# **Gravity Wave Parameterizations in Earth system models**

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- I. Introduction and basic equations
- II. Representations in models
- III.Gravity wave tuning in models
- IV.Effects on key science questions



## **Introduction and Basic Equations**

**Gravity Waves (GWs):** waves in Earth's atmosphere for which buoyancy is the restoring force.



**In water: on boundary of a denser fluid (water) and air above**



## **Atmosphere:** continuously stratified -> propagation in vertical and horizontal



**GW Sources:** any process that produces perturbations of air parcels

**Primary Sources:** Orography, Convection (including TCs), Fronts

**Other:** polar vortex edge, secondary wave generation (from wave breaking)

## **Basic characteristics:**

Horizontal wavelengths: 10's to 100's km

Vertical wavelengths: 3 to 30 km

Periods: 10 min to hours



#### **Why do GW's matter?**

- GW propagate upwards
- Amplitude grows exponentially with height (wave energy flux is conserved - air density decreases with altitude)
- They deposit momentum when they encounter critical levels or break



*Shading: Vertical velocity (m/s)*





3D simulation of convectively generated gravity waves using the Complex Geometry Compressible Atmospheric Model (CGCAM) [\[Felton and Lund \(2006\)\].](https://www.sciencedirect.com/science/article/pii/S0021999105004997?via%3Dihub) Latent heating is used as a proxy for convection.

https://www2.cgd.ucar.edu/staff/jrichter/animations.html *Animations: Christopher Heale* 



GW characteristics are governed by a **'dispersion relationship'**: relates frequency (period), horizontal and vertical wavenumbers (wavelengths)

Linearized Boussinesq equations: ->



$$
\omega^{*2} = (\omega - Uk)^2 = \frac{N^2k^2}{(k^2+m^2)}.
$$

- $\omega^*$ Intrinsic frequency
- Frequency relative to the ground  $\omega$
- $k = 2\pi/\lambda_x$ horizontal wavenumber
- $m = 2\pi/\lambda_z$ vertical wavenumber

$$
N^2 = \frac{g}{\theta} \frac{\partial \theta}{\partial z}
$$

buoyancy frequency



**Key wave properties:** horizontal phase speed, group velocity

$$
\omega^{*2} = (\omega - Uk)^2 = \frac{N^2k^2}{(k^2+m^2)} \qquad \qquad \longrightarrow \qquad m^2 = \frac{k^2(N^2-\omega^{*2})}{\omega^{*2}}
$$





$$
c_{px} = \frac{\omega}{k}
$$

Horizontal wave phase speed: Horizontal intrinsic wave phase speed:

$$
c_{px}^* = \frac{\omega^*}{k} = c_{px} - U = \pm \frac{N}{(k^2 + m^2)^{\frac{1}{2}}}
$$

$$
C_{gx} = c_{gx}^* = \frac{\partial \omega^*}{\partial k} = c_{px}^* \left( 1 - \frac{k^2}{k^2 + m^2} \right)
$$

Horizontal Group Velocity:

Vertical Group Velocity: (speed GW energy

$$
c_{gz} = \frac{\partial \omega}{\partial m} = \frac{\omega^*}{m} \left( \frac{m^2}{m^2 + k^2} \right)
$$







**Mountain Waves: Linear theory can predict the general features of MWs when the mountain** height is small in comparison to the vertical wavelength of the wave.



Small scale ridges, with intrinsic frequency higher than buoyancy frequency:

 $Uk > N \Rightarrow m$  imaginary  $\Rightarrow$  exponential decay with height

Flow over wider ridges -> **propagation with height**  $f < Uk < N$ 

#### Mountains can produce low level blocking and downslope windstorms



- $t = 0.00$  hr an a baile ann  $100$  $t = 0.00$  hr,  $z = 65$  km 900 60 80 56 52 600 48 Zonal Wind (m s<sup>-1</sup>) 44 Height (km) 300 60 40 y-Distance 36 32 CONSTANT FIELD - VALUE IS 30  $\mathsf{O}$ 28 24 40 20  $-300$ 16  $12$  $-600$ 8 20  $\overline{4}$  $\Omega$  $-900$ 1200  $-600$  $-300$ 300 600 900  $\Omega$  $\pmb{0}$ x-Distance 800  $-800$  $-400$ 400 Distance (km)
- 200-km-wide, 1000-m-high isotropic compact-cosine mountain
- WRF model
- 30 m/s wind  $(-5 0 at 24 hrs)$

https://www2.cgd.ucar.edu/staff/jrichter/animations.html



#### **Convectively generated gravity waves:**



600

 $X$  (km)

