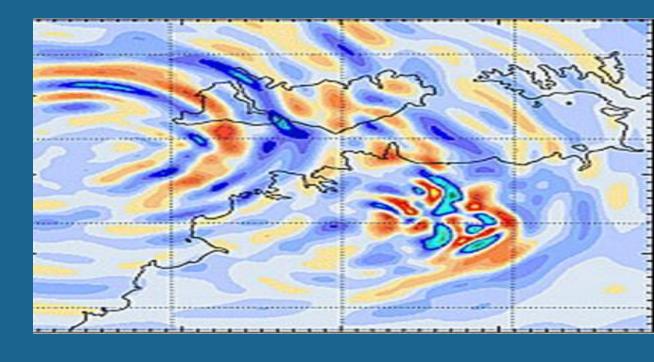
# **Gravity Wave Parameterizations in Earth system models**

Jadwiga (Yaga) Richter



October 22, 2024



#### **Outline**

I. Introduction and basic equations

II. Representations in models

III.Gravity wave tuning in models

IV.Effects on key science questions



# Introduction and Basic Equations

**Gravity Waves (GWs):** waves in Earth's atmosphere for which buoyancy is the restoring force.

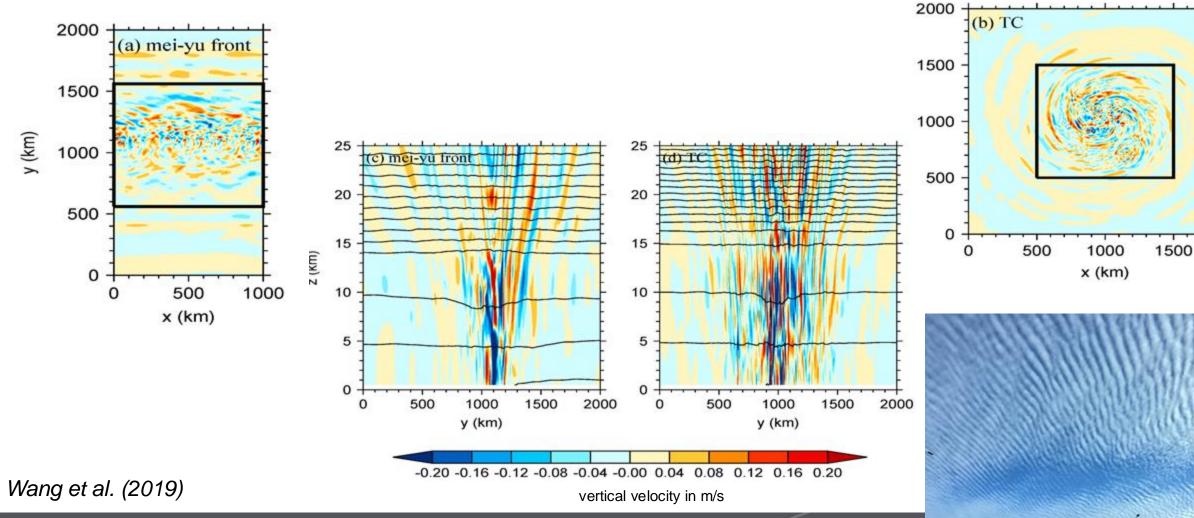




In water: on boundary of a denser fluid (water) and air above



# Atmosphere: continuously stratified -> propagation in vertical and horizontal



2000



**GW Sources:** any process that produces perturbations of air parcels

Primary Sources: Orography, Convection (including TCs), Fronts

Other: polar vortex edge, secondary wave generation (from wave breaking)

#### **Basic characteristics:**

Horizontal wavelengths: 10's to 100's km

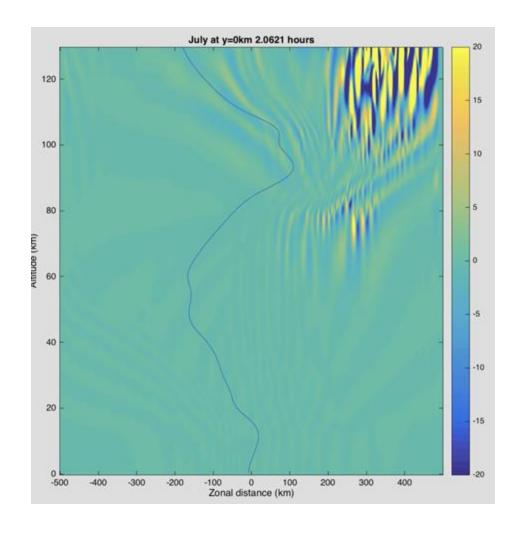
Vertical wavelengths: 3 to 30 km

Periods: 10 min to hours



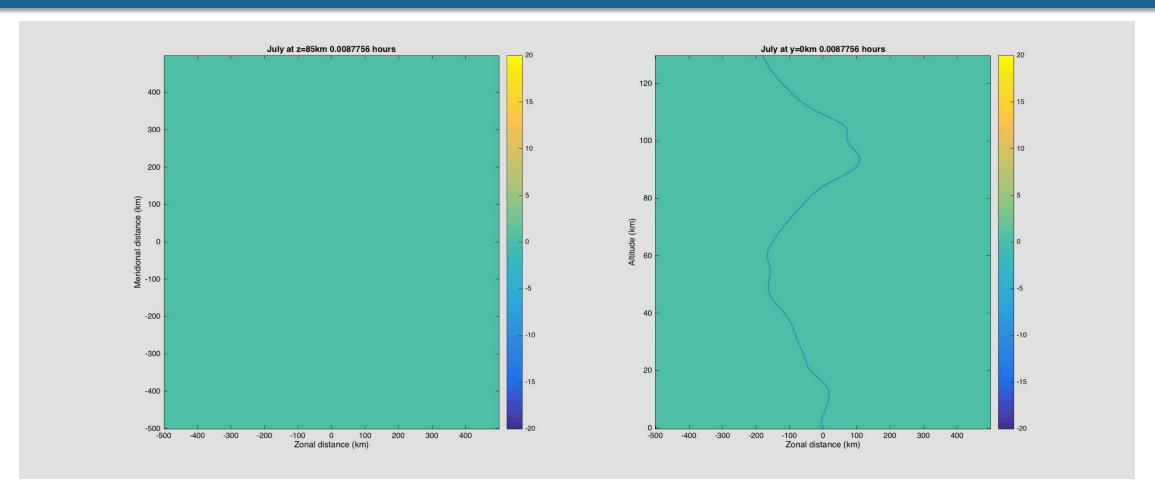
#### Why do GW's matter?

- GW propagate upwards
- Amplitude grows exponentially with height (wave energy flux is conserved - air density decreases with altitude)
- They deposit momentum when they encounter critical levels or break



Shading: Vertical velocity (m/s)





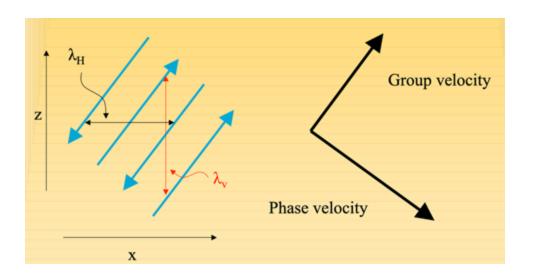
3D simulation of convectively generated gravity waves using the Complex Geometry Compressible Atmospheric Model (CGCAM) [Felton and Lund (2006)]. Latent heating is used as a proxy for convection.

https://www2.cgd.ucar.edu/staff/jrichter/animations.html



GW characteristics are governed by a 'dispersion relationship': relates frequency (period), horizontal and vertical wavenumbers (wavelengths)

