ECMWF Overview

- The European Centre for Medium-Range Weather Forecasts is an international organisation supported by 23 European States.

- The center was established in 1973 by a Convention and the real-time medium-range forecasts were made in June 1979. The Centre has been producing operational medium-range weather forecasts since 1 August 1979.

- The Centre has three Fujitsu systems, a 100-processor VPP5000, a 116-processor VPP700 and a 48-processor VPP700E. The aggregate sustained performance of these three machines is about 400 Gflops (400 thousand million floating point operations per second).

- ECMWF model is a global model which predicts the behavoiur of the atmosphere in the medium-range up to ten days ahead.

- The resolution of the discretization is equivalent to having gridpoints separated by about 40 km around the globe and at 60 levels in the vertical.The model forecasts the wind, the temperature and the humidity at 20,911,680 points throughout the atmosphere. With this resolution it is possible, for example, to distinguish clearly the French Massif Central from the Alps.

- The ECMWF model is also a spectral model with a triangular truncation of 213 waves in the horizontal and 31 levels in the vertical (T213/L31).

- The ECMWF model has a hybrid coordinate in the vertical , $\eta(p, p_{surf})$ with $\eta(0, p_{surf}) = 0$ and $\eta(p_{surf}, p_{surf}) = 1$.

- Physics of the model include:

  Radiation

  Turbulent diffusion and interaction with the surface

  Subgrid-scale orographic drag

  Convection

  Clouds and large-scale precipitation

  Land surface parameterization

  Methane oxidation

  Climatological data

The Continuous equations in (λ, θ, η)

- The momentum equations

$$\frac{\partial U}{\partial t} + \frac{1}{a\cos\theta}\left\{U\frac{\partial U}{\partial \lambda} + V\cos\theta\frac{\partial U}{\partial \theta}\right\} + \dot{\eta}\frac{\partial U}{\partial \eta} - fV$$

$$+ \frac{1}{a}\left\{\frac{\partial \Phi}{\partial \lambda} + R_{dry}T_v\frac{\partial}{\partial \lambda}(\ln p)\right\} = P_U + K_U \qquad (1)$$

$$\frac{\partial V}{\partial t} + \frac{1}{a\cos^2\theta}\left\{U\frac{\partial V}{\partial \lambda} + V\cos\theta\frac{\partial V}{\partial \theta} + \sin\theta(U^2 + V^2)\right\} + \dot{\eta}\frac{\partial V}{\partial \eta} + fU$$

$$+ \frac{\cos\theta}{a}\left\{\frac{\partial \Phi}{\partial \theta} + R_{dry}T_v\frac{\partial}{\partial \theta}(\ln p)\right\} = P_V + K_V \qquad (2)$$

a is the radius of the Earth, $\dot{\eta}$ is the η-coordinate vertical velocity ($\dot{\eta} = d\eta/dt$),

Φ is geopotential, R_{dry} is the gas constant for dry air, and T_v is the virtual temperature defined by $T_v = T\{1 + [R_{vap}/(R_{dry} - 1)]q\}$.

P_U and P_V represent contributions of parameterized physical processes, while K_U and K_V are the horizontal diffusion terms.

- The thermodynamic equation

$$\frac{\partial T}{\partial t} + \frac{1}{a\cos^2\theta}\left\{U\frac{\partial T}{\partial \lambda} + V\cos\theta\frac{\partial T}{\partial \theta}\right\} + \dot{\eta}\frac{\partial T}{\partial \eta} - \frac{\kappa T_v \omega}{\{1 + (\delta - 1)q\}p} = P_T + K_T \qquad (3)$$

$\kappa = R_{dry}/c_{p,dry}$, ω is the p-coordinate vertical velocity $\omega = dp/dt$ and $\delta = c_{p,vap}/c_{p,dry}$.