855 855 1500



*Pahlavan et al. 2023 (ERA5, 30 km resolution; 50 km vertical velocity)*

**Horizontal wavelengths: 10's to 100's km Vertical wavelengths: few to 40 km Horizontal phase speeds: up to 100 m/s**



505 500

 $X$  (km)

*Beres et al. 2002*



#### **Convectively generated GWs - mechanisms:**



- **1) Thermal or diabatic forcing:** temporal and spatial variations of convective heating produce perturbations that force a spectrum of GWs (Bretherton et al. 1998, Chun and Baik 1998, Pandya and Alexander (1999)
- **2) Mechanical oscillator:** oscillating updrafts and downdrafts perturb the stably stratified atmosphere at and above the top of convective motion (Clark et al. 1986, Fovell et al. 1992)
- **3) Moving mountain:** top of a convective elements acts as a barrier to the background mean flow, producing upstream propagating waves in a manner similar to flow over a mountain (Clark et al. 1986, Pfister et al. 1993)



#### **Frontally generated gravity waves:** dominant GW source in mid-latitudes

Gravity wave study based on the idealized baroclinic wave simulations: Weak moist run at 24hr



Yellow: temperature or potential temperature Turquoise: dynamic tropopause (PV = 1.5 PVU ) Black: horizontal wind <mark>blue</mark>: horizontal divergence



**Frontally generated gravity waves:** dominant GW source in mid-latitudes



**1) Spontaneous imbalance adjustment** (generalization of geostrophic adjustment): GW are generated as imbalance flow comes back to balance

Emission of large amplitude inertia gravity waves in regions of strong horizontal curvature

- **2) Adjustment emission:** well-balance flow continuously radiates GWs during the course of near-balance evolution
- **3) Shear instability:** nonlinear interaction between Kelvin-Helmholtz instability and propagating modes; May occur in very intense shear layers near the surface or at upper levels, above tropopause jets



**Waves generated by polar vortex:** 



12UT on 13 Jan at z=70.km, T:-42.,54.K. U<sub>H</sub> at z=70.km, mx(U<sub>H</sub>)=111.m/s



Colors: T'; Vectors (mean U, V)

From HIAMCM (Becker & Vadas 2020): high-resolution, whole atmosphere, spectral model, effective resolution ~ 52 km; top at  $\sim$  450 km

*Vadas et al. 2024*



#### **Secondary wave generation:**

- First proposed using theoretical arguments by **Vadas et al. (2002):** Mechanism for the Generation of Secondary Waves in Wave Breaking Regions
- Deep **3D body forces (GW breaking)**, which generate secondary waves very efficiently, create high-frequency waves with large vertical wavelengths that possess large momentum fluxes.

**Primary and Secondary Gravity Waves and Large-Scale** Wind Changes Generated by the Tonga Volcanic Eruption on 15 January 2022: Modeling and Comparison With **ICON-MIGHTI Winds** 

Sharon L. Vadas<sup>1</sup> **.** Erich Becker<sup>1</sup> **.** Cosme Figueiredo<sup>2</sup> **.** Katrina Bossert<sup>3</sup> **.** Brian J. Harding<sup>4</sup> **.** 



**Secondary waves generated:** a continuum of medium to largescale secondary GWs with *τr* ∼ 20 min to 7 hr, *λH* ∼ 400–7,500 km, *cH* ∼ 100–600 m/s, and *u*′, *v*′ ∼ 100–200 m/s.



#### **GW propagation and dissipation:**

 $m^2 = \frac{k^2(N^2 - \omega^{*2})}{\omega^{*2}}$ 





**GW momentum flux: constant with height (till wave breaks/dissipates)**

$$
\bar{\rho}u'w'
$$

**density decreases -> amplitude increases**



$$
GWD = \frac{1}{\overline{\rho}} \frac{\partial}{\partial z} (\overline{\rho} \overline{u'w'}, \overline{\rho} \overline{v'w'})
$$

is the background atmospheric density,  $u'$ ,  $v'$ , and w'are the horizontal and vertical velocity perturbations





$$
GWD = \frac{1}{\overline{\rho}} \frac{\partial}{\partial z} (\overline{\rho} \overline{u'w'}, \overline{\rho} \overline{v'w'})
$$

is the background atmospheric density,  $u'$ ,  $v'$ , and w'are the horizontal and vertical velocity perturbations