Linearized Boussinesq equations: ->



$$\omega^{*2} = (\omega - Uk)^2 = \frac{N^2k^2}{(k^2+m^2)}$$

 $\omega^*$  Intrinsic frequency

 $\omega$  Frequency relative to the ground

$$k = 2\pi/\lambda_x$$
 horizontal wavenumber

$$m=2\pi/\lambda_z$$
 vertical wavenumber

$$N^2 = \frac{g}{\theta} \frac{\partial \theta}{\partial z}$$
 buoyancy frequency

Key wave properties: horizontal phase speed, group velocity

$$\omega^{*2} = (\omega - Uk)^2 = \frac{N^2 k^2}{(k^2 + m^2)}.$$
  $m^2 = \frac{k^2 (N^2 - \omega^{*2})}{\omega^{*2}}$ 

Horizontal wave phase speed:

$$c_{px} = \frac{\omega}{k}$$

Horizontal intrinsic wave phase speed:

$$c_{px}^* = \frac{\omega^*}{k} = c_{px} - U = \pm \frac{N}{(k^2 + m^2)^{\frac{1}{2}}}$$

Horizontal Group Velocity:

$$c_{gx} = c_{gx}^* = \frac{\partial \omega^*}{\partial k} = c_{px}^* \left( 1 - \frac{k^2}{k^2 + m^2} \right)$$

Vertical Group Velocity: (speed GW energy propagates in the vertical)

$$c_{gz} = \frac{\partial \omega}{\partial m} = \frac{\omega^*}{m} \left( \frac{m^2}{m^2 + k^2} \right)$$

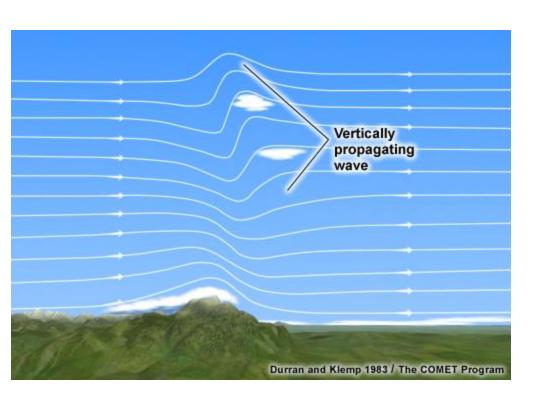
Vertical Group Velocity (hydrostatic limit):

$$c_{gz} \approx \pm \frac{(c_{px}-U)^2 k}{N}$$

Large k or large  $c_{px}$  - > fast  $c_{gz}$ 



#### **Mountain Waves:**



Linear theory can predict the general features of MWs when the mountain height is small in comparison to the vertical wavelength of the wave.

$$m^2 = \frac{k^2(N^2 - \omega^{*2})}{\omega^{*2}}$$
  $\omega = 0$ 

$$m^2 = k^2 \left( \frac{N^2 - (Uk)^2}{(Uk)^2} \right).$$

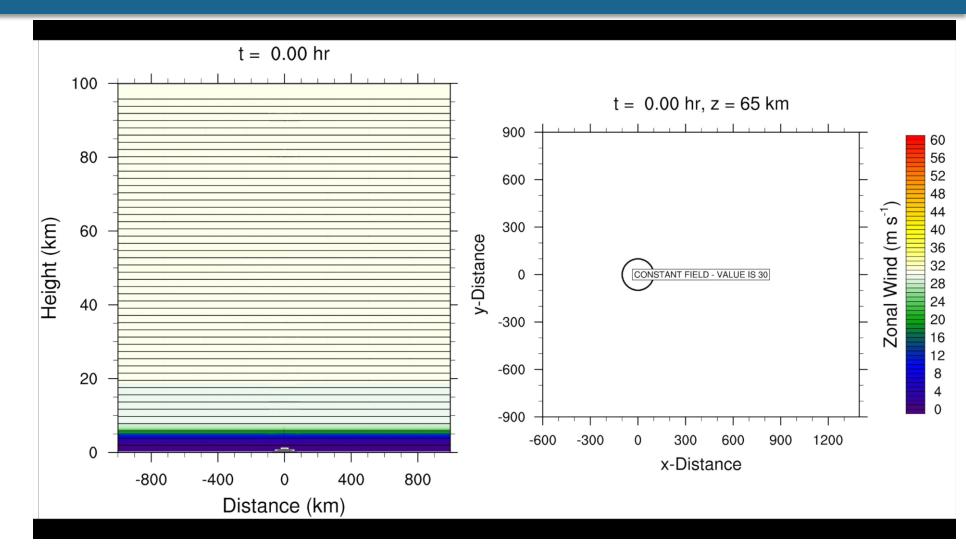
Small scale ridges, with intrinsic frequency higher than buoyancy frequency:

 $Uk > N \rightarrow m$  imaginary -> exponential decay with height

Flow over wider ridges -> propagation with height

Mountains can produce low level blocking and downslope windstorms

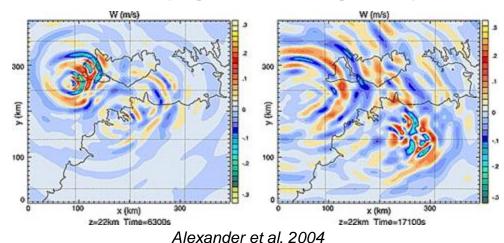
- 200-km-wide, 1000-m-high isotropic compact-cosine mountain
- WRF model
- 30 m/s wind (-> 0 at 24 hrs)

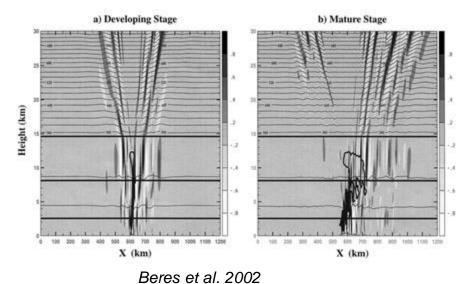


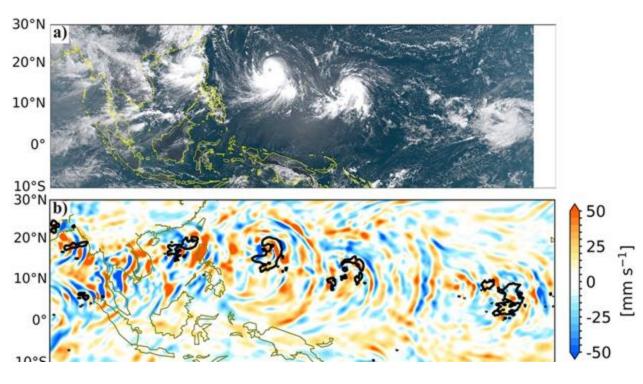
https://www2.cgd.ucar.edu/staff/jrichter/animations.html