- The moist equation

$$\frac{\partial q}{\partial t} + \frac{1}{a\cos^2\theta}\left\{U\frac{\partial q}{\partial \lambda} + V\cos\theta\frac{\partial q}{\partial \theta}\right\} + \dot{\eta}\frac{\partial q}{\partial \eta} = P_q + K_q \qquad (4)$$

- The continuity equation

$$\frac{\partial}{\partial t}\left(\frac{\partial p}{\partial \eta}\right) + \nabla\bullet\left(V_H\frac{\partial p}{\partial \eta}\right) + \frac{\partial}{\partial \eta}\left(\dot{\eta}\frac{\partial p}{\partial \eta}\right) = 0 \qquad (5)$$

$V_H = (U, V)$ is the horizontal wind.

- The hydrostatic equation

$$\frac{\partial \Phi}{\partial \eta} = \frac{R_{dry} T_v}{p} \frac{\partial p}{\partial \eta} \tag{6}$$

- The vertical velocity ω is given by

$$\omega = -\int_0^{\eta} \nabla \bullet \left(V_H \frac{\partial p}{\partial \eta} \right) d\eta + V_H \bullet \nabla p \tag{7}$$

Expressions for the rate of change of surface pressure and for vertical velocity $\dot{\eta}$,

are obtained by integrating (5), using the boundary conditions, $\dot{\eta} = 0$ at $\eta = 0$ and $\eta = 1$,

$$\frac{\partial}{\partial t}(\ln p_s) = -\frac{1}{p_s}\int_0^1 \nabla\bullet\left(V_H\frac{\partial p}{\partial \eta}\right)d\eta \qquad (8)$$

$$\dot{\eta}\frac{\partial p}{\partial \eta} = -\frac{\partial p}{\partial t} - \int_0^\eta \nabla\bullet\left(V_H\frac{\partial p}{\partial \eta}\right)d\eta \qquad (9)$$

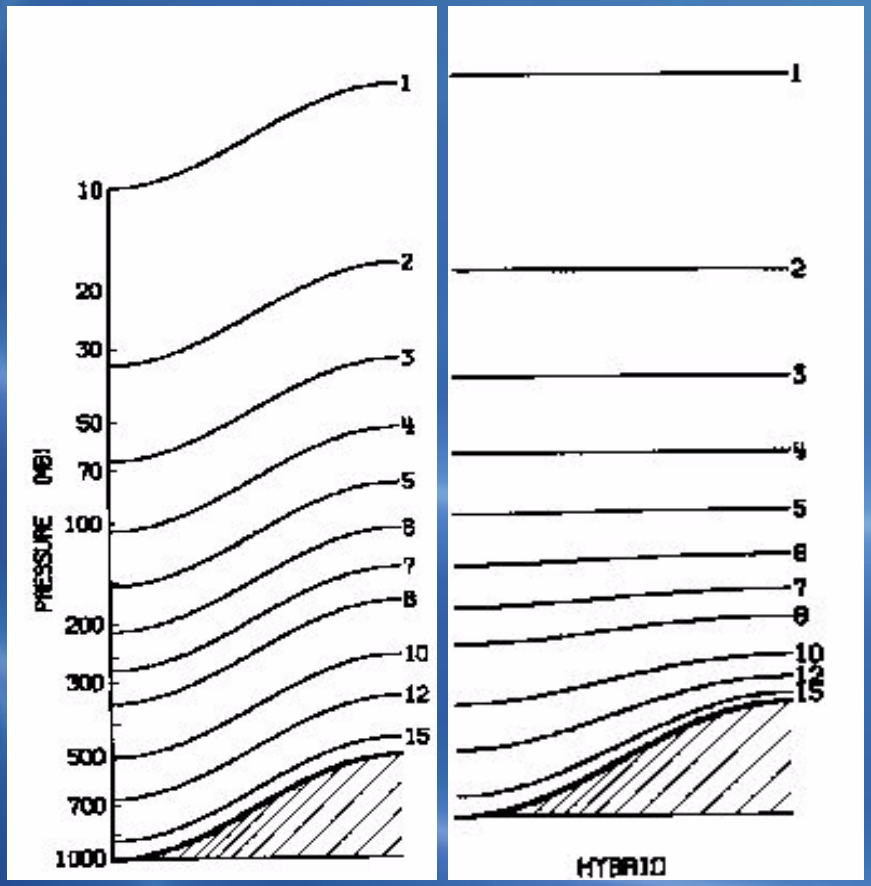
Vertical discretization

- To represent the vertical variation of the dependent variables U, V, T and q the atmosphere is divided into $NLEV$ layers. These layers are defined by the pressure at the interfaces between them (the 'half-levels'), and these pressures are given by