- **Need for GWs:** recognition that there was a 'missing drag' in middle atmosphere GCMs
- Without drag, stratospheric winter jet would be much stronger, and mesopause would not be warm
- **Early GCMs, used Rayleigh friction (e.g.: Boville 1986)**
- **First implementation of GWs:** orographic parameterizations (Boer et al. 1984, **Palmer et al 1986, McFarlane 1987**)
- **Non-orographic:** (Rind et al. 1988, Fritts and Lu 1993,

Medvedev and Klaasen 1995, **Hines 1997a,b, Alexander and Dunkerton, 1999,** Warner and McIntyre 2001)







#### **Byron Boville**

## "Garbage In. Garbage Out.

# It's as simple as that"



#### **GW parameterization components:**

1) Specification of waves at **source level**: wavenumbers, phase speeds, propagation direction, source height

2) **Wave propagation with height:** typically in column and instant!

(Except for Amemiya and Sato (2010), and Eckermann et al. 2015)

3) **Wave dissipation** -> momentum deposition to the mean flow; Plane wave assumption: Flux and force along same direction as at source; Force applied to the vector momentum equations



### **Orographic Parameterizations: Non-orographic Parameterizations:**





#### **Wave dissipation:**

Foundation: **Lindzen's (1981)** saturation theory; mods by Holton (1982)

#### **Assumption:**

parameterized waves are individual, steady, monochromatic plane waves



When  $> 1$ : linary theory  $\rightarrow$  static instability

Lindzen scheme: Keep amplitude at or below 1

if  $\hat{u} > 1$  -> MF is reduced till  $\hat{u} = 1$ 

d MF/dz -> force to the mean flow



**Other parameterizations:**

- Alexander and Dunkerton (1999): discrete spectrum of monochromatic waves **deposition of all momentum flux at breaking level** works with any source spectrum
- **Hines (1997 a,b)**: proposed a **"Doppler Spread"** mechanism -> nonlinear interactions among waves in the spectrum reshape the spectrum with altitude.
- **Warner and McIntyre (2001):** ~ Hines-like spectrum reshaping with altitude (based on shape) + Lindzen's wave reshaping
- Both **Hines and Warner and McIntyre** assume a particular vertical wavenumber spectrum shape



#### **Source parameterizations: Orography**



- **First formulations** (and what's still used most of the time): single, monochromatic vertically propagating wave with c=0 (Boer et al. 1984, Palmer et al. 1986, McFarlane 1987)
- Based on **2D theory** assuming hydrostatic, steady, horizontally uniform flow over an obstacle
- Amplitude at source level: based on subgrid-scale orographic variance
- **Surface stress vector:** parallels to and opposite of the mean flow at the lowest level of the model, assuming **isotropic topography (single length scale)**



#### **Source parameterizations: Orography**



$$
Froude #: \quad F_r = \frac{v}{N h_m}
$$

- **Lott and Miller (1997)**: incorporated impact of nearsurface nonlinearities (blocking, flow splitting)
- $\bullet$  when  $h_m$  exceeds a critical value -> portion of the flow is diverted or blocked
- **Scinocca and McFarlane (2000):** employs two vertically propagating waves - to provide azimuthal distribution (using elliptical barrier model)
- Also includes representation of low-level drag

Fr > 1: linear; upward propagating waves Fr < 1: non-linear flow; blocked flow or diverting around obstacle -> Momentum flux of upward propagating waves is reduced



#### **Turbulent orographic Form Drag:**

Typically representing scales < 5 km Drag exerted by hills/mountains through generation of turbulence

#### **Implementation in IFS:**

$$
\frac{\partial U}{\partial t} = \frac{\partial}{\partial z} \frac{\tau_x}{\rho} = -C_{\text{tofd}} |\mathbf{U}(z)| U(z),
$$
  

$$
C_{\text{tofd}} = -\alpha \beta C_{\text{md}} C_{\text{coor}} 2.109 e^{-(z/1500)^{1.5}} a_2 z^{-1.2}
$$