#### **Convectively generated gravity waves:**







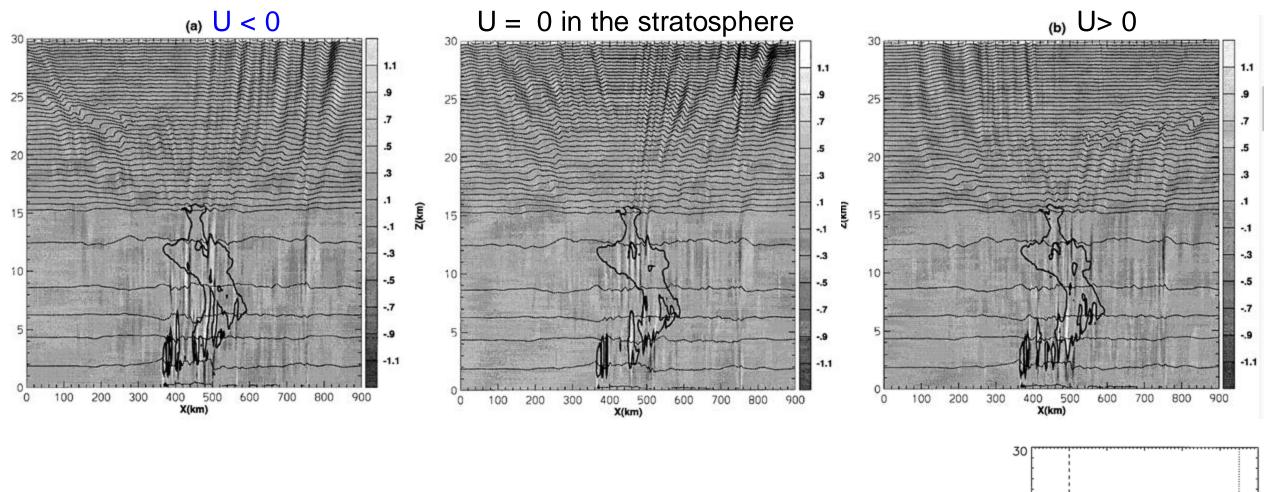
Pahlavan et al. 2023 (ERA5, 30 km resolution; 50 km vertical velocity)

Horizontal wavelengths: 10's to 100's km

Vertical wavelengths: few to 40 km

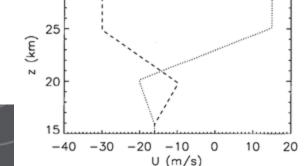
Horizontal phase speeds: up to 100 m/s



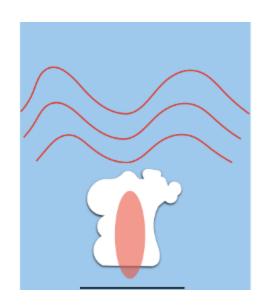


Alexander and Holton (1997)





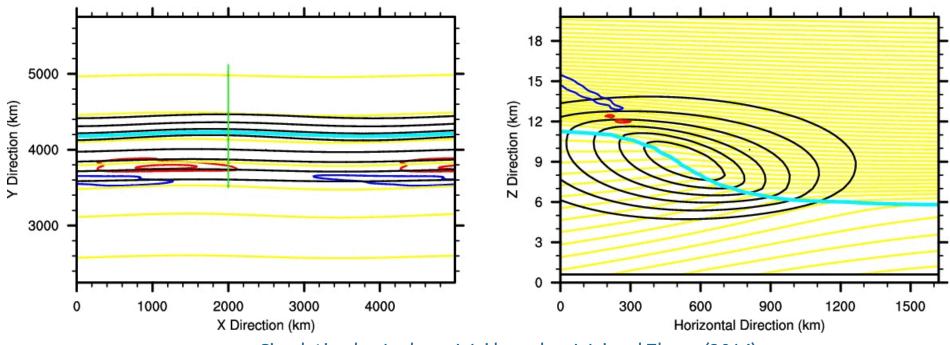
#### **Convectively generated GWs - mechanisms:**



- 1) Thermal or diabatic forcing: temporal and spatial variations of convective heating produce perturbations that force a spectrum of GWs (Bretherton et al. 1998, Chun and Baik 1998, Pandya and Alexander (1999)
- 2) Mechanical oscillator: oscillating updrafts and downdrafts perturb the stably stratified atmosphere at and above the top of convective motion (Clark et al. 1986, Fovell et al. 1992)
- 3) Moving mountain: top of a convective elements acts as a barrier to the background mean flow, producing upstream propagating waves in a manner similar to flow over a mountain (Clark et al. 1986, Pfister et al. 1993)

#### Frontally generated gravity waves: dominant GW source in mid-latitudes

Gravity wave study based on the idealized baroclinic wave simulations: Weak moist run at 24hr

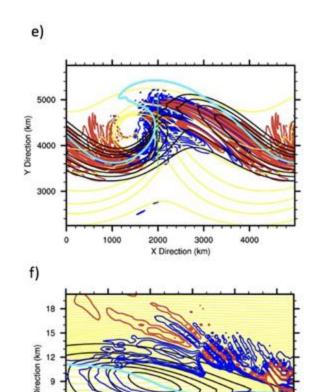


Simulation by Junhong Wei based on Wei and Zhang (2014).

Yellow: temperature or potential temperature Turquoise: dynamic tropopause (PV = 1.5 PVU) Black: horizontal wind Red/blue: horizontal divergence

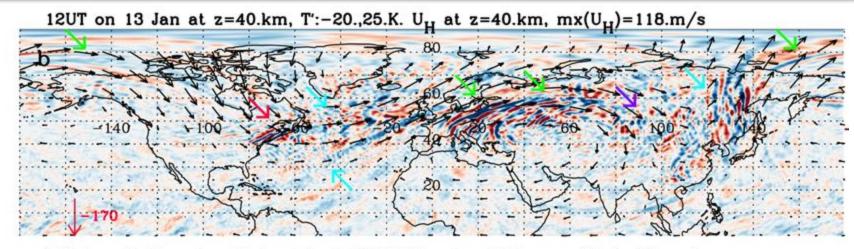


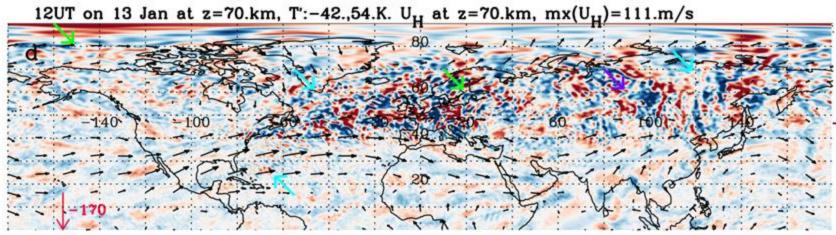
#### Frontally generated gravity waves: dominant GW source in mid-latitudes



- 1) Spontaneous imbalance adjustment (generalization of geostrophic adjustment): GW are generated as imbalance flow comes back to balance
  - Emission of large amplitude inertia gravity waves in regions of strong horizontal curvature
- 2) Adjustment emission: well-balance flow continuously radiates GWs during the course of near-balance evolution
- 3) Shear instability: nonlinear interaction between Kelvin-Helmholtz instability and propagating modes; May occur in very intense shear layers near the surface or at upper levels, above tropopause jets

# Waves generated by polar vortex:





Colors: T'; Vectors (mean U, V)

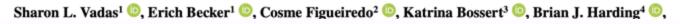
From HIAMCM (Becker & Vadas 2020): high-resolution, whole atmosphere, spectral model, effective resolution ~ 52 km; top at ~ 450 km

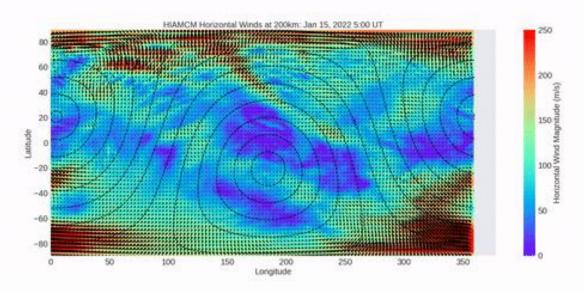


## Secondary wave generation:

- First proposed using theoretical arguments by Vadas et al. (2002): Mechanism for the Generation of Secondary Waves in Wave Breaking Regions
- Deep 3D body forces (GW breaking), which generate secondary waves very efficiently, create high-frequency waves with large vertical wavelengths that possess large momentum fluxes.

Primary and Secondary Gravity Waves and Large-Scale Wind Changes Generated by the Tonga Volcanic Eruption on 15 January 2022: Modeling and Comparison With ICON-MIGHTI Winds





**Secondary waves generated:** a continuum of medium to largescale secondary GWs with  $\tau r \sim 20$  min to 7 hr,  $\lambda H \sim 400-7,500$  km,  $cH \sim 100-600$  m/s, and u',  $v' \sim 100-200$  m/s.