$$p_{k+1/2} = A_{k+1/2} + B_{k+1/2} p_s \qquad (10)$$

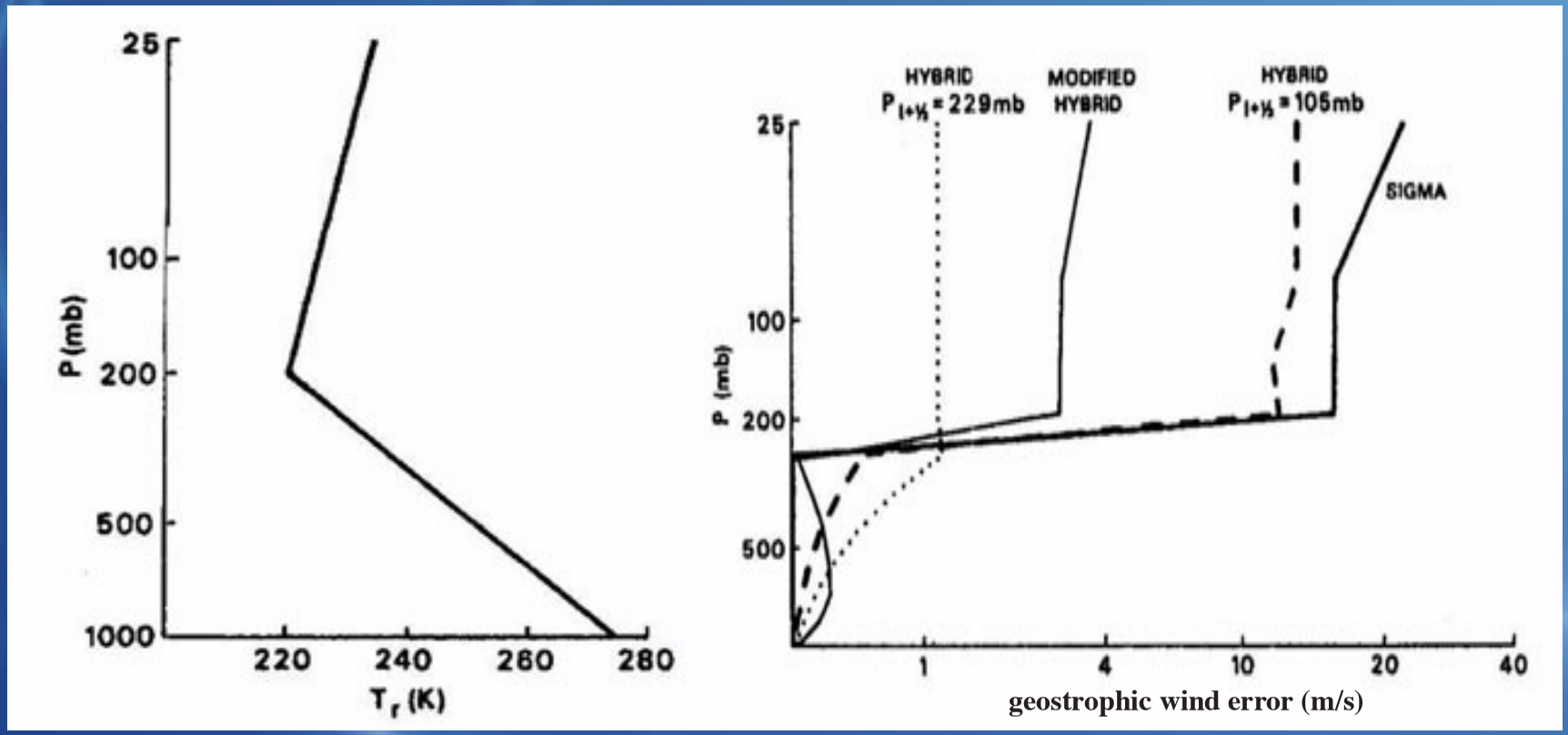
for $0 \le k \le NLEV$ and $\left(\frac{\partial p}{\partial p_s}\right)_{k+1/2} = B_{k+1/2}$. The $A_{k+1/2}$ and $B_{k+1/2}$ are constants whose values effectively define the vertical coordinate. They are determined by requiring that half-level pressures coincide with those of sigma coordinates for a surface pressure of 1013.2mb, that at the top interior levels ($1^1/_2$

