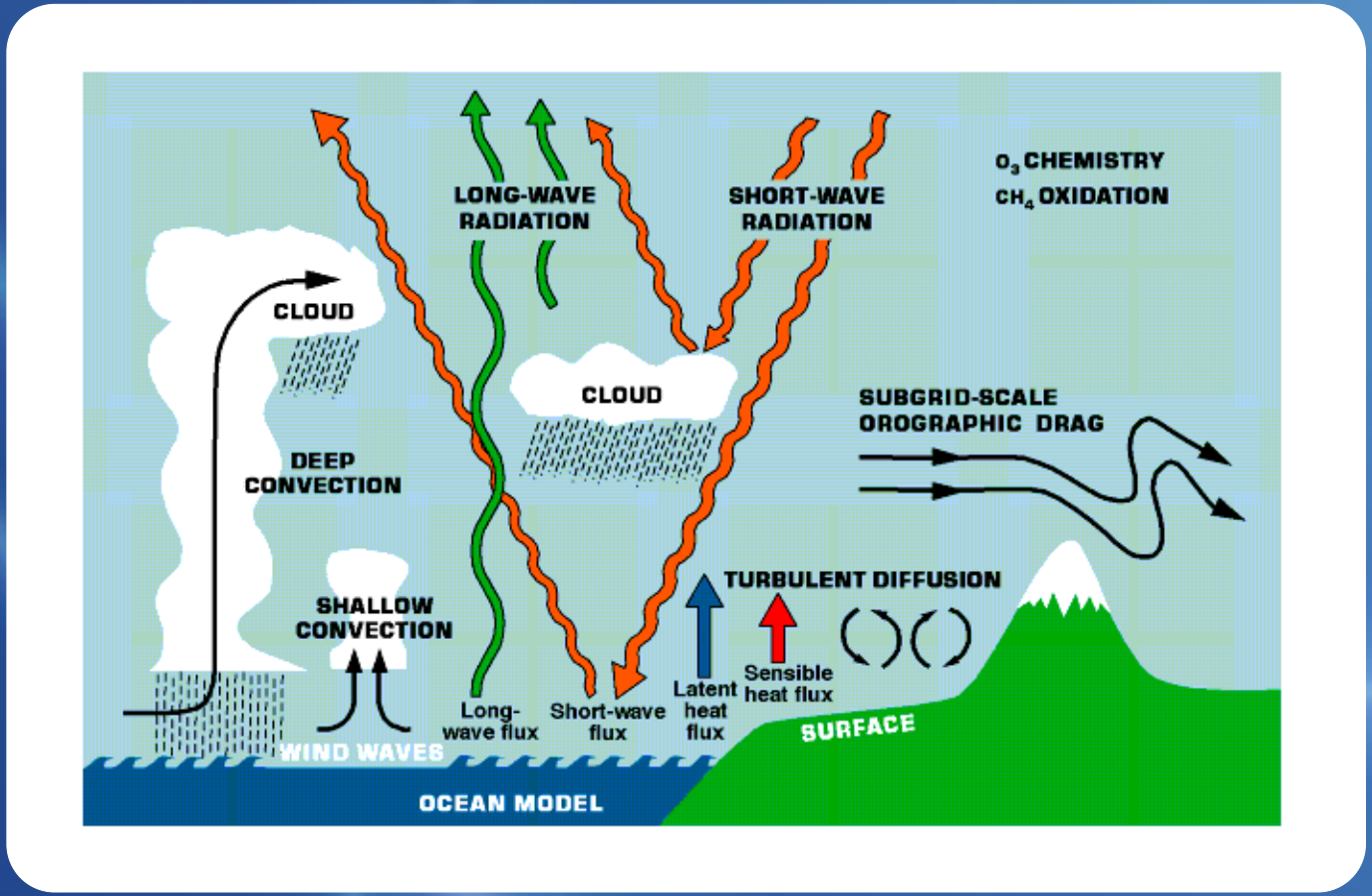
The PBL parameterization in ECMWF model



- The parameterization scheme represents the turbulent transfer of heat, momentum and moisture between the surface and the lowest model level and the turbulent transport of the same quantities between model levels. These tendencies are obtained as differences between the results of an implicit time-step from t to $t+1$.

- The scheme computes the physical tendencies of the four prognostic variables $(u, v, T$ and $q)$ due the vertical exchange by turbulent (non-moist) processes.

- The surface boundary condition is formulated separately for 8 different tiles: water, ice, wet skin, low vegetation, exposed snow, high

vegetation, snow under vegetation and bare soil. The different tiles have their own surface balance and their own skin temperature.