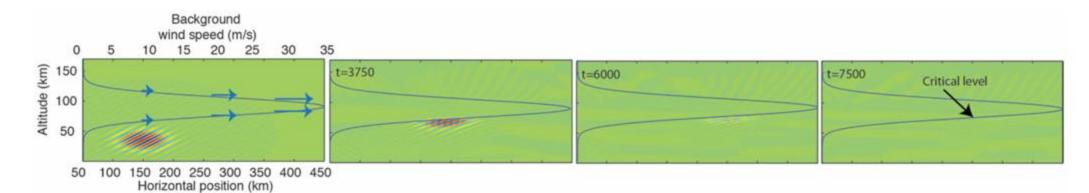
Vadas et al. 2023



#### **GW** propagation and dissipation:

$$m^2 = \frac{k^2 (N^2 - \omega^{*2})}{\omega^{*2}}$$

Critical level -> momentum deposition to the mean flow



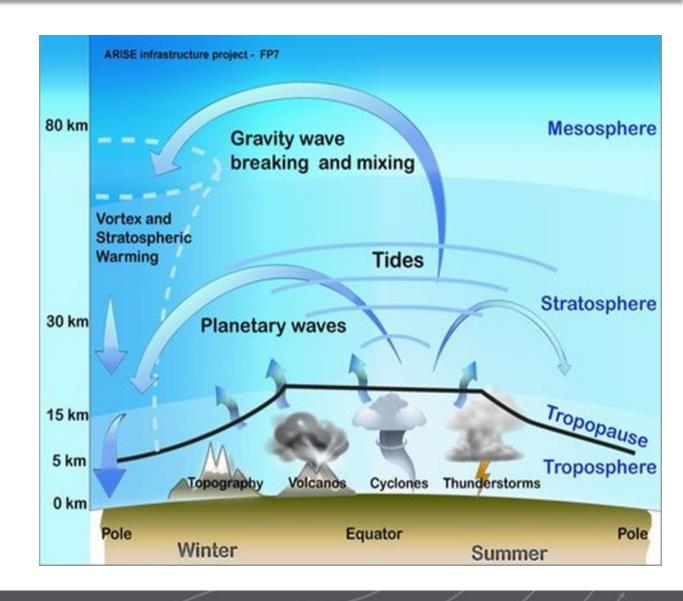
GW momentum flux: constant with height (till wave breaks/dissipates)

$$\bar{\rho}\overline{u'w'}$$

density decreases -> amplitude increases

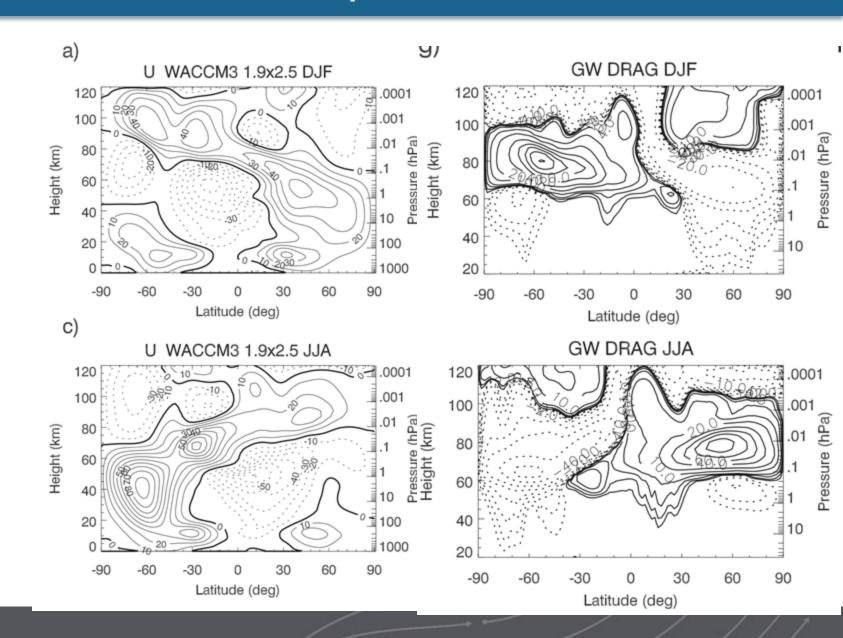
$$\boldsymbol{GWD} = \frac{1}{\overline{\rho}} \frac{\partial}{\partial z} (\bar{\rho} \overline{u'w'}, \bar{\rho} \overline{v'w'})$$

is the background atmospheric density, u', v', and w' are the horizontal and vertical velocity perturbations

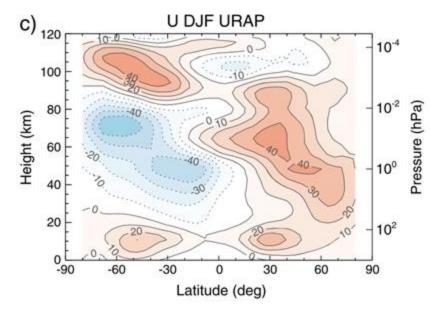


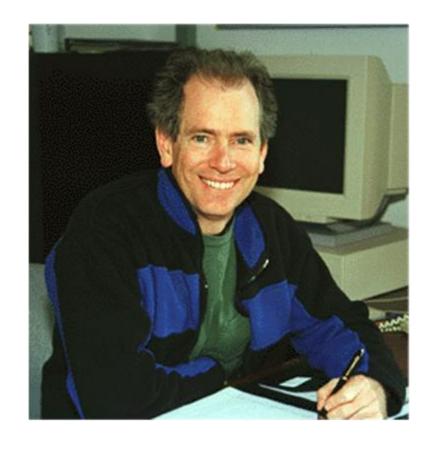
$$\boldsymbol{GWD} = \frac{1}{\overline{\rho}} \frac{\partial}{\partial z} (\overline{\rho} \overline{u'w'}, \overline{\rho} \overline{v'w'})$$

is the background atmospheric density, u', v', and w' are the horizontal and vertical velocity perturbations



- Need for GWs: recognition that there was a 'missing drag' in middle atmosphere GCMs
- Without drag, stratospheric winter jet would be much stronger, and mesopause would not be warm
- Early GCMs, used Rayleigh friction (e.g.: Boville 1986)
- First implementation of GWs: orographic parameterizations (Boer et al. 1984, Palmer et al 1986, McFarlane 1987)
- Non-orographic: (Rind et al. 1988, Fritts and Lu 1993, Medvedev and Klaasen 1995, Hines 1997a,b, Alexander and Dunkerton, 1999, Warner and McIntyre 2001)





Byron Boville

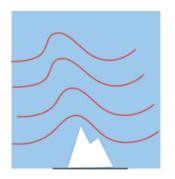
"Garbage In. Garbage Out.