- **Wood and Mason 1993: represented with an effective** roughness length approach -> enhances roughness proportionally to orographic height
	- **Beljaars et al. 2004: explicitly distributed form drag**

Applies drag explicitly on model levels

with  $\tau_x$  being the stress,  $\rho$  the density, z the height above the surface,  $\alpha = 35$ ,  $\beta = 1$ ,  $C_{\text{md}} = 0.005$ ,  $C_{\text{coor}} = 0.6$ ,  $a_2 = a_1 k^{n_1 - n_2}$ ,  $a_1 = \sigma_{\text{fit}}^2 (I_H k_{\text{fit}}^{n_1})^{-1}$ ,  $k_1 = 0.003 \,\text{m}^{-1}$ ,  $n_1 = -1.9$ ,  $n_2 = -2.8$ ,  $k_{\text{fit}} = 0.00035 \,\text{m}^{-1}$ ,  $I_h = 0.00102 \text{ m}^{-1}$  and  $\sigma_{\text{fit}}$  is the standard deviation of filtered subgrid orography (to remove scales larger than 5 km). A corresponding equation is used for the meridional wind  $V$ . For numerical stability, these

*Kanehama et al. 2022 (ECMWF Technical note)*



#### Towards a more "scale-aware" orographic gravity wave drag parametrization: Description and initial testing

A. van Niekerk<sup>®</sup> | S.B. Vosper

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- DOI: 10.1002/qj.4126
	- Uses **linear theory** for hydrostatic GWs
	- **Fourier description of subgrid orography** -> produces a MF vector that accounts for anisotropy of the topography; eliminates monochromatic assumption (computed from 1-km source orography dataset offline)
	- Also accounts for **flow-blocking**
	- Better behaved for model grid spacing from 32 to 2 km
	- Parameterized GW fluxes increase with coarser resolution
	- as resolved GW MF decrease)
	- **Total momentum flux is the same**



#### **Non-orographic gravity waves:**

- Typically all lumped together
- Source specified somewhere in the troposphere
- Emitted with the same properties at all times
- Sometimes a latitudinal dependence is specified



**Source parameterizations: Convective gravity waves**

- **First non-orographic source spectrum parameterization: Rind et al. (1988):** convection and wind shear: used in **NASA GISS** model
- Convective GW MF related to convective mass flux Phase speed: U avg over convective region +/- 10 m/s; for deeper convection additional waves +/- 20 m/s, 40 m/s
- **Kershaw et al. (1995); Chun and Baik (1988)** parameterization of the obstacle effect;
- Beres et al. 2004: based on linear theory and models: used in CESM, E3SM & now NASA
- Song and Chun (2005): similar to above: more complex U/N structure
- Bushell et al. 2015: UK Model
- Lott and Guez (2013): LMDz model



**Beres et al. (2004):** based on linear theory, thermal forcing (steady and oscillating component); verified on mesoscale model simulations

$$
\frac{\partial u'}{\partial t} + \overline{U} \frac{\partial u'}{\partial x} + \frac{1}{\rho_0} \frac{\partial p'}{\partial x} = 0,
$$
\n
$$
\frac{\partial w'}{\partial t} + \overline{U} \frac{\partial w'}{\partial x} + \frac{1}{\rho_0} \frac{\partial p'}{\partial z} - g \frac{\theta'}{\theta_0} = 0,
$$
\n
$$
\frac{\partial u'}{\partial x} + \frac{\partial w'}{\partial z} = 0,
$$
\n
$$
\frac{g}{\theta_0} \left( \frac{\partial \theta'}{\partial t} + \overline{U} \frac{\partial \theta}{\partial x} \right) + w'N^2 = \frac{g}{\theta_0} J',
$$
\n
$$
q_z(z) = \begin{cases} \sin(\pi z/h) & \text{for } 0 \le z \le h \\ 0 & \text{for } z > h. \end{cases}
$$
\n
$$
q_z(z) = \begin{cases} \sin(\pi z/h) & \text{for } 0 \le z \le h \\ 0 & \text{for } z > h. \end{cases}
$$
\n
$$
\text{Heat source: horizontal scale } 2\sigma_x \text{ vertical scale: } h
$$



#### **Beres et al. 2004:**

- GW MF dependent primarily on vertical scale of heating and wind
- Also on horizontal scale and dominant frequency (need to assume)