☞ Each grid box is divided into 8 fractions: two vegetated fractions (high and low vegetation without snow), one bare soil fraction, three snow/ice fractions (snow on bare ground/low vegetation, high vegetation with snow beneath, and sea-ice, respectively), and two water fractions (interception reservoir, ocean/lakes). A mixture of land and water (ocean/inland water) tiles is not allowed, a grid box is either 100% ocean (water + ice) or 100% land.

The Surface Layer

The surface layer approximation is applied between the lowest model level (~10 m about the surface) and the surface for each tile separately. It is assumed that the turbulent fluxes are constant with height and equal to the surface values. They can be expressed, using Monin-Obukhov theory, in terms of the gradient of wind, dry static energy and specific humidity, which are assumed to be proportional to universal gradient function of a stability parameter.

$$\frac{\kappa z}{u_*}\frac{\partial u}{\partial z} = \Phi_M\left(\frac{z}{L}\right) \qquad (1)$$

$$\frac{\kappa z}{s_*}\frac{\partial s}{\partial z} = \Phi_H\left(\frac{z}{L}\right)$$

$$\frac{\kappa z}{q_*}\frac{\partial q}{\partial z} = \Phi_Q\left(\frac{z}{L}\right) \qquad (2)$$

The scaling parameters u_* , s_* and q_* are expressed in terms of surface fluxes:

$$\rho u_*^2 = J_M$$

$$\rho u_* s_* = J_s \qquad (3)$$

$$\rho u_* q_* = J_q$$

The stability parameter L is the Obukhov length and κ is the Von Karman constant (= 0.4).

☞ In the surface layer the horizontal wind, dry static energy and specific humidity profiles are:

$$u = \frac{\tau_x}{\kappa \rho u_*}\left\{\log\left(\frac{z_l + z_{0M}}{z_{0M}}\right) - \Psi_M\left(\frac{z_l + z_{0M}}{L}\right) + \Psi_M\left(\frac{z_{0M}}{L}\right)\right\} \quad (4)$$

$$v = \frac{\tau_y}{\kappa \rho u_*}\left\{\log\left(\frac{z_l + z_{0M}}{z_{0M}}\right) - \Psi_M\left(\frac{z_l + z_{0M}}{L}\right) + \Psi_M\left(\frac{z_{0M}}{L}\right)\right\} \quad (5)$$

$$s - s_{surf} = \frac{J_s}{\kappa \rho u_*}\left\{\log\left(\frac{z_l + z_{0M}}{z_{0M}}\right) - \Psi_M\left(\frac{z_l + z_{0M}}{L}\right) + \Psi_M\left(\frac{z_{0M}}{L}\right)\right\} \quad (6)$$

$$q - q_{surf} = \frac{J_q}{\kappa \rho u_*}\left\{\log\left(\frac{z_l + z_{0M}}{z_{0M}}\right) - \Psi_M\left(\frac{z_l + z_{0M}}{L}\right) + \Psi_M\left(\frac{z_{0M}}{L}\right)\right\} \quad (7)$$

where Ψ is the universal profile stability function.

Surface Fluxes

☞ Surface fluxes for heat and moisture are computed separately for the different tiles.

☞ The surface fluxes are expressed in terms of differences between parameters at level z_l and surface quantities:

$$\begin{aligned} J_M &= \rho C_M |U|^2 \\ J_s &= \rho C_H |U| (s_l - s_{surf}) \\ J_q &= \rho C_Q |U| (\alpha_l q_l - \alpha_{surf} q_{surf}) \end{aligned} \qquad (8)$$

where $q_{surf} = q_{sat}(T_{surf})$, α_l and α_{surf} are provided by the land scheme $s_{surf} = c_{p,dry}(1 + \delta q_{sa}) T_{surf}$, and q_{sa} also provided by the land scheme.

☞ The transfer coefficients are expressed as

$$C_M = \frac{\kappa^2}{\left[\log\left(\frac{z_l + z_{0M}}{z_{0M}}\right) - \Psi_M\left(\frac{z_l + z_{0M}}{L}\right) + \Psi_M\left(\frac{z_{0M}}{L}\right)\right]^2} \tag{9}$$

$$C_H = \frac{\kappa^2}{\left[\log\left(\frac{z_l + z_{0M}}{z_{0M}}\right) - \Psi_M\left(\frac{z_l + z_{0M}}{L}\right) + \Psi_M\left(\frac{z_{0M}}{L}\right)\right]} \times \frac{1}{\left[\log\left(\frac{z_l + z_{0M}}{z_{0H}}\right) - \Psi_H\left(\frac{z_l + z_{0M}}{L}\right) + \Psi_H\left(\frac{z_{0M}}{L}\right)\right]} \tag{10}$$

$$C_H = \frac{\kappa^2}{\left[\log\left(\frac{z_l + z_{0M}}{z_{0M}}\right) - \Psi_M\left(\frac{z_l + z_{0M}}{L}\right) + \Psi_M\left(\frac{z_{0M}}{L}\right)\right]} \times \frac{1}{\left[\log\left(\frac{z_l + z_{0M}}{z_{0Q}}\right) - \Psi_Q\left(\frac{z_l + z_{0M}}{L}\right) + \Psi_Q\left(\frac{z_{0M}}{L}\right)\right]} \quad (11)$$