It's as simple as that"

#### **GW** parameterization components:

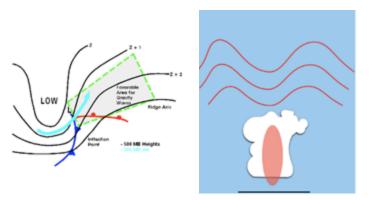
- 1) Specification of waves at **source level**: wavenumbers, phase speeds, propagation direction, source height
- 2) Wave propagation with height: typically in column and instant! (Except for Amemiya and Sato (2010), and Eckermann et al. 2015)
- 3) **Wave dissipation** -> momentum deposition to the mean flow; Plane wave assumption: Flux and force along same direction as at source; Force applied to the vector momentum equations

#### **Orographic Parameterizations:**



c=0

#### **Non-orographic Parameterizations:**



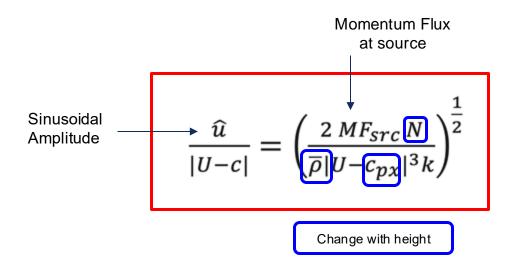
-100 < c< 100 m/s

#### Wave dissipation:

Foundation: Lindzen's (1981) saturation theory; mods by Holton (1982)

#### **Assumption:**

parameterized waves are individual, steady, monochromatic plane waves



When > 1: linary theory -> static instability

Lindzen scheme: Keep amplitude at or below 1

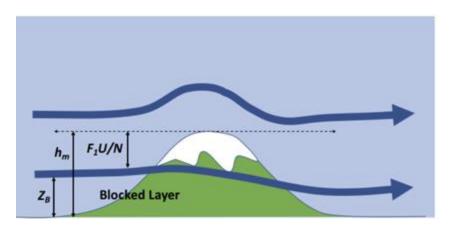
if  $\hat{u} > 1 -> MF$  is reduced till  $\hat{u} = 1$ 

d MF/dz -> force to the mean flow

#### Other parameterizations:

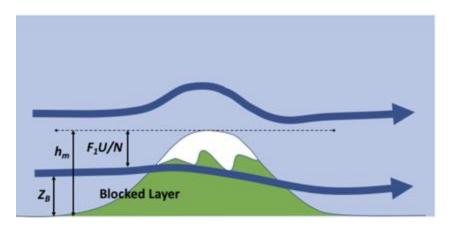
- Alexander and Dunkerton (1999): discrete spectrum of monochromatic waves
   deposition of all momentum flux at breaking level
   works with any source spectrum
- Hines (1997 a,b): proposed a "Doppler Spread" mechanism -> nonlinear interactions among waves in the spectrum reshape the spectrum with altitude.
- Warner and McIntyre (2001): ~ Hines-like spectrum reshaping with altitude (based on shape)
   + Lindzen's wave reshaping
- Both Hines and Warner and McIntyre assume a particular vertical wavenumber spectrum shape

#### **Source parameterizations: Orography**



- **First formulations** (and what's still used most of the time): single, monochromatic vertically propagating wave with c=0 (Boer et al. 1984, Palmer et al. 1986, McFarlane 1987)
- Based on 2D theory assuming hydrostatic, steady, horizontally uniform flow over an obstacle
- Amplitude at source level: based on subgrid-scale orographic variance
- Surface stress vector: parallels to and opposite of the mean flow at the lowest level of the model, assuming isotropic topography (single length scale)

#### **Source parameterizations: Orography**



Froude #:  $F_r = \frac{U}{Nh_m}$ 

- Lott and Miller (1997): incorporated impact of nearsurface nonlinearities (blocking, flow splitting)
- when h<sub>m</sub> exceeds a critical value -> portion of the flow is diverted or blocked
- Scinocca and McFarlane (2000): employs two vertically propagating waves - to provide azimuthal distribution (using elliptical barrier model)
- Also includes representation of low-level drag
- Fr > 1: linear; upward propagating waves
- Fr < 1: non-linear flow; blocked flow or diverting around obstacle
- -> Momentum flux of upward propagating waves is reduced

#### **Turbulent orographic Form Drag:**

Typically representing scales < 5 km Drag exerted by hills/mountains through generation of turbulence

# Wood and Mason 1993: represented with an effective roughness length approach -> enhances roughness proportionally to orographic height

#### Implementation in IFS:

$$\frac{\partial U}{\partial t} = \frac{\partial}{\partial z} \frac{\tau_x}{\rho} = -C_{\text{tofd}} |\mathbf{U}(z)| U(z),$$

$$C_{\text{tofd}} = -\alpha \beta C_{\text{md}} C_{\text{coor}} 2.109 e^{-(z/1500)^{1.5}} a_2 z^{-1.2}$$

Beljaars et al. 2004: explicitly distributed form drag

Applies drag explicitly on model levels

with  $\tau_x$  being the stress,  $\rho$  the density, z the height above the surface,  $\alpha = 35$ ,  $\beta = 1$ ,  $C_{\text{md}} = 0.005$ ,  $C_{\text{coor}} = 0.6$ ,  $a_2 = a_1 k^{n_1 - n_2}$ ,  $a_1 = \sigma_{\text{flt}}^2 (I_H k_{\text{flt}}^{n_1})^{-1}$ ,  $k_1 = 0.003 \, \text{m}^{-1}$ ,  $n_1 = -1.9$ ,  $n_2 = -2.8$ ,  $k_{\text{flt}} = 0.00035 \, \text{m}^{-1}$ ,  $I_h = 0.00102 \, \text{m}^{-1}$  and  $\sigma_{\text{flt}}$  is the standard deviation of filtered subgrid orography (to remove scales larger than 5 km). A corresponding equation is used for the meridional wind V. For numerical stability, these

# Towards a more "scale-aware" orographic gravity wave drag parametrization: Description and initial testing

A. van Niekerk® | S.B. Vosper

Received: 29 April 2021 | Revised: 10 July 2021 | Accepted: 13 July 2021 | Published on: 17 August 2021

DOI: 10.1002/qj.4126

- Uses linear theory for hydrostatic GWs
- Fourier description of subgrid orography -> produces a MF vector that accounts for anisotropy of the topography; eliminates monochromatic assumption (computed from 1-km source orography dataset offline)
- Also accounts for flow-blocking
- Better behaved for model grid spacing from 32 to 2 km
- Parameterized GW fluxes increase with coarser resolution
- ( as resolved GW MF decrease)
- Total momentum flux is the same

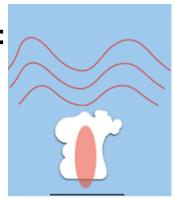


#### Non-orographic gravity waves:

- Typically all lumped together
- Source specified somewhere in the troposphere
- Emitted with the same properties at all times
- Sometimes a latitudinal dependence is specified

#### Source parameterizations: Convective gravity waves

- First non-orographic source spectrum parameterization: Rind et al. (1988):
   convection and wind shear: used in NASA GISS model
- <u>Convective GW</u> MF related to convective mass flux Phase speed: U avg over convective region +/- 10 m/s; for deeper convection additional waves +/- 20 m/s, 40 m/s



- Kershaw et al. (1995); Chun and Baik (1988) parameterization of the obstacle effect;
- Beres et al. 2004: based on linear theory and models: used in CESM, E3SM & now NASA
- Song and Chun (2005): similar to above: more complex U/N structure
- Bushell et al. 2015: UK Model
- Lott and Guez (2013): LMDz model



**Beres et al. (2004):** based on linear theory, thermal forcing (steady and oscillating component); verified on mesoscale model simulations

$$\frac{\partial u'}{\partial t} + \overline{U}\frac{\partial u'}{\partial x} + \frac{1}{\rho_0}\frac{\partial p'}{\partial x} = 0,$$

$$\frac{\partial w'}{\partial t} + \overline{U}\frac{\partial w'}{\partial x} + \frac{1}{\rho_0}\frac{\partial p'}{\partial z} - g\frac{\theta'}{\theta_0} = 0,$$

$$\frac{\partial u'}{\partial x} + \frac{\partial w'}{\partial z} = 0,$$

$$\frac{g}{\theta_0}\left(\frac{\partial \theta'}{\partial t} + \overline{U}\frac{\partial \theta}{\partial x}\right) + w'N^2 = \frac{g}{\theta_0}J',$$

$$\frac{g}{\theta_0}J' = q_x(x)q_z(z)q_t(t),$$

$$q_x(x) = Q_0 \exp\left[-\frac{(x - x_0)^2}{\sigma_x^2}\right],$$

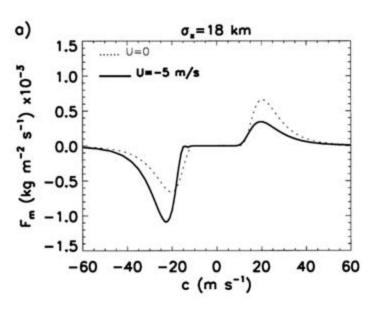
$$q_z(z) = \begin{cases} \sin(\pi z/h) & \text{for } 0 \le z \le h \\ 0 & \text{for } z > h. \end{cases}$$

