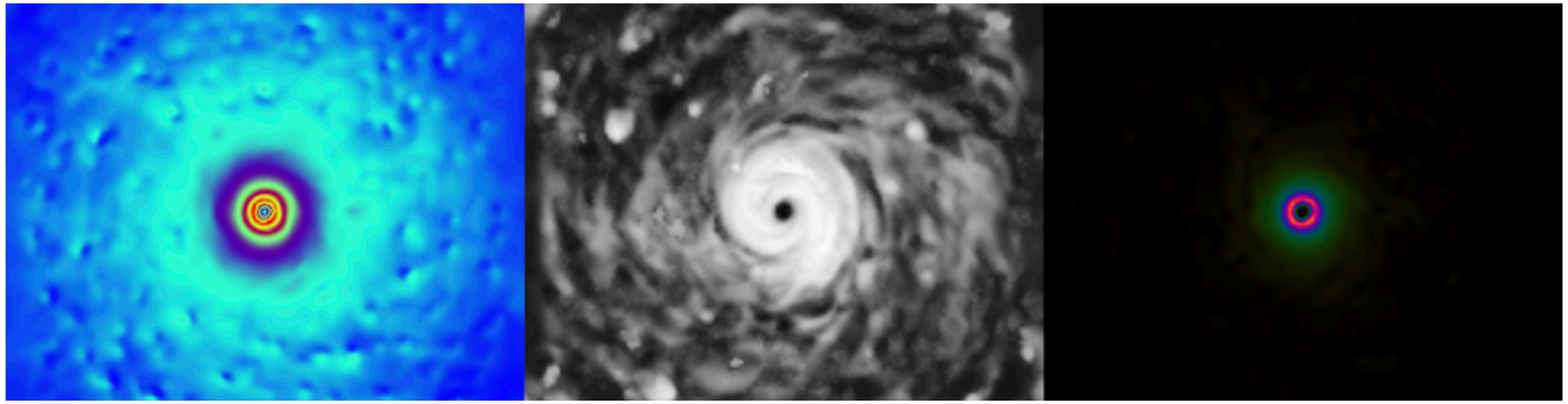
Momentum

$T, q,$ and w are not enough.

What about the momentum fluxes?



Improving Modeled Momentum Flux in the Atmospheric Boundary Layer



ABOUT THE PROJECT

The flux of momentum in the lowest levels of the atmosphere plays a critical role for both weather and climate, impacting extreme storms such as tropical cyclones and severe convection, as well as serving as a key physical process associated with radiatively-important shallow cumulus layers in trade-wind regimes. All of these phenomena have historically been challenging to simulate, with biases arising from required approximation of boundary layer processes.

With climate models being more frequently utilized at smaller grid spacings capable of resolving societally-relevant features with near-surface wind maxima and sharp momentum gradients, improving model treatment of these processes is critical for enhancing the understanding and predictability of the climate system on seasonal, annual, decadal, and centennial time scales.

Our Climate Process Team (CPT), jointly supported by NSF and NOAA, was created in order to better understand the importance of momentum flux in global models and subsequently improve its representation in order to advance the next generation of Earth system models. We do this by spanning a comprehensive hierarchy, ranging from observations to process-based modeling to complex global simulations.

- BACKGROUND
- PEOPLE
- PUBLICATIONS
- SOFTWARE

Project led by Colin Zarzycki at Penn State.