and $2^1/_2$) are levels of constant pressure, and that the lowest two layers have half the thicness for a surface pressure of 500mb that they have for the surface pressure of 1013.2mb.



- The form of this hybrid coordinate is efficient from a computational viewpoint, and allows a direct control over the “flattening” of coordinate surfaces as pressure decrease, since the A‘s and B‘s may be determined by specifying the distribution of model pressure levels for a mean sea-level pressure and for a surface pressure typical of the lowest expected to be attained in the model.

- One disadvantage of sigma coordinate is cancellation of geopotential and pressure gradient terms over steep orography. Simmons and Burridge (1981) have compared the error in the representation of pressure gradient over steep orography for hybrid and sigma coordinates. They considered a temperature field which is a function of pressure alone and calculated an error function.


25
100
200
500
1000
P(mb)
220
240
260
280
T_r (K)
HYBRID
$P_{l+½}$ = 229mb
MODIFIED
HYBRID
HYBRID
$P_{l+½}$ = 105mb
SIGMA
25
100
200
500
P (mb)
1
4
10
20
40
geostrophic wind error (m/s)

- The prognostic variables are represented by their values at 'full-level' pressures p_k.

  Values for p_k are not explicitly required by the model's vertical finite-difference scheme and they are calculated as

$$p_k = 1/2(p_{k+1/2} + p_{k-1/2}) \tag{11}$$

- The discrete analogue of the surface pressure tendency equation (8) is

$$\frac{\partial}{\partial t}(\ln p_s) = -\frac{1}{p_s}\sum_{k=1}^{NLEV} \nabla\bullet(V_H \Delta p_k) \tag{12}$$

  where $\Delta p_k = p_{k+1/2} - p_{k-1/2}$. Substituting $\Delta p_k = \Delta A_k + \Delta B_k p_s$

$$\frac{\partial}{\partial t}(\ln p_s) = -\sum_{k=1}^{NLEV} \frac{1}{p_s}\{D_k(\Delta p_k) + (V_k \Delta \ln p_s)(\Delta B_k)\} \tag{13}$$

where D_k is the divergence at level k,

$$D_k = \frac{1}{a\cos^2\theta}\left\{\frac{\partial U_k}{\partial \lambda} + \cos\theta\frac{\partial U_k}{\partial \theta}\right\}, \text{ and} \tag{14}$$

$$\Delta B_k = B_{k+1/2} - B_{k+1/2} \tag{15}$$

- The discrete analogue of (9) is

$$\left(\dot{\eta}\frac{\partial p}{\partial \eta}\right)_{k+1/2} = -\frac{\partial p_{k+1/2}}{\partial t} - \sum_{j=1}^{k} \nabla\bullet(V_j \Delta p_j) \qquad (16)$$

and from (10) we obtain

$$\left(\dot{\eta}\frac{\partial p}{\partial \eta}\right)_{k+1/2} = -p_s\left\{B_{k+1/2}\frac{\partial}{\partial t}(\ln p_s) + \sum_{j=1}^{k}\frac{1}{p_s}[D_j \Delta p_j + (V_j \bullet \nabla \ln p_s)\Delta B_j]\right\} \qquad (17)$$

In evaluating $\left(\dot{\eta}\frac{\partial p}{\partial \eta}\right)_{k+1/2}$ at time t_1, $\frac{\partial p_s}{\partial t}$ should first be used to compute values of p_s at the following time step $t + \Delta t$. The new value of $p_{k+1/2}$ may then be calculated, and thus $\partial p_{k+1/2}/\partial t$ found using, for example a centered difference in time.