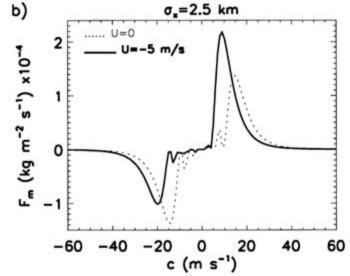
Heat source: horizontal scale  $2\sigma_x$  vertical scale: h

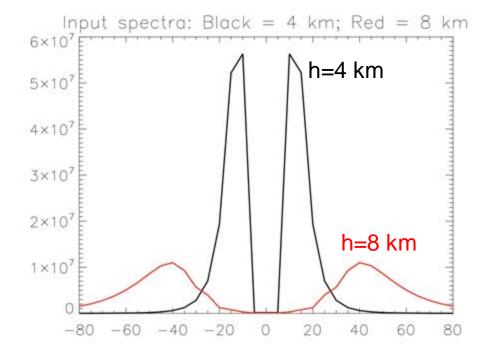
## **II.** Representation in Models

#### Beres et al. 2004:

- GW MF dependent primarily on vertical scale of heating and wind
- Also on horizontal scale and dominant frequency (need to assume)







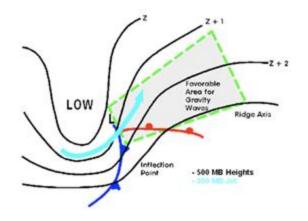
$$\rho_0 \overline{u'w'} = \frac{1}{\sqrt{2\pi}} \frac{\rho_0}{L\tau} \int_0^{+\infty} \int_{-\infty}^{+\infty} \operatorname{sgn}(\hat{\nu}) \left[ \left( \frac{N}{\hat{\nu}} \right)^2 - 1 \right]^{1/2} |B_{k\nu}|^2 dk d\nu.$$
(28)

The coefficient 
$$B_{k\nu}$$
 is

$$B_{k\nu} = \frac{k^2}{\hat{\nu}^2} G_k Q_0 Q_i(\nu) \left( \frac{\pi}{m_{k\nu} h} \right) \frac{\sin(m_{k\nu} h)}{(m_{k\nu}^2 - \pi^2/h^2)}. \quad (23)$$

## **II.** Representation in Models

### Frontally/Shear generated waves:



- Rind et al. (1988)
   Shear-generated GWs: launched at jet stream level; assigned a single wavenumber and phase speed dependent on the direction of the shear and wind velocity in shear layers
- Charron and Manzini (2002): using 'frontogenesis function' to diagnose location of fronts (Miller 1948, Hoskins 1982)
- if the frontogenesis function exceeds a critical threshold ->
   GWs launched at a fixed level of 600 hPa with high amplitude
- otherwise: small amplitude spectrum

# **Gravity Wave Tuning**

## **GW parameterizations in CESM:** upcoming CAM versions (80 km top) and WACCM (150 km top)

## 1. Orographic GWs:



#### **McFarlane** (1987):

1 wave with c=0
Amplitude dependent on orography height
& mean wind Tunable parameter: Efficiency
Beliaars et al. (2004)

### 2. Frontal GWs:

#### modified Charron and Manzini (2002):

40 waves with -100 < c < 100 m/s

Gaussian distribution in phase speed centered at 600 hPa Constant wave amplitude

Tunable parameters: Efficiency, amplitude, phase speed distribution, frontal threshold

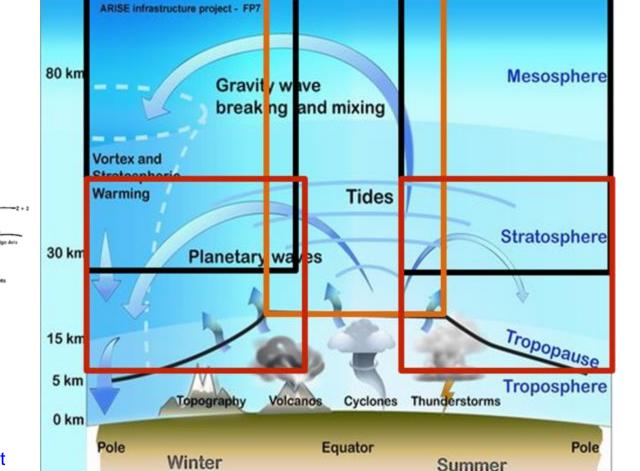
### 3. Convectively generated GWs:

#### Beres et al. (2004):

40 waves with -100 < c < 100 m/s

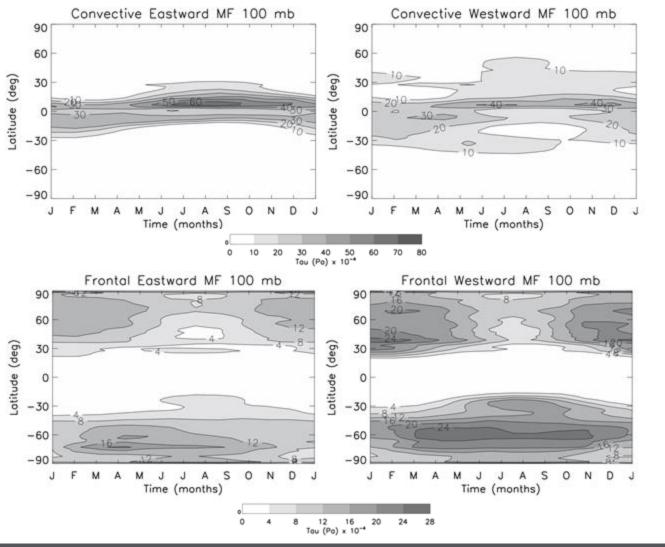
Dominant c related to h; Amplitude proportional to Q<sup>2</sup>

Tunable parameters: Efficiency, amplitude conversion (assumptions about scale/frequency)

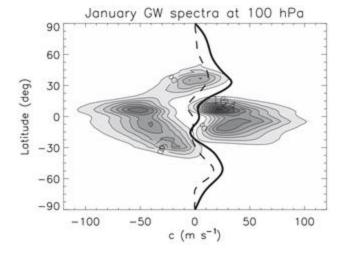




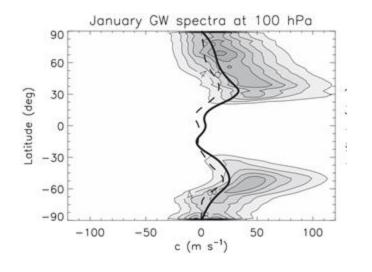
## Gravity waves in CESM(WACCM): Richter et al. (2010)