☞ The wind speed U is expressed as

$$|U|^2 = u^2 + v^2 + w_*^2 \quad (12)$$

with w_* the free convection velocity defined by

$$w_* = \left(z_i \frac{g}{T_l} Q_{0v}\right)^{1/3} \quad (13)$$

☞ When the surface is heated, this term guarantees a finite surface wind-forcing even for vanishing u and v, and prevents $|U|$ and L from becoming zero.

Diagnostic Boundary Layer Height

Boundary layer height h_{BL} is defined as the level where the bulk Richardson number, based on the difference between quantities at that level and the lowest model level, riches the critical value $R_{i,\,cr} = 0.25$. The bulk Richardson number is computed from the following set of equation

$$R_{i_b} = h_{bl}\frac{2g(s_{vhbl} - s_{vl} - \Delta s)}{(s_{vhbl} + s_{vl} - gh_{bl} - gz_l)|\Delta U|^2} \qquad (14)$$

$$
\begin{aligned}
|\Delta U|^2 &= (u_{hbl} - u_l)^2 + (v_{hbl} - v_l)^2 \\
s_{vl} &= c_{p,dry} T_l (1 + \varepsilon q_l) + g z_l \\
s_{vhbl} &= c_{p,dry} T_{hbl} (1 + \varepsilon q_{hbl}) + g h_{bl} \\
\Delta s &= 8.5 c_{p,dry} u_* Q_{0v} / w_s
\end{aligned}
\qquad (15)
$$

$$
\begin{aligned}
w_s &= \{u_*^3 + 0.6(g/T) Q_{0v} h_{bl}\}^{1/3} \qquad unstable \\
w_s &= u_* \qquad stable
\end{aligned}
\qquad (16)
$$

index l indicates the lowest model level. The virtual dry static energy from the lowest level, s_{vl} is increased with a turbulent part Δs and compared to the virtual dry static energy at boundary layer height h_{bl}. The boundary layer height is found by a vertical scan from the surface upwards. If the boundary layer is found to be between two levels a linear interpolation is done to find the exact position.

☞ Advantages of bulk Richarson method (*Troen and Mart*, 1986):

- *does not require resolution of the capping inversion, when it exists;*
- *allows for a continuous transition between the stable and unstable boundary layer;*
- *describes the near-neutral case when the surface heat flux is unimportant;*
- *removes certain inconsistencies between the application of surface similarity theory and the "countergradient" flux corrections;*
- *consider the important problem of boundary-layer clouds.*

Equation for vertical diffusion

The equation for the vertical diffusion of any conservative quantity ψ is:

$$\frac{\partial \psi}{\partial t} = \frac{1}{\rho}\frac{\partial}{\partial z}\left(\rho K_{\psi}\frac{\partial \psi}{\partial z}\right) = \frac{1}{\rho}\frac{\partial J_{\psi}}{\partial z} \qquad (17)$$

The vertical turbulent flux J_{ψ} (positive upward) is written using a first-order closure, where K_{ψ} is the exchange coefficient. The goal of the vertical diffusion parameterization is to define the exchange coefficients and then solve equation (17) with the following boundary conditions:

$$K_\psi \frac{\partial \psi}{\partial z} = 0 \qquad at \qquad p = p_{top}$$

$$K_\psi \frac{\partial \psi}{\partial z} \rightarrow \sum_{i=1}^{N_T} F_i C_{\psi,i} |U(z)| (\psi(z) - \psi_{surf,i}) \qquad as \qquad z \rightarrow 0 \tag{18}$$

where p_{top} is the pressure at the top of the atmosphere. For heat and moisture the surface boundary condition is provided tile by tile and fluxes are averaged over N_T tiles, weighted by their fraction F_i. For the wind, a single tile is used with a no slip condition at the surface.