- Conservation of energy is preserved by the vertical advection terms if the advection of a variable F is such as to satisfy the finite-difference analogue of the relations

$$\int_0^1 \dot{\eta}\frac{\partial F}{\partial \eta}\frac{\partial p}{\partial \eta}d\eta = -\int_0^1 F\frac{\partial}{\partial \eta}\left(\dot{\eta}\frac{\partial p}{\partial \eta}\right)d\eta \qquad (18)$$

$$\int_0^1 \dot{\eta}F\frac{\partial F}{\partial \eta}\frac{\partial p}{\partial \eta}d\eta = -\int_0^1 \frac{1}{2}F^2\frac{\partial}{\partial \eta}\left(\dot{\eta}\frac{\partial p}{\partial \eta}\right)d\eta \qquad (19)$$

This is achieved by choosing

$$\left(\dot{\eta}\frac{\partial F}{\partial \eta}\right)_k = \frac{1}{2\Delta p_k}\left[\left(\dot{\eta}\frac{\partial p}{\partial \eta}\right)_{k+1/2}(F_{k+1}-F_k)+\left(\dot{\eta}\frac{\partial p}{\partial \eta}\right)_{k-1/2}(F_k-F_{k-1})\right] \qquad (20)$$

- The discrete analogue of the hydrostatic equation is

$$\Phi_{k+1/2} - \Phi_{k-1/2} = -R_{dry}(T_v)_k \ln\left(\frac{p_{k+1/2}}{p_{k-1/2}}\right) \tag{21}$$

which gives

$$\Phi_{k+1/2} = \Phi_s + \sum_{j=k+1}^{NLEV} R_{dry}(T_v)_j \ln\left(\frac{p_{j+1/2}}{p_{j-1/2}}\right) \tag{22}$$

where Φ_s is the geopotential at the surface. The full-level values of the geopotential, as requiered in the momentum equations (1) and (2), are given by

$$\Phi_k = \Phi_{k+1/2} + \alpha_k R_{dry}(T_v)_k \tag{23}$$

The value of α_k must be specified, but is not necessary to specify it in order to assure conservation of energy. One choise could be to require cancellation of error in the sum of the geopotential and pressure gradient terms for a reference temperature distribution which is a function of pressure alone. Therefore,

$$\alpha_1 = \ln 2 \text{ and } \alpha_k = 1 - \frac{p_{k-1/2}}{\Delta p_k} \ln\left(\frac{p_{k+1/2}}{p_{k-1/2}}\right) \tag{24}$$

- Connected with the form chosen for the integration of the hydrostatic equation is the expression for full level values of the pressure gradient term $(R_{dry} T_v) \nabla \ln p$, in (1) and (2), in terms of the known 'half-level' values $p_{k+1/2}$. This expression is determined by requiring that the vertical difference scheme preserves the conservation of angular momentum

$$\int_0^{2\pi}\left[\int_0^1\left(\frac{\partial\Phi}{\partial\lambda}+\frac{RT}{p}\frac{\partial p}{\partial\lambda}\right)\left(\frac{\partial p}{\partial\eta}\right)d\eta+\Phi_s\frac{\partial p_s}{\partial\lambda}\right]d\lambda \tag{25}$$

This requires in finite-difference form,

$$\sum_{k=1}^{NLEV}\Phi_k\frac{\partial}{\partial\lambda}\Delta p_k=\Phi_s\frac{\partial p_s}{\partial\lambda}+\sum_{k=1}^{NLEV}R\left(\frac{T}{p}\frac{\partial p}{\partial\lambda}\right)_k\Delta p_k \tag{26}$$

which is satisfied if

$$R_{dry}(T_v\nabla\ln p)_k=\frac{R_{dry}(T_v)_k}{\Delta p_k}\left\{\ln\left(\frac{p_{k+1/2}}{p_{k-1/2}}\right)+\alpha_k\nabla(\Delta p_k)\right\} \tag{27}$$

Angular momentum conservation is hereby achieved without the artificial dependence of geopotential on the temperature of all model levels. This is a

consequence of using ‘half-level’ values of Φ in the sumation of the hydrostatic equation.