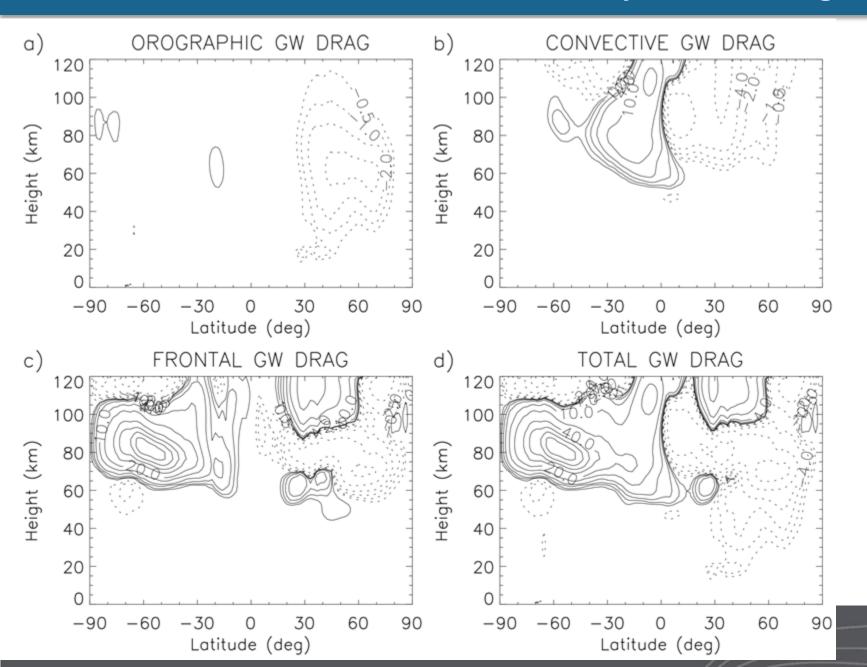
### Convective



**Frontal** 







Gravity wave drag in WACCM3.5

## Quasi-biennial Oscillation: U (10S to 10N):

Beres et al. (2004) Tunable parameters:

CF: Convective Fraction (tunable)

Eff<sub>gw</sub> (multiplies the GWD)

$$\mathsf{Eff}_{\mathsf{gw}} = 0.4$$

$$CF = 5\%$$

$$Q_0 = \frac{Q_{\text{GCM}}}{CF}$$

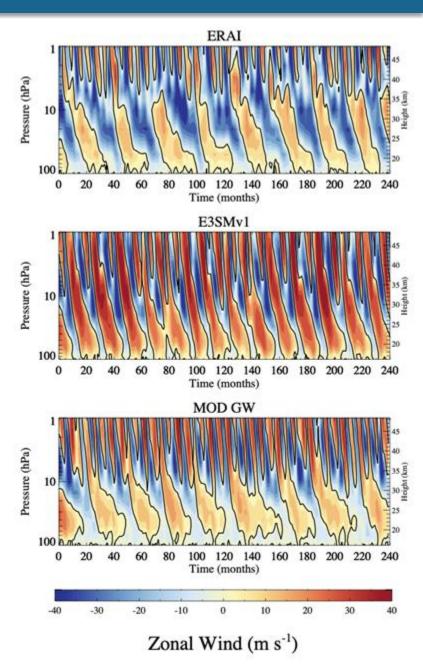
# **GW** parameterizations can't fix all deficiencies in the model

$$Eff_{gw} = 0.35$$

$$CF = 8\%$$

Richter et al. 2019

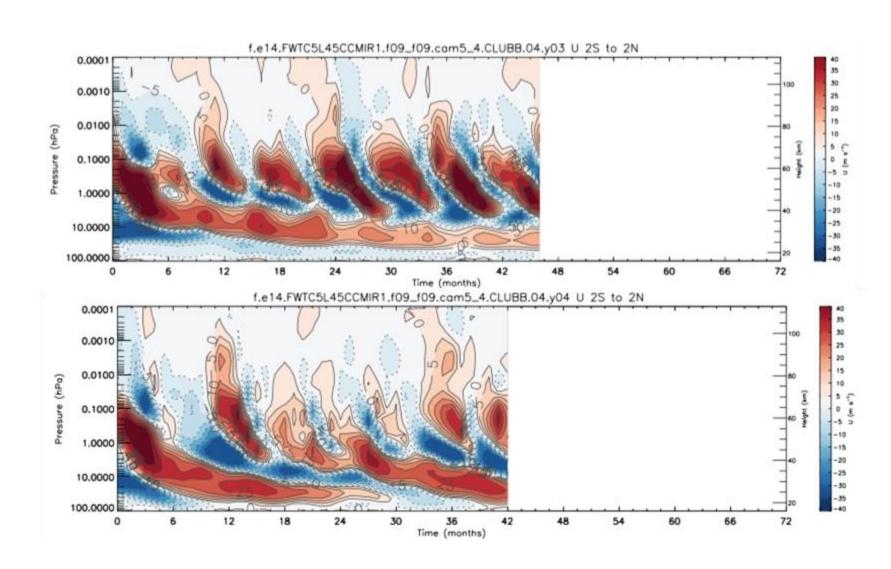


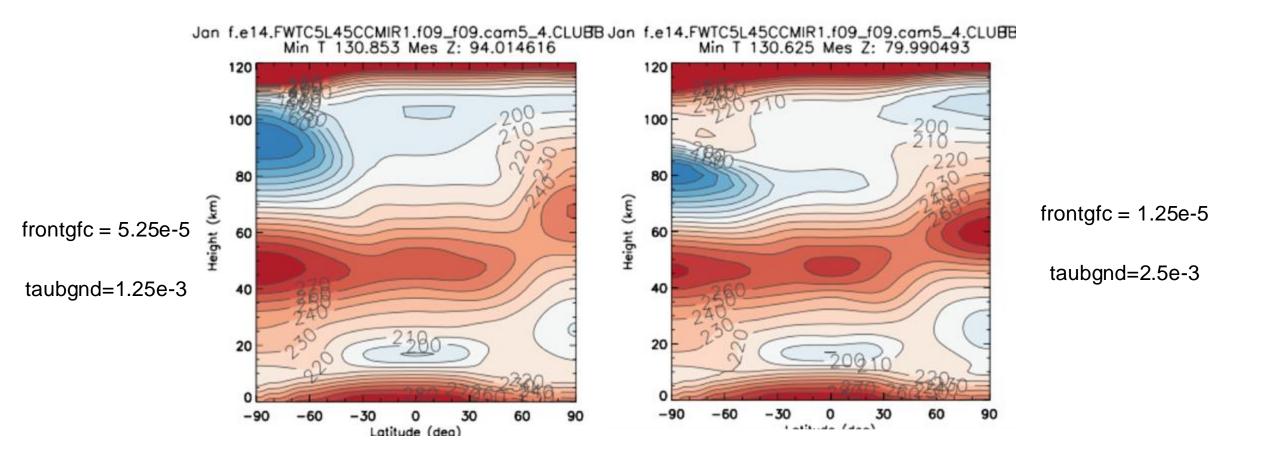


## CESM(WACCM5) tuning:

$$Eff_{gw} = 0.3$$

$$Eff_{gw} = 0.6$$





Need to get mesopause temperature and height right



Gravity wave tuning in a high-top model:

## WACC-M-OLE





 $\sim$  3 -6 months

"If there was a hell, gravity wave tuning would surely be one of the key activities there", Rolando Garcia

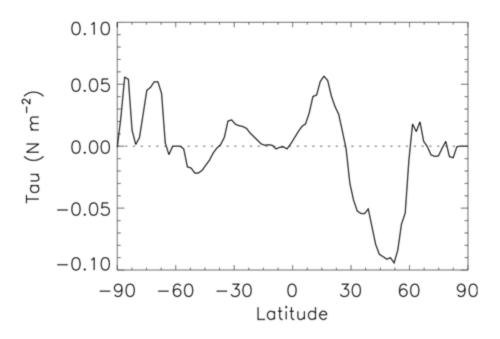


# Why does any of this matter?

## **Effect on Sudden Stratospheric Warmings (SSWs):**

Simulations with turbulent mountain stress (TMS): SSW freq NDJFM: **0.6** (same as ERAI) Simulations without TMS: SSW freq NDJFM: **0.25** 

TMS: adds surface drag term



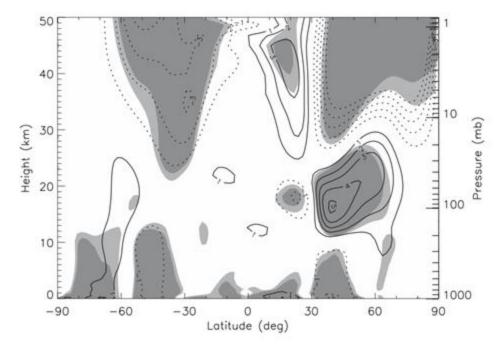


FIG. 14. DJF zonal wind difference: WACCM3.5 – WACCM3.5ntms. Contours are  $\pm (1, 2, 3, 4, 5, 10, 15, 20, 25)$  m s<sup>-1</sup>. Light and dark shading represent regions with Student's *t*-test values at the 95% and 99% levels, respectively.



Received: 7 March 2019 Revised: 11 October 2019

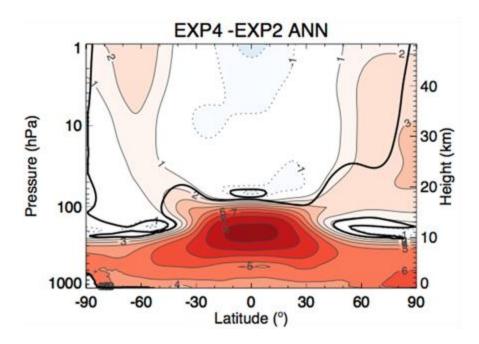
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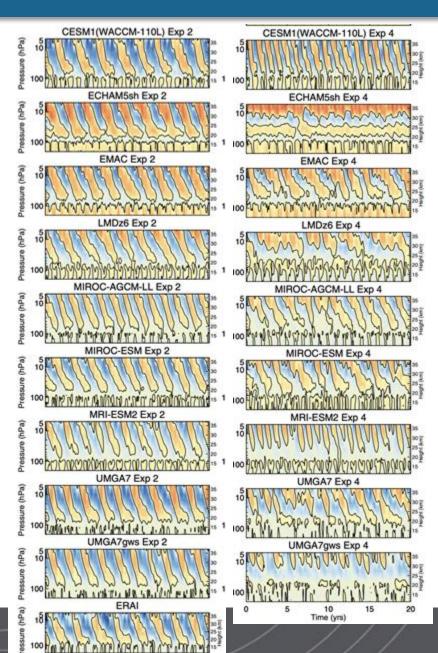
SPECIAL SECTION QBO MODELLING INTERCOMPARISON

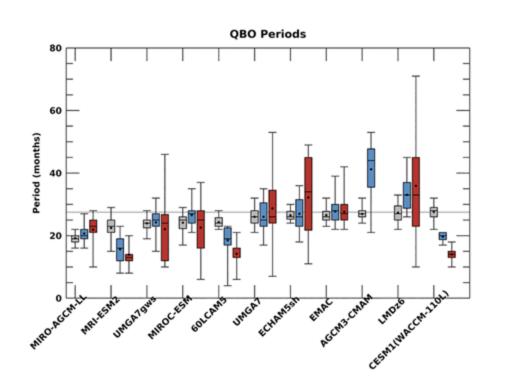
# Response of the Quasi-Biennial Oscillation to a warming climate in global climate models

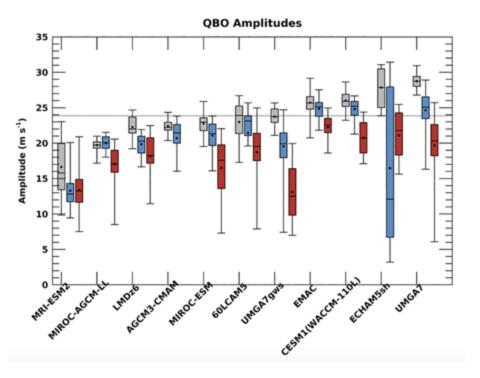


EXP4: annually-repeating SSTs + 4K, 4 X CO<sub>2</sub>









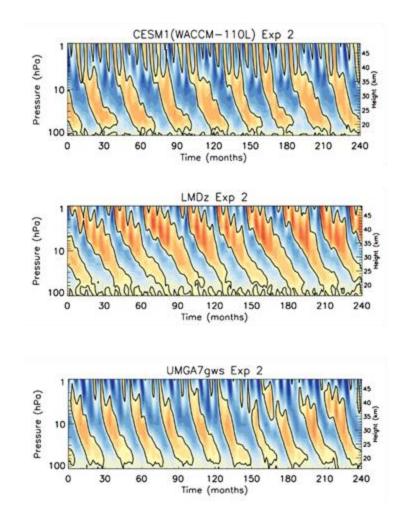
No consistency in how the QBO period will change in future climate; Consistent decrease in QBO amplitude.

Models with interactive GW sources: Present:

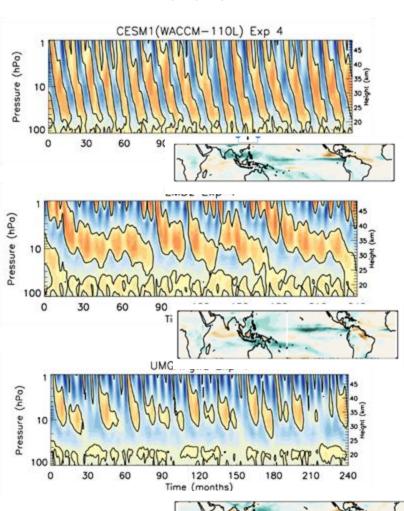
Wave **amplitude** is related to the **square of convective heating** 

Wave amplitude is related to **square of precipitation**, which is converted into heating rate

Wave amplitude is related to the square root of precipitation (based on empirical relationship)



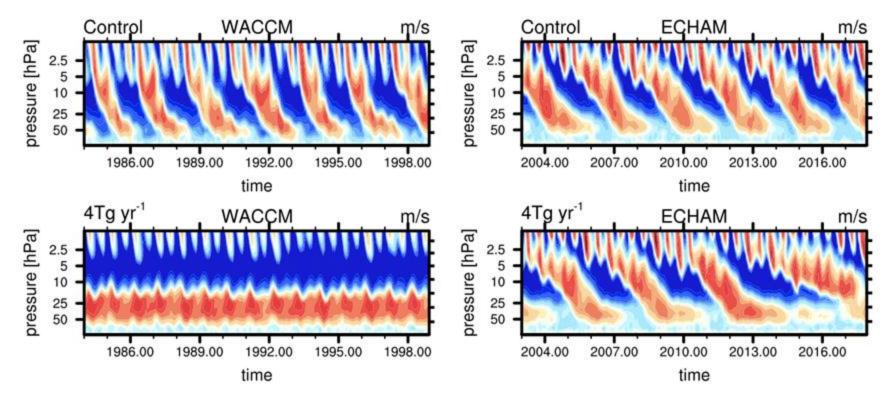
### Future:



Richter et al. 2020, QJRMS



Climate Intervention: How will the QBO respond to injection of aerosols into the stratosphere? (at 60 hPa)



Difference due to tropical w\* (partially driven by GWs)



### Conclusions

- Gravity wave parameterizations are still very much needed to simulate the stratosphere and the MLT region correctly in Earth system models
- For models with non-orographic source GW parameterizations, errors from the troposphere will carry up to the middle atmosphere - large uncertainties in formulation still exist
- Not all sources of waves are accounted for
- Lots of room still for improvement especially need for scale-aware parameterizations