☞ The equations for turbulent transfer are solved with the tendencies from adiabatic and radiative processes as source terms in the RHS:

$$\frac{\partial \psi}{\partial t} = g \frac{\partial J_\psi}{\partial p} + \left(\frac{\partial \psi}{\partial t}\right)_{ad} + \left(\frac{\partial \psi}{\partial t}\right)_{rad} \tag{19}$$

☞ By applying a "fractional-step" method, the discretization of equation (19) becomes, for $1 < k < l$,

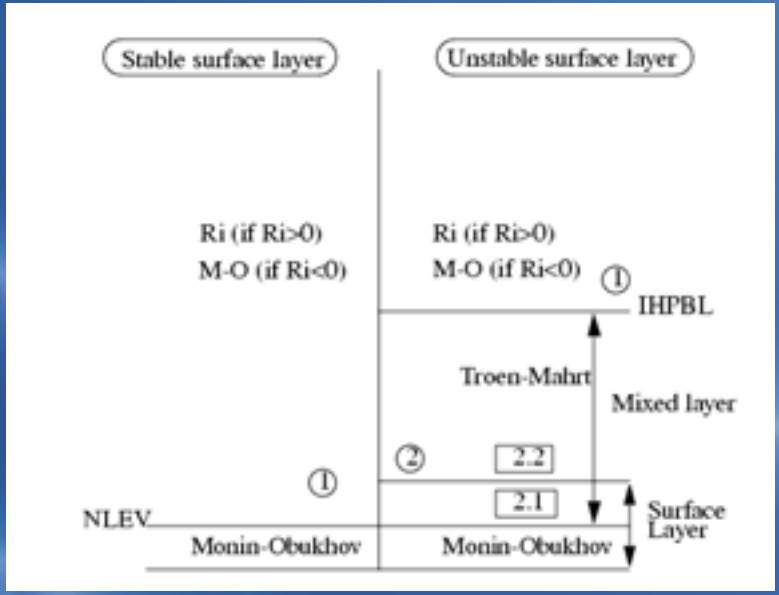
$$\left.\frac{\partial \psi}{\partial t}\right|_{diff} = \frac{\psi^{t+1} - \{\psi^{t} + \psi_{ad} + \psi_{rad}\}}{\Delta t}$$

$$= \frac{g}{p_{k+1/2} - p_{k-1/2}} \left\{ \rho_{k+1/2} K^{t}_{k+1/2} \left(\frac{\hat{\psi}_{k+1} - \hat{\psi}_{k}}{z_{k+1} - z_{k}} \right) \right. \qquad (20)$$

$$\left. - \rho_{k-1/2} K^{t}_{k-1/2} \left(\frac{\hat{\psi}_{k} - \hat{\psi}_{k-1}}{z_{k} - z_{k-1}} \right) \right\}$$

where $\hat{\psi} = \alpha \psi^{t+1} + (1-\alpha)\psi^{t}$, the parameter α determines the implicitness of the scheme. For $\alpha = 0$ the scheme is explicit, for $\alpha = 0.5$ we have a Crank-Nicholson and for $\alpha = 1$ we have an implicit backward scheme. In the model $\alpha = 1.5$ to avoid non-linear instability from K.

☞ The exchange coefficients K_ψ are estimated at half model levels. The computation of the exchange coefficients depends on the stability regimes (locally and at the surface) and on the vertical location above the surface.



☞ Equation (20) can be written as

$$-\frac{K^{*}_{k-1/2}}{\Delta p_k}\left(\frac{\hat{\psi}_{k-1}}{\alpha}\right)+\left[1+\frac{K^{*}_{k+1/2}}{\Delta p_k}+\frac{K^{*}_{k-1/2}}{\Delta p_k}\right]\left(\frac{\hat{\psi}_k}{\alpha}\right)-\frac{K^{*}_{k+1/2}}{\Delta p_k}\left(\frac{\hat{\psi}_{k+1}}{\alpha}\right) \qquad (21)$$

$$=\frac{\hat{\psi}^{t}_{k}}{\alpha}+\Delta\psi_{ad}+\Delta\psi_{rad}$$

☞ At the lowest level ($k = l$) the equation includes the surface fluxes which are obtained over N_T tiles:

$$\frac{\hat{\psi}_l}{\alpha}=\frac{\hat{\psi}^{t}_{l}}{\alpha}+\Delta\psi_{ad}+\Delta\psi_{rad}-K^{*}_{l-1/2}\frac{\hat{\psi}_l-\hat{\psi}_{l-1}}{\alpha\Delta p_l} \qquad (22)$$

$$\sum_{i=1}^{N_T}F_i\frac{C^{*}_{\psi,i}}{\alpha\Delta p_l}\{A_{l,i}\hat{\psi}_l-A_{surf,i}\hat{\psi}_{surf,i}\}$$

with $C^{*}_{\psi,i} = C^{t-1}_{\psi,i}|U|g\rho\alpha\Delta t$.