- To obtain a form of the energy conversion term $\kappa T\omega/\{1+(1-\delta)q\}p$ in the thermodynamic equation (3) we use the definition of vertical velocity ω to write

$$\frac{\kappa T_v \omega}{\{1+(1-\delta)q\}p} = -\frac{\kappa T_v}{\{1+(1-\delta)q\}p}\int_0^{\eta} \nabla\bullet\left(V\frac{\partial p}{\partial \eta}\right)d\eta + \frac{\kappa T_v}{\{1+(1-\delta)q\}}\left(\frac{1}{p}\nabla p\right) \qquad (28)$$

Full-level values of this expression are then determined by the requirment that the difference scheme conserves the total energy of the model atmosphere for adaiabatic, frictionless motion. This is achieved by evaluating the first term at level

k by

$$-\frac{\kappa(T_v)_k}{1+(1-\delta)q_k}\frac{1}{\Delta p_k}\left[\ln\left(\frac{p_{k+1/2}}{p_{k-1/2}}\right)\sum_{j=1}^{k-1}\nabla\bullet(V_j\Delta p_j)+\alpha_k\nabla\bullet(V_k\Delta p_k)\right] \tag{29}$$

and the second term as

$$\frac{\kappa(T_v)_k}{1+(1-\delta)q_k}\left(\frac{1}{p}\nabla p\right)_k \tag{30}$$

after substitute for α_k, $(\Delta p)_k$ and $\left(\frac{1}{p}\nabla p\right)_k$ we obtain

$$\frac{\kappa T_v \omega}{\{1+(1-\delta)q\}p} = \frac{\kappa (T_v)_k}{1+(1-\delta)q_k}\left\{-\frac{1}{\Delta p_k}\left[\ln\left(\frac{p_{k+1/2}}{p_{k-1/2}}\right)\sum_{j=1}^{k-1}(D_j\Delta p_j + p_s(V_j \bullet \nabla \ln p_s)\Delta B_j) + \alpha_k(D_k\Delta p_k + p_s(V_k \bullet \nabla \ln p_s)\Delta B_k)\right) + \frac{p_s}{\Delta p_k}\left(\Delta B_k + \frac{C_k}{\Delta p_k}\ln\left(\frac{p_{k+1/2}}{p_{k-1/2}}\right)\right)(V_k \bullet \nabla \ln p_s)\right]\right\} \quad (31)$$

where $C_k = A_{k+1/2}B_{k-1/2} - A_{k-1/2}B_{k+1/2}$.

- Full-level values of pressure

Two different full level values of p are suggested by the finite-diference scheme.

For this we go back to first term in (28) $-\frac{\kappa T}{p}\int_0^1 \nabla \bullet \left(V\frac{\partial p}{\partial \eta}\right)d\eta$ which may be generally written

$$-\frac{\kappa(T_v)_k}{\Delta p_k}\ln\left(\frac{p_{k+1/2}}{p_{k-1/2}}\right)\left[\sum_{j=1}^{k-1}\nabla\bullet(V_j\Delta p_j)+\left(\frac{1}{2}+O\left(\frac{\Delta p_k}{p_{k-1/2}}\right)\right)\nabla\bullet(V_k\Delta p_k)\right] \tag{32}$$

For $k = 1$ the simplest equivalent is

$$-\frac{kT_1}{\Delta(p_1/2)}\left[\frac{1}{2}\nabla\bullet(V_1\Delta p_1)\right] \tag{33}$$

These expressions suggest the full-level values

$$\begin{cases} p_k = \dfrac{\Delta p_k}{\ln(p_{k+1/2}/p_{k-1/2})}, k>1 \\ p_1 = \dfrac{1}{2}\Delta p_1 \end{cases} \tag{34}$$

Away from the upper boundary (34) yields

$$p_k = \frac{1}{2}(p_{k+1/2} + p_{k-1/2}) + O\left[\frac{(\Delta p_k)^2}{p_{k-1/2}}\right] \tag{35}$$

but top-level values are given by $\Delta p_1/2$ and $\Delta p_1/e$.