#### **Frontally/Shear generated waves:**



#### ● **Rind et al. (1988)**

Shear-generated GWs: launched at jet stream level; assigned a single wavenumber and phase speed dependent on the direction of the shear and wind velocity in shear layers

#### **Charron and Manzini (2002):**

using 'frontogenesis function' to diagnose location of fronts (Miller 1948, Hoskins 1982)

- if the frontogenesis function exceeds a critical threshold -> GWs launched at a fixed level of 600 hPa with high amplitude
- otherwise: small amplitude spectrum



**Gravity Wave Tuning**

**GW parameterizations in CESM:** upcoming CAM versions (80 km top) and WACCM (150 km top)

## 1. Orographic GWs:



**McFarlane (1987):** 1 wave with  $c=0$ Amplitude dependent on orography height & mean wind Tunable parameter: Efficiency **Beljaars et al. (2004)**

## 2. Frontal GWs:

#### **modified Charron and Manzini (2002):**

40 waves with  $-100 < c < 100$  m/s LOW Gaussian distribution in phase speed centered at 600 hPa Constant wave amplitude

Tunable parameters: Efficiency, amplitude, phase speed distribution, frontal threshold

## 3. Convectively generated GWs:

#### **Beres et al. (2004):**

40 waves with -100 < c < 100 m/s Dominant c related to h; Amplitude proportional to Q<sup>2</sup> Tunable parameters: Efficiency, amplitude conversion (assumptions about scale/frequency)











#### Gravity wave drag in WACCM3.5

**Quasi-biennial Oscillation: U (10S to 10N):**

**Beres et al. (2004)** Tunable parameters:

CF: Convective Fraction (tunable)

Eff<sub>gw</sub> (multiplies the GWD) Eff<sub>gw</sub> = 0.4

$$
Eff_{gw} = 0.4
$$
  
CF = 5%



#### $Eff<sub>gw</sub> = 0.35$  $CF = 8%$ **GW parameterizations can't fix all deficiencies in the model**

*Richter et al. 2019*





#### CESM(WACCM5) tuning:

$$
Eff_{gw} = 0.3
$$

 $Eff<sub>gw</sub> = 0.6$ 







Need to get mesopause temperature and height right



Gravity wave tuning in a high-top model:

## **WACC-M-OLE**





 $\sim$  3 -6 months

*"If there was a hell, gravity wave tuning would surely be one of the key activities there", Rolando Garcia*



## **Why does any of this matter?**

**Effect on Sudden Stratospheric Warmings (SSWs):**

Simulations **with turbulent mountain stress** (TMS): SSW freq NDJFM: **0.6** (same as ERAI) Simulations **without TMS**: SSW freq NDJFM: **0.25**

TMS: adds surface drag term





FIG. 14. DJF zonal wind difference: WACCM3.5 WACCM3.5ntms. Contours are  $\pm$  (1, 2, 3, 4, 5, 10, 15, 20, 25) m s<sup>-1</sup>. Light and dark shading represent regions with Student's t-test values at the 95% and 99% levels, respectively.



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Quarterly Journal of the RMetS **Royal Meteorological Society** 

SPECIAL SECTION QBO MODELLING INTERCOMPARISON

Response of the Quasi-Biennial Oscillation to a warming climate in global climate models



**EXP4: annually-repeating SSTs + 4K, 4 X CO**<sub>2</sub>







No consistency in how the QBO period will change in future climate; Consistent decrease in QBO amplitude.

*Richter et al. 2020, QJRMS*



### Models with interactive GW sources: Present: Future: Future:

CESM1(WACCM-110L) Exp 2

Wave **amplitude** is related to the **square of convective heating**

Wave amplitude is related to **square of precipitation**, which is converted into heating rate

Wave amplitude is related to the **square root of precipitation** (based on empirical relationship)

#### *Richter et al. 2020, QJRMS*











**Climate Intervention: How will the QBO respond to injection of aerosols into the stratosphere? (at 60 hPa)**



Difference due to tropical w\* (partially driven by GWs)

*Niemeier, Richter, Tilmes 2020*



### **Conclusions**

- Gravity wave parameterizations are still very much needed to simulate the stratosphere and the MLT region correctly in Earth system models
- For models with non-orographic source GW parameterizations, errors from the troposphere will carry up to the middle atmosphere - large uncertainties in formulation still exist
- Not all sources of waves are accounted for
- Lots of room still for improvement especially **need for scale-aware parameterizations**