☞ At the top of the atmosphere ($k = 1$) turbulent fluxes are set to zero and we have

$$\frac{\hat{\psi}_l}{\alpha} = \frac{\hat{\psi}^t_l}{\alpha} + \Delta\psi_{ad} + \Delta\psi_{rad} - K^{*}_{-1/2}\frac{\hat{\psi}_1 - \hat{\psi}_0}{\alpha\Delta p_1} \qquad (23)$$

Boundary layer clouds

- This part of the scheme considers *stratocumulus clouds* at the top of convective boundary layers. They are distinguished from shallow cumuli by making the assumption, that the cloud depth must not exceed one model-layer depth. All clouds deeper than one layer are represented as convective clouds by the cumulus convection scheme.

- The scheme follows the mass-flux approach, so that the cloud transport for moisture is written as

$$F_q = \rho \hat{w}(q_{up} - q_{down}) \qquad (24)$$

where q_{up} and q_{down} are the updraft and downdraft specific humidity, respectively, and $\rho\hat{w}$ ($=\rho\alpha_{up}w_{up}$) is the cloud mass flux and α_{up} is the cloud fractional area of updrafts, typically about 0.2-0.3.

☞ Updrafts start near the surface with the properties of the mean flow at the lowest model level and downdrafts start at the first model level about cloud base, again with the properties of the mean.

☞ Updraft and downdraft values of thermodynamical variables are determined by dry-adiabatic (below cloud base) and moist-adiabatic ascent/descent (above cloud base).

☞ Downdrafts can be saturated or unsaturated depending on the existence of clouds.

☞ In the case of a solid cloud deck, all downdrafts become saturated and stay saturated right down to the cloud base if the cloud water content equals the updraft value. The condensation in updrafts is then totally compensated by evaporation in downdrafts and as a consequence, cloud water is now longer produced and the net liquid water flux becomes zero. This agrees with the finding of *Randall* (1987) that liquid water fluxes and buoyancy production by clouds are largest when the cloud cover is small and become smaller with increasing cloud cover.

☞ For the case of partial cloud cover the downdrafts occupy cloudy and cloud-free areas at even distribution and are saturated or unsaturated accordingly.

☞ The cloud-mass flux is determined by reformulating the turbulent moisture transport at cloud base produced by the boundary layer parameterization J_q into the mass-flux concept. The assumption is that the turbulent fluxes in the interior of the boundary layer are entirely due to convective circulation

$$(\rho\hat{w})_{base} = \frac{J_q}{q_0 - \{\alpha(q_{sat} + l_{cld})_{top} + (1-\alpha)q_{top}\}} \qquad (25)$$

The subscripts “0” and “top” refer to model levels near the surface and close to the cloud top (i.e. next level above cloud base), respectively, indicating that the updrafts starts close to the surface and the downdrafts cloud top.

☞ Stratocumulus cloud decks are strongly affected by large-scale subsidence, radiative transfer and turbulent entrainment of warm and dry air above the inversion.

☞ For the entrainment, the negative buoyancy flux at cloud top is assumed to be proportional to the average buoyancy flux within the mixed layer (*Deardorff*, 1976)

$$-(\overline{w's_v'})_H = \frac{\alpha}{H}\int \overline{w's'_v}dz \tag{26}$$

where H is the height of the mixed layer and $\alpha = 0.5$. Introducing the entrainment velocity

$$w_e = -\frac{(\overline{w's'_v})_H}{\Delta s_v} \tag{27}$$

the entrainment fluxes of s, q and l become

$$\begin{aligned}(\overline{w's'})_H &= -w_e\Delta s \\ (\overline{w'q'})_H &= -w_e\Delta q \\ (\overline{w'l'})_H &= -w_e\Delta l\end{aligned} \tag{28}$$

where Δs_v, Δs, Δq and Δl are jumps across the inversion.

☞ Above cloud base the assumption is made that $\hat{w}$ decreases linearly to zero at cloud top. The net generation of cloud water/ice due to condensation in updrafts and evaporation in downdrafts becomes

$$S_{bl} = -\frac{1}{\rho}\frac{\partial}{\partial z}(\rho\hat{w})(l_{up} - \alpha l_{down}) \qquad (29)$$

and the source of cloud air in terms of cloud cover is

$$\delta\alpha_{bl} = -\frac{1}{\rho}\frac{\partial}{\partial z}(\rho\hat{w})(1-\alpha) \qquad (30